A DOUBLE AUCTION MARKET: TEACHING, EXPERIMENT AND THEORY

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Abstract

A simultaneous double auction market with bid and offer cards was utilized in classes on the theory and history of money and financial institutions and occasionally in classes on the theory of games. The prime purpose in using this game was to teach the students how to construct process models of economic phenomena. The second purpose was to consider the properties of the double auction market. The third purpose was to interpret the experimental results an link them to theory.

Keywords: Double auctions, Experimental games, Allocation games, Noncooperative equilibria

JEL Classification: C7, D44, C92, G1

Contents

1 Introduction

From 1981 to 2003 in teaching a seminar on the theory and history of money and financial institutions and courses in game theory I have employed a simple double auction game in class or in other lectures both to illustrate the problems in the construction of formal models and to heighten student participation in investigating process models in the study of economics. The respondents were not paid and, with one exception, played only once in a class or lecture. Most of the use of the games was in classes at Yale where the students were, for the most part, undergraduates (seniors) or Master's degree students.

Although these double auction games were run over twenty times, part of the data files were lost. The results noted here are based on the data for the remaining fifteen games.

Beyond noting the reconfirming but modest results obtained from the informal experiments, the purpose of this note is to stress the use of teaching and theory combined with informal experimentation. In particular the relevance of the results on the double auction raises many problems in gaming for teaching and experimentation; in the development of financial models of price movement and in basic noncooperative and cooperative game theory. These links are explored in some depth in a separate paper. The goal here is limited to pointing out the links and their potential meaning.

2 The Double Auction Market

The double auction market is a well known mechanism and has been studied in a simultaneous and sequential form in many ways from the viewpoints of pure theory, stock market practises and experimentation. From the viewpoint of noncooperative game theory applications the double auction market is one of the three simplest price formation mechanisms which can be constructed [8], [11]. When viewed from cooperative game theory the double auction mechanism can be utilized to calculate a characteristic function which, in turn, can be utilized to investigate the game as a form of assignment game [25].

In the sequential or continuous variation the double auction market is a fairly good approximation of the trading mechanism employed by the New York Stock Exchange and several other exchanges. The sequential form illustrates the intimate relationship between this market mechanism and the handling of situations where the sequencing of information is of considerable concern and where there is a lack of common knowledge of the valuations placed by others. The simultaneous move game minimizes concern for information conditions and is best described as a model of a form of a sealed bid.

It has been a source of wonder as to how well this simple mechanism forms a price that is quickly close to the theoretical competitive price. The informal teaching experiments noted here reconfirm this observation. The volume of trade, in these "one shot games,", however is significantly lower than called for by the equilibrium analysis. Prior to discussing the specific games considered here, a brief summary of other experimental findings is noted

2.1 Comments of Some of the Experimental Results and Theory

The literature and experiments on the double auction markets is extensive and to some extent surprisingly separate. There are at least three strands to be noted. There is the considerable literature on experimentation and what can be described as more or less central economic theory as well as a growing literature verging somewhat into behavioral and information economics; there is a growing literature in applied finance on price movements in the stock market with a continuous sequential double auction market and there is a small literature on the pure game theory of the double auction market.

2.2 Experimentation, simulation behavior and information

Chamberlin's [5] more or less informal class experiment is possibly the originator of the experimental game in economics. His class apparently motivated Vernon Smith [27], [28], [30] to consider experimental gaming as a valuable tool in economics and in particular to consider the double auction market. There is now an active study of variations on the double auction mechanism. A bibliography by Holt [18] notes around seventy references. The book edited by Friedman and Rust [14] is motivated by the need to reconcile economic theory with price formation mechanisms. This is possibly the first book to attempt to deal with the role of institutions, how they interface with current theory and experimental evidence from gaming. The theme developed is consistent with the basic approach in the development of strategic market games [32], [33] that general equilibrium theory is noninstitutional and does not deal with process, yet price formation requires process. Following the methods of game theory and experimental gaming, in order to fully specify a playable game the rules must be adequately described. But the rules themselves can be interpreted as the elementary institutions which carry process.

There are four sources in the last decade which provide a reasonably thorough coverage of much of the work on double auctions, both from the viewpoint of experimental games and some economic theory. They are Klemperer's two volume [20] coverage of all variants of auctions; Friedman and Rust's special edited volume devoted exclusively to all aspects of the double auction [14]; the encompassing *Handbook of Experimental Economics* by Kagel and Roth [19] covering all forms of gaming and the work on Experimental Economics by Davis and Holt [7] supplemented by a more recent bibliography on the Web by Holt [18].

As there are myriads of institutional variations and sensitivity analysis is virtually an art form, no attempt is made here to be either exhaustive or to regurgitate in any detail the material covered in these careful references. The key features over which there appears to be some clear consensus is that the double auction price under many variations converges quickly to the competitive equilibrium price. The convergence of the trading volume is not as immediate. The level of information required and knowledge of the rules of the game appear to be considerably less than the requirements of formal game theory. A distinction is found in the behavior in a call market where trades do not take place until some extra condition which may be longer than a first crossover, as compared with the double auction with trade as soon as there is a willing pair [35]. The full import of this difference is not fully understood.

The influence of heterogeneous agents following different policies in not well understood. The work of Arthur et al. [1] points to the possibility that locally successful market strategies of any level of simplicity or complexity carry within them the seeds of their own destruction as they are destroyed and replaced by other temporarily successful strategies which in turn are imitated.

2.3 The Finance Literature on Price Formation

Almost disconnected from both the gaming and the game theory literature is the finance literature on the movement of market prices in the New York Stock Exchange and other exchanges as well as the growing work that can be described as eco-physics. This work has its origins in the seminal work of Bachelier [2]. Recent modern interest started possibly with the work of Mendelson [21] and has picked up considerably in the last twenty years. The models tend to be variations of the stock market computerized sequential double auction market or closely related continuous trading physics analogies such as Bak et al. [3]. The program of Farmer and colleagues has been devoted to both the empirical data and to micromodeling process with an attempt to formalize "market pressure" which from an economist's point of view must be related to the surplus perceived by the individuals. The recent work of E. Smith et al. [26] on the statistical theory of the double auction and of Farmer et al. [13] on an empirical test of Zero-Intelligence Agents operating on the data of the London Stock Exchange indicate that this work is beginning to connect with the economics literature.

2.4 Some Game Theory Observations

The game theory literature directly relevant to the double auction includes Wilson [37], and Satterthwaite and Williams [24]connection between the simultaneous move game and the various sequential games is made.

The single simultaneous move double auction game is the third and most complex of the three simplest one-market-price formation mechanisms which can be constructed. Details and modeling considerations as to how to consider "simplicity" are discussed elsewhere ([33], Chapters 6–9).

From the viewpoint of theory, practise and experimentation the distinction between the single simultaneous move double auction and multistage versions, especially the continuous sequential move market is considerable in information and memory aspects, but when individual agents are deemed to have little memory this may be used to indicate why sequential auctions with apparently near-zero intelligent agents may be reasonably efficient.

2.5 The Single Simultaneous Move Double Auction

Much of the experimentation in auctions has been performed with considerable attention paid to the formation of the experimental market price which is then contrasted with the competitive market price derived from the standard economic theory of the competitive market. When dealing with the double auction mechanism it is instructive to contrast the solution set of noncooperative equilibrium points with the competitive equilibrium. A simple example will serve to illustrate both the delicacy of the error and the information aspects of the auction mechanism.

We consider an auction with traders each with three valuations on either side of the market, as is shown in Figure 1. For simplicity we assume on the supply side that there are n individuals each of whom has a single unit for sale with a reservation price of 1. There are r individuals with a reservation price of p where $1 \leq p \leq 5$ and there are n individuals with a reservation price of 5. On the demand side we assume that there are n individuals each of whom wishes to buy a single unit that he values at 5 . There are r buyers with a valuation price of p where $1 \leq p \leq 5$ and there are n buyers with a valuation price of 1

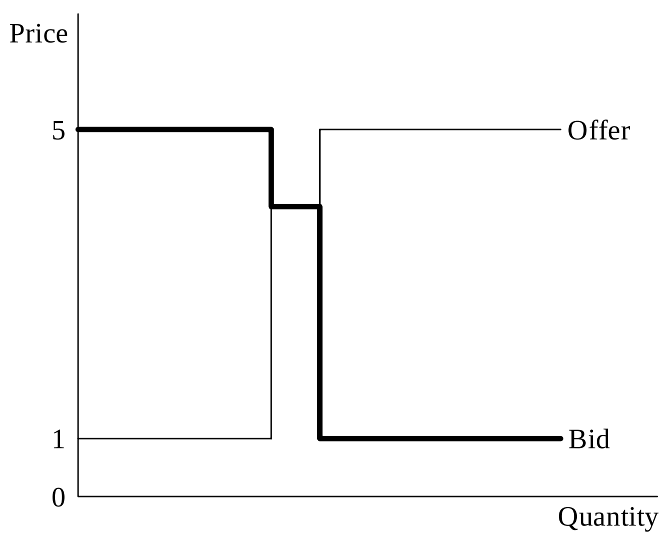

Figure 1

It is easy to observe that the market competitive equilibrium price is p. The volume of trade will be $n+r$ units and there will be n extra-marginal traders on each side of the market. Consumer and seller surplus will be $4n$ in total which will be split with each of the extra-marginal buyers obtaining $(5-p)$ and each seller obtaining obtaining $(p-1)$. The marginal traders obtain zero profit.

Except for some possible indeterminacy in price or quantity depending on how the marginal pair of traders determine price and the size of the marginal sale, the market price will essentially be unique. This is not so for the Noncooperative equilibria (NEs). Any combination of strategies in which an individual seller i offers to sell for an amount $\hat{p} \leq p^i \leq R^i$ where R^i is $i's$ reserve price, $\hat{p}$ is expected market price and p^i is the price named by i. The rational expectations hypothesis is equivalent to the noncooperative equilibrium condition. This implies that the actual market price $p = \hat{p}$. Similarly any buyer j may bid a price p^jsuch that $\hat{p} \leq p^j \leq V^j$.

NEs and truth revelation Taking any of these combinations we observe that each one satisfies the weak-no-improvement property of an NE. Furthermore the strategy under which each reveals her true preferences by bidding to buy at $p^j = V^j$ or offering to sell at $p^i = R^i$ is an NE where the histogram of strategies and preferences coincide.[1]

Common knowledge In a fully defined game the players are assumed to know the number of competitors and the strategy sets and payoffs of their competitors. In this market mechanism this game is playable without this information. In the first four games reported below the players were not even given information on the possibility that a price had been formed yesterday.

One way in which the arbitrariness of playing the game once without common knowledge is avoided is to instruct the students that in a previous play of the game with conditions similar to theirs, a price was formed and they are informed of this price. This might be a specific number or a range. If a range is given, then does the referee specify the probabilities over the range or is this left to the players?

In the other games reported either a specific number was given or a range without a distribution was provided. A reasonable approximation of market reality is that if the market has existed before, yesterday's prices are probably known.

A sensitivity analysis Suppose we have either a completely well defined game or game where the individuals are told that the previous market price was p. A reasonable question to consider is how does the game change with changes in the parameters n and r? We observe that the individual strategy sets do not change and that qualitatively even if there are many marginal pairs and extra marginal pairs neither of these have any influence on the game. This changes radically when a multistage or sequential game is considered.

A multistage simultaneous move double auction market The games reported here are single simultaneous move games where minimal information conditions are encountered. Each individual has only one information set. The bulk of the literature is devoted to sequential bidding where the information conditions are of considerable importance and can have many institutional variations. A half way house between these two conditions is to consider the simultaneous move game

[1] If there are two or more marginal pairs of agents trading in more than one unit each a rationing scheme may be required.

played T times with the price and volume of trade reported on each occasion. The constraint requiring all to bid simultaneously has considerable strategic influence.

Buying or selling pressure How important are the levels of the gains from trade in promoting the size and timing of bidding? Although there has been some discussion and experimentation, as yet there appears to be no clear answers, however a natural candidate for quantification is the concept of "buying or selling pressure." The recent work of Farmer appears to be concerned with this.

Price and quantity adjustment When individuals are constrained to buying or selling one unit (as in the original Chamberlin market) this is, from the viewpoint of game theory, restriction to a one dimensional strategy set, i.e. to price alone. When they have more than one unit to buy or sell a new host of difficulties appear together with the two dimensional strategy set. Not only must quantities as well as prices be selected, but constraints on cash flows must be calculated and observed if short selling is not permitted.

Although in the games run and reported here only one period was investigated, it was found that price converged essentially immediately, but the quantity traded was lower than predicted by general equilibrium. I conjecture that the convergence to close to efficient trade takes place sequentially, first to price and then with a follow through to quantity of trade.

The sequential move double auction The sequential move double auction market opens up a vast set of different information conditions and trading arrangements. Even if we restrict ourselves to some of the simpler formulations strikingly different information conditions are encountered in contrast with the simultaneous move games. Suppose that in the game described with $4n + 2r$ traders the activity were sequential with a pairwise trade following every cross, then it is straightforward to observe that traders who would be strictly excluded from trade in the simultaneous bid-offer game might trade and the volatility of price would depend on who traded with a few marginal players.

The role of middlemen Empirically brokers, dealers and other forms of middlemen have played an important role in many markets [4], [33]. They have served as aggregation, disaggregation and information processing devices. Are they a necessity or is it this layer in the trading process that can be replaced by computerization?

2.6 Random Search for Trading Pairs

Random search for a trading partner can be regarded as a premarket precursor of a more organized and aggregated form of trading. There is a small literature both in the pure combinatorics of trade and in random matching of pairs as is evinced by Starr [34], Rubinstein [22] and Gale [15]. The type of behavior consider is where a seller and a buyer meet and decide to either trade (for example at a point of split the difference) or to hunt for someone else. This literature is not far from the queuing rules used in the double auction.

3 Results from a Single Play Double Auction

In the games considered here each student was given either a seller or buyer card as are shown in Figures 2a and b.

SELLER #1	*Round off your offer to nearest 10¢ **Whole units

Reserve Price = $____________

Unit for Sale = ____________

OFFER: p = price* = $___________
q = quantity offered** = ___________

No "FUTURES" SELLING
Offer not valid unless q ________ ≤ ________
YOUR PROFIT
= (Market price – Reserve Price) x (quantity sold)

(a)

BUYER #1	*Round off your offer to nearest 10¢ **Whole units

Unit Maximum Worth = $____________

Money Available = ____________

OFFER: p = price* = $___________
q = quantity offered** = ___________

No "FUTURES" SELLING
Bid not valid unless q x p ________ ≤ ________
YOUR PROFIT
= (Unit max worth – Market Price) x (quantity sold)

(b)

Figure 2

An important part of the value of the exercise comes in the discussion of the details required to fully define the market price formation mechanism. Even for the single simultaneous move double auction these items include:

- Do you know the number of players?
- Do you share common knowledge?
- What history do you have of the market?
- How does it matter?
- Does the name of the good you are trading in matter?
- What is a reserve price?
- Does the unit size of the bid matter?
- Can one sell fractions of a unit?
- Are short sales permitted
- What are the credit restrictions?
- How are ties resolved?

- Can there be a range of outcomes?
- If so how is one selected?
- How is profit or gain measured?

  If we play the game more than once new problems concerning information conditions and many other time dependent features appear.

  After the game has been played, in the class discussion the students are asked to explain their behavior and the discussion stresses the relationship between the market mechanism structure and individual behavior. This leads into the consideration of what constitutes a solution and to the difference between "objective supply and demand curves" given by the reservation and valuation prices and the actual bids and offers made.

- A feature of "unreality" concerning the game noted by several students over the years is the motivation for the individual to bid or offer anything if expected profits are zero. The argument given is invariably a positive transactions cost argument. If an individual is a member of a marginal pair where exchange yields zero profit to each why should they incur the transactions costs of bothering to fill in numbers and turn in the bidding costs? In the experiments as class rules required the turning in of the cards the quantity offered or bid by some of the marginal traders was 0.

3.1 Procedure

The reserve price and the amount for sale by the sellers and the worth of the commodity and money available to the seller were specified and entered on the cards by the instructor. The way price is formed by the intersection of the bid and offer curves was explained.

After the game was played, but before it had been analyzed the distinction between the objective supply and demand curves and the strategically determined bid and offer curves was discussed.

As the games were played in class primarily for their expository value there was no monetary reward to the individuals. In some of the post game discussions, some of the students noted that they might have behaved differently if "real money"been riding on the outcome.

The Appendix shows a briefing and the results of a play.

3.2 The Games and Results

The first four games were played without giving the students information about any history. The remaining games were played with the students being informed that the game had been played previously under "more or less the same conditions" and that the price formed had been "fairly close to p" where p was the actual competitive price.

In the first two games as there were very few players they acted only as buyers with the referee supplying the sell side strategies. The last game was played twice by the same students a week apart.

Expt	n	Sell	Buy	p(expt)	p(CE)	q(expt)	q(CE)	$p/p(C)$	$q/q(C)$
1	5	C	5	48	48	100	150	1	.667
2	7	C	7	48	48	175	210	1	.833
3	12	6	6	50	48	140	180	1.04	.778
4	23	13	10	50	48	205	300	1.04	.683
5	22	122	11	6.15	6	25	45	1.025	.556
6	12	6	6	4	4	28	30	1	.933
7	23	12	11	2.5	2.5	28	40	1	.700
8	15	7	8	13	13	14	30	1	.467
9	35	20	15	10	10	97	140	1	.693
10	19	9	10	12	12	20	39	1	.513
11	14	7	7	12	12	13	30	1	.433
12	19	8	11	6	6-7	4	32	1	.125
13	19	10	9	6	6-7	20	36	1	.556
14	18	10	8	11.8	12	10	40	.983	.25
15a	23	9	14	15	15	113	210	1	.538
15b	22	10	12	15.3	15	150	180	1.02	.833

Table 1

We observe from Table 1 that even in a one play game the power of the presence of one or more marginal pairs appears to be sufficient to obtain immediate convergence to the competitive price. From the viewpoint of the game theory two traders of each type trading in one commodity are required in the market for competition in both supply and demand.

Although a price close to the competitive price was obtained in one play, there was considerable undertrading in comparison with the competitive equilibrium volume. Game #15 was run a second time (with two fewer bidders) to see if the volume of trade would increase. In the first run trade was at the level of 54% of the CE; in the second trade was at 83% of the CE. Considerable replication is required to attach much statistical significance to this single observation, but at least it is consistent with the conjecture that trading will increase with repetition and with the results of Vernon Smith both concerning slight variations in the number of market participants and the lack of monetary payments.

4 Simultaneous or Sequential Markets?

The games considered here, except for one, have been the single simultaneous move game which is the simplest possible, involving essentially no flow of information to the players beyond the initial conditions. Once all moves have been selected, the game is over. In contrast the continuous time auction with bids and offers flowing in sequentially and price moving from moment to moment stresses information flow and is a far closer approximation to actual trading on an exchange.

4.1 Auctions or Markets?

The term auction appears to be derived from the Latin *auctio* (increase); which seems to imply a dynamic process with individuals with the ability to change their bids. More properly the simultaneous or one shot bid-offer strategic market game which differs only from the sequential double-auction mechanism in information conditions can best be described as the mathematical equivalent of the sealed bid rather than as an open auction where individuals can change their action flexibly.

4.2 Zero-intelligence Models: A game Theoretic Interpretation

The stimulating set of papers by Sunder and associates [16], [17], [4] over the last few years has been based on simulations and games with live players. The strength of the convergence properties of simple models of behavior where the individuals either bid randomly or bid with weak constraints obtained from their valuations and budget constraints has been of note.

4.2.1 The NEs of a simultaneous move bid offer market model

A simple simultaneous move bid offer market model where each individual seller has one unit for sale has essentially[2] a unique competitive equilibrium; but it has a set of noncooperative equilibria all giving the same joint payoff. The strategy set is precisely that of the Sunder Zero-Intelligence agents constrained to bid in a range that prevents losses. The bidding constraints guarantee that price will be determined by the marginal pairs and the bid offer mechanism is completely insensitive to the cardinal properties of the other bids.

The Truth Revelation Noncooperative Equilibrium Suppose that all individuals bid their true reserve and valuation prices. This means that the histograms drawn are precisely the conventional supply and demand curves of standard microeconomic theory. It could be, as is shown in Figure 3 that there is some small indeterminacy in price, an *ad hoc* rule must be supplied to resolve this.

[2] A way must be specified for resolving some indeterminacy in price caused by the integer property of the goods. In actual markets rules such as "split the difference" or satisfy the offeror or bidder first or toss a fair coin may be used. A tie breaking rule must also be specified if there is a some excess supply or demand at the market price.

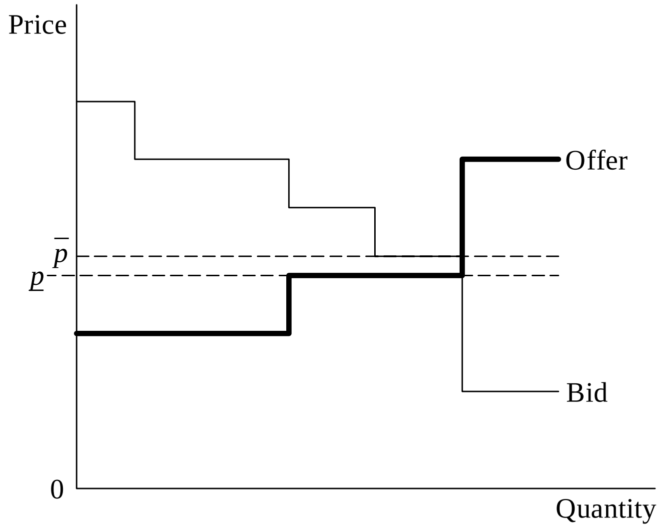

Figure 3. The other Noncooperative Equilibria

Select any set of strategies generated by the Sunder Zero-Intelligence constrained traders.[3] Replace any one of the traders by a standard rational economic agent. His payoff remains the same regardless of his selection of strategies within his no-loss range.

We now observe that the randomization without memory in effect obliterates the major difference between the simultaneous bid offer model and the sequential double auction as in the formal sequential game the size of an individual strategy set depends on the number of information sets of the individual. But with these agents lack of memory prevents them from using information different from that used in the bid offer model.[4] There still will be a difference in payoffs and prices as there is only one price in the simultaneous bid offer model but price is continuously being formed by pairs in the Zero-Intelligence model.

5 Local and Global Structure and Behavior

The investigations of game theory contain considerably more than the structure of a game in normal or strategic form and the solution concept of a Nash equilibrium point. Von Neumann and Morgenstern presented three different primitive structures for the consideration of a game. They were the extensive form, the strategic form and the coalitional form. There are more. The games originally presented had a fixed finite number of players, yet many models, including those of markets are better represented by structures which have a continuous flow of individuals entering and exiting with no specific start or end to the process. These require yet another

[3] The constraint prevents extra-marginal traders from trading. If the traders are merely fully random within the same range, as noted by Sunder extra marginal trade may occur.

[4] For a discussion of the influence of varying information sets on the same economic market structure see Dubey and Shubik [12].

formulation. Consideration of subjective probability as suggested by Harsanyi and lack of common knowledge as investigated by Aumann and others call for even more basic models.

Even if we limit ourselves to fully defined games with common knowledge and no subjective probability the links among the different primitive forms of a game of strategy have been scarcely studied. It is well known that there is a many to one mapping from the extensive form to the strategic form. Furthermore there usually is some extra modeling required and there may be a many to one mapping from the strategic form to the coalitional form.

The general equilibrium analysis, by throwing away process, managed to rid itself of the institutions which carry process. To a certain extent the coalition form of a game does the same. But any attempt to link the coalitional form with strategic or extensive form requires a linking of the specific rules of the game in such a manner that the coalitional form is no longer taken as a primitive concept but is derived from a far more detailed (and essentially institutional) form.

The grand theorists, defending both general equilibrium theory and cooperative game theory might argue that they are after the great invariants of the economic system and do not wish to have their thinking beclouded by institutional detail. To some extent there is merit in trying to avoid detail provided that the loss of detail is not fatal to understanding the questions at hand. But the price paid by both general equilibrium theory and cooperative game theory was to totally wipe out process in the analysis.

The reason why I switched from the study of market games utilizing the coalitional form, to strategic market games utilizing the strategic or extensive forms of the game was that the study of economies using money has to involve process. Markets and price formation form a critical part of the model. Disequilibrium positions as well as equilibrium positions must be considered. That is what money, cash flow positions, credit and bankruptcy and reorganization are all about.

Initially I was concerned that there was an insurmountable gap between the non-institutional models of general equilibrium and cooperative games and the intrinsically institutional models of games in coalitional form. However there is at least one approach which indicates that the choice is not one or the other. It may be that although associated with each game in strategic form there is a different game in cooperative form, there is some form of measure that indicates that the different games are "close enough" that for the purposes at hand they can be regarded as more or less equivalent. One way of studying this possibility is to consider the behavior of various game theoretic solutions to games with large numbers of players. Dubey, Mas-Colell and Shubik [10] show that there are large classes of mechanisms whose equilibrium positions are the same when the number of agents is large. Although this result was established and helps to answer many questions concerning the existence of equilibrium prices it gives us no insight whatsoever into the dynamics of price formation.

The understanding of the links between specific games in extensive form and related games in coalitional form has hardly begun. The natural and possibly easiest candidate is the sequential double auction model in its many variations and the simultaneous bid-offer game. As is indicated above in Section 4.2.1. the special structure of the mechanism makes it extremely robust to strategic variations in all but the strategies of the marginal pairs. Several variations of the sequential continuous time double auction with “open book” i.e., complete information concerning bids and offers appear to be amenable and from the point of view of games in coalition form may be utilized together with the simultaneous bid-offer game to construct upper and lower bounding characteristic functions which reflect the differences in the information conditions between the games with minimal and maximal information. If the memories of the traders are limited the distinction between games with many information sets and few information sets is limited.

5.1 Expertise and Due Diligence and/or Market Psychology

There is often a great temptation to try to explain some phenomenon on an either/or basis, when often the explanation may be more consistent with a mixture of both. In particular along with the growth of behavioral finance there has been a tendency to stress the importance of herd and irrational behavior in the markets. Experiments have been run using both businessmen [31] and students [29] which display bubbles being formed. If this is so, then where does *homo oeconomicus* or the rational optimizing economic agent fit into the picture? A possible explanation is that both behaviors are consistent with the functioning of the market system. The basic force in the achieving of allocational efficiency given valuations and reserve prices is provided by the structure of the market mechanism. Thus the stress is intelligent design of the mechanism so that societal goals can be achieved by simple local optimizers; but the analytical depth of conscious economic behavior is primarily directed towards valuation which in turn serves as a bound on simple market behavior.

It is possible that there exists a small class of experts in social psychology who profit over the long term while basing their actions on phenomena such as the Elliot wave studies or other forms of market charting analysis. The simulations of Arthur et al. [1] and elementary game theory considerations indicate that no behavioral strategy is going to work consistently in the short run market once a sufficiently large number of entrants have adopted it. If there is an artificial or a natural barrier to entry, such as a short supply of ingenious social-psychologists or stochastic process modelers a small set of behavioral experts might earn their long term rents.

Even more likely than the presence of the behavioral experts, and consistent with both the experimental evidence and basic finance, due diligence and securities analysis [36] is that there exists a set of fundamental value analysts whose time is devoted to deeper and more complex valuation of the assets than most of the other individuals in the markets. Good due diligence calls for both high economic rationality and both macro and micro-economic forecasting, but it is essentially an individual economic valuation and not a “beauty contest” as Keynes described the short term stock market. There is no reason to expect that the “market timing” or ability of the economic valuation experts is much better than a random agent in ”beating the market” in the short term.

Both the optimization of evaluation and allocation play a basic role in the functioning of a market economy. They are interlinked but different activities. Both from the individual and societal point of view the productivity of deep analysis is most likely to be in a one-person evaluation of real assets, even with stochastic income forecasts than in an attempt to estimate strategies and valuations of competitive agents in a stock market.

The financial system and price formation mechanisms of markets attempt to provide an efficient allocation mechanism for an economy in constant motion. A basic consideration of the sequencing of moves is sufficient to show that individuals with superior information or valuation can benefit from this advantage [9]. The choice is not local or global behavior. Nor is it simple algorithms for hill-climbing or for complex optimization. The limited capacity agent has to choose where to utilize complexity if she is capable and motivated and where to stay with simplicity. For most, but not all individuals with time and intelligence to invest actively[5] the better bet is probably to aim for simplicity in the market and economic sophistication in evaluation.

5.2 An Example with the Cost of Due Diligence in a Bayesian Game

A sketch of a simple example illustrates the remarks in Section 5.1. Suppose that there is a double market with many buyers and sellers. The rules are such that the book is built up until there is a cross, at which point a sale is made and the information is announced to all. If the bid is over the offer, price is determined by split the difference. Ties are resolved by randomization.

Suppose that the reserve price for each seller is i.i.d. and is drawn from a distribution which attaches a valuation of 6, 4 or 2 with probabilities of 1/3. The valuations of the buyers are 4, 2 or 0 with probabilities of 1/3.

We consider that each agent is willing to bid randomly any price (between a lower bound of 0 and some large upper bound which avoids an expected loss. The expected average price for trading will be $p = 3$ as can be seen from the 3×3 trading table for price formation.

Sell\Buy	6	4	2
4	5	4	–
2	4	3	2
0	3	2	1

All the traders are ill-informed about the actual valuation of the asset. In this market the Zero-Intelligence results of Sunder et al. should hold.

Suppose that we consider the presence of a small set of sellers who follow the Zero-Intelligence policy in the market, but are able, by means of due diligence or

[5] In the stock market there may be some highly intelligent niche players operating on the details of special instruments and arbitrages. There are also, brokers and specialists earning commissions and something off the top who do not care which way the market goes as long as volume remains high.

basic securities analysis to reduce their uncertainty in valuation. Let the cost of this work be c. The payoff to an individual without doing due diligence is

$$\text{Payoff} = \frac{1}{3}(6-3) + \frac{1}{3}(4-3) + \frac{1}{3}(2-3) = 1. \tag{1}$$

If ,she had an accurate valuation, she would adjust the range of her bid accordingly and her payoff would be

$$\text{Payoff} = \frac{1}{3}(6-3) + \frac{1}{3}(4-3) - c. \tag{2}$$

If $c < 1/3$ it pays the individual to do the basic work. If the population of experts is not large the modification of their bidding range will not have a significant influence on market price

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6 Appendix

6.1 Game 7

Your are either a buyer or seller in a market for a single good. This is the second time that the market has met and the only information you have (beyond your knowledge of your own resources and their value to you) is the price in the last market and the volume of trade

Last period the price was: $2.50

The volueme of trade was: 60

and that there may be a few more or a few less participants in the market this time.

If you are a seller you must select a price p at, or above which you are willing to sell up to an amount q which you must also select. The price can be any nonnegative number. The quantity must not be greater than the amount of the good that you have.

If you are a buyer, you must select a price p^* at, or below which you are willing to buy up to an amount q^* which you must also select. p^*q^* must not be greater than the amount of cash on hand that you have.

The bids will all be lined up in descending order of price bid and the offers will be lined up in ascending order of price offered. Where these two histograms cross will determine the ruling market price and the volume of trade.

If there are any ties or indeterminacies, the referee will prorate or take a mean.

6.2 Analysis of Game 7

You played in a simple game representing a simultaneous bid low information double auction market. The theoretical equilibrium solution is given on the graph. The information you were given was based on there being 15 traders on each side of the market. You actually played with 12 on one side and 11 on the other.

Your were given the previous price and volume as context for your decision. It appears that the price information was used as an important cue.

The volume transacted was considerably less than the equilibrium theory would predict.

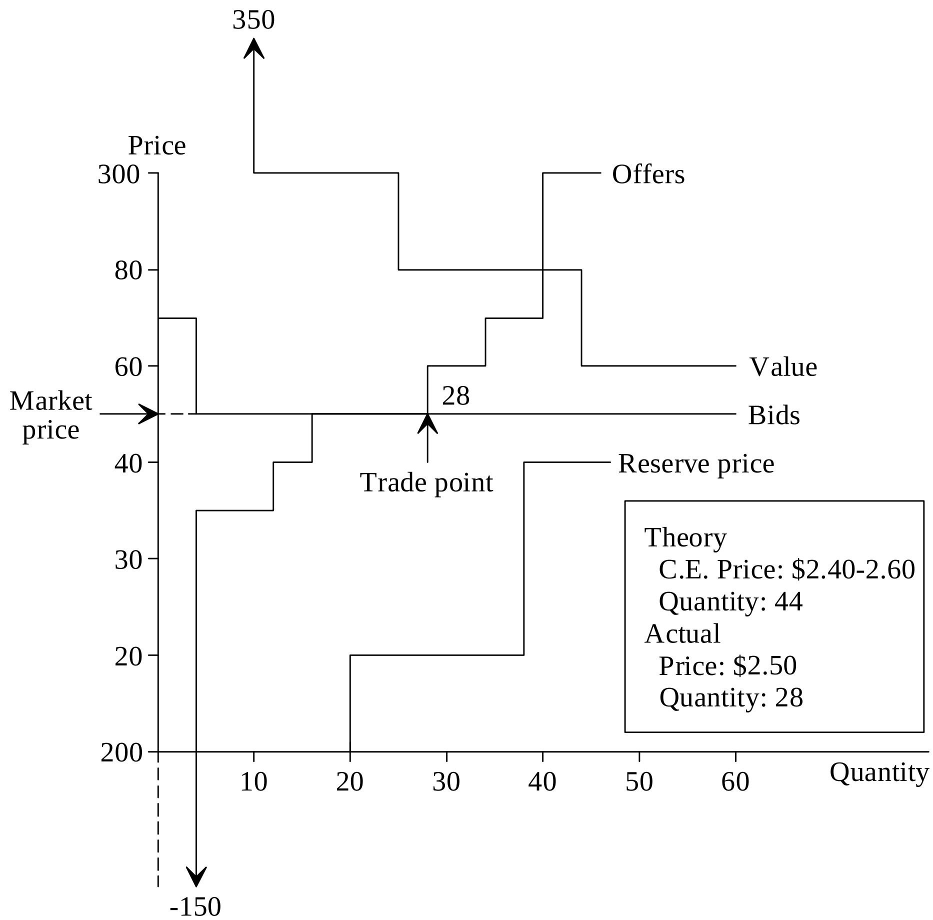

Figure 4. Bid–offer Histograms