The CNBC Effect:
Welfare Effects of Public Information

By

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The CNBC Effect: Welfare Effects of Public Information*

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Abstract

What are the welfare effects of enhanced dissemination of public information through the media and disclosures by market participants with high public visibility? For instance, is it always desirable to have frequent and timely publications of economic statistics by government agencies and the central bank? We examine the impact of public information in a setting where agents take actions appropriate to the underlying fundamentals, but they also have a coordination motive arising from a strategic complementarity in their actions. When the agents have no private information, greater provision of public information always increases welfare. However, when agents also have access to independent sources of information, the welfare effect of increased public disclosures is ambiguous.

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"The history of speculative bubbles begins roughly with the advent of newspapers. One can assume that, although the record of these early newpapers is mostly lost, they regularly reported on the first bubble of any consequence, the Dutch tulipmania of the 1630s. Although the news media - newspapers, magazines, and broadcast media, along with their new outlets on the Internet - present themselves as detached observers of market events, they are themselves an integral part of these event. Significant market events generally occur only if there is similar thinking among large groups of people, and the news media are essential vehicles for the spread of ideas." Shiller (2000).

"Finally, there is the CNBC effect.... now that CNBC can no longer run breathless tales about the economic boom, it has started to feature equally breathless (and potentially self-fulfilling) speculations about the economic crisis." Krugman (2001a).

"... some IMF officials fear that a published list of nations that are mismanaging their economics would lead any rational foreign investors to pull out their capital - touching off the kind of crisis the IMF is trying to avoid." New York Times (2001).

1. Introduction

In single person decision making, more information always leads to higher ex ante utility. But is more information socially desirable when the prevailing conventional wisdom or consensus impinge on people's decision making process? The susceptibility to conventional wisdom need not imply any wishful thinking or irrationality. Take, for example, someone who has a stake in a bet on the financial markets. The media (whether it be 17th century newspapers or 21st century cable news channels) convey information on the underlying fundamentals, but their overall influence runs deeper. The very fact that the news reaches a large audience also tells the recipient that many others have also just learned this piece of news. Since asset prices react to the decisions of market participants, the recipients of news will try to anticipate the reactions of other traders. Markets may therefore behave in ways that seem like "overreaction" relative to what would be justified by the face value of the news.

The role of the media in influencing economic outcomes is one aspect of a more general question concerning the role of shared knowledge in economics. Keynes's (1936) example of the beauty contest in which the objective is to outguess what the "average opinion expects average opinion to be" drives home the importance of shared knowledge in decision making in strategic settings. Arguably, the role of shared knowledge goes far beyond economics. Chwe (2001) provides a compelling discussion of the importance of shared knowledge in a wide variety of settings. For example, he documents the high per unit cost of reaching a viewer when the audience is large, and shows that goods that have a prominent 'social' dimension are more likely to receive the benefit of such high cost advertising.

More narrowly, we can also apply the same criteria to assess the disclosure policy of central banks and other organizations that command high visibility in the market. Central bank officials have learned to be wary of public utterances that may unduly influence financial markets, and have developed their own respective strategies for communicating with the market. In formulating their disclosure policies, central banks and government agencies face a number of interrelated issues concerning how much they should disclose, in what form, and how often. Frequent and timely dissemination would aid the decision making process by putting current information at the disposal of all economic agents, but this has to be set against the fact that provisional estimates are likely to be revised with the benefit of hindsight. By their nature, economic statistics are imperfect measurements of sometimes imprecise concepts, and no government agency or central bank can guarantee flawless information. This raises legitimate concerns about the publication of preliminary or incomplete data, since the benefit of early release may be more than outweighed by the disproportionate impact of any error. This trade-off between timely but noisy information and slow but more accurate information is a familiar theme, as witnessed by the debate in Japan about whether preliminary GDP figures should be published. Australia moved from a monthly calendar in reporting its balance of trade figures to a quarterly calendar because it was felt that the noise in the monthly statistics were injecting too much volatility into the price signals from financial markets[1]. Sometimes, the damage done by the "noise" in official statistics can be significant. The flaws in the U.K.'s earnings data has been credited with provoking unjustifiably tight credit conditions in the U.K. in the spring and summer of 1998[2].

Our paper investigates the effect of public information on economic welfare in a model reminiscent of Keynes's beauty contest example. We show how public information may be socially damaging when individuals have to reconcile the motive of taking the action that best fits the underlying fundamentals with other, strategic motives. Of course, public information might lead to irrational fads and

[1] We are grateful to Philip Lowe for this example.

[2] See, for instance, "Garbage in, garbage out" *Economist magazine*, October 15th 1998.

some CNBC viewers may not be good at distinguishing real news from breathless speculation; however, we want to note that irrationality is not necessary for public information to be damaging.

In our model, a large population of agents have access to public and private information. The agents aim to take actions appropriate to the underlying state, but they also face a spillover effect arising from other agents' actions. When there is perfect information concerning the underlying state, the unique equilibrium in the game between the agents also maximizes social welfare. However, when there is imperfect information, the welfare effects of increased public information are more equivocal. In particular,

- when the agents have no private information - so that the only source of information for the agents is the public information - then greater precision of the public information always increases social welfare.

- However, if the agents have access to some private information, it is not always the case that greater precision of public information is desirable. Over some ranges, increased precision of public information is detrimental to welfare. Specifically, the greater the precision of the agents' *private* information, the more likely it is that increased provision of public information lowers social welfare.

The challenge for central banks and other public organizations is to strike the right balance between providing timely and frequent information to the private sector so as to allow it to pursue its goals, but to recognize the inherent limitations in any disclosure and to guard against the potential damage done by noise. This is a difficult balancing act at the best of times, but this task is likely to become even harder. As central banks' activities impinge more and more on the actions of market participants, the latter have reciprocated by stepping up their surveillance of central banks' activities and pronouncements. The intense spotlight trained on the fledgling European Central Bank and the ECB's delicate relationship with the media and private sector market participants illustrate well the difficulties faced by policy makers. Winkler (2000) presents one of the few surveys of central bank transparency that is sympathetic to the ECB's predicament.

In the highly sensitized world of today's financial markets populated with Fed watchers (and watchers of other central banks and their personalities), economic analysts, and other commentators of the economic scene, disclosure policy assumes great importance. Our results suggest that private sources of information may actually *crowd out* the public information by rendering the public information detrimental to the policy maker's goals. The heightened sensitivities of the market could magnify any noise in the public information to such a large extent that public information ends up by causing more harm than good. If the information provider anticipates this effect, then the consequence of the heightened sensitivities of the market is to push it into reducing the precision of the public signal. In effect, private and public information end up being *substitutes*, rather than complements.

To the extent that agents forecast others' actions in our model, the results of this paper may also be useful in the related literature on the strategic effects in macroeconomic forecasting. Ottaviani and Sorensen (2000) note that the strategies of professional forecasters may be influenced in subtle ways by the incentives at work. Both excessive conformity and excessive differentiation can arise, depending on the payoffs faced by the forecasters. See also Lamont (1995) and Keane and Runkle (1998).

There is a small literature examining the value of information in strategic settings, following Hirshleifer's (1971) paper showing that public information may be damaging because it removes insurance possibilities. Raith (1996) reviews and unifies a literature on private and public information in oligopoly. The mechanisms in this paper are quite different. Messner and Vives (2000) is closer to our framework. They examine the welfare properties of a rational expectations equilibrium in which the price serves as a public signal of the distribution of costs among producers, and show how this information may be detrimental to welfare.

Our conclusion that public information and private information end up being substitutes has parallels in the herding literature initiated by Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992). In both cases, access to not very accurate public information results in socially valuable private information being lost. However, the mechanisms are very different. An attractive feature of the simple static coordination model of this paper is that the results do not depend on fine details of the timing structure. The model also delivers a simple message about when there may be excessive reliance on noisy public information: this will occur when (1) there is significant private information; and (2) agents have a strong coordination motive.

There is a literature examining the impact of public information in binary action coordination games where agents have both private and public signals about some underlying state (see Morris and Shin (1999, 2000), Metz (2000) and Hellwig (2000)). In this work, if private information is sufficiently accurate relative to public information, there is a unique equilibrium in a setting where multiple equilibria would exist with common knowledge. But when there is a unique equilibrium, it is possible to examine the relative impact of private and public signals and - as in this paper - public signals have a strategic impact above their informational impact. However, it is hard to quantify the strategic impact of public information and it is entangled with the equilibrium selection issue. In this paper, we are able to examine the impact of public signals and obtain cleaner welfare implications in a much simpler model where there is a unique equilibrium with or without private information.

2. Model

There is a continuum of agents who face the problem of tailoring their action to the underlying state θ, but also face an externality arising from the actions of other agents. The externality induces behaviour akin to the "beauty contest" example of Keynes (1936) in which an individual tries to second guess the decisions of other individuals in the economy. Moreover, this spillover effect is socially inefficient in that they are of a zero-sum nature across the individual agents.

More formally, there is a continuum of agents, indexed by the unit interval $[0, 1]$. Agent i chooses an action $a_i \in \mathbb{R}$, and we write $\mathbf{a}$ for the action profile over all agents. The payoff function for agent i is given by

$$u_i(\mathbf{a}, \theta) \equiv -(1-r)(a_i - \theta)^2 - r\left(L_i - \bar{L}\right) \tag{2.1}$$

where r is a constant, with $0 < r < 1$ and

$$\begin{aligned} L_i &\equiv \int_0^1 (a_j - a_i)^2 \, dj \\ \bar{L} &\equiv \int_0^1 L_j dj \end{aligned}$$

The loss function for individual i has two components. The first component is a standard quadratic loss in the distance between the underlying state θ and his action a_i. The second component is the "beauty contest" term. The loss L_i is increasing in the average distance between i's action and the action profile of the whole population. Thus, the actions of others introduce an externality in which an individual tries to second-guess the decisions of other individuals in the economy. The parameter r gives the weight on this second-guessing motive. The larger is r, the more severe is the externality. Moreover, this spillover effect is socially inefficient in that it is of a zero-sum nature. In the game of second-guessing, the winners gain at the expense of the losers. Social welfare, defined as the (normalized) average of individual utilities is

$$\begin{aligned} W(\mathbf{a},\theta) &\equiv \frac{1}{1-r}\int_0^1 u_i(\mathbf{a},\theta)\,di \\ &= -\int_0^1 (a_i-\theta)^2\,di. \end{aligned}$$

so that a social planner who cares only about social welfare seeks to keep all agents' actions close to the state θ. From the point of view of agent i, however, his action is determined by the first order condition:

$$a_i = (1-r)\,E_i(\theta) + rE_i(\overline{a})$$

where $\overline{a}$ is the average action in the population (i.e., $\overline{a} = \int_0^1 a_j dj$) and $E_i(\cdot)$ is the expectation operator for player i. Thus each agent puts positive weight on the expected state and the expected actions of others. Note, however, that if θ is common knowledge, the equilibrium entails $a_i = \theta$ for all i, so that social welfare is maximized at equilibrium. So, when there is perfect information, there is no conflict between individually rational actions and the socially optimal actions. We now examine the case where θ is not known with certainty.

2.1. Public Information Benchmark

Consider now the case where agents face uncertainty concerning θ, but they have access to public information. The state θ is drawn from an (improper) uniform prior over the real line, but the agents observe a *public signal*

$$y = \theta + \eta \tag{2.2}$$

where η is normally distributed, independent of θ, with mean zero and variance σ_η^2. The signal y is 'public' in the sense that the actual realization of y is common knowledge to all agents. They choose their actions after observing the realization of y. The expected payoff of agent i at the time of decision is then given by the conditional expectation:

$$E\left(u_i \mid y\right) \tag{2.3}$$

where $E\left(\cdot \mid y\right)$ is the common expectation operator. Conditional on y, both agents believe that θ is distributed normally with mean y and variance σ_η^2. Hence, the best reply of i is

$$a_i\left(y\right) = \left(1-r\right) E\left(\theta \mid y\right) + r \int_0^1 E\left(a_j \mid y\right) dj \tag{2.4}$$

where $a_i\left(y\right)$ denotes the action taken by agent i as a function of y. Since $E\left(\theta \mid y\right) = y$ and since the strategies of both agents are measurable with respect to y, we have $E\left(a_j \mid y\right) = a_j\left(y\right)$, so that in the unique equilibrium,

$$a_i\left(y\right) = y \tag{2.5}$$

for all i; expected welfare, conditional on θ, is

$$\begin{aligned} E\left(W \mid \theta\right) &= -E\left[\left(y-\theta\right)^2 \mid \theta\right] \\ &= -\sigma_\eta^2 \end{aligned}$$

Thus, the smaller the noise in the public signal, the higher is social welfare. We will now contrast this with the general case in which agents have private information as well as public information.

2.2. Private and Public Information

Consider now the case where, in addition to the public signal y, agent i observes the realization of a *private signal*:

$$x_i = \theta + \varepsilon_i \tag{2.6}$$

where noise terms ε_i of the continuum population are normally distributed with zero mean and variance σ_ε^2, independent of θ and η, so that $E(\varepsilon_i \varepsilon_j) = 0$ for $i \neq j$. The private signal of one agent is not observable by the others. This is the sense in which these signals are private.

As before, the agents' decisions are made after observing the respective realizations of their private signals as well as the realization of the public signal. Denote by

$$a_i(\mathcal{I}_i) \tag{2.7}$$

the decision by agent i as a function of his information set $\mathcal{I}_i$. The information set $\mathcal{I}_i$ consists of the pair (y, x_i) that captures all the information available to i at the time of decision. The notation in (2.7) makes explicit that the *strategy* of agent i in the imperfect information game is a function that maps the information $\mathcal{I}_i$ to the action a_i. For any given strategy, a_i is therefore a random variable that is measurable on the partition generated by $\mathcal{I}_i$.

Let us denote by α the precision of the public information, and denote by β

the precision of the private information, where

$$\begin{cases} \alpha = \dfrac{1}{\sigma_\eta^2} \\ \beta = \dfrac{1}{\sigma_\varepsilon^2} \end{cases} \tag{2.8}$$

Then, based on both private and public information, agent i's expected value of θ is:

$$E_i(\theta) = \frac{\alpha y + \beta x_i}{\alpha + \beta} \tag{2.9}$$

where we have used the shorthand $E_i(\cdot)$ to denote the conditional expectation $E(\cdot|\mathcal{I}_i)$.

2.3. Linear Equilibrium

We will now solve for the unique equilibrium. We do this in two steps. We first solve for a linear equilibrium in which actions are a linear function of signals. We will follow this with a demonstration that this linear equilibrium is the unique equilibrium. Thus, as the first step, suppose that the population as a whole is following a linear strategy of the form

$$a_j(\mathcal{I}_j) = \kappa x_j + (1-\kappa) y. \tag{2.10}$$

Then agent i's conditional estimate of the average expected action across all agents is:

$$\begin{aligned} E_i(\overline{a}) &= \kappa \left(\frac{\alpha y + \beta x_i}{\alpha + \beta} \right) + (1-\kappa) y \\ &= \left(\frac{\kappa\beta}{\alpha+\beta} \right) x_i + \left(1 - \frac{\kappa\beta}{\alpha+\beta} \right) y \end{aligned}$$

Thus agent i's optimal action is

$$a_i(\mathcal{I}_i) = (1-r) E_i(\theta) + r E_i(\overline{a}) \tag{2.11}$$

$$= (1-r)\left(\frac{\alpha y+\beta x_i}{\alpha+\beta}\right)+r\left(\left(\frac{\kappa\beta}{\alpha+\beta}\right)x_i+\left(1-\frac{\kappa\beta}{\alpha+\beta}\right)y\right)$$
$$= \left(\frac{\beta\left(r\kappa+1-r\right)}{\alpha+\beta}\right)x_i+\left(1-\frac{\beta\left(r\kappa+1-r\right)}{\alpha+\beta}\right)y$$

Comparing coefficients in (2.10) and (2.11), we therefore have

$$\kappa=\frac{\beta\left(r\kappa+1-r\right)}{\alpha+\beta}$$

from which we can solve for κ.

$$\kappa=\frac{\beta\left(1-r\right)}{\beta\left(1-r\right)+\alpha}.$$

Thus, the equilibrium action a_i is given by

$$a_i\left(\mathcal{I}_i\right)=\frac{\alpha y+\beta\left(1-r\right)x_i}{\alpha+\beta\left(1-r\right)} \tag{2.12}$$

2.4. Uniqueness of Equilibrium

The argument presented above establishes the existence of a linear equilibrium. We will follow this by showing (through a "brute force" solution method) that the linear equilibrium we have identified is the unique equilibrium. In doing so, we establish the role of higher order expectations in this model. In particular, we note that if someone observes a public signal that is worse than her private signal, then her expectation of others' expectations of θ is lower than her expectation of θ, i.e., it is closer to the public signal than her own expectation. This in turn implies that if we look at nth order expectations about θ, i.e., someone's expectation of others' expectations of others' expectations of (n times) of θ, then this approaches the public signal as n becomes large. Higher order expectations depend only on public signals.

Recall that player i's best response is to set

$$a_i=\left(1-r\right)E_i\left(\theta\right)+rE_i\left(\overline{a}\right)$$

Substituting and writing $\overline{E}(\theta)$ for the average expectation of θ across agents we have

$$\begin{aligned} a_i &= (1-r) E_i(\theta) + (1-r) r E_i\left(\overline{E}(\theta)\right) + (1-r) r^2 E_i\left(\overline{E}^2(\theta)\right) + ... \\ &= (1-r) \sum_{k=0}^{\infty} r^k E_i\left(\overline{E}^k(\theta)\right) \end{aligned} \tag{2.13}$$

In order to evaluate this expression, and check that the infinite sum is bounded, we must solve explicitly for $E_i\left(\overline{E}^k(\theta)\right)$. Recall that player i's expected value of θ is:

$$E_i(\theta) = \frac{\alpha y + \beta x_i}{\alpha + \beta} \tag{2.14}$$

Thus the average expectation of θ across agents is

$$\overline{E}(\theta) = \int_0^1 E_i(\theta)\, di = \frac{\alpha y + \beta \theta}{\alpha + \beta}$$

Now player i's expectation of the average expectation of θ across agents is

$$\begin{aligned} E_i\left(\overline{E}(\theta)\right) &= E_i\left(\frac{\alpha y + \beta \theta}{\alpha + \beta}\right) \\ &= \frac{\alpha y + \beta\left(\frac{\alpha y + \beta x_i}{\alpha + \beta}\right)}{\alpha + \beta} \\ &= \frac{\left((\alpha+\beta)^2 - \beta^2\right) y + \beta^2 x_i}{(\alpha+\beta)^2} \end{aligned}$$

and the average expectation of the average expectation of θ is

$$\begin{aligned} \overline{E}^2(\theta) &= \overline{E}\left(\overline{E}(\theta)\right) \\ &= \frac{\left((\alpha+\beta)^2 - \beta^2\right) y + \beta^2 \theta}{(\alpha+\beta)^2} \end{aligned}$$

More generally, we have the following lemma.

Lemma 2.1. *For any k, $\overline{E}^k(\theta) = \left(1-\mu^k\right) y + \mu^k \theta$ and $E_i\left(\overline{E}^k(\theta)\right) = \left(1-\mu^{k+1}\right) y + \mu^{k+1} x_i$ where $\mu = \beta/(\alpha+\beta)$.*

The proof is by induction on k. We know from (2.14) that the lemma holds for $k = 1$. Suppose that it holds for $k - 1$. Then,

$$E_i\left(\overline{E}^{k-1}(\theta)\right) = \left(1 - \mu^k\right) y + \mu^k x_i;$$

so

$$\overline{E}^k(\theta) = \left(1 - \mu^k\right) y + \mu^k \theta$$

and

$$\begin{aligned} E_i\left(\overline{E}^k(\theta)\right) &= \left(1 - \mu^k\right) y + \mu^k \left(\frac{\alpha y + \beta x_i}{\alpha + \beta}\right) \\ &= \left(1 - \mu^{k+1}\right) y + \mu^{k+1} x_i \end{aligned}$$

which proves lemma 2.1. Now substituting the expression from lemma 2.1 into equation (2.13), we obtain

$$\begin{aligned} a_i &= (1 - r) \sum_{k=0}^{\infty} r^k \left[\left(1 - \mu^{k+1}\right) y + \mu^{k+1} x_i\right] \\ &= \left(1 - \frac{\mu(1 - r)}{1 - r\mu}\right) y + \left(\frac{\mu(1 - r)}{1 - r\mu}\right) x_i \\ &= \frac{\alpha y + \beta(1 - r) x_i}{\alpha + \beta(1 - r)} \end{aligned}$$

This is exactly the unique linear equilibrium we identified earlier.

3. Welfare Effect of Public Information

We are now ready to address the main question of the paper. How is welfare affected by the precisions of the agents' signals? In particular, will welfare be increasing in the precision α of the public signal? From the solution for a_i, we can solve for the equilibrium strategies in terms of the basic random variables θ, η and $\{\varepsilon_i\}$.

$$a_i = \theta + \frac{\alpha\eta + \beta(1 - r)\varepsilon_i}{\alpha + \beta(1 - r)} \tag{3.1}$$

If $r = 0$, the two types of noise (private and public) would be given weights that are commensurate with their precision. That is, η would be given weight equal to its relative precision $\alpha/(\alpha+\beta)$ while ε_i would be given weight equal to its relative precision $\beta/(\alpha+\beta)$. However, the weights in (3.1) deviate from this. The noise in the public signal is given relatively more weight, and the noise in the private signal is given relatively less weight. This feature reflects the coordination motive of the agents, and reflects the disproportionate influence of the public signal in influencing the agents' actions. The magnitude of this effect is greater when r is large. What effect does this have on welfare? Expected welfare at θ is given by

$$\begin{aligned} E\left[W(\mathbf{a},\theta)\middle|\,\theta\right] &= -\frac{\alpha^2 E(\eta^2) + \beta^2 (1-r)^2 \left[E(\varepsilon_i^2)\right]}{(\alpha+\beta(1-r))^2} \\ &= -\frac{\alpha+\beta(1-r)^2}{(\alpha+\beta(1-r))^2} \end{aligned} \tag{3.2}$$

By examining (3.2), we can answer the comparative statics questions concerning the effect of increased precision of private and public information.

Welfare is always increasing in the precision of the private signals. We can see this by differentiating (3.2) with respect to β, the precision of the private signals. We have:

$$\frac{\partial E(W|\,\theta)}{\partial\beta} = \frac{(1-r)\left((1+r)\alpha + (1-r)^2\beta\right)}{(\alpha+\beta(1-r))^3} > 0 \tag{3.3}$$

Thus, increased precision of private information enhances welfare unambiguously. The same cannot be said of the effect of increased precision of the public signal. The derivative of (3.2) with respect to α is:

$$\frac{\partial E(W|\,\theta)}{\partial\alpha} = \frac{\alpha - (2r-1)(1-r)\beta}{(\alpha+\beta(1-r))^3} \tag{3.4}$$

so that

$$\frac{\partial E(W|\,\theta)}{\partial\alpha} \geq 0 \quad \text{if and only if} \quad \frac{\beta}{\alpha} \leq \frac{1}{(2r-1)(1-r)} \tag{3.5}$$

When $r > 0.5$, there are ranges of the parameters where increased precision of public information is detrimental to welfare. Increased precision of public information is beneficial only when the private information of the agents is not very precise. If the agents have access to very precise information (so that β is high), then any increase in the precision of the public information will be harmful. Thus, as a rule of thumb, when the private sector agents are already very well informed, the official sector would be well advised not to make public any more information, unless they could be confident that they can provide public information of very great precision. If a social planner were choosing ex ante the optimal precision of public information and increasing the precision of public information is costly, then corner solutions at $\alpha = 0$ may be common.

Even if greater precision of public information can be obtained relatively cheaply, there may be technical constraints in achieving precision beyond some upper bound. For instance, the social planner may be restricted to choosing α from some given interval $[0, \bar{\alpha}]$. In this case, even if the choice of α entails no costs, we will see a "bang-bang" solution to the choice of optimal α in which the social optimum entails either providing no public information at all (i.e. setting $\alpha = 0$), or providing the maximum feasible amount of public information (i.e. setting $\alpha = \bar{\alpha}$). The better informed is the private sector, the higher is the hurdle rate of precision of public information that would make it welfare enhancing.

Figure 3.1 illustrates the social welfare contours in (α, β)-space. The curves are the set of points that satisfy $\mathrm{E}(W|\theta) = C$, for constants C. As can be seen from figure 3.1, when $\beta > \alpha / [(2r-1)(1-r)]$, the social welfare contours are upward sloping, indicating that welfare is *decreasing* in the precision of public information.

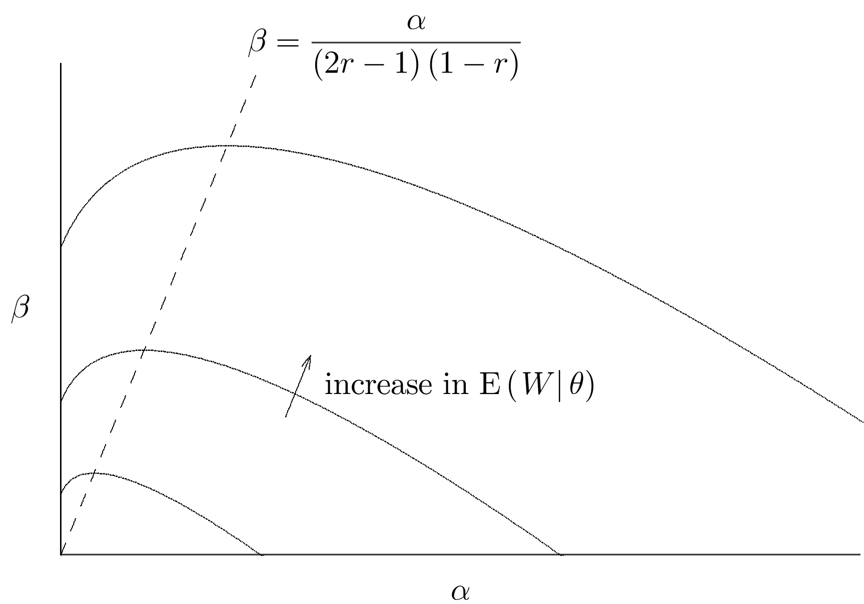

Figure 3.1: Social Welfare Contours

What is the intuition for this result? Observe that equation (2.12) can be re-written as

$$\begin{aligned} a_i &= \frac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)} \\ &= \frac{\alpha y + \beta x_i}{\alpha + \beta} + (y - x_i)\left(\frac{\alpha}{\alpha+\beta}\right)\frac{\beta r}{\alpha + \beta(1-r)} \end{aligned} \tag{3.6}$$

This equation shows well the added impact of public information in determining the actions of the agents. In addition to its role in forming the conditional expectation of θ, there is an additional (positive) term involving the public signal y, while there is a corresponding negative term involving the private signal x_i. Thus, the agents "overreact" to the public signal while suppressing the information content of the private signal. The impact of the noise η in the public signal is given more of an impact in the agents' decisions than it deserves.

Having established the possibility that public information may be detrimental, we now address a number of extensions and variations of our model. The purpose is both to gauge the robustness of our conclusions, and also to delve deeper into the results.

4. Finite Number of Players

The framework with a continuum of small players that we have used has the virtue of simplicity, but may give rise to the suspicion that our results are somehow dependent on the zero measure nature of individual agents. We show that this is not the case by deriving our results as the limit of a finite player framework as the number of agents become large. The finite player framework has a more substantial advantage in that it is a good base to examine alternative payoffs and welfare functions.

Thus, suppose that there are $n \geq 2$ players, each with payoff function

$$u_i = -(1-r)(a_i-\theta)^2 - r\left(L_i - \bar{L}\right)$$

where $0 < r < 1$ and

$$\begin{aligned} L_i &= \frac{1}{n}\sum_{j=1}^{n}(a_j - a_i)^2 \\ \bar{L} &= \frac{1}{n}\sum_{j=1}^{n} L_j \end{aligned}$$

These loss functions are the finite-player analogues of our basic model. The normalized social welfare is

$$-\frac{1}{n}\sum_{i=1}^{n}(a_i-\theta)^2 .$$

The optimal action for player i is given by

$$a_i = (1-\rho)\,E_i(\theta) + \rho E_i(\overline{a}_{-i}) \tag{4.1}$$

where $\overline{a}_{-i}$ is the average action of all players except i, and ρ is the constant

$$\rho = r\tfrac{(n-1)(n-2)}{n^2}$$

Note that $\rho \to r$ when n becomes large, but $\rho = 0$ when $n = 2$. Thus, the strategic interaction is absent when there are only two players, but we have approximately the same action as for the continuum case when there are many players. We can solve for the equilibrium by solving directly for the higher order beliefs. For any sequence of names for the individuals $s = (s_1, s_2, \cdots, s_k)$, where consecutive terms refer to different individuals (i.e. $s_m \neq s_{m+1}$ for all m), define the iterated expectations operator:

$$\mathrm{E}_s^k(\cdot) \equiv \mathrm{E}_{s_1}\left(\mathrm{E}_{s_2}\left(\cdots \mathrm{E}_{s_k}(\cdot)\cdots\right)\right) \tag{4.2}$$

Thus, $\mathrm{E}_s^k(z)$ is s_1's expectation of s_2's expectation of ... s_k's expectation of z. Finally, denote by S_i^k the set of name sequences of length k such that the first term is i, and any two consecutive terms are distinct. That is,

$$S_i^k \equiv \left\{(s_1, s_2, \cdots, s_k) \,|s_1 = i \text{ and } s_m \neq s_{m+1} \text{ for all } m\right\}$$

Using this notation, the iterated substitution of the equations in (4.1) yields the following characterization of the equilibrium strategy for i.

$$a_i = (1-\rho)\sum_{k=1}^{\infty}\sum_{s\in S_i^k}\left[\tfrac{\rho}{n-1}\right]^{k-1}\mathrm{E}_s^k(\theta) \tag{4.3}$$

Using the same induction argument as before, we can show that for any $s \in S_i^k$,

$$E_s^k(\theta) = \left[1-\mu^k\right]y + \mu^k x_i \tag{4.4}$$

where $\mu = \beta/(\alpha+\beta)$. Substituting (4.4) into (4.3), we obtain

$$a_i = \frac{\alpha y + \beta(1-\rho)x_i}{\alpha + \beta(1-\rho)}.$$

Since $\rho \rightarrow r$ as n becomes large, we can see that the equilibrium action a_i tends to the equilibrium of the continuum model we examined earlier. The corresponding expression for equilibrium welfare is also analogous to the continuum player case (3.2), and converges to it as n becomes large.

5. Alternative Welfare Definitions

So far in the paper, we have taken for granted that social welfare is the sum of individual payoffs. In some cases, however, it may be more fruitful to consider formulations where we allow a more general definition of welfare. We could consider a principal who has the ability to choose the degree of precision of the public signal, but whose objectives may differ in subtle ways from the agents. For instance, if we pursue our macroeconomic interpretation of the model as the interaction between a central bank and the private sector agents, one natural way to formulate the principal's objective function is in terms of the deviation of the *aggregate* level of activity from the true state θ. How will our results be affected by this alternative formulation of the welfare function? Suppose that the principal's objective is to minimize

$$\left(\frac{1}{n}\sum_{j=1}^{n} a_j - \theta\right)^2$$

so that the objective for the principal is to set the *average* action as close as possible to θ. Suppose that all agents follow a linear strategy and set their action according to

$$a_i = \kappa x_i + (1 - \kappa)\, y$$

where y is the public signal, and x_i is i's private signal. Then the expected loss for the principal at θ is

$$\begin{aligned} & \mathrm{E}\left(\left(\frac{\kappa}{n}\sum_{j=1}^{n} x_j + (1-\kappa)\,y - \theta\right)^2 \middle| \, \theta\right) \\ = \; & \mathrm{E}\left(\left(\frac{\kappa}{n}\sum_{j=1}^{n} \varepsilon_j + (1-\kappa)\,\eta\right)^2 \middle| \, \theta\right) \\ = \; & \frac{\kappa^2}{n\beta} + \frac{(1-\kappa)^2}{\alpha} \end{aligned}$$

The value of κ that minimizes the principal's loss is

$$\kappa = \frac{n\beta}{\alpha + n\beta}$$

Note that when n is large, the principal would like the agents to put small weight on the public signal, and base their decision largely on the private signal. Whereas the noise terms $\{\varepsilon_i\}$ in the private signals of the agents tend to cancel each other out, the noise term η in the public signal remains in place irrespective of the number of agents. Thus, if the welfare function places weight on some aggregate activity variable, the overweighting of the public signal by the agents would cause an even greater social welfare loss.

This example is clearly rather simplistic in the way that it exploits the i.i.d. private noise terms, and it would be important to explore the more realistic case where the noise terms are correlated across individuals. We will do this in the next section. First, we will investigate how general our results are by examining a general welfare formulation that allows a large number of different motivations for the players. For this, we revert to our continuum player framework. Thus, suppose that player i seeks to maximize the general payoff:

$$u_i(\mathbf{a},\theta) \equiv \left\{ \begin{array}{l} -r_1 \int_0^1 (a_j - a_i)^2 \, dj \\ -r_2 (a_i - \theta)^2 \\ -r_3 \left(a_i - \int_{j\in[0,1]} a_j dj \right)^2 \\ +r_4 \int_0^1 \int_0^1 (a_j - a_k)^2 \, dj dk \\ -r_5 \left(\int_{j\in[0,1]} a_j dj - \theta \right)^2 \end{array} \right\}. \tag{5.1}$$

The speficiation of payoffs allows differing weights to the losses arising from the distances between a_i, θ, and the average actions. From the first order condition, the optimal action for i is given by

$$\widehat{r} \int_{j\in[0,1]} E_i(a_j) + (1-\widehat{r}) E_i(\theta).$$

where

$$\widehat{r} = \frac{r_1 + r_3}{r_1 + r_2 + r_3}.$$

We can solve for the equilibrium in the same way as before, yielding equilibrium actions:

$$a_i = \frac{\alpha y + \beta(1-\hat{r}) x_i}{\alpha + \beta(1-\hat{r})}$$

In deriving an expression for welfare, note that

$$\begin{aligned} a_i - a_j &= \frac{1}{\beta(1-\widehat{r}) + \alpha} \beta(1-\widehat{r})(\varepsilon_i - \varepsilon_j) \\ a_i - \int_{j\in[0,1]} a_j dj &= \frac{1}{\beta(1-\widehat{r}) + \alpha} \beta(1-\widehat{r}) \varepsilon_i \\ a_i - \theta &= \frac{1}{\beta(1-\widehat{r}) + \alpha} [\beta(1-\widehat{r}) \varepsilon_i + \alpha\eta] \\ \int_{i\in[0,1]} a_i di - \theta &= \frac{1}{\beta(1-\widehat{r}) + \alpha} \alpha\eta \end{aligned}$$

Normalized welfare is then

$$W \equiv \frac{1}{1-\widehat{r}} \int_0^1 u_i(\mathbf{a},\theta) \, di$$

$$= -\frac{1}{1-\widehat{r}}\left[\frac{1}{\beta(1-\widehat{r})+\alpha}\right]^2\left[\beta\left(1-\widehat{r}\right)^2\left(2\left(r_1-r_4\right)+r_2+r_3\right)+\alpha\left(r_2+r_5\right)\right]$$

Then, the derivative $\frac{dW}{d\alpha}$ is given by

$$\frac{1}{1-\widehat{r}}\left[\frac{1}{\beta(1-\widehat{r})+\alpha}\right]^3\left\{\begin{array}{l}-\beta\left(1-\widehat{r}\right)\left(r_2+r_5-2\left(1-\widehat{r}\right)\left(2\left(r_1-r_4\right)+r_2+r_3\right)\right)\\+\alpha\left(r_2+r_5\right)\end{array}\right\}$$

Thus public information is always valuable if $\beta = 0$. Public information can sometimes be damaging (when $\beta > 0$ and α is low) when

$$r_2+r_5 \geq \frac{2r_2\left(2\left(r_1-r_4\right)+r_2+r_3\right)}{r_1+r_2+r_3}.$$

Our leading model in section 2 is the special case of this when $r_1 = r_4 = r$, $r_2 = 1-r$ and $r_3 = r_5 = 0$. In this case, this condition reduces to $r \geq \frac{1}{2}$.

We conclude this section with one other example. Let each player i have the following payoff function:

$$u_i\left(\mathbf{a},\theta\right) \equiv \left(1-\varepsilon\right)V\left(\theta,\bar{a}\right)-\varepsilon\left(1-r\right)\left(a_i-\theta\right)^2-\varepsilon r\int_0^1\left(a_j-a_i\right)^2 dj \qquad (5.2)$$

for some small $\varepsilon > 0$. Equilibrium is unaltered by this change in payoffs, since the first term is an externality that no individual player can influence. However, for small ε, social welfare will be approximately equal to $V\left(\theta,\bar{a}\right)$. For some choice of $V\left(\cdot\right)$, public information may be damaging *even in the absence of private information.* For example, suppose that

$$V\left(\theta,\overline{a}\right)=\left\{\begin{array}{l}1,\text{ if }\overline{a}\geq a^*\\0,\text{ if }\overline{a}\leq a^*\end{array}\right\}$$

Suppose that the only information is the public signal $y = \theta + \eta$, where η is normally distributed with mean zero and precision α. Each player will set his action equal to y. So conditional on state θ, expected welfare (for small ε) is

$$1-\Phi\left(\sqrt{\alpha}\left(a^*-\theta\right)\right).$$

This is decreasing in α if $a^* > \theta$. In other words, if players would do something socially inefficient if there were perfect information, then reducing the accuracy of public information should be expected to improve social welfare.[3]

6. Correlated Private Signals

So far, we have dealt with conditionally independent noise across agents. This paints a rather unrealistic picture of private signals as being uncontaminated by imperfections that the players' private information may share in common. More realistically, we would expect that private signals have shared raw ingredients across the population that impart complex correlation structures across private signals. As a simple example, private signals that have the structure $x_i = \theta + \xi + \varepsilon_i$, where ξ is a common noise term that enters into all players' private signals will impart correlations into the private signals, even if we condition on the true state θ.

In the appendix, we present quite a general analysis of a two player version of our model where the players can observe many signals, where the signals are multivariate normal with a general correlation structure. Here, we confine ourselves to presenting a special case of that analysis. Thus, suppose there are two players $i = 1, 2$. There is a public signal $y = \theta + \eta$, where η is normally distributed with mean 0 and precision α. In addition, each player i observes two private signals $x_{i1} = \theta + \varepsilon_{i1}$ and $x_{i2} = \theta + \varepsilon_{i2}$. While ε_{i1} and ε_{i2} are assumed to be independent, we assume that $(\varepsilon_{1j}, \varepsilon_{2j})$ are jointly normally distributed with zero means and covariance matrix

$$\begin{pmatrix} \frac{1}{\beta_j} & \frac{\rho_j}{\beta_j} \\ \frac{\rho_j}{\beta_j} & \frac{1}{\beta_j} \end{pmatrix}.$$

[3]This effect occurs in the analysis of transparency in currency markets in Metz (2000).

Thus each signal j has precision β_j, and the correlation coefficient between the two players' jth signals is $\rho_j \in [0,1)$.

In this setting, we show in the appendix that while the socially optimal action for player i (minimizing $-(a_i-\theta)^2$) is

$$a_i = \frac{\alpha y + \beta_1 x_{i1} + \beta_2 x_{i2}}{\alpha + \beta_1 + \beta_2},$$

the equilibrium action for player i is

$$a_i = \frac{\alpha y + \beta_1 \left(\frac{1-r}{1-r\rho_1}\right) x_{i1} + \beta_2 \left(\frac{1-r}{1-r\rho_2}\right) x_{i2}}{\alpha + \left(\frac{1-r}{1-r\rho_1}\right)\beta_1 + \left(\frac{1-r}{1-r\rho_2}\right)\beta_2}.$$

This expression concretely captures the trade-off between the accuracy of a signal and its ability to coordinate the players. The corresponding expression for welfare is

$$-\frac{\alpha + \left(\frac{1-r}{1-r\rho_1}\right)^2 \beta_1 + \left(\frac{1-r}{1-r\rho_2}\right)^2 \beta_2}{\left(\alpha + \left(\frac{1-r}{1-r\rho_1}\right)\beta_1 + \left(\frac{1-r}{1-r\rho_2}\right)\beta_2\right)^2}.$$

In fact, the conclusions of this paper do not rely on the normality assumptions. Samet (1998) showed that iterated expectations of the type defined in equation (4.2) always converge (as the number of expectations k tends to infinity) to the expectation of θ conditional only on public (i.e., common knowledge) information. Combined with equation (4.3), this implies that equilibrium actions in the finite action game must ignore private information in the limit as $r \to 1$. Similar results could also be proved for the continuum player case with more general signal structures. In Morris and Shin (2001), we have verified that we obtain qualitatively similar conclusions in a binary state binary signal model, although in that example private information must be very accurate in order for public information to damage welfare.

7. Concluding Remarks

Public information has attributes that make it a double-edged instrument. The strategic interactions between agents mean that its impact is larger than could be justified by the face value of its content. Thus, although public information is extremely effective in influencing actions, the danger arises from the fact that it is *too effective* at doing so. Agents overreact to public information, and thereby magnify any noise which inevitably creeps in.

Commentators such as Krugman (2001b) have raised the possibility that the parameter r in our model - indicating the strength of the strategic motive - may have become larger in recent years. Commenting on the recent downturn in economic activity in the United States, he suggests that

> "firms making investment decisions are starting to emulate the hair-trigger behavior of financial investors. That means a growing part of the economy may be starting to act like a financial market, with all that implies - like the potential for bubbles and panics. One could argue that far from making the economy more stable, the rapid responses of today's corporations make their investment in equipment and software vulnerable to the kind of self-fulfilling pessimism that used to be possible only for investment in paper assets."

In terms of the framework of our paper, the increased vulnerability mentioned by Krugman is an entirely rational response by individual actors, but is socially inefficient. Such effects serve to emphasize the double-edged nature of public information.

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Appendix: Multivariate Normal Signals

A state θ is drawn from a uniform distribution on the real line. There is a public signal $y = \theta + \eta$. Each agent $i = 1, 2$ observes n private signals; the kth signal of agent i is $x_{ik} = \theta + \varepsilon_{ik}$. We write $\boldsymbol{\varepsilon}_i$ for the n-vector of agent i's noise terms. We assume that the $(2n+1)$ vector

$$\boldsymbol{\xi} = \begin{pmatrix} \eta \\ \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \end{pmatrix}$$

is normally distributed with mean

$$\begin{pmatrix} 0 \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

(we are writing $\mathbf{0}$ for a vector of 0s of arbitrary length) and covariance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{00} & \boldsymbol{\Sigma}_{01} & \boldsymbol{\Sigma}_{02} \\ \boldsymbol{\Sigma}_{10} & \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{20} & \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{pmatrix}.$$

We are interested in

$$\mathbf{z} = \begin{pmatrix} \theta \\ \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} - y\mathbf{1}.$$

(we are writing $\mathbf{1}$ for a vector of 1s of arbitrary length). Setting

$$\mathbf{A} = \begin{pmatrix} -1 & 0 & 0 & 0 & \cdot & \cdot \\ -1 & 1 & 0 & 0 & \cdot & \cdot \\ -1 & 0 & 1 & 0 & \cdot & \cdot \\ -1 & 0 & 0 & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

we have

$$\mathbf{z} = \mathbf{A}\boldsymbol{\xi}$$

so z is normally distributed with mean

$$\begin{pmatrix} 0 \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

and covariance matrix

$$\begin{aligned} \widehat{\mathbf{\Sigma}} &= \mathbf{A\Sigma A}' \\ &= \begin{pmatrix} \widehat{\mathbf{\Sigma}}_{00} & \widehat{\mathbf{\Sigma}}_{01} & \widehat{\mathbf{\Sigma}}_{02} \\ \widehat{\mathbf{\Sigma}}_{10} & \widehat{\mathbf{\Sigma}}_{11} & \widehat{\mathbf{\Sigma}}_{12} \\ \widehat{\mathbf{\Sigma}}_{20} & \widehat{\mathbf{\Sigma}}_{21} & \widehat{\mathbf{\Sigma}}_{22} \end{pmatrix}. \end{aligned}$$

We will often be interested in the case where the public signal is conditionally independent of the various private signals. In this case,

$$\mathbf{\Sigma} = \begin{pmatrix} \tau^2 & 0 & 0 \\ 0 & \mathbf{\Sigma}_{11} & \mathbf{\Sigma}_{12} \\ 0 & \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} \end{pmatrix}$$

and

$$\widehat{\mathbf{\Sigma}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathbf{\Sigma}_{11} & \mathbf{\Sigma}_{12} \\ 0 & \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} \end{pmatrix} + \tau^2 \mathbf{M}$$

where $\mathbf{M}$ is a $(2n+1) \times (2n+1)$ matrix of 1's.

By standard properties of the multivariate normal (e.g., Spanos (1986), ch 14, p 317),

$$\begin{aligned} E_1(\theta - y) &= E_1(\theta - y\,|\mathbf{x}_1 - y\mathbf{1}) = \widehat{\mathbf{\Sigma}}_{01}\widehat{\mathbf{\Sigma}}_{11}^{-1}(\mathbf{x}_1 - y\mathbf{1}) \\ E_2(\theta - y) &= E_2(\theta - y\,|\mathbf{x}_2 - y\mathbf{1}) = \widehat{\mathbf{\Sigma}}_{02}\widehat{\mathbf{\Sigma}}_{22}^{-1}(\mathbf{x}_1 - y\mathbf{1}) \end{aligned}$$

Thus player 1's *optimal action* is

$$\begin{aligned} a_1 &= E_1(\theta) \qquad (7.1) \\ &= y + E_1(\theta - y) \\ &= y + \widehat{\mathbf{\Sigma}}_{01}\widehat{\mathbf{\Sigma}}_{11}^{-1}(\mathbf{x}_1 - y\mathbf{1}) \end{aligned}$$

Also

$$
\begin{aligned}
E_1\left(\mathbf{x}_2 - y\mathbf{1}\right) &= E_1\left(\mathbf{x}_2 - y\mathbf{1}\,|\mathbf{x}_1 - y\mathbf{1}\right) = \widehat{\boldsymbol{\Sigma}}_{21}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\left(\mathbf{x}_1 - y\mathbf{1}\right) \\
E_2\left(\mathbf{x}_1 - y\mathbf{1}\right) &= E_2\left(\mathbf{x}_1 - y\mathbf{1}\,|\mathbf{x}_2 - y\mathbf{1}\right) = \widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\left(\mathbf{x}_2 - y\mathbf{1}\right)
\end{aligned}
$$

Now

$$
\begin{aligned}
E_1E_2\left(\mathbf{x}_1 - y\mathbf{1}\right) &= E_1\left(\widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\left(x_2 - y\right)\right) \\
&= \widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\left(E_1\left(\mathbf{x}_2 - y\mathbf{1}\right)\right) \\
&= \widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\widehat{\boldsymbol{\Sigma}}_{21}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\left(\mathbf{x}_1 - y\mathbf{1}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
E_2E_1E_2\left(\mathbf{x}_1 - y\mathbf{1}\right) &= E_2\left(E_1E_2\left(\mathbf{x}_1 - y\mathbf{1}\right)\right) \\
&= E_2\left(\widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\widehat{\boldsymbol{\Sigma}}_{21}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\left(\mathbf{x}_1 - y\mathbf{1}\right)\right) \\
&= \widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\widehat{\boldsymbol{\Sigma}}_{21}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\left(E_2\left(\mathbf{x}_1 - y\mathbf{1}\right)\right) \\
&= \widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\widehat{\boldsymbol{\Sigma}}_{21}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\left(x_2 - y\right)
\end{aligned}
$$

Thus by induction

$$
\left[E_1E_2\right]^n\left(\mathbf{x}_1 - y\mathbf{1}\right) = \left[\widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\widehat{\boldsymbol{\Sigma}}_{21}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\right]^n\left(\mathbf{x}_1 - y\mathbf{1}\right)
$$

and

$$
\begin{aligned}
\left[E_1E_2\right]^n\left(E_1\left(\theta - y\right)\right) &= \left[E_1E_2\right]^n\left(\widehat{\boldsymbol{\Sigma}}_{01}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\left(\mathbf{x}_1 - y\mathbf{1}\right)\right) \\
&= \widehat{\boldsymbol{\Sigma}}_{01}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\left[\widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\widehat{\boldsymbol{\Sigma}}_{21}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\right]^n\left(\mathbf{x}_1 - y\mathbf{1}\right)
\end{aligned}
$$

Also

$$
\begin{aligned}
E_2\left[E_1E_2\right]^n\left(E_1\left(\theta - y\right)\right) &= \widehat{\boldsymbol{\Sigma}}_{02}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\left[\widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\widehat{\boldsymbol{\Sigma}}_{21}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\right]^n\widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\left(\mathbf{x}_2 - y\mathbf{1}\right) \\
\left[E_2E_1\right]^n\left(E_2\left(\theta - y\right)\right) &= \widehat{\boldsymbol{\Sigma}}_{02}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\left[\widehat{\boldsymbol{\Sigma}}_{21}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\right]^n\left(\mathbf{x}_2 - y\mathbf{1}\right) \\
E_1\left[E_2E_1\right]^n\left(E_2\left(\theta - y\right)\right) &= \widehat{\boldsymbol{\Sigma}}_{01}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\left[\widehat{\boldsymbol{\Sigma}}_{21}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\widehat{\boldsymbol{\Sigma}}_{12}\widehat{\boldsymbol{\Sigma}}_{22}^{-1}\right]^n\widehat{\boldsymbol{\Sigma}}_{21}\widehat{\boldsymbol{\Sigma}}_{11}^{-1}\left(\mathbf{x}_1 - y\mathbf{1}\right)
\end{aligned}
$$

Now player 1's equilibrium action is:

$$
\begin{aligned}
a_1 &= (1-r)\,E_1(\theta) + (1-r)\,rE_1E_2(\theta) + (1-r)\,r^2E_1E_2E_1(\theta) + ... \qquad (7.2)\\
&= y + \left\{ \begin{array}{l}
(1-r)\,\widehat{\mathbf{\Sigma}}_{01}\widehat{\mathbf{\Sigma}}_{11}^{-1}(\mathbf{x}_1 - y\mathbf{1}) \\
+(1-r)\,r\widehat{\mathbf{\Sigma}}_{01}\widehat{\mathbf{\Sigma}}_{11}^{-1}\widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1}(\mathbf{x}_1 - y\mathbf{1}) \\
+(1-r)\,r^2\widehat{\mathbf{\Sigma}}_{01}\widehat{\mathbf{\Sigma}}_{11}^{-1}\left[\widehat{\mathbf{\Sigma}}_{12}\widehat{\mathbf{\Sigma}}_{22}^{-1}\widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1}\right](\mathbf{x}_1 - y\mathbf{1}) \\
+(1-r)\,r^3\widehat{\mathbf{\Sigma}}_{01}\widehat{\mathbf{\Sigma}}_{11}^{-1}\left[\widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1}\widehat{\mathbf{\Sigma}}_{12}\widehat{\mathbf{\Sigma}}_{22}^{-1}\right]\widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1}(\mathbf{x}_1 - y\mathbf{1}) \\
+(1-r)\,r^4\widehat{\mathbf{\Sigma}}_{01}\widehat{\mathbf{\Sigma}}_{11}^{-1}\left[\widehat{\mathbf{\Sigma}}_{12}\widehat{\mathbf{\Sigma}}_{22}^{-1}\widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1}\right]^2(\mathbf{x}_1 - y\mathbf{1}) \\
+....
\end{array} \right\} \\
&= y + (1-r)\,\widehat{\mathbf{\Sigma}}_{01}\widehat{\mathbf{\Sigma}}_{11}^{-1}\left[I + r^2\widehat{\mathbf{\Sigma}}_{12}\widehat{\mathbf{\Sigma}}_{22}^{-1}\widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1} + r^4\left[\widehat{\mathbf{\Sigma}}_{12}\widehat{\mathbf{\Sigma}}_{22}^{-1}\widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1}\right]^2 + ...\right] \\
&\quad \times \left(I + r\widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1}\right)(\mathbf{x}_1 - y\mathbf{1}) \\
&= y + (1-r)\,\widehat{\mathbf{\Sigma}}_{01}\widehat{\mathbf{\Sigma}}_{11}^{-1}\left[I - r^2\widehat{\mathbf{\Sigma}}_{12}\widehat{\mathbf{\Sigma}}_{22}^{-1}\widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1}\right]^{-1}\left(I + r\widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1}\right)(\mathbf{x}_1 - y\mathbf{1})
\end{aligned}
$$

In the symmetric case, where

$$\mathbf{N} = \widehat{\mathbf{\Sigma}}_{12}\widehat{\mathbf{\Sigma}}_{22}^{-1} = \widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1},$$

this formula becomes

$$a_1 = y + (1-r)\,\widehat{\mathbf{\Sigma}}_{01}\widehat{\mathbf{\Sigma}}_{11}^{-1}\left[I - r\mathbf{N}\right]^{-1}(\mathbf{x}_1 - y\mathbf{1}) \qquad (7.3)$$

In the two signal example discussed in section 6, we have

$$\mathbf{\Sigma} = \begin{pmatrix} \frac{1}{\alpha} & 0 & 0 & 0 & 0 \\ 0 & \sigma_1^2 & 0 & \rho_1\sigma_1^2 & 0 \\ 0 & 0 & \frac{1}{\beta_2} & 0 & \rho_2\frac{1}{\beta_2} \\ 0 & \rho_1\sigma_1^2 & 0 & \sigma_1^2 & 0 \\ 0 & 0 & \rho_2\frac{1}{\beta_2} & 0 & \frac{1}{\beta_2} \end{pmatrix}$$

and

$$\widehat{\mathbf{\Sigma}} = \begin{pmatrix} \frac{1}{\alpha} & \frac{1}{\alpha} & \frac{1}{\alpha} & \frac{1}{\alpha} & \frac{1}{\alpha} \\ \frac{1}{\alpha} & \frac{1}{\alpha} + \frac{1}{\beta_1} & \frac{1}{\alpha} & \frac{1}{\alpha} + \rho_1\frac{1}{\beta_1} & \frac{1}{\alpha} \\ \frac{1}{\alpha} & \frac{1}{\alpha} & \frac{1}{\alpha} + \frac{1}{\beta_2} & \frac{1}{\alpha} & \frac{1}{\alpha} + \rho_2\frac{1}{\beta_2} \\ \frac{1}{\alpha} & \frac{1}{\alpha} + \rho_1\frac{1}{\beta_1} & \frac{1}{\alpha} & \frac{1}{\alpha} + \frac{1}{\beta_1} & \frac{1}{\alpha} \\ \frac{1}{\alpha} & \frac{1}{\alpha} & \frac{1}{\alpha} + \rho_2\frac{1}{\beta_2} & \frac{1}{\alpha} & \frac{1}{\alpha} + \frac{1}{\beta_2} \end{pmatrix}$$

Now

$$\begin{aligned}
\widehat{\mathbf{\Sigma}}_{01} &= \begin{pmatrix} \frac{1}{\alpha} & \frac{1}{\alpha} \end{pmatrix} \\
\widehat{\mathbf{\Sigma}}_{11} &= \begin{pmatrix} \frac{1}{\alpha}+\frac{1}{\beta_1} & \frac{1}{\alpha} \\ \frac{1}{\alpha} & \frac{1}{\alpha}+\frac{1}{\beta_2} \end{pmatrix}
\end{aligned}$$

and

$$\mathbf{\Sigma}_{11}^{-1} = \frac{1}{\alpha+\beta_1+\beta_2}\begin{pmatrix} \beta_1(\alpha+\beta_2) & -\beta_1\beta_2 \\ -\beta_1\beta_2 & \beta_2(\beta_1+\alpha) \end{pmatrix}$$

so

$$\widehat{\mathbf{\Sigma}}_{01}\mathbf{\Sigma}_{11}^{-1} = \frac{1}{\alpha+\beta_1+\beta_2}\begin{pmatrix} \beta_1 & \beta_2 \end{pmatrix}.$$

Now by (7.1), player 1's optimal action is

$$\begin{aligned}
a_1^* &= y + \widehat{\mathbf{\Sigma}}_{01}\mathbf{\Sigma}_{11}^{-1}\begin{pmatrix} x_{11}-y \\ x_{12}-y \end{pmatrix} \\
&= y + \frac{\beta_1 x_{11} - \beta_1 y + \beta_2 x_{12} - \beta_2 y}{\alpha+\beta_1+\beta_2} \\
&= \frac{\alpha y + \beta_1 x_{11} + \beta_2 x_{12}}{\alpha+\beta_1+\beta_2}
\end{aligned}$$

Also

$$\mathbf{\Sigma}_{12} = \mathbf{\Sigma}_{21} = \begin{pmatrix} \frac{1}{\alpha}+\rho_1\frac{1}{\beta_1} & \frac{1}{\alpha} \\ \frac{1}{\alpha} & \frac{1}{\alpha}+\rho_2\frac{1}{\beta_2} \end{pmatrix}$$

so

$$\begin{aligned}
\mathbf{N} &= \widehat{\mathbf{\Sigma}}_{21}\widehat{\mathbf{\Sigma}}_{11}^{-1} = \widehat{\mathbf{\Sigma}}_{12}\widehat{\mathbf{\Sigma}}_{22}^{-1} = \frac{1}{\alpha+\beta_1+\beta_2}\begin{pmatrix} \beta_1+\rho_1(\alpha+\beta_2) & \beta_2(1-\rho_1) \\ \beta_1(1-\rho_2) & \beta_2+\rho_2(\alpha+\beta_1) \end{pmatrix} \\
\mathbf{I}-r\mathbf{N} &= \frac{1}{\alpha+\beta_1+\beta_2}\begin{pmatrix} (1-r)\beta_1+(1-r\rho_1)(\alpha+\beta_2) & -r\beta_2(1-\rho_1) \\ -r\beta_1(1-\rho_2) & (1-r)\beta_2+(1-r\rho_2)(\alpha+\beta_1) \end{pmatrix}
\end{aligned}$$

Thus by (7.3), player 1's equilibrium action is

$$\begin{aligned}
a_1 &= y + (1-r)\widehat{\mathbf{\Sigma}}_{01}\widehat{\mathbf{\Sigma}}_{11}^{-1}[I-r\mathbf{N}]^{-1}\begin{pmatrix} x_{11}-y \\ x_{12}-y \end{pmatrix} \\
&= \frac{\alpha y + \beta_1\left(\frac{1-r}{1-r\rho_1}\right)x_{11} + \beta_2\left(\frac{1-r}{1-r\rho_2}\right)x_{12}}{\alpha + \beta_1\left(\frac{1-r}{1-r\rho_1}\right) + \beta_2\left(\frac{1-r}{1-r\rho_2}\right)}
\end{aligned}$$