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RETHINKING MULTIPLE EQUILIBRIA IN MACROECONOMIC MODELLING

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Abstract

Are beliefs as indeterminate as suggested by models with multiple equilibria? Multiplicity of equilibria arise largely as the unintended consequence of two modelling assumptions - the fundamentals are assumed to be common knowledge, and economic agents know others' actions in equilibrium. Both are questionable. When others' actions are not known with certainty, such as when actions rely on noisy signals, self-fulfilling beliefs lead to a unique outcome determined by the fundamentals and the knowledge that others are rational. This paper illustrates this approach in the context of a model of bank runs and other similar applications. Such an approach places comparative statics and policy analyses on a firmer footing. It also suggests that public information has a disproportionately larger impact on the outcome than private information.

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1. Introduction

It is a commonplace that actions are motivated by beliefs, and so economic outcomes are influenced by the beliefs of individuals in the economy. In many examples in economics, there seems to be an apparent indeterminacy in beliefs in the sense that one set of beliefs motivate actions which bring about the state of affairs envisaged in the beliefs, while another set of self-fulfilling beliefs bring about quite different outcomes. In both cases, the beliefs are logically coherent, consistent with the known features of the economy, and are borne out by subsequent events. However, they are not fully determined by the underlying description of the economy, leaving a role for "sunspots".

Models that utilize such apparent indeterminacy of beliefs have considerable intuitive appeal, since they provide a convenient and economical prop in a narrative of unfolding events. However, they are vulnerable to a number of criticisms. For a start, the shift in beliefs which underpins the switch from one equilibrium to another is left unexplained. As such, it runs counter to our theoretical scruples against indeterminacy. More importantly, it runs counter to our intuition that bad fundamentals are somehow "more likely" to trigger a financial crisis, or to tip the economy into recession. In other words, sunspot explanations do not provide a basis for exploring the *correlation* between the underlying fundamentals and the resultant economic outcomes. Finally, comparative statics analyses and the policy implications that flow from them are only as secure as the equilibrium chosen for this exercise.

The literature on multiple equilibria is large and diverse. The recent book by Cooper (1999) provides a taxonomy for a selection of examples from macroeconomics. Technological complementarities (such as Bryant (1983)), demand spillovers (such as the "big push" model of Murphy, Shleifer and Vishny (1989)), and thick market externalities (illustrated by Diamond's (1982) search model) are some of the examples. Models of financial crises, encompassing both banking crises and attacks on currency pegs have a similarly large and active research following. Obstfeld and Rogoff (1997) and Freixas and Rochet (1997) are good stepping off points for this literature.

Our objective in this paper is to encourage a re-examination of the theoretical basis for multiple equilibria. We doubt that economic agents' beliefs are as indeterminate as implied by the multiple equilibrium models. Instead, the apparent indeterminacy of beliefs can be seen as the consequence of two modelling assumptions introduced to simplify the theory. First, the economic fundamentals

are assumed to be common knowledge; and second, economic agents are assumed to be certain about others' behavior in equilibrium. Both assumptions are made for the sake of tractability, but they do much more besides. They allow agents' actions and beliefs to be perfectly co-ordinated in a way that invites multiplicity of equilibria. We will describe an approach where agents have a small amount of idiosyncratic uncertainty about economic fundamentals. Even if this idiosyncratic uncertainty is small, agents will be uncertain about each other's behavior in equilibrium. This uncertainty allows us as modellers to pin down which set of self-fulfilling beliefs will prevail in equilibrium.

To elaborate on this point, it is instructive to contrast a single person decision problem with a game. In a single person decision problem, payoffs are determined by one's action and the state of the world. When a decision maker receives a message which rules out some states of the world, this information can be utilized directly by disregarding these states in one's deliberations. However, the same is not true in an environment where payoffs depend on the actions of other individuals, as well as the state of the world. Since my payoff depends on your actions and your actions are motivated by your beliefs, I care about the range of possible beliefs you may hold. So, when I receive a message which rules out some states of the world, it may not be possible to disregard these states in my deliberations, since some of these states may carry information concerning your beliefs. Even for small disparities in the information of the market participants, uncertainty about others' beliefs may dictate a particular course of action as being the uniquely optimal one. In this way, it may prove possible to track the shifts in beliefs as we track the shifts in the economic fundamentals. There is no longer a choice in what beliefs to hold. One's beliefs are dictated by the knowledge of the fundamentals and the knowledge that other agents are rational.

In this paper, we provide an elementary demonstration of why adding noise to a game with multiple equilibria removes the multiplicity. The analysis builds on the game theoretic analysis of Carlsson and van Damme (1993) for two player games and the continuum player application to currency attacks of Morris and Shin (1998). We develop a very simple continuum player example to illustrate the argument, and show by example why this is a flexible modelling approach that can be applied to many of the macroeconomic models with multiplicity discussed above. In doing so, we hope to show that the indeterminacy of beliefs in multiple equilibrium models is an artifact of simplifying assumptions that deliver more than they are intended to deliver; and that the approach described here is not merely a technical curiosity, but represents a better way of understanding the role

of self-fulfilling beliefs in macroeconomics.

We also outline the principal benefits of the approach. One is in generating comparative statics, which in turn aids policy analysis. The other is in suggesting observational implications. Here we summarize those benefits in a general way; in the text, we will discuss them in the context of particular applications.

Multiple equilibrium models in macroeconomics are often used as a starting point for policy analysis, despite the obvious difficulties of any comparative static analysis with indeterminate outcomes. The unique equilibrium in the approach described here is characterized by a marginal decision maker who, given his uncertainty about others' actions, is indifferent between two actions. Changing parameters in the model then delivers intuitive comparative static predictions and implications for optimal policy. In general, we show that inefficiencies are unavoidable in equilibrium. The question is how large such inefficiencies are. The answer turns on the underlying fundamentals of the economy, as well as the information structure of the economic agents. Thus, the notion of a "solvent but illiquid borrower" can be given a rigorous treatment and the extent of the welfare losses associated with such illiquidity can be calculated.

The theory offers a different perspective on existing empirical work. One traditional approach in the literature is to attempt to distinguish empirically between multiple equilibrium models and fundamentals-driven models. These ultimately reduce to tests of whether observed fundamentals are sufficient to explain outcomes or whether there is a significant unexplained component that must be attributed to self-fulfilling beliefs. We argue that correlation between fundamentals and outcomes is exactly what one should expect even when self-fulfilling beliefs are playing an important role in determining the outcome. One will be pessimistic about others' beliefs exactly when fundamentals are weak. The standard sunspot approach, by constrast, offers no theoretical rationale as to why good outcomes should be correlated with good fundamentals (although admittedly this is consistent with the theory and often assumed).

We also suggest one distinctive observational implication. Consider an environment where agents' actions are driven by their beliefs about fundamentals and others' actions. Suppose agents are slightly uncertain about some "fundamental" variable when they make their decisions, but that ex post the econometrician is able to observe the actual realization of that fundamental variable as well as some public signal concerning that fundamental variable that was available to agents at the time. The theory suggests the prediction that the public signals will have an apparently disproportionate impact on outcomes, even controlling for the real-

ization of fundamentals, precisely because it signals information to agents about other agents' equilibrium beliefs.

We start in the next section by analyzing a simple model of bank runs, in the spirit of Diamond and Dybvig (1983), to illustrate the approach in the context of a particular application. Goldstein and Pauzner (1999) have developed a richer model; we abstract from a number of complications, in order to bring out our methodological message. In section 3, we show how the insights are more general and can be applied in a variety of contexts. In particular, we discuss models of currency crises and pricing debt in the presence of liquidity risk.

2. Bank Runs

There are three dates, $\{0, 1, 2\}$, and a continuum of consumers, each endowed with 1 unit of the consumption good. Consumption takes place either at date 1 or date 2. There is a measure λ of *impatient* consumers who derive utility only from consumption in date 1, and measure 1 of *patient* consumers for whom consumption at dates 1 and 2 are perfect substitutes. The consumers learn of their types at date 1. At date 0, the probability of being patient or impatient is proportional to the incidence of the types. Thus, there is probability

$$\frac{\lambda}{1+\lambda}$$

of being an impatient consumer, and complementary probability of being the patient consumer. All consumers have the log utility function, and the utility of the impatient type is

$$u(c_1) = \log c_1$$

where c_1 is consumption at date 1, while the utility of the patient type is

$$u(c_1 + c_2) = \log(c_1 + c_2)$$

where c_2 is consumption at date 2.

The consumers can either store the consumption good for consumption at a later date, or they can deposit it in the bank. For those consumers who have invested their wealth in the bank, they have a decision at date 1, after learning of their type. They can either leave their money deposited in the bank, or they can withdraw the sum permitted in the deposit contract (to be discussed below).

The bank can either hold the deposits in cash (with rate of return 1) or invest the money in an illiquid project, with gross rate of return $R > 1$ obtainable at date 2. We assume that this technology is only available to the bank. If proportion ℓ of the resources invested in the illiquid investment are withdrawn at date 1, then the rate of return is reduced to $R \cdot e^{-\ell}$, reflecting the costs of premature liquidation. Writing $r \equiv \log R$, this rate of return can be written as $e^{r-\ell}$. We assume that $0 < r < 1$.

2.1. Optimal Contract

We proceed to solve for the optimal contract in this context. The aim is to maximize the ex ante expected utility

$$\tfrac{\lambda}{1+\lambda} u(c_1) + \tfrac{1}{1+\lambda} u(c_2) \tag{2.1}$$

by choosing the amount c_1 that can be withdrawn on demand at date 1. We assume that the bank is required to keep sufficient cash to fund first period consumption under the optimal contract. Thus, the first constraint is

$$\lambda c_1 + \frac{c_2}{R} \leq 1 + \lambda \tag{2.2}$$

which requires that the amount held in cash (λc_1) plus the amount invested in the project (c_2/R) cannot exceed the total resources. The second is the incentive compatibility constraint

$$u(c_1) \leq u(c_2) \tag{2.3}$$

which requires that patient consumers will, indeed, choose to leave their money in the bank. Ignoring the incentive compatibility constraint, we obtain $c_1 = 1$ and $c_2 = R$. Then,

$$u(c_1) = 0 < r = u(c_2)$$

so that the incentive compatibility constraint is satisfied strictly. Thus, the optimal deposit contract stipulates that any depositor can withdraw the whole of their 1 unit deposit at date 1. Because the investment is assumed to be available only to the bank, such a contract can only be implemented through the bank. Under such a contract, it is a weakly dominant action for every consumer at date 0 to deposit their wealth in the bank. At worst, they will get their money back at date 1, and possibly do better if the consumer turns out to be a patient type. Thus, at date 0, all consumers deposit their money in the bank.

2.2. The Coordination Game between Patient Consumers

Diamond and Dybvig (1983) observed that, unfortunately, the optimal contract gives rise to multiple equilibria at date 1. At date 1, the impatient consumers will clearly have a dominant strategy to withdraw. Given this behavior, the patient consumers are playing a coordination game. If a patient consumer withdraws, he gets a cash payoff of 1, giving utility of $0 = u(1)$. This payoff is independent of the number of patient consumers who withdraw. If a patient consumer does not withdraw, then the payoff depends on the proportion of patient consumers who withdraw. If proportion ℓ withdraw, his cash payoff to leaving money in the bank is $e^{r-\ell}$, which gives utility $r - \ell$. Thus, utility is linearly decreasing in the proportion of patient consumers who withdraw. If a patient consumer expects all other consumers *not* to withdraw (i.e., $\ell = 0$), then his utility from not withdrawing is $r > 0$. Thus there is an equilibrium where all patient consumers conform to the optimal deposit contract and leave their money in the bank. But if a patient consumer expects all other patient consumers to withdraw (i.e., $\ell = 1$), then his utility from not withdrawing is $r - 1 < 0$. Thus there is also an equilibrium where all patient consumers withdraw.

2.3. Uncertain Return and Unique Equilibrium

Postlewaite and Vives (1987) and Chari and Jagannathan (1988) both examine how bank runs become a unique equilibrium when asymmetric information is added to the model. We follow Goldstein and Pauzner (1999) in introducing a small amount of uncertainty concerning the log return r, holding fixed the deposit contract described above. It should be noted that as soon as we depart from the benchmark case, there is no guarantee that the existing deposit contract is optimal. Neither the portfolio choice of the bank, nor the amount that can be withdrawn at date 1 need be optimal in the new context. The objective here is to examine the equilibrium outcome and the welfare losses that result when the benchmark contract is imposed on an environment with noisy signals.

Suppose that r is a normal random variable, and that r has mean $\overline{r}$ and precision α (i.e., variance $\frac{1}{\alpha}$). We carry forward the assumption that the return is neither too small nor too large - we assume that $\bar{r}$ lies in the range:

$$0 < \bar{r} < 1$$

The depositors have access to very precise information about r before they make their withdrawal decisions, but the information is not perfect. Depositor i observes

the realization of the signal

$$x_i = r + \varepsilon_i \tag{2.4}$$

where ε_i is normally distributed with mean 0 and precision β, and independent across depositors.

With the introduction of uncertainty, we need to be explicit in what is meant by equilibrium in the bank run game. At date 1, depositor i not only observes his type, but also observes his signal x_i, and forms the updated belief concerning the return r, and the possible signals obtained by other depositors. Based on this information, depositor i decides whether to withdraw or not. A *strategy* for a depositor is a rule of action which prescribes an action for each realization of the signal. A profile of strategies (one for each depositor) is an equilibrium if, conditional on the information available to depositor i and given the strategies followed by other depositors, the action prescribed by i's strategy maxmizes his conditional expected utility. Treating each realization of i's signal as a possible "type" of this depositor, we are solving for the Bayes Nash equilibria of the imperfect information game. To economize on the statement of the results, we assume that if withdrawal yields the same expected utility as leaving money in the bank, then the depositor prefers to leave money in the bank. This assumption plays no substantial role in what follows.

Since both r and x are normally distributed, a depositor's updated belief of r upon observing signal x is

$$\rho = \frac{\alpha \bar{r} + \beta x}{\alpha + \beta} \tag{2.5}$$

In contrast to the benchmark case in which there is no uncertainty, the introduction of uncertainty eliminates multiplicity of equilibrium if private signals are sufficiently accurate. The result depends on the prior and posterior precision of r. Specifically, let

$$\gamma \equiv \frac{\alpha^2 (\alpha + \beta)}{\beta (\alpha + 2\beta)}, \tag{2.6}$$

and write $\Phi(.)$ for the standard normal distribution function. Our main result states that there is a unique equilibrium in this context, provided that γ is small enough.

Theorem. *Provided that $\gamma \leq 2\pi$, there is a unique equilibrium. In this equilibrium, every patient consumer withdraws if and only if $\rho < \rho^*$ where ρ^* is the unique solution to*

$$\rho^* = \Phi\left(\sqrt{\gamma}\,(\rho^* - \bar{r})\right).$$

In the limit as γ tends to zero, ρ^* tends to $\frac{1}{2}$.

Provided that the depositors' signals are precise enough (β is high relative to α), every depositor follows the switching strategy around the critical value ρ^*. This critical value is obtained as the intersection of a cumulative normal distribution function with the 45° line, as depicted in figure 2.1. In the limiting case when the noise becomes negligible, the curve flattens out and the critical value ρ^* tends to 0.5. The critical value ρ^* then divides the previously indeterminate region $[0, 1]$ around its mid point.

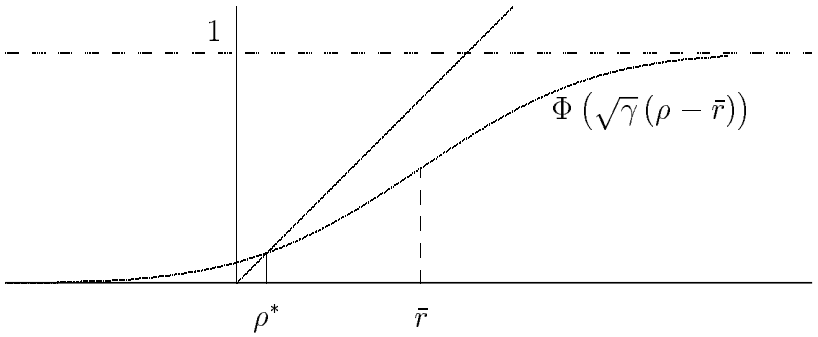

Figure 2.1: Switching point ρ^*

Let us sketch the argument behind this result. For ρ^* to be an equilibrium switching point, a depositor whose updated belief is exactly ρ^* ought to be indifferent between leaving his money deposited in the bank and withdrawing it. The utility of withdrawing is zero, and is non-random. The utility of leaving money in the bank is

$$r - \ell \tag{2.7}$$

which is random and depends on ℓ, the proportion of the patient depositors that withdraw. At the switching point ρ^*, the expectation of $r - \ell$ conditional on ρ^*

must therefore be zero. The expectation of r conditional on ρ^* is simply ρ^* itself. Thus, consider the expectation of ℓ conditional on ρ^*. Since noise is independent of the true return r, the expected proportion of patient depositors who withdraw is equal to the probability that any particular depositor withdraws. And since the hypothesis is that every depositor follows the switching strategy around ρ^*, the probability that any particular depositor withdraws is given by the probability that this depositor's updated belief falls below ρ^*.

When patient depositor i has posterior belief ρ_i, what is the probability that i attaches to some other depositor j having posterior belief lower than himself? Figure 2.2 illustrates the reasoning.

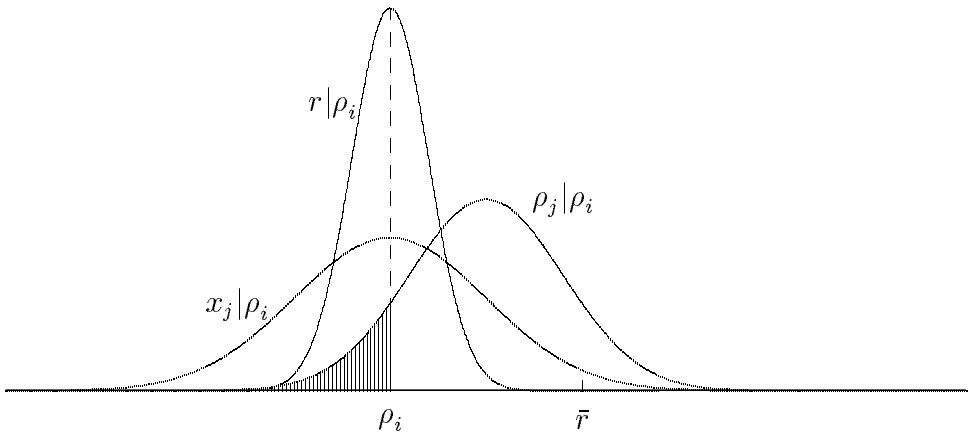

Figure 2.2: Beliefs conditional on ρ_i

Conditional on ρ_i, return r is normal with mean ρ_i and precision $\alpha + \beta$. Since $x_j = r + \varepsilon_j$, the distribution of x_j conditional on ρ_i is normal with mean ρ_i and precision:

$$\frac{1}{\frac{1}{\alpha+\beta} + \frac{1}{\beta}} = \frac{\beta(\alpha+\beta)}{\alpha+2\beta}. \tag{2.8}$$

But $\rho_j = (\alpha\bar{r} + \beta x_j)/(\alpha+\beta)$, so that the distribution of $\rho_j|\rho_i$ is as depicted in figure 2.2, and the probability that ρ_j is less than ρ_i conditional on ρ_i is given by the shaded area. Moreover,

$$\rho_j < \rho_i \Leftrightarrow \frac{\alpha\bar{r} + \beta x_j}{\alpha+\beta} < \rho_i \Leftrightarrow x_j < \rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r}) \tag{2.9}$$

so the question of whether ρ_j is smaller than ρ_i can be reduced to the question of whether x_j is smaller than $\rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r})$. Hence,

$$\begin{aligned}
\text{Prob}\left(\rho_j < \rho_i | \rho_i\right) &= \text{Prob}\left(x_j < \rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r}) \middle| \rho_i\right) \\
&= \Phi\left(\sqrt{\tfrac{\beta(\alpha+\beta)}{\alpha+\beta}}\left(\rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r}) - \rho_i\right)\right) \\
&= \Phi\left(\sqrt{\gamma}(\rho_i - \bar{r})\right). \qquad (2.10)
\end{aligned}$$

So, the shaded area in figure 2.2 can be represented in terms of the area under a normal density which is centered on the ex ante mean $\bar{r}$. Figure 2.3 illustrates.

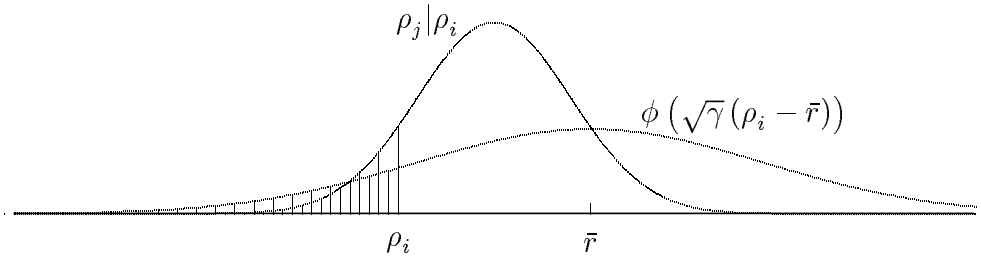

Figure 2.3: Density $\phi\left(\sqrt{\gamma}(\rho_i - \bar{r})\right)$

If ρ^* is an equilibrium switching point, the expectation of $r - \ell$ conditional on ρ^* must be zero. Since

$$\text{E}\left(r - \ell | \rho^*\right) = \rho^* - \Phi\left(\sqrt{\gamma}(\rho^* - \bar{r})\right), \qquad (2.11)$$

ρ^* must be the point at which $\Phi\left(\sqrt{\gamma}(\rho - \bar{r})\right)$ intersects the 45° line, exactly as depicted in figure 2.1. Provided that γ is small enough, the slope of $\Phi\left(\sqrt{\gamma}(\rho - \bar{r})\right)$ is less than one, so that there can be at most one point of intersection. Since the slope of the cumulative normal is given by the corresponding density function (which has the maximum value of $\sqrt{\gamma/2\pi}$), we can guarantee that there is a unique intersection point provided that γ is less than 2π. All that remains is to show that if there is a unique symmetric equilibrium in switching strategies, there can be no other equilibrium. Appendix A completes the argument.

2.4. Comparative Statics and Policy Analysis

The uniqueness of equilibrium makes it possible to perform secure comparative statics analysis. We will illustrate this with the simple exercise in our example, where an early withdrawal penalty t is imposed on consumers who withdraw in period 1.

In order to set a benchmark to measure our results against, consider the case with no uncertainty. The log return r is commonly known, and there is multiplicity of equilibria. The introduction of the early withdrawal penalty has little effect in this case. The only effect is to shift the range of returns where multiple equilibria exist from $[0, 1]$ to $[\log(1-t), \log(1-t)+1]$. Without a theory guiding us as to which outcome results in the game, it is hard to evaluate the welfare consequences of this policy. The most we can say is that when r is close to 1 (i.e., in the marginal interval $(\log(1-t)+1, 1)$), the tax will remove the multiplicity of equilibrium and the efficient outcome that consumers do not withdraw will occur for sure. When r is slightly less than 0 (i.e., in the marginal interval $(\log(1-t), 0)$, the tax will allow multiple equilibria.

In contrast to the lack of meaningful comparative statics when r is common knowledge, we can say much more when r is observed with noise. In particular, contrast the case with no uncertainty with the case in which noise is negligible (i.e. the limiting case where $\gamma \to 0$). The theorem tells us that patient consumers will withdraw if and only if $\rho < \log(1-t) + \frac{1}{2}$. This allows us to calculate the incidence of withdrawals at any realized value of r. Policy affects outcomes for interior values of the parameters, by shifting the boundary of the two populations, not merely at extremal parameter values.

We can also use this unique equilibrium to examine policy trade-offs. Recall that the efficient outcome at date 1 is for withdrawal by patient consumers to take place only if $r < 0$. If noise concerning r is very small, we achieve this outcome with very high probability by setting $t = 1 - e^{-\frac{1}{2}}$ (so that $\log(1-t) + \frac{1}{2} = 0$). But of course achieving efficiency in the withdrawal decision comes at the cost of reducing the value of the contract to consumers. The explicit form for the equilibrium allows us to calculate the ex ante expected utility of consumers. For any given t, it is $1/(1+\lambda)$ times

$$\left[\lambda + \Phi\left(\sqrt{\alpha}\left(\log\left(1-t\right) + \tfrac{1}{2} - \overline{r}\right)\right)\right]\log\left(1-t\right) + \int_{\log(1-t)+\frac{1}{2}}^{\infty} r\phi\left(\sqrt{\alpha}\left(r-\overline{r}\right)\right)\sqrt{\alpha}dr$$

while the revenue from the penalty is

$$\left[\lambda + \Phi\left(\sqrt{\alpha}\left(\log\left(1-t\right) + \frac{1}{2} - \overline{r}\right)\right)\right]t$$

An increase in the penalty can be welfare enhancing for consumers (even if they derive no benefit from the tax revenue). Goldstein and Pauzner (1999) examine contracts where early withdrawal penalties are received by consumers who leave their money until date 2. This further enhances the desirability of early withdrawal penalties from the consumers' point of view.

2.5. Observable Implications

We presented a highly simplified model of bank runs. Even in this highly simplified model, we can start thinking about observable implications of this theory. The main prediction is that despite the self-fulfilling aspect of the bank run, each depositor will withdraw his money exactly when his beliefs about the riskiness of bank deposits crosses some threshold implying that the size of equilibrium bank runs will be negatively correlated with returns. Consider the incidence of deposit withdrawals as given by the equilibrium value of ℓ. This incidence is a random variable that depends on the realized return r. A depositor withdraws whenever his posterior belief falls below the critical value ρ^*, which happens whenever

$$\frac{\alpha\overline{r} + \beta x_i}{\alpha + \beta} < \rho^*.$$

In other words, a depositor withdraws whenever the realization of his signal x_i falls below the critical value

$$x^*\left(\rho^*, \overline{r}\right) \equiv \frac{\alpha+\beta}{\alpha}\rho^* - \frac{\alpha}{\beta}\overline{r} \tag{2.12}$$

Since $x_i = r + \varepsilon_i$, the incidence of withdrawal is a function of the realized return r, and is given by

$$\ell(r) = \Phi\left(\sqrt{\beta}\left(x^*(\rho^*, \bar{r}) - r\right)\right). \tag{2.13}$$

Figure 2.4 illustrates.

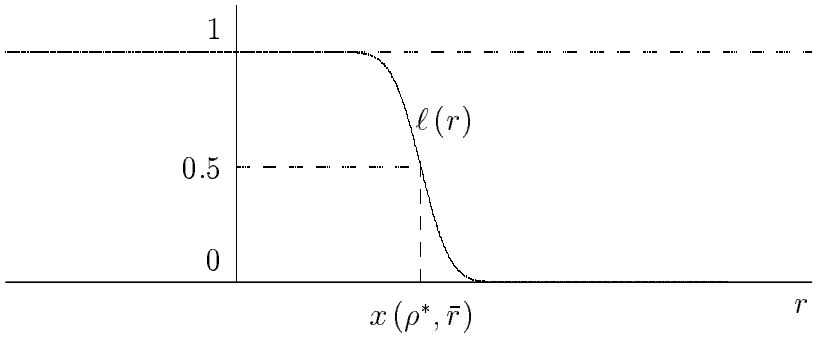

Figure 2.4: Proportion $\ell(r)$ of withdrawals

Clearly, the incidence of withdrawal is high when return is low. Fundamentals play a key explanatory role. Gorton (1988) studies bank panics in the US National Banking Era (1863-1914). He interprets the data in the light of the traditional dichotomy between fundamentals and sunspots as a cause of panics:

> "A common view of panics is that they are random events, perhaps self-confirming equilibria in settings with multiple equilibria, caused by shifts in the beliefs of agents which are unrelated to the real economy. An alternative view makes panics less mysterious. Agents cannot discriminate between the riskiness of various banks because they lack bank-specific information. Aggregate information may then be used to assess risk, in which case it can occur that all banks may be perceived to be riskier. Consumer then withdraw enough to cause a panic.... (This latter) hypothesis links panics to occurences of threshold value of some variable predicting the riskiness of bank deposits."

He concludes that the latter theory performs well. The highly simplified model of bank runs presented here suggests a re-interpretation of the evidence. The theory suggests that depositors will indeed withdraw their money when the perceived

riskiness of deposits crosses a threshold value. But nonetheless, the banking panic is self-fulfilling in the sense that individual investors only withdraw because they expect others to do so. The theory suggests both that banking panics are correlated with poor fundamentals and that inefficient self-fulfilling panics occur. Of course, it is possible to make assumptions about sunspots that mimic these predictions; but the theory presented here places tighter restrictions on outcomes than sunspot theory.

One would like to come up with distinctive implications that are harder to mimic with judiciously chosen sunspots. We will suggest one example in this bank deposit context. Suppose that we were about to observe both the prior mean of the log return $\overline{r}$ and the realized log return r; the prior mean $\overline{r}$ is a public signal that is observable by all depositors when they make their withdrawal decisions. Our theory predicts that for any given level of fundamentals r, the proportion of consumers running would be increasing in $\overline{r}$. This is apparent from our theorem, since a fall in the ex ante mean $\bar{r}$ shifts the curve $\Phi\left(\sqrt{\gamma}\left(\rho-\bar{r}\right)\right)$ to the left, so that its intersection with the 45° degree line is shifted to the right. Figure 2.5 illustrates this shift. Thus, when the fundamentals are commonly known to be weak (i.e. $\bar{r}$ is low), the equilibrium strategy dictates much more aggressive withdrawals, even controlling for one's posterior belief about r.

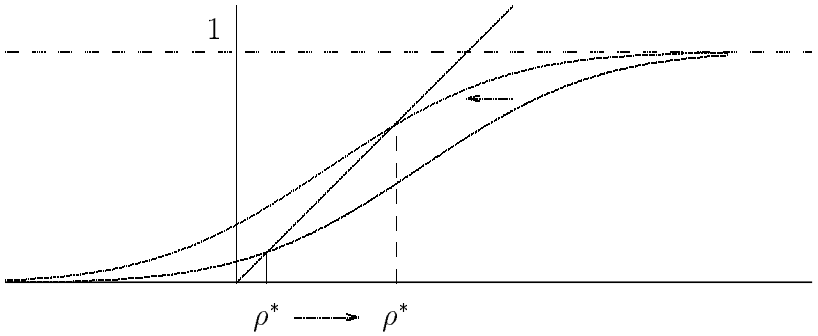

Figure 2.5: Shift in ρ^*

A prediction of the model, then, would be that if we could divide the fundamentals variables in Gorton's analysis into those that were most readily available to depositors contemporaneously, and those that were not, we should expect the most readily available variables to have the biggest impact. We will come across

another instance of the impact of public information below.

3. Complementarities and Macroeconomics

The example above was constructed around a simple coordination game played by a continuum of players. Much of the macroeconomics literature on complementarities, multiple equilibria and sunspots similarly reduces in the end to coordination games played by large populations. In this section, we illustrate how other issues can be addressed using similar methods.

Consider the following class of problems. A continuum of individuals must choose between a safe action and a risky action. If an individual chooses the safe action, his payoff is a constant. If he chooses the risky action, his payoff is an increasing function of the "state of fundemantals" r but a decreasing function of the proportion of the population who choose the safe action, ℓ. In the bank run example above, the payoff was linear in both r and ℓ. This linearity allowed us to give simple characterizations of the equilibrium. But as long as the payoff to the safe action is increasing in r and decreasing in ℓ, there will be a unique equilibrium of the type described above when information is sufficiently accurate. We will give an informal description of two applications that fit this general set-up that we have analyzed elsewhere.

3.1. Currency Crises

A continuum of speculators must decide whether to attack a fixed exchange rate. The cost to the monetary authority of defending the peg depends on the fundamentals of the economy and the proportion of speculators who attack the currency. If the monetary authority has some fixed benefit of maintaining the peg, then for each realization of fundamentals, there will be some critical mass of speculators sufficient to induce abandonment of the currency. If the peg is abandoned, the exchange rate will float to some level that depends on the fundamentals. A speculator may choose to attack by selling a fixed amount of the currency short. If he attacks, he must pay a transaction cost but receives the difference between the peg and the floating rate if the attack is successful and there is a devaluation.

This stylized model is in the spirit of the self-fulfilling attacks literature (see, for example, Obstfeld (1996)). If the state of fundamentals is common knowledge, there are three ranges of fundamentals to consider. If fundamentals are sufficiently low, devaluation is guaranteed. If fundamentals are sufficiently high, there will

be no devaluation. But for some intermediate range of fundamentals, there are multiple equilibria. Morris and Shin (1998) show how if there is a small amount of noise concerning fundamentals, there is a unique equilibrium.

Now consider a policy that makes it harder for an attack to be successful. For example, the monetary authority might accumulate reserves. A naive calculation of the value of those reserves might involve calculating the likelihood of contingencies in which those extra reserves would make the difference in the authority's ability to defend against an attack. This is analogous to seeing when a tax on early withdrawals would remove the existence of a withdrawal equilibrium in the bank run model. But taking into account the strategic analysis, we see that the true benefit of accumulating reserves is as a confidence building measure. If the accumulation of reserves is publicly observed, speculators will anticipate that other speculators will be less aggressive in attacking the currency. So in regions of fundamentals where a self-fulfilling attack is in fact feasible, it will not occur.

The theory also generates intuitive predictions about which events lead to currency attacks. Deteriorating fundamentals, even if observed by most participants, will have less impact if the fact that fundamentals are deteriorating is not common knowledge. Very public signals that fundamentals have deteriorated only a small amount may have a large impact. This is because a speculator observing a bad signal not only anticipates that the monetary authority will have a harder time defending against an attack, but also anticipates that other speculators will be attacking. This explanation is quite commonplace. But the theoretical model that we have described captures this argument exactly.

3.2. Pricing Debt

Our methods may also help us to understand some of the anomalies noted in the empirical literature on the pricing of defaultable debt. One influential approach has been to note that a lender's payoff is analogous to the payoff that arises from holding a short position in a put option on the borrower's assets. Hence, option pricing techniques can be employed to price debt, as shown in the classic paper by Merton (1974). Nevertheless, the empirical success of this approach has been mixed, with the usual discrepancy appearing in the form of the overpricing (by the theory) of the debt, and especially of the lower quality, riskier debt. The anomaly would be explained if it can be shown that the default trigger for asset values actually *shifts* as the underlying asset changes in value, and shift in such a way that disadvantages lower quality debt. The incidence of inefficient liquidation

seen in our bank run example suggests that similar inefficiencies might arise in the coordination problem between creditors facing a distressed borrower. This would give us a theory of *solvent* but *illiquid* borrowers, enabling us to address the empirical anomalies. This is attempted in Morris and Shin (1999).

When the fundamentals are bad, coordination to keep a solvent borrower afloat is more difficult to achieve, and the probability of inefficient liquidation is large. This is another manifestation of the importance of public information in achieving coordination alluded to in the previous section. The disproportionate impact of public information can be illustrated in the following example of a borrower in distress.

Consider a group of lenders who are funding a project. Time is discrete, and advances by increments of $\Delta > 0$. The fundamentals of the project at date t are captured by random variable r_t. Conditional on its current realization, the next realization of r_t is i.i.d., normally distributed around its current realization, with variance Δ. In other words, $\{r_t\}$ is a sequence of snapshots of a simple Brownian motion at time intervals of Δ. To economize on notation, we denote by r the current value of the fundamentals, and by r_+ its value in the next period. At each date, every lender chooses whether or not to continue funding the project. The project fails if and only if

$$\ell > r,$$

where ℓ is the proportion of creditors who pull out of the project. Hence, when $r > 1$ the project is viable irrespective of the actions of the creditors. If $r < 0$, the project fails irrespective of the actions of the creditors. However, when r lies between 0 and 1, the fate of the project depends on how severe the creditor run is. At each date, a lender receives a payment of 1 if the project has survived. When the project fails, a lender receives zero. By pulling out, a lender receives an intermediate payoff λ, where $0 < \lambda < 1$. We also suppose that a creditor who withdraws when the project is still viable rejoins the project in the next period (having missed a single payment of 1). This assumption ensures that the creditors face a sequence of one-shot games.

None of the creditors observe the current fundamentals perfectly. Each has signal:

$$x_i = r + \varepsilon_i,$$

where ε_i and ε_j are independent for $i \neq j$, ε_i is normal with mean 0 and variance Δ^2. The noise in the signal x is thus small as compared to the underlying

uncertainty. The lenders, however, observe the previous realization of r perfectly. This will serve as the public information on which much of the analysis will hinge. As the time interval Δ becomes small, the noise disappears at a faster rate than the overall uncertainty governing r. Each lender chooses an action based on the realized signal x and the (commonly known) previous realization of r.

This game has a unique equilbrium (the proof is sketched in appendix B) in which there is a critical value of fundamentals r_+^* for which the project fails next period whenever $r_+ < r_+^*$. We call r_+^* the *collapse point* for the project. It is given by the (unique) solution to

$$r_+^* = \Phi\left(r_+^* - r + \Phi^{-1}(\lambda)\sqrt{1+\Delta}\right) \tag{3.1}$$

The collapse point is obtained as the intersection between the 45 degree line and the distribution function for a normal with unit variance centred on $r - \Phi^{-1}(\lambda)\sqrt{1+\Delta}$. The following points are worthy of note.

- r_+^* is a function of the current realization r. Hence, public information plays a crucial role in determining the trigger point for collapse.
- The continuous time limit as $\Delta \to 0$ is well-defined.
- r_+^* is a *decreasing* function of r. So, when fundamentals deteriorate, the probability of collapse increases both because the fundamentals are worse, but also because the trigger point has moved unfavourably.

This last feature is possibly quite significant. For an asset whose fundamentals are bad (i.e. r is low), the probability of default is higher than would be the case in the absence of coordination problems among creditors. Such a pattern would explain why one would misprice such an asset in a model that assumes a fixed default point. The mispricing takes the form of *overpricing* the riskier bonds - exactly the empirical anomaly discussed in the literature.

There is a more general lesson. The onset of financial crises can be very rapid, and many commentators note how the severity of a crisis is disproportionate to the deteriorating fundamentals. In our account, such apparently disproportionate reactions arise as an essential feature of the model. When fundamentals deteriorate, coordination is less easy to achieve. We can explore this effect further by examining the comparative statics of the probability of collapse. The probability

of collapse next period conditional on the current fundamentals r is

$$\Phi\left(\frac{r_+^* - r}{\sqrt{\Delta}}\right)$$

As r falls, the probability of collapse increases at the rate:

$$\frac{\phi}{\sqrt{\Delta}\,(1-\phi)}$$

where ϕ is the standard normal density at $\left(r_+^* - r\right)/\sqrt{\Delta}$. The increase in the probability of collapse can be quite large when r hovers close to the collapse point, and the onset of failure can thus be quite rapid. As compared to the naive model which does not take into account the dependence of the collapse point on the current fundamentals, this is larger by a factor of $1/(1-\phi)$. When r is close to the collapse point r_+^*, this is roughly $\sqrt{2\pi}/\left(\sqrt{2\pi}-1\right) \approx 1.66$.

The inverse relationship between the current value of fundamentals and the collapse point is suggestive of the precipitous falls in the prices of defaultable securities during financial crises.

The continuous time limit of the model makes possible further simplifications in the analysis. Taking the limit as $\Delta \to 0$, the fundamentals r evolve as a simple Brownian motion, and the collapse point r_+^* for the next period converges to the collapse point in the current period. So, (3.1) can be written

$$r^* = \Phi\left(r^* - r + \Phi^{-1}(\lambda)\right)$$

Collapse occurs when r hits r^*, i.e. at $r^* = \lambda$.

3.3. How Special is the Analysis?

In this paper, we described stylized examples with normally distributed states and signals, binary choices by a symmetric continuum of players and payoffs linear in the state and proportion of players choosing each action. These assumptions allowed us to give simple characterizations of the unique equilibrium. However, the analysis is arguably quite general. If one is only interested in the limiting case where noise in signals is very small, the exact shape of the noise or prior beliefs over the state do not matter. Asymmetries among the players can also be incorporated. Corsetti, Morris and Shin (1999) examine the role of a large trader in currency markets in an asymmetric game. The qualitative features of

the analysis are very similar between continuum and finite player cases. Indeed, in the special case of the payoffs in the bank runs model, where only the proportion of other players choosing each action matters, the analysis is literally unchanged. That is, if we had a finite number of depositors, with proportion $\frac{\lambda}{1+\lambda}$ impatient and proportion $\frac{1}{1+\lambda}$ impatient, the unique equilibrium would have impatient consumers using the same cutoff point for withdrawals. Dealing with many actions is more delicate (see Frankel, Morris and Pauzner (2000)) although the analysis extends straightforwardly in some instances. Carlsson and Ganslandt (1998) describe what happens when noise is added to Bryant's (1983) model of technological complementarities.

4. Concluding Remarks

We draw two conclusions from our analysis. The first is that applied theorists should be wary of selecting an arbitrary outcome for further attention when conducting comparative statics exercises and in drawing policy implications. The mere fact that an outcome is Pareto superior to another is no good reason for it to be selected, and we should expect to see some inefficiencies as a rule. The notion of a "solvent but illiquid bank" can be given a rigorous treatment, and we hope that our discussion can contribute to policy debates in the area.

Our second conclusion is a methodological one. Contrary to the impression given by multiple equilibrium models of the apparent autonomy of beliefs to float freely over the fundamentals, we believe that such autonomy of beliefs is largely illusory when information is modelled in a more realistic way. No doubt, some researchers may find this regrettable since one degree of freedom is lost in the exercise of providing a narrative of unfolding events. However, there are compensations for this loss, and we hope that these benefits will be recognized by researchers. One promising line of inquiry is to explore the correlations between the underlying fundamentals and the degree of optimism of the economic agents. Empirical investigations will then have a much firmer basis.

Appendix A

When there is a unique symmetric equilibrium in switching strategies, there can be no other equilibrium. An argument is sketched here. Denote by $u(\rho, \hat{\rho})$ the expected utility from leaving one's money in the bank conditional on posterior ρ when all other patient depositors follow a switching strategy around $\hat{\rho}$. Conditional on ρ, the expected proportion of depositors who withdraw is given by the probability that any particular depositor receives a signal lower than the critical value $\hat{\rho}$. From the argument in the text, this probability is given by

$$\Phi\left(\sqrt{\tfrac{\beta(\alpha+\beta)}{\alpha+2\beta}}\left(\widehat{\rho}+\frac{\alpha}{\beta}\left(\widehat{\rho}-\overline{r}\right)-\rho\right)\right)=\Phi\left(\sqrt{\gamma}\left(\widehat{\rho}-\bar{r}+\frac{\beta}{\alpha}\left(\widehat{\rho}-\rho\right)\right)\right). \tag{4.1}$$

Hence, $u(\rho, \widehat{\rho})$ is given by

$$u\left(\rho,\widehat{\rho}\right)=\rho-\Phi\left(\sqrt{\gamma}\left(\widehat{\rho}-\bar{r}+\frac{\beta}{\alpha}\left(\widehat{\rho}-\rho\right)\right)\right). \tag{4.2}$$

If r is negative, the utility to withdrawing is higher than that from leaving money in the bank *irrespective* of what the other depositors decide. So, if the posterior belief ρ is sufficiently unfavourable, withdrawing is a dominant action. Let $\underline{\rho}_1$ be the threshold value of the belief for which withdrawal is the dominant action. Any belief $\rho < \underline{\rho}_1$ will then dictate that a depositor withdraws. Both depositors realize this, and rule out strategies of the other depositor which leaves money in the bank for signals lower than $\underline{\rho}_1$. But then, leaving money in the bank cannot be optimal if one's signal is lower than $\underline{\rho}_2$, where $\underline{\rho}_2$ solves

$$u\left(\underline{\rho}_2,\underline{\rho}_1\right)=0 \tag{4.3}$$

This is so, since the switching strategy around $\underline{\rho}_2$ is the best reply to the switching strategy around $\underline{\rho}_1$, and even the most optimistic depositor believes that the incidence of withdrawals is higher than that implied by the switching strategy around $\underline{\rho}_1$. Since the payoff to withdrawing is increasing in the incidence of withdrawal by the other depositors, any strategy that leaves money in the bank for signals lower than $\underline{\rho}_2$ is dominated. Thus, after *two* rounds of deletion of dominated strategies, any strategy that leaves money in the bank for signals lower than $\underline{\rho}_2$ is eliminated. Proceeding in this way, one generates the increasing sequence:

$$\underline{\rho}_1<\underline{\rho}_2<\cdots<\underline{\rho}_k<\cdots \tag{4.4}$$

where any strategy that leaves money in the bank for signal $\rho < \underline{\rho}_k$ does not survive k rounds of deletion of dominated strategies. The sequence is increasing since $u(\cdot,\cdot)$ is increasing in its first argument, and decreasing in its second. The smallest solution $\underline{\rho}$ to the equation $u(\rho,\rho) = 0$ is the least upper bound of this sequence, and hence its limit. Any strategy that leaves money in the bank for signal lower than $\underline{\rho}$ does not survive iterated dominance.

Conversely, if $\overline{\rho}$ is the largest solution to $u(\rho,\rho) = 0$, there is an exactly analogous argument from "above", which demonstrates that a strategy that withdraws for signals larger than ρ does not survive iterated dominance. But if there is a *unique* solution to $u(\rho,\rho) = 0$, then the smallest solution just *is* the largest solution. There is precisely one strategy remaining after eliminating all iteratively dominated strategies. Needless to say, this also implies that this strategy is the only *equilibrium* strategy.

Appendix B

The posterior belief of the current value of r is normal with mean

$$\rho = \frac{x_i + \Delta r_-}{1 + \Delta}$$

and precision $(1 + \Delta)/\Delta^2$, where r_- denotes the previous realization of r. Denote by $U(\rho)$ the payoff to continuing with the project conditional on ρ, when all creditors are following the ρ-switching strategy. It is given by

$$U(\rho) = \Phi\left(\frac{\sqrt{1+\Delta}\,(r^* - \rho)}{\Delta}\right) \tag{4.5}$$

where r^* is the trigger value of fundamentals at which the project collapses. r^* satisfies $r^* = \ell$. But if other speculators follow the ρ-switching strategy, ℓ is the proportion of creditors whose signal is lower than the marginal value of x that implies the switching posterior ρ. This gives

$$r^* = \Phi\left(\rho - r_- + \frac{\rho - r^*}{\Delta}\right) \tag{4.6}$$

From these two equations, we can show by implicit differentiation that $U'(\rho) > 0$. There is a unique solution to $U(\rho) = \lambda$, and equilibrium is unique for the same reasons as cited for the main theorem. To solve explicitly for the collapse point r^*, we solve the pair of equations given by (4.6) and $U(\rho) = \lambda$. This gives

$$r^* = \Phi\left(r^* - r_- + \Phi^{-1}(\lambda)\sqrt{1+\Delta}\right),$$

as required.

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