The Uses of Teaching Games in Game Theory Classes And Some Experimental Games

By

Martin Shubik

January 2001

COWLES FOUNDATION DISCUSSION PAPER NO. 1289



COWLES FOUNDATION FOR RESEARCH IN ECONOMICS YALE UNIVERSITY Box 208281 New Haven, Connecticut 06520-8281

http://cowles.econ.yale.edu/

The Uses of Teaching Games in Game Theory Classes and Some Experimental Games

Martin Shubik¹

1. Preamble

The impetus to finally get around to writing this paper comes from three sources. First and foremost are the stimulating papers by Ariel Rubinstein (1999) and Charles Holt (1999) as well as the feature on classroom games in Economic Perspectives and the handbook of experimental economics edited by John Kagel and Alvin Roth(1995).

Much of current interest among economists has been focused on experimental economics somewhat less on noncooperative game theory and even less on cooperative game theory. There is also a large and allied literature in social psychology and some in political science, as well as a large related, but more specialized literature on business games.

My interest in designing and using games in the study of game theory dates from the early 1950s at Princeton where Hausner, McCarthy, Minsky, Nash, Shapley and Shubik as graduate students informally discussed the use of games in the study of game theory and some of us devised "so long sucker" to illustrate the vital roles of coalition formation and double-crossing (Hausner, Nash, Shapley and Shubik, 1964). In 1956, by chance, I met Sidney Siegel in the High Sierra and he convinced me of the importance of doing controlled experiments in economics and psychology (Fouraker, Shubik and Siegel, 1965); a little later Richard Bellman and, independently, George Feeney stressed the possibilities of the utilization of computers to construct business games for teaching and training. The era of the use of the computer was beginning to dawn and the potentials for its utilization in long range planning and simulation as well as gaming were already being considered (and probably overestimated) by both the military and the few enthusiasts in academia. In spite of the considerable expenditures by the military in gaming and simulation,² the connections between the operational and experimental gaming communities appears to be and have been slight. Between strict operational gaming and the uses of gaming to verify more or less context free theory, the work of Plott, Ledyard and others has been devoted to exploring the utilization of specific economic mechanisms.

¹Cowles Foundation for Research in Economics, Department of Economics, Yale University, New Haven, CT 06511. email: martin.shubik@yale.edu

² See Brewer and Shubik (1979) for a general survey of war gaming.

2. Context, Control and Calculation

A safe rule of thumb in examining the uses of gaming is that there is no universal right type of experimental gaming to fit all circumstances. The disconnect between the experimental and operational gaming community is distressing, especially when one realizes that the expenditures in operational gaming are orders of magnitude greater. However the discussion here is constrained to teaching and experimentation. Social scientists often suffer from "real science" envy. They remember the elementary physics lab and dream of doing properly controlled experiments in sociology, economics or social psychology. Unfortunately there are many aspects of the enculturation and memories of individual humans which limit the analogies we can make with "hard science." In particular the atoms in economics experiments are individuals who come equipped with social conditioning, language and memory of individual experience. When they come to an experiment their memories and conditioning cannot be wiped out. In their work on game theory Shapley and Shubik suggested that the condition of "external symmetry" be made explicit in game models (see Shubik, 1982, p.16). This condition indicates that unless explicitly modeled otherwise, all aspects of the players are assumed to be the same. In fact, they never are. At best the experimenter can suggest that as a first order approximation the individuals are sufficiently similar.

The mere fact that individuals come to an experiment with the baggage of enculturation and personality may have a perverse influence on the "dust free" experiment. If the context is made too clean or too impoverished the influence of the controls may be perverse. Because the environment is context free or too stark individual history and memory may be used by the players to supply their own cues. The introduction of the computer console and the gaming lab impose considerable and not necessarily desirable special structure on games. Certain environments such as the command center in an aircraft carrier or a stock market trading desk are so artificial that they can be simulated reasonably accurately in any game. But even there the influence of two months at sea or six months of trading in a down market are left out. Little details such as the differences in how data are entered, how bad the time compression is when compared with the "real thing" may easily introduce artifacts. When the environment is impoverished the influence of differences in wording instructions is magnified.

The importance to motivation on how much individuals are paid (in contrast to a Hawthorne effect) is still an open question. The best way to study risk behavior in gambling might be to set up observations of individuals using their own money in Las Vegas.

These observations are made, not as an argument against those who like to perform heavily controlled laboratory experiments with paid subjects, but to suggest that one might be able glean

useful insights from other and less formal uses of gaming. One might even be able to learn from asking individuals for their opinions concerning how certain games should be played.

3. The Games and their Purpose

The paper of Rubinstein(1999 recounted his experiences with two undergraduate courses in game theory in 1998 and 1999. Most of the games were variations of noncooperative games, several were zero sum games; several ultimatum games; some were games played in extensive form as well as several others. The stress was on teaching game theory. Context was minimal; some were pure abstract games, others had minimal stories such as the battle of the sexes or prisoner's dilemma; a few involved economic situations with bargaining; another considered an ambush and another the manipulation of an election.

Rubinstein (1999) makes a case for the value of the crude in class experiments as a source of valuable experimental insights. He states: "it is argued that the crude experimental methods produced results which are not substantially different from those obtained at much higher cost using stricter experimental methods."

Holt, in contrast to Rubinstein, both in his paper on classroom experiments (Holt, 1999) and in his special feature in Economic Perspectives concentrates on games in teaching economics. In each instance the game has an economic scenario.

My own use of gaming in teaching has differed somewhat from both the uses of Rubinstein and Holt. The games reported on were used in various classes in teaching game theory from 1961 to 1992. For the most part the students were undergraduates (seniors) or MBA students at Yale. Some games were run in lectures given elsewhere in the United States, Austria, Australia, Canada, Chile, India and Hong Kong. One game on a double auction market (as indicated below) was run in classes on the history and theory of money and financial institutions. Another game, "a business game for teaching and experimental purposes" was utilized (in several variations) both in formal experiments and in several classes on industrial organization, oligopoly theory and corporate planning.

The games, results and interpretations are discussed below. Several of the games involved the examination of cooperative game theory solutions. Both the various solutions to games in coalitional form and the Nash equilibrium to a game in strategic form present a mathematical analysis of a static situation. Frequently a verbal description with a dynamical story is attached.

Although much of the current teaching of game theory in economics departments stresses noncooperative theory, there is a large body of cooperative theory stressing solutions such as the core and value and other cooperative solutions. The original mathematical work on noncooperative equilibria was directed at games in strategic form, which usually meant games represented by matrices of payoffs. Much of the discussion concerning the noncooperative equilibrium attributes behavioral justification to the equilibrium. There is little evidence that this is so in general. How a one shot matrix game is played does not appear to conform with any generality to the noncooperative equilibrium. Harsanyi and Selten (1988) have devoted considerable ingenuity in attempting to provide an axiomatic and normative justification for the noncooperative equilibrium. The cooperative solution concepts are openly normative and in some instances such as the value (Shapley, 1953) have been used as the recommended way to solve problems in the imputation of joint economic usage. Concepts such as joint optimality, fairness and symmetry are all utilized as axioms in constructing these solution concepts.

Much of the experimental work has been directed at multistage games. The pioneering work of Rapoport, Guyer and Gordon (1976) records experiments with all of the 78 ordinally different 2×2 games (without tied entries).

When one considers the playing of multistage games the dichotomy between cooperative and noncooperative theories is no longer clear and one may wish to consider the meaning of reputation, coordination, implicit collusion and quasi-cooperation.

Even at the level of two-person zero-sum games context and professional training appear to be relevant. An early PhD thesis of R. I. Simon (1967) utilized business school and military science majors playing a two person zero sum game with three scenarios for the quantitatively same game. One scenario was based on business, one was military and one was abstract. Different results were obtained in all instances.

Various experiments with many players and an economic market scenario indicate the emergence of a competitive price system. Possibly one of the extreme powers of the model of the individualistic competitive market is that it expressly destroys the game theoretic aspects of behavior. The ideal large competitive market, in the limit, reduces the strategic power of the individual to zero, thus the individual plays against "the market" or "nature." In the infinite limit the competitive equilibrium in economic theory may be regarded as a form of noncooperative equilibrium; but both the maxmin solution of two person zero sum games and the competitive equilibrium are solutions in their own right and a conceptually different from the noncooperative equilibrium.

My uses of games in class were directed to exploring the relationships among various game theory assumptions and solutions and student opinion and behavior. Their purpose was both to teach and to "sweeten the intuition." The games utilized in class or in lectures were as follows:

List of Experimental Games

Games in Coalitional Form

- 1a. Coalitional form a fat core
- 1b. Coalitional form 1 point core
- 6. Coalitional form no core

The Name of the Game

2. Three games with names: Prisoner's .Dilema., Battle of Sexes, Chicken

Psychological Considerations

- 3. Interpersonal comparisons
- 4. Prominence or symmetry
- 5. Uncertain prize Bernouilli Utility sealed bid for uncertain prize

Bidding

- 7a. A simple bidding game resources 3,9
- 7b. A simple bidding game resources 9, 3
- 7c. A simple bidding game resources 9,9
- 8. Double auction market
- 13. Bidding problem with nonsymmetric information

Other

- 9. Two zero sum games fair, unfair
- 10. Allais paradox
- 11. Rational behavior in games in extensive form
- 12. An analysis of threats
- 14. Two 3×3 games

A business game for teaching and experimental purposes was also utilized both in classes and lectures and in formally controlled experiments from 1960 to 1978.

Only the first six games are noted here. Two further parts cover the remaining games.

4. A Report on the Games

The results from several of these games have been published and reported in detail elsewhere. When this is so, the references are given and only a brief discussion is noted here.

Games in Coalitional Form (1a, 1b and 6):

The first three exercises were used in teaching cooperative game solutions. The three cooperative game solution concepts considered were the core, the value, the nucleolus as well as the price system (which requires an economic story to locate it if the core is large) as well as the symmetric or even-split point. Intuitively, *the core* is the set of imputations against which no coalition can propose an alternative which they prefer and could achieve if they acted independently. *The value* of a game gives each individual his *a priori* expected combinatoric marginal value considering the worth of an individual's entry into all coalitions. *The nucleolus* is that imputation for which the difference between what it offers and what any coalition can achieve is minimized. *The competitive equilibrium* is obtained by constructing an associated economy which is represented by the game and solving for the efficient price system. The *even split point* can be regarded as an egalitarian solution which ignores the influence or power of subgroups (see Shubik. 1986).

My conjectures were that when the students were asked to play judge, normatively deciding how the proceeds should be split among the players a point in the core would be most probably selected when the core was large; however, with a one point highly non-symmetric core the frequency of selection would be considerably attenuated. It would, however be higher if an economic scenario were supplied. I had no clear conjecture for the game (#6) with no core whatsoever. I wished to use the game as a means for gaining some insight into the choices made. In all instances I asked the students to provide a verbal description or rationalization of their choice.

The predominant set of games consisted of plays by nine classes at Yale. There were also data obtained from seven games in India, seven games in Australia and games in Hong Kong and elsewhere in the United States.

The information on the gain through cooperation was displayed by means of a characteristic function. The characteristic functions for games 1a, 1b and 6 are shown below.

Game 1a

$$v(A) = v(B) = v(C) = 0$$

 $v(AB) = 100, v(AC) = 200, v(BC) = 300$
 $v(ABC) = 400.$

Game 1b

$$v(A) = v(B) = v(C) = 0$$

 $v(AB) = 0$, $v(AC) = 400$, $v(BC) = 400$
 $v(ABC) = 400$.

Game 6

$$v(A) = v(B) = v(C) = 0$$

 $v(AB) = 250$, $v(AC) = 300$, $v(BC) = 350$
 $v(ABC) = 400$.

Where A, B and C are the names of the players and v(AC) indicates the amount that players A and C can obtain by collaborating.

Game 1a has a "fat core" (i.e., there are many divisions of 400 units which satisfy all inequalities). Game 1b has a single point core) with all gain going to player C, this is also the competitive equilibrium of any associated economy. The imputation is (0, 0, 400). Game 6 has no core whatsoever. There is no way of reconciling subgroup and group rationality.

Date	1980	1981	1983	1984	1985	1988	1989	91S	91F	Avg.
%1a	86	86.8	87	89.5	93	92	88.6	100	96.7	91
%1b	28	5	22	19	20	11	29	30	28	21

Points in the Core

Table 1

Table 1 shows the percentage of points in the core contrasting the relatively large core (3/16 of the payoff simplex) with the one point core (which was also the unique competitive equilibrium). The total sample size was 321 individuals for 1a and 340 for 1b.

A detailed analysis, which includes an international comparison as well as students contrasted with game theorists, is given in Shubik (1986). Here we observe that the distinction between the percentage of points selected in the core for the two games is striking and significant. In the second game 187/340 = 55% chose the imputation (100, 100, 200) which could be arrived at by considering the symmetric game with a union of (1, 2) against 3 where 1 and 2 split the proceeds of an even split of 200 each with 3.

In game 6 with no core the most popular choice was the even split (133.3, 133.3, 133.3), but only 13.4% made this selection. There was considerable variety with little support for either the value or the nucleolus.

In the discussion and debriefing it appeared that the value and nucleolus were not intuitively easy to grasp. The core was more or less "naturally comprehended," but was rejected when it was

too biased or non-symmetric. The one point core was chosen deliberately to be "unfair" and was perceived as such by the students. When there was no core to provide a cue symmetry provided a weak focal point.

The Name of the Game

The basic reason why most of the matrix game experimentation and didactic examples have been confined to 2×2 matrix games is because they provide the simplest set of examples of matrix games with which one can investigate anonymous or non face-to-face interactive behavior.

There are 78 strategically different 2×2 games with no ties in an individual's preferences. If ties are included the number of games is of the order of 700. When we extend the analysis to 3×3 matrices we find that there are around 65×10^9 strategically different games. The simplest three person matrix game (with no strategic dummies) involves a $2 \times 2 \times 2$ matrix and there are of the order of over 800 strategically different games.

If we try to extend the idea of the classical Prisoner's Dilemma game (illustrated in Table 2) to a 3×3 matrix game we need to count those games which have the two properties of the Prisoner's Dilemma:

- 1. The game is ordinally symmetric with respect to the players
- 2. The outcome reached by all players choosing the dominant strategy is inferior to that reached by all of them choosing another strategy. Imelda Powers has calculated that there are a total of 1408 strictly ordinal Prisoner's Dilemma games. For the three person $2 \times 2 \times 2$ game there is a total of 160 strictly ordinal Prisoner's Dilemma games.

Because the 2×2 games are relatively few in number and can be used to illustrate several of the paradoxes involving the relationship between individual isolated and interactive behavior, some of the 2×2 games have been given names. Each of the games utilized here has a name and story attached to it.

The three games considered are the Prisoner's Dilemma game, the Battle of the Sexes and Chicken. The stories told about, or briefings presented for each of the games vary somewhat. But the three presented are more or less typical.

The Prisoner's Dilemma

Two suspects are taken into custody and questioned separately. They are strongly suspected of a crime, but the prosecution does not have sufficient evidence. If both remain silent they are able to get off with a minor charge. If one confesses and the other does not, the one who confessors the

informant will obtain lenient treatment for turning state's evidence, while the other will receive a harsh penalty. If both confess both will receive a less harsh penalty than the hold out when the other confesses.

The Battle of the Sexes

A couple want to go out for an evening's entertainment. He wants to go to a basketball game and she wants to go to the ballet. They both prefer going out somewhere over not going out at all.

Chicken

Two California hot-rodders drive in different cars down an empty road at high speed, towards each other. Each has one set of the wheels of the car on the center strip markings. If they both stay on course they will crash. If one or both of them veer from the center strip the crash will be avoided, but whoever veers will deemed to be chicken

Examples of payoffs for the games associated with these are shown in 2, 3 and 4.

	1	2
1	5, 5	-9, 10
2	10, –9	0, 0

The Prisoner's Dilemma **Table 2**

	1	2
1	-10, -10	5, -5
2	-5, 5	0, 0

Chicken **Table 3**

	1	2
1	2, 1	0, 0
2	0, 0	1, 2

The Battle of the Sexes **Table 4**

The games were presented to the students in the form above with no titles and no story. They were presented as abstract games. They played the games and were then given the three titles and were asked to match the games with the titles. The game was played eight times.

Table 5 shows the percentage of the class who correctly identified the name with the game.

	1980	1981	1983	1984	1985	1988	1989	1991
Chicken	38	64.7	60.6	29.4	47.5	26.9	0	43.3
B. of S.	70.6	55.9	62.5	70.6	73.7	41.4	50	60.9
P. D.	47	55.9	61.3	29.4	46.2	34.6	10	26.3

Percentage of correct associations **Table 5**

There appears to be some support that "numbers tell a story" for the Battle of the Sexes but little for the other two. After the students were told of the names they for the most part agreed that the names were "reasonable." But the percentage of correct guesses indicates at best a fairly weak association.

Psychological Considerations

3. <u>Interpersonal comparisons</u>

When games are played once by anonymous players a frequent game theory assumption is that only the ordering of the payoffs of the competitor matters. In order to examine this assumption among naive players, in 1980, 1981 and 1983 three matrices were considered. In 1984 and subsequently a fourth (zero-sum) matrix game was added as a check. The games were utilized nine times in total.

The three noncooperative games A, B and C, utilized are shown in Tables 6-8. They are games with two Pareto optimal noncooperative equilibria, selected so that one equilibrium favors one player and the other favors the other player.

	1	2
1	1, 2	0, 0
2	0, 0	2, 1

Table 6

	1	2
1	100, 2	0, 0
2	0, 0	200, 1

Table 7

	1	2		
1	1, 200	0, 0		
2	0, 0	2, 100		

Table 8

The matrix in Table 7 is constructed from Table 6 by multiplying the payoffs to Player 1 by 100. The matrix in Table 8 is constructed from Table 7 by multiplying the payoffs to Player 2 by 100.

If the size of the numbers and interpersonal comparisons between the payoffs to Player 1 and Player 2 do not matter then all three games should be played the same way. Table 9 shows the choices made. The symbol 111 indicates that the same choice (strategy 1) was made in each game. If the respondents were uninfluenced by a comparison of payoffs they should choose either 111 or 222.

	1980	1981	1983	1984	1985	1988	1989	1991s	1991f Total		Percentage
111	5	4	1	2	3	7	4	4	3	33	13.15%
112	1	2	3	2	3	1	0	2	2	16	6.37%
121	3	7	4	3	0	1	4	4	1	27	10.76%
122	3	1	0	0	2	0	0	0	0	6	2.39%
211	4	1	3	0	0	0	0	0	3	11	4.38%
212	6	1	5	4	5	11	3	2	6	43	17.13%
221	8	10	4	2	3	8	6	8	2	51	20.32%
222	3	5	8	7	8	13	10	6	4	64	25.50%
Total	33	31	28	20	24	41	27	26	21	251	

Table 9Selection of 111 or 222 amounted to 38% of the choices.

A simple zero sum game was utilized six times with the above games, merely as a comprehension or rationality check. The results showed that error or irrational behavior was slight in the play of the zero sum game.

On analyzing the responses for 1980, 1981 and 1983 it was observed that many of the respondents treated the three games as if they were playing the same (unknown) competitor, thus there was possibility for quid pro quo, as was indicated explicitly in many of the verbal descriptions.

From 1983 onwards I changed the instructions to read "You are going to play each game once, against a different unknown competitor." This removed the possibility for quid pro quo.

4. Prominence or symmetry

This brief note revisits and extends the data for an elementary game related to the experiment and analysis of Stone(1958) and Schelling's (1960) comments on prominence and focal points. The results for the Yale runs from 1980 to 1991 have been reported in a previous publication (Shubik, 1991).

Figure 1 shows an elementary game which was handed out in class or during a lecture. The points N (the Nash bargaining point solution) and E (the equal split solution) were not labeled. The instructions to the respondents were as follows.

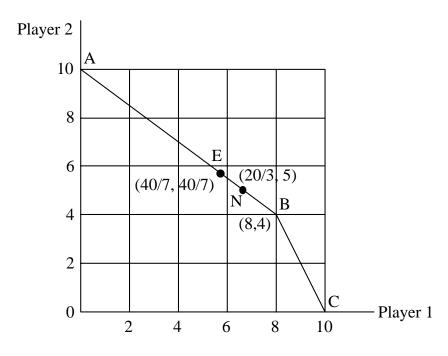


Figure 1

Player 1 and 2 move simultaneously without knowledge of each other's moves. You are Player 2, your strategy is to draw a horizontal line. Player 1 will draw a vertical line. If the two lines intersect within (or on the boundary of) the diagram, that is your payoff. If it is outside you both obtain zero. The hypothesis was that the distribution of responses would be bimodal with the split being between the point of prominence (8,4) and the fair division or equal payoff point of (40/7, 40/7) each.

Table 10 shows the Yale data plus the addition of the Santa Fe Institute and the Hong Kong Polytechnic.

Feb 20 1998 1980 1981 1983 1984 1985 1988 19891991s1991FSFI Point HKP Total <4.0 4. 5. >6.0 Total

Table 10

5. Uncertain prize — Bernouilli Utility — sealed bid for uncertain prize

Students submitted bids for an uncertain prize. Special dice were utilized to generate a random number between 1 and 1000. The payoff was in cents, thus 1000 was \$10. Thus the expected payoff was 500.5 cents

The games were run seven times. Table 11 summarizes the distribution of the strategies.

	1980	1983	1984	1985	1988	1989	1991
Mean bid	3.73	3.58	3.29	3.87	3.09	3.35	3.87
Median bid	4.00	3.34	3.00	4.00	3.50	5.00	4.05
% above \$5	31.8	17	37.5	27.8	22.6	17.39	0
% \$3.33-\$5.00	31.8	35	12.5	27.8	29.0	39.13	80.9
% Below \$3.33	36.4	48	50	44.4	48.4	43.48	19.1
Sample size	29	23	16	14	31	23	21

Table 11

We note a that considerable percentage of the individuals bid more than the expected value of the prize. Even, with small sums of money changing hands one must be careful in interpreting the link between "utility" and money. For example, in 1983 there was a bid of \$10 for the prize with expected value of \$5.05. I was concerned with this "irrational behavior" so I asked the individual who bid the \$10 for an explanation. He noted that his goal was to "win" the bid under all circum-

stances and figured that a bid of \$10 would certainly be sufficient as he estimated that no one would be as heavily motivated for a "win."

5. Gaming for Teaching and Research

Games in class appear to be of considerable aid in involving the students in actively trying to utilize or challenge the concepts they are being taught. They are "sloppy" and hardly controlled.. They are also a source for insight, for suggesting other types of experiments and for sweetening the intuition. Roth (Kagel and Roth, 1995, p. 22) suggests several different styles in experimental gaming including: "Speaking to Theorists," "Searching for Facts (and Meaning)" and "Whispering into the Ears of Princes." All of these have their purposes and different weaknesses.

They should be regarded as complementary approaches with different purposes and costs.

The use of gaming in teaching, in a certain sense comes at a cost of less than zero. Because it is an effective teaching device and can yield experimental (or at least preliminary experimental) insights as a part of a joint product in rhe class room, it provides an economical source for considerable replication and variation in the investigation of both individual behavior and opinion.

References

Brewer, G. and M. Shubik, 1979. *The War Game*, Cambridge: Harvard University Press.

Harsanyi, J.C. and R. Selten, 1988. *A General Theory of Equilibrium Selection in Games*. Cambridge Mass: MIT Press.

Hausner, M., J. Nash, L. Shapley and M. Shubik, 1964. "So Long Sucker — A Four-Person Game" in M. Shubik (ed.), *Game Theory and Related Approaches to Social Behavior*. New York: Wiley, pp. 359–361.

Holt, C., 1999. "Teaching economics with classroom experiments," *Southern Economic Journal*, 65(3), 603–610.

Fouraker, L.E., M. Shubik and S. Siegel, 1963. "Oligopoly Bargaining: The Quantity Adjuster Models," partially reported in Siegel and Siegel, *Bargaining Behavior*. Hightstown, NJ: McGraw-Hill.

Roth, A.E., 1995. *The Handbook of Experimental Economics*, edited by J.H. Kagel. and A.E. Roth. Princeton: Princeton University Press, pp.3–109

Rubinstein, A., 1999. "Experience from a Course in Game Theory," *Ames and Economic Behavior*, 28, 155–170.

Schelling, T.C. 1960. The Strategy of Conflict. Cambridge, MA: Harvard Univ. Press.

Shapley, L. S., 1953. "A Value for *n*-person Games," in R.D. Luce and A.W. Tucker (eds.), *Annals of Mathematical Studies*. Princeton, NJ: Princeton University Press, pp. 307–317.

Shubik, M., 1986. "Cooperative Game Solutions: Australian, Indian and U.S. Opinions," *Journal of Conflict Resolution*, 30, 1: 63–76.

Shubik, M., 1978. Opinions on how to play some simple games. Simulation and Games. 9: 67-88

Shubik, M. 1994. "Prominence, Symmetry, or Other?" *Games and Economic Behavior*, 6: 328–332.

Simon. R. I., 1967. "The Effects of Different Encodings on Complex Problem Solving," Ph.D. dissertation, Yale University.

Stone, J. J. 1958. "An Experiment in Bargaining Games," Econometrica 26: 286–296.