

International Finance in General Equilibrium

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Abstract

Our purpose in this paper is to unify international trade and finance in a single general equilibrium model. Our model is rich enough to include multiple commodities (including traded and nontraded goods), heterogeneous consumers in each country, multiple time periods, multiple credit markets, and multiple currencies. Yet our model is simple enough to be effectively computable. We explicitly calculate the financial and real effects of changes in tariffs, productivity, and preferences, as well as the effects of monetary and fiscal policy.

We maintain agent optimization, rational expectations, and market clearing (i.e., perfect competition with flexible prices) throughout. But because of the important role money plays, and because of the heterogeneity of markets and agents, we find that fiscal and monetary policy both have real effects. The effects of policy on real income, long-term interest rates, and exchange rates are qualitatively identical to those suggested in Mundell–Fleming (without the small country hypothesis), although our equilibrating mechanisms are different. However, because the Mundell–Fleming model ignores expectations and relative price changes, our model predicts different effects on the flow of capital, the balance of trade, and real exchange rates in some circumstances.

Keywords: Currency, cash, fiscal policy, monetary policy, money, trade

JEL Classification: E5, E6, F1, F2, F3

1 Introduction

International trade is most commonly analyzed via general equilibrium theory (see e.g., [2], [13], [34]), with its three-legged apparatus of agent optimization, market clearing (i.e., perfect competition with flexible prices), and rational expectations. International finance, on the other hand, has traditionally been studied via a potpourri of models and methodologies (see e.g., [3], [7], [8], [12], [27]), in which some markets clear and others do not, some prices are flexible and others are not, some expectations are rational and others are formed as if prices were flexible even when they are not, and agent activity is described not by optimization but by behavioral equations.

The traditional international finance literature following Mundell and Fleming recognizes the fundamental importance of interactions among multiple markets, and

its general equilibrium character. But, like the mainstream, Keynesian macroeconomics literature from which it derives, it usually repudiates one (and sometimes all) of the three legs of a genuine, full-bodied general equilibrium approach. The alternative international finance literature inspired by Lucas ([18], [19], [25], [27]) does maintain all three hypotheses of agent optimization, market clearing, and rational expectations. But by adopting the auxiliary hypothesis of an exchange economy with a single “representative agent” who is obliged to put his entire endowment up for sale (to himself!), this literature *a priori* eliminates many interactions between the financial and real sectors of the economy.

Our purpose in this paper is to unify international trade and finance in a genuine general equilibrium model. Our model is rich enough to include multiple commodities (partly to allow for relative price changes and partly in order to distinguish between traded and nontraded goods), heterogeneous consumers in each country, multiple time periods, short-term and long-term assets, and multiple currencies. Yet our model is simple enough to be effectively computable.

We believe that international finance cannot properly be separated from international trade because the most interesting financial questions invariably turn on the interactions of real and financial variables. For example, if a country reduces its tariffs, or becomes more productive, or more impatient, will its currency appreciate or depreciate? What will happen to income, to short- and long-term interest rates, to price levels and to the rate of inflation, to the real terms of trade, and to the balance of trade? What will happen if a country’s government, or a trading partner’s government, spends more, or prints more money, or increases the rate at which it expands its money supply?

If the real sector is important, then international finance depends on international trade, and if the latter requires a general equilibrium analysis, then logically speaking, so must the former. Moreover, there are direct advantages to a full-bodied general equilibrium approach to international finance. Foremost among these is the accounting clarity which comes from explicitly modeling every transaction: we understand the demand for money better when we see where and why each agent spends each dollar he obtains. For example, if policy essentially changes the regime in which an agent acts, the model will specify behavioral changes that are logical for him given his preferences. In contrast, models which exogenously specify behavior that is not derived from underlying preferences can seem plausible in some regimes and absurd in other regimes. By deriving behavior from utility maximization, general equilibrium also makes welfare analysis, and especially distributional questions, amenable to rigorous analysis. Finally, without general equilibrium one must resort to reduced form behavioral equations in which various indirect effects are *a priori* ignored. In general equilibrium one sees all the indirect effects and can judge whether, and under what conditions, they can safely be ignored.

The mainstream literature in international finance deriving from the Mundell–Fleming extension of the Keynesian IS–LM model has avoided what we call full-bodied general equilibrium for the same reasons that Keynesian macroeconomics has. First, there is a powerful Keynesian intuition that goods markets are slower

to clear than asset markets, and therefore in the short-run goods market do not clear. In Keynesian macroeconomics this meant that commodity and labor prices were taken to be sticky in the short run. Although we do not wish to dogmatically reject the hypothesis of fixed prices (and we are glad to see others investigate its consequences), we feel that it is also worthwhile to fully analyze the consequences of flexible prices. Second, it is more convenient to work with a single period model, as in IS–LM. This, however, leaves expectations about future price levels and exchange rates to arbitrarily pre-specified behavioral rules. Some theorists are comfortable knowing that expectations can be systematically biased; this outlook is represented by temporary equilibrium models, such as [14], [15], [17], and [16]. By contrast, we work with an explicit multi-period model so that expectations (particularly resulting from policy changes) must conform to the subsequent reality. The effects of policy, if there are any, cannot then be attributed to irrational expectations. The effect of policy changes on long-term interest rates, inflation, and exchange-rate trajectories is one of the most important features of our model. The Mundell–Fleming model also simplifies the picture by imposing the small country hypothesis, which fixes foreign (and therefore with perfect capital mobility, domestic) interest rates. We work with multiple countries of arbitrary relative size; an extended example is computed for two roughly equal countries.

Third, and most importantly, it has not been clear how to maintain agent optimization and a positive value for fiat money in a finite horizon model. In the last period no optimizing agent will accept worthless fiat money, and therefore optimizing agents calculating backward to the beginning will immediately set the price of money equal to zero. This puzzle is not avoided at all by postulating cash-in-advance constraints. We overcome the problem by also adding the possibility of borrowing (selling bonds) to the central bank; then we prove that in our model money always has positive value, if there are potential gains to trade.

Thus in spite of these Keynesian doubts, we present a finite horizon model in which all prices (including exchange rates) are flexible and all markets clear, in which expectations are rational, and in which money always has positive value. We work out an elaborate example to show that our model is easily computable. We have not tested our comparative statics predictions by estimating parameters, but we believe our conclusions are sensible and generally in accordance with the stylized facts without being obvious at first glance. Indeed there are so many endogenous variables, including trades in multiple commodities, exchange rates, real exchange rates, inflation rates, short and long nominal interest rates, and commodity prices, that it is inconceivable that one could automatically work out all their changes without any analysis.

Our model resembles the real business cycle literature in the sense that all markets clear all the time. But our comparative statics results are compatible with Keynesian analysis. In particular, monetary and fiscal policy are not neutral. Expansionary monetary policy leads to higher income and a fall in nominal interest rates, higher domestic prices, a currency depreciation, a fall in the real terms of trade and an increase in net exports. All these effects (except the price changes) are consistent

with the Mundell–Fleming model. A short burst of expansionary fiscal policy leads to higher domestic economic activity, higher long-term real and nominal interest rates, a temporarily higher exchange rate, and higher expected inflation following a temporary drop in prices. Again, most of these effects are consistent with the Mundell–Fleming model.

Though most of the effects of monetary and fiscal policy in our model are identical to the Mundell–Fleming model, important differences remain because the Mundell–Fleming model implicitly assumes that expectations about future exchange rates never change, and that the real terms of trade move in lock step with the exchange rate. In the Mundell–Fleming story, expansionary fiscal policy tends to increase output and interest rates; the latter causes an influx of foreign capital, which tends to appreciate the domestic currency; this in turn encourages imports and discourages exports, increasing the balance of trade deficit. In our multi-period model, however, we notice at once that this story misses several important elements.

In the first place, when the expansionary fiscal policy eventually reverts to its normal levels, the currency will also tend to revert to its previous exchange level. Rational investors therefore expect a currency depreciation following the appreciation, and hence it is no longer clear that there will be an influx of foreign capital, despite the higher long term interest rates. “Overshooting” of exchange rates is a fundamental property of our equilibrium, as it must be in any flexible exchange rate model where short run policy has short run effects on exchange rates. In combination with rational expectations, this imposes significant restrictions on equilibrium behavior.¹ In the second place, government expenditures might be proportioned quite differently from private demand, and this too could affect the balance of trade.

In our model the effect of expansionary fiscal policy on the balance of trade therefore depends on a more precise description of the policy. For instance, if the government expenditures are financed by contemporary taxes (a balanced budget expansion), and if the government spends its money exclusively on domestic nontraded goods, which does not seem a completely unreasonable assumption, then we find that the balance of trade improves rather than worsens. The reason is clear: the balanced budget multiplier (with flexible prices) turns out to be less than 1 (in our example it is .4). Hence the government expenditure crowds out some domestic expenditure, part of which would have been on foreign goods. Even if the real terms of trade move in the same direction as the exchange rate, this is a sign of flagging demand for imports, not a stimulus to extra imports. If on the other hand, the government buys all goods in the same proportion as the economy as a whole, then we find ambiguous effects on net imports in our example. If in addition we suppose the expenditures are financed by new bonds, which will be paid off much later by raising taxes on agents not active at the time of the government expenditures, then our example confirms an increase in

¹Incidentally, the same is also true of price levels; flexible commodity price models in which short run policy has short run effects on the price levels will also display overshooting of price levels, which with rational expectations, will impose restrictions on the equilibrium behavior. Dornbusch [7] first brought attention to overshooting of exchange rates. But he deduced it as a consequence of a model in which markets did not instantly adjust. We note here that it must arise when all markets do immediately clear.

the balance of trade deficit depending on how the tax revenue to pay off the bonds is raised. The only program which is guaranteed to yield the Mundell–Fleming effects on output, interest rates, exchange rates, real terms of trade, and the balance of trade is government expenditure that is targeted entirely at domestic goods that are also exported. (In this case the extra demand raises their prices relative to imports, which is the real terms of trade, and chokes off exports).

The domestic effects of monetary policy are also identical in our model and in Mundell–Fleming, but again the underlying mechanisms are different and so are some of the international effects. In Mundell–Fleming, short term open market operations raise output and lower interest rates, so capital flees, depreciating the currency and thus causing an increase in net exports. In our model we again observe overshooting, so that with rational expectations, the fall in the exchange rate is accompanied by an expected appreciation, rendering the direction of capital mobility ambiguous. Furthermore, in our example, the real terms of trade turn against the domestic country because the increased economic efficiency stemming from lower interest rates increases the demand for imports and the willingness to export. The real balance of trade, as it were, is thus not much different, but measured in (depreciated) dollars, starting from an original deficit position, the balance of trade deficit increases in our example, rather than decreasing as predicted by Mundell–Fleming. As with fiscal policy, the effect of monetary policy depends on the precise nature of the policy. Does the open market operation involve long term bonds or short term bonds? Is the policy instantaneous, or is it anticipated? Is it expected to continue into the future, or will the government allow the money supply to contract (compared to what it would have been absent the open market operations) when it comes time for the bonds to pay off? We defer our discussion of these topics to Section 11.

A fundamental reason why the international effects of fiscal and monetary policy are different in our model and in the Mundell–Fleming model is that in the latter model there is essentially one channel by which policy affects the currency exchange rates: they are determined largely by the flow of capital controlled by agents who weigh the marginal benefits of money invested domestically against money invested abroad. In our model agents also weigh the marginal benefits of money spent on commodities domestically or abroad. This opens an entirely new channel for the determination of exchange rates. For example, if there is a burst of domestic economic activity, so that the same money chases more transactions (and if velocity does not change to make up all of the difference), domestic prices for commodities will go down, attracting foreign money aimed at domestic commodity purchases, and the currency will appreciate.

In summary, our model of flexible prices, rational expectations, and explicit agent optimization makes exactly the same qualitative predictions about output, interest rates, and currency exchange rates as the Mundell–Fleming model (modified to allow for two equal-sized countries), while it differs with respect to expectations, and possibly also with respect to real terms of trade, and the balance of trade.

We believe any sensible general equilibrium model should agree with Keynesian predictions about the effects of policy on output and interest rates. Balanced budget

fiscal policy transfers wealth (say via lump sum taxes) from private agents who would have transacted only a portion of it and places it in the hands of the government who transacts all of it. This action is bound to stimulate economic activity and measured income, even if it does not raise welfare. Moreover the government spends this wealth on markets that meet earlier than the agents would on average have spent the wealth themselves, raising the relative price of current consumption over future consumption. Expansionary fiscal policy is thus also bound to raise interest rates. Expansionary open market operations make it easier to borrow money, and thus reduce nominal interest rates. Lower nominal interest rates on money must improve the efficiency of trade in any economy where money plays a fundamental role in trade.

Keynesians often wave the banner of involuntary unemployment and variable velocity to argue for their view that policy matters. We do not wish to discount the importance of either. But our analysis shows that the qualitative features of domestic Keynesian policy analysis hold in a world which has no involuntary unemployment and a fixed velocity of money. Involuntary unemployment (or more generally fixed prices), variable velocity of money, and irrational expectations might increase the magnitude of domestic Keynesian effects, but they are not responsible for their qualitative features.

We might well ask, why did not Lucas detect these Keynesian features in his general equilibrium analysis of the macroeconomy and international finance? The answer is surprisingly simple. In his early papers, Lucas postulated a world in which each agent is obliged to sell the whole of his endowment in each period. Real income, which by definition is the aggregate of sales in a period, is thus exogenously fixed, independent of any government monetary or fiscal policy that does not directly create new commodities.

In cash-in-advance economies, trade is usually inefficient because the positive interest rate on money discourages transactions (assuming the nominal interest rate exceeds the real interest rate). Agents cannot always time their sales so as to instantaneously deposit the money and earn the maximal rate of interest. As a result they make fewer transactions. In our stylized model, agents who sell goods in period 0 cannot spend the money or deposit it in banks until period 1. An agent who has no cash and wishes to trade his \$1 apple in period 0 for a \$1 orange in period 0 must go to the bank and borrow the \$1 to buy the orange, but since the interest rate is positive, he will need to sell more than 1 apple in order to repay the loan. If he values the orange only slightly more than the apple, he will forego the entire set of transactions; in particular, he will not sell the apple. A similar effect obtains for intertemporal trade if it is costly to make trips to the bank, even if sales receipts can be used for contemporaneous purchases. An agent who wants to trade his apple today for an orange tomorrow could sell the apple today, but if there were inflation so that the orange tomorrow cost more than \$1, he would have to go to the bank to deposit the \$1 in order to have enough to pay for the orange. But if it were costly to go to the bank, he might forego the entire set of transactions.

Keynesian IS–LM models with variable velocity implicitly incorporate similar effects. When interest rates go up, agents are assumed to demand less money. The

implicit justification is that each agent makes more frequent trips to the bank, partially mitigating the trading inefficiency from higher interest rates but substituting “shoe leather” costs. In the early Lucas model these inefficiencies do not affect the number of transactions because the agent is obliged to sell all his apples anyway, no matter what the interest rate.

Our methodological approach has parallels in the overlapping generations literature and in the work of Lucas and his followers. Both of these literatures maintain all three legs of the general equilibrium paradigm. But both avoid the backward induction value of money puzzle by working in an infinite horizon which has no last period from which to start the backward induction. The infinite horizon introduces several inconvenient features to the modeling which we have sought to avoid. In overlapping generations models, there can be a continuum of different equilibria, and one country can run a balance of trade deficit for all time (see [23]). A further difficulty is that the infinite horizon makes the model almost completely intractable from a computational point of view. Indeed, in order to make his model tractable, Lucas makes the heroic assumption that each country is represented by a single agent, and that all these representatives are identical. In order to motivate trade inside each country, the representative agent is given a split personality that trades with itself at some moments, and pools all its resources at other moments. By contrast, we develop a computationally tractable model in which there is genuine diversity between agents. Thus although our methodological approach is akin to Lucas, our policy conclusions are not.

Our closest methodological precursor is Martin Shubik [28], [31] who introduced a central banking sector with exogenously specified stocks of money, and cash-in-advance constraints. Shubik ([28], [29]) also emphasized the virtues of modeling each transaction. Grandmont ([14], [15], [17], [16]) also introduced a banking sector and he emphasized the inefficiency of trade with money. (The cash-in-advance constraint can be traced at least as far back as Clower [5], and has been used by Lucas.) Though they had most of the individual ingredients, neither Grandmont nor Shubik combined a central bank which makes loans, with cash in advance constraints and with private money. It is this triple combination which is crucial to make equilibrium determinate and to separate our international monetary equilibrium from competitive equilibrium. Neither Grandmont nor Shubik saw the need for a gains to trade hypothesis, and neither proved the existence of a monetary equilibrium distinct from competitive equilibrium in a finite horizon. Neither focused on international finance.

Our analysis owes much to the framework developed by Dubey and Geanakoplos [9] in a one-period general equilibrium model, and then extended by them [10] in unpublished work to multiple periods in order to combine macroeconomics and general equilibrium. In particular we owe to [10] the proof that monetary and fiscal policy are not neutral and to [9] the proof that money can have positive value in a finite horizon model. We extend those models by considering many countries and international finance (though we drop uncertainty). In the Dubey–Geanakoplos framework, as in ours, agents begin with stocks of cash which they own free and clear with no obligations. But they also have the option of voluntarily borrowing more money from

the central bank, at endogenously determined interest rates, which, if positive, will cause them to owe more money than they borrowed. (These bank loans are available at different moments and for different periods of time, and therefore agents will be repaying them throughout the time horizon of the model.) In equilibrium all the money left in the agents' hands in the last period (including their private endowments) will be owed to the bank, so money will have value (i.e., some agents will give up goods for money) in the last period because they need the cash to pay off their debts. The central bank is regarded as an arm of the government which provides exogenously fixed quantities of money for loans of varying lengths at various time periods, and then collects on its debts.² Equivalently, the bank executes open market transactions, buying bonds from the public. By varying the quantities of money available at the bank (i.e., the value of the open market purchases), the government can control the stocks of money in the economy.

We show in our model, that under a gains to trade hypothesis, “international monetary equilibrium” (IME) always exists, and money has value, and exchange rates are well-defined. (If there were no gains to trade, there would be no reason to obtain money, and money would have no value.) Moreover, although we do not prove it here, there are typically only a finite number of equilibria, so one can speak of *the* exchange rate or price level determined in equilibrium, in contrast to models of international trade based on overlapping generations economies. IME allocations are typically not Pareto efficient because purchases must be made with money, and money is scarce. The scarcity of money gives money value (i.e., it makes the relative price between commodities and money less than infinity) and it makes interest rates positive (i.e., it makes the relative price of future money, or bonds, to present money less than 1).

In Sections 2–4 we describe the model. In Section 5 we note that the Keynesian sources of demand for money apply even though we derive behavior from utility maximization. We also observe that our simple choice of the order of markets requires the velocity of money to be 1. However, in our model the “real” velocity of money, by which we mean the stock of money divided by the amount of real transactions (somehow aggregated) is variable and endogenous. If we allowed for simultaneous markets the velocity would also be variable and endogenous.

In Section 6 we state our existence theorem and note that it depends on potential gains to trade. It is curious that previous authors have not found it necessary to invoke such a hypothesis in discussing money. We note that if the ratio of government deficits to central bank loans and open market operations exceeds the gains to trade, then equilibrium will not exist. At some finite level of debt, prices will explode in a hyperinflation. In Section 7 we note that as government expenditures and private money shrink to 0, our international monetary equilibrium approaches competitive equilibrium.

In Section 8 we show that in the presence of private money or government deficits,

²No explanation is given why the central bank is willing to take the money in the last period (or what happens to the profits of the bank if interest rate payments are positive). However, it is possible to extend the model to allow the bank to be owned by shareholders instead of the government, and still support a positive value of money as in Shubik–Tsomocos [30].

international monetary equilibrium is not Pareto efficient, and monetary policy necessarily has real effects.

In Sections 9 and 10 we derive straightforwardly many of the standard relationships of international finance, including the uncovered interest rate parity, purchasing power parity, the Fisher effect, long run international trade balance, and a version of the quantity theory of money.

Armed with these general principles we turn in Section 11 to analyzing concrete comparative statics changes in computable general equilibrium models. Often the general principles can point us to the correct directions of change, but other times it is only after tracing out the new equilibrium that we can rationalize the outcome. For example, in general equilibrium the exchange rate must simultaneously satisfy purchasing power parity, uncovered interest parity, and it must guarantee a long-run trade balance. Some shocks to the economy will move these three requirements in different directions, apparently leaving the direction of change of the exchange rate ambiguous. Only by simultaneously equilibrating all the equations and variables can one determine the direction of movement of each.

The comparative statics conclusions we come to depend of course on the parameterization of tastes and endowments that we have chosen. It has been suggested that general equilibrium is not useful because in such models anything can happen when policy parameters are changed, if the initial data of the economy is chosen fortuitously. We regard this not as a criticism of general equilibrium, but as its vindication. One cannot hope to fully understand any policy change until one knows how *all* its effects depend on the behavior of the agents.

In order to give a more comprehensive treatment of issues arising in international finance, we would need to extend the model we have described here to incorporate production, uncertainty, and incomplete markets. Such an elaborate framework might also permit us to reconsider some of the traditional international trade results in a financial context. We believe that incorporating money is the crux of the problem, and therefore that this extension is within our grasp.³ Finally, it seems evident that after we gain some experience with these models, we will be able to derive our comparative statics results in a more general context than our example.

2 The Model

2.1 The International Economy

We consider countries $\alpha \in C = \{1, 2, \dots, C\}$ and the time horizon $t \in T = \{0, 1, \dots, T\}$. Each household $h \in H = \{1, \dots, H\} = \bigcup_{\alpha \in C} H^\alpha$ belongs to a single country; we write $h \in H^\alpha$ if household h belongs to country α .

The commodities $\ell \in L = \{1, \dots, L\}$ are all perishable and cannot be inventoried between periods.⁴ We also associate each commodity with a single country, and we

³Dubey–Geanakoplos [10] includes money, production, uncertainty, and incomplete markets in a one-country model, and this suggests that our international framework can also be so extended.

⁴Perishability is assumed for convenience only; it reduces notation by half. Dubey–Geanakoplos [10] show how to include durable commodities.

write for example $\ell \in L^\alpha$. (America and Japan may both sell cars, but they are American cars and Japanese cars.) Furthermore, we divide the goods $\ell \in L^\alpha = D^\alpha \cup F^\alpha$ in each country α into domestic goods which can only be purchased by households in country α , and international (foreign) goods which can be purchased by all agents.

Agent $h \in H^\alpha$ has initial endowment $e^h \in \mathbb{R}_+^{TL^\alpha}$ of country α commodities⁵ and utility function $u^h : \mathbb{R}_+^{TL} \rightarrow \mathbb{R}$. Consumption of good $t\ell$ by agent h is written $x_{t\ell}^h$. We assume that every commodity is actually present in the international economy, i.e., for each $\alpha \in C$ and $t \in T$ and $\ell \in L^\alpha$,

$$\sum_{h \in H^\alpha} e_{t\ell}^h > 0.$$

Moreover, no agent has the null endowment of commodities in any time period, i.e., for each $t \in T$, $h \in H$,

$$e_{t\ell}^h > 0 \text{ for some } \ell \in L^\alpha.$$

Let A be the maximum amount of any commodity $t\ell$ that exists in the international economy and let 1 denote the unit vector in \mathbb{R}^{TL} . Then we assume that each u^h is continuous, concave, strictly increasing in each variable and each commodity matters, i.e.,

$$\exists Q > 0 \text{ such that } u^h(0, \dots, Q, \dots, 0) > u^h(A1),$$

for Q in an arbitrary component. (Our results remain unaltered if instead of the previous condition we assume smoothness of u^h .)

2.2 Governments and Central Banks

Each country $\alpha \in C$ is run by a government which has the capacity to act on markets (perhaps through an agency called its central bank or treasury or Federal Reserve). We shall take these government actions as exogenously specified, and analyze their consequences.

We shall also allow each government to buy commodities and to buy and sell bonds and foreign currency. We interpret government purchases of (domestic) commodities as part of fiscal policy; transactions in the bond market are regarded as open market operations for monetary policy; and transactions in foreign currencies are thought of as efforts to control exchange rates.

2.3 The Time Structure of Markets

In each period $t = 0, 1, \dots, T$, four markets meet: first the three financial markets, beginning with the short-term (intra-period) bond market, followed by the foreign exchange market and then the long-term (inter-period) bond market. The commodity markets are the last to meet in the period. At the end of the period tax revenue is redistributed to the public. Long-term bonds come due after the short-term bond

⁵So as not to clutter our notation, we shall sometimes write $e^h \in [\mathbb{R}]_+^{TL}$ when we should write $(e^h, 0) \in [\mathbb{R}]_+^{TL}$, since $L^\alpha \subset L$.

market meets, but before the foreign exchange market meets. Short-term bonds come due at the end of the period. Each period thus has seven transaction moments: short bonds, long bond deliveries, foreign exchange, long bonds, commodities, government transfers, short bond deliveries.

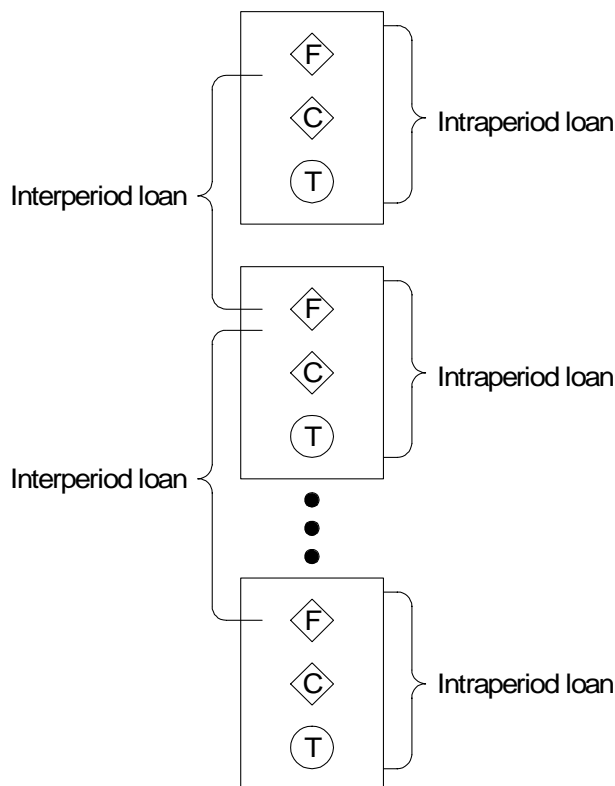


Figure 1

Figure 1 indicates our time line, including the moments at which the various loans come due. We make the sequence precise when we formally describe the budget set. The ordering of the markets is unimportant if the time horizon T is large (and if goods are durable). We have chosen the order (with the financial markets first) in order to maximize the amount of possible trading activity per period.

2.4 Money and Markets

We postulate a cash-in-advance requirement for any purchase. To buy a commodity or bond or foreign currency, an agent must pay cash, which he can obtain from his private monetary endowments, or out of inventories from previous market transactions. Since fiat money is the stipulated means of exchange, transactions in each market have a convenient physical interpretation according to which money is traded for equal value of commodities or bonds or foreign currency.

We regard a market as a symmetric exchange between two instruments. In standard Arrow–Debreu terminology, a market bears only one name. Since in our model, money is always one of the instruments, we too can give each market a single name, but one could easily imagine other markets (e.g., for credit card purchases) where neither side was cash: for more discussion see Dubey–Geanakoplos [10]. Just as agents cannot “sell” money they do not have on a market, so in our model agents cannot sell commodities or foreign exchange they do not have. The only exception is credit markets, where we allow agents to write their own promises (bonds).

The necessity to have cash in order to purchase commodities destroys the “classical” dichotomy between the real and financial sectors of the economy if interest rates are positive. (It also explains why we have placed the credit markets before the foreign exchange and commodity markets on our time line.)

Each country $\alpha \in C$ has its own fiat money with which transactions of its domestic and international goods take place. We call the money of country α , α -money.

Money enters the international economy in three ways. First, it may be present in the private endowments of agents. Agent $h \in H^\alpha$ has an endowment $m_{t\alpha}^h$ of α -money, for each $t \in T$.⁶ Second, when any government α (perhaps through a central bank) purchases commodities, bonds, and foreign currency with the appropriate currency, it injects money into the economy. Third, when the α -government repays previously issued α -government bonds, it also injects money into the economy. The channel by which money enters the economy influences the effect the money has on the economy, as we shall see in great detail later on.

Money exits the system in three ways: as taxes, in purchases of government bonds (or government supplied foreign currency), and as payment on bonds (promises) sold to the government. Of these, the last is the most important for our paper.

We now discuss the various markets in reverse chronological order.

2.5 The Commodity Markets

Commodity prices $p_{t\ell}$ are taken by the agents as fixed. Let $b_{t\ell}^h \equiv$ amount of α -money sent by agent h to trade in the market of commodity $\ell \in L^\alpha$. A good from country α can only be purchased with α -money. Agents cannot sell commodities they do not own, so $q_{t\ell}^h \leq e_{t\ell}^h$. Note that if $\ell \in D^\alpha$ and $b_{t\ell}^h > 0$, then $h \in H^\alpha$, whereas, if $\ell \in F^\alpha$ then $h \in H$. Let $M_{t\ell}^\gamma$ be the amount of α -money the γ government puts up to buy commodity $\ell \in L^\alpha$. These government expenditures are taken as exogenous. Let $q_{t\ell}^h \equiv$ amount of good ℓ offered for sale by $h \in H^\alpha$. In equilibrium, at positive levels of trade, $0 < p_{t\ell} < \infty$,

$$p_{t\ell} = \frac{\sum_{h \in H} b_{t\ell}^h + \sum_{\gamma \in C} M_{t\ell}^\gamma}{\sum_{h \in H^\alpha} q_{t\ell}^h}.$$

⁶This is “outside money” in the macroeconomics jargon, because it is owned free and clear by the public with no offsetting debt, or “liquid wealth,” in the sense that it can be used immediately to purchase commodities and assets. We can also interpret $m_{t\alpha}^h$ as the face value of previously acquired zero-coupon government bonds maturing at the beginning of period t , or as government transfers.

All the commodity markets meet simultaneously; hence cash obtained from the sale of commodity $t\ell$ cannot be used for the purchase of another commodity $t\ell'$ in the same period t .

2.6 Stocks and Flows

In the abstract Arrow–Debreu model it is difficult to distinguish stocks and flows, especially when public production is involved. Even when they can be distinguished formally, there is no importance in the distinction; one might as well assume that every good is bought and sold each period. In the Lucas (exchange economy) model, agents are required to put everything up for sale in each period, so there can be no distinction between stocks and flows.

In our model the sales $q_{t\ell}^h$ of commodities by agents are endogenous; agents are not required to put anything up for sale. In fact, since sales equal purchases in equilibrium, the total quantity of sales is a good measure of aggregate economic activity. (In Lucas it is essentially exogenous.) Indeed we shall call

$$Y_t^\alpha \equiv \sum_{\ell \in L^\alpha} \sum_{h \in H^\alpha} p_{t\ell} q_{t\ell}^h \text{ and } \bar{Y}_t^\alpha \equiv \sum_{\ell \in L^\alpha} \sum_{h \in H^\alpha} q_{t\ell}^h$$

the nominal and real incomes of agents in country α in period t . They are endogenous variables.⁷

2.7 The Foreign Exchange Markets

The foreign exchange markets meet once every period, and endogenous exchange rates $\pi_{t\alpha\beta}$, $\alpha, \beta \in C$, $t \in T$, are determined at equilibrium, though agents regard them as fixed. There is a foreign exchange market for each pair of currencies, and therefore the number of such markets in the world economy is $TC(C-1)/2$. In these markets individual agents with their domestic currency, or foreign exchange they have accumulated in previous periods, can participate. Let $b_{t\alpha\beta}^h \equiv$ amount of α -money offered by h to trade against β -money at $t \in T$. Let $M_{t\alpha\beta}^\gamma$ be the amount of α -money government γ puts up at time t to buy β -money. In equilibrium, at positive levels of foreign exchange trade, $0 < \pi_{t\alpha\beta} < \infty$,

$$\pi_{t\alpha\beta} = \frac{\sum_{h \in H} b_{t\beta\alpha}^h + \sum_{\gamma \in C} M_{t\beta\alpha}^\gamma}{\sum_{h \in H} b_{t\alpha\beta}^h + \sum_{\gamma \in C} M_{t\alpha\beta}^\gamma}.$$

Note that $\pi_{t\alpha\beta} = (\pi_{t\beta\alpha})^{-1}$.

For example, if agent h of country α is willing to offer an amount of, say, $b_{t\alpha\beta}^h$ dollars to acquire foreign exchange, say of Germany, and h of country β offers $b_{t\beta\alpha}^h$ marks, then $h \in H^\alpha$ acquires $f_{t\beta}^h = \pi_{t\alpha\beta} \cdot b_{t\alpha\beta}^h$ marks and $h \in H^\beta$ acquires $f_{t\alpha}^h = (\pi_{t\alpha\beta})^{-1} \cdot b_{t\beta\alpha}^h$ dollars.

⁷The real income is an arbitrary number since it depends on the units in which we measure goods. Some policy changes, however, will increase \bar{Y}_t^α no matter how we measure the units.

2.8 Bond Markets = Credit Markets = Loan Markets

We distinguish two kinds of bond (equivalently credit or loan) markets. Short-term (intraproduct or overnight) α -bonds promise 1 unit of α -money at the end of the same period in which they are taken. Long-term (interperiod) α -bonds promise 1 unit of α -money at the beginning of the next period, but after the next period short loan begins. Note that the long-term loans can be thought of as one-period zero coupon bonds. By rolling over a long loan to the next short loan to the next long loan, agents can borrow against any future time period. Let $r_{t\alpha}, \bar{r}_{t\alpha}$ be the interest rates on the short-and long-term loans in country α respectively (we do not consider interperiod loans of longer duration for simplicity, since with rollover and the absence of uncertainty, our results would remain essentially the same). An agent who borrows ε in the intraproduct (interperiod) loan market owes $(1 + r_{t\alpha})\varepsilon$ ($(1 + \bar{r}_{t\alpha})\varepsilon$) at the end of period t (at the beginning of period $t + 1$). Agents regard the interest rates as fixed.

The bank of country γ has exogenously specified positive quantities of α -money $M_{t\alpha}^\gamma, \bar{M}_{t\alpha}^\gamma)_{t \in T}$ which it auctions off in the short- and long-term α -bond market, respectively, in each period. The bank also sells bonds $(\mu_{t\alpha}^\gamma, \bar{\mu}_{t\alpha}^\gamma)$ in the same markets.

Agents are permitted to sell and buy bonds, i.e., to borrow and deposit money in the loan markets in all countries. Borrowing and depositing occur in local currencies. (Acquisition of foreign currency occurs via the foreign exchange market whose function is explained in Section 2.7.) Let $\mu_{t\alpha}^h$ (or $\bar{\mu}_{t\alpha}^h$) be the amount of zero-coupon bonds issued by agent h , or equivalently, the amount of money agent h chooses to owe on the short loan (or long loan) taken in period t in country α , for each $h \in H$. Let $d_{t\alpha}^h$ and $\bar{d}_{t\alpha}^h$ be the amounts of α -money agent h deposits in period t , or equivalently, the amount of money h spends on purchases of short-term and long-term bonds. If all agents pay exactly what they owe, then we must have that

$$(1 + r_{t\alpha}) = \frac{\sum_{h \in H} \mu_{t\alpha}^h + \sum_{\gamma \in C} \mu_{t\alpha}^\gamma}{\sum_{h \in H} d_{t\alpha}^h + \sum_{\gamma \in C} M_{t\alpha}^\gamma} \text{ and } (1 + \bar{r}_{t\alpha}) = \frac{\sum_{h \in H} \bar{\mu}_{t\alpha}^h + \sum_{\gamma \in C} \bar{\mu}_{t\alpha}^\gamma}{\sum_{h \in H} \bar{d}_{t\alpha}^h + \sum_{\gamma \in C} \bar{M}_{t\alpha}^\gamma}.$$

We do not allow agents to default, hence all bonds of the same maturity and currency are perfect substitutes, so it is no loss in generality to aggregate them into one market.

Bonds are the only instrument agents are allowed to sell without owning. Essentially any agent can write out his own promise, which then becomes a bond. However, he is limited in how many he can sell by the condition he does not default in equilibrium.

Bonds and money can be inventoried; they are the stores of value in our model. Neither provides utility directly to the agents. We denote by $\hat{m}_{t\alpha}^h$ the amount of α -money that agent h chooses to carry over from period t to period $t + 1$.

2.9 Tariffs

As often is the case in international trade, the government imposes tariffs on imported goods. We model tariffs as a per unit fiat α -money tax $\varphi_{t\ell}^\alpha$, $\ell \in \hat{L}^\alpha = \bigcup_{\beta \in C} F^\beta \setminus F^\alpha$, levied on each commodity imported into country α . (An alternative way to model tariffs would be as percentages of the prices formed at equilibrium for the internationally traded goods.) Tariffs are exogenously imposed and inter-country strategic considerations for the imposition of tariffs do not take place. Tariff revenues in period t are equal to $R_t^\alpha = \sum_{h \in H^\alpha} \sum_{\ell \in \hat{L}^\alpha} \varphi_{t\ell}^\alpha \cdot x_{t\ell}^h$. The revenues from tariffs collected in period t by government α are redistributed back to the agents of country α at the end of period t in lump-sum transfers. Since the government does not know a priori what it will accumulate from tariffs, redistribution is modeled as a vector of percentages $(w_t^h)_{h \in H^\alpha, t \in T}$, $\sum_{h \in H^\alpha} w_t^h = 1$, of the realized revenues. In equilibrium, realized revenues correspond to expected revenues. We note that tariffs, besides their price effect and impact on exchange rates, also have wealth effects in the economy through the redistribution of revenues via lump-sum transfers.

Finally, note that tariffs are levied in terms of domestic money whereas payments for imported goods occur in the currency of the country which exports them.⁸

For simplicity of notation we do not allow for any other taxes. In our comparative statics calculations we investigate the effect of lump sum taxes by subtracting Δ from some agent's endowment of money.

2.10 Summary of Government Actions

The government actions of country $\gamma \in C$ are given by the vector

$$(M^\gamma, \mu^\gamma) \equiv (M_{t\alpha}^\gamma, \mu_{t\alpha}^\gamma, M_{t\alpha\beta}^\gamma, \bar{M}_{t\alpha}^\gamma, \bar{\mu}_{t\alpha}^\gamma, M_{t\ell}^\gamma)$$

for $t \in T$, $\alpha, \beta \in C$, and $\ell \in L$. Note that we do not inquire how government γ gets currency α to spend. If $\gamma = \alpha$, it can simply print it. For $\gamma \neq \alpha$, it comes out of currency reserves. For readers uncomfortable with this, nothing is harmed by supposing $M_{t\alpha}^\gamma = 0$ if $\alpha \neq \gamma$. We also do not necessarily impose a government budget constraint, such as requiring that the government spends no more than it borrows and taxes. (The existence of equilibrium is perfectly compatible with the government printing money to finance its expenditures.) In our examples in Section 11, however, we shall indeed suppose that the government does have such a budget constraint.

3 The Budget Set for Agents

For simplicity, we have supposed that the commodities are perishable, and that each market meets only once each period. Aside from putting an upper bound on the velocity of money, which is easily corrected in more complicated models, the drawback of this simplicity is that we must carefully choose the order in which the markets

⁸Alternatively, we could have modeled tariffs to be paid in foreign currency and subsequently to be levied from the exporter.

meet.⁹ Accordingly, we suppose that at the beginning of each period, intraperiod bank loans are available so that agents can borrow the cash to make purchases. The foreign exchange markets must meet before the long bond and commodity markets in order for agents with currency α to obtain β -money to buy long bonds and goods in country β in the same time period. The revenue from tariffs is assumed to be distributed before the end of the period in which they are collected, and before the intraperiod loans come due. The timing of the interperiod loan does not much matter, as long as it is such that agents can roll over loans by alternating short and long loans. Our sequence of moves is indicated in Figure 1. Note that there are seven transaction moments in each period. The endowments, the prices, and the timing of markets imposes various constraints on the cash balances of agents which we now indicate.

Denote the macro variables which are determined in equilibrium, and which every agent regards as fixed, by $\eta = (p, \pi, r, \bar{r}, R)$. Denote the choices of agent h by $\sigma^h \in \Sigma^h(\eta)$, where $\sigma^h = (x^h, \mu^h, \bar{\mu}^h, d^h, \bar{d}^h, b^h, q^h, \hat{m}^h) \in \mathbb{R}_+^{LT} \times \mathbb{R}_+^T \times \mathbb{R}_+^{T-1} \times \mathbb{R}^{CT} \times \mathbb{R}_+^{C(T-1)} \times \mathbb{R}_+^{LT+C(C-1)T} \times \mathbb{R}_+^{LT} \times \mathbb{R}_+^T$ is the vector of all his market decisions. The budget set $B^h(\eta) = \{\sigma^h \in \Sigma^h : (\text{ta1} - \text{ta7}) \text{ below}\}$, where $\Delta(\text{i})$ represents the difference between the RHS and LHS of inequality (i).

For all $t < 0$, we suppose that every choice variable is 0, and we suppose $\bar{\mu}_{T\alpha}^h = 0 \forall h \in H, \alpha \in C$. For $0 \leq t \leq T$, and all $\alpha \in C$, we must have:

$$d_{t\alpha}^h \leq m_{t\alpha}^h + \hat{m}_{t-1,\alpha}^h \quad (\text{ta1})$$

$$\bar{\mu}_{t-1,\alpha}^h \leq \Delta(\text{ta1}) + \frac{\mu_{t\alpha}^h}{(1+r_{t\alpha})} \quad (\text{ta2})$$

$$\sum_{\beta \in C} b_{t\alpha\beta}^h \leq \Delta(\text{ta2}) + \bar{d}_{t-1,\alpha}^h (1 + \bar{r}_{t-1,\alpha}) \quad (\text{ta3})$$

$$\bar{d}_{t\alpha}^h \leq \Delta(\text{ta3}) + \sum_{\beta \in C} b_{t\beta\alpha}^h \pi_{t\beta\alpha} \quad (\text{ta4})$$

$$\left. \begin{aligned} \sum_{\ell \in F^\alpha} b_{t\ell}^h &\leq \Delta(\text{ta4}) \text{ if } h \notin H^\alpha, \\ \sum_{\ell \in D^\alpha} b_{t\ell}^h + \sum_{\ell \in \hat{L}^\alpha} \varphi_{t\ell}^\alpha \frac{b_{t\ell}^h}{p_{t\ell}} &\leq \Delta(\text{ta4}) + \frac{\bar{\mu}_{t\alpha}^h}{1 + \bar{r}_{t\alpha}} \text{ if } h \in H^\alpha \end{aligned} \right\} \quad (\text{ta5})$$

$$q_{t\ell}^h \leq e_{t\ell}^h \quad (\text{ta5}')$$

$$x_{t\ell}^h \leq \Delta(\text{ta5}') + \frac{b_{t\ell}^h}{p_{t\ell}} \quad (\text{ta5}'')$$

⁹If we had allowed for durable goods, as in [10], we could have avoided both of these problems. In that case the periods could be regarded as arbitrarily close together and the order of market transactions inside a period would become irrelevant. If market α preceded market β inside each period, an agent could always trade on market β in period t and then wait a nanosecond to trade in market α in period $t + 1$ if that is the order he preferred.

$$\mu_{t\alpha}^h \leq \Delta(\text{ta5}) + \sum_{\ell \in L^\alpha} p_{t\ell} q_{t\ell}^h + w_t^h R_t^\alpha \quad (\text{ta6})$$

$$\hat{m}_{t\alpha}^h \leq \Delta(\text{ta6}) + (1 + r_{t\alpha}) d_{t\alpha}^h \quad (\text{ta7})$$

The period begins with h simultaneously buying and selling short bonds. Condition (ta1) says that h can only spend money on short term bonds (i.e., deposit) that he already has, inventoried from previous periods or newly endowed. Next h must pay off his previous long bond debts, and simultaneously receive payments on long bonds he holds. Note that these conditions hold for all $\alpha \in C$, so that h might be holding bonds in many different countries. Condition (ta2) says these payments must come out of money on hand, including the money borrowed in step 1. In step 3, h spends currency α buying other currencies β . Condition (ta3) says these expenditures must come out of money on hand, including the receipts from bonds in step 2.

In step 4 h buys and sells long bonds. Condition (ta4) says that he cannot spend more α -money than he has, including his α -currency purchases in step 3. In step 5 h buys and sells goods out of money on hand, including the receipts from bonds sold in step 4. Recall that h can sell bonds only in his home country, and that h pays a tariff in α -money on foreign purchases. Condition (ta5') says that h cannot sell goods he does not have. Agent h then consumes the goods he has (ta5'').

In step 6 h receives transfers from the government from money raised by tariffs. In step 7 h delivers on his short term bond promises, and simultaneously receives deliveries on his holdings of short bonds. Condition (ta6) says the deliveries by h come out of money on hand, including that acquired by commodity sales in step 5 and government transfers in step 6. Condition (ta7) says that the money carried over to the next period must be the money on hand, including receipts from deliveries in step 7.

4 International Monetary Equilibrium

We say that¹⁰ $(\eta, (\sigma^h)_{h \in H})$ is an *international monetary equilibrium* (and denote it IME) for the world economy $E = ((u^h, e^h, m^h)_{h \in H}, M^\gamma, \mu^\gamma, \varphi_{t\ell}^\alpha, w_t^h)_{\gamma \in C, \alpha \in C, t \in T, \ell \in \hat{L}^\alpha, h \in H^\alpha}$ iff:

¹⁰Recall that, by assumption p, π are different from 0 and ∞ in each component.

- (i) $p_{t\ell} \sum_{h \in H^\alpha} q_{t\ell}^h = \sum_{h \in H} b_{t\ell}^h + \sum_{\gamma \in C} M_{t\ell}^\gamma, \forall t \in T, \ell \in L;$
- (ii) $\pi_{t\alpha\beta} \left[\sum_{h \in H} b_{t\alpha\beta}^h + \sum_{\gamma \in C} M_{t\alpha\beta}^\gamma \right] = \sum_{h \in H} b_{t\beta\alpha}^h + \sum_{\gamma \in C} M_{t\beta\alpha}^\gamma, \forall t \in T, \alpha \in T, \alpha \in C, \beta \in C;$
- (iii) $\sum_{h \in H} \frac{\mu_{t\alpha}^h}{1+r_{t\alpha}} + \sum_{\gamma \in C} \frac{\mu_{t\alpha}^\gamma}{1+r_{t\alpha}} = \sum_{\gamma \in C} M_{t\alpha}^\gamma + \sum_{h \in H} d_{t\alpha}^h, \forall t \in T, \alpha \in C;$
- (iv) $\sum_{h \in H^\alpha} \frac{\bar{\mu}_{t\alpha}^h}{1+\bar{r}_{t\alpha}} + \sum_{\gamma \in C} \frac{\bar{\mu}_{t\alpha}^\gamma}{1+\bar{r}_{t\alpha}} = \sum_{\gamma \in C} \bar{M}_{t\alpha}^\gamma + \sum_{h \in H} \bar{d}_{t\alpha}^h, 0 \leq t < T, \alpha \in C;$
- (v) $R_t^\alpha = \sum_{h \in H^\alpha} \sum_{\ell \in \bar{L}^\alpha} \varphi_{t\ell}^\alpha \cdot x_{t\ell}^h;$
- (vi) $(x^h, \mu^h, \bar{\mu}^h, d^h, \bar{d}^h, b^h, q^h, \hat{m}^h) \in \underset{\sigma^h \in B^h(p, \pi, r, \bar{r}, R)}{\text{Argmax}} u^h(x^h).$

Condition (i) says that all commodity markets clear, or equivalently, that price expectations are correct, (ii) says that all currency exchange markets clear, or equivalently, that currency exchange forecasts are correct, (iii) says that all short-term credit markets clear, or equivalently, that predictions of short-term interest rates are correct, (iv) says that all long-term credit markets clear, or equivalently, that predictions of long-term interest rates are correct, (v) says tariff revenue forecasts are rational, and (vi) says that all agents optimize.

If an IME exists, fiat money has positive value in a finite horizon economy (for an extensive discussion on this see [9], [10]). Government actions are exogenously fixed and are not deduced from optimizing behavior.

5 Money Demand, Interest Rates, and the Quantity Theory

At each meeting of the market, money is exchanged for another instrument, which can be either commodities or bonds or foreign exchange. It is customary to regard such market exchanges as the interaction of demand and supply for the other instrument. But logically speaking, one might just as well speak of each of these exchanges as the interaction of demand and supply for money. The price of money at a moment in time depends on the instrument against which it trades at that moment, as does the reason for the demand. When the other instrument is a commodity, the price of money is the inverse of commodity prices. When it is a short-term bond, the price of money is the short-term interest rate $(1+r)$, and so on.

Money differs from other instruments in that it is never wanted in and of itself, but only so that it can be exchanged later for another instrument, or to pay back a debt. The demand for money thus always depends on the need for transactions. When these transactions are a long way off, we can speak of the store of value demand for money. In a model with uncertainty, such as in [10], we could speak of speculative and precautionary demands for money.

When the short-term bonds are sold for money, we speak of the transactions demand for money. The higher the interest rate, the fewer transactions will be

desired, and the less will be the demand for money. The higher the nominal value of purchases desired (and presumably the higher the money value of sales desired), the greater will be the demand for money, given the same interest rate. When long-term bonds are sold for money, the desire for money is often a desire to transfer wealth from the future to the present. In this case the demand for money will depend on the difference between the long-term nominal rate and the rate of inflation, that is on the real rate of interest. Finally, the sale of commodities at date t , beyond what is necessary to repay debts at the end of date t , is motivated by a wish to transfer wealth forward. That source of demand for money is adversely affected by higher inflation.

Thus we see that the standard Hicksian IS/LM determinants of money demand, namely interest rates and income, are at work in our model. Nevertheless, it is also easy to see that with our simple specification of an economy (without assets except for money and one-period bonds), if all the α -money interest rates are positive, then all the α -money will be spent in each period in the α -commodity markets. An agent (from any country β) who has α -cash that he does not wish to spend will not hold it idle, but will lend it out to somebody who will spend it.

Quantity Theory of Money Proposition. *In IME, if $\bar{r}_{t\alpha} > 0$, then the aggregate income of country α at date t , namely the value of all domestic commodity sales $\sum_{h \in H^\alpha} \sum_{\ell \in L^\alpha} p_{t\ell} q_{t\ell}^\alpha$ plus tariff revenue at date t , R_t^α , is equal to the total stock of bank money $M_t^\alpha + \bar{M}_t^\alpha$ and remaining private money.*

Proof. If $\bar{r}_{t\alpha} > 0$, then agents (whether α nationals or not) who hold α -money just before the long loan market meets at date t will deposit (i.e., loan) the money and earn the interest if they do not plan on spending it. The borrowers will spend the money, or else they should have waited and borrowed later. Finally, since there are no other assets, all the α -money being spent must be for α -commodities, or on the tariffs for non α -goods purchased by α -nationals. ■

It follows from the foregoing that if all interest rates are positive, then in equilibrium the quantity theory of money must hold, with velocity of money fixed at one. At any moment the stock of money will be equal to the value of nominal income, as we have defined it. Given the level of real economic activity, price levels will move in the same direction as the stock of money (as more money chases the same goods). Yet as we have been at pains to point out, this is no crude quantity theory in which the demand for money is independent of interest rates; quite the opposite is the case. For example, the “real” velocity of money, that is how many real transactions can be moved by money per unit time, is endogenous. We also hasten to add that in richer versions of the model, in which there are other durable assets and perhaps transactions costs from depositing and withdrawing money from the bank, money might be held without being spent, depending on interest rates and income, or else spent on assets (which would not count as aggregate income).

In equilibrium the quantity of economic activity, by which we mean the quantity of real goods traded in a period, is endogenous. By contrast, in the model of Lucas

and his followers ([25], [26], [19]), the amount of real economic activity is exogenously specified by the requirement that each agent sell the whole of his endowment in each period. A corollary of the quantity theory of money in our model is that, all other things being equal, increases in trading activity in period t , due perhaps to more productivity or lower interest rates, will result in lower period t price levels, given the same money supply at date t .

The money injected into the economy by the government, and in money endowments, is exogenous in our model. In equilibrium, over the T -period horizon, the money flowing out of the system in interest payments to the central bank must equal the money flowing into the economy. Otherwise agents would default (which is ruled out in equilibrium), or else they would be stuck in the last period with worthless paper, and the value of money would be zero (which is also ruled out in IME). However, at times $t < T$ the supply of money is endogenous. Letting $M_{t\alpha} \equiv \sum_{\gamma \in C} M_{t\alpha}^\gamma$ and $\bar{M}_{t\alpha} = \sum_{\gamma \in C} \bar{M}_{t\alpha}^\gamma$, and $\mu_{t\alpha} \equiv \sum_{\gamma \in C} \mu_{t\alpha}^\gamma$ and $\bar{\mu}_{t\alpha} = \sum_{\gamma \in C} \bar{\mu}_{t\alpha}^\gamma$, and $M_{t\beta\alpha} \equiv \sum_{\gamma \in C} M_{t\beta\alpha}^\gamma$ and $M_{t\ell} = \sum_{\gamma \in C} M_{t\ell}^\gamma$, we have:

Proposition 1. *At any IME, for all $0 \leq \tau \leq T$, for all $\alpha \in C$, and defining $m_{T+1}^h \equiv 0 \forall h \in H$,*

$$\begin{aligned} & \sum_{\tau=0}^t \left\{ \frac{\mu_{\tau\alpha}}{1+r_{\tau\alpha}} + \bar{M}_{\tau-1,\alpha}(1+\bar{r}_{\tau-1,\alpha}) + \sum_{\beta \in C} M_{\tau\beta\alpha} \pi_{\tau\beta\alpha} + \frac{\bar{\mu}_{\tau\alpha}}{1+\bar{r}_{\tau\alpha}} + M_{\tau\alpha}(1+r_{\tau\alpha}) \right\} \\ & \leq \sum_{\tau=0}^t \left\{ M_{\tau\alpha} + \bar{\mu}_{\tau-1,\alpha} + \bar{M}_{\tau\alpha} + \sum_{\beta \in C} M_{\tau\alpha\beta} + \sum_{\ell \in L^\alpha} M_{t\ell} + \mu_{\tau\alpha} + \sum_{h \in H^\alpha} m_{\tau}^h \right\}, \end{aligned}$$

with equality at $t = T$, or equivalently,¹¹

$$\begin{aligned} \star(\text{T}) \quad & \sum_{\tau=0}^t \left\{ \bar{M}_{\tau-1,\alpha} \bar{r}_{\tau-1,\alpha} + \sum_{\beta \in C} M_{\tau\beta\alpha} \pi_{\tau\beta\alpha} + M_{\tau\alpha} r_{\tau\alpha} \right\} + \frac{\bar{\mu}_{t\alpha}}{1+\bar{r}_{t\alpha}} \\ & \leq \sum_{\tau=0}^t \left\{ \frac{\bar{r}_{\tau-1,\alpha} \bar{\mu}_{\tau-1,\alpha}}{1+\bar{r}_{\tau-1,\alpha}} + \sum_{\beta \in C} M_{\tau\alpha\beta} + \sum_{\ell \in L^\alpha} M_{t\ell} + \frac{r_{\tau-1,\alpha} \mu_{\tau-1,\alpha}}{1+r_{\tau-1,\alpha}} + \sum_{h \in H} m_{\tau}^h \right\} + \bar{M}_{t\alpha}, \end{aligned}$$

with equality at $t = T$.

The RHS represents the money flowing into the system, and the LHS represents the money flowing out of the system. The interest rates must satisfy a sequence of inequalities, but only one equality. Therefore, in the one-period case, the interest rate can be specified a priori, independent of the “real” data of the economy. But in a multiperiod setting, one equation per country cannot determine all $(2T-1)C$ interest rates; they depend on the real data of the economy, including that of the agents in foreign countries. The exception is where all government deficit spending is zero, in which case all $r_{t\alpha} = \bar{r}_{t\alpha} = 0$, for all $\alpha \in C$.

¹¹ Recall $\bar{M}_{T\alpha} = 0 = \bar{\mu}_{T\alpha}$.

The money stock at the end of date $1 \leq t \leq T$ is given by $\Delta(t)$, the difference between the RHS and LHS of this inequality, which is endogenous. Given the exogenous monetary injections, and assuming that $\mu_{t\alpha} \leq M_{t\alpha}$ and $\bar{\mu}_{t\alpha} \leq \bar{M}_{t\alpha} \forall t \in T$, we see that the lower the interest rates, the higher the money supply.

As we mentioned in the introduction, if we think of the α -government choosing the m_t^h (i.e., money financed fiscal transfers), then $D_t \equiv \left\{ \frac{\bar{r}_{\tau-1,\alpha} \bar{\mu}_{\tau-1,\alpha}}{1+\bar{r}_{\tau-1,\alpha}} + \sum_{\beta \in C} M_{\tau\alpha\beta} + \sum_{\ell \in L^\alpha} M_{t\ell} + \frac{r_{\tau-1,\alpha} \mu_{\tau-1,\alpha}}{1+r_{\tau-1,\alpha}} + \sum_{h \in H} M_\tau^h \right\}$ represents total deficit spending of α -money in period t , including interest on the debt, and $\{\bar{M}_{t-1,\alpha} \bar{r}_{t-1,\alpha} + \sum_{\beta \in C} M_{t\beta\alpha} \pi_{t\beta\alpha} + M_{t\alpha} r_{t\alpha}\}$ represents the profit the central banks lending operations achieve. As deficit spending increases, interest rates must on average increase unless open market operations $M_{t\alpha}$ and $\bar{M}_{t\alpha}$ also increase (assuming expenditures on foreign currency are zero). If there are no private monetary endowments, and government deficit spending is known to always be zero, then all interest rates must be zero, as can be seen from $\star(T)$. This is the extreme case where interest rates are not endogenous. We state a version of this for use in the next section:

Corollary. *At any IME, for all $t \in T$ and $\alpha \in C$,*

$$r_{t\alpha} \leq \frac{\sum_{\tau=0}^T D_\tau}{M_{t\alpha}} \leq \frac{\sum_{\tau=0}^T \{\bar{\mu}_{\tau-1,\alpha} + \sum_{\beta \in C} M_{\tau\alpha\beta} + \sum_{\ell \in L^\alpha} M_{\tau\ell} + \mu_{\tau-1,\alpha} + \sum_{h \in H} m_\tau^h\}}{M_{t\alpha}}.$$

If the government tries to “spend” too much money without printing enough bank money, it will bring trade to a complete halt, as our existence proof makes clear.

6 The Existence of Equilibrium: Gains to Trade

Agents in our model are not required to trade. They always have the option of simply consuming their endowment. Therefore there is no guarantee that agents will need money, and therefore no guarantee that the credit or foreign exchange markets will be active, or indeed that money will have positive value.

Lucas circumvents this problem by forcing each agent to put up his entire endowment for sale against money in each period, even if he himself wants to consume it. Agents need money by assumption, since they cannot even eat their own endowments without it.

When some private endowments of money are positive (and the central banks are not acting on the foreign exchange markets), Proposition 1 shows that in equilibrium some interest rates must be positive. Otherwise agents would be left at the end of period T holding worthless paper money, and then by rational expectations and backward induction, money would have zero value. When interest rates are positive, the agents might be willing to give up goods even in period T in order to obtain the cash needed to repay the loans they voluntarily took with the banks.

However, when interest rates are positive, agents may not be willing to borrow from the bank. By borrowing \$1 from the bank, an agent can buy \$1 worth of commodities. But subsequently he must sell $\$(1+r) > \1 worth of commodities in

order to repay the loan. The agent will only agree to such a deal if he is sufficiently anxious to trade.

We are thus led to make a crucial assumption that there are sufficient gains to trade available to the agents, to justify their giving up interest payments. The formal description of the necessary gains to trade turns out to be surprisingly simple to state. Before turning to the description, let us note that by dropping the artificial assumption that agents must sell everything, we require heterogeneity between agents so that they will want to trade. The simplest heterogeneity is intraperiod heterogeneity in endowments, so we also require at least two goods per country. Our approach thus stands in sharp contrast to Lucas, who postulated identical consumers and only one good per country.

Let $x^h \in \mathbb{R}_+^{TL}$ for each $h \in J \subset H$, and let $x = \sum_{j \in J} x^j$. Debreu suggested that the allocation $(x^h)_{h \in J}$ is not δ -Pareto efficient for the agents in J if it is possible to costlessly reallocate the commodity bundle $(1 - \delta)x$ among those agents and make them all better off than they were at $(x^h)_{h \in J}$. Following Dubey–Geanakoplos [9], we suggest a different definition. We say that the allocation $(x^h)_{h \in J}$ allows for at least δ -gains to trade if starting from that allocation, it is possible to make everybody in J better off by transferring commodities, even though a fraction δ of every transfer is lost. Debreu threw out a fraction $\delta/(1 + \delta)$ of the original bundle, but allowed for costless transfers. We keep the whole of the original bundle, but throw out a fraction $\delta/(1 + \delta)$ of every transfer. Below we give a formal definition, restricting the transfers to a particular period t .

Definition. For any $\delta \geq 0$, we will say that $(x^h)_{h \in J}$ permits at least δ -gains to trade in period t if there exist feasible net trades $(\tau^h)_{h \in J}$ (i.e., $\tau^h \in \mathbb{R}^L$, $x^h + \tau^h \in \mathbb{R}_+^L$ for all $h \in J$, and $\sum_{h \in J} \tau^h = 0$) such that,

$$u^h(\bar{x}^h) > u^h(x^h) \text{ for all } h \in J \text{ where,}$$

$$\bar{x}_{\nu\ell} = \begin{cases} x_{\nu\ell}^h & \text{if } \nu \in T \setminus \{t\} \\ x_{\nu\ell}^h + \min\{\tau_\ell^h, \tau_\ell^h/(1 + \delta)\} & \text{for } \ell \in L \text{ and } \nu = t. \end{cases}$$

Note that when $\delta > 0$, $\bar{x}_{t\ell}^h < x_{t\ell}^h + \tau_\ell^h$ if $\tau_\ell^h > 0$ and $\bar{x}_{t\ell}^h = x_{t\ell}^h + \tau_\ell^h$ if $\tau_\ell^h \leq 0$. Formally, the condition we impose on the world economy for sufficient gains to trade is:

(G to T) For each $\alpha \in C$, the initial endowment $(e^h)_{h \in H^\alpha}$ permits at least $\delta_{\alpha t}$ -gains to trade in at least one period $t \in T$, where $\delta_{t\alpha} = \sum_{\tau=0}^T \{\bar{\mu}_{\tau-1,\alpha} + \sum_{\beta \in C} M_{\tau\alpha\beta} + \sum_{\ell \in L^\alpha} M_{\tau\ell} + \mu_{\tau-1,\alpha} + \sum_{h \in H} m_\tau^h\} / M_{t\alpha}$.

Condition (G to T) needs to be valid in just one period in each country; it is not necessary for the other periods. Also, (G to T) precludes the case where $L = 1$. Moreover, if the initial endowment at one period t is not Pareto optimal (fixing $x_\tau^h = e_\tau^h$ for $\tau \neq t$), then, holding all other government actions fixed, as $M_{t\alpha} \rightarrow \infty$, (G to T) is automatically satisfied. The following theorem is proved in the Appendix.

Theorem. *If in the world economy $E = ((u^h, e^h, m^h)_{h \in H}$,*

$(\mathcal{M}_t^\alpha, \mu_t^\alpha, \varphi_{t\ell}^\alpha, w_t^h)_{\alpha \in C, t \in T, \ell \in \hat{L}^\alpha, h \in H^\alpha}$ (1) (Gains to Trade) holds, (2) $M_{t\alpha} > 0$ for all $t \in T$ and $\alpha \in C$, and (3) for all $\alpha \in C$ there is some $h \in H_\alpha$ and $t \in T$ with $m_t^h > 0$, then an international monetary equilibrium exists.

6.1 Hyperinflation

The proof of the theorem also indicates that the gains to trade necessary for the existence of equilibrium get arbitrarily large as the ratio of government deficits (including private money) to open market purchases goes to infinity. In fact, as the deficits rise, eventually prices must rise, converging rapidly to infinity as the deficits reach the finite limit beyond which equilibrium cannot be sustained.

7 International Monetary Equilibrium (IME) and Competitive Equilibrium (CE)

We say that $(p, (x^h)_{h \in H})$ where $p \in \mathbb{R}_+^{TL}$ is an *international competitive equilibrium* (and denote it CE) for the world economy $E = ((u^h, e^h)_{h \in H})$ iff:

- (i) $\sum_{h \in H} x_{t\ell}^h = \sum_{h \in H} e_{t\ell}^h$, for all $t \in T, \ell \in L$;
- (ii) $x^h = \operatorname{argmax}\{x^h \in \mathbb{R}_+^{TL}, px^h = pe^h, x_{t\ell}^h = 0 \text{ if } \ell \in D^\alpha \text{ and } h \notin H^\alpha\}$ for all $h \in H$.

The CE ignores all monetary phenomena both domestic as well as international, and therefore collapses to a standard international trade model. Neither money markets nor currency exchange markets exist, since all transactions take place via an international clearing house which matches receipts from sales and payments for purchases. Then one can immediately see from the definition of IME:

Proposition 2: Let $M_{t\beta\alpha} = M_{t\ell} = \varphi_{t\ell}^\alpha = w_t^h = 0$ for all $t \in T, \ell \in \hat{L}^\alpha, \alpha, \beta \in C, h \in H^\alpha$. Let $\mu_{t\alpha} < M_{t\alpha}$ and $\bar{\mu}_{t\alpha} < \bar{M}_{t\alpha}$ for all $t \in T$ and $\alpha \in C$. Suppose $\sum_{h \in H} \sum_{t \in T} m_t^h = 0$. Then IME and CE coincide with respect to prices (normalized by exchange rates) and allocations. Furthermore, even if $\sum_{h \in H} \sum_{t \in T} m_t^h > 0$ then as M_t^α and $\bar{M}_t^\alpha \rightarrow \infty$ for all $\alpha \in C, t \in T$, IME coincide with CE in the limit.

Proposition 2 follows from the first inequality in Proposition 1 which shows that when either private monetary endowments are zero or money supplies become sufficiently large then interest rates are driven to zero.

8 The Non-Neutrality of Money

We saw in the last section that when governments are not spending money and private stocks of money are very small (or zero) relative to bank money, IME is very close to CE. In this section we show that as long as some private endowment of money is positive, and IME is different from CE, government monetary policy (open market

operations, i.e., changing the stock of bank money) necessarily has real effects on consumption. Similarly, government transfers of money to agents, no matter how it is distributed across the population, also necessarily has real effects.

These non-neutrality conclusions are contrary to those derived by Lucas. The explanation is that in our model, IME is not Pareto efficient because of the distortion caused by trading via money borrowed at positive interest rates. When the government eases credit (by putting more money up at the banks) it facilitates borrowing, reduces interest rates, and increases real activity.

Consider the world economy $E = ((u^h, e^h, m^h)_{h \in H}, (M^\gamma, \mu^\gamma, \varphi^\gamma, w^h)_{\gamma \in C, h \in H})$ in which $\sum_{h \in H} m_t^h > 0$ for all $t \in T$. We show elsewhere that IME are typically finite. Therefore, prices, exchange rates and interest rates are almost always determinate with respect to the data of the world economy. This allows us to determine the impact of monetary changes. Let

$$(M^\gamma, \mu^\gamma, \varphi^\gamma)_{\gamma \in C} \equiv (M_{t\alpha}^\gamma, \bar{M}_{t\alpha}^\gamma, \mu_{t\alpha}^\gamma, \bar{\mu}_{t\alpha}^\gamma, M_{t\alpha\beta}^\gamma, \gamma_{t\ell}, \varphi_{t\ell}^\gamma)_{\gamma \in C, \alpha \in C, \beta \in C, \ell \in L, t \in T}$$

be the financial data of the economy. The “no-money illusion” property easily follows:

Proposition 3. *A proportional increase of all $(m^h)_{h \in H}^\gamma$ and $(M^\gamma, \mu^\gamma, \varphi^\gamma)$ for any fixed $\gamma \in C$ does not affect consumption in IME, assuming governments $\alpha \neq \gamma$ are not acting in γ -markets.*

This proposition says that if we change the units of account in a proportional manner in a country then nothing will change except the price levels, currency exchange rates, and tariff revenues which will absorb the change in units. For example, if the U.S.A. switches from dollars to cents while England sticks to pounds and Germany sticks to D.M.’s then American prices and dollar exchange rates alone will change.

Monetary policy usually changes the ratio of bank money to private endowments of money. In our model we interpret monetary policy as a change in the bank money $M_{t\alpha}^\gamma$ or $\bar{M}_{t\alpha}^\gamma$. A change in the m_t^h is called a money-financed fiscal transfer. These policy instruments typically have real effects because they change nominal interest rates. As we explained in the introduction, higher nominal short rates $r_{t\alpha}$ reduce the efficiency of intraperiod trade, and higher nominal long rates $\bar{r}_{t\alpha}$ reduce the efficiency of interperiod trade. The following proposition demonstrates the non-neutrality of policy in a case which is easy to analyze via the first order conditions of equilibrium. In the applications sections we give a robust example satisfying all the conditions of the proposition.

We call an IME *indecomposable* if for any $t \in T$ and any partition of the goods L^α into disjoint sets L_1^α and L_2^α there is some agent $h \in H^\alpha$ who transacts in at least one commodity from each set in period t .

Proposition 4. *Suppose all u^h differentiable, and $m_t^h > 0$ for all $h \in H, t \in T$. Suppose at an indecomposable IME of E that at every t , all α -agents consume positive amounts of all α -goods, that $\bar{r}_{t\alpha} \geq r_{t\alpha}$, that all α -agents spend more α -money at t on commodities than they get by selling foreign exchange, that some α -agent borrows on $\bar{M}_{t\alpha}$, and that some α -agent carries some α -money from t into $t + 1$. Then any*

change in \mathcal{M} which violates the government budget constraint (i.e., the equation of Proposition 1) at fixed $(r_{t\alpha}, \bar{r}_{t\alpha})_{t \in T}$ and results in a new IME satisfying all the above conditions must also cause different consumption for some agent.

Proof. Under the maintained hypotheses, if at the original IME agent h buys $x_{t\ell}$ and sells x_{tk} , then

$$\frac{\frac{\partial u^h(x)}{\partial x_{t\ell}}}{p_{t\ell}} = \frac{\frac{\partial u^h(x)}{\partial x_{tk}}}{p_{tk}}(1 + r_{t\alpha})$$

If LHS > RHS, then the agent should simply have borrowed $\varepsilon p_{t\ell}$ on $M_{t\alpha}$, bought ε units of $t\ell$, sold $(\varepsilon p_{t\ell}/p_{tk})(1 + r_{t\alpha})$ of tk to defray the loan and been better off. If LHS < RHS, the agent should have spent $\varepsilon p_{t\ell}$ less on good $t\ell$, deposited the money on $\bar{M}_{t\alpha}$ (or borrowed $\varepsilon p_{t\ell}$ less), sold $(\varepsilon p_{t\ell}/p_{tk})(1 + \bar{r}_{t\alpha})$ less of good tk , and ended up better off. Note that this last option was feasible, since by hypothesis the agent had α -money beyond that obtained in the exchange market before the meeting of the commodity markets.

After the change in \mathcal{M}_α , at least one interest rate must change. Suppose $r_{t\alpha}$ becomes higher for some t , yet all agents continue to buy and sell the same amount of each commodity. Since every agent is buying, and nothing can be bought unless it is sold, some agent is selling as well as buying. For any pair of commodities $t\ell$ and tk that are bought and sold, respectively, by the same agent, we must have that $p_{t\ell}/p_{tk}$ falls. But $t\ell$ must have a seller, who buys another good tn . So $p_{tn}/p_{t\ell}$ must also fall. Continuing in this fashion, we will eventually reach a commodity ta that we already mentioned, and then we would have $(p_{ta}/p_{tb})(p_{tb}/p_{tc}) \cdots (p_{td}/p_{ta}) = 1$ falling; a contradiction.

Suppose instead that some $\bar{r}_{t\alpha}$ increases. If agent $h \in H^\alpha$ is a borrower on $\bar{M}_{t\alpha}^\alpha$ (i.e., a seller of long bonds at time t), then for each pair of goods $t\ell$ and $t+1, k$ that he buys, we must have

$$\frac{\frac{\partial u^h(x^h)}{\partial x_{t\ell}}}{p_{t\ell}} = \frac{\frac{\partial u^h(x^h)}{\partial x_{t+1,k}}}{p_{t+1,k}}(1 + \bar{r}_{t\alpha}).$$

Hence $p_{t\ell}/p_{t+1,k}$ must fall if h maintains his consumption. From indecomposability and the argument given in the last paragraph, if all consumption and $r_{t\alpha}$ and $r_{t+1,\alpha}$ stay the same, then all relative prices at time t and all relative prices at time $t+1$ must stay the same. We conclude that for every pair of commodities $t\ell$ and $t+1, k$, $p_{t\ell}/p_{t+1,k}$ must fall. But for the agent h' who carries money over from t to $t+1$, let ti and $t+1, j$ be commodities he buys. If h' does not alter his consumption, and if $r_{t\alpha}$ stays fixed, then $p_{ti}/p_{t+1,j}$ must stay fixed, a contradiction. ■

Since increases in the stock of bank money (that is, expansionary open market operations) ultimately move the economy closer to a competitive equilibrium, and hence closer to Pareto efficiency, one question is why doesn't the government drastically increase this expansion? We present three explanations.

First, there is a political reason. Expansionary open market operations increase the effective money supply, causing prices to rise. (Rich) agents who own money, or money denominated assets, will find their real wealth reduced. Since rising domestic prices also reduce the exchange rate, all citizens with assets denominated in α -money (rich and poor) will find their international real wealth reduced.

Second, the government doesn't really have the freedom to increase bank money while leaving the private endowments of money fixed. As we have said earlier, much of the private endowment of money is due to receipts from government transfers such as social security, pension plans, and welfare, and a great deal is due to government expenditures on salaries for government workers. All of these rise as the price level rises.

Third, increasing $M_{t\alpha}^\alpha$ probably would make agents think that the government was also going to increase $M_{t+1,\alpha}^\alpha$. If agents thought the latter increase might be bigger than the former, then they would anticipate an inflation, which would *raise* the long rate $\bar{r}_{t\alpha}$, making equilibrium less efficient.

9 International Capital Mobility and the Balance of Trade

Agents in our model can exchange their α -money for β -money in any period. They can also deposit foreign currency holdings into foreign banks, hence we may say that there is perfect capital mobility. Multiple time periods and the opportunity to undertake international financial investments makes the balance of trade interesting. We define the balance of trade surplus for country α with respect to country β in period t by

$$B(\alpha, \beta, t) \equiv \sum_{h \in H^\beta} \sum_{\ell \in F^\alpha} p_{t\ell} x_{t\ell}^h - \pi_{t\beta\alpha} \sum_{h \in H^\alpha} \sum_{\ell \in F^\beta} p_{t\ell} x_{t\ell}^h .$$

The α -balance of trade surplus is given by

$$B(\alpha, t) \equiv \sum_{\beta \in C \setminus \{\alpha\}} B(\alpha, \beta, t) .$$

With floating exchange rates such as we have, the only way this number could be nonzero is if an α agent exchanged α -money for β -money, and instead of spending it on commodities, deposited in a β bank, or vice versa. In particular, when there is only one time period, every country must have a zero balance of trade surplus. Multiple time periods make it possible for countries to run balance of payment deficits in some time periods.

10 Uncovered Interest Parity, Multiperiod Balance of Trade, Purchasing Power Parity, and the Fisher Effect

In this section we show that the standard propositions of international finance have interpretations in our general equilibrium model. Our first proposition is reminiscent of the capital mobility equilibrium conditions stipulated in the Mundell–Fleming model. It relies on agents who optimize their investments across countries. Note that these agents will not simply put their money in the country with the highest interest rate (as presumed in the Mundell–Fleming model), but, as is well-known, will take into account the expected appreciation of each currency. Note also that the familiar formula we derive depends on some agent who is both investing and consuming in the foreign country.

Uncovered Interest Parity Proposition. *Suppose some agent h in country α exchanges α -money for β -money in period t and does not spend all the β -money in period t . Suppose further that he does not spend all the α -money he has on hand when it is time to send α -money to the foreign exchange market in periods t and $t + 1$. Then if we are in IME, we must have that*

$$\frac{1 + \bar{r}_{t\alpha}}{1 + \bar{r}_{t\beta}} = \frac{\pi_{t+1\beta\alpha}}{\pi_{t\beta\alpha}} .$$

Proof. Agent h can transform a unit of α -money in period t just before the currency markets meet into $\pi_{t\alpha\beta}(1 + \bar{r}_{t\beta})$ units of β -money, or into $(1 + \bar{r}_{t\alpha})$ units of α -money, just before the currency markets meet in period $t+1$. If $\pi_{t\alpha\beta}(1 + \bar{r}_{t\beta}) < (1 + \bar{r}_{t\alpha})\pi_{t+1\alpha\beta}$, the agent made a mistake acting in the currency markets and then not spending all his β -money on commodities in period t . If the opposite inequality holds, then the agent should have sent an extra unit of α -money into the currency market in period t and invested the resulting β -money in the period t β -long loan and then transformed the resulting β -money back into α -money in period t , replaced the period t α -money by borrowing on the period t α long loan, repaid the loan out of the cash he has on hand just before the period $t + 1$ currency markets meet, and he would have money to spare. ■

The proposition tells us that if α is paying higher interest rates than β , and if there are rational agents participating in both α and β long loan markets, then the currency of β must be appreciating with respect to α . Note that if there is no agent who is active in the long loan markets of both countries, then the uncovered interest parity relationship would depend on shorter rates as well as the long rate. We can also deduce the following proposition.

Multiperiod Balance of Trade Proposition. *Suppose that in every period t , and for every pair of countries α and β , there is some agent who behaves as described in the previous theorem, and $B(\alpha, -1) = 0$. Then we must have that for each country*

α , the discounted sum of α 's balance of trade surpluses is zero:

$$B(\alpha, 0) = \sum_{t=1}^T \prod_{\tau=1}^t (1 + \bar{r}_{\tau-1, \alpha}^{-1}) B(\alpha, t) = 0 .$$

Proof. Suppose that at time t an agent h exchanges a unit of his domestic currency α for $\pi_{t\alpha\beta}$ of β -money, and does not plan to spend the β -money at time t . This raises the α -discounted balance of trade surplus by 1. Agent h will then deposit the money in the long loan in β if $\bar{r}_{t\alpha} > 0$. (If $\bar{r}_{t\alpha} = 0$, it makes no difference if he deposits the money or not.) If he spends the β -money at time $t + 1$, it will reduce the discounted balance of trade surplus by

$$\frac{1}{(1 + \bar{r}_{t\alpha})} [\pi_{t\alpha\beta}(1 + \bar{r}_{t\beta})] \pi_{t+1, \beta\alpha} = 1 ,$$

from the uncovered interest parity proposition, as was to be shown. If agent h does not spend the money on β -commodities at time $t + 1$, then we proceed by induction (even if agent h transforms the money into γ -money), repeating the same argument. Eventually the agent must purchase commodities, or else transform the money back into α -money to repay a loan. But this last step also reduces the balance of trade surplus by the requisite amount. Adding over the finite number of traders $h \in H^\alpha$ concludes the proof. ■

This proposition shows, among other things, that it is impossible to run a balance of trade deficit forever. (We should note that we have relied on the finite horizon nature of the economy. In an overlapping generations world with an infinite horizon, the same proof would not necessarily hold.) In particular, any policy that causes a balance of trade deficit in the short run must lead to a change in exchange rates or future interest rates that in turn causes a balance of trade surplus in the future. We emphasize that the multiperiod balance of payments proposition depends on the uncovered interest parity, which in turn depends on some agent acting simultaneously in different long loan markets. In general, there will be no simple equation.

In our model agents weigh the marginal benefit of domestic and foreign consumption. In the Mundell–Fleming model, behavioral functions suggest that consumers change their spending depending on exchange rate changes. But of course it is the real terms of trade (i.e., local prices corrected by exchange rates) which should motivate spending: the marginal utility of a dollar spent in the U.S. should be the same as a dollar spent in Germany (first to get marks, then to get German goods). This equation is one of the most important determinants in our model. It implies, for instance, that when money supplies increase in one country, causing prices to increase, the currency of that country will typically depreciate. Purchasing power parity has not fared well in empirical tests. But those tests usually check whether the same basket of goods would cost the same amount in two countries (corrected for the currency exchange rates). An American, however, typically has no interest in purchasing a basket of goods in Germany equal to the basket he is consuming in

America, simply because he lives in America. On the margin, however, he will carefully consider whether to buy an imported Volkswagen as opposed to a domestically produced Chrysler. The implied equality of marginal utilities is crucial to understanding international equilibrium; the empirical failure of purchasing power parity is no excuse for dropping this fundamental equation.

For our next two propositions, which follow from the optimization conditions, we suppose that each agent has differentiable utility.

Purchasing Power Parity Proposition. *Let $\varphi_{t\ell}^\alpha = w_t^h = 0$ for all $t \in T$, $\ell \in \hat{L}^\alpha$, $\alpha \in C$, $h \in H^\alpha$. At any IME, for any $\ell \in L^\alpha$, $k \in F^\beta$, $\alpha, \beta \in C$, if some $h \in H^\alpha$ chooses $b_{t\ell}^h > 0$, $b_{tk}^h > 0$, and if agent h does not spend all his α -money on hand on foreign currency in period t , then*

$$\frac{\partial u^h(x)}{\partial x_{t\ell}} / \frac{\partial u^h(x)}{\partial x_{tk}} = \frac{p_{t\ell} \pi_{t\alpha\beta}}{p_{tk}}.$$

The purchasing power parity theorem tells us that if an agent is purchasing a commodity in Germany and another in the United States that give the same marginal utility, then they must have the same real price. (If the commodities have the same name, this is called the law of one price.) Note, however, that he could be buying tk and selling $t\ell$ in equilibrium, in which case we could have $[\partial u^h(x)/\partial x_{t\ell}]/[\partial u^h(x)/\partial x_{tk}] = (p_{t\ell} \pi_{t\alpha\beta}/p_{tk})(1 + r_{t\alpha})$. Moreover, there is no reason why the same good which is nontraded in both Germany and the United States should satisfy the purchasing power parity condition.

Fisher Effect Proposition. *Suppose that some agent h in country α chooses $b_{t\ell}^h > 0$ and $b_{t+1,k}^h > 0$, where ℓ and k are in L^α . Suppose further that h has α -money left over the moment period t long loans come due in period $t + 1$. Then at IME we must have:*

$$(1 + \bar{r}_{t\alpha}) = \left(\frac{\partial u^h(x)}{\partial x_{t\ell}} / \frac{\partial u^h(x)}{\partial x_{t+1,k}} \right) \left(\frac{p_{t+1,k}}{p_{t\ell}} \right).$$

Taking the logarithm of both sides and interpreting loosely, this says that the nominal rate of interest is equal to the real rate of interest plus the (expected) rate of inflation.

11 Applications

We show now that our theoretical framework is computationally tractable in practice. Given a change in a policy parameter, or the physical data of the economy, we can calculate the effects on all the variables in the economy. We do not need to resort to a stationary state, or to a representative agent, or to ignore some of the equilibrium conditions. Moreover, the comparative statics can be interpreted, and usually predicted, on the basis of the principles that we derived through our propositions in Sections 6 through 10. The calculations have all been done on a home computer using a version of Newton's method in *Mathematica*.

We are most interested in comparative statics that involve the interaction of the real and financial sectors of the global economy. For example, if country α raises

tariffs, what happens to the value of the α -currency? If country α becomes more productive, either with respect to traded or nontraded goods, what happens to its currency? If country α develops a taste for foreign cars, what happens to its currency, to the balance of trade? Does it matter whether the change in taste is expected to be permanent, or if it is expected that domestic cars will eventually be regarded with the same fondness as foreign cars? What effects do fiscal and monetary policy have on the economy?

Hereafter, we specialize the general model we presented earlier to the case of two countries, say US and Germany, two consumers, and two goods for each, say Jeans and Apples for US = α , and cars (Volkswagens) and Beers for Germany = β . We suppose Jeans and Volkswagens are internationally traded, while Apples and Beers are only domestically traded.

Consider a two-period model with the following utilities:¹²

$$\begin{aligned}
u_\alpha^1(J_0, J_1, V_0, V_1, A_0, A_1) &= (14J_0 - \frac{1}{2}J_0^2) + (\frac{61}{4}V_0 - \frac{1}{2}V_0^2) + (18A_0 - \frac{1}{2}A_0^2) \\
&\quad + (8J_1 - \frac{1}{2}J_1^2) + (\frac{17}{2}V_1 - \frac{1}{2}V_1^2) + (\frac{21}{2}A_1 - \frac{1}{2}A_1^2), \\
u_\alpha^2(J_0, J_1, V_0, V_1, A_0, A_1) &= (17J_0 - \frac{1}{2}J_0^2) + (\frac{61}{4}V_0 - \frac{1}{2}V_0^2) + (15A_0 - \frac{1}{2}A_0^2) \\
&\quad + \left(\frac{19}{2}J_1 - \frac{1}{2}J_1^2\right) + (\frac{17}{2}V_1 - \frac{1}{2}V_1^2) + (9A_1 - \frac{1}{2}A_1^2), \\
u_\beta^3(J_0, J_1, V_0, V_1, B_0, B_1) &= (\frac{45}{2}J_0 - \frac{1}{2}J_0^2) + (14V_0 - \frac{1}{2}V_0^2) + (\frac{35}{2}B_0 - \frac{1}{2}B_0^2) \\
&\quad + (13J_1 - \frac{1}{2}J_1^2) + (10V_1 - \frac{1}{2}V_1^2) + (13B_1 - \frac{1}{2}B_1^2), \text{ and} \\
u_\beta^4(J_0, J_1, V_0, V_1, B_0, B_1) &= (\frac{51}{2}J_0 - \frac{1}{2}J_0^2) + (18V_0 - \frac{1}{2}V_0^2) + (\frac{82}{5}B_0 - \frac{1}{2}B_0^2) \\
&\quad + (15J_1 - \frac{1}{2}J_1^2) + (14V_1 - \frac{1}{2}V_1^2) + (\frac{63}{5}B_1 - \frac{1}{2}B_1^2).
\end{aligned}$$

The endowments of the agents are given in Table II. The private monetary endowments and the government bank monies are also given.¹³ Notice that government open market purchases are concentrated on short-term credit instruments (M_0^i and M_1^i are much bigger than \bar{M}_0^i in both countries $i = \alpha, \beta$). Table II also indicates the initial IME. Notice that the marginal utilities of a dollar's worth of commodities that an agent buys (whether domestic or international), are the same (as the purchasing power parity proposition maintains), while the marginal utility of a dollar's worth of a commodity that he sells at the same time is lower, since he incurs no interest loss in selling. Observe that the short rates are lower than the long rate. An agent is willing to pay a higher interest on the long loan because he has more time to repay; in particular, he can pay out of next period's money endowment, or roll over the loan and sell a good in the final period to repay his loan. A possible exception may occur in the initial period where the agent might desire to enter the foreign exchange market and, therefore, could not wait for the long-term market to open.

¹²Note that the American agents 1 and 2 have utilities that depend on the internationally traded goods, Jeans and Volkswagens, and on the domestic American good Apples but not on the German domestic good, Beers. Similarly, the German agents 3 and 4 have utilities that depend on all the goods except Apples.

¹³The utilities and endowments were chosen more or less at random, and then perturbed in order to make the initial equilibrium prices all be 1, and the interest rates simple numbers.

TABLE I

	J ₀	V ₀	A ₀	B ₀	J ₁	V ₁	A ₁	B ₁	m ₀	m ₁
e ¹	8	0	0	0	10	0	0	0	0	0
e ²	0	0	6	0	0	0	6	0	8/5	13/5
e ³	0	10	0	0	0	6	0	0	3/5	21/20
e ⁴	0	0	0	9/2	0	0	0	6	0	13/5
Money supplies	$M_0^\alpha = 36/5, \bar{M}_0^\alpha = 1/5, \bar{M}_1^\alpha = 44/5,$ $M_0^\beta = 42/5, \bar{M}_0^\beta = 1/5, \bar{M}_1^\beta = 28/5$									
Prices	1	1	1	1	1	1	1	1	1	1
Exchange rates	$\pi_{0\alpha\beta} = 4/3$				$\pi_{1\alpha\beta} = 1$					
Interest rates	$r_{0\alpha} = 1/4, \bar{r}_{0\alpha} = 1,$ $r_{0\beta} = 1/2, \bar{r}_{0\beta} = 1/2$				$r_{1\alpha} = 1/4$ $r_{1\beta} = 1/4$					
Trade balances	$B(\alpha, \beta, 0) = -2$				$B(\alpha, \beta, 0) = 4$					
x ¹	2	4	3	0	2	1	3	0		
x ²	2	4	3	0	2	1	3	0		
x ³	5/2	2	0	5/2	3	2	0	3	$d_{0\alpha}^3 = 2$	
x ⁴	3/2	0	0	2	3	2	0	3		

The American long rate is higher than the German long rate, and the first German agent is depositing in the American bank. The higher American interest rate is compensated (as the uncovered interest rate parity condition assures us it must be) by the expected appreciation of the DM. Notice also that the US is running a balance of trade deficit in the initial period, and a surplus in the last period. Finally, notice that the real rates of interest in the US and Germany are equal to the nominal long-term interest rate, in the respective countries, as the Fisher relation assures us it must be, given the lack of inflation.

Now we consider the effects of changes in the parameters of the model. Table II describes the directional effects on endogenous variables of increasing various parameters listed in the first column. The first row corresponds to an American tariff in period 0 on German Volkswagens.

11.1 Tariffs

Across the world, from South America to Eastern Europe to Japan, economic leaders are embracing free market strategies and reducing tariffs. To see the effect of such a regime change, we study the effects of a temporary, unilateral tariff increase. (A tariff reduction would of course produce the opposite effects.) We find that a temporary, unilateral tariff (starting from a no-tariff regime) will improve the balance of trade of a country, appreciate its currency, raise its long-term interest rates, reduce the long-term interest rates of the taxed country, and also reduce the latter's welfare.

A tariff levied by the American government in dollars on Americans who import Volkswagens ought, from the first principles of elementary partial equilibrium economics, to raise the relative price of Volkswagens inclusive of the tariff, and reduce the relative price of Volkswagens net of the tariff. From the entry of row 1 in Table II

under the column $P_{0,V}/\pi_{0\alpha\beta}$ we see that the tariff does indeed reduce the dollar price of Volkswagens net of the tariff, while at the same time it raises the dollar price of Volkswagens inclusive of the tariff. Indeed, the relative prices all move as we would expect.

The most interesting effect of the (temporary) tariff is that it leads to an appreciation of the American dollar, as can be seen from the + under $\pi_{0\alpha\beta}$. In general, the example suggests that a country that (conversely) follows a free market strategy of reducing tariffs will see its currency depreciate, as has in fact been the case in Latin America and Eastern Europe. With the tariff increase in our example, less Volkswagens are imported, and each brings the German seller fewer American dollars (the tariff is collected by the American government), hence fewer dollars are spent on Volkswagens, and the dollar appreciates. This argument has nothing to do with elasticities, and hence is independent of any Marshall–Lerner conditions.

The same argument suggests that the American balance of trade will improve in the first period (that is, the net dollar value of Volkswagens purchased by Americans will go down compared to the dollar value of Jeans purchased by Germans.) This in turn must cause German holdings of American financial assets to decrease (deposits $d_{0\alpha}^3$ do indeed decrease) since, with floating exchange rates, the dollars leaving America must equal the dollar value of marks entering the country, and the latter are spent either on American goods, or on American assets, i.e., on deposits in American banks. In the last period, nobody will hold assets, hence the balance of trade is determined entirely by the interest payments from the previous periods. The reduced German deposits show up in the last period as a reversal of the trade balance. This effect is even stronger because the tariff is lifted for the last period.¹⁴

Note, incidentally, that the real terms of trade in both periods go in the same direction as the balance of trade. A reduced form model which tried to link trade balances with the terms of trade would be misleading.

The improvement in the American balance of trade goes hand in hand with fewer German deposits, as we have seen. Let us consider the other side of the long term credit market. Americans face higher dollar prices of goods (inclusive of the tariff), and they know they can anticipate more revenue in the future from the tariff redistribution. Thus if the long term interest rate remained the same, we would expect more demand for borrowing from America. Taking this together with less deposits from Germany, we must expect the long term American interest rate to go up. Indeed it does. Since the relative supplies of private and bank money is unchanged, the rise in the long rate implies that some American short rate goes down slightly (which happens in period 1).

The tariff reduces the sale of Volkswagens, and the welfare of Volkswagen sellers u_{β}^3 . Their loss in income reduces their demand for Beers in both periods, so we see that trade of all goods is reduced in Germany, and welfare falls for all Germans, as

¹⁴Even with a permanent increase in tariffs, we would see a trade balance improvement in America in the first period. The second period surplus would tend to decline unless the long interest rate increased sufficiently far. We did not carry out the calculations in our example, but we conjecture that when a country running a deficit imposes a permanent tariff instead of a temporary tariff, it will increase the long interest rate even more.

does national real income.

Given that the same amount of German money chases fewer German transactions, German deutschmark prices generally rise. Since the rise in German transactions is partly alleviated in period 1 by the elimination of the tariff, the fall in German prices is greatest in period 0. Hence the German expected inflation is lower after the temporary tariff is imposed, and by the Fisher effect, we should expect the German long interest rate to fall, which it does.

The fall in German income reduces their demand for Jeans, so the American seller of Jeans gains only slightly from the tariff revenue he receives from the government at the end of period 0. The American seller of Apples gains substantially. Since some dollars are diverted to pay the tariffs, we should expect general nontaxed American prices to fall in period 0, and indeed they do (though not by as much as the dollar equivalent price of Volkswagens). American nominal income goes down, although counting transfer payments even nominal income is as high as before. In period 1 prices return to their normal levels, and even slightly higher, because there are fewer Jeans sold to the poorer Germans, and hence the same money chases slightly fewer transactions. Thus the temporary tariff temporarily reduces American prices, then leads to a much higher inflation. This is another reason to have expected the long term American interest rate to have increased.

11.2 Productivity Increases

A strong American dollar is certainly beneficial, other things equal, to the holders of American financial assets, because they then have more purchasing power over foreign goods. We investigate here whether a strong dollar is also a sign of a strong American economy. Suppose American productivity (endowment) increases, say in the nontraded sector. What effect does that have on the dollar, on the balance of trade, and on long term American interest rates? What if the productivity increase were in the traded sector? What effect would it have on Germany?

The Mundell–Fleming model implies in both situations that American income and interest rates go up, the dollar appreciates, and net exports go down. We agree with the first three propositions, but not the fourth. This thought experiment illustrates what we regard as a fundamental problem with the Mundell–Fleming model: it treats output as a single homogeneous commodity, it ignores changes in the real terms of trade, and it ignores changes in the expected appreciation of the currencies, thereby compromising its predictions about international capital flows and the balance of trade. In line 2 of Table II, we show the effects of an increase in the period 0 American endowment of Apples, and in line 4 we show the effects of an increase in the period 0 American endowment of Jeans.

The effect of an increase in the domestically traded Apple endowment A_0 is to increase real wealth in America. Since the endowment of apples is concentrated in part of the economy (agents of type 2), increases in their endowment increase the gains to trade in America. There will be far more apples traded, and the relative price of apples to jeans and to Volkswagens will fall. In our example, the first American agent's elasticity of demand for apples is slightly more than 1, hence he will give up

more Jeans than before to get the extra apples. Thus trade of both commodities (and hence real income) unambiguously rises in period 0, as we see in columns 13 and 14 of Table III, as does the welfare of both American agents, as we see in columns 7 and 8 of Table III. The increased wealth of the apple endowed agent carries over even to the last period, when his endowment has returned to normal, increasing the trade in jeans even in period 1 (see column III 17), and so increasing real American income in both periods.

In order for more transactions to be carried out with the same money, American prices must fall in period 0 (and apple prices fall more than Jean prices), and again in period 1, but not by as much, since there isn't a huge apple stock that needs trading anymore. The rate of expected inflation is thus increased, and from the Fisher effect, there is a rise in the American long nominal rate (column II 11).

From purchasing power parity, and the fall of nominal American prices, we know that there must be an appreciation of the dollar against the mark in the initial period. (In the last period the dollar returns nearly to its former level.) We confirm that a stronger American economy does lead to a stronger dollar, if monetary policy is unchanged. American financial wealth owners are thus able to purchase slightly more German goods, which they do. But we also note that the temporary productivity increase has given rise to a temporary appreciation. Rational investors therefore anticipate in period 0 that the dollar will depreciate in period 1 back to its former level, hence they do not necessarily move capital from Germany to America. Indeed capital moves slightly in the other direction (see column II 15).

Although American wealth increases, the real relative price of Apples falls, so that the demand for Volkswagens is influenced in contrary directions by the income and substitution effects. In the new equilibrium, the real relative price of Jeans and Volkswagens is not much changed, as can be seen in column 1 of Table III. Hence German trade is not much affected, and price levels in Germany remain almost where they were before.

Conventional models of international finance presume that an increase in American income must raise net American imports, as well as appreciating the American dollar and raising the long term American interest rate. But that is because they do not allow for changes in relative prices. An appreciation of the dollar does not necessarily imply an improvement in the real terms of trade. Indeed, we see in our example that the dollar does unambiguously appreciate, while the terms of trade stay approximately the same (see column III 1). The balance of trade actually slightly improves (see column III 3). In other words, an increase in American interest rates does not necessarily make it more attractive to invest in America, because of the counterbalancing expected depreciation of the dollar. The Mundell–Fleming effect on net exports fails because of relative price changes.

If the increased productivity comes instead in the export sector of the American economy (see row 4), the situation is similar, and shows even more clearly the vulnerability of models like Mundell–Fleming that ignore relative price changes. The fall in relative prices is now in Jeans, i.e., precisely in the real terms of trade. (Hence German welfare also improves.) So we have an example with an appreciating Amer-

ican dollar, but worsening terms of trade. The American balance of trade improves: though the Americans are richer, and therefore import more, the extra endowment of Jeans makes the price of American exports more attractive, so exports also rise. Thus our model rejects one of the cornerstones of the Mundell–Fleming analysis of international trade.

11.3 Preferences

Consider the consequences, displayed in line 6, if the Americans develop a sudden taste for German Volkswagens which is known to be temporary (perhaps because American products will be as good as German cars in the future, or perhaps because the novelty of German cars will wear off). We find exactly the effects we would expect. There is a surge of international trade. The American balance of trade worsens in the initial period, and the dollar depreciates. The real terms of trade turn against America. Real income increases worldwide. Perhaps the only subtlety is that the German long rate goes up.

With the same dollars chasing more trades, there is a general fall in prices. Since the Germans are most affected, German real income increases the most in period 0. Therefore prices fall the most in that period. The Fisher effect then guarantees that the German long rate must rise.

11.4 Balanced Budget Fiscal Policy

The essence of fiscal policy is a transfer of wealth from a low marginal propensity to transact and consume agent to a high marginal propensity to transact and consume agent. Government expenditure on the domestic commodity market, i.e., on Apples in period 0, financed by lump sum¹⁵ taxes raised on say agent 2, transfers wealth from an agent who (1) does not spend all his wealth in the market (since he consumes a significant part of his endowment, as most people consume a significant portion of their labor endowment as leisure) and (2) directs a significant portion of his expenditures to period 1. The tax revenue is transferred to the government, which spends it all in period 0.

Fiscal policy necessarily has real effects, as we shall see in a moment. In the domestic economy these effects are consistent with Keynesian economics. The international effects, however, turn out to be different from those forecast by Mundell and Fleming because we do not pretend that all output can be aggregated into a single homogeneous good; in particular, it is not the case that increases in domestic output increase imports but do not affect exports. The Mundell–Fleming model predicts that expansionary American fiscal policy will increase American interest rates, appreciate the dollar, but decrease net exports and the balance of trade. As with productivity increases, we agree with the first two predictions but not with the third. Our analysis is summarized in line 8 of Tables II and III.

¹⁵In reality, temporary income tax surcharges are a mixture of lump sum and distortionary taxation. By treating taxes as lump sum we overestimate the multiplier we calculate below.

The most immediate impact is in the Apple sector, which experiences a large increase in trade on account of higher government demand. Trade in Jeans goes down since the government does not buy Jeans, and the American consumers are now poorer on account of the tax. But the decline in Jean activity is smaller than the rise in Apple activity, since the marginal propensity of the government to transact is higher than that of the agent from whom the government raised the tax revenue. Thus on the whole, aggregate activity goes up in America in period 0. In our example, the balanced budget multiplier is approximately $4/10$ (i.e., government expenditure worth 1 unit of apples increases apple sales by .55 and reduces the sales of jeans by .15, where units are normalized so that the price of apples and jeans are the same). In period 1, activity returns almost to where it had been before, although it is slightly different since the distribution of wealth is altered by the government tax.

The transfer of spending from period 1 back to period 0 must increase the long term real interest rate in America. (Since the government is consuming some of the period 0 goods, the consumers must consume less of them, and so period 0 marginal utilities rise compared to period 1 marginal utilities.)

Since American trading activity increases in period 0, the same money chases more transactions, and prices must fall. In period 1 prices are almost where they were before. Hence expected inflation is increased by balanced budget fiscal policy. The rise of both inflation and the real interest rate implies that the long term nominal interest rate must rise in America, which we indeed observe in our example (II 11). (Since the total money stock is constant, there must be some countervailing fall in some interest rate. We observe a small drop in the period 1 short rate, as shown in II 10.)

In the conventional analysis of international finance based on a single aggregate good, the increase in American real income would by assumption increase the demand for imports at the old prices. But here we see that the government is taxing wealth, part of which would have been spent on Volkswagens, and spending it all on Apples. Hence the demand for imports actually goes down (also because of the rise in interest rates). The real terms of trade turn in America's favor (see III 1); but, this is the *result* of weak demand for imports not a cause of strong demand for imports. The falling relative price of imports compensates for the weaker demand, and imports stay almost the same (II 19). Since the dollar price of Volkswagens goes down (the DM price holds the same, while the dollar appreciates), the American balance of trade deficit improves.

We can put the same point in a different way. The rise in the long-term American interest rate implies in the Mundell–Fleming model that capital must flow into America. But the temporary appreciation of the dollar implies that rational investors expect a future depreciation, which actually causes them to move capital back toward Germany and to reduce the American deficit (see II 15).

If the U.S. government expenditures were in the same proportion as private sector expenditures, there is still a slight improvement in the initial period balance of trade. The only way to get the Mundell–Fleming result on the balance of trade is if the government expenditures are concentrated on the export sector Jeans. That would

raise the price of Jeans, choking off exports and increasing imports, just as predicted in Mundell–Fleming.

11.5 Bond Financed Fiscal Policy

The Reagan budget deficit of the 1980s led to a huge balance of trade deficit. We can see why in our example.

Had the government raised money by issuing bonds instead of raising taxes, agents would realize the government would need money to redeem the bonds later. If the government made it clear that this would be achieved by a tax on the same agent 2 in the last period (instead of the first period), then we can easily see that agent 2 would buy the government bonds in period 0 and use the payments to meet its tax obligations in period 1. The equilibrium would be exactly the same as that described in the last section on balanced budget government expenditures. Barro’s principle of neutrality would be confirmed.

Suppose, however, that the government planned on redeeming its bonds by raising taxes on agents who did not have access to the bond markets of period 0. For example, in 1985 agents might rationally have assumed that the government would not raise taxes for a long time, and that when it did there would be a new generation of taxpayers that would have to bear a significant part of the burden. We can imagine adding another American household who is born in period 1, and who, at the equilibrium prices did not wish to trade even when he was alive. We calculate, in row 9 of Tables II and III, the effects of government expenditures in period 0 that are financed by bonds that will be redeemed by a tax on this new agent in period 1.

We see that the effects are similar to the balanced budget fiscal expansion, except that now the American balance of trade worsens, the long German interest rate also rises, and the increase in value of the American currency is smaller than before. The American demand for imports is almost unchanged by the fiscal policy because private wealth for those alive in period 0 is not reduced by the government taxation in period 1: the effects on imports of a higher interest rate (implying less consumption of all goods in period 0) and increased terms of trade (implying more consumption of foreign goods) just cancel. But the improved terms of trade decrease American exports, thus worsening the balance of trade.¹⁶

11.6 Monetary Policy (Open Market Operations)

In standard Keynesian models, an expansionary open market operation increases output and lowers interest rates. In the Mundell–Fleming model of international finance, expansionary open market operations also depreciate the currency, thereby increasing net exports and thus further increasing output, but returning interest rates to

¹⁶Part of the reason for the worsening balance of trade in period 0 is that the extra agent is forced to put jeans on the market in the last period in order to pay his taxes. Had he been a seller and consumer of Apples instead, we would have gotten results very similar to the balanced budget fiscal policy. Note that the forced sale of jeans reduces the price of jeans, and hence the terms of trade. It tends to make the currency appreciate in the last period as Germans try to buy more jeans. In order to maintain interest rate parity, the German long rate must also rise.

their original levels. Our analysis confirms most of these predictions, although via a different dynamic. Since we do not make the Mundell–Fleming small country hypothesis, we find that interest rates are indeed lowered by open market operations. Furthermore, although in our example exports rise and imports stay approximately the same, the depreciating dollar actually increases the American trade deficit (measured in dollars). Other differences also emerge depending on the maturity of the bonds the government bank tries to buy.

Suppose the government prints more money, making it available at the central bank to buy outstanding bonds or newly issued bonds. We trace out the effects when the American government operates at the short end of the yield curve, increasing M_0 , in line 11 of Tables II and III. We see that short interest rates in America decline, and with the increased efficiency, income increases. The increased activity in America leads to a higher demand for imports, worsening the real terms of trade for America. That in turn stimulates Germany to demand more imports itself, and chokes off the potential increase in American imports of German goods.

Because of the extra money, American price levels rise in period 0, although this is partly ameliorated by the increased activity of trade. Since the increased money supply is assumed to be short-lived (we suppose the open market operations are only in period 0), prices will not rise in period 1, and hence (compared to the original equilibrium), there is an expected deflation at period 0. Thus from the Fisher effect, the long interest rate in America also drops.

The rise in American prices, coupled with the increased activity in Germany, which causes a decrease in German prices (as the same money chases more goods) necessarily implies a depreciation in the dollar in period 0 from purchasing power parity. The real terms of trade also turn against America, since the increased efficiency of American trade leads to a greater demand for German goods, as we mentioned. All this is exactly consistent with the Mundell–Fleming model (noting that the drop in interest rates would have been discovered in Mundell–Fleming had they not made the small country hypothesis).

However, the differences between our model and the Mundell–Fleming model are revealed clearly in the balance of trade and the mechanisms for achieving equilibrium. In the Mundell–Fleming models, the drop in American interest rates necessarily implies that capital will flow out of America into Germany until capital markets equilibrium is reestablished. But that conclusion depends on the implicit hypothesis that exchange rates will not change further. In our example, the temporary nature of the monetary injections implies that agents should rationally anticipate a return of exchange rates to nearly their previous levels, and hence an expected appreciation of the dollar. This makes German investment in American bonds much more attractive, and in fact in our example, German investment actually increases after expansionary open market operations. The American trade deficit thus decreases, instead of increasing.

On account of the worsening terms of trade, American exports increase and American imports stay approximately the same (they are bolstered by increasing American income). This conclusion is consistent with the deteriorating American balance of

trade since that is measured in dollars, and the dollar has fallen in value, so the same imports cost more money.

We also note that the fall in the American long rate and the expected dollar appreciation just cancel, so that there is no reason for the German long rate to move. In the last period American prices rise very slightly, since most of the extra money has left the system, while German prices fall no more than in period 0.

The rise in American prices in the original period shifts the distribution of wealth in America away from the second agent, who began with large stocks of money, to the first agent, who did not lose any wealth from the price rise. As usual, welfare closely follows income. One can easily imagine that the second agent would use his potential influence to prevent the government from undertaking expansionary monetary policy.

A puzzling aspect of monetary policy is the question of what effect more money has on interest rates: on the one hand looser money should make it easier to borrow and hence lower interest rates; on the other hand more money means higher prices, more inflation, and hence by the Fisher effect, higher interest rates. This puzzle is easily resolved by distinguishing between increases in the stock of money, and increases in the expected growth rate of money. Both ultimately increase price levels, but the former does so all at once, lowering interest rates, while the latter does so gradually, and thereby increases nominal interest rates.

Had the monetary injections come at date 1, instead of date 0, and been foreseen back at date 0, which is tantamount to a policy of faster growth in the money supply, then we would again find nonneutral effects, but with some differences. The most obvious consequence would be a rise in American price levels in period 1, and therefore an increase in period 0 in expected inflation. From the Fisher equation, this would increase the long term American interest rate, even as the period 1 short interest rate was falling.

Row 12 of Tables II and III complete the analysis. The period 1 interest rate drops, American income increases as trade gets more efficient, and the dollar depreciates and the real terms of trade turn against America in period 1. Exports increase and imports stay about the same in period 1, so the American balance of trade surplus increases.

An increase in the amount of bank money \bar{M}_0 to purchase long bonds has effects which are similar to those of increasing M_0 except for one interesting difference, which is probably just an artifact of the timing we have chosen for our markets. Although the long rate goes down as expected, the short American interest rate goes up, so that the term structure of interest rates is twisted. The reason is that as before, the increased stock of money causes American prices to rise in period 0 and the dollar to depreciate. This increases the transactions demand for cash, especially for international purchases. But the extra money M is not available in time for the purchase of foreign currency. Hence the short rate is driven up by American agents intent on borrowing dollars in the short term market to purchase German marks. That reduces American trading efficiency, and indeed lowers the welfare of both the American agents. We could easily imagine both American agents resisting plans for expansionary monetary policy if they thought it might involve purchases of the long

bond without parallel purchases of short term bonds.

11.7 Money Financed Fiscal Transfers

In rows 13 and 14 of Tables II and III we trace out the consequences when the American government prints money and transfers it to the second agent. When the transfer comes in period 1 and is anticipated from the beginning of period 0, American prices rise in period 1, but not by much in period 0. Thus there is an immediate expected inflation in period 0, and so by the Fisher effect, the long nominal interest rate must rise in America. From purchasing power parity, the dollar must fall in value in period 1, but not in period 0. Hence there is also an expected depreciation. The increase in interest rates and the expected depreciation have countervailing effects on the flow of capital in period 0, hence from general principles we cannot anticipate the direction of capital flow.

The main real effect of the transfer of newly printed money in period 1 is to raise the short interest rate in period 1 in America. This reduces trading efficiency without changing real wealth (because of the inflation). Hence real income drops in America in period 1. This tends to reduce the demand for imports and improves the period 1 terms of trade for America. In equilibrium, after the price effects are taken into account, exports decrease and imports decrease slightly in period 1. However, the American balance of trade surplus further increases in period 1 since the price effect outweighs the quantity change.

A transfer of money in period 0 to the second agent also has negative effects on income and welfare since it increases the short interest rate $r_{0\alpha}$. Even though the money is durable, it increases contemporaneous prices more than future prices, thus tending to lower long term interest rates (row 13, column II 11).

Unless we allow for missing agent markets (as in [10]), printing money and transferring it to agents lowers the general welfare, except of course for the recipient of the transfer.

11.8 International Transfers

The United States has very often been called upon to bail other countries out of financial crises, most recently in Mexico. The consequences of printing American dollars and giving them to agents of the foreign country can be trace out in row 15 of our tables.¹⁷ The results are as expected. German welfare improves, American welfare declines, German balance of trade improves and America's worsens, and the dollar depreciates. As with transfers to Americans, American trading activity declines because short interest rates rise. American prices rise more in period 0 than in period 1, so the long American interest rate falls.

¹⁷An identical scenario obtains if, say, Iran succeeds in counterfeiting American dollars.

11.9 Foreign Exchange Intervention

Governments often act on the foreign exchange markets to prop up the values of their own currency, by selling reserves of the foreign currency. The Bundesbank is probably the most famous actor in this direction. In row 16 we see that the effects of such actions are unambiguous. Leaving aside the question of how the Americans got their reserves of DM in the first place, row 16 shows that a U.S. government sale of DM would indeed increase the value of the dollar, improve American welfare, and worsen German welfare. The situation is similar to the case discussed in Section 11.9, where the “gift” of DM has been to the American government instead of to American people.

Table II

	P _{0,J}	P _{0,A}	P _{0,V}	P _{0,B}	P _{1,J}	P _{1,A}	P _{1,V}	P _{1,B}	r _{0α}	r _{1α}	r̄ _{0α}	r _{0β}	r _{1β}	r̄ _{0β}	d _{0α} ³	π _{0αβ}	π _{1αβ}	EX ₀	IM ₀	EX ₁	IM ₁
φ _{0,V} ^α	-	-	≈+	+	≈-	+	+	+	≈	≈-	+	-	+	-	-	+	≈+	-	-	-	-
A ₀	-	-	≈+	≈+	-	-	≈+	≈-	≈	≈-	+	≈	≈-	≈+	≈-	+	≈+	≈	≈	≈	≈
A ₁	≈	≈	≈	≈	-	-	≈	≈	≈	≈+	-	≈	≈	≈	+	≈	+	≈	≈	≈	≈
J ₀	-	-	≈-	-	≈+	+	≈-	-	≈	≈-	+	≈+	≈-	≈	-	+	≈-	+	≈+	≈-	+
J ₁	≈+	≈+	≈-	≈-	-	-	≈	≈-	≈	≈+	-	≈+	≈-	≈	+	≈-	+	-	≈+	+	+
MU(V ₀)	-	-	-	-	≈	≈-	≈-	≈-	≈	≈	≈	+	-	+	+	-	≈-	+	+	≈+	≈-
MU(V ₀) and MU(V ₁)	≈-	≈-	-	-	≈-	-	≈-	-	≈-	≈-	≈+	+	+	-	-	-	-	+	+	+	+
Fiscal policy tax financing	-	-	≈	≈	≈	≈	≈	≈	≈	-	+	≈	≈	≈	-	+	≈	≈	≈	≈	≈
Fiscal policy bond financing	-	-	≈	≈	≈+	+	≈-	≈-	≈	≈-	+	≈	≈-	+	+	≈+	-	-	≈	+	≈+
M	+	+	≈	≈+	-	-	≈	≈	+	-	-	≈	≈	≈	+	-	+	-	≈-	+	≈-
M ₀	+	+	≈	≈-	≈+	≈+	≈	≈	-	≈+	-	≈+	≈-	≈	+	-	≈-	≈+	≈	≈	≈+
M ₁	≈	≈+	≈	≈-	+	+	≈	≈	≈	-	+	≈	≈	≈	≈+	≈	-	≈	≈	≈+	≈
m ₀ ²	+	+	≈+	≈+	≈	≈	≈	≈	+	≈+	-	≈-	≈	≈+	+	-	≈	≈-	≈	≈+	≈-
m ₁ ²	≈	≈	≈	≈	+	+	≈	≈	≈	+	+	≈	≈	≈	≈-	≈	-	≈+	≈	-	≈-
International transfer	+	+	≈+	≈+	≈	≈-	≈+	≈+	+	≈+	-	-	≈	≈+	+	-	≈+	-	≈-	+	≈-
Foreign exchange intervention	-	-	≈	≈	≈	≈	≈	≈	-	-	+	≈+	≈	≈-	-	+	≈	-	-	-	+
	+ (-): Substantial increase (decrease) + (-): Weak increase (decrease) ≈ : Approximately equal								EX: Exports from US IM: Imports to US T: Terms of trade of US MU: Marginal utility in US						CA: Current account in US TB ₁ : Trade balance at t = 1 in US TB: Total trade balance in US						

Table III

	T ₀	T ₁	CA ₀	CA ₁	TB ₁	TB	U ₁	U ₂	U _{US}	U ₃	U ₄	U _{GER}	q _{0,J} ¹	q _{0,A} ²	q _{0,V} ³	q _{0,B} ⁴	q _{1,J} ¹	q _{1,A} ²	q _{1,V} ³	q _{1,B} ⁴	$\frac{P_{0,V}}{\pi_{b,\alpha\beta}}$
$\varphi_{0,V}^\alpha$	+	-	+	-	-	-	≈	+	+	-	-	-	-	+	-	-	-	≈	+	-	-
A ₀	≈	-	-	-	≈	+	+	+	+	-	-	-	+	+	≈	≈	≈	-	≈	≈	≈
A ₁	≈	+	-	+		-	+	+	+	-	-	-	≈	-	≈	≈	≈	+	≈	≈	≈
J ₀	-	+	+	-	-	+	+	+	+	+	+	+	+	+	+	+	-	+	+	+	-
J ₁	≈	-	-	+	+	-	+	+	+	+	+	+	-	+	+	≈	+	≈	≈	+	+
MU(V ₀)	-	+	-	+	+	+	+	+	+	+	+	+	+	≈	+	+	+	≈	≈	≈	+
MU(V ₀) and MU(V ₁)	-	-	+	-	-	-	+	+	+	+	+	+	+	-	+	≈	-	-	+	+	+
Fiscal policy tax financing	+	-	+	-	≈	+							-	+	≈	≈	-	+	≈	≈	≈
Fiscal policy bond financing	+	-	-	+	+	+	+						+	≈	≈	-	+	≈	-	+	+
M	-	-	-	+	-	-	-	-	-	≈	≈	≈	+	+	≈	≈	-	≈	≈	≈	≈
M ₀	-	+	-	+	≈	-	+	-	≈	≈	≈	≈	+	+	≈	≈	-	+	≈	≈	≈
M ₁	≈	-	-	+	+	+	+	-	≈	≈	≈	≈	-	+	≈	≈	+	+	≈	≈	≈
m ₀ ²	-	≈	-	+	+	-	-	+	-	≈	≈	≈	-	-	-	-	≈	≈	-	≈	≈
m ₁ ²	≈	+	+	-	+	+	-	+	≈	≈	≈	≈	≈	≈	≈	≈	-	-	≈	≈	≈
International transfer	+	+	-	+	+	-	-	-	-	+	+	+	-	-	-	≈	≈	≈	≈	≈	≈
Foreign exchange intervention	-	≈	-	-	-	-	+	+	+	-	-	-	+	+	-	-	-	≈	≈	-	-

References

- [1] Bhagwati, J. N., 1958. "Immiserizing Growth: A Geometrical Note," *Review of Economic Studies*, 25: 201–205.
- [2] Bhagwati, J. N. and T. N. Srinivasan, 1983. *Lectures on International Trade*. Cambridge: M.I.T. Press.
- [3] Blanchard, O. and S. Fisher, 1989. *Lectures on Macroeconomics*. Cambridge: M.I.T. Press.
- [4] Bruno, M., 1973. "Protection and Tariff Change under General Equilibrium," *Journal of International Economics*, 3: 205–226.
- [5] Clower, B., 1967. "A Reconsideration of the Microeconomic Foundations of Monetary Theory," *Western Economic Journal*, 6: 1–8.
- [6] Deaton, A., 1991. "Savings and Liquidity Constraints," *Econometrica*, 59: 1221–1248.
- [7] Dornbusch, R., 1976. "Expectations and Exchange Rate Dynamics," *Journal of Political Economy*, 84: 1161–1176.
- [8] Dornbusch, R., 1988. *Exchange Rates and Inflation*. Cambridge: M.I.T. Press.
- [9] Dubey, P. and J. Geanakoplos, 1992. "The Value of Money in a Finite-Horizon Economy: A Role for Banks," in P. Dasgupta, D. Gale *et al.* (eds.), *Economic Analysis of Market and Games*. Cambridge: M.I.T. Press.
- [10] Dubey, P. and J. Geanakoplos, 1993. "Monetary Equilibrium with Incomplete Markets," forthcoming CFDP.
- [11] Dubey, P. and M. Shubik, 1978. "The Non-cooperative Equilibria of a Closed Trading Economy with Market Supply and Bidding Strategies," *Journal of Economic Theory*, 17:1–20.
- [12] Flemming, J. M., 1962. "Domestic Financial Policies under Fixed and under Floating Exchange Rates," *IMF Staff Papers* 9, 3: 369–379.
- [13] Frenkel, J. A. and A. Razin, 1992. *Fiscal Policies and the World Economy*. Cambridge: M.I.T. Press.
- [14] Grandmont, J. M., 1983. *Money and Value*. Cambridge: Cambridge University Press.
- [15] Grandmont, J. M. and G. Laroque, 1973. "On Money and Banking," *Review of Economic Studies*, 207–236.
- [16] Grandmont, J. M. and Y. Younes, 1973. "On the Efficiency of a Monetary Equilibrium," *Review of Economic Studies*, 149–165.

- [17] Grandmont, J. M. and Y. Younes, 1972. "On the Role of Money and the Existence of a Monetary Equilibrium," *Review of Economic Studies*, 39: 355–372.
- [18] Grilli, V. and N. Roubini, 1991. "Financial Intermediation and Monetary Policies in the World Economy," N.B.E.R., Technical Working Paper No. 104.
- [19] Grilli, V. and N. Roubini, 1992. "Liquidity and Exchange Rates," *Journal of International Economics*, 32: 339–352.
- [20] Groden, N. M. and F. Gruen, 1970. "A Tariff That Worsens the Terms of Trade," in J. A. McDougall and R. H. Snape, *Studies in International Economics*. North Holland.
- [21] Hahn, F. H., 1977. "The Monetary Approach to the Balance of Payments," *Journal of International Economics*, 7: 231–249.
- [22] Helpman, E., 1973. "The Exact Measurement of Welfare Losses Which Result from Trade Taxes," *International Economic Review*.
- [23] Kareken, J. and N. Wallace, 1981. "On the Indeterminacy of Equilibrium Exchange Rates," *Quarterly Journal of Economics*, 96: 207–222.
- [24] Kirman, A. P., 1992. "Whom or What Does the Representative Individual Represent?" *Journal of Economic Perspectives*, 6, 2: 117–136.
- [25] Lucas, R. E., Jr., 1982. "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics*, 10: 335–359.
- [26] Lucas, R. E., Jr., 1990. "Liquidity and Interest Rates," *Journal of Economic Theory*, 50: 237–264.
- [27] Mundell, R., 1968. *International Economics*. New York: Macmillan.
- [28] Shubik, M., 1973. "Commodity Money, Oligopoly, Credit and Bankruptcy in a General Equilibrium Model," *Western Economic Journal*, 11: 24–38.
- [29] Shubik, M., 1993. *The Theory of Money and Financial Institutions*, manuscript.
- [30] Shubik, M. and D. P. Tsomocos, 1992. "A Strategic Market Game with a Mutual Bank with Fractional Reserves and Redemption in Gold," *Journal of Economics*, 55(2): 123–150.
- [31] Shubik, M. and C. Wilson, 1977. "The Optimal Bankruptcy Rule in a Trading Economy Using Fiat Money," *Journal of Economics*, 37: 337–354.
- [32] Stockman, A., 1980. "A Theory of Exchange Rate Determination," *Journal of Political Economy*, 88(41): 673–698.
- [33] Stockman, A., 1987. "The Equilibrium Approach to Exchange Rates," *Richmond Federal Reserve Bank Economic Review*, 12–30.

- [34] Svensson, L. and A. Razin, 1983. “The Terms of Trade and the Current Account: The Harberger–Laursen–Metzler Effect,” *Journal of Political Economy*, 91: 97–125.

APPENDIX I: PROOF OF THEOREM

Let $M_\alpha^* \equiv \sum_{t=0}^T (M_{t\alpha} + \sum_{\beta \in C} \bar{M}_{t\alpha} + M_{t\beta\alpha} + \mu_{t\alpha} + \bar{\mu}_{t\alpha} \sum_{h \in H} m_{t\alpha}^h)$ be the total quantity of money ever appearing in country α . For each $h \in H$, $\alpha \in C$, and $\varepsilon > 0$ let

$$\sum_\varepsilon^h = \left\{ (x^h, \mu^h, \bar{\mu}^h, d^h, \bar{d}^h, b^h, q^h, \hat{m}^h) \in \mathbb{R}_+^{LT} \times \mathbb{R}_+^T \times \mathbb{R}_+^{T-1} \times \mathbb{R}_+^{CT} \times \mathbb{R}_+^{C(T-1)} \right. \\ \left. \times \mathbb{R}_+^{[L+C(C-1)]T} \times \mathbb{R}_+^{LT} \times \mathbb{R}_+^T : 0 \leq x^h \leq 2A1, \varepsilon m_t^h \leq \mu_{t\alpha}^h \leq \frac{1}{\varepsilon}, \varepsilon m_t^h \leq \bar{\mu}_{t\alpha}^h \leq \frac{1}{\varepsilon}, \right. \\ \left. 0 \leq d_{t\alpha}^h \leq M_\alpha^*, 0 \leq \bar{d}_{t\alpha}^h \leq M_\alpha^*, \varepsilon e_{t\ell}^h \leq q_{t\ell}^h \leq e_{t\ell}^h, \varepsilon m_t^h \leq b_{t\ell}^h \leq \frac{1}{\varepsilon}, 0 \leq \hat{m}_t^h \leq M_\alpha^* \right\}$$

which is compact and convex. Let the typical element of \sum_ε^h be $\sigma^h \in \sum_\varepsilon^h$. Define $B_\varepsilon^h(\eta) = B^h(\eta) \cap \sum_\varepsilon^h$. Also, $\sigma = (\sigma^1, \dots, \sigma^H) \in \sum_\varepsilon = \prod_{h \in H} \sum_\varepsilon^h$. Define the map:

$$\Psi_\varepsilon : \sum_\varepsilon \rightarrow N, \text{ where}$$

$$N = \{ \eta = (p, \pi, r, \bar{r}, R) \in (\mathbb{R}_{++}^{LTC} \times \mathbb{R}_{++}^{TC(C-1)}) \times (-1, \infty)^{TC} \times (-1, \infty)^{(T-1)C} \times \mathbb{R}_+^{TC} \}.$$

and Ψ_ε is defined by equations (i)–(v). In addition define (η, σ) to be an ε -IME iff $\eta = \Psi_\varepsilon(\sigma)$, and (vi)' $\sigma^h \in \text{Argmax}_{\sigma^h \in B_\varepsilon^h(\eta)} u^h(x^h(\sigma^h))$. Note also that all elements of $\Psi_\varepsilon(\sigma) = \eta$ are continuous functions of σ , since on each market some agents are bidding (offering) strictly positive amounts.

Furthermore, define

$$G : N \rightrightarrows \prod_{h \in H} \sum_\varepsilon^h = \sum_\varepsilon, \text{ where}$$

$$G^h = \text{Argmax}_{\sigma^h \in B_\varepsilon^h(\eta)} u^h(x^h(\sigma^h)) \text{ and } G = \prod_{h \in H} G^h.$$

Finally, let $F = G \circ \Psi : \sum_\varepsilon \rightrightarrows \sum_\varepsilon$. G is convex-valued since $\sigma^h \rightarrow u^h(x^h(\sigma^h))$ is concave. Recall, $\sigma^h \rightarrow x^h(\sigma^h)$ is linear, and that $B_\varepsilon^h(\eta)$ is convex. Since Ψ is a function, $F = G \circ \Psi$ is also convex-valued. Moreover, if ε is sufficiently small, G is non-empty, since all actions can be financed out of initial endowments. When $\varepsilon > 0$, the prices are all positive, and since $e^h \neq 0$, $B_\varepsilon^h(\eta)$ is a continuous correspondence. Hence by the Maximum Theorem, G is compact valued and upper semi-continuous, and therefore so is F . Note that since we have restricted the domain of Ψ to \sum_ε , and since for each good and money, some agent has a strictly positive endowment, we legitimately restricted the range of Ψ to strictly positive prices, and interest rates strictly greater than -1 .

Step 1. An ε -IME exists for any sufficiently small $\varepsilon > 0$.

The map F satisfies all the conditions of the Kakutani fixed point theorem, and therefore admits a fixed point $F(\sigma) \ni \sigma$ which is easily verified to be an ε -IME.

For every small $\varepsilon > 0$, let $(\eta(\varepsilon), \sigma_\varepsilon)$ denote the corresponding ε -IME.

Step 2. At any ε -IME, $r_{t\alpha}(\varepsilon), \bar{r}_{t\alpha}(\varepsilon) \geq 0, \forall t \in T, \alpha \in C$.

W.l.o.g., suppose $r_{t\alpha}(\varepsilon) < 0$ for some $t \in T$, $\alpha \in C$. Then for each h , $\mu_{t\alpha}^h(\varepsilon) < 1/\varepsilon$, otherwise $1 + r_{t\alpha} \geq (\sum_{h \in H^\alpha} \mu_{t\alpha}^h(\varepsilon))/M_\alpha \geq 1$. But then any agent h could have increased $\mu_{t\alpha}^h(\varepsilon)$ by $\Delta > 0$, spent $-r_{t\alpha}(\varepsilon)\Delta$ on any good at time t and strictly increased his utility, and then returned $[1 + r_{t\alpha}(\varepsilon)]\Delta$ to the bank without being insolvent and disturbing his other choices, a contradiction. Similarly, $\bar{r}_{t\alpha}(\varepsilon) \geq 0$, $\forall t \in T$, $\alpha \in C$.

Step 3. At any ε -IME, there exists some $Z < \infty$ such that $r_{t\alpha}(\varepsilon), \bar{r}_{t\alpha}(\varepsilon) < Z$, and $\mu_{t\alpha}^h(\varepsilon) \leq M_\alpha^*$, $\bar{\mu}_{t\alpha}^h \leq M_\alpha^*$, and $b_{t\ell}^h \leq M_\alpha^*$ for all $t \in T$, $\alpha \in C$, and $h \in H$.

Since no more than

$$M_\alpha^* = \sum_{t=0}^T (M_{t\alpha} + \bar{M}_{t\alpha} + M_{t\beta\alpha} + \sum_{h \in H} m_{t\alpha}^h)$$

can ever be borrowed or returned on α -loans at any $t \in T$,

$$\sum_{h \in H} (\mu_{t\alpha}^h(\varepsilon) + \bar{\mu}_{t\alpha}^h(\varepsilon)) \leq M_\alpha^* \text{ and } \sum_{h \in H} \sum_{\ell \in L^\alpha} b_{t\ell}^h(\varepsilon) \leq M_\alpha^*$$

if every agent chooses in his budget set. Hence,

$$1 + r_{t\alpha}(\varepsilon) \leq \frac{M_\alpha^*}{M_{t\alpha}}, \text{ and } 1 + \bar{r}_{t\alpha} \leq \frac{M_\alpha^*}{\bar{M}_{t\alpha}}.$$

Step 4. For any ε -IME, there exists $k > 0$ such that $p(\varepsilon) \geq k1$, where $1 = (1, \dots, 1)$ of suitable dimension. Furthermore, for any countries, $\alpha, \beta \in C$, $\pi_{t\beta\alpha}(\varepsilon)p_{t\ell}(\varepsilon) \geq k1$, for $\ell \in L^\beta$.

For each country $\alpha \in C$, there is some agent $h \in H^\alpha$ with $m_{0\alpha}^h > 0$. In any ε -IME, agent h consumes no more than the aggregate world endowment of each good. Hence $x^h(\varepsilon) \leq A$ for some constant A . But from our hypothesis on u^h , $u^h(0, \dots, K, \dots, 0) > u^h(A1)$ for large enough K , where the K appears in the $t\ell$ commodity place, for any $\ell \in L^\alpha$, $t \in T$. Hence $p_{t\ell} \geq m_{0\alpha}^h/K$, otherwise agent h would hoard his $m_{0\alpha}^h$ money until period t , spend all of it on good $t\ell$, and improve his utility, contradicting the fact that we are at an ε -IME.

Similarly, an agent in country α with $m_{0\alpha}^h > 0$ can purchase $m_{0\alpha}^h/\pi_{t\beta\alpha}(\varepsilon)p_{t\ell}(\varepsilon)$ units of good $t\ell$, where $\ell \in L^\beta$, by hoarding his α -money until period t , then exchanging it for $m_{0\alpha}^h/\pi_{t\beta\alpha}(\varepsilon)$ of β -money, and then purchasing $m_{0\alpha}^h/\pi_{t\beta\alpha}(\varepsilon)p_{t\ell}(\varepsilon)$ units of good $t\ell$.

Take a sequence of ε -IME with $\varepsilon \rightarrow 0$.

Step 5. If for some good $\bar{t}\bar{\ell}$ with $\bar{\ell} \in L^\alpha$, $p_{\bar{t}\bar{\ell}}(\varepsilon) \rightarrow \infty$, then $p_{t\ell}(\varepsilon) \rightarrow \infty$ for all $t \in T$, $\ell \in L^\alpha$; furthermore, $\pi_{t\beta\alpha}(\varepsilon)p_{t\ell}(\varepsilon) \rightarrow \infty$ for all $\beta \in C/\{\alpha\}$ and $\ell \in L^\beta$.

Consider first commodity $t\ell$, for $t \leq \bar{t}$, and $\ell \in L^\alpha$. Some agent owns $e_{\bar{t}\bar{\ell}}^h > 0$. If $p_{t\ell}(\varepsilon)$ stays bounded on some subsequence, then by choosing very large $\bar{\mu}_{t\alpha}^h(\varepsilon)$ agent h can borrow a large amount of money at time t and use it to buy more than K units of good $t\ell$ (see Step 4). Agent h can then roll over his loan until period \bar{t} by taking out another loan at time $t+1$ to pay off the loan from period t and so on. Since

the interest rates are uniformly bounded by Step 3, the agent will have a bounded amount of money to repay at time \bar{t} . Since $p_{\bar{t}\bar{\ell}}(\varepsilon) \rightarrow \infty$, for small ε he will be able to repay it all out of sales of commodity $\bar{t}\bar{\ell}$, contradicting the optimality of his choice at the ε -IME.

If $t > \bar{t}$, and $p_{t\ell}(\varepsilon)$ stays bounded from above, then agent h could have sold his endowment $e_{\bar{t}\bar{\ell}}^h > 0$, gained $p_{\bar{t}\bar{\ell}}(\varepsilon)e_{\bar{t}\bar{\ell}}^h \rightarrow \infty$, inventoried the money and purchased $p_{t\ell}^h(\varepsilon)e_{\bar{t}\bar{\ell}}^h/p_{t\ell}(\varepsilon) \rightarrow \infty$ units of good $t\ell$, again contradicting the optimality of the ε -IME.

The same argument applied to foreign goods $\ell \in L^\beta$ yields the rest of Step 5.

Step 6. There is $K > 0$ such that $p_{t\ell}(\varepsilon) < K$ for all t, ℓ, ε .

Let $\bar{t} \in T$ and $\bar{\ell} \in L^\alpha$, and suppose that $p_{\bar{t}\bar{\ell}}(\varepsilon) \rightarrow \infty$. Then $p_{t\ell}(\varepsilon) \rightarrow \infty$ for all $t \in T, \ell \in L^\alpha$, and $\pi_{t\beta\alpha}(\varepsilon)p_{t\ell}(\varepsilon) \rightarrow \infty$ for all $t \in T, \beta \neq \alpha, \ell \in L^\beta$. Since

$$p_{t\ell}(\varepsilon) = \frac{\sum_{h \in H} b_{t\ell}^h(\varepsilon)}{\sum_{h \in H^\alpha} q_{t\ell}^h(\varepsilon)} \leq \frac{M_\alpha^*}{\sum_{h \in H^\alpha} q_{t\ell}^h(\varepsilon)},$$

we must have that $q_{t\ell}^h(\varepsilon) \rightarrow 0$ for all $\ell \in L^\alpha, h \in H^\alpha$. Hence $x_{t\ell}^h(\varepsilon) \xrightarrow{\varepsilon \rightarrow 0} e_{t\ell}^h$ for all $\ell \in L^\alpha$. We now show that $x_{t\ell}^h(\varepsilon) \xrightarrow{\varepsilon \rightarrow 0} 0 = e_{t\ell}^h$ for all $\ell \notin L^\alpha$. Suppose to the contrary that $x_{t\ell}^h(\varepsilon)$ stayed bounded away from 0 for some $\ell \in L^\beta$. Somehow agent h must have acquired $p_{t\ell}(\varepsilon)x_{t\ell}^h(\varepsilon)$ of β -money by period t . But the only way for $h \in H^\alpha$ to acquire β -money is to exchange α -money for β -money at period t or earlier. Since interest rates are uniformly bounded, agent h must have acquired at least $\bar{k}p_{t\ell}(\varepsilon)x_{t\ell}^h(\varepsilon)$ of β -money at some period $t' \leq t$, where $\bar{k} > 0$ does not depend on ε . But that would have required $\pi_{t'\beta\alpha}(\varepsilon)\bar{k}p_{t\ell}(\varepsilon)x_{t\ell}^h(\varepsilon) \xrightarrow{\varepsilon \rightarrow 0} \infty$ of α -money, contradicting the finiteness of M_α^* . Thus, we see that $x^h(\varepsilon) \rightarrow e^h$.

At any ε -IME, $r_{t\alpha}(\varepsilon) \leq \delta_{t\alpha}$ for all t . Hence at any ε -IME, there are less than δ_α -gains to trade in country α at any t . By continuity, there are less than δ_α -gains to trade at the allocation $(e^h)_{h \in H^\alpha}$. But the gains to trade hypothesis guarantees that there are more than δ_α -gains to trade in some period t at the initial endowment $(e^h)_{h \in H^\alpha}$, a contradiction.

Thus we have shown $p_{t\ell}(\varepsilon)$ is bounded for all $\ell \in L^\alpha$. But α was arbitrary, so Step 6 is verified.

Step 7. There are $0 < k < K$ such that the exchange rates are bounded:

$$k < \pi_{t\alpha\beta}(\varepsilon) < K \text{ for all } t, \alpha, \beta.$$

We have shown that $\pi_{t\alpha\beta}(\varepsilon)p_{t\ell}(\varepsilon) \geq k$ for all $\ell \in L^\beta$ and all ε . Since from Step 6, $p_{t\ell}(\varepsilon)$ is bounded above, we know that $\pi_{t\alpha\beta}(\varepsilon)$ is bounded from below for all α, β . But $\pi_{t\alpha\beta}(\varepsilon) = \pi_{t\beta\alpha}^{-1}(\varepsilon)$, so $\pi_{t\alpha\beta}(\varepsilon)$ is also bounded above.

Now we know that $(\eta(\varepsilon), \sigma_\varepsilon)$ is bounded above and below in every coordinate, independent of ε . Hence we can select a convergent subsequence $(\eta(\varepsilon), \sigma_\varepsilon) \rightarrow (\eta, \sigma)$.

A standard argument shows that (η, σ) is an IME. In particular, the artificial upper and lower bounds on choices are irrelevant since they are not binding and utilities are concave in actions. ■