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EVALUATING THE PROBABILITY OF FAILURE OF A BANKING FIRM

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# Evaluating the Probability of Failure of a Banking Firm\*

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## Abstract

We develop a dynamic model in which the probability of failure of an infinitely lived financial intermediary (bank) is determined endogenously as a function of observable state and policy variables. The bank takes into account the effect of the optimal policy (the interest on deposits, dividend payouts, risky investments) on the probability of failure, which in turn affects the bank's ability to extract deposits. With the aid of simulations we study the effect of variables such as bank size, the riskiness of the bank's investment opportunities, and reserve requirements on the bank's optimal policy and on its probability of failure. A major finding is that small banks choose policies that render them more risky than large banks. As the risks are correctly priced by depositors, rates offered by small banks incorporate substantial risk premia. Another interesting finding is that a tighter reserve requirement induces banks of all sizes to take fewer risks.

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# 1 Introduction

Assessing the probability of failure of banking firms is of great importance whether one believes in “market discipline” or in “hands on” bank regulation. We develop a model in which the probability of failure of a financial intermediary (bank) is determined endogenously. The model contributes to a better understanding of the economic forces at play in the determination of the equilibrium probability of failure of depository institutions in the absence of full deposit insurance. It sheds light on the effectiveness of market discipline in controlling the riskiness of banks’ portfolios, and hence on their probability of failure. The model is also important for regulators, as it may help in developing tools for identifying “problem banks” and for evaluating regulatory measures such as risk-based deposit insurance schemes.<sup>1</sup>

In the model we develop, an infinitely lived bank accepts deposits each period for a specified interest rate, and it invests the total available funds in riskless bonds, risky securities, and risky projects. The depositors take into account the probability of failure when supplying funds to the bank. The bank takes into account the effect of its policy decisions on its own probability of failure, and hence on the willingness of depositors to supply funds. In equilibrium, depositors’ beliefs are correct in that their perceived probability of failure is true. We can, therefore, express the probability of failure as a function of the bank’s observable policy variables and its equity capital, which we shall call, for simplicity, the bank’s size. Under fairly general conditions we show that there exists a unique value function for the bank’s stochastic dynamic program, and we provide a set of conditions that ensures that the bank’s policy function is unique.

The paper is related to the portfolio selection models of financial intermediaries (e.g. Hart and Jaffe (1974)), where intermediaries are modeled as risk averse agents, and asset returns are exogenously determined. Flannery (1989) incorporates in this framework deposit insurance and capital adequacy regulation in the form of a permissible leverage ratio, which decreases with bank riskiness.<sup>2</sup> We depart from the traditional portfolio selection

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<sup>1</sup>There is empirical literature that attempts to develop an early warning system for “problem banks” by identifying measures of risk that best predict actual bank failure rates (e.g., Santomero and Vinso (1977); Lane, Looney, and Wansley (1986)).

<sup>2</sup>In Flannery’s model the bank can select the desired degree of riskiness; by increasing it, the bank

model in several ways. First, our model is an infinite horizon, dynamic model of optimization. Second, it incorporates an explicit mechanism of market discipline. Third, and most important, the probability of failure is determined endogenously.

An important feature of the model is that in equilibrium the interest rate offered by the bank incorporates a premium above the risk free rate (e.g., the rate on U.S. Treasury bills of a similar maturity.) This is consistent with Matutes and Vives (1994) where two banks are engaged in a one-shot game, each selecting both its riskiness and the interest rate on deposits. Their model predicts that the interest rate chosen by each bank increases with its riskiness.<sup>3</sup> Recent empirical work is consistent with this prediction, indicating that the rates on large, uninsured certificates of deposit (CD's) include significant default risk premia.<sup>4</sup> See, for example, James (1988), Hannan and Hanweck (1988), Keeley (1990), and Ellis and Flannery (1992).

To address a series of policy related issues we simulate the model for a variety of parameter values. In the base case simulation we study the optimal policy—dividends, risky investments, and the interest rate on bank CD's—for banks of various sizes. As a result, we can determine the probability of failure for banks of different sizes. We then study the effect of several parameter changes on the optimal policy and the probability of failure.

An interesting question addressed in the simulations is whether a high interest rate on uninsured CD's reflects bank riskiness or, rather, simply indicates a pressing (possibly temporary) need for funds. In the base case simulation high interest rates are associated with large deposits *and* a high probability of failure; high rates reflect both a high demand for deposits and a risk premium.

It is well known that when there is deposit insurance banks tend to become less safe, regardless of size. The "too big to fail" principle suggests that the moral hazard problem

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increases the value of the put option embedded in deposit insurance, at the price of decreasing the permissible leverage ratio. Flannery shows that, for suitable parameter values, the bank's problem is concave with an interior solution, despite the fact that the bank is risk neutral.

<sup>3</sup>For a related model in the asset pricing tradition see Crouhy and Galai (1991).

<sup>4</sup>An exception is Crane (1976) who finds no significant relation between CD rates and measures of bank risk. Crane reports, though, that bank CD rates tend to decrease with total demand deposits in the bank. According to Crane total deposits can be regarded as a proxy for bank size, which is an indication for the soundness of the bank. We do not agree with this interpretation as a large amount of deposits (relative to the equity base, for example) may entail—depending, of course, on the bank's behavior on the asset side—a large risk of default.

arising from the introduction of deposit insurance is most severe for large banks. Therefore, other things equal, we should expect large banks to be less safe. Boyd and Runkle (1993) observe that according to modern financial intermediation theory larger banks can afford to invest in many projects (whose returns are not perfectly correlated) and hence are more diversified and safer. They provide empirical evidence suggesting that the investment portfolios of large banks are less risky, although they find no evidence that these banks have a lower probability of failure. Also, they are not able to disentangle empirically the “too big to fail” and the cost of diversification effects.

In our model there is no deposit insurance, and banks of all sizes have access to the same number of risky technologies. Therefore, neither of the above effects is present. The focus is on the interest rate policy, the investment policy, and the dividend consumption decisions of banks of different sizes. The simulations indicate that small banks (i.e., banks with little equity capital) are less safe than large banks. They do not reduce dividends and risky investments sufficiently to become as safe as larger banks. The implication for regulators is obvious—small banks should be regulated more tightly, possibly by limiting dividend distribution.<sup>5</sup>

Next, we study the effect of a reserve requirement on the portfolio choice and the probability of failure of banks. Monetary theorists have traditionally focused on the role reserve requirements play in controlling the money supply, in raising revenues for the Treasury (being a tax on deposits), and in insuring banks against adverse liquidity shocks. Our analysis suggests that reserve requirements may also influence the dividend and investment policies of banks, thereby affecting their probability of survival. The main finding is that a tighter reserve requirement induces smaller banks to consume more dividends but to offer lower interest rates and raise fewer deposits; overall, the banks become safer.

When the riskiness of the investment opportunities that banks face increases, banks of all sizes respond by shifting funds away from the most risky investments, but not sufficiently to prevent the probability of failure from increasing. This is the main finding from simulations where we vary the dispersion of the returns on risky investments, keeping all other

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<sup>5</sup>The US General Accounting Office has recently proposed to prohibit dividends by under-capitalised banks. Horne (1991) provides further details as well as a discussion of other factors affecting bank dividend policy.

parameters constant. Small banks increase the interest rate on CD's but do not reduce the consumption of dividends. This is a further indication that the risk-taking behavior of small (under-capitalized) banks should be monitored.

We study the effect of changes in the risk aversion of the bank owner-manager and find that more risk averse banks distribute fewer dividends. Interpreting higher risk aversion as more concentrated ownership, the simulations yield an interesting testable prediction—other things equal, banks with concentrated ownerships pay fewer dividends.

To evaluate the effect of “market discipline” on banks' policies and the probability of failure, we study the effect of changes in the sensitivity of the supply of deposits to the interest rate offered by the bank as well as the sensitivity of deposits to the probability of failure of the bank. The results indicate a clear role for market discipline. Finally, we check the effect of small changes in the riskless interest rate and find that they result in changes in policy and negligible changes in the probability of failure.

For every simulation we follow, for thirty periods, a cohort of one thousand banks that start operating at the same time and are initially of equal size. We allow the banks to select optimal policies. As they are exposed to idiosyncratic uncertainty, the banks gradually begin to differ in their size. We portray the evolution of the cohort as time goes by along two dimensions—the number of banks left operating and the size distribution of these banks. The shape of the distribution typically becomes almost stationary after thirty periods, and in most cases, the limit distribution is skewed to the left with a small number of very large banks. An exception is the simulation that studies the effect of the riskiness of the investment opportunities the bank faces. As the riskiness increases, the size distribution of the population of surviving banks becomes less skewed to the left, with many large banks—those that make it, make it big.<sup>6</sup>

In the next section we lay out the basic ingredients of the model and describe the environment in which the bank operates. Section 3 is devoted to a formal presentation and analysis of the model. In section 4 we present the simulation results and discuss their policy implications. Section 5 concludes the paper.

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<sup>6</sup>Of related interest is the literature on the growth rate and the size distribution of business firms; see, e.g., Segal and Spivak (1989) and Lucas (1978) respectively.

## 2 Overview of the Model

An infinitely lived, risk averse owner of a bank derives utility from the consumption of dividend income.<sup>7</sup> Each period the bank invests the money at its disposal, the newly acquired deposits, minus the payments to the depositors from the previous period. The bank can purchase risky securities on the stock exchange (the market portfolio), as well as invest in projects such as loans to new businesses, funding of R&D ventures, or the extension of mortgages, all of which involve risk. The bank can also purchase riskless government bonds. Short positions are allowed (up to a limit) in risky securities on the stock exchange but not on riskless government bonds. That is to say, the bank cannot borrow at the riskless rate. Furthermore, the bank can raise money from firms only by issuing certificates of deposit.<sup>8</sup>

We abstract from the optimization problem of individual depositors by introducing a supply function of deposits (that is, a demand function for CD's) which the bank faces. The amount of funds depositors are willing to supply increases with the interest rate offered by the bank and with its probability of survival. These assumptions are discussed further below.

The size of the bank is defined as the monetary value of its equity capital. This value can become negative, up to a limit. If the monetary value of the bank's capital plus the maximal amount the bank can raise by short selling securities is negative at the beginning of a period, then the bank is declared bankrupt and ceases to operate. We assume that banks cannot rely on funds from new depositors to avoid bankruptcy; that is, a bank can raise funds from depositors only if it is able to survive without them. This restriction can be interpreted as part of the bank closure policy of regulators or as the depositors' response to the bank's precarious situation. Each period, provided it has not failed, the bank chooses the following policy variables: (i) dividends to be distributed to shareholders; (ii) the interest rate to be offered on deposits; (iii) the amount to be invested in risky

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<sup>7</sup>We do not distinguish between owners and managers, assuming that their interests are perfectly aligned.

<sup>8</sup>The specification of the model is sufficiently flexible to allow for restrictions of the Glass-Steagall type, which prohibit commercial banks from owning the equity of firms. For example, it can be postulated that investments in risky projects must take the form of loans. The possibility of short sales, which is intended to capture the ability of banks to raise money on public securities markets, can be prohibited altogether without altering the nature of the analysis.

securities; and (iv) the amount to be invested in risky projects. The bank's remaining funds are invested in riskless government bonds, and they are required to exceed an exogenously specified fraction of deposits (a reserve requirement).<sup>9</sup>

The policy variables, together with the bank's size at the beginning of the period, determine its probability of survival for the next period. Depositors take the probability of survival into account when they choose the amount of funds they are willing to supply. In equilibrium, depositors' beliefs regarding the probability of survival are correct. That is, given the supply-of-funds function induced by the depositors' beliefs, the policy chosen by the bank induces precisely the probability of survival conjectured by the depositors. The size of the bank at the end of each period (conditional on survival) is determined by its initial size, the policy it chooses, and the realizations of the returns from risky investments.

A crucial assumption of the model is that the bank behaves as a risk averse agent. When markets are complete, every consumer who owns a share in a productive firm wants the firm to maximize expected profits, which is equivalent to maximizing the value of the firm's shares on the stock market.<sup>10</sup> The assumption of complete markets is, of course, an abstract benchmark. Our model subsumes a certain degree of incompleteness in financial markets, whereby depositors are unable fully to diversify their portfolios by depositing in several banks and by hedging on securities markets against bank default risk. When markets are incomplete banks may contribute to financial innovation by introducing CD's whose probability of default depends on the types of projects the bank finances.

Drèze (1987) discusses various approaches to the choice of an objective function for a firm when markets are incomplete. One approach relates the firm's decision criterion to the preferences of its shareholders. Another approach endows the firm with preferences of its own. Although there are sensible criticisms of the latter approach, we find it quite suitable for our purposes.<sup>11</sup> When a bank is owned by a small group of shareholders who are also

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<sup>9</sup>The bank in our model cannot use the Central Bank discount window to raise funds, even though the interest rate on such loans is often set below the market rate. This assumption is based on the general belief that the discount window cannot be simply regarded as an inexpensive source of funds. Loans at the window are very short term (15 days in the U.S.), require collateral, and often entail tighter supervision on the part of the regulators. Hence, these loans are in fact more costly than they may seem at first. See Poole (1992) for an extensive discussion.

<sup>10</sup>See Drèze (1987) and the references therein.

<sup>11</sup>An interesting criticism of this approach is that "although the axioms of consistent behavior may be



active in its management (for example, “family banks”), it is likely that the personal wealth of the shareholders will be tied closely to the performance of the bank. The assumption of a risk averse owner-manager captures this situation rather well. For banks whose ownership is less concentrated, this effect is weaker, and hence the degree of risk aversion is likely to be smaller. In the simulations of the model, we parameterize the utility function of the bank and study the effect of the degree of risk aversion on the bank’s choice of policy and probability of failure.

#### **The market structure:**

Banks are thought of as selling differentiated commodities: Their branches are situated in different geographical locations, some have more branches than others, some specialize in particular services (foreign currency transactions, import and export services), some securitize loans and mortgages while others do not, etc. Depositors differ in their tastes and needs, and therefore—other things equal—prefer to do business with a particular bank. By “other things equal” we mean price and the probability of failure. The former is captured by the interest rate on CD’s. The latter is a summary statistic of several features that typically differ across banks: bank size, the portfolio of the bank’s investments, and the bank’s dividend policy. When all banks offer the same interest rate on CD’s and have the same probability of failure, each depositor chooses to work with the bank whose other features are most preferred by the depositor (location, number of branches, etc.).

We focus on the differentiation across banks in the interest rate offered on CD’s and the probability of failure, while abstracting from other features. We view depositors as choosing an optimal savings portfolio and allocating funds between bank deposits and other assets. Each bank faces an elastic supply of deposits that depends on the characteristics of the bank. The policy choices made by the bank (the interest rate on CD’s, the dividends to be distributed, and the investment portfolio) affect the probability of failure. Banks may differ in size and in the policies they choose, and hence may differ in the probability of failure. As a consequence, bank CD’s are differentiated commodities that trade at different prices.

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equally cogent for a firm as for an individual, the decisions of the firm are in the nature of group decisions; institutional rules of group decision easily result in violation of the axioms—as illustrated by the Condorcet paradox of majority voting.” Drèze (1987, p.315)

Our purpose is to study the relation between the interest rate on CD's, the dividend and investment policies, and the (endogenously determined) probability of failure for banks of various sizes. We accomplish this by solving the dynamic stochastic program of a bank that, each period, faces an elastic supply of deposits. For every contingency—and in particular, for every possible bank size—the solution prescribes an optimal policy, and yields a probability of failure, which is perceived correctly by depositors and affects the amount of funds they are willing to supply.

When deposit insurance is not explicit, as in the U.S. for deposits exceeding \$100,000, it is likely that the behavior of depositors will be affected by the possibility of bank failure even though there is a (possibly high) probability that the government will cover losses, at least partially.<sup>12</sup> The studies by James (1988), Hannan and Hanweck (1988), Keeley (1990), and Ellis and Flannery (1992) confirm this hypothesis. Strahan (1992) provides evidence that depositors in the U.S. respond to bank risk even when their deposits are fully insured. This may reflect less than full confidence in the solvency of the insurer, or an anticipation of personal loss even if the insurer is solvent, due to delay in payment and foregone interest during the period of delay. This evidence motivates us to assume the amount of funds that depositors are willing to supply decreases with the probability of failure, even when depositors are fully insured.

### 3 The Model

A bank maximizes expected life-time utility from an infinite stream of consumption of dividend income. In each period, taking into consideration its financial situation, the bank chooses an optimal policy. We shall set up and solve the bank's dynamic program.

The state and policy variables:

The state of the dynamic program in period  $t$  is characterized by: (i)  $M_t$ , the monetary value (possibly negative) of the bank's portfolio at the beginning of period  $t$ , which we shall call the bank's size;<sup>13</sup> and (ii)  $Z_t^a$  and  $Z_t^l$ , the realized gross (and hence non-negative)

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<sup>12</sup>For a comparison of deposit insurance practices across OECD countries, see Frankel and Montgomery (1991).

<sup>13</sup> $M_t$  may take on negative values, while still allowing the bank to operate. This will become clearer when

one-period returns on investments made in period  $t - 1$  in securities and in risky projects, respectively.<sup>14</sup>  $Z_t^a$ , the return on the market portfolio, is an aggregate random variable that affects the entire economy, whereas  $Z_t^l$  represents the returns to the bank's idiosyncratic investments in projects. In the cohort simulations, all banks will face the same realization of  $Z_t^a$  and different realizations of  $Z_t^l$ . The returns on investments from period  $t - 1$  to  $t$  (as well as the bank's policy variables) determine  $M_t$ , so that the state of the dynamic program in period  $t$  can in fact be described by  $M_t$  alone. For notational convenience we define the returns vector  $Z_t \equiv (Z_t^a, Z_t^l)$ .

In period  $t$  the bank chooses the following policy variables: (i)  $R_t$ , the gross, one period rate of return on deposits—the principal plus interest will be paid to depositors at the beginning of period  $t + 1$ ; (ii)  $d_t$ , the dividend to be distributed (and consumed) at the beginning of period  $t$ ; (iii)  $a_t$ , the amount invested in securities; and (iv)  $\ell_t$ , the amount invested in projects. For notational convenience we define the policy vector  $c_t \equiv (R_t, d_t, a_t, \ell_t)$ .

The (gross) realized returns on investments made in period  $t$  accrue at the beginning of period  $t + 1$ . The amount invested in riskless government bonds can be thought of as a residual, since the total amount invested by the bank is subject to a budget constraint. The principal plus interest,  $R^B$ —an exogenously specified parameter of the model—will be repaid to the bank in the beginning of period  $t + 1$ .<sup>15</sup> We use the convention that variables indexed by  $t$  are known with certainty in period  $t$ , while the random variables in period  $t$  are indexed by  $t + 1$ . Thus, the size of the bank at the beginning of period  $t$ ,  $M_t$ , and the realized returns on risky investments made in period  $t - 1$ ,  $Z_t$ , are known to the bank in period  $t$ . The policy variables  $c_t$ , chosen by the bank in period  $t$ , are also indexed by  $t$ . The returns on risky investments made in period  $t$ ,  $Z_{t+1}$ , are random variables as of period  $t$ . The joint probability distribution of  $Z_{t+1}^a$  and  $Z_{t+1}^l$  is assumed to be stationary, with

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we provide the definition of bankruptcy.

<sup>14</sup>To economize on notation we assume that there is one risky security—the market portfolio—and similarly one risky project. For reasons of tractability we abstract from the distinction between long- and short-term projects. A possible interpretation is that, although projects may take more than one period to complete, they can be liquidated at any time, yielding  $Z_t^l$  if held from period  $t - 1$  to period  $t$ . Any amount can then be re-invested into the project, yielding  $Z_{t+1}^l$  in period  $t + 1$  if liquidated, and so forth.

<sup>15</sup>A more general formulation would be that in period  $t$  the return on riskless government bonds is known with certainty, but future riskless rates are stochastic. The model can easily accommodate such a specification. It would complicate, however, the proof of uniqueness of the optimal policy.

strictly increasing and continuously differentiable cumulative distribution function, which is known to the bank.<sup>16</sup>

#### The probability of survival:

Let  $q_t$  denote the probability as of period  $t$  that the bank will survive to period  $t + 1$ . We impose an equilibrium requirement that there be no divergence of opinion between the bank and depositors regarding  $q_t$ . Below, we shall explain the precise meaning of the term “survive” and show how  $q_t$  is (uniquely) determined. As the probability of survival in period  $t + 1$  is known in period  $t$  (by the bank and by depositors), it is indexed by  $t$ , in accord with our convention.

#### The supply of deposits:

In period  $t$  the bank faces a supply-of-deposits function,  $S(R_t, q_t)$ . The dependence of the supply function on the interest rate and on the probability of survival should be regarded as a reduced form of a complex set of considerations on the part of depositors regarding their payoff in the event of bankruptcy, e.g., the amount of money they will actually lose, the amount of money that regulators will reimburse despite the absence of explicit deposit insurance, delays, transaction costs. We do not model these contingencies explicitly. To keep the analysis tractable we assume that depositors care about only the promised interest rate and the probability of survival.

The supply function of deposits satisfies the following properties (see Figure 1). For any  $q_t \in (0, 1]$  and  $R_t \in [\underline{R}(q_t), \bar{R}(q_t)]$ ,  $S(R_t, q_t)$  is continuously differentiable and strictly increasing in both arguments. The supply of deposits is not stochastic, namely the bank does not face liquidity shocks. The riskiness of the bank is due entirely to its activities on the asset side.<sup>17</sup> The bounds  $\underline{R}(q_t)$  and  $\bar{R}(q_t)$  are continuous, differentiable, and decreasing in  $q_t$ —that is, the higher the probability of survival, the easier it becomes for the bank to induce depositors to supply the first dollar, as well as the last dollar they are willing

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<sup>16</sup>The assumption of stationarity can be dispensed with by introducing other assumptions, such as compact and convex supports for all the random variables, and the *Feller property*; see, e.g., Stokey and Lucas (1989, p.220).

<sup>17</sup>In Figure 1,  $S(R_t, q_t)$  is depicted as being strictly concave in the region  $R_t \in [\underline{R}(q_t), \bar{R}(q_t)]$ . This assumption will be added later (when we establish uniqueness of the policy function), but for now it is not needed.

to deposit with a bank of “quality”  $q_t$ . Furthermore, we assume that  $\underline{R}(q_t) > R^B$  for all  $q_t \in (0, 1]$ , i.e., the bank must offer a premium above the riskless rate to raise deposits; the premium must be positive even if the probability of survival is unity (i.e.  $\underline{R}(1) > R^B$ ) because CD’s are less liquid than government bonds. It is also assumed that  $\lim_{q_t \rightarrow 0} \underline{R}(q_t) = \lim_{q_t \rightarrow 0} \bar{R}(q_t) = \bar{R}$ .

For  $R_t < \underline{R}(q_t)$ ,  $S(R_t, q_t) = 0$ , and for  $R_t > \bar{R}(q_t)$ ,  $S(R_t, q_t) = S(\bar{R}(q_t), q_t)$ , which is the maximal amount of funds that a bank of “quality”  $q_t$  can raise. We make the simplifying assumption that  $S(\bar{R}(q_t), q_t)$  decreases as  $q_t$  decreases, approaching zero as  $q_t$  approaches zero. An interpretation of this assumption is that the set of potential clients that a bank faces becomes smaller as the “quality” of the bank deteriorates. We also assume that for any  $R_t$ ,  $S(R_t, 0) = 0$ . From the above assumptions it follows that  $S(R_t, q_t)$  is right-continuous in  $q_t$  at  $q_t = 0$ , and that it is bounded below by zero and above by  $S(\bar{R}, 1)$ .

With these assumptions we can restrict attention without loss of generality to policies such that  $R_t \in [\underline{R}(q_t), \bar{R}(q_t)]$ , as it is never optimal for the bank to offer  $R_t > \bar{R}(q_t)$ , and the bank is indifferent between offering  $R_t < \underline{R}(q_t)$  and offering  $R_t = \underline{R}(q_t)$ .

#### The law of motion of bank size:

The law of motion governing the evolution of the bank’s size is

$$M_{t+1} = [M_t + S(R_t, q_t) - d_t - a_t - \ell_t]R^B + a_t Z_{t+1}^a + \ell_t Z_{t+1}^\ell - S(R_t, q_t)R_t. \quad (1)$$

That is, in period  $t$  the bank offers  $R_t$  to depositors, raising  $S(R_t, q_t)$ . The resources  $M_t + S(R_t, q_t)$  are then allocated, subject to constraints described below, to their various uses: dividends ( $d_t$ ), investments in risky securities ( $a_t$ ), investments in risky projects ( $\ell_t$ ), and investments in riskless government bonds. At the beginning of period  $t + 1$  the bank collects the gross return from its investments (in securities, projects, and bonds), and pays off the depositors. The remaining funds constitute the bank’s monetary value or size,  $M_{t+1}$ , in period  $t + 1$ .

#### The constraint on short sales of securities:

The bank is allowed to short sell risky securities up to a finite limit  $A(\cdot)$ , which is a function of the bank’s monetary value net of dividends, namely  $a_t \geq A(M_t - d_t)$ . The constraint  $A(M_t - d_t)$  takes the following form: Let  $M^o$  be a non-positive number. Then,

(i) for  $M_t - d_t \leq M^0$ ,  $A(M_t - d_t) = 0$ , that is no short sales are allowed; and (ii) for  $M_t - d_t > M^0$ ,  $A(M_t - d_t)$  is strictly negative, finite, continuous, twice differentiable, strictly decreasing, and convex. A constant cap on short sales, a cap on short sales which is proportional to  $M_t - d_t$ , and a no short sales constraint, are all special cases of this specification. Figure 2 shows the constraint for  $d_t = 0$ .

The interpretation of the constraint on short sales of securities is as follows. The securities market observes the size of the bank and the amount of dividends it distributes. The difference  $M_t - d_t$  (the bank's size net of dividends) is a measure of the bank's soundness. The larger the size, the more funds the securities market will be willing to supply to the bank, with the cap increasing at a non-increasing rate. The constraint captures the idea that banks cannot raise funds on public securities markets independently of their size. This can be a regulatory constraint, or it may simply reflect the reluctance of investors on securities markets to supply funds to a bank whose net monetary value  $M_t - d_t$  is too low. Notice that in addition to its size in period  $t$ ,  $M_t$ , the bank's behavior in period  $t$  (i.e. the choice of  $d_t$ ) affects its ability to raise funds on the stock market.

#### **Bankruptcy—definition:**

If at the beginning of period  $t + 1$ ,  $M_{t+1} < 0$ , then the bank cannot repay its debt to depositors (see equation (1)). It does not, however, necessarily follow that the bank is bankrupt. The bank may still be able to raise funds from external sources, use part to repay depositors, invest the rest, and survive. We postulate that whenever the bank's monetary value is negative, the bank is not allowed to distribute dividends; it is allowed to raise money, however, by short selling securities. The maximal amount the bank can raise in this manner is the absolute value of  $A(M_{t+1})$ . If  $M_{t+1} - A(M_{t+1}) \geq 0$  the bank can repay depositors, raise money from new depositors, and continue to operate. On the other hand, if  $M_{t+1} - A(M_{t+1}) < 0$  the regulators will not allow the bank to acquire new deposits and will declare it bankrupt. The bank then ceases to operate.<sup>18</sup> Letting  $M^*$  denote the (unique) solution to  $M_{t+1} - A(M_{t+1}) = 0$ , the condition for bankruptcy can be written as

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<sup>18</sup> Any remaining funds, which by assumption are less than the principal plus interest promised to depositors, are confiscated by the regulators. These funds may be distributed in part or in full to depositors. As explained above, we do not model the bankruptcy procedure in detail, as it is not central to the analysis.

$M_{t+1} < M^*$  (Figure 2). In period  $t$  the quantity  $M_{t+1} - A(M_{t+1})$  is a random variable. For notational convenience we define the following indicator variable:

$$\delta_t = \begin{cases} 1 & \text{if } M_t - A(M_t) \geq 0 \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

That is,  $\delta_t = 0$  denotes bankruptcy in period  $t$ , while  $\delta_t = 1$  indicates that the bank will continue to operate for at least one more period.

### The determination of the probability of survival:

We turn to the determination of  $q_t$ , the probability of survival in period  $t+1$  as perceived in period  $t$ . Note that  $q_t$  is the probability that  $\delta_{t+1} = 1$ , i.e. that  $M_{t+1} - A(M_{t+1}) \geq 0$ . In light of (1), the c.d.f. of  $M_{t+1} - A(M_{t+1})$ , which will be denoted by  $F(\cdot; M_t, c_t, q_t)$ , is induced by the c.d.f. of  $Z_{t+1}$ , and is parameterized by  $M_t$ ,  $c_t$ , and  $q_t$ . By the properties of the c.d.f. of  $Z_{t+1}$ , for any  $M_t$ ,  $c_t$ , and  $q_t$ ,  $F(\cdot; M_t, c_t, q_t)$  is continuously differentiable and strictly increasing in its argument. The probability of bankruptcy in period  $t+1$ , as of period  $t$ , is

$$\Pr\{M_{t+1} - A(M_{t+1}) < 0\} = F(0; M_t, c_t, q_t).$$

We further assume that  $F(0; M_t, c_t, q_t)$  is twice continuously differentiable in  $M_t$ ,  $c_t$ , and  $q_t$ , with bounded first derivatives and continuous second (and cross) derivatives.<sup>19</sup> Since the probability of bankruptcy in period  $t+1$  is defined as  $1 - q_t$ , for the model to be internally consistent we require that  $q_t$  be a solution to the equation

$$1 - q_t = F(0; M_t, c_t, q_t). \quad (3)$$

The significance of this requirement is that the belief depositors hold regarding the probability of survival—which affects their inclination to supply funds—induces the bank to take actions that make this belief correct. Thus, equation (3) can be regarded as a condition for a rational expectations equilibrium.<sup>20</sup> In the following lemma we show that given the state

<sup>19</sup>These additional properties regarding the derivatives of  $F(0; M_t, c_t, q_t)$  are needed to establish uniqueness of the optimal policy.

<sup>20</sup>We do not require depositors to be able to compute  $q_t$  themselves—we only impose that the probability of survival which they take into account when supplying deposits be correct. If some (or all) depositors happen to know  $c_t$ ,  $M_t$  and also know the model and the stochastic processes, we can think of them as computing  $q_t$  themselves.

and policy variables in period  $t$ , there is a unique

$$q_t = q(M_t, c_t) \quad (4)$$

satisfying equation (3).

**Lemma 1** *For any state  $M_t$  and policy  $c_t$ : (i) there is a unique  $q_t \in [0, 1]$  that satisfies equation (3); (ii) the function  $q_t = q(M_t, c_t)$  is continuous in its arguments; and (iii) whenever  $q(M_t, c_t) \in (0, 1)$ , it is differentiable.*

The proof is in the appendix. When solving its dynamic program, the bank takes into account the effect of the policy  $c_t$  on its own probability of survival (according to the function in (4)) and hence also on the willingness of depositors to supply funds.<sup>21</sup>

**The feasible policy correspondence:**

The set of feasible policy choices for the bank when the state is  $M_t$ , denoted by  $\Gamma(M_t)$ , is characterized by the following conditions:

$$0 \leq d_t \leq M_t - A(M_t - d_t), \quad (5)$$

$$a_t \geq A(M_t - d_t), \quad (6)$$

$$\ell_t \geq 0, \quad (7)$$

$$M_t + S(R_t, q_t) - d_t - a_t - \ell_t \geq \lambda S(R_t, q_t), \quad (8)$$

$$\underline{R}(q_t) \leq R_t \leq \bar{R}(q_t), \quad (9)$$

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<sup>21</sup>Multiplicity of self-fulfilling equilibria is common in the literature on banking (e.g. Postlewaite and Vives (1987)) if depositors believe that a bank is risky they will require a high interest rate which will make it optimal for the bank to engage in risky activities, whereas if depositors believe that a bank is safe, they will not require a high interest rate, and the bank will indeed be safe. This well known coordination problem should not be confused with the result in Lemma 1, which says that for a *given* policy and a given bank size there is a unique value of  $q_t$  for which this policy results in the probability of survival  $q_t$ . The essence of the proof of uniqueness can be understood by considering the law of motion of bank size, equation (1). Fix  $M_t$  and  $c_t \equiv (R_t, d_t, a_t, \ell_t)$ , suppose that equation (3) holds, and consider an increase in the probability of survival  $q_t$ . This induces an increase in the supply of deposits at the given interest rate  $R_t$ , and since all other things are equal the extra funds the bank raises are invested in riskless bonds. Since by assumption  $R^B < R_t$  this induces a leftward shift in the distribution of  $M_{t+1}$ , namely the probability of survival decreases. Similarly, if  $q_t$  decreases the distribution of  $M_{t+1}$  shifts to the right and the probability of survival increases. Hence, there can be at most one equilibrium that, as shown in the proof of Lemma 1, always exists.



where  $q_t$  is determined by (4). Condition (5) states that the owner-manager can neither infuse new capital into the bank nor distribute dividends in excess of its monetary value plus the amount it can raise by short selling securities. Note that as long as  $M_t \geq M^*$ , i.e.  $M_t - A(M_t) \geq 0$ , the bank is not bankrupt and is allowed to distribute dividends provided (5) is not violated. A further implication of (5) is that the bank cannot distribute dividends from funds supplied by depositors. Condition (6) defines a cap on short sales of securities. Condition (7) precludes short sales of projects since, by definition, these are loans extended by the bank. Condition (8) is a reserve requirement—the bank is required by law to keep at least a fraction  $\lambda \in (0, 1)$  of the deposits in riskless bonds. Condition (9) restricts the range of interest rates on CD's that the bank can offer. As was explained above, in light of the assumptions we have made regarding the supply function of deposits, condition (9) does not limit the generality of our analysis.

We say that a policy  $c_t$  is feasible in state  $M_t$  if conditions (5)–(9) are satisfied, and  $q_t$  is determined by (4); we write  $c_t \in \Gamma(M_t)$ . In period  $t$ , the bank chooses a feasible plan  $\{c_k\}_{k=t}^{\infty}$ , which consists of a feasible policy  $c_t \in \Gamma(M_t)$ , and feasible policies  $c_k \in \Gamma(M_k)$  for every possible state  $M_k$  in periods  $k = t + 1, t + 2, \dots$

#### Existence of the value function:

The bank's utility function from consumption of dividends,  $u(\cdot)$ , is assumed to be continuous, strictly increasing, concave, twice differentiable, and bounded. It is normalized so that  $u(0) = 0$ . In case of bankruptcy, the bank's utility is zero in all subsequent periods.<sup>22</sup> In period  $t$  the bank chooses a feasible plan  $\{c_k\}_{k=t}^{\infty}$  to maximize  $E \left[ \sum_{k=t}^{\infty} \beta^{k-t} u(d_k) \delta_k \right]$ , subject to the equation of motion (1), where  $\beta \in (0, 1)$  is the discount factor, and the expectation is taken with respect to the sequence of random variables  $\{Z_k\}_{k=t+1}^{\infty}$ . The Bellman functional equation corresponding to the bank's optimization problem is

$$V(M_t) = \begin{cases} \sup_{c_t \in \Gamma(M_t)} \{u(d_t) + \beta E[V(M_{t+1})]\} & \text{if } \delta_t = 1 \\ 0 & \text{if } \delta_t = 0. \end{cases} \quad (10)$$

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<sup>22</sup>We think of the utility function of an owner-manager of a bank as being additively separable, where  $u(\cdot)$  is the utility derived from the consumption of dividends from this bank. A failed bank cannot re-open, and hence the utility from the consumption of dividends from this bank is zero forever. Alternatively, we can imagine that an owner-manager of a failed bank suffers a reputational damage which prevents him from opening another bank. His utility in subsequent periods is normalized to zero.

**Proposition 1** *Subject to the above assumptions: (i) there is a unique continuous and bounded function  $V(\cdot)$  satisfying (10); and (ii) if  $\delta_t = 1$  then  $V(\cdot)$  is strictly increasing.*

The proof is in the appendix.

**Uniqueness of the optimal policy:**

This section is devoted to establishing conditions under which for every state  $M_t$  there is a unique optimal policy. More specifically, we prove that for a particular functional form of the supply of deposits and under assumptions on the distribution of  $Z_{t+1}$ , there is an open region of the model's parameters for which uniqueness of the optimal policy is guaranteed. The proof requires showing, among other things, that for any realization of  $Z_{t+1}$ , the right hand side of (1) is strictly concave in  $(M_t, c_t)$ , taking into account that  $q_t$  is a function of  $(M_t, c_t)$ , and that the correspondence  $\Gamma(\cdot)$  is convex in a sense to be defined (see Lemma 5 in the appendix). To meet these requirements we add the following assumption regarding the joint distribution of  $Z_{t+1}^a$  and  $Z_{t+1}^l$ :

**Assumption D:** Let  $Y_k = k_0 + k_1 Z_{t+1}^a + k_2 Z_{t+1}^l$  and let  $Y_m = m_0 + m_1 Z_{t+1}^a + m_2 Z_{t+1}^l$ , where  $k_i$  and  $m_i$ ,  $i = 0, 1, 2$ , are arbitrary constants. Then for any  $\theta \in [0, 1]$ ,

$$\Pr[\theta Y_k + (1 - \theta) Y_m \leq 0] \leq \theta \Pr(Y_k \leq 0) + (1 - \theta) \Pr(Y_m \leq 0).$$

The meaning of Assumption D is best understood by glancing at Figure 3. The less positively correlated are  $Z_{t+1}^a$  and  $Z_{t+1}^l$ , the more likely the assumption is to hold.

From now on we restrict attention to the following particular form of the supply function of deposits,

**Assumption S:**  $S(R_t, q_t) = \gamma_R (R_t - R^B)^{\alpha_R} + \gamma_q q^{\alpha_q}$ ,  $\gamma_R > 0$ ,  $\gamma_q > 0$ ,  $\alpha_q > 1$ ,  $\alpha_R \in (0, 1]$ .

We also need

**Assumption R:** The bounds  $\underline{R}(q_t)$  and  $\bar{R}(q_t)$  satisfy the following properties: (a)  $\bar{R}(q_t)$  is concave in  $q_t$ ; and (b)  $\underline{R}(q_t)$  is sufficiently convex in  $q_t$  so that  $\underline{R}(q(M_t, c_t))$  is convex in  $(M_t, c_t)$ .

Assumption R, which is made for technical reasons, has the following economic interpretation. The concavity of  $\bar{R}(q_t)$  reflects the fact that the rate at which it becomes easier

to induce depositors to invest the “last dollar” increases as the bank becomes safer. On the other hand, although it also becomes easier to induce depositors to supply the “first dollar” as  $q_t$  increases, this occurs at a decreasing rate. We now have

**Proposition 2** *If Assumptions D, S, and R are satisfied, then for any  $\alpha_R \in (0, 1]$  there is an open region of  $\alpha_q$ ,  $\gamma_R$ , and  $\gamma_q$  such that for every  $M_t$  the optimal policy is unique and is a continuous function of  $M_t$ .*

The proof is in the appendix.

## 4 Simulation Results

We begin by presenting our specific choices for the utility function of the bank owner-manager, parameter values, and the stochastic processes. We proceed with a presentation of the base case simulation. Then we present simulations in which we change parameters of the model—in each simulation one parameter value is different from the base case simulation. The economic interpretation and the policy implications of the various simulations are discussed as we proceed.

In all the simulations the utility function of the owner-manager of the bank is

$$u(x) = \left( \frac{\log(1+x)}{1+\log(1+x)} \right)^\rho, \quad (11)$$

where  $\rho \in (0, 1]$ . The function  $u(\cdot)$  is bounded, strictly concave, and exhibits decreasing absolute risk aversion; the absolute risk aversion coefficient decreases with the parameter  $\rho$ . The intertemporal discount factor  $\beta$  is set at  $\beta = .95$  and remains unchanged throughout the simulations.

For the distributions of the returns on the market portfolio and on investments in projects we use the following log-normal, stationary distributions:

$$\begin{aligned} \log Z_t^a &\sim N(1.10, 1) \\ \log Z_t^l &\sim N(1.13, \sigma_l^2). \end{aligned} \quad (12)$$

As a lower bound on short sales of the market portfolio we use the function

and as bounds on  $R_t$  we use

$$\underline{R}(q_t) = 2 - (1.995 - R^B)q_t^{0.25}$$

for the lower bound, and

$$\bar{R}(q_t) = 2 - 0.5q_t^2$$

for the upper bound. Note that as  $q_t$  approaches zero, both bounds approach the same limit, and  $R^B < \underline{R}(q_t) < \bar{R}(q_t)$  for all  $q_t \in (0, 1)$ , for reasonable values of  $R^B$ .<sup>23</sup> We restrict attention to the following supply function of deposits:

$$S(R_t, q_t) = 9(R_t - R^B)^{\alpha_R} + 2q_t^{\alpha_q}. \quad (13)$$

In the various simulations we vary the parameters  $\rho$ ,  $\alpha_R$ ,  $\alpha_q$ ,  $\lambda$ , and  $\sigma_t$ , one at a time, to study the behavior of a bank in different environments. We turn to a description of the simulations.

#### The base case simulation:

In the base case simulation we use the following values for the parameters:  $R^B = 1.03$  (i.e., 3%) for the riskless interest rate,  $\sigma_t = 2$  for the standard deviation of the returns on projects,  $\alpha_R = 0.50$  and  $\alpha_q = 1.1$  for the parameters of the supply function of deposits,  $\lambda = 0.1$  for the reserve requirement, and  $\rho = 1$  for the risk aversion of the bank owner-manager. The choice of  $\alpha_R$  and  $\alpha_q$  is in accord with the sufficient conditions for uniqueness of the optimal policy (see Assumption S above).

The results for the base case simulation are presented in Figure 4. A central finding is that the probability of survival increases with bank size.<sup>24</sup> Small banks have a low probability of survival and therefore must offer a high interest rate to depositors. The high interest rate incorporates, no doubt, a risk premium. However, small banks also raise more deposits than large banks. Thus, the high interest rate on deposits also reflects a desire on the part of small banks to raise funds and “escape from poverty.” This is particularly

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<sup>23</sup>  $R^B < 1.415$  is sufficient for these inequalities to hold; in the simulations we shall use much smaller values for the risk free interest  $R^B$ .

<sup>24</sup> We remind the reader that the term “bank size” stands for the amount of equity capital of the bank.

evident from the U-shape of the schedule of investment in risky projects in Figure 1f; the very small banks “gamble” by investing a large fraction of their funds in these projects.

Had they wanted to, small banks could have offered a lower interest rate, raised less deposits, and invested less in risky projects, increasing their probability of survival. Taking the argument to the extreme, we find that even though small banks can reduce their riskiness entirely by raising no deposits and investing their capital only in riskless securities, they choose to do precisely the opposite, taking large risks in the hope of growing big. Stated somewhat differently, our conclusion is that small banks prefer to adopt a strategy that will increase the probability of default in the short run, in order to increase the probability of getting larger. Since bankruptcy entails  $u(0) = 0$  forever, the bank’s payoff is essentially the maximum between zero and a positive value. Indeed, the value function is zero for  $M_t < M^*$ , but is strictly concave for  $M_t \geq M^*$ , that is it is locally *convex* in the neighborhood of  $M^*$ . Therefore, small banks have an incentive to take big risks in order to be able to become larger if the realization of the investment turns out to be good. This behavior increases the probability of default.<sup>25</sup>

We also see that, roughly speaking, banks of different sizes allocate the same proportion of their equity capital to dividends, to investments in the market portfolio, and to investments in risky projects. In particular, despite their higher probability of failure small banks allocate the same proportion of equity capital to dividend consumption as large banks. The inclination of small banks to take risks is particularly evident in Figure 4f—the very small banks (including those with negative equity) invest heavily in risky projects as a desperate attempt to restore their equity base.

*Cohort simulation:*

We explore the results of the base case simulation (as well as the results of the simulations reported below) by considering a cohort of one thousand banks of equal size, who operate in the same environment as described above. In each period all the banks of the cohort face the same return on the market portfolio,  $Z_t^a$ , both ex-ante and ex-post. In contrast, the returns on projects,  $Z_t^l$ , have the same underlying distribution, but ex-post

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<sup>25</sup> An analogous phenomenon is what has become known in the Corporate Finance literature as “asset substitution”—shareholders (in our case the bank owner-manager) take excessive risk in order to extract surplus from debtholders (in our case the depositors).

the realizations differ. For simplicity we assume that the  $Z_t^i$ 's across banks are i.i.d. Using the results generated by the above simulation, we let each bank choose its optimal policy. Being identical, all the banks choose the same policy in the first period. However, as the banks face idiosyncratic uncertainty in their investments in projects, they gradually begin to differ in their size, and hence also in their optimal policies. We follow the cohort for thirty periods, portraying the evolution of the cohort as time goes by along two dimensions—the number of banks left operating, and the size distribution of these banks. The results are presented in Figure 5. Figure 5a depicts the size distribution of the surviving banks after 5, 10, 20, and 30 periods, while Figure 5b depicts the number of surviving banks as a function of time. The shape of the size distribution becomes almost stationary after thirty periods, and is skewed to the left with a small number of very large banks.

#### **The reserve requirement simulation:**

Our objective is to evaluate the effect of the level of the reserve requirement  $\lambda$  on the endogenous variables. As pointed out in the introduction, economists have traditionally focused on the role reserve requirements play in controlling the money supply, in raising revenues for the Treasury (being a tax on deposits), and in insuring banks against adverse liquidity shocks. The simulations indicate that reserve requirements may also influence the dividend and investment policies of banks, affecting their probability of survival.

We simulate the model for  $\lambda = 0.2$  and  $\lambda = 0.4$ . The results, compared with those for the base case ( $\lambda = 0.1$ ), are presented in Figure 6. The major finding of this set of simulations is that in response to an increase in the reserve requirement, small banks offer lower interest rates on deposits, raise less deposits, and become safer. A possible interpretation is that a large reserve requirement makes deposits more costly, reducing the effective return on investments (i.e. the return relative to the cost of funds), rendering “gambling” on the part of small banks less attractive.

Interestingly, the optimal policy with respect to dividend distribution and investment in both the market portfolio and projects is affected very little by the imposition of different reserve requirements; the graphs detailing these results are therefore omitted. Nonetheless, we detect a tendency on the part of small banks to slightly decrease dividend distributions when  $\lambda$  is higher, whereas for large banks the opposite is true.

The interpretation is again related to the fact that a large reserve requirement reduces the effective return on investments. When  $\lambda$  increases, large banks respond by substituting away from investment and increasing dividend consumption. By contrast, small banks who want to grow and are reluctant to be perceived as risky, reduce dividend consumption. This interpretation suggests that small banks are more sensitive to market discipline, and that the market takes into account the effect of regulatory policies (in this case the reserve requirement) on the risk taking behavior of banks.

#### **The riskiness of investment opportunities simulation:**

Here we simulate the model for two alternative standard errors for the return on investments in projects,  $\sigma_\ell = 1$  and  $\sigma_\ell = 3$ , i.e., lower and higher riskiness relative to the base case ( $\sigma_\ell = 2$ ). The results are presented in Figure 7. Banks of virtually all sizes respond to an increase in the riskiness of the investment in projects by shifting resources from investment in projects to investment in the less risky market portfolio. In and of itself the shift away from investment in risky projects increases the probability of survival in any single period. This notwithstanding, the probability of survival decreases; that is, even though the bank decreases its investment in the more risky assets, it is still the case that for any given size the ex-ante probability of survival is lower. This is particularly true for small banks—the differences in the probability of survival across the three simulations are on the order of 10 percentage points. However, as the bank becomes larger the probabilities of survival across the simulations converge.

The explanation for this phenomenon can be found in the bank's dividend and interest rate policy. Since an increase in  $\sigma_\ell$ , other things kept equal, constitutes a worsening of the investment opportunities, banks respond by shifting funds into increased dividend consumption, which contributes to a lower probability of survival. Furthermore, the increase in the variance of the return on risky projects renders "gambling" more attractive for small banks who increase the interest offered on deposits, raising more deposits than before, which contributes to a lower probability of survival.

Thus, overall, when the environment becomes more risky banks respond by becoming more risky themselves. Instead of acting as a buffer, by adopting sufficiently prudent policies that offset the increased riskiness of the investment opportunities, individual banks

(especially the small ones) transmit the risk to depositors.

This might lead one to believe that there would be more bank failures the more risky the environment. However, the cohort simulation reveals an interesting phenomenon. The size distributions depicted in Figure 8 indicate that when the riskiness of the investment opportunities increases, the size distribution of the banks becomes less skewed to the left, since more banks succeed in “making it big.” Although for a given size, banks are less safe when the riskiness of the investment opportunities is high, the overall riskiness of the cohort becomes lower after several periods have gone by, as can be seen from the survival function schedules. The reason for this phenomenon is quite simple. The survival probability at each bank size is lower in the more risky environment, but the larger variance of the return on investments increases the probability of “hitting the jackpot.” Consequently, the number of banks who become very large is bigger when the environment is more risky. Since large banks are less risky this results in a higher average survival rate for the cohort.

In sum, even though individual banks transmit the increased risk of the investment opportunities to depositors, the overall riskiness of the banking system gradually becomes smaller. We can therefore imagine the following scenario. If a bad shock hits the economy, rendering investment opportunities more risky, we shall have a more risky banking system in the short run (as the size distribution is unchanged and each bank adopts policies which are riskier than before), but in the long run we should expect an endogenous change in the size distribution of the banks (ignoring the possibility of entry of new banks), with a bigger fraction of large, safe banks, and an overall sounder banking system.

#### **The risk aversion simulation:**

In this set of simulations we increase the risk aversion of the bank's owner-manager relative to the base case ( $\rho = 1$ ) by simulating the model for  $\rho = 0.75$  and  $\rho = 0.50$ . As can be seen in Figure 9, a higher risk aversion induces the banks to consume less dividends. Interpreting higher risk aversion as more concentrated ownership, we have a testable hypothesis: Banks with more concentrated ownership distribute less dividends. Consistent with this behavior, higher risk aversion induces small banks to offer a lower interest rate on deposits. An immediate consequence is that higher risk aversion entails a higher probability of survival.



The investment decisions of banks of all sizes remain virtually unchanged with respect to the base case. The results for the interest rate on deposits and the amount of deposits raised are straightforward: A more risk averse bank offers a lower interest rate and raises less deposits. These results are therefore not shown.

The cohort simulation indicates (figures are omitted) that the size distribution of banks changes very little when banks are more risk averse. The results do indicate, however, that the distribution becomes more skewed to the left as risk aversion increases. Also, in line with the higher probability of survival (Figure 9b), the fraction of remaining banks in each year is higher when banks are more risk averse, that is when ownership is more concentrated.

#### **The riskless interest rate simulation:**

Interest rate policy, namely setting the riskless interest rate ( $R^B$  in our model) by the monetary authorities has always been the subject of heated debates. In this set of simulations we try to evaluate the effects of changes in  $R^B$  on the behavior of banks. We simulate the model for  $R^B = 1.02$  and  $R^B = 1.04$  (lower and higher riskless interest rates compared to the base case,  $R^B = 1.03$ ).

The bank's optimal policy and probability of survival change very little. As the supply of deposits depends positively on  $R_t - R^B$ , banks are inclined to increase  $R_t$  when  $R^B$  increases in order not to lose depositors. This contributes to a decrease in the probability of survival. The only meaningful changes that can be observed are the investments practices, as shown in Figure 10. Note that as the interest on riskless bonds increases banks of all sizes shift away from the more risky investments (projects) into less risky investments (the market portfolio). This is a response to the higher cost of raising deposits which, other things constant, tends to decrease the probability of survival. The net effect of the increase in the interest rate offered on deposits and the shift away from risky investments is that survival probabilities remain almost unchanged.

The effect of changes in the riskless rate on the size distribution and the survival function are minimal (figures are omitted).

### The $\alpha_R$ simulation:

Recall that in the supply function of deposits the part related to the interest rate is  $\gamma_R(R_t - R^B)^{\alpha_R}$ , where the parameter  $\alpha_R$  measures the sensitivity of supply with respect to  $R_t$ ; the higher  $\alpha_R$  the stronger is the response of depositors to changes in the offered interest rate  $R_t$ .<sup>26</sup>

A priori, the effect of  $\alpha_R$  on the bank's policy and probability of failure is ambiguous. A higher value of  $\alpha_R$  means that depositors react more vigorously to changes in the interest rate on deposits. Banks may respond by behaving more aggressively, raising the rates offered on CD's and increasing the share allocated to the high return risky investments (in order to meet the promised interest payments to depositors). However, banks may realize that a more aggressive (interest rate and investment) policy reduces the probability of survival. Since depositors take this probability into account, the bank may choose to behave more prudently, not more aggressively.

We simulate the model for  $\alpha_R = 0.3$  and  $\alpha_R = 0.1$  (in the base case  $\alpha_R = 0.5$ ). The simulation results (graphed in Figure 11) indicate that, typically, the market discipline effect prevails. As  $\alpha_R$  increases, banks of all sizes reduce the consumption of dividends significantly. Although the very small banks (especially those with negative equity) offer a high interest rate on deposits and exhibit a low probability of survival, as banks become larger they can afford to reduce the interest rate on deposits, increasing their probability of survival.

We turn to the cohort simulation. The short run effect of the increase in the probability of survival for banks of most sizes is a higher survival rate for the cohort. However, as we saw in the  $\sigma_I$  (riskiness of investment opportunities) simulation, the endogenous change in the size distribution of banks may affect the survival rate in the longer run. As  $\alpha_R$  increases the size distribution becomes more skewed to the left (see Figure 12). This is a consequence of the lower investment in risky projects which reduces the number of banks that succeed in becoming very big. This change in the size distribution contributes to a decrease in the survival rate, since small banks are less safe than larger ones. However, the effect is not

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<sup>26</sup>More precisely, the elasticity of the supply of savings with respect to  $R_t$ , holding  $q_t$  constant, strictly increases with  $\alpha_R$ .

sufficiently strong to overturn the increase in  $q_t$  for every bank size (Figure 11a), as can be seen from the survival rates (Figure 12d).

**The  $\alpha_q$  simulation:**

Recall that in the supply function of deposits the term that relates to the probability of survival is  $\gamma_q q_t^{\alpha_q}$ , where the parameter  $\alpha_q$  measures the sensitivity of supply with respect to  $q_t$ ; the higher  $\alpha_q$  the stronger is the response of depositors to changes in the probability of survival  $q_t$ .<sup>27</sup> Changes in  $\alpha_q$  have negligible effects on  $q_t$ ,  $R_t$ , and hence on the amount of deposits raised. The results for the dividend distributions and the investment policies are depicted in Figure 13. As  $\alpha_q$  increases, the bank invests less in risky projects and more in the (less risky) market portfolio. This can be interpreted as a desire on the part of the bank to avoid increasing the interest on deposits. Had the bank not reduced the riskiness of the investment portfolio, the probability of survival would have decreased, forcing the bank to raise the interest rate on CD's in order not to lose deposits.

The cohort simulations (figures are omitted) indicate that the size distribution changes very little with changes in  $\alpha_q$ . The effect on the survival function is also very small, in line with the small changes in  $q_t$  and in the size distribution.

## 5 Concluding Remarks

We conclude with a policy note. The finding that small banks take advantage of low bankruptcy costs by taking large risks suggests that if bank failures constitute a negative externality for the economy, regulatory policies that differ by size (or by equity capital) should be considered seriously. For example, the dividend distribution and risky investments of small, undercapitalized banks can be restricted to a specified fraction of assets. Disclosure of the banks' policy is not sufficient. In our model depositors know precisely what the bank is doing, and they price the risk correctly; nevertheless, a small bank chooses to raise a large amount of deposits and take big risks despite the high price of deposits.

If, however, bank failures are not viewed as detrimental to the economy then since we

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<sup>27</sup>The elasticity of the supply of savings with respect to  $q_t$ , holding  $R_t$  constant, strictly increases with  $\alpha_q$ .

assume that the bank and depositors have the same information and depositors are rational, the bank's risks are priced correctly and there is no room for government intervention. We should simply be aware of the central prediction of the model—small banks rationally choose to take large risks and fail more often.

## 6 Appendix—Proofs of Lemmas and Propositions

**Lemma 1** For any state  $M_t$  and policy  $c_t$ : (i) there is a unique  $q_t \in [0, 1]$  which satisfies equation (3); (ii) the function  $q_t = q(M_t, c_t)$  is continuous in its arguments; and (iii) whenever  $q(M_t, c_t) \in (0, 1)$ , it is differentiable.

*Proof of Lemma 1.*

*Step 1.* We first establish the continuity and differentiability of  $F(\cdot; M_t, c_t, q_t)$  in  $q_t$ . Fix  $M_t$  and  $c_t$ . By the differentiability of  $S(R_t, q_t)$  in  $q_t$  it follows from (1) that  $M_{t+1}$  is differentiable in  $q_t$ . By the differentiability of  $A(\cdot)$  in  $M_{t+1}$ , and the continuity and differentiability of  $F(\cdot; M_t, c_t, q_t)$  in its argument, the continuity and differentiability of  $F(\cdot; M_t, c_t, q_t)$  in  $q_t$  is established. In a similar manner, using the differentiability of  $M_{t+1}$  in  $M_t$  and  $c_t$ , we have that  $F(\cdot; M_t, c_t, q_t)$  is continuous and differentiable in  $M_t$  and  $c_t$ .

*Step 2.* We now establish that  $F(\cdot; M_t, c_t, q_t)$  is increasing in  $q_t$ . Rewrite (1) as

$$M_{t+1} = (M_t - d_t)R^B + a_t(Z_{t+1}^a - R^B) + \ell_t(Z_{t+1}^l - R^B) - S(R_t, q_t)(R_t - R^B).$$

Since  $S(R_t, q_t)$  is increasing in  $q_t$ , and  $R^B < R_t$ , we have that  $M_{t+1}$  is decreasing in  $q_t$ . By the assumption that  $A(\cdot) \leq 0$  and  $A'(\cdot) \leq 0$ , it follows that  $M_{t+1} - A(M_{t+1})$  is non-increasing in  $q_t$  and hence its c.d.f. is non-decreasing in  $q_t$ .

*Step 3.* Rewrite (3) as

$$H(M_t, c_t, q_t) = 1, \tag{14}$$

where  $H(M_t, c_t, q_t) \equiv F(0; M_t, c_t, q_t) + q_t$ . Extend the function  $H(M_t, c_t, q_t)$  so that for  $q_t \leq 0$ ,  $H(M_t, c_t, q_t) = F(0; M_t, c_t, 0) + q_t$ , and for  $q_t \geq 1$ ,  $H(M_t, c_t, q_t) = F(0; M_t, c_t, 1) + q_t$ . By step 2,  $F(0; M_t, c_t, q_t)$  is non-decreasing in  $q_t$ . Therefore, holding  $M_t$  and  $c_t$  fixed,  $H(M_t, c_t, q_t)$  is strictly increasing in  $q_t$ . Since  $F(0; M_t, c_t, q_t)$  is bounded, it follows that  $\lim_{q_t \rightarrow \infty} H(M_t, c_t, q_t) = \infty$  and  $\lim_{q_t \rightarrow -\infty} H(M_t, c_t, q_t) = -\infty$ . By the continuity and the strict monotonicity of  $H(M_t, c_t, q_t)$  in  $q_t$ , there is a unique value of  $q_t$  for which (14) holds. Since  $F(0; M_t, c_t, q_t) \in [0, 1]$ , this value of  $q_t$  must also be in the interval  $[0, 1]$ .

*Step 4.* Let  $q_t \in (0, 1)$ . By arguments analogous to those in step 2 it follows that  $F(0; M_t, c_t, q_t)$  is strictly increasing in  $q_t$ . As for any  $q_t \in (0, 1)$  the function  $H(M_t, c_t, q_t)$  is differentiable in  $(M_t, c_t)$ , we can apply the Implicit Function Theorem to (14), obtaining the

continuity and differentiability of  $q(M_t, c_t)$  as desired. By the continuity of  $F(0; M_t, c_t, q_t)$ , it follows from (14) that  $q(M_t, c_t)$  is continuous at any point  $(M_t, c_t)$  where  $q(M_t, c_t)$  equals one or zero.

Q.E.D.

**Proposition 1** *Subject to the above assumptions: (i) there is a unique continuous and bounded function  $V(\cdot)$  satisfying (10); and (ii) if  $\delta_t = 1$  then  $V(\cdot)$  is strictly increasing.*

*Proof of Proposition 1.*

When  $\delta_t = 0$ , the functional equation (10) is satisfied by construction.

Let  $\delta_t = 1$ . From the law of motion (1), and the continuity of  $S(R_t, q_t)$  and  $q_t = q(M_t, c_t)$  (Lemma 1), it follows that

$$\begin{aligned} \phi(M_t, c_t, Z_{t+1}) &\equiv (M_t - d_t)R^B \\ &+ a_t(Z_{t+1}^a - R^B) + \ell_t(Z_{t+1}^\ell - R^B) - S(R_t, q(M_t, c_t))(R_t - R^B) \end{aligned} \quad (15)$$

is continuous in its arguments. The functional equation takes the form

$$V(M_t) = \sup_{c_t \in \Gamma(M_t)} \{u(d_t) + \beta E[V(M_{t+1})]\}. \quad (16)$$

Let  $C(\mathbf{R})$  be the space of bounded and continuous functions  $f : \mathbf{R} \rightarrow \mathbf{R}$ , with the sup norm. Define the operator  $T$  on  $C(\mathbf{R})$  by

$$(Tf)(M_t) = \sup_{c_t \in \Gamma(M_t)} \{u(d_t) + \beta E[f(\phi(M_t, c_t, Z_{t+1}))]\}. \quad (17)$$

The operator  $T$  maps  $C(\mathbf{R})$  into itself. This can be seen as follows. Fix  $M_t$ . Using (5)–(9) it is readily shown that  $\Gamma(M_t)$ —the set of policy vectors satisfying the constraints—is closed and convex, and hence compact. For any realization of  $Z_{t+1}$ , the function  $\phi(\cdot)$  is continuous in  $c_t$ . Hence, by the continuity of  $f$  in its argument and the continuity of the c.d.f. of  $Z_{t+1}$ , it follows that  $E[f(\phi(M_t, c_t, Z_{t+1}))]$  is continuous in  $c_t$ . As  $u$  is continuous, it follows that the supremum in equation (17), denoted by  $u(d_t^*(M_t)) + \beta E[f(\phi(M_t, c_t^*(M_t), Z_{t+1}))]$ , is achieved. By the continuity of  $\phi(\cdot)$  in  $M_t$ , and the Theorem of the Maximum (e.g., Stokey and Lucas 1989, p.62) we have that the function  $(Tf)(M_t) = u(d_t^*(M_t)) + \beta E[f(\phi(M_t, c_t^*(M_t), Z_{t+1}))]$  is continuous in  $M_t$ . Since  $u$  and  $f$  are bounded,  $(Tf)(M_t)$  is bounded.

It is easy to show that  $(Tf)(M_t)$  satisfies monotonicity and discounting (e.g., Stokey and Lucas, 1989, p.54), and hence, by the Contraction Mapping Theorem, is a contraction with a unique fixed point. This completes the proof of (i).

Since  $u(\cdot)$  is strictly increasing, part (ii) of the proposition follows from the fact that a plan that is feasible for  $M_t$  is also feasible for  $M'_t > M_t$ .

Q.E.D.

**Proposition 2** *If Assumptions D, S, and R are satisfied, then for any  $\alpha_R \in (0, 1]$  there is an open region of  $\alpha_q$ ,  $\gamma_R$ , and  $\gamma_q$  such that for every  $M_t$  the optimal policy is unique and is a continuous function of  $M_t$ .*

*Proof of Proposition 2.*

We start by proving several auxiliary results.

**Lemma 2** *If (i) Assumption D is satisfied; and (ii)  $S(R_t, q_t)(R^B - R_t)$  is strictly concave in  $R_t$  and  $q_t$ , then (i)  $1 - F(0; M_t, c_t, q_t)$  is strictly concave in  $(M_t, c_t, q_t)$ ; and (ii)  $q(M_t, c_t)$  is strictly concave in  $(M_t, c_t)$ .*

The concavity of the supply of funds function in the interest rate on deposits, and its (strict) convexity in the probability of survival can be interpreted as follows. For a given probability of survival it is becoming increasingly harder to induce depositors to supply funds through increases in the interest rate. On the other hand, for a fixed interest rate, equal increases in the probability of survival result in increasingly larger increments in the supply of funds, reflecting the importance depositors attribute to the bank being "very safe."

*Proof of Lemma 2.*

*Step 1.* Let  $\psi(M_t, c_t, q_t) \equiv (M_t - d_t)R^B + a_t(Z_{t+1}^a - R^B) + l_t(Z_{t+1}^l - R^B) - S(R_t, q_t)(R_t - R^B)$ . Let  $M_t^\theta \equiv \theta M_t + (1 - \theta)M'_t$ ,  $c_t^\theta \equiv \theta c_t + (1 - \theta)c'_t$ ,  $q_t^\theta \equiv \theta q_t + (1 - \theta)q'_t$ , and  $R_t^\theta \equiv \theta R_t + (1 - \theta)R'_t$ . Then, for any  $\theta \in (0, 1)$  we have that

$$\psi(M_t^\theta, c_t^\theta, q_t^\theta) = \theta\psi(M_t, c_t, q_t) + (1 - \theta)\psi(M'_t, c'_t, q'_t) + \Delta,$$

where

$$\Delta \equiv S(R_t^\theta, q_t^\theta)(R^B - R_t^\theta) - [\theta S(R_t, q_t)(R^B - R_t) + (1 - \theta)S(R'_t, q'_t)(R^B - R'_t)] > 0$$

by the strict concavity of  $S(R_t, q_t)(R^B - R_t)$ .

*Step 2.* Using by the strict concavity of  $S(R_t, q_t)(R^B - R_t)$ , the properties of  $A(\cdot)$ , and Assumption D, we have that

$$\begin{aligned}
& 1 - F(0; M_t^\theta, c_t^\theta, q_t^\theta) \\
&= 1 - \Pr\{\psi(M_t^\theta, c_t^\theta, q_t^\theta) - A[\psi(M_t^\theta, c_t^\theta, q_t^\theta)] \leq 0\} \\
&> 1 - \Pr\{\theta\psi(M_t, c_t, q_t) + (1 - \theta)\psi(M_t', c_t', q_t') \\
&\quad - A[\theta\psi(M_t, c_t, q_t) + (1 - \theta)\psi(M_t', c_t', q_t')] \leq 0\} \\
&\geq 1 - \Pr\{\theta\psi(M_t, c_t, q_t) + (1 - \theta)\psi(M_t', c_t', q_t') \\
&\quad - \theta A[\psi(M_t, c_t, q_t)] - (1 - \theta)A[\psi(M_t', c_t', q_t')] \leq 0\} \\
&\geq 1 - \theta \Pr\{\psi(M_t, c_t, q_t) - A[\psi(M_t, c_t, q_t)] \leq 0\} \\
&\quad - (1 - \theta) \Pr\{\psi(M_t', c_t', q_t') - A[\psi(M_t', c_t', q_t')] \leq 0\} \\
&= \theta[1 - F(0; M_t, c_t, q_t)] + (1 - \theta)[1 - F(0; M_t', c_t', q_t')],
\end{aligned} \tag{18}$$

which completes the proof of part (i).

*Step 3.* Using part (i) of this lemma, we have

$$\begin{aligned}
& \theta q(M_t, c_t) + (1 - \theta)q(M_t', c_t') \\
&= \theta[1 - F(0; M_t, c_t, q(M_t, c_t))] + (1 - \theta)[1 - F(0; M_t', c_t', q(M_t', c_t'))] \\
&< 1 - F(0; M_t^\theta, c_t^\theta, \theta q(M_t, c_t) + (1 - \theta)q(M_t', c_t')).
\end{aligned} \tag{19}$$

Since  $1 - F(0; M_t, c_t, q_t)$  is decreasing in  $q_t$  (see the proof of Lemma 1) it follows that the value of  $q_t$  which solves equation (3) must satisfy  $q(M_t^\theta, c_t^\theta) > \theta q(M_t, c_t) + (1 - \theta)q(M_t', c_t')$ .

**Q.E.D.**

**Lemma 3** *If  $q(M_t, c_t)$  is strictly concave in  $(M_t, c_t)$ , then there exists  $\bar{\alpha}_q > 1$  such that for all  $\alpha_q \in (1, \bar{\alpha}_q]$ ,  $[q(M_t, c_t)]^{\alpha_q}$  is strictly concave in  $(M_t, c_t)$ .*

*Proof of Lemma 3.*

*Step 1.* In the proof of Lemma 1 the continuity and differentiability of  $q(M_t, c_t)$  were established by applying the Implicit Function Theorem to (14). For instance,

$$\frac{\partial q(M_t, c_t)}{\partial R_t} = \frac{\partial F(0; M_t, c_t, q_t)/\partial R_t}{1 + \partial F(0; M_t, c_t, q_t)/\partial q_t}. \tag{20}$$



By our assumptions on  $F(0; M_t, c_t, q_t)$  this derivative is bounded. Furthermore, since, by assumption,  $F(0; M_t, c_t, q_t)$  is twice continuously differentiable with respect to its arguments, the derivatives of the derivative in equation (20) exist and are continuous. Analogous properties hold for the other derivatives of  $q(M_t, c_t)$ .

*Step 2.* Let  $h(\cdot) \equiv q(\cdot)^{\alpha_q}$ ,  $\alpha_q > 1$ , where  $(\cdot)$  stands for  $(M_t, c_t)$ . Taking derivatives we get  $h_k(\cdot) = \alpha_q q(\cdot)^{\alpha_q-1} q_k(\cdot)$ , and  $h_{kk}(\cdot) = \alpha_q [q(\cdot)^{\alpha_q-1} q_{kk}(\cdot) + (\alpha_q - 1) q_k(\cdot)^2 q(\cdot)^{\alpha_q-2}]$ , where the subscripts  $k$  and  $kk$  denote first and second partial derivatives with respect to  $k = M_t, R_t, d_t, a_t, \ell_t$ . As the first derivatives of  $q(\cdot)$  are bounded (step 1), we have that for  $\alpha_q$  sufficiently close to 1,  $h_{kk}(\cdot)$  and  $q_{kk}(\cdot)$  are of the same sign. An analogous reasoning holds for minors of higher dimension.

**Q.E.D.**

**Lemma 4** *Let  $S(R_t, q_t)$  satisfy Assumption S. Then, for any  $\alpha_q > 1$  and any  $\alpha_R \in (0, 1]$ , there is an open region of the parameters  $\gamma_R$  and  $\gamma_q$  such that  $S(R_t, q_t)(R^B - R_t)$  is strictly concave in  $(R_t, q_t)$ .*

*Proof of Lemma 4.*

Define  $\rho(R_t, q_t) \equiv S(R_t, q_t)(R^B - R_t)$ . Using  $(\cdot)$  to denote  $(M_t, c_t)$ , and subscripts to denote partial derivatives, we get

$$\rho_{RR}(\cdot) = -\alpha_R(\alpha_R + 1)\gamma_R(R_t - R^B)^{\alpha_R-1} < 0,$$

$$\rho_{qq}(\cdot) = -\alpha_q(\alpha_q - 1)\gamma_q q_t^{\alpha_q-2}(R_t - R^B) < 0,$$

and

$$\begin{aligned} \rho_{Rq}(\cdot) &= q_t^{\alpha_q-2} \gamma_q \alpha_q [\alpha_R(\alpha_R + 1)(\alpha_q - 1)\gamma_R(R_t - R^B)^{\alpha_R} - \alpha_q \gamma_q q_t^{\alpha_q}] \\ &> q_t^{\alpha_q-2} \gamma_q \alpha_q [\alpha_R(\alpha_R + 1)(\alpha_q - 1)\gamma_R(\underline{R}(1) - R^B)^{\alpha_R} - \alpha_q \gamma_q], \end{aligned}$$

which is strictly positive if and only if

$$\frac{\gamma_R}{\gamma_q} > \frac{\alpha_R(\alpha_R + 1)(\alpha_q - 1)}{\alpha_q(\underline{R}(1) - R^B)^{\alpha_R}}. \quad (21)$$

For any  $\alpha_R$  and  $\alpha_q$  satisfying the assumptions of the lemma, the inequality in (21) holds for an open region of the parameters  $\gamma_R$  and  $\gamma_q$ .

**Q.E.D.**

**Lemma 5** *If Assumptions D, S, and R are satisfied, then there is  $\bar{\alpha}_q > 1$  such that for all  $\alpha_q \in (1, \bar{\alpha}_q]$ , the correspondence  $\Gamma(\cdot)$  is convex in the following sense: Let  $M_t, M'_t \geq M^*$ , where  $M^*$  is the solution to  $M_t = A(M_t)$  (see Figure 2), and let  $c_t \in \Gamma(M_t)$  and  $c'_t \in \Gamma(M'_t)$ . Then  $c_t^\theta \in \Gamma(M_t^\theta)$  for all  $\theta \in (0, 1)$ , where  $c_t^\theta = \theta c_t + (1 - \theta)c'_t$  and  $M_t^\theta = \theta M_t + (1 - \theta)M'_t$ .*

*Proof of Lemma 5.*

We consider each of the constraints defining the correspondence of feasible policies  $\Gamma(\cdot)$ .

*Dividend constraint.* Let  $0 \leq d_t \leq M_t - A(M_t - d_t)$  and  $0 \leq d'_t \leq M'_t - A(M'_t - d'_t)$ . Obviously,  $d_t^\theta \geq 0$ . Using the fact that  $A(\cdot)$  is negative and convex, we have

$$d_t^\theta \leq \theta[M_t - A(M_t - d_t)] + (1 - \theta)[M'_t - A(M'_t - d'_t)] \leq M_t^\theta - A(M_t^\theta - d_t^\theta).$$

*Stock market constraint.* Let  $a_t \geq A(M_t - d_t)$  and  $a'_t \geq A(M'_t - d'_t)$ . Again, by the convexity of  $A(\cdot)$ ,

$$a_t^\theta \geq \theta A(M_t - d_t) + (1 - \theta)A(M'_t - d'_t) \geq A(M_t^\theta - d_t^\theta).$$

*Projects constraint.* Since  $\ell_t \geq 0$  and  $\ell'_t \geq 0$  it follows that  $\ell_t^\theta = \theta \ell_t + (1 - \theta)\ell'_t \geq 0$ .

*Reserve requirement constraint.* Since  $S(R_t, q_t)$  satisfies Assumption S, it follows from Lemma 4 that for any  $\alpha_q > 1$  there is an open region of the parameters  $\gamma_R$  and  $\gamma_q$  such that  $S(R_t, q_t)(R^B - R_t)$  is strictly concave in  $(R_t, q_t)$ . As Assumption D also holds, it follows from Lemma 2 that  $q(M_t, c_t)$  is strictly concave in  $(M_t, c_t)$ . By lemma 3 there is  $\bar{\alpha}_q > 1$  such that for all  $\alpha_q \in (1, \bar{\alpha}_q]$ ,  $[q(M_t, c_t)]^{\alpha_q}$  is strictly concave in  $(M_t, c_t)$ . Recalling that  $\alpha_R \in (0, 1]$ , it follows that for  $\alpha_q \in (1, \bar{\alpha}_q]$ ,  $S[R_t, q(M_t, c_t)] = \gamma_R(R_t - R^B)^{\alpha_R} + \gamma_q q(M_t, c_t)^{\alpha_q}$  is strictly concave in  $(M_t, c_t)$ , implying that

$$S[R_t^\theta, q(M_t^\theta, c_t^\theta)] > \theta S[R_t, q(M_t, c_t)] + (1 - \theta)S[R'_t, q(M'_t, c'_t)].$$

It is then straightforward to verify that if condition (8) is satisfied for  $(M_t, c_t)$  and  $(M'_t, c'_t)$ , then it is also satisfied for  $(M_t^\theta, c_t^\theta)$ .

*Supply of funds constraint.* By Assumption R,  $\bar{R}[q(M_t, c_t)]$  is concave in  $(M_t, c_t)$ , and  $\underline{R}[q(M_t, c_t)]$  is convex in  $(M_t, c_t)$ . Therefore, if condition (9) is satisfied for  $(M_t, c_t)$  and

$(M'_t, c'_t)$ , then it is also satisfied for  $(M_t^\theta, c_t^\theta)$ .

**Q.E.D.**

We turn to the proof of the Proposition: *Step 1.* Since  $S(R_t, q_t)$  satisfies Assumption S, we know by Lemma 4 that for any  $\alpha_q > 1$  and  $\alpha_R \in (0, 1]$  there is an open region of the parameters  $\gamma_R$  and  $\gamma_q$  such that  $S(R_t, q_t)(R^B - R_t)$  is strictly concave in  $(R_t, q_t)$ . By Lemma 2 and Assumption D,  $q(M_t, c_t)$  is strictly concave in  $(M_t, c_t)$ . Consequently  $S(R_t, q(M_t, c_t))(R^B - R_t)$  is strictly concave in  $(M_t, c_t)$ , which in turn implies that  $\phi(M_t, c_t, Z_{t+1})$  is strictly concave in  $(M_t, c_t)$ . *Step 2.* Consider the operator in (17), and let  $f(\cdot)$  be a concave function. Then,  $f(\phi(M_t^\theta, c_t^\theta, Z_{t+1}))$  is concave, and hence so is  $E[f(\phi(M_t^\theta, c_t^\theta, Z_{t+1}))]$ . Since  $u(\cdot)$  is strictly concave we have

$$\begin{aligned}
(Tf)(M_t^\theta) &\geq u(d_t^\theta) + \beta E[f(\phi(M_t^\theta, c_t^\theta, Z_{t+1}))] \\
&> \theta u(d_t) + \theta \beta E[f(\phi(M_t, c_t, Z_{t+1}))] \\
&\quad + (1 - \theta)u(d'_t) + (1 - \theta)\beta E[f(\phi(M'_t, c'_t, Z_{t+1}))] \\
&= \theta(Tf)(M_t) + (1 - \theta)(Tf)(M'_t),
\end{aligned} \tag{22}$$

where  $c_t^\theta = \theta c_t + (1 - \theta)c'_t$  and  $M_t^\theta = \theta M_t + (1 - \theta)M'_t$ . Therefore,  $(Tf)(\cdot)$  is a strictly concave function, which implies that the fixed point  $V(\cdot)$  must also be strictly concave.

*Step 3.* Fix  $M_t$ . Since  $\Gamma(M_t)$  is compact, the supremum in equation (16) is achieved. By the strict concavity of  $\phi(\cdot)$  in  $c_t$ , and of  $V(\cdot)$  in its argument, it follows that there is a unique  $c_t$  which achieves the maximum.

*Step 4.* By the Theorem of the Maximum, the maximizing policy vector  $c_t$  is a continuous function of  $M_t$ .

**Q.E.D.**

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Figure 1: Supply function of CD deposits

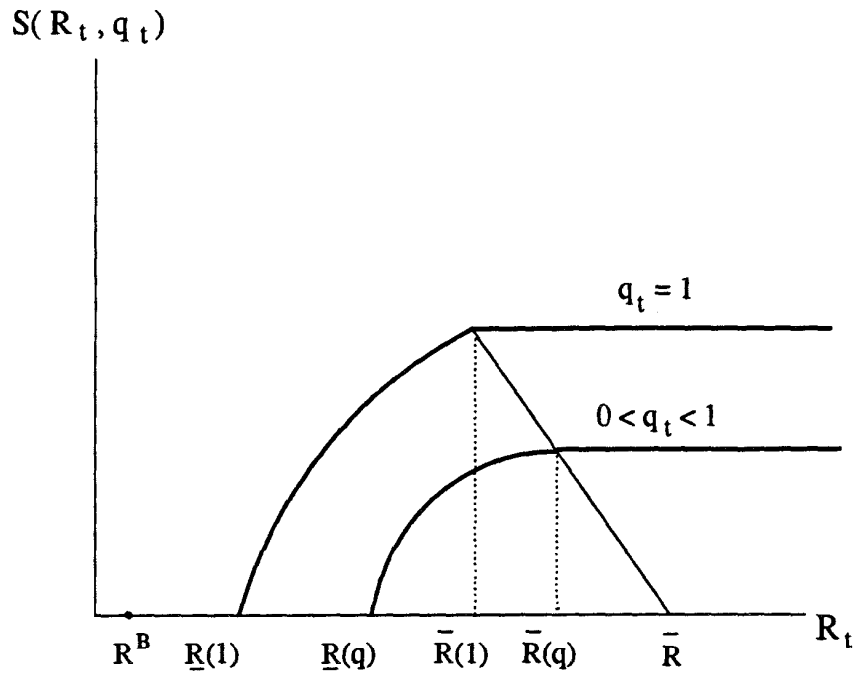


Figure 2: Lower bound on securities short sale

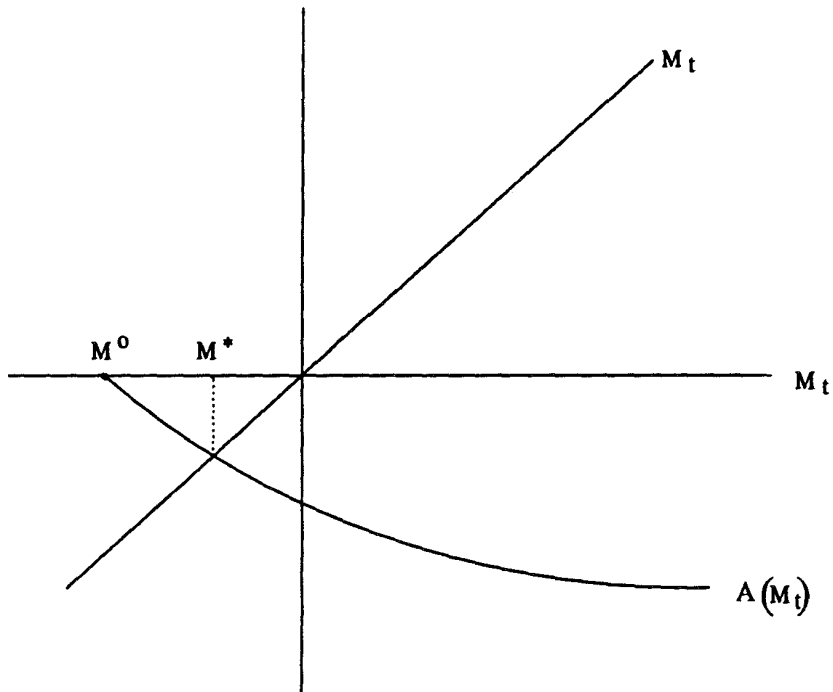
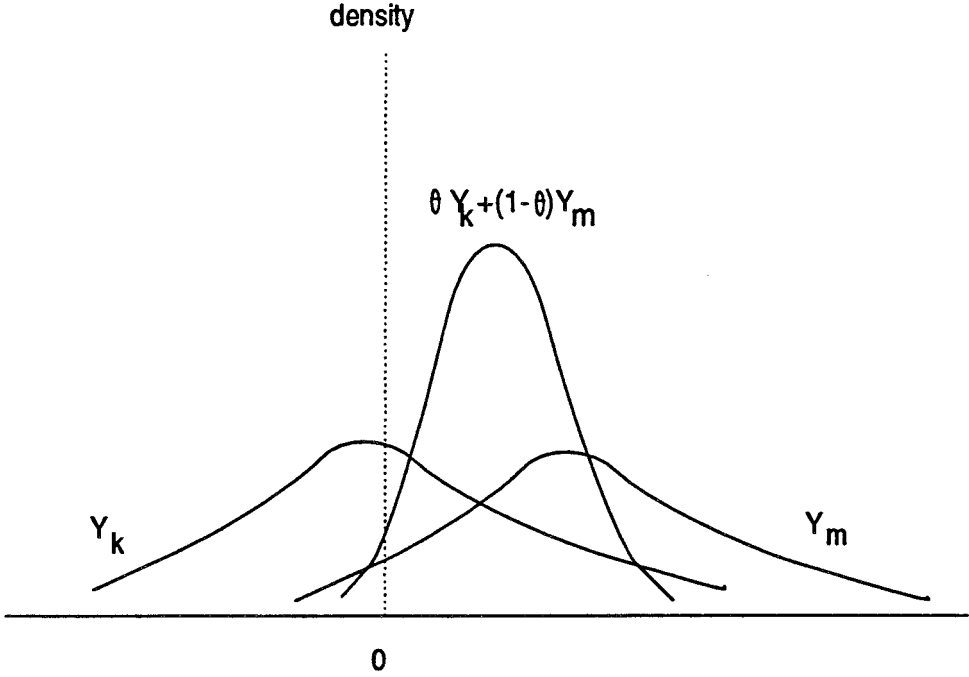
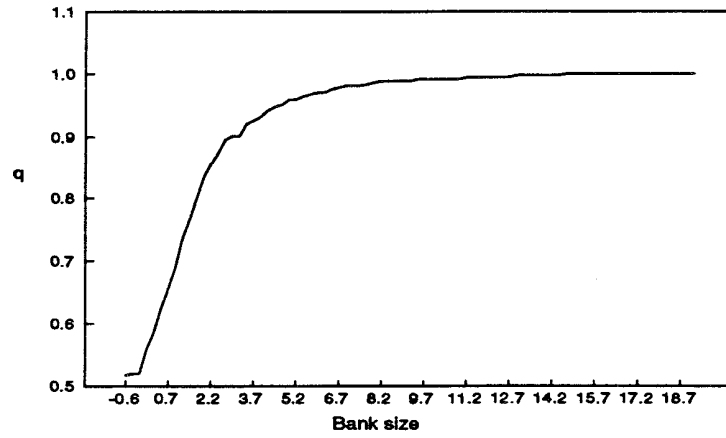


Figure 3: Density function under Assumption D

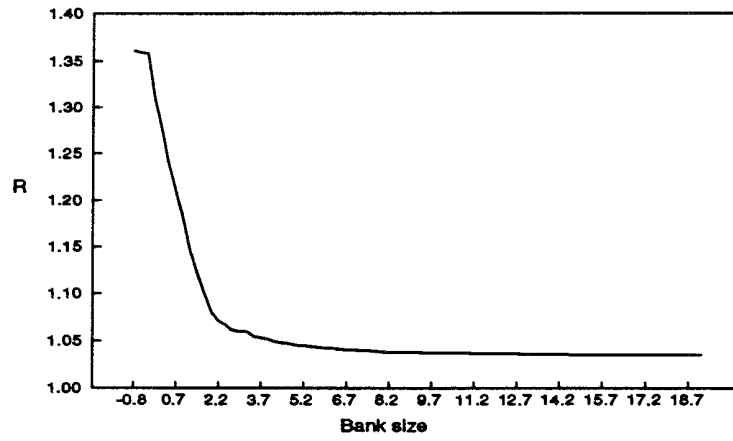


**Figure 4: Base Case Simulation**

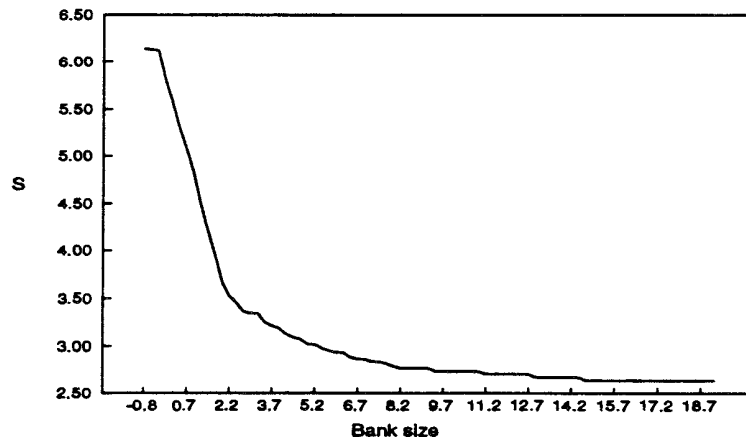
**a. Probability of Survival ( $q$ )**



**b. Interest on CD ( $R$ )**

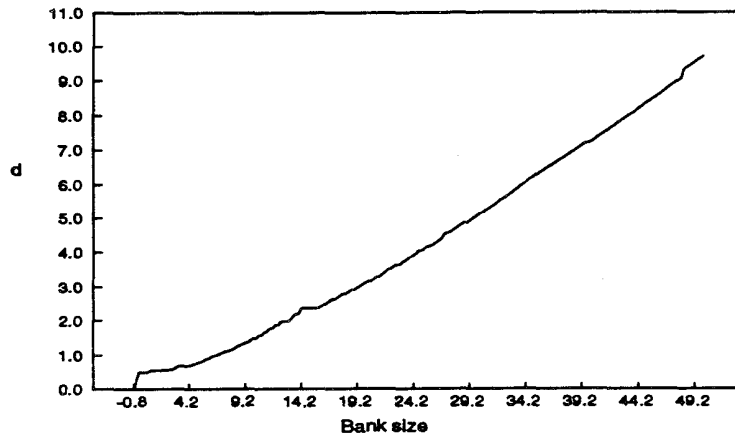


**c. CD Deposits ( $S$ )**

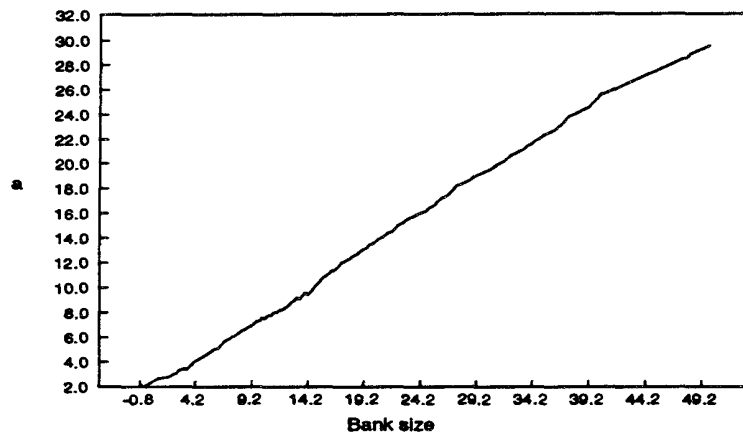




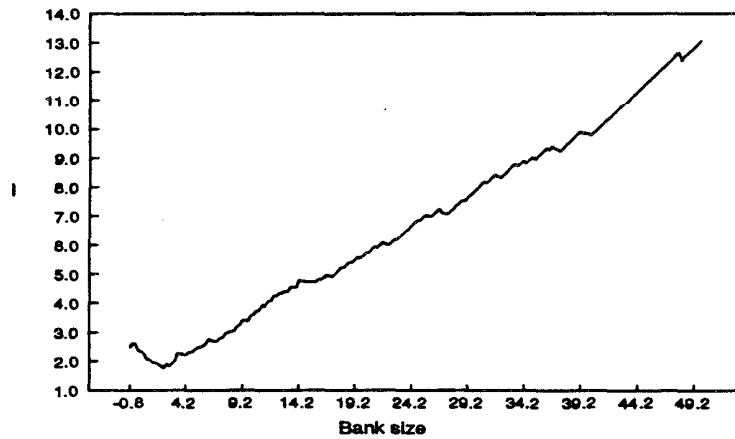
**Figure 4 (Continued)**  
**d. Dividend Distribution (d)**



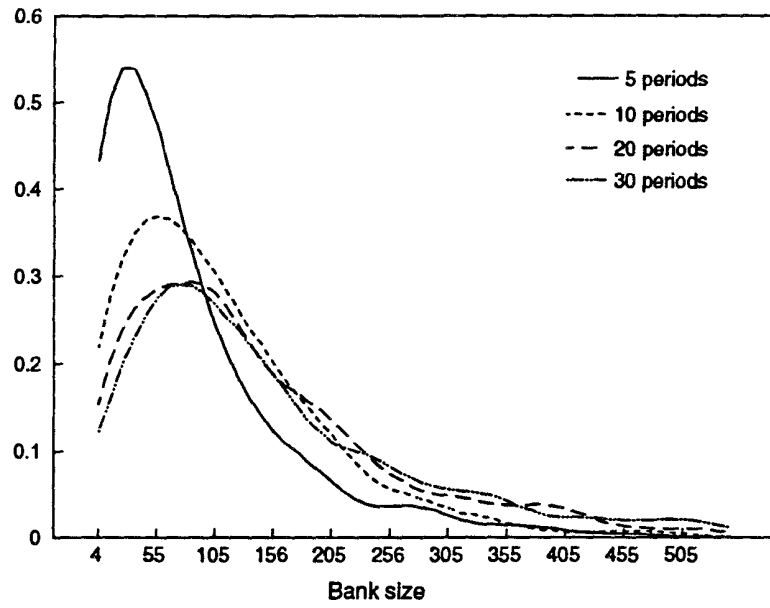
**e. Investment In Market Portfolio (a)**



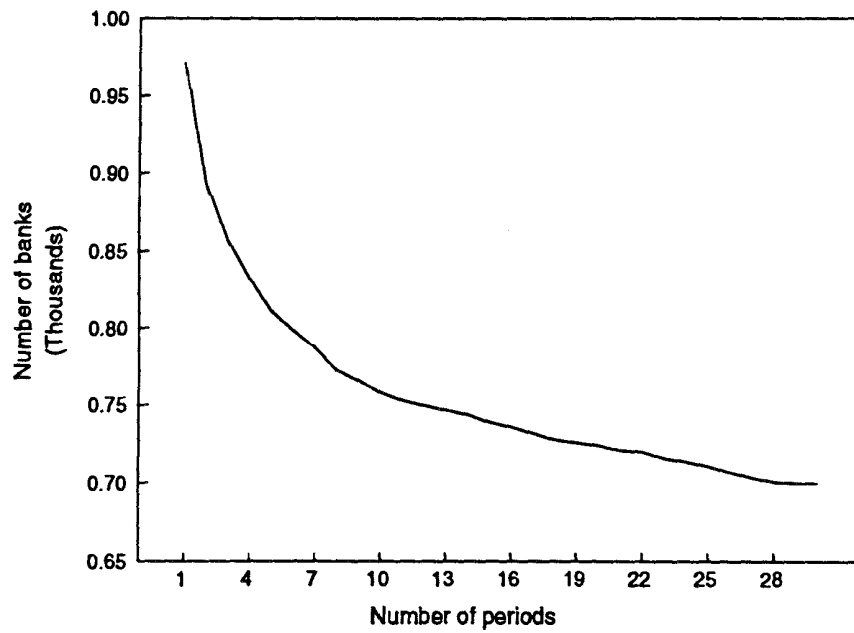
**f. Investment In Projects (I)**



**Figure 5: Base Case—Cohort Simulation**  
**a. Banks Distribution by Periods**

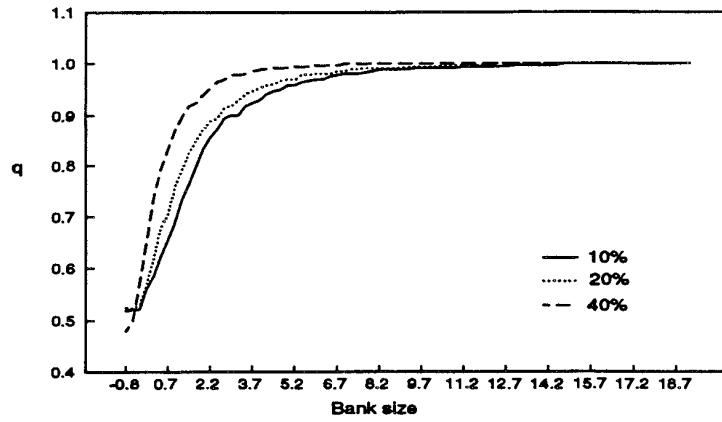


**b. Number of Surviving Banks by Period**

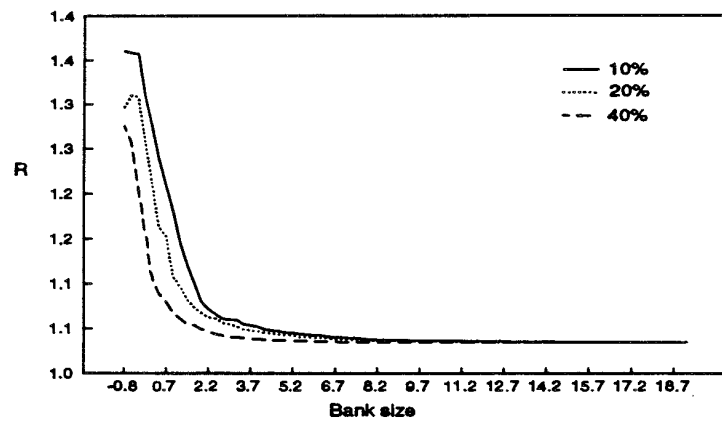


**Figure 6: Reserve Requirement Simulation**

**a. Probability of Survival ( $q$ )**



**b. Interest Rate on CD ( $R$ )**



**c. CD Deposits ( $S$ )**

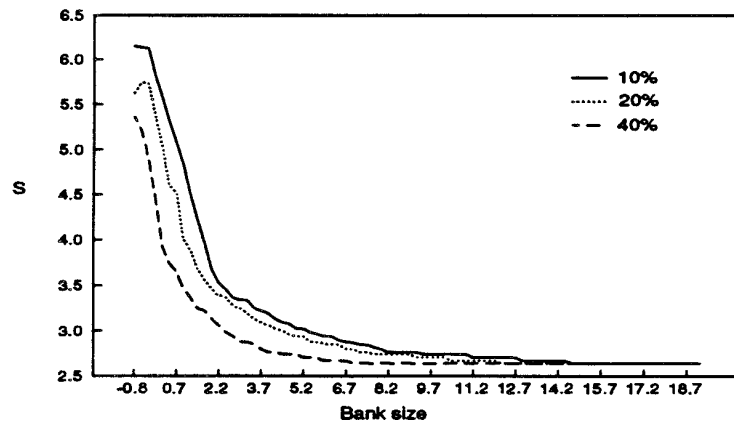
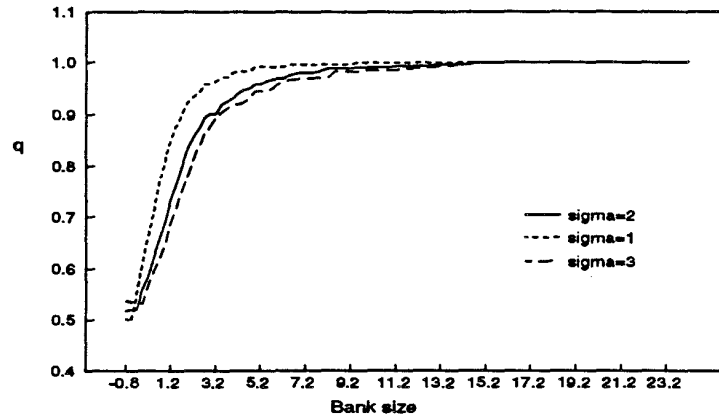
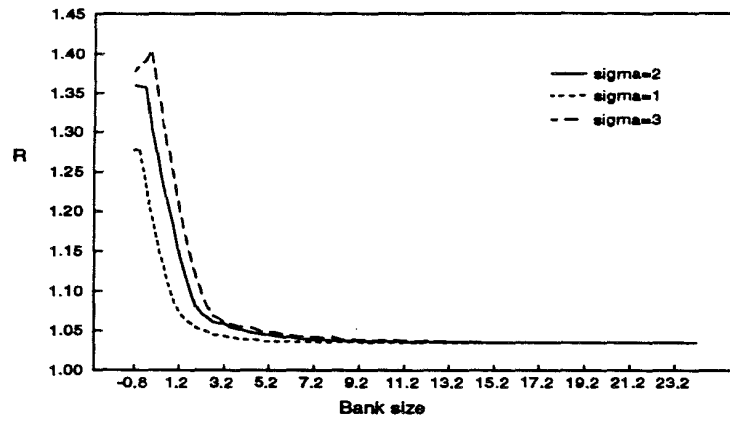


Figure 7: Riskiness of Investment in Project Simulation

a. Probability of Survival ( $q$ )



b. Interest Rate on CD ( $R$ )



c. CD Deposits ( $S$ )

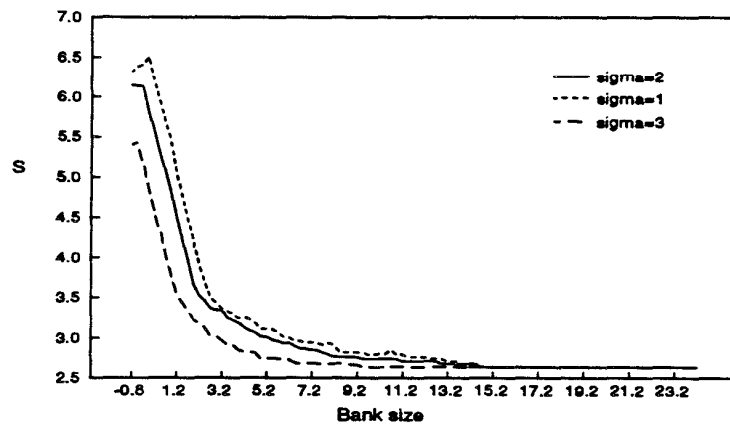
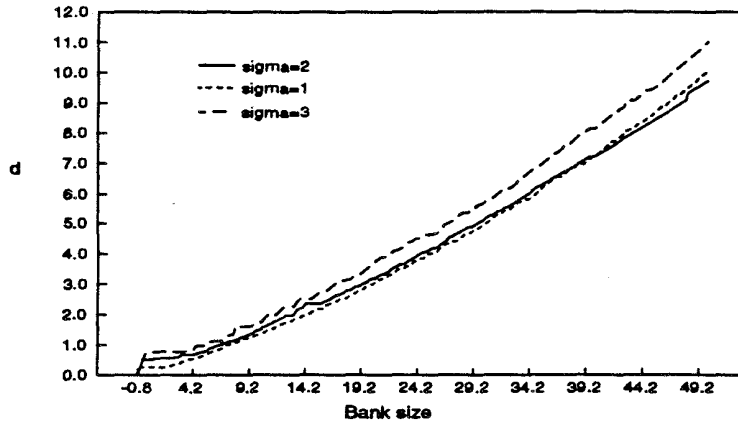
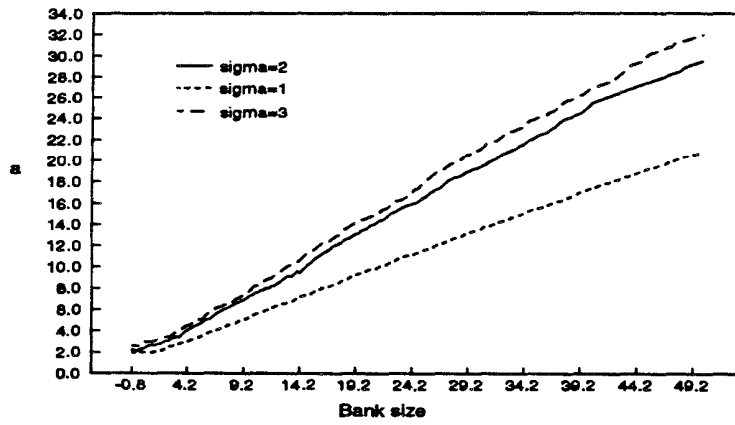


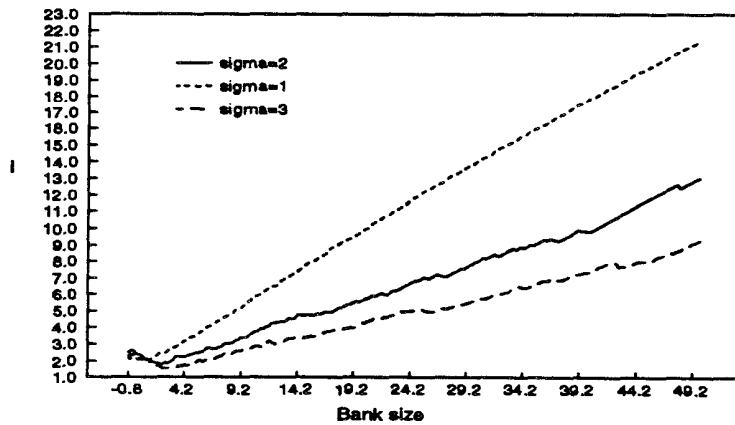
Figure 7 (Continued)  
d. Dividends Distribution (d)



e. Investment in Market Portfolio (a)

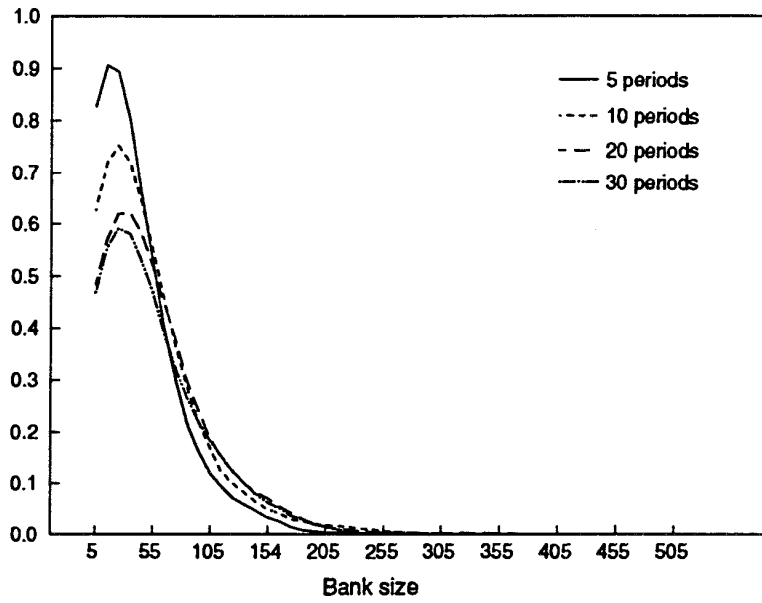


f. Investment in Projects (I)



**Figure 8: Riskiness of Investment in Project Cohort Simulation**

**a. Banks Distribution by Periods (Sigma=1)**



**b. Banks Distribution by Periods (Sigma=2)**

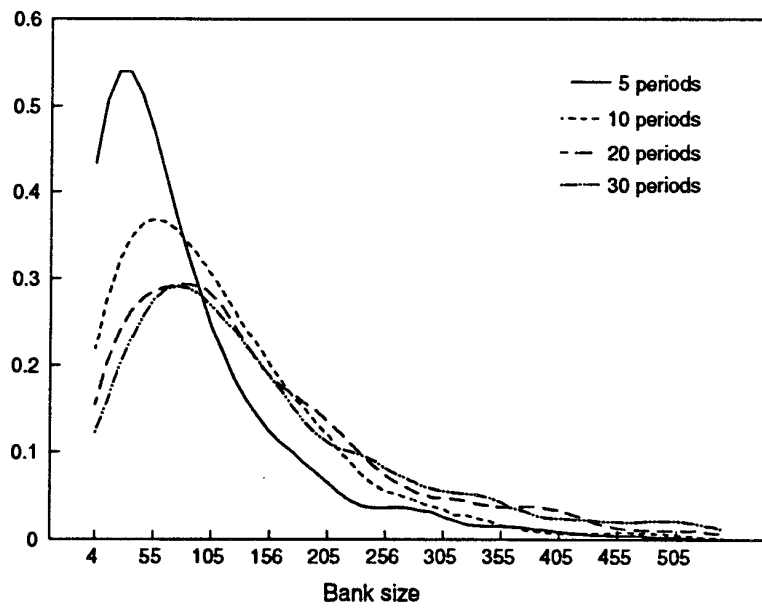
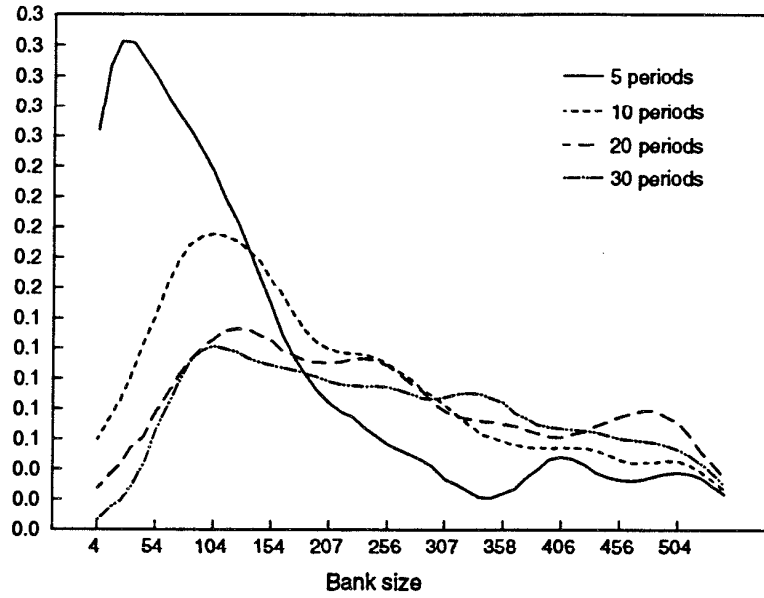
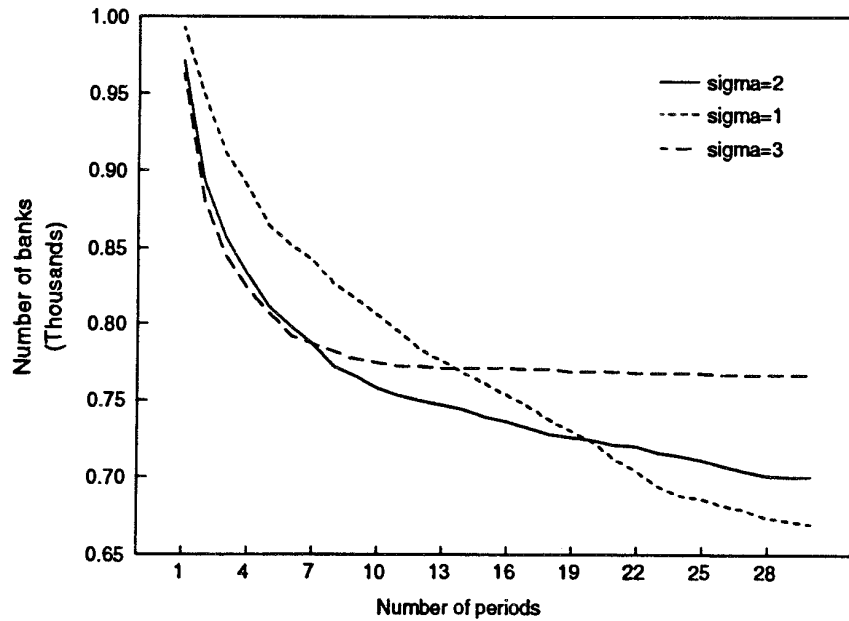


Figure 8 (Continued)

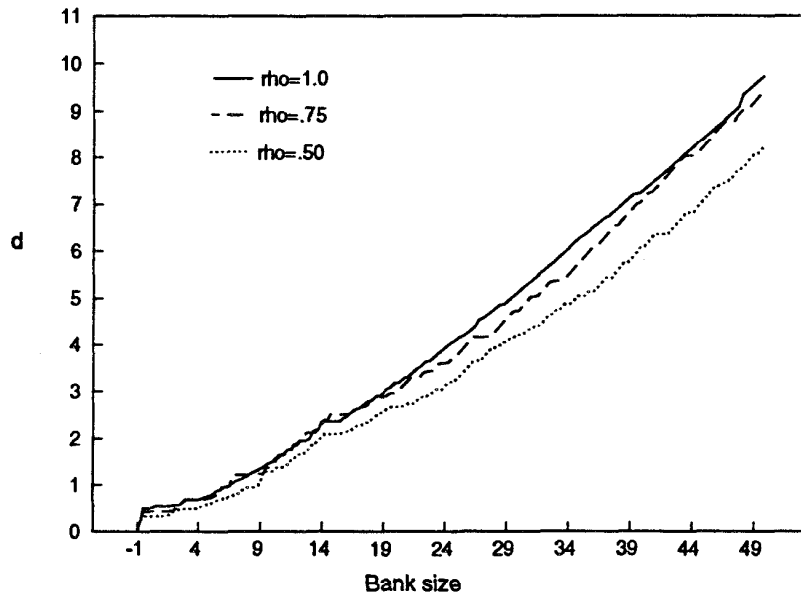
c. Banks Distribution by Periods (Sigma=3)



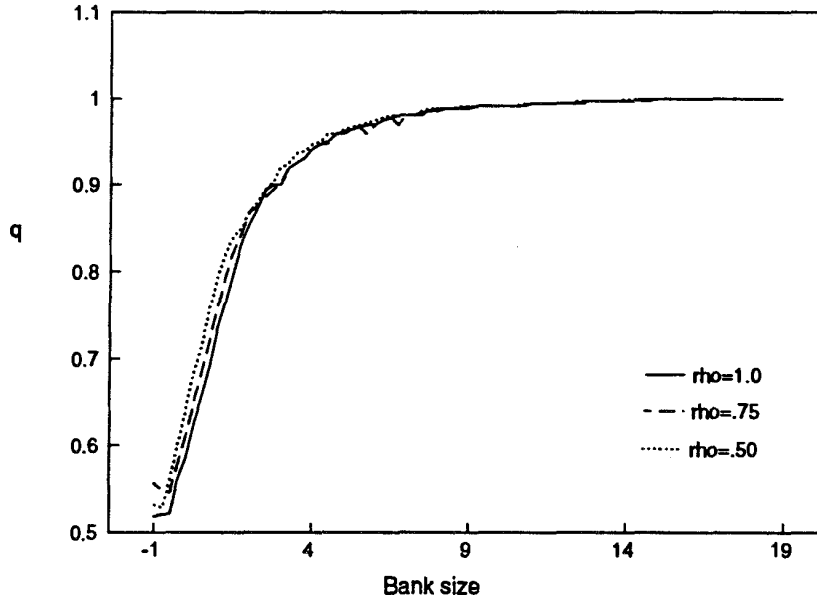
d. Number of Surviving Banks by Period



**Figure 9: Risk Aversion (of Manager) Simulation**  
**a. Dividends Distribution (d)**



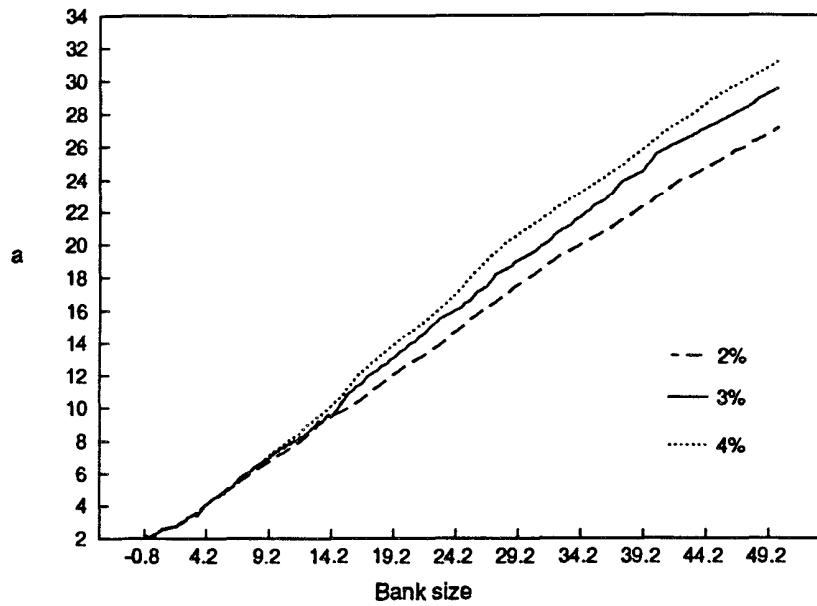
**b. Probability of Survival (q)**



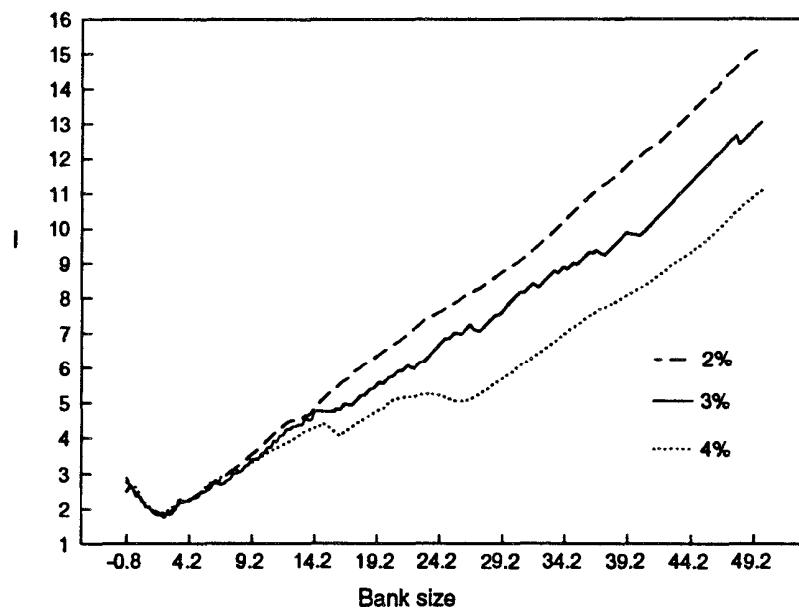


**Figure 10: Riskless Interest Rate Simulation**

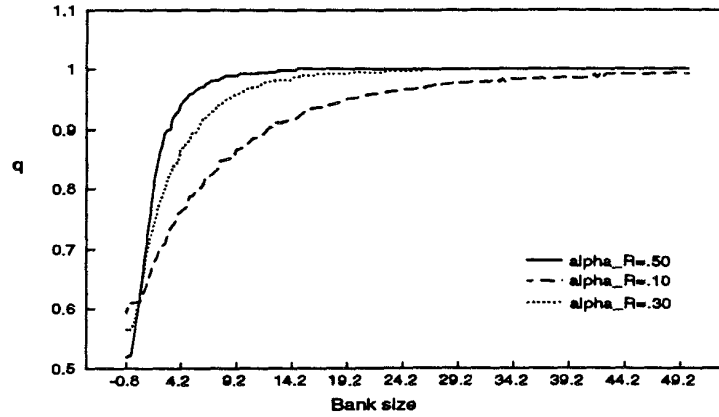
**a. Investment in Market Portfolio (a)**



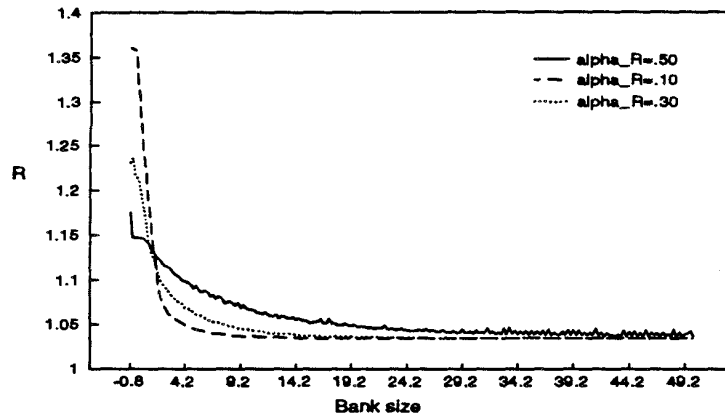
**b. Investment in Projects (I)**



**Figure 11: Deposits Sensitivity to CD Rate Simulation**  
**a. Probability of Survival (q)**



**b. Interest Rate on CD (R)**



**c. CD Deposits (S)**

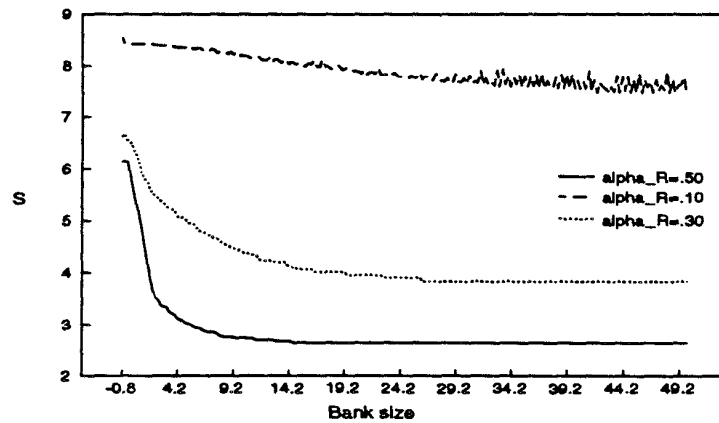
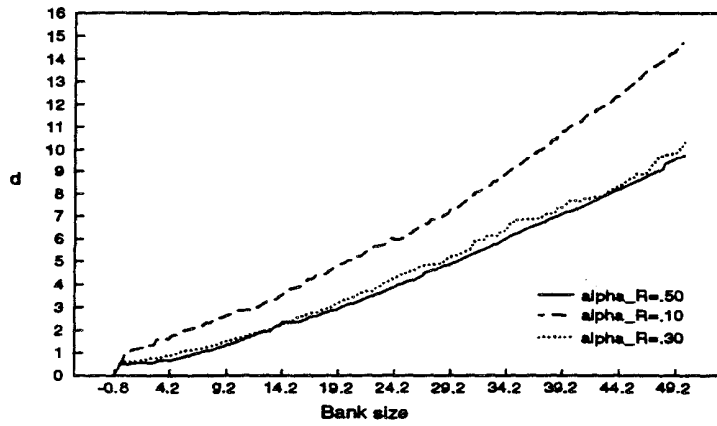
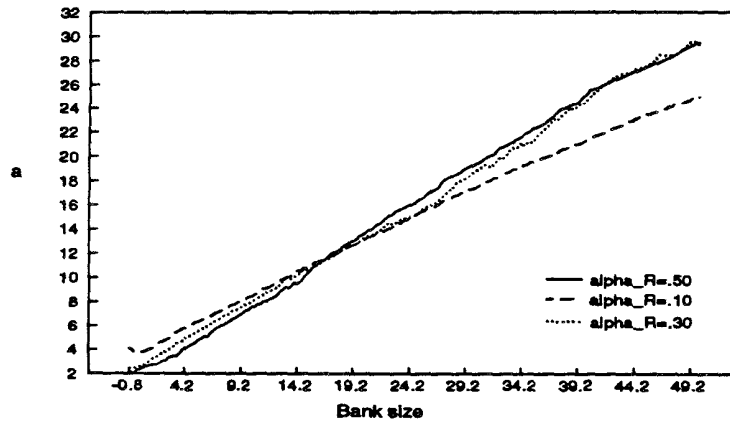


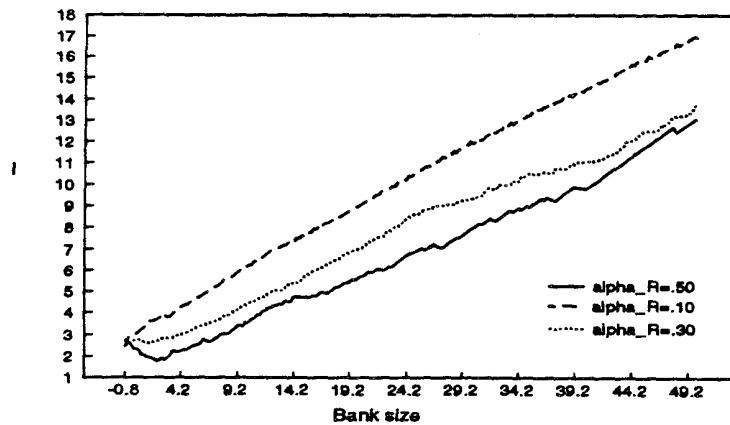
Figure 11 (Continued)  
d. Dividends Distribution (d)



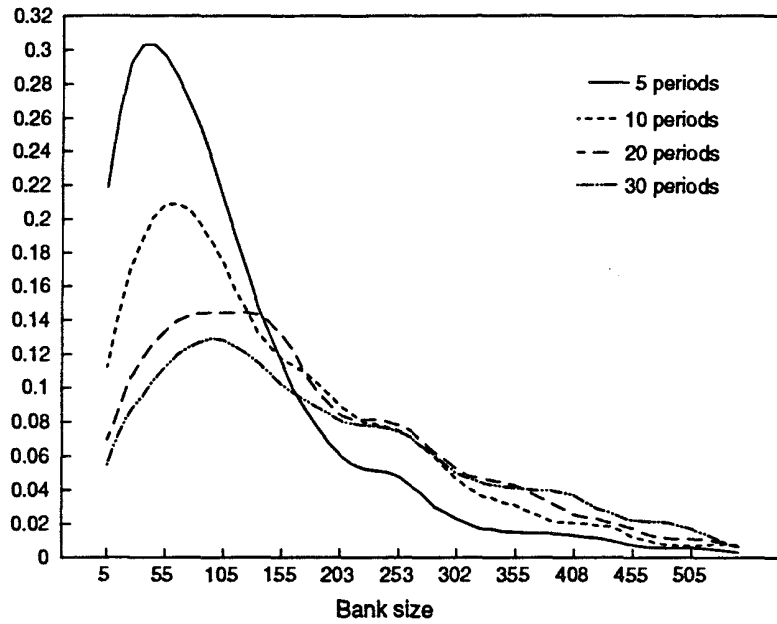
e. Investment In Market Portfolio (a)



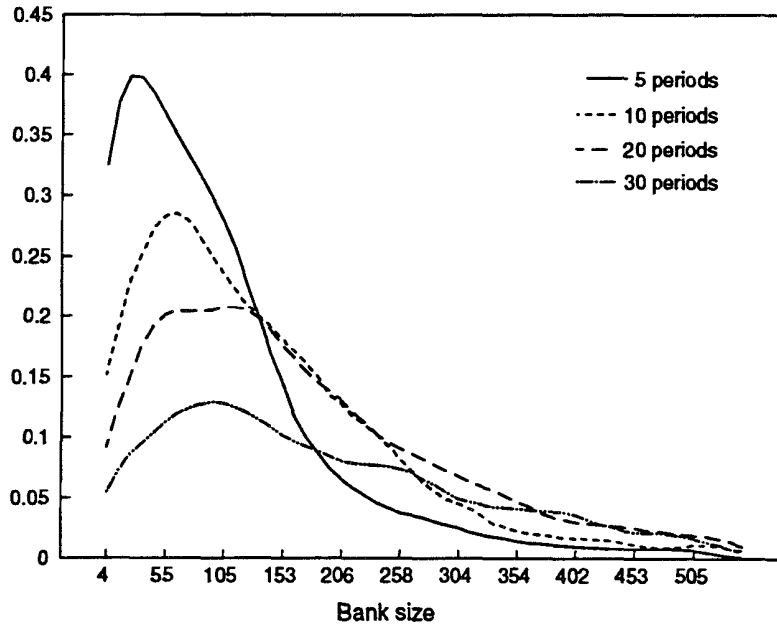
f. Investment In Projects (I)



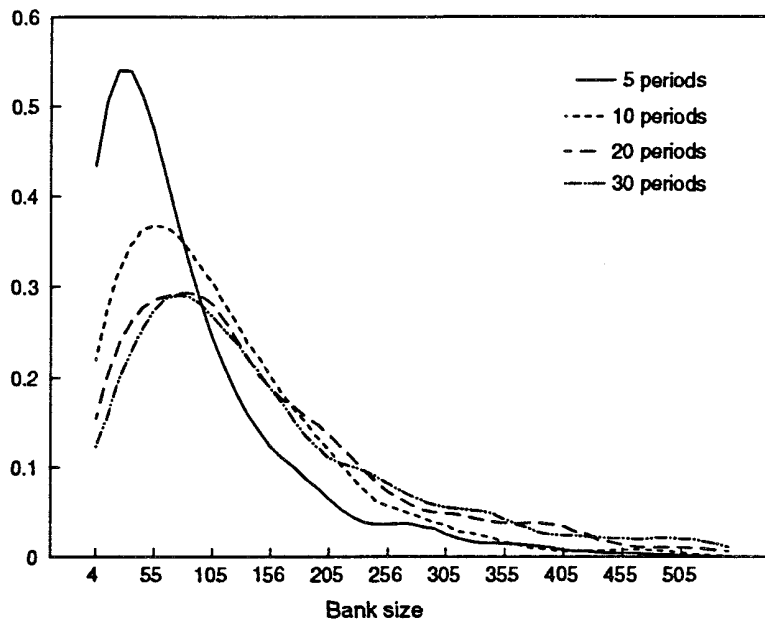
**Figure 12: Deposits Sensitivity to CD Rate—Cohort Simulation**  
**a. Banks Distribution by Periods (alpha\_R=.10)**



**b. Banks Distribution by Periods (alpha\_R=.30)**



**Figure 12 (Continued)**  
**c. Banks Distribution by Periods ( $\alpha_R=50$ )**



**d. Number of Surviving Banks by Period**

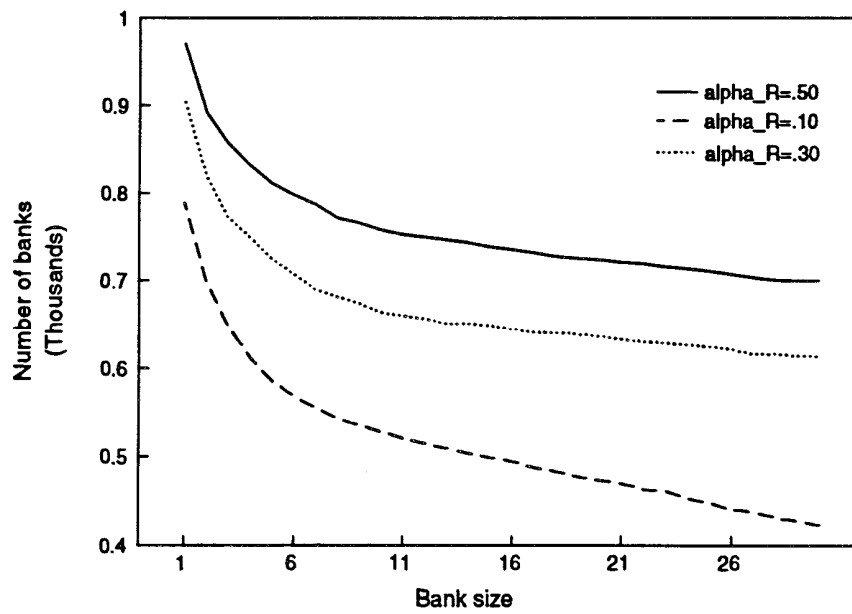
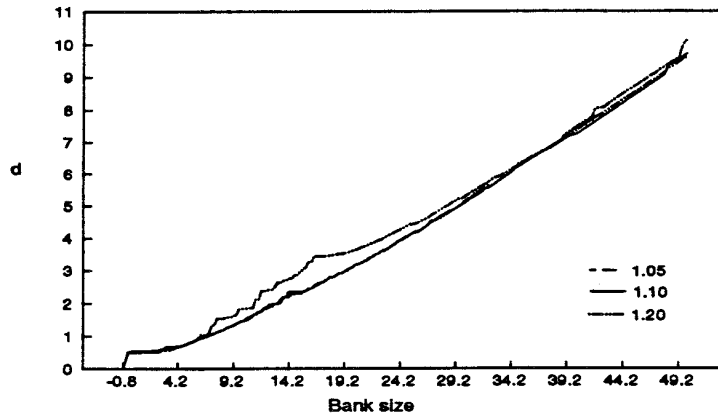
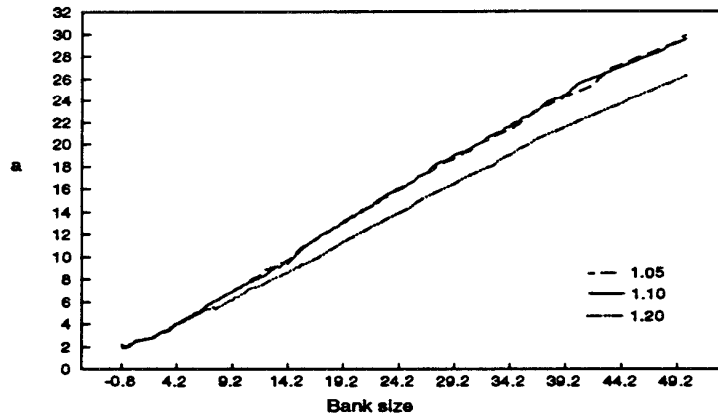


Figure 13: Deposits Sensitivity to Survival Probability Simulation

a. Dividends Distribution (d)



b. Investment in Market Portfolio (a)



c. Investment In Projects (I)

