

# **Coordination Risk and the Price of Debt**

**By**

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# Coordination Risk and the Price of Debt\*

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## Abstract

Creditors of a distressed borrower face a coordination problem. Even if the fundamentals are sound, fear of premature foreclosure by others may lead to pre-emptive action, undermining the project. Recognition of this problem lies behind corporate bankruptcy provisions across the world, and it has been identified as a culprit in international financial crises, but has received scant attention from the literature on debt pricing. Without common knowledge of fundamentals, the incidence of failure is uniquely determined provided that private information is precise enough. This affords a way to price the coordination failure. Comparative statics on the unique equilibrium provides several insights on the role of information and the incidence of inefficient liquidation.

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## 1. Introduction

Our premise in this paper is that creditors face a coordination problem when facing a borrower in distress, and that this will be reflected in the price of debt. The problem faced by creditors is akin to that faced by depositors of a bank which is vulnerable to a run. Even if the project is viable, so that the value at maturity is enough to pay all the creditors in full, a creditor may be tempted to foreclose on the loan or seize any assets it can, fearing similar actions by other creditors. Such fears would be self-fulfilling, since the disorderly liquidation of assets and the consequent disruption to the project is more likely to lead to failure of the project.

It is hard to overstate the importance of coordination failures. The recognition of this problem - known as the “common pool problem” among lawyers - lies at the heart of corporate bankruptcy provisions across the world, taking on its most elaborate form in the chapter 11 provisions of the U.S. bankruptcy code (Baird and Jackson (1990), Jackson (1986)). Also, coordination failure among creditors has figured prominently in the accounts of emerging market financial crises in recent years, mostly notably the Asian crisis of 1997 (Radelet and Sachs (1998), Fischer (1999)). The effects on commitment arising from coordination failure have also been recognized as an important determinant of financial contracts (Bolton and Scharfstein (1996)).

Given the importance of this problem, it seems incongruous that it has received such scant attention from the literature on asset pricing. The main difficulty in incorporating coordination failure in a pricing theory for debt is that coordination problems lead to multiple equilibria, in the manner of Diamond and Dybvig (1983). Without quantifiable information on the incidence of coordination failure, it is impossible to incorporate this into the ex ante price of the debt. In this respect, our aim in this paper can be achieved only if we can provide a theory which explains the incidence of cases where a solvent borrower is forced into failure. In other words, we must first have a theory of *solvent* but *illiquid* borrowers, akin to the theory of solvent but illiquid banks<sup>1</sup>. Extending the methods of Carlsson and van Damme (1993) on ‘global games’, and our earlier work on currency attacks (Morris and Shin (1998)), we develop such a theory here.

Our study is motivated by three main objectives. The first is theoretical. The global games method shows how a departure from common knowledge of the fun-

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<sup>1</sup>Goldstein and Pauzner (2000) and Rochet and Vives (2000) use similar methods to that used in our paper to develop a theory of solvent but illiquid banks.

damentals may generate a unique equilibrium even in those games that typically have multiple equilibria under common knowledge. However, in economically relevant settings with informative public signals, uniqueness of equilibrium rests on detailed features of the environment, as noted in Morris and Shin (1999), which extends the earlier currency attack model to a dynamic environment with normally distributed fundamentals. Equilibrium is unique provided that private information is precise enough, relative to the public information. Our first objective is to provide a set of conditions that are necessary and sufficient for uniqueness of equilibrium.

Policy analysis and other comparative statics exercises are made possible when equilibrium is unique, and our second objective in this paper is to address a number of important policy debates concerning the role of information in crisis episodes. “Transparency” has become a touchstone of the policy response following recent financial crises. The notion of transparency is multi-faceted and touches on a wide range of issues such as accountability, legitimacy, and the efficacy of the legal infrastructure in enforcing contracts. However, one narrow interpretation of transparency focuses on the provision of more accurate and timely information to market participants. Our framework allows us to subject the arguments to more rigorous scrutiny, and reveal some of the subtleties in the interaction between uncertainty concerning the fundamentals and uncertainty over the actions of others. Any shift in the informativeness of signals or underlying distribution of fundamentals affect both types of uncertainty, and the resulting effect on the equilibrium rests on the complex interplay between these two types of uncertainty. We outline the main factors at work. Heinemann and Illing (1999), Hellwig (2000), Metz (2001) and Tarashev (2001) have provided further insights into this question and address policy issues on the provision of information. Also, in a notable empirical investigation for the 1997-8 period, Prati and Sbracia (2001) show how enhanced disclosure of public information have an ambiguous effect on the speculative pressure against a currency. The strength of fundamentals determine whether disclosures are beneficial or detrimental. These conclusions are consistent with our theoretical predictions.

Our third objective is to contribute to the empirical debate on the pricing of defaultable securities. A classic reference in the theory of the pricing of defaultable debt is Merton (1974), who models company asset value as a geometric Brownian motion, and assumes that bankruptcy occurs when asset value reaches some given fixed level relative to liabilities. Then, the price of debt can be obtained from

option pricing techniques<sup>2</sup>. However, the empirical success of this approach has been mixed. One early study is Jones, Mason and Rosenfeld (1984), which uses data from 1975 to 1981 and finds that the actual observed prices of corporate bonds are below those predicted by the theory, and that the prediction error is larger for lower rated bonds. For investment grade bonds, the error is around 0.5%, while for non-investment grade bonds, the error is much larger, at around 10%. Subsequent work has suggested that over-pricing is resilient to various refinements of the theory, and alternatives have been proposed<sup>3</sup>. By means of comparative statics analysis on the default point, we can understand how the default trigger levels for asset values actually *shift* as the underlying asset changes in value. It thus provides a possible theoretical framework that accommodates the empirical anomalies of the Merton model. Bruche (2001) develops a formal continuous time extension of our model embedding the creditor coordination problem, and compares the empirical performance of his model against the alternatives.

More generally, studies of financial restructuring of firms under distress suggest that instances of disorderly liquidation and deviations from priority of debtors play a significant role (see, for instance, Franks and Torous (1994)). Notably, Brunner and Krahn (2000) document evidence from the loan books of the major German banks that the pivotal factor which determines the success of the reorganization of a distressed firm is the formation of a “creditor pool” that coordinates the interests of the creditors (see also Hubert and Schäfer (2000)). The role of creditor coordination in sovereign debt crises is equally important, if not more so. We believe that further refinements of the framework in our paper may contribute to a better understanding of the empirical literature.

The plan of the paper is as follows. We present the model in the next section, and solve for the equilibrium in section 3. A discussion of the conditions for uniqueness follows in section 4, after which we go to the main part of our paper on the comparative statics questions concerning the role of information and the empirical implications for the pricing of debt.

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<sup>2</sup>More refined treatments of this approach include Leland (1994) - recognizing debt level as a decision by the firm - and Longstaff and Schwartz (1995) - which allows interest rate risk. Anderson and Sundaresan (1996) adjusts for varying bargaining power between the firm and creditors.

<sup>3</sup>An alternative approach is to assume that default is an exogenous event which follows some hazard rate process. Then, the default risk is reflected in a higher discount rate. Duffie and Singleton (1999) develop this approach.

## 2. The Model

A group of creditors are financing a project. Each creditor is small in that an individual creditor's stake is negligible as a proportion of the whole. At the end of its term, the project yields a liquidation value  $v$ , which is uncertain at the time of investment. The financing is undertaken via a standard debt contract. The face value of the repayment is  $L$ , and each creditor receives this full amount if the realized value of  $v$  is large enough to cover repayment of debt.

At an interim stage, before the final realization of  $v$ , the creditors have an opportunity to review their investment. The loan is secured on collateral, whose liquidation value is  $K^* < L$  if it is liquidated at the interim stage, but has the lower value  $K_*$  if it is liquidated following the project's failure, so that

$$K_* < K^* < L$$

At the interim stage, each creditor has a choice of either rolling over the loan until the project's maturity, or seizing the collateral and selling it for  $K^*$ . The value of the project at maturity depends on two factors - the underlying state  $\theta$ , and the degree of disruption caused to the project by the early liquidation by creditors. Denoting by  $\ell$  the proportion of creditors who foreclose on the loan at the interim stage, the realized value of the project is given by

$$v(\theta, \ell) = \begin{cases} V & \text{if } z\ell \leq \theta \\ K_* & \text{if } z\ell > \theta \end{cases} \quad (2.1)$$

where  $V$  is a constant greater than  $L$ , and  $z > 0$  is a parameter that indicates the severity of disruption caused by the inability to coordinate.

We can give the following interpretation to the payoff function  $v$ . The underlying fundamental  $\theta$  is a measure of the ability of the firm to meet short term claims from creditors, while  $z$  is the mass of the creditor group. Thus, when proportion  $\ell$  of the creditors foreclose, the total incidence of foreclosure is  $z\ell$ . The firm remains in operation provided that  $\theta$  is large enough to meet this foreclosure. Otherwise, the firm is pushed into default. The simple form of our payoff function implies that the recovery rate conditional on default does not depend on  $\theta$ . A richer model aimed at empirical investigations would need to relax this feature of our framework.

By normalizing the payoffs so that  $L = 1$  and  $K_* = 0$ , the payoffs to a creditor

are given by the following matrix, where  $\lambda \equiv (K^* - K_*) / (L - K_*)$ .

	Project succeeds	Project fails
Roll over loan	1	0
Foreclose on loan	$\lambda$	$\lambda$

The bold lines in figure 2.1 depict the payoff to a creditor arising from the loan when proportion  $\ell$  foreclose on the loan.

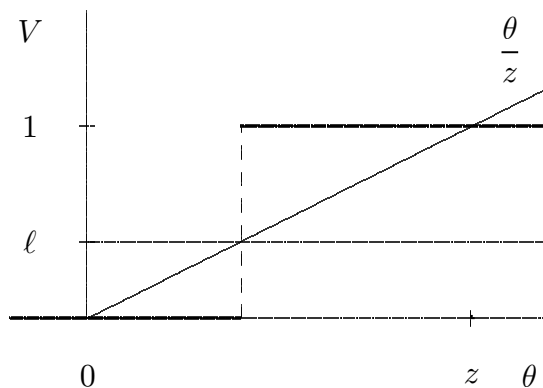


Figure 2.1: Payoff to Roll-over

To avoid notational clutter, we assume that if rolling over the loan yields the same expected payoff as foreclosing on the loan, then a creditor prefers to foreclose. This assumption plays no substantial role.

If the creditors know the value of  $\theta$  perfectly before deciding on whether to roll over the loan, their optimal strategy can be analysed thus. If  $\theta > z$ , then it is optimal to continue with the project, irrespective of the actions of the other creditors. This is so, since even if every other creditor forecloses, the project survives. Conversely, if  $\theta < 0$ , then it is optimal to foreclose on the loan irrespective of the actions of the others. Even if all other creditors roll over their loans, the project fails.

When  $\theta$  lies in the interval  $(0, z)$ , there is a coordination problem among the creditors. If all other creditors roll over their loans, then the payoff to rolling over the loan is 1, so that rolling over the loan yields more than the premature liquidation value  $\lambda$ . However, if everyone else recalls the loan, the payoff is  $0 < \lambda$ ,

so that early liquidation is optimal. This type of coordination problem among creditors is analogous to the bank run problem (Diamond and Dybvig (1983)), and leads to multiple equilibria in the simple perfect information game in which creditors choose their actions when  $\theta$  is common knowledge<sup>4</sup>.

The value of  $\theta$  is normally distributed with mean  $y$ , and precision  $\alpha$  (that is, with variance  $1/\alpha$ ). At the interim stage, when each creditor decides on whether to roll over the loan, each creditor receives information concerning  $\theta$ , but this information is imperfect. Creditor  $i$  observes the realization of the noisy signal

$$x_i = \theta + \varepsilon_i \tag{2.2}$$

where  $\varepsilon_i$  is normally distributed with mean 0 and precision  $\beta$ . For  $i \neq j$ ,  $\varepsilon_i$  and  $\varepsilon_j$  are independent. A *strategy* for creditor  $i$  is a decision rule which maps each realization of  $x_i$  to an action (i.e. to roll over the loan, or to foreclose). An *equilibrium* is a profile of strategies - one for each creditor - such that a creditor's strategy maximizes his expected payoff conditional on the information available, when all other creditors are following the strategies in the profile. We now solve for equilibrium strategies of this game.

### 3. Equilibrium

Our task of solving for equilibrium will be divided into two parts. We first solve for equilibria in which the creditors use a simple “switching strategy” in which they roll over whenever their estimate of the underlying fundamentals is higher than some given threshold level, and foreclose if their estimate of the fundamentals falls below this threshold. In the next section, we will identify conditions for the uniqueness of equilibrium. It turns out that there is no loss of generality by confining our attention to switching strategies.

When creditor  $i$  observes the realization of the signal  $x_i$ , his posterior distribution of  $\theta$  is normal with mean

$$\xi_i \equiv \frac{\alpha y + \beta x_i}{\alpha + \beta} \tag{3.1}$$

and precision  $\alpha + \beta$ . When creditors use a switching strategy, they have a threshold level  $\xi$  (the ‘switching point’) for their switching strategies, and roll over the loan

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<sup>4</sup>We do not have much to add to the debate on whether a secondary market will mitigate inefficiencies, except to note that any attempt to internalize the externalities are confronted by coordination/free-rider problems at a higher level. See Gertner and Scharfstein (1991).



if and only if the private signal  $x$  is greater than

$$x(\xi, y) \equiv \frac{\alpha + \beta}{\beta} \xi - \frac{\alpha}{\beta} y. \quad (3.2)$$

The critical value of the fundamentals at which the project is on the margin between failing and succeeding is at the state  $\theta$  for which  $\theta = z\ell$ , where  $\ell$  is the proportion of creditors who foreclose resulting from the switching strategy around  $\xi$ . We denote by  $\psi$  the critical state  $\theta$  at which the project is on the margin of success and failure. Since  $\psi = z\ell$ , we have

$$\begin{aligned} \psi &= z\Phi\left(\sqrt{\beta}(x - \psi)\right) \\ &= z\Phi\left(\sqrt{\beta}\left(\frac{\alpha + \beta}{\beta}\xi - \frac{\alpha}{\beta}y - \psi\right)\right) \\ &= z\Phi\left(\frac{\alpha}{\sqrt{\beta}}(\xi - y) + \sqrt{\beta}(\xi - \psi)\right) \end{aligned} \quad (3.3)$$

This gives us our first equation in terms of  $\xi$  and  $\psi$ .

For our second equation, we appeal to the fact that at the switching point  $\xi$ , a creditor is indifferent between rolling over and foreclosing. The payoff to foreclosure is  $\lambda$ , while the expected payoff to rolling over is the probability that the project succeeds. Since the project succeeds whenever  $\theta > \psi$ , and since the conditional density over  $\theta$  is normal with mean  $\xi$  and precision  $\alpha + \beta$ , this indifference condition is given by

$$1 - \Phi\left(\sqrt{\alpha + \beta}(\psi - \xi)\right) = \lambda \quad (3.4)$$

which implies

$$\psi - \xi = \frac{\Phi^{-1}(1 - \lambda)}{\sqrt{\alpha + \beta}}. \quad (3.5)$$

This gives us our second equation. From this pair of equations, we can solve for our two unknowns,  $\psi$  and  $\xi$ . Solving for  $\psi$ , we have

$$\psi = z\Phi\left(\frac{\alpha}{\sqrt{\beta}}\left(\psi - y + \Phi^{-1}(\lambda)\frac{\sqrt{\alpha + \beta}}{\alpha}\right)\right). \quad (3.6)$$

The failure point  $\psi$  is obtained as the intersection between the 45° line and a scaled-up cumulative normal distribution whose mean is  $y - \Phi^{-1}(\lambda)\frac{\sqrt{\alpha + \beta}}{\alpha}$ , and whose precision is  $\alpha^2/\beta$ . The interval  $[0, \psi]$  represents the incidence of inefficient

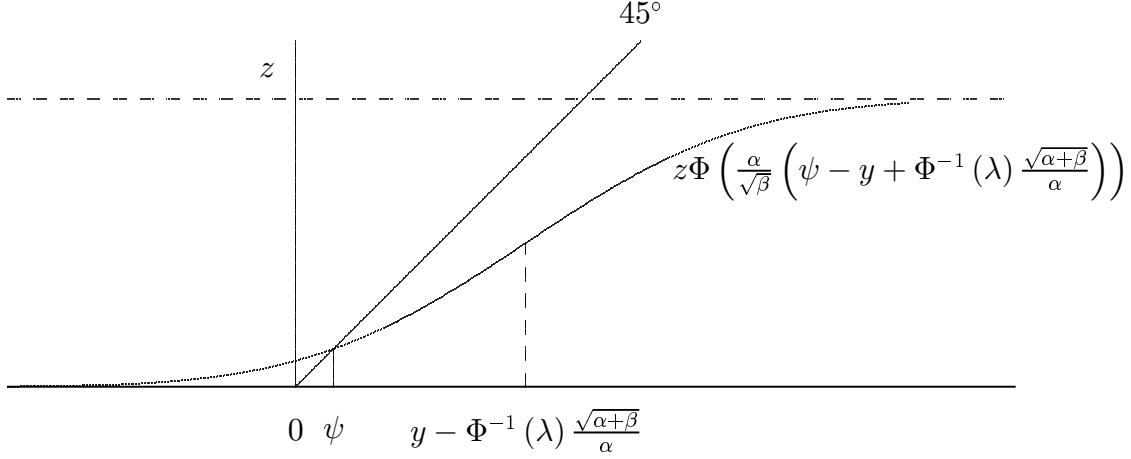


Figure 3.1: Default point  $\psi$

liquidation. These are the states at which liquidation is inefficient, but liquidation is forced on the borrower. In this sense, the interval  $[0, \psi]$  represents those states where the borrower is solvent, but illiquid. When  $z$  is large, the destruction of value can be very substantial.

Equation (3.6) has a unique solution if the expression on the right hand side has a slope that is less than one everywhere. The slope of the right hand side is given by  $z\phi \cdot \frac{\alpha}{\sqrt{\beta}}$  where  $\phi$  is the density of the standard normal evaluated at the appropriate point. Since  $\phi \leq 1/\sqrt{2\pi}$ , a sufficient condition for a unique solution for  $\psi$  is given by

$$\frac{\alpha}{\sqrt{\beta}} \leq \frac{\sqrt{2\pi}}{z} \quad (3.7)$$

Since  $\alpha$  is the precision of the ex ante distribution of  $\theta$ , while  $\beta$  is the precision of the creditors' signals, condition (3.7) is satisfied whenever the private signals are precise enough relative to the underlying uncertainty. We postpone an interpretation of this condition until section 5, when we will be able to explain the significance of information precision in terms of their effects on two types of uncertainty - concerning fundamentals, and concerning actions of others. First, we will show why (3.7) is the key to uniqueness of equilibrium, whether in switching strategies or in any other strategy.

## 4. Uniqueness of Equilibrium

We can state our result on uniqueness as follows.

**Theorem.** If  $\alpha/\sqrt{\beta} \leq \sqrt{2\pi}/z$ , there is a unique equilibrium. Conversely, if  $\alpha/\sqrt{\beta} > \sqrt{2\pi}/z$ , there is a value of  $\lambda$  such that there is more than one equilibrium. When equilibrium is unique, the equilibrium strategy is the only strategy that survives the iterated deletion of dominated strategies.

In other words, the condition  $\alpha/\sqrt{\beta} \leq \sqrt{2\pi}/z$  is necessary and sufficient for the uniqueness of equilibrium - whether in switching strategies or in any other strategy. Furthermore, the unique equilibrium is dominance solvable. The argument for our theorem can be presented in several smaller steps. Consider the expected payoff of rolling over the loan when one's posterior belief is  $\xi$ , when all other creditors are using the switching strategy around  $\xi$ . Denote this payoff by  $U(\xi)$ . Our result on uniqueness is a consequence of the following pair of results.

**Lemma 1.** If  $\xi$  solves  $U(\xi) = \lambda$ , then there is an equilibrium in which everyone uses the switching strategy around  $\xi$ . If there is a unique  $\xi$  that solves  $U(\xi) = \lambda$ , then the switching strategy around  $\xi$  is the only strategy that survives the iterated deletion of dominated strategies.

**Lemma 2.**  $U'(\xi) \geq 0$  for all  $\xi$  if and only if  $\alpha/\sqrt{\beta} \leq \sqrt{2\pi}/z$ .

The first lemma draws on the work on supermodular games of Vives (1990) and Milgrom and Roberts (1990). The general structure of our model conforms to a particularly simple class of supermodular games in which the iterated deletion of dominated strategies yields the unique equilibrium. The details are presented in the appendix. Heinemann and Illing (1999) have also explored dominance arguments in a currency crisis setting.

As for the second lemma, we give the proof here. The payoff  $U(\xi)$  is given by

$$\begin{aligned} U(\xi) &= \int_{\psi}^{\infty} \phi\left(\sqrt{\alpha+\beta}(\theta-\xi)\right) d\theta \\ &= 1 - \Phi\left(\sqrt{\alpha+\beta}(\psi-\xi)\right) \end{aligned} \tag{4.1}$$

so that

$$U'(\xi) = -\sqrt{\alpha+\beta} \cdot \phi\left(\sqrt{\alpha+\beta}(\psi-\xi)\right) \cdot \left(\frac{\partial\psi}{\partial\xi} - 1\right) \tag{4.2}$$

where  $\phi(\cdot)$  is the density of the standard normal. Hence  $U'(\xi) \geq 0$  if and only if  $\frac{\partial \psi}{\partial \xi} \leq 1$ . The critical state  $\psi$  satisfies

$$\psi = z\Phi\left(\sqrt{\beta}(x - \psi)\right) \quad (4.3)$$

By implicit differentiation of (4.3) with respect to  $\xi$ ,

$$\frac{\partial \psi}{\partial \xi} = z\sqrt{\beta}\left(\frac{\alpha + \beta}{\beta} - \frac{\partial \psi}{\partial \xi}\right)\phi\left(\sqrt{\beta}(x - \psi)\right)$$

Solving for  $\frac{\partial \psi}{\partial \xi}$ , we have  $\frac{\partial \psi}{\partial \xi} = \frac{\phi\sqrt{\beta}}{\frac{1}{z} + \phi\sqrt{\beta}} \cdot \frac{\alpha + \beta}{\beta}$ . Thus,  $\frac{\partial \psi}{\partial \xi} \leq 1$  if and only if

$$\frac{\phi\sqrt{\beta}}{\frac{1}{z} + \phi\sqrt{\beta}} \leq \frac{\beta}{\alpha + \beta} \quad (4.4)$$

Since the left hand side is maximized at  $\phi = 1/\sqrt{2\pi}$ , a sufficient condition for  $\frac{\partial \psi}{\partial \xi} \leq 1$  is  $\sqrt{\beta}/\left(\frac{\sqrt{2\pi}}{z} + \sqrt{\beta}\right) \leq \beta/(\alpha + \beta)$  which boils down to  $\alpha/\sqrt{\beta} \leq \sqrt{2\pi}/z$ . Conversely, suppose  $\alpha/\sqrt{\beta} > \sqrt{2\pi}/z$ . Then, from (4.4),  $\frac{\partial \psi}{\partial \xi} > 1$  when  $x = \psi$ . This proves the lemma.

## 5. Transparency

Having dealt with the technical issues concerning uniqueness, we are now in a position to address policy questions and other comparative statics issues. We begin with the role of information in influencing the equilibrium. In addressing this issue, we hope to highlight the importance of the interplay between two types of uncertainty - *fundamental uncertainty* versus *strategic uncertainty*. Fundamental uncertainty refers to uncertainty concerning the payoff relevant state of nature, which in our model is given by  $\theta$ . Strategic uncertainty refers to the uncertainty concerning the actions of other creditors. When the informational environment of the economy changes for some reason, both types of uncertainty will undergo changes. The net effect on the equilibrium outcome depends on the complex interplay between the two types of uncertainty.

In order to focus our investigation, let us first see what happens in the limit when the private signals of the creditors become very precise, and noise becomes negligible. This corresponds to the case where  $\beta \rightarrow \infty$ . From (3.6), the failure point  $\psi$  satisfies

$$\psi \rightarrow z\Phi\left(\Phi^{-1}(\lambda)\right) = z\lambda \quad (5.1)$$

For large  $z$ , the efficiency loss is sizeable. This result may seem puzzling at first, since the private signal  $x_i$  now reveals what the underlying state  $\theta$  is. In other words, there is no longer any *fundamental uncertainty*. However, the key to understanding this result is to note that strategic uncertainty is not resolved even when the private signal becomes very precise.

One way to pose the question is to ask what is the subjective probability distribution over  $\ell$ , the proportion of creditors who foreclose. From the point of view of any creditor, the equilibrium  $\ell$  is a random variable, and has a density over the unit interval  $[0, 1]$ . One could say something about the degree of strategic uncertainty in terms of the shape of the density over  $[0, 1]$ . For instance, if the density is a degenerate spike at 0, this would suggest that there is no strategic uncertainty, since no-one forecloses. Similarly if the density is the degenerate spike at 1, then everyone forecloses, so that again, there is no strategic uncertainty. However, if the density is more diffuse, then there is uncertainty over the what the other creditors will do. For the case where  $\beta \rightarrow \infty$ , we have the remarkable result that the subjective density for  $\ell$  held by a player at the equilibrium switching point is given by the *uniform density*. Since the uniform density is the most diffuse of all densities over the unit interval, this suggests that strategic uncertainty is at its greatest when  $\beta \rightarrow \infty$ . In our survey paper of global games (Morris and Shin (2000, section 2)), we show that this feature arises as a general feature of binary action global games, not merely those that assume normally distributed fundamentals and signals. Here, we will show this result for the normally distributed case.

Suppose that a creditor is indifferent between rolling over and foreclosing, having received signal  $x$ . Consider the probability that proportion  $\kappa$  or less have a signal higher than this critical signal. Then  $\kappa$  is the proportion of creditors who roll over. Let  $\theta^*$  be the marginal state such that, at  $\theta^*$ , the proportion of players with a signal higher than the critical level is  $\kappa$ . That is

$$\Phi\left(\sqrt{\beta}(x - \theta^*)\right) = 1 - \kappa$$

implying

$$\theta^* = x - \frac{\Phi^{-1}(1 - \kappa)}{\sqrt{\beta}}$$

Now, what is the probability that the true  $\theta$  lies below this  $\theta^*$  conditional on signal  $x$ ? It is given by

$$P = \Phi \left( \sqrt{\alpha + \beta} (\theta^* - \xi) \right)$$

where  $\xi = \frac{\alpha y + \beta x}{\alpha + \beta}$ . Substituting in, we have

$$P = \Phi \left( \frac{\alpha}{\sqrt{\alpha + \beta}} (x - y) - \sqrt{\frac{\alpha + \beta}{\beta}} \cdot \Phi^{-1} (1 - \kappa) \right) \quad (5.2)$$

As  $\beta \rightarrow \infty$ , we have

$$\begin{aligned} P &\rightarrow \Phi \left( -\Phi^{-1} (1 - \kappa) \right) \\ &= 1 - \Phi \left( \Phi^{-1} (1 - \kappa) \right) \\ &= \kappa \end{aligned}$$

So that  $P$  is the identity function. Since  $P$  is the cumulative distribution function over  $\kappa$ , this implies that the density function over  $\kappa$  is uniform. Since  $\ell = 1 - \kappa$ , this means that  $\ell$  is also uniformly distributed.

The uniform density over  $\ell$  gives a direct interpretation of why we have  $\psi = z\lambda$  in the limit as  $\beta \rightarrow \infty$ . The payoff to foreclosure is  $\lambda$ , while the payoff to rolling over is 1 if  $\ell \leq \theta/z$ , and is zero if  $\ell > \theta/z$ . Since  $\ell$  is uniformly distributed over  $[0, 1]$  for the marginal creditor who is indifferent between rolling over and foreclosing, the expected payoff to rolling over is given by

$$\frac{\theta}{z}$$

Indifference between rolling over and foreclosure implies  $\lambda = \theta/z$ , from which we can deduce the critical state as being  $z\lambda$ .

The analysis of this limiting case demonstrates quite starkly how even when information concerning the underlying fundamentals becomes very precise, the strategic uncertainty concerning the actions of other players may, nevertheless, be very severe. It is the interplay between these two types of uncertainty that determines the equilibrium outcome, and this interplay can be quite subtle.

We can pursue this theme a little further. Let us now examine a rather different formalization of the improvement in information. Here, we envisage the quality of the *public information* improving without bound in the sense that the

underlying uncertainty on the fundamentals becomes very small. Formally, we let the precision  $\alpha$  of the ex ante distribution of  $\theta$  become large, but where  $\beta$  keeps pace fast enough so as to ensure uniqueness of equilibrium. In particular, we let  $\beta$  increase at the rate of  $\alpha^2$ . This keeps the ratio  $\alpha/\sqrt{\beta}$  constant - where the constant is small enough to guarantee uniqueness of equilibrium. Thus, consider a sequence of pairs  $(\alpha, \beta)$  where, for constant  $c$ ,

$$\alpha \rightarrow \infty, \quad \beta \rightarrow \infty, \quad \text{but} \quad \frac{\alpha}{\sqrt{\beta}} = c \leq \frac{\sqrt{2\pi}}{z}$$

Then,  $\sqrt{\beta/(\alpha + \beta)} = 1/\sqrt{1 + c/\sqrt{\beta}} \rightarrow 1$ , so that the limit of  $\psi$  is

$$\psi = z\Phi\left(c\left(\psi - y + \frac{\Phi^{-1}(\lambda)}{c}\right)\right) \quad (5.3)$$

Notice the appearance of the ex ante mean  $y$  in this expression. Even though the private signal is extremely precise so that there is no fundamental uncertainty, the ex ante mean of  $\theta$  still matters for equilibrium. Again, this may appear somewhat puzzling, since the information contained in the ex ante mean ought to be dominated by the extremely accurate private signal. However, this is to neglect the distinction between fundamental uncertainty and strategic uncertainty. The ex ante mean  $y$  is valuable in making inferences concerning the beliefs of *other* creditors, and hence on what they do. It is this which is crucial in strategic situations such as ours. We will discuss some policy related issues that arise from our analysis in the concluding section.

## 6. Pricing of Defaultable Debt

We will now turn to another set of comparative statics questions and draw some lessons on the pricing of defaultable securities. The relevant comparison is between the case where there is a single creditor (so that there is no coordination failure) versus the equilibrium outcome in our model where there is inefficient liquidation due to coordination problems. From (3.6) we note the following comparative statics features of our model.

- $\psi$  is increasing in  $z$
- $\psi$  is decreasing in  $y$

The interpretation given to  $z$  in setting up the model was in terms of the mass of the group of creditors, and hence this result can be interpreted as saying that the project is more vulnerable to default when there are more creditors. The static nature of our model cannot distinguish between short and long term claims on the project, but to the extent that the creditors have a choice between rolling over and foreclosing, we can regard the creditors as being short term claim holders. This suggests the hypothesis that the greater the proportion of short term debt in the capital structure, the more fragile is the project to creditor runs. This has a counterpart in the practical use of the Merton model in the industry. The Merton model has been implemented by consulting firms such as KMV Corporation who use the Merton model in their estimation of default probabilities. In practice, KMV have found it optimal to define the default point (the reorganization point) by adding up the whole of short term debt, and half of the long term debt (see Crouhy, Galai and Mark (2000)). This industry practice seems to be consistent with our comparative statics result for  $z$ .

More interesting is the comparative statics of  $\psi$  with respect to  $y$ . Let us begin by verifying that the failure point  $\psi$  does indeed move in the opposite direction to  $y$ . Differentiate (3.6) to obtain

$$\frac{\partial \psi}{\partial y} = z \frac{\alpha}{\sqrt{\beta}} \left( \frac{\partial \psi}{\partial y} - 1 \right) \phi$$

so that

$$\frac{\partial \psi}{\partial y} = - \frac{\frac{\alpha}{\sqrt{\beta}} z \phi}{1 - \frac{\alpha}{\sqrt{\beta}} z \phi} \quad (6.1)$$

But the denominator is positive, since uniqueness implies (by our theorem)  $\alpha/\sqrt{\beta} \leq \sqrt{2\pi}/z$ , and  $\phi$  is bounded above by  $1/\sqrt{2\pi}$ . Hence,

$$\frac{\partial \psi}{\partial y} < 0.$$

The following figure illustrates this comparative statics effect.

The fact that the failure point moves up as the fundamentals deteriorate reflects the fact that strategic uncertainty becomes more and more important in the reasoning of the creditors. Intuitively, when  $y$  is low, a low signal observed by me is more likely to imply low signals observed by other creditors. This makes it more likely that other creditors will foreclose, and thereby undermine the project.

The ex ante price of secured debt will reflect the recovery rate in the case of default as well as the probability of default. In our model, there is the added



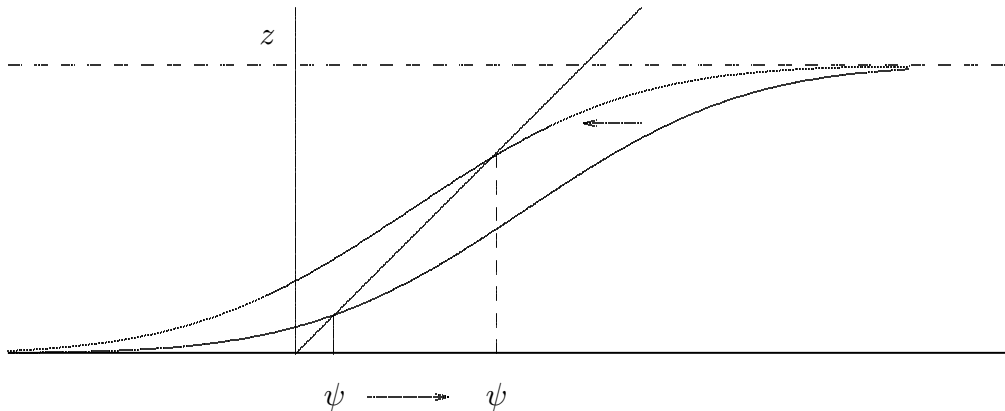


Figure 6.1: Shift in  $\psi$

complication of whether a creditor forecloses or not when the project fails<sup>5</sup>. In order to illustrate the comparative statics effects more succinctly, we consider the value of a long term *unsecured* loan to the project with face value 1. The lender in this case is essentially passive. The lender receives 1 whenever  $\theta \geq \psi$ , but receives zero when  $\theta < \psi$ . The ex ante price of such a loan given ex ante mean  $y$  is

$$W(y) = \int_{\psi}^{\infty} \phi(\sqrt{\alpha}(\theta - y)) d\theta = 1 - \Phi(\sqrt{\alpha}(\psi - y)) \quad (6.2)$$

The change in the asset value of the loan to a shift in the mean of the distribution is

$$\frac{\partial W}{\partial y} = \sqrt{\alpha} \cdot \phi - \frac{\partial \psi}{\partial y} \sqrt{\alpha} \cdot \phi \quad (6.3)$$

The first term could be dubbed the *conventional effect* in that it reflects the change in the weight of the left tail of the distribution due to a shift in the centre of the distribution. The second term is the novel feature. It arises from the fact that the threshold for the tail also shifts. We could call this the *coordination effect*. Since  $\partial \psi / \partial y < 0$ , the coordination effect reinforces the conventional effect.

<sup>5</sup>The ex ante payoff of a particular creditor depends on the probability of four events: project fails and he forecloses, project fails but he does not foreclose, the project succeeds but he forecloses and the project succeeds and he does not foreclose.

For the creditor, a deterioration in the fundamentals in terms of a fall in  $y$  implies that the asset value of the loan is falling at a rate *more than proportional* to the fall in  $y$ . Thus, it is precisely when risk management is most important - when  $y$  is falling - that it is important not to neglect the coordination effect. By neglecting the coordination effect, the creditor is underestimating the true value at risk. Note also that such an effect is capable of reconciling the pricing data for defaultable debt and the Merton model. The greater incidence of coordination failure for lower quality debt implies a higher default trigger. We can illustrate this by means of the following numerical example.

Defining the yield on the unsecured loan as

$$\text{Yield} = \frac{\text{Par} - \text{Price}}{\text{Price}},$$

we can compare the yields generated by the true model (with failure occurring at  $\psi$ ) with the yields given by the naive model which assumes away coordination risk. The following table is generated from the case where

$$\alpha = 1, \quad \beta = 5, \quad z = 1, \quad \lambda = 0.5.$$

Note that the recovery rate  $\lambda$  is relevant only in determining the failure point  $\psi$ . The unsecured lender receives zero in the case where the project fails. The first column gives the ex ante mean of the payoff distribution and the second column gives the yield on the loan for the naive model (no coordination risk). The naive model applies to the case where there is a single large creditor, so that there is no coordination failure. The third column gives the yield arising from the true model, and the value of the break point  $\psi(y)$  appears in the last column. Since  $\alpha = 1$ , the values of  $y$  are in units of standard deviations. So, the first row of the table pertains to the case where the ex ante mean  $y$  is three standard deviations from zero. The last element of this row tells us that the true failure point is  $\psi(3) = 0.097$ , and the true yield is 0.19%, rather than the yield given by the naive model of 0.14%. This difference in yield is not large, since the loan is a very safe one - the mean being three standard deviations away from zero.

Ex ante mean $y$	Yield from naive model	Yield from true model	Failure point $\psi$
$y = 3$	0.0014	0.0019	0.097
$y = 2$	0.0233	0.0383	0.212
$y = 1.5$	0.0716	0.1288	0.295
$y = 1.25$	0.1181	0.2226	0.342
$y = 1$	0.1886	0.3735	0.393
$y = 0.75$	0.293	0.6143	0.446
$y = 0.5$	0.4462	1.0	0.5
$y = 0.25$	0.6703	1.6279	0.554
$y = 0$	1.0	2.6774	0.607

However, as  $y$  falls, we can see that the yield difference becomes large. At one standard deviation away from zero (i.e. for  $y = 1$ ), the naive model predicts a yield of 19%, but the true yield is actually almost double that number, at 37%. This corresponds to the break point of  $\psi = 0.393$ . Thus, the interval  $[0, 0.393]$  represents the size of inefficient liquidation. For even lower values of  $y$ , the yield difference is even higher. When  $y = 0$ , the naive model predicts a yield of 100%, but the true yield is 268%.

Such a pattern of discrepancies between the benchmark model and the true model is quite suggestive. The overpricing of defaultable bonds relative to market prices (and the underpricing of its yield), as well as the fact that such discrepancies are larger for lower quality bonds, has been one of the persistent problems with empirical implementations of the Merton model. Our theory predicts that the default point will actually be a function of the quality of the bond, and that the default point will be higher for lower quality bonds. When this additional effect is taken into account, the apparent anomalies can be accommodated. Bruche (2001) develops a continuous time version of our model that provides a more standardized platform for comparing the pricing implications of alternative models.

Furthermore, although the parameters  $\alpha$  and  $\beta$  are rather abstract quantities pertaining to information, they have an indirect empirical counterpart in terms of the relationship between the fundamental value  $y$  and the failure point  $\psi$ . Thus, in principle, it would be possible to extract some information on  $\alpha$  and  $\beta$  if we

had sufficiently detailed data on the relationship between the distance to default and the default probability.

## 7. Concluding Remarks

The term “transparency” has taken on great significance in the policy debate after the market turmoil of 1997/8, and has figured prominently in numerous official publications (such as IMF (1998b), BIS (1999)). The notion of transparency has many subtleties, and it would be wrong to give too simplistic an interpretation of it. However, to the extent that improved transparency can be given a straightforward interpretation in terms of provision of more information, our analysis suggests that the effects can be counterintuitive. This observation holds some important lessons for the conduct of public policy in dissemination of information. When calling for improved transparency, it is important to be clear as to *how* the improved information will improve the outcome. The mere provision of information will not be enough. However, if the improved information is one element of better coordination of the disparate market participants, the information may have some beneficial effect. This suggests that concrete institutional changes must accompany the provision of information if the information is to be effective.

In spite of the acknowledged simplicity of the model, we may nevertheless draw some lessons for the current debate concerning the reform of the international financial system. With the benefit of theoretical hindsight, it is perhaps not surprising that the provision of more information to market participants does not mitigate the problem. After all, we should draw a distinction between a single-person decision problem and a strategic situation. In a single-person decision problem, more information is always more valuable. When I debate whether to carry an umbrella into work, an accurate weather forecast will minimize both the inconvenience of carrying a bulky umbrella on a sunny day, and also the opposite inconvenience of getting caught in a shower without shelter. In such instances, “transparency” works.

However, it is far from clear whether better information will mitigate a coordination problem. There is little guidance from economic theory that better information about payoffs to players of a coordination game leads to greater incidence of successful coordination. Indeed, the intuition conveyed by existing theory is of a much more prosaic sort - typified by the debate on the Coase Theorem - in which all the emphasis is placed on the impediments to efficient bargaining. When the interested parties are diffuse and face uncertainty both about the fundamentals

and the information of others, it would be overly optimistic to expect ex post efficient bargains to be struck.

We have already noted how instances of successful coordination by creditors - such as the reorganization of Long Term Capital Management in September 1998 - have had a forceful facilitator orchestrating the rescue. In the case of LTCM, this role was played by the New York Fed. The U. S. Treasury has also played a key role in a number of episodes in recent years (Brazil in 1999, Korea in 1997/8). Although governments and central banks are best placed to play such a role, there is no reason why a non-governmental party cannot play a similar role. The account of J. P. Morgan's role in coordinating the 1907 bailout is an instructive example<sup>6</sup>. Proponents of more elaborate multilateral institutions would do well to pause for thought on how the new institution will fare in the role of facilitator.

## APPENDIX

In this appendix, we provide an argument for lemma 1. Consider first the expected payoff to rolling over the loan conditional on  $\xi$  when all others are using the switching strategy around some point  $\hat{\xi}$ . Denote this payoff as

$$u(\xi, \hat{\xi}) \tag{7.1}$$

This payoff is given by  $1 - \Phi(\sqrt{\alpha + \beta}(\psi - \xi))$ , where  $\psi$  is the failure point defined as the unique solution to the equation  $\psi = z\Phi(\sqrt{\beta}(x(\hat{\xi}, y) - \psi))$ . The conditional payoff  $u(\xi, \hat{\xi})$  can be seen to satisfy the following three properties.

**Monotonicity.**  $u$  is strictly increasing in its first argument, and is strictly decreasing in its second argument.

**Continuity.**  $u$  is continuous.

**Full Range.** For any  $\hat{\xi} \in \mathbb{R} \cup \{-\infty, \infty\}$ ,  $u(\xi, \hat{\xi}) \rightarrow 0$  as  $\xi \rightarrow -\infty$ , and  $u(\xi, \hat{\xi}) \rightarrow 1$  as  $\xi \rightarrow \infty$ .

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<sup>6</sup>See, for instance, New Yorker Magazine, Nov 23, 1998 (page 62). We are grateful for Arijit Mukherji for this reference.

By appealing to these features, we can define two sequences of real numbers. First, define the sequence

$$\underline{\xi}^1, \underline{\xi}^2, \dots, \underline{\xi}^k, \dots \quad (7.2)$$

as the solutions to the equations:

$$\begin{aligned} u(\underline{\xi}^1, -\infty) &= \lambda \\ u(\underline{\xi}^2, \underline{\xi}^1) &= \lambda \\ &\vdots \\ u(\underline{\xi}^{k+1}, \underline{\xi}^k) &= \lambda \\ &\vdots \end{aligned}$$

In an analogous way, we define the sequence

$$\bar{\xi}^1, \bar{\xi}^2, \dots, \bar{\xi}^k, \dots \quad (7.3)$$

as the solutions to the equations:

$$\begin{aligned} u(\bar{\xi}^1, \infty) &= \lambda \\ u(\bar{\xi}^2, \bar{\xi}^1) &= \lambda \\ &\vdots \\ u(\bar{\xi}^{k+1}, \bar{\xi}^k) &= \lambda \\ &\vdots \end{aligned}$$

We can then prove:

**Lemma A1.** Let  $\xi$  solve  $U(\xi) = \lambda$ . Then

$$\underline{\xi}^1 < \underline{\xi}^2 < \dots < \underline{\xi}^k < \dots < \xi \quad (7.4)$$

$$\bar{\xi}^1 > \bar{\xi}^2 > \dots > \bar{\xi}^k > \dots > \xi \quad (7.5)$$

Moreover, if  $\underline{\xi}$  and  $\bar{\xi}$  are, respectively, the smallest and largest solutions to  $U(\xi) = \lambda$ , then

$$\underline{\xi} = \lim_{k \rightarrow \infty} \underline{\xi}^k \quad \text{and} \quad \bar{\xi} = \lim_{k \rightarrow \infty} \bar{\xi}^k. \quad (7.6)$$

**Proof.** Since  $u(\underline{\xi}^1, -\infty) = u(\underline{\xi}^2, \underline{\xi}^1) = \lambda$ , monotonicity implies  $\underline{\xi}^1 < \underline{\xi}^2$ . Thus, suppose  $\underline{\xi}^{k-1} < \underline{\xi}^k$ . Since  $u(\underline{\xi}^k, \underline{\xi}^{k-1}) = u(\underline{\xi}^{k+1}, \underline{\xi}^k) = \lambda$ , monotonicity implies  $\underline{\xi}^k < \underline{\xi}^{k+1}$ . Finally, since  $U(\xi) = u(\xi, \xi) = u(\underline{\xi}^{k+1}, \underline{\xi}^k)$ , and  $\underline{\xi}^k < \underline{\xi}^{k+1}$ , monotonicity implies that  $\underline{\xi}^k < \xi$ . Thus,  $\underline{\xi}^1 < \underline{\xi}^2 < \dots < \underline{\xi}^k < \dots < \xi$ . An exactly analogous argument shows that  $\bar{\xi}^1 > \bar{\xi}^2 > \dots > \bar{\xi}^k > \dots > \xi$ . Now, suppose  $\underline{\xi}$  is the smallest solution to  $u(\xi, \xi) = \lambda$ . By (7.4) and the monotonicity of  $u$ ,  $\underline{\xi}$  is the smallest upper bound for the sequence  $\{\underline{\xi}^k\}$ . Since  $\{\underline{\xi}^k\}$  is an increasing, bounded sequence, it converges to its smallest upper bound. Thus  $\underline{\xi} = \lim_{k \rightarrow \infty} \underline{\xi}^k$ . Analogously, if  $\bar{\xi}$  is the largest solution to  $u(\xi, \xi) = \lambda$ , then (7.5) and monotonicity of  $u$  implies that  $\bar{\xi} = \lim_{k \rightarrow \infty} \bar{\xi}^k$ . This proves the lemma.

**Lemma A2.** If  $\sigma$  is a strategy which survives  $k$  rounds of iterated deletion of interim dominated strategies, then

$$\sigma(\xi) = \begin{cases} F & \text{if } \xi < \underline{\xi}^k \\ R & \text{if } \xi > \bar{\xi}^k \end{cases} \quad (7.7)$$

The argument is as follows. Let  $\sigma^{-i}$  be the strategy profile used by all players other than  $i$ , and denote by  $\tilde{u}^i(\xi, \sigma^{-i})$  the payoff to  $i$  of rolling over the loan conditional on  $\xi$  when the others' strategy profile is given by  $\sigma^{-i}$ . The incidence of failure is minimized when everyone is rolling over the loan irrespective of the signal, and the the incidence of failure is *maximized* when everyone is foreclosing on the loan irrespective of the signal. Thus, for any  $\xi$  and any  $\sigma^{-i}$ ,

$$u(\xi, \infty) \leq \tilde{u}^i(\xi, \sigma^{-i}) \leq u(\xi, -\infty) \quad (7.8)$$

From the definition of  $\underline{\xi}^1$  and monotonicity,

$$\xi < \underline{\xi}^1 \implies \text{for any } \sigma^{-i}, \tilde{u}^i(\xi, \sigma^{-i}) \leq u(\xi, -\infty) < u(\underline{\xi}^1, -\infty) = \lambda. \quad (7.9)$$

In other words,  $\xi < \underline{\xi}^1$  implies that rolling over the loan is strictly dominated by foreclosing. Similarly, from the definition of  $\bar{\xi}^1$  and monotonicity,

$$\xi > \bar{\xi}^1 \implies \text{for any } \sigma^{-i}, \tilde{u}^i(\xi, \sigma^{-i}) \geq u(\xi, \infty) > u(\bar{\xi}^1, \infty) = \lambda. \quad (7.10)$$

In other words,  $\xi > \bar{\xi}^1$  implies that foreclosing on the loan is strictly dominated by rolling over. Thus, if strategy  $\sigma^i$  survives the initial round of deletion of dominated strategies,

$$\sigma^i(\xi) = \begin{cases} F & \text{if } \xi < \underline{\xi}^1 \\ R & \text{if } \xi > \bar{\xi}^1 \end{cases} \quad (7.11)$$

so that (7.7) holds for  $k = 1$ .

For the inductive step, suppose that (7.7) holds for  $k$ , and denote by  $U^k$  the set of strategies which satisfy (7.7) for  $k$ . We must now show that, if player  $i$  faces a strategy profile consisting of those drawn from  $U^k$ , then any strategy which is not in  $U^{k+1}$  is dominated. Thus, suppose that player  $i$  believes that he faces a strategy profile  $\sigma^{-i}$  consisting of strategies from  $U^k$ . Given this, the incidence of failure is minimized when  $\sigma^{-i}$  is the (constant) profile consisting of the  $\bar{\xi}^k$ -trigger strategy, and the incidence of failure is *maximized* when  $\sigma^{-i}$  is the (constant) profile consisting of  $\underline{\xi}^k$ -trigger strategy. Thus, for any  $\xi$  and any strategy profile  $\sigma^{-i}$  consisting of those from  $U^k$ ,

$$u(\xi, \bar{\xi}^k) \leq \tilde{u}^i(\xi, \sigma^{-i}) \leq u(\xi, \underline{\xi}^k) \quad (7.12)$$

From the definition of  $\underline{\xi}^k$  and monotonicity, we have the following implication for any strategy profile  $\sigma^{-i}$  drawn from  $U^k$ .

$$\xi < \underline{\xi}^{k+1} \implies \tilde{u}^i(\xi, \sigma^{-i}) \leq u(\xi, \underline{\xi}^k) < u(\underline{\xi}^{k+1}, \underline{\xi}^k) = \lambda. \quad (7.13)$$

In other words, when  $\xi < \underline{\xi}^k$  and when all others are using strategies from  $U^k$ , rolling over the loan is strictly dominated by foreclosing. Similarly, from the definition of  $\bar{\xi}^k$  and monotonicity, we have the following implication for any strategy profile  $\sigma^{-i}$  consisting of those from  $U^k$ .

$$\xi > \bar{\xi}^{k+1} \implies \tilde{u}^i(\xi, \sigma^{-i}) \geq u(\xi, \bar{\xi}^k) > u(\bar{\xi}^{k+1}, \bar{\xi}^k) = \lambda. \quad (7.14)$$

In other words, when  $\xi > \bar{\xi}^{k+1}$  and all others are using strategies from  $U^k$ , foreclosing on the loan is strictly dominated by rolling over. Thus, if strategy  $\sigma^i$  survives  $k + 1$  rounds of iterated deletion of dominated strategies,

$$\sigma^i(\xi) = \begin{cases} F & \text{if } \xi < \underline{\xi}^{k+1} \\ R & \text{if } \xi > \bar{\xi}^{k+1} \end{cases} \quad (7.15)$$



This proves the lemma.

With these preliminary results, we can complete the proof of Lemma 1. First, let us show that if  $\xi$  solves  $U(\xi) = \lambda$ , then there is an equilibrium in trigger strategies around  $\xi$ . Since  $U(\xi) = u(\xi, \xi) = \lambda$ , if everyone else is using the  $\xi$ -trigger strategy, the payoff to rolling over conditional on  $\xi$  is the same as that for foreclosing. Since  $u$  is strictly increasing in its first argument,

$$\xi_* < \xi < \xi^* \iff u(\xi_*, \xi) < \lambda < u(\xi^*, \xi)$$

so that the  $\xi$ -trigger strategy is the strict best reply.

Finally, let us show that if  $\xi$  is the unique solution to  $U(\xi) = \lambda$ , then there is no other equilibrium. From Lemma A1, we know that

$$\xi = \lim_{k \rightarrow \infty} \xi^k = \lim_{k \rightarrow \infty} \bar{\xi}^k \tag{7.16}$$

so that the only strategy which survives the iterated deletion of dominated strategies is the  $\xi$ -trigger strategy. Among other things, this implies that the equilibrium in  $\xi$ -trigger strategies is the unique equilibrium.

The basic properties of our model conform to the class of supermodular games examined by Milgrom and Roberts (1990) and Vives (1990), in that the payoffs exhibit strategic complementarities, and the strategy set can be seen as a lattice for the appropriate ordering of strategies. The following features of our model echo the general results obtained in this literature.

- There is a “smallest” and “largest” equilibrium, corresponding to the smallest and largest solutions to the equation  $U(\xi) = \lambda$ .
- Any strategy other than those lying between the smallest and largest equilibrium strategies can be eliminated by iterated deletion of dominated strategies. Thus, if  $\underline{\xi}$  and  $\bar{\xi}$  are, respectively, the smallest and largest solutions to  $U(\xi) = \lambda$ , then rationalizability removes all indeterminacy in a player’s strategy except for the interval  $[\underline{\xi}, \bar{\xi}]$ .
- If there is a unique solution to  $U(\xi) = \lambda$ , then there is a unique equilibrium, and this is obtained as the uniquely rationalizable strategy.

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