COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

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COWLES FOUNDATION DISCUSSION PAPER NO. 358

SOME PROBLEMS OF CENTRALIZED ECONOMY

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April 5, 1973

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by

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During the past several years, at the Computer Center of the USSR Academy of Sciences, they have been studying program oriented methods of planning in the context of a socialist economy. This research is of a pragmatic nature. We are seeking to find rational organizational forms as well as procedures for decision taking, and also planning algorithms, that will make it possible to implement methods of program planning on a comprehensive scale.

These studies were devoted to a rather wide range of problems and this paper deals with those of them which were the subject of the authors' research.

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*The research described in this paper was supported under a grant from the Ford Foundation. This paper is a report delivered by the authors to a Cowles Foundation Seminar on April 9, 1973.

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1. The Historical Origins of Program Planning in the Soviet Union

The planning and implementation of individual programs has a long history in our country. When the Civil War ended in 1920 its government confronted a problem for which there was no precedent, namely, how does one guide the entire economic organism of a country of the dimensions of the Soviet Union? The development of a country pursues simultaneously a variety of objectives and is influenced by political, social and economic doctrines. It is in these conditions of multiple-objective development that the program-oriented method of planning was born.

The first program in our country was the program of electrification. Subsequently we came to know many other programs. Most recently a program for the technical re-equipment of agriculture has been adopted. Programs of this kind served as a base for building national economy plans. The efforts of researchers, however, are currently concerned with extending the methods of program planning into an integrated instrument of comprehensive planning and management that will encompass not only specifically economic programs, but also social programs, programs for the protection of the natural environment, for the attaining of pre-determined levels of the standard of living, and for all other measures that require expenditures of the country's resources. Such an approach requires considerable development of the traditional methods of planning and specification of accepted terms. The method of programs requires, in particular, a more concrete definition of the expression "optimal plan."

As we shall see below, the program method is not the alternative of the optimization theories, but includes them as an auxiliary apparatus.

2. Two Postulates Concerning a Comprehensive Extension of Program Planning

In this connection we would like to dwell on several aspects of the research activities in this area that are being carried out at our Institute, concerning both the theory and the technical basis of program planning.

Our research is based on the conception that the economic organism is a developing, controllable dynamic system that confronts continuously changing problems in the course of time.

The process of development of the country takes place under continuously changing conditions (political environment, technical progress, pollution of the environment, pecularities of the demography processes, etc.). Hence the society's goals are also changing with time.

From this postulate, it follows directly that neither the theory of marginal utility nor the conception of a general equilibrium of the economy can serve as a point of departure for the elaboration of the required procedures. Yet this assertion should not be interpreted to mean that we reject the formal framework of optimal programming altogether, for it has proved to be a reliable means for solving specific problems. The procedures of program planning contain various optimization problems.

Our research is also based on a second postulate, namely that the role of markets in the distribution of products will continuously decline. It seems to me that such a postulate is self-evident for socialist countries, since even from the philosophical point of view the notion of combining a socialist form of ownership with capitalist methods of distribution is an eclectic one.

Just now in socialist countries market relationships are practically limited only by sphere of personal consumption and services. But it also appears to be true that a reduction of the role of markets in the distribution of industrial products is taking place in the non-socialist countires of our planet. Long term agreements and planning begin to play an increasing role in these countries.

3. Four Stages in Program Planning

Speaking very generally, the information that is required by program planning is produced in four stages, namely: 1) forecasting; 2) the construction of programs; 3) proper planning; and 4) the implementation of programs. This last step presupposes its integration with the procedures of short term planning.

The division of the program planning system on 4 stages is connected with the fact that all these stages are methodologically different: they require the use of models of the different nature and different degree of detailization. Besides that the implementation of every stage requires the competence of various institutions.

The forecast is done mainly by scientific institutions and its goal is to create the necessary informational base for the next stage.

The construction of the program requires the activity of state and political decision-makers. The scientists and the specialists can only play the role of consultant here.

The proper planning in the USSR is made through special state institutions united in the state planning commission (GOSPLAN). The implementation of the program is made (or has to be made) by the use of economic mechanisms allowing the high degree of the decision making decentralization. The problem of projecting and improving these mechanisms is a special one.

Let us consider briefly each of these in turn.

4. Forecasting

This term is employed to refer to a system of procedures that makes it possible to represent possible alternative paths in the development of the economy, of international situations, of the pollution of the natural environment, of technological progress, etc. Similar forecasts may be carried out at the level of individual regions.

Since many of the processes occurring in society are steerable or controllable in some measure, it is the purpose of forecasts to establish feasible boundaries of change for those parameters that may in fact be altered by policy measures.

Such forecasts rely both on aggregated models of the economy and on a variety of procedures based on the processing of expert estimates. Unlike in models of planning the task of forecasting is concerned with identifying a certain set of trajectories for the economic system rather than with finding a single trajectory. In the terminology of mathematical control theory it may be said that the task of such a forecast is to construct an attainability set.

In one of the following sections we will give an example of the model which was worked out by our colleagues from the Computing Center (Dr. A. A. Petrov and Dr. Jn. P. Ivanilov) for solving forecast and long term planning problems.

Here we limit ourselves by only one note. The realization of any solution in social, political and economic spheres requires a certain amount of resources. For example, the improvement of the quality of life in big cities depends on quantity of clean water consumed by the population, i.e. on those investments which are necessary for the development of water supply network and refinement system. Hence the achievement of any goals at the state scale requires a certain level of economic development of the country.

This requirement leads to the system of economic goals (criteria). Let us denote them by $J_1, J_2, \ldots, J_n$. For example, J_1 is the level of consumption per capita, J_2 is the energy production, etc.

Let us denote the minimum feasible magnitudes of criteria J_i by J_i^*. Their values are given by the experts.

We can then solve a number of optimization problems for every criteria:

$$\max J_j \qquad j = 1, 2, \ldots, N$$

$$J_i \geq J_i^* \qquad i \neq j .$$

In the space of criteria the solutions of these problems determine some rectangular domain (see Figure 1). This domain is the attainability domain. One can call it by domain of realistic forecasts or by domain of possible alternatives.

By changing J_i^* we will obtain a more exact picture of this domain of possible alternatives (see Figure 2).

The construction of this domain is the most important problem of the forecast theory. However the solution of these problems is only a part of

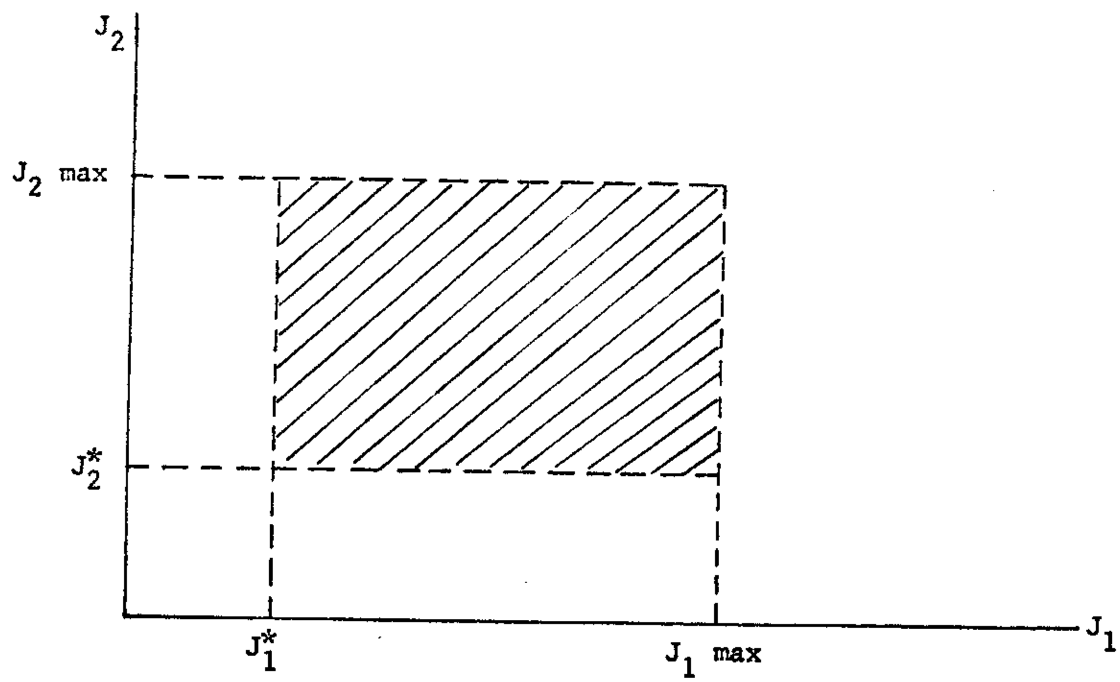

FIGURE 1

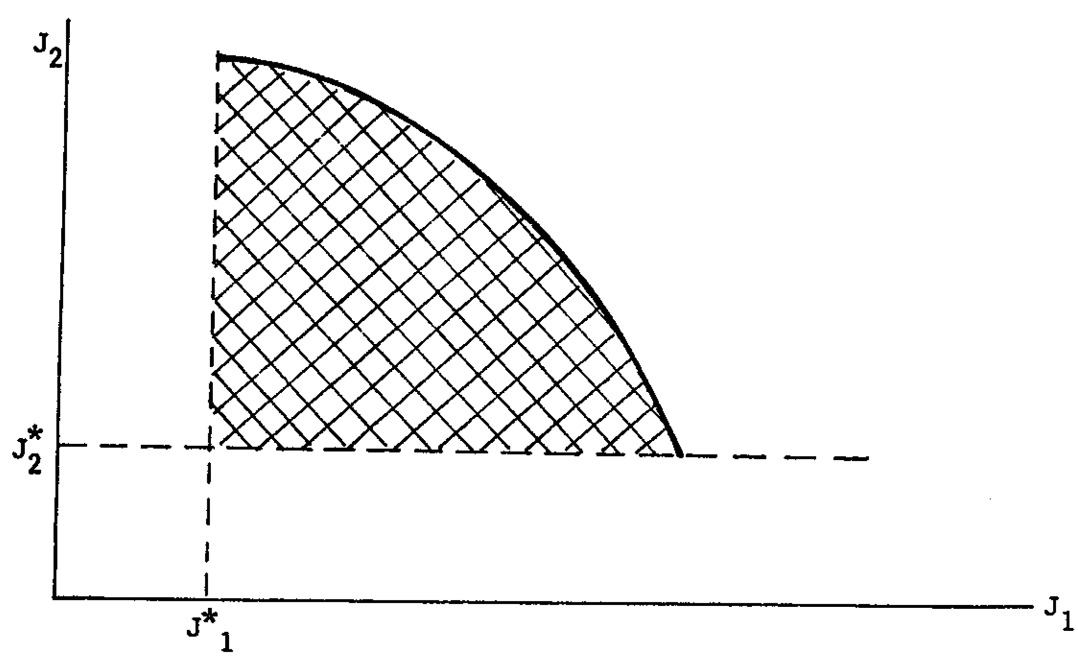

FIGURE 2

the forecast problem. Under real conditions there exist many more constraints of nonformal character which we also must take into account. There are, for example, distribution of specialists of different qualification in the country, traditions, national pecularities and so on.

At this stage an important role is played by the compression of the number of available alternatives. We have borrowed this conception from the American mathematician, Wald. In this connection it may be demonstrated that Wald's scheme of sequential analysis in fact encompasses dynamic programming as well as branch-and bound methods, and many other methods for the analysis of alternatives. In problems of forecasting this method is modified in such a way that it may be extended to the processing of non-formal expert estimates. The use of branch-and-bound methods allows us to develop new techniques of expert data processing.

Thus the procedure of forecasting could be interpreted as the union of non-formal expertizes using various methods of expert estimate processing and the solution of the numerous optimization problems for aggregate models of economics.

Real execution of this procedure requires rather developed software. Some examples of that will be given below.

The completion of the forecast work is the description of feasible alternatives domain, nonformal constraints being taken into account. This domain could be called "minimum set of feasible alternatives."

The forecasting stage of program planning is carried out by persons who do not have the right to make decisions. These are mathematicians, economists, lawyers, and other specialists. Their responsibility is to prepare information in a way that is appropriate for the subsequent design of individual programs. Thus, the activities of the forecasting service produce a certain "minimal set" within which each state of the system is shown to be attainable.

5. The Design of the Programs

The minimum set of feasible alternatives consists usually of more than one point, so the process of specifying and designing a program is not a formal one. It is ultimately determined by social norms and doctrines in relation to whose realization the programs represent intermediate steps. The specification and designing of programs must be carried out by a special branch of the government. In several socialist countries there already exist governmental bodies that are organs for the making of long-term planning decisions.

The effective design of programs requires a differentiated and complex software. First of all, we must have a system of procedures which allows us to check whether the alternative under investigation belongs to the minimum set.

For the solution of this problem, the same system of models and programs which was made (or has to be made) for forecast goals can be used.

After we are convinced that the alternative under investigation could, in principle, be realized, we proceed to the study of it in more detail. This requires the use of dynamic models of the multi-sectoral type. Within these models normative values are employed to represent delays in construction, in the initiation of production, and in the assimilation of new production processes. Their instrumental variables include the initiation of the construction of new capacities (or else their conversion from other forms);

the utilization of reserve capacities; the level of utilization of the labor force; and the coefficient of utilization of existing capacities.

An interesting optimization problem arises in testing for the feasibility of programs. We refer to it as the problem of the "indicated duration" of implementation. In it the minimal time is determined during which programs can be fully implemented. Should the resulting "indicated duration" turn out to be exceedingly long then a reexamination of the programs and a reduction in their scope is undertaken. In addition, trial distributions of the budget are carried out in relation to alternative variants of program priorities.

We have already discussed that the programs are formulated only in terms of the end product. However, there is one exclusion: this is the energy.

Of course, a part of energy plays a role as an end product because it is used for the satisfaction of citizens' needs. But the most part of the energy is an intermediate product.

Nevertheless, the construction of the modern energetical complexes requires that amount of time which exceeds the usual planning time interval. Hence the energitical complex construction of which was started at the beginning of the planning period will not give production by its end. That is why the construction of the energy base requires choosing it as a special program, as for example, a program of regional development, a program of refinement of environment, etc.

Programs of regional development occupy a special position, since our research organization is currently participating in the elaboration of two such programs, namely, the development of Moldavia, and the complex development of north-west Siberia.

While these are very different problems, it is necessary in both cases to render consistent extremely contradictory requirements. In the case of Siberia it is necessary to carry out the exploration, exploitation, refining and transportation of petroleum and natural gas in such a way that the ecological balance of the Siberian rivers will not be disturbed. At the same time it is necessary to meet a series of social problems that result from the extremely unfavorable environment of the far north, as well as to identify appropriate technical methods for the exploitation of forest resources of this region.

Similarly in the case of Moldavia, where the basic problem (namely the shortage of water) is very difficult, not only is a principle for its economical distribution required, but it is also necessary to avoid its pollution.

The creation of the efficient analysis and regional development program projecting is independent and difficult problem. It is not purely an economical one. That is why the analysis of the development project (for example, for Moldavia) requires the mutual analysis of the economic models system, different sectors, the demography model, the model of the water usage (rivers and subsurface sources), etc.

Thus the final design of programs is not a formal act. It is the result of an entire series of iterations and compromises. Beyond this, even after it is accepted, a program is not regarded as an absolute law, for the program planning approach is by its very principle adaptive. Alterations in the external setting, that is of political, social, technological, and other conditions, will require an alteration of the programs. How frequently one should carry out such adaptations of programs is a separate matter.

6. Planning

The final stage, namely proper planning, is carried out by the planning bodies of the government. Its central task may be described as the superposition of a plan on capacities.

During the forecasting process and during the design of programs, highly aggregated models were employed. Planning, however, belongs to the next hierarchical level in the procedure associated with the making of decisions. Its output results in the identification of production tasks for individual groups of production units, and in some cases for individual plants.

A number of difficult questions are encountered at this level that are still not resolved and that may not be reduced to simply mathematical ones.

One of them concerns the level of detail associated with planning. Another concerns the stability of plans. Just as in the case of programs a plan must change in the course of time. Without this no adaptation to changing external conditions is possible. Nevertheless, there must be a certain minimal time period during which the plan must be stable, and during which it has the status of legal obligation. Without this it would not be possible to conceive a functioning of the country's economic organism.

In economic science the term "optimal plan" is widely used. However, we can speak about the optimal plan only in the case when objective function is known. The problem of choosing the objective function (utility function) for planning purposes on a country scale is the independent and difficult problem. The program method gives us the opportunity to find reasonable technique for the objective function construction.

Thus let us assume that the program has already existed and planning horizon is determined. That means that general "planning indicators" are determined. Let us denote them by $\hat{y}_i(T)$. These could be the summary (integral) quantities of the end product and outputs for the separate years.

Now the problem of planning is: how to allocate resources, how to determine "planning indicators" to industries and separate firms in order to fulfill state planning indicators in some sense best way.

The problem would be relatively simple if we could comeasure these indicators. In this case we would proceed to additive objective function of the following type:

$$J = \Sigma c_i y_i(T) .$$

However, to obtain this is difficult (if possible in general). That is why at the Computer Center of the USSR Academy of Sciences we have been elaborating on another conception. We consider the following form as an objective function:

$$J = \min_i \frac{y_i(T)}{\hat{y}_i(T)} . \qquad (!)$$

In this case the problem of optimal planning is reduced to such an allocation of control actions which supplies

$$\max \min_i \frac{y_i(T)}{\hat{y}_i(T)} . \qquad (!!)$$

The realization of the numerical scheme for the functional of type (!!) is sufficiently complex. The general idea of methods developed is connected with the fact that economics does not allow the sharp changes and plan for the previous planning period could always be accepted as a first approximation. This assumption permits us to develop the efficient methods in the theory of disturbances.

Today we have the standard optimization program ready for the π-model with criteria (!!) in the case of 28 industries. In order to evaluate the complexity of the problem it is sufficient to say that for 10 years period each iteration is equivalent to the solution of linear programming problem with 2,500 variables.

If we use standard models obtaining of the solution of this problem may seem to be a hopeless task. However, the use of the specific features of the π-model and pecularities of inner connections between industries allowed to develop the special method. It has the independent interest and in the near future will be published in the Journal of Computing Mathematics and Mathematical Physics.

7. The Description of the π-Model

One of the main models in the system of models which is under investigation at the Computer Center of the USSR Academy of Sciences is the π-model proposed by A. A. Petrov and Jn. P. Ivanilow in 1965.

We have already mentioned it several times. Now we will briefly describe one of its variants.

The base of any model is balance equation (conservation law). In the π-model this balance equation has the form of a Leontief equation

$$z_t = x_t - Ax_t - w \qquad \forall t$$

where x = gross product

z = part of end product which is used for investment

w = consumption

A = technology matrix (matrix of direct input).

Let us denote capacity of the i^{th} industry by $\xi_i(t)$. By this term we call that quantity of the product which could be produced during the t^{th} year if during this period the introduction of new capital does not take place.

Comment. Knowing the technology of the production and the volume of the capital in the i^{th} industry it is possible to calculate out its capacity. On the contrary, knowing the capacity of the industry and its technology we can determine the necessary capital.

Let us denote by $\zeta_i(t)$ the increase of the capacity for the t^{th} year. Thus at the beginning of $(t+1)^{th}$ year (at the end of the t^{th} year) the i^{th} industry will have a capacity

$$\xi_i(t) = \xi_i(t-1) + \zeta_i(t) \; . \qquad (!)$$

The equation (!) is the main dynamic equation of the model.

Let us denote by $\theta_i(t)$ that part of capacity in the i^{th} industry the creation of which began in the t^{th} year. By n_i we denote the magnitude of production lag. This is some industry characteristic. Now we have

$$\zeta_i(t) = \theta_i(t - n_i) \; .$$

However the plant or the industrial complex begins to output the production before the final completion. The introduction of its capacities into the work is taking place gradually beginning with some year after beginning of construction. Let us introduce the function $\alpha_i(t)$ which shows what part of the industrial complex's capacity will output the production in t years after the beginning of its construction. The typical function of this kind is shown in Figure 3.

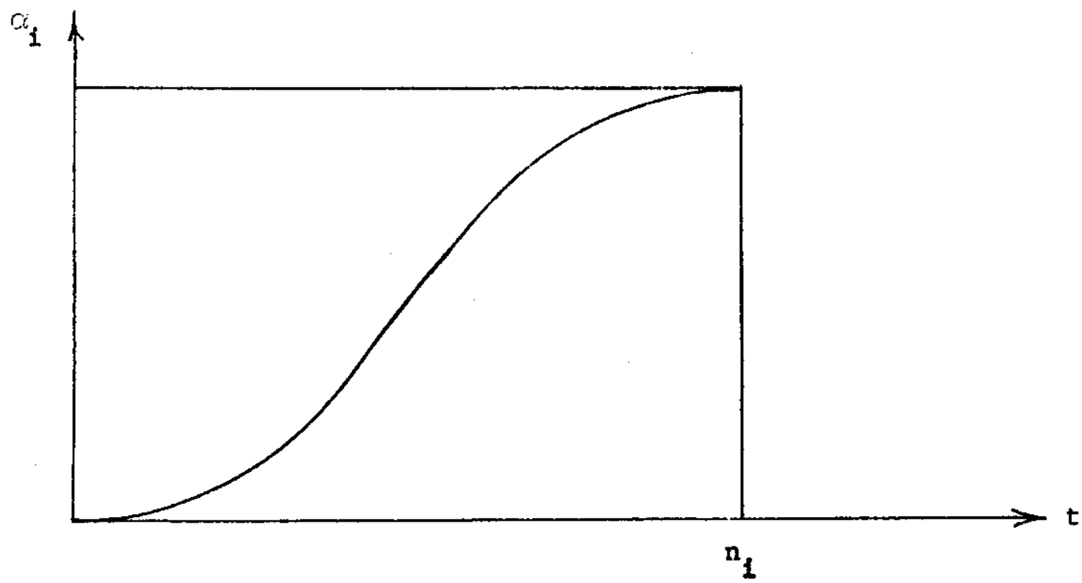

FIGURE 3

Thus the limit possibilities of the production in the i^{th} industry in the t^{th} year are

$$\bar{\xi}_i(t) = \xi_i(t) + \bar{\zeta}_i(t)$$

where $\bar{\xi}_i(t)$ is the increase of efficient capacity

$$\bar{\zeta}_i(t) = \sum_{s=0}^{n_i-1} \alpha_i(s)\theta_i(t-s) \ .$$

Hence $\bar{\zeta}_i(t)$ determines that amount of the product which the i^{th} industry can produce in those plants whose construction is not completely finished.

Using relations introduced we can write out the natural constraints

$$0 \leq x_i(t) \leq \bar{\xi}_i(t) \qquad \forall t . \tag{1}$$

A vector $z(t)$ we will represent as a sum of two terms

$$z(t) = \hat{z}(t) + \Delta q . \tag{2}$$

Here $\hat{z}(t)$ is that part of free product which goes on investment during the t^{th} year. $q(t)$ is a level of stock

$$\Delta q(t) = q(t) - q(t-1) . \qquad (!!)$$

The components of the vector Δq can have any sign but

$$q(t) \geq 0 \qquad \forall t . \tag{3}$$

The equation (!!) is a second dynamic equation of the π-model.

The following constraint joins the gross product vector $x(t)$ and the models of demography growth and education. In the simplest case this constraint has the form

$$(c, x(t)) \leq L(t) \qquad \forall t \tag{4}$$

where c_i is the average cost of labor per unit of production x_i and $L(t)$ is given the amount of the labor force.

The last constraint of the model is the constraint on the consumption

$$w(t) \geq w^-(t) \qquad \forall t \tag{5}$$

where $w^-(t)$ is some minimum feasible consumption.

In order to make the model functional, it is necessary to describe the relationship between the product which is used for the building of new capacities and the capacities to be introduced.

For the description of this relationship the authors of the model introduce an important <u>hypothesis of normative capital investment</u>: a vector $\hat{z}(t)$ is uniquely determined by the volume of capacities which are under construction

$$\hat{z}_i(t) = \sum_{j=1}^{n} \sum_{s=0}^{n_i-1} K^i_{js}\theta_j(t-s)$$

where K^i_{js} = matrix of normatives (some fixed numbers).

The sense of this hypothesis is completely evident: if the construction of the plant is started it means that it has to be continuously financed. The character of these capital investments is strictly determined.

The π-model contains the whole series of free functions (control variables) which we could choose:

a) the volume of the gross product $x(t)$;

b) the volume of consumption $w(t)$;

c) the fraction of free product which goes on the creation of stock Δq ;

d) the volume of the capacities $\theta_i(t)$ construction of which will be started in the t^{th} year.

The π-model was and is used for the solution of various problems. Let us count some of the problems which were solved by its use.

I. <u>The problem about directive time interval</u>. The problem is: how to find the minimum time T which is sufficient for the achievement of a given volume of the end product:

$$\int_0^T w_i(t)dt = \delta_i$$

where δ_i are the given numbers, constraint (5) takes place.

II. The problem of planning: To find the magnitude of the control functions which maximize the functional

$$J = \min_i \frac{\int_0^T w_i(t)dt}{\delta_i} .$$

We have already discussed the significance of these problems for planning and forecasting.

Besides these problems other important problems were studied for the construction of the problem. One of them is the problem of conversion. Let us suppose that the proportion of capacities for the end of the planning period T is given

$$t = T , \quad \xi_1(T) : \xi_2(T) : \dots : \xi_n(T) = \lambda_1 : \lambda_2 : \dots : \lambda_n$$

or

$$\xi_i = \mu_i \xi_1 , \quad \mu_i = \frac{\lambda_1}{\lambda_i} . \qquad (6)$$

Then we could formulate the problem on the achievement of the maximum production under given proportions. This problem is equivalent to maximization of ξ_1 subject to the condition (6).

One of the problems of this kind is the achievement of the given proportions at the maximum rate of growth.

8. Centralized and Decentralized Aspects of Implementation

The observations in this paper are concerned with the problems of a centralized economic system. We use this term to refer to an organism whose component elements are able to concentrate their efforts for the attainment of common objectives and tasks. A centralization of the steering of an economic system, however, does not presuppose a full centralization of all decisions. It is not difficult to see that no matter how advanced the level of the techniques for data processing and data transmission may be, a certain level of decentralization in management will always be necessary. Indeed the volume of information that is necessary to steer an economic organism is so large that even if one should possess a technological capacity to transmit and process all necessary data it will still not be possible to prepare the information that is necessary for the steering of an economic organism without significant delays. This would cause fully centralized decisions to be taken on the basis of obsolete or inaccurate information. The results of such decisions will then be characterized by a high level of uncertainty. The only way to overcome this difficulty is to carry out a parallel processing of information at lower levels of a formally specified data processing hierarchy.

In assigning data processing functions to individual lower levels of the hierarchy, however, a part of the rights to make decisions is also inevitably transferred to them as well. Thus individual elements of the system are transformed into partly autonomous organisms. But once they are transformed into autonomous organisims, they also acquire their own objectives, which in the general case are not identical with those of the system as a whole. Thus the decisions that are taken by individual elements will not be the best ones from the point of view of the central objectives, and a theory of decentralized management and of hierarchies must take this fact into account.

From the point of view of overall management such a transfer of the right to make decisions to lower levels has, therefore, two aspects. On the one hand, it improves the quality of data processing, and this improves the quality of overall management. On the other hand the appearance of new objectives leads to a deterioration of the quality of overall management. It follows that there exists a certain optimal measure of centralization and decentralization that depends on the technical level of the processing and transmission of data.

Today there already exists a substantially comprehensive and rapidly developing theory of hierarchical systems that includes evaluations of the quality of hierarchical structures.*

One of the central problems of this theory is the definition of the objectives of the individual levels of a hierarchy. Here the most important task is to determine juridical norms in such a way that it is to the advantage of individual organisms entering into the system to strive for the particular objectives that are placed before them in accordance with the program method. This is above all a conceptual problem. In our opinion it is not possible to solve it without turning to the conceptions of R. Ashby. Once it is accepted

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*In the following paragraph we give an example of planning in a system with a hierarchical structure.

as a postulate, however, the possibility appears of formalizing it in terms of a comprehensive mathematical theory. This entire complex of problems may be called the design of economic mechanisms. It is in such a context that the problems of stimulation and sanctions for the non-fulfillment of the norms of a plan emerges, as well as their relation to market mechanisms, and many other similar problems.

It follows that the implementation of program planning is thus achieved with the help of economic mechanisms rather than directly through detailed centralized instructions even though they represent elements of a central plan. The organs of central planning may thus introduce plans only at a sufficiently aggregated level, while lower level activities are defined by the context of a hierarchical production structure. The associated system of economic mechanisms must be such, however, that it will also evaluate the activity of individual sub-organisms in terms of their ultimate results, that is, in terms of the extent to which they correspond with plan indicators. At the present time the problem of designing such economic mechanisms occupies a central position in studies concerned with control processes in the country.

9. The Dynamic Two-Level Model

In order to give some idea about the direction of research which goes on in connection with the problem of economic mechanism, we will consider the simplest model of industrial union of firms.

We would assume that this union consists of the Center and Production-Makers.

Each Production-Maker produces a product $p_i(t)$ $(i = 1, 2, \ldots, N)$.

$p_i(t)$ is the quantity of the product which the i^{th} Production-Maker outputs during one year. $p_i(t)$ is some vector. Output p_i is determined by production function of the plant

$$p_i = f_i(x_i, L_i, \omega_i)$$

where x_i is a volume of capital

L_i is a labor force.

The difference of these production functions from more conventional ones is that we take into account here a dependence of the production volume of wage ω_i. We will not discuss here in detail any concrete form of this dependence which a subject of complex sociology research. We indicate only that $p_i(\omega_i)$ is a concave function which, for small ω_i takes the form shown in Figure 4.

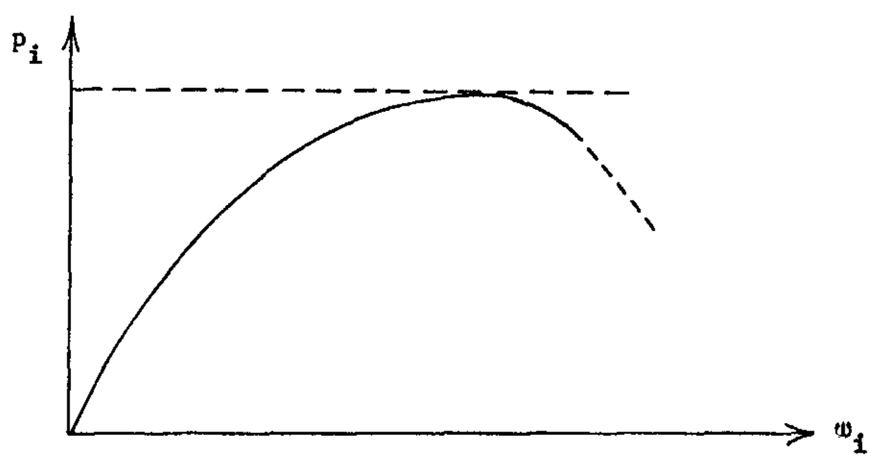

FIGURE 4

<u>Note</u>: There is some information to think that for large ω_i the curve $p_i(\omega_i)$ will follow the dotted line. However, this is a discussion question and we will not consider it here.

The capital's change is described by the equation

$$\dot{x}_i = -k_i x_i + u_i v_i$$

where k_i is an amortization coefficient

$u_i(t)$ is a flow of Production-Maker's investment

$v_i(t)$ is a flow of Center investment (exogenous resource).

The result of the production activity of the plant will be described by the summary value of the whole output (c_i, p_i). Naturally we will call this value by gross income. It must be spent on

1. payments concerning production cost $g_i(p_i)$
2. wage bill $\omega_i L_i$
3. endogenous investment u_i
4. payments to the Center $F_i(p_i)$.

Let us introduce

$$z(t) = (c_i, p_i) - g_i(p_i) - F_i(p_i) - \omega_i L_i - u_i \quad .$$

Naturally we call z by nonproduction capital of Production-Maker.

Let us describe now functioning of Production-Maker. Suppose Center corresponds to Production-Maker the volume of v_i and the rule of payment to Center--a function $F(p_i)$. Besides we will assume that Production-Maker knows prices c_i and production costs.

The result of his production activity will depend on how he chooses his control variables which are $L_i(t)$, $\omega_i(t)$ and $u_i(t)$.

Note. Under real conditions of our country the volume of wage of the plant is limited

$$\omega_i L_i \leq Q_i(t) \qquad i = 1, 2, \ldots, N \ . \tag{3}$$

Thus Production-Maker has to make decision subject to (3). How will Production-Maker choose his control variables? It is a difficult problem and the following answer is of discussion nature. We will assume that Production-Maker tends to maximize nonproduction capital. Under conditions of our country this capital could be spent on material incentives and social needs (house construction, clubs, etc.).

Some preliminary sociology studies which were done in our institution show that this assumption is not far from the truth. Nevertheless, we will call this assumption by <u>hypothesis of behavior</u> Γ_i .

According to this hypothesis Γ_i , the i^{th} Production-Maker chooses his control variables as follows:

$$\max_{L_i, \omega_i, u_i} \int_0^T z_i(t)dt \ . \tag{4}$$

Let us describe now Center's action. Center can choose u_i , Q_i and the form of the function $F_i(p_i)$ which we call incentive function (if $F < 0$) or penalty function (if $F > 0$). These values are not of course arbitrary: they must satisfy various constraints.

Let us denote by V that resource which Center receives from some other organization (for example, government subsidies). Then

$$V + \Sigma\, F_i(p_i) - \Sigma\, u_i \geq 0 \ . \tag{5}$$

The volume of wages is also limited

$$\Sigma\, Q_i \leq Q(t) \tag{6}$$

where $Q(t)$ is a summary volume of wages of given Center (industrial union, industry). The magnitude $Q(t)$ is determined by financial institutions of the country while taking into account that summary volume of wage should be equal to the summary volume of consumption. We will not consider this procedure.

Thus (and this is close to reality) the management board of the industrial union (Center) can only redistribute the volume of wage between different plants.

Center is control organization. It does not produce any production output and its activity efficiency is determined by the results of the plant's actions, i.e., $p_1, p_2, \ldots, p_N$. That means that its objective function depends only on these values. Let us assume that

$$J = \int_0^T \Phi(p_1, p_2, \ldots, p_n)dt \, . \tag{7}$$

Because, unlike Production-Makers, Center is a bureaucratic organization its objective function depends on proportions planning outputs of products with real ones.

Hence the model of planning in this case is reduced to some differential game. From one side there are Production-Makers who tend to maximize a functional of type (4). From the other side of Production-Makers, there is Center maximizing functional (7). However, this game is not symmetrical one. Center has the right of the first move. It corresponds to Production-Maker's incentive function $F_i(p_i)$, resource u_i and the volume of wage. Thus Center has the right to assume that Production-Makers will know these magnitudes and will choose their control variables in accordance to hypothesis of behavior Γ_i.

Hence, the magnitude of p_i will be determined by only the control action of Center, i.e.

$$p_i = R_i[F_i(p_i), u_i, Q_i] \qquad (8)$$

R_i is a functional of the complex nature.

Now Center has only to choose the control variables $F_i(p)$, u_i , and Q_i in order to maximize the functional (7).

Thus we have shown that planning in hierarchy system under consideration is reduced to the solution of the sequence of the optimization problems.

The problems which are raised here are sufficiently complex. However during two past years a certain success is achieved in the development of necessary numerical methods, as well as the qualitative analysis.

Note. The problem under consideration is a new subject of research in theory of control. So far in this theory only two types of control were studied: program control when control actions are functions of time and control of the synthesis type when control is a function of the phase variables. In a given case both types of control are represented. There are exogenous resource $u_i(t)$ and the incentive function $F_i(p_i)$. Besides that a new type of control appears in this case. This is $Q_i(t)$, i.e. control by use of the change of constraints; in other words, this is a possibility of the limitation imposing on the activity of the subsystem. Thus we proceed here to a new class of the problems of the mathematical control theory.

As it was said in the previous section, the use of the hierarchy structure is approved in that case when different levels have different information. For example, Production-Maker knows exactly the structure of his production function while Center has no sufficient information on this subject.

10. The Use of Simulation Systems as a Framework for Research

Because the problems that arise in the formalization of program planning and management, and especially in the design of economic mechanisms are extremely complex, it is not possible to reduce them to ordinary mathematical problems. Their analysis must rely instead on the utilization of man-machine simulation systems. We introduced this term in the middle of the 1960's to refer to a certain integrated system of models, experts, and data banks, together with procedures for their functioning and for their relationship to machines and also to an integrating master program that makes it possible for participants to address the system of models and of information data banks and to consult auxiliary algorithms. For example in the simulation system relating to the development of north western Siberia that is at the present being developed in our Institute an important role is played by programs that produce solutions for problems in such areas as underground hydrodynamics and the transportation of petroleum.