COURAGE TO CAPITAL?
A MODEL OF THE EFFECTS OF RATING AGENCIES
ON SOVEREIGN DEBT ROLL-OVER

By

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Courage to Capital?

A Model of the Effects of Rating Agencies on Sovereign Debt Roll–over

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Abstract

We propose a model of rating agencies that is an application of global game theory in which heterogeneous investors act strategically. The model allows us to explore the impact of the introduction of a rating agency on financial markets. Our model suggests that the addition of the rating agency affects the probability of default and the magnitude of the response of capital flows to changes in fundamentals in a non–trivial way, and that introducing a rating agency can bring multiple equilibria to a market that otherwise would have the unique equilibrium.

JEL classification: F34, G14, G15

Key words: credit rating, rating agency, sovereign debt, global game

1 Introduction

Credit ratings are an important component of both international and domestic financial markets and their role has been steadily growing over time. The number of countries that have received a credit rating increased from about a dozen in 1980 to about a hundred in 2002.[1] Similar growth has occurred in U.S. markets: In 2002, Moody's rated about 20,000 public and private bond issuers, up from 5,500 in 1975 (Partnoy 2002). In this paper, we take a step towards understanding the effect of the increasing importance of rating agencies on financial markets by applying a theory of coordination amongst investors. In particular, we analyze the effect of the introduction of a rating agency on the probability of default by the borrower, the reaction of financial markets to changing fundamentals in short and long run, and the set of parameters supporting a unique equilibrium in financial markets.

Empirical studies of rating agencies have found that one way that rating agencies affect financial markets is by affecting borrowers' costs. Several studies, such as Ammer (1998) and Cantor and Packer (1996) who analyze yields of a cross–section of bonds, and Clark and Lakshmi (2003) who focus on a particular episode in detail, have found a significant impact of credit rating announcements on sovereign bond spreads. Additionally, movements in sovereign credit ratings have been found to affect the prices of other financial instruments, including equity markets (Kaminsky and Schmukler 2002), and the price of sovereign debt of other countries (Gande and Parsley 2002). Studies of corporate debt markets in the United States have also found evidence of the effect of actions by rating agencies. One example is Kliger and Sarig (2000), who find that the Moody's announcement of refinements of its ratings system in 1982 raised or lowered the price of debt of companies depending on whether their rating was increased or decreased by the refinement.

Credit ratings can also influence investors' portfolio holdings. Some institutional borrowers, such as mutual funds, are constrained to hold only securities that rating agencies have certified as investment grade (International Monetary Fund 1999). Under the Basel II capital accords, the effect of rating agencies on investor portfolios is likely to grow further as the role credit ratings play

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[1]As of today, Moody's and Standard and Poors provide ratings for over 100 countries each, while twenty years ago only 11 sovereigns were rated by Standard and Poors (Ferri, Liu, and Majnoni 2000).

in risk management practices increases.[2]

Some scholars have discussed whether an increase in the role of credit ratings might make lending more procyclical as downgraded debt would require more capital and a creditor's ability to lend would be curtailed at times when monetary authorities are trying to ease financial conditions (Blum and Hellwig 1995, Carpenter, Whitesell, and Zakrajsek 2001). There has also been a debate in the empirical literature about whether the credit ratings issued by rating agencies themselves move procyclically and might exacerbate a boom or a bust (Ferri, Liu, and Stiglitz 1999, Reisen and Maltzan 1998).

There has been some theoretical work involving rating agencies, but, to the best of our knowledge, most of this literature does not deal with the effects of rating agencies on the equilibria reached by financial markets. The majority of work involving rating agencies has focused on the development of markets for information. Millon and Thakor (1985) develop a model explaining why information gatherers would group together in a firm and why this firm would have a finite size. Allen (1990) presents a model exploring why markets for information might exist. Laster, Bennett, and Geoum (1999) discuss the incentives for analysts to announce their true forecasts. One recent paper that does explore the effect of rating agencies on financial markets is by Boot, Milbourn, and Schmeits (2004). In their paper, the authors argue that, if some investors base their lending decisions largely on the announcements of rating agencies, then rating agencies can act to discipline firms. However, in their paper, it is not always clear that investors are taking full advantage of the information available to them or how economic fundamentals affect rating agencies' announcements and thus the equilibrium.

In this paper, we analyze the effects of rating agencies on financial markets using a global game model.[3] In the model, investors receive imperfect and heterogeneous private information regarding the ability of a borrower to repay its debt and make their decision about whether to extend a new loan based on their information and their expectations about the behavior of other investors. When the rating agency announces its assessment of the borrower's creditworthiness, investors incorporate

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[2]Concerns about using credit rating to assess risks have been raised in the literature, by Danielsson, Embrechts, Goodhart, Keating, Muennich, Renault, and Shin (2001) and Kraussl (2003), among others.

[3]See survey by Morris and Shin (2003) and also Goldstein and Pauzner (2003).

the agency's assessment into their own forecast. Thus, the agency provides a focal point toward which the investors' beliefs gravitate and also causes investors to revise their expectations regarding what other investors will do. As a result, the agency affects the amount of debt the borrower is able to roll-over and the probability of default. This global games setup provides a useful framework because it specifies an explicit role for an economic fundamental, investor's private information about the fundamental and beliefs about other agents, and the public signal provided by the rating agency. Thus it is relatively easy to observe the interactions of these elements.

This paper is primarily an application of the global games theory to a particular situation. We analyze the implications of this model in ways not fully explored in the literature and show how it can be useful in understanding the role rating agencies play in financial markets. We refer to the borrower in the model as a sovereign, and most of the empirical work cited in the paper relates to international financial markets and sovereign debt ratings; however, the general structure and implications of the model are applicable to corporate markets as well.

The model indicates that the effect the rating agency on financial markets depends, among other parameters, on the characteristics of the borrower. Specifically, for high quality borrowers, the introduction of a rating agency tends to increase the average probability of default. This is because a bad rating announcement worsens investors' beliefs on average by more than a good rating announcement improves them. The opposite is true for lower quality borrowers. Our model also predicts that the addition of the rating agency is stabilizing in a sense that it decreases swings in capital flows in response to long–run changes in fundamentals. This is due to the fact that, without the rating agency, posterior beliefs about fundamentals are more disperse and, for the relevant range of parameters, this implies that a larger share of investors will change their behavior in response to changes in fundamentals. The effects of short–run changes in fundamentals are more complex.

Finally, the model suggests that a credit rating is likely to increase the likelihood of multiple equilibiria, i.e. the constraint that ensures the uniqueness of equilibrium is tighter if the rating agency is introduced. In the case of multiple equilibria, if the rating agency announces some measure of default probability, which is not unique, this signal can be used by investors to coordinate on

one equilibrium or the other. However, if the rating agency is announcing its belief about the fundamentals, as in the main setup of the model, the equilibrium selection is still undetermined.

It should be recognized that our model does not take into account some important attributes of rating agencies that also likely affect the impact of rating agencies on market outcomes, such as the fact that they reduce the cost of information acquisition and thus increase the pool of potential borrowers. Nevertheless, we believe that our model provides a useful framework for examining the effect that rating agencies have on financial markets and does point to some potential trade-offs connected with the use of credit ratings.

The paper is organized as follows. In part two, we present a model of debt roll–over without a rating agency. In part three, we introduce the rating agency and analyze how its introduction affects the market equilibrium, especially default probability, the response of capital flows to shifts in fundamentals, and the likelihood of multiple equilibria. Part four concludes.

2   Model

We start by presenting a benchmark model without a credit rating agency in which imperfectly informed investors decide whether to roll over the debt of a sovereign government based on their own assessments of the sovereign's ability to repay (which reflects economic fundamentals) and on their expectations of the actions of the other investors. This model is a basic version of a global game model, as described in Morris and Shin (2004).

We assume that the sovereign government has an outstanding amount of one period debt that it wishes to rollover. We normalize the size of this stock to be equal to 1. The government debt is held by N risk–neutral investors with mass c each, thus $Nc = 1$. We assume here that investors are atomistic. We will later assume that $N \to \infty$ and analyze the version of the model with continuum of investors with mass 1.[4]

The government can and is willing to repay an exogenous share θ of this debt while the remaining amount of debt $(1 - \theta)$ needs to be rolled over. One could think of θ as a measure of the economic

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[4]For a model with big players, see Corsetti, Dasgupta, Morris, and Shin (2004).

fundamentals of the sovereign country's economy. Stronger fundamentals lead to more economic growth providing the government with a larger revenue base and bolstering its ability to repay its debts.

Each investor decides individually whether or not to roll over her unit of debt and there is no cooperation among investors.[5] Denote investor i's decision d_i: $d_i = 1$ if investor i decides to roll over her unit of debt and $d_i = 0$ otherwise. The total amount of debt that will be rolled over is then

$$D = c \sum_{i=1}^{N} d_i,$$

which implies that $D \in [0, 1]$.

If the investor decides not to roll over her unit of debt, she can withdraw it without any premium or punishment and invest it in a risk–free asset with gross return normalized to 1.[6] If she decides to roll over her debt, she forgoes the opportunity of withdrawing money and investing in a risk–free assets and instead receives a gross return of $R > 1$ if there is no default.[7,8]

If an insufficient number of investors roll over their debt holdings, i.e. $D < 1 - \theta$, the country is forced to default on its debt and none of the investors that rolled over their holdings of debt are paid. This implicitly assumes that a country bears costs of default that are fixed and independent of the amount of debt defaulted, and that the country does not build long–run relationships with investors. Under these conditions it will be optimal for the country to always default on the entire

[5]To keep the emphasis of the paper on the rating agency, we do not discuss bargaining or cooperation between the government and investors, or among the investors. See Bulow and Rogoff (1989) for a model with bargaining. The assumption that debt roll–over is financed by investors that currently hold the stock of debt is not essential. The model can be easily reinterpreted for the case when new investors choose between sovereign debt and risk–free assets.

[6]In this model we assume that those who want to withdraw their money can always do it before the draw of θ occurs. We will not focus on how this is financed.

[7]Again, we do not specify how the premium is financed. Potentially economic growth, as reflected by θ, would allow the country to repay part of the debt while the remainder is financed by future debt rollovers. This is sustainable as long as the growth of revenue is greater than or equal to the growth of interest payments.

[8]In our model it is implicit that default is not economically desirable, which is in contrast with Chang (2002).

debt stock.[9] Thus, the payoff structure for investors is as follows:

$$u_i(d_i, d_{-i}, \theta) = \begin{cases} R & \text{if } d_i = 1 \ \& \ D \geq (1 - \theta), \\ 0 & \text{if } d_i = 1 \ \& \ D < (1 - \theta), \\ 1 & \text{if } d_i = 0, \end{cases} \qquad (1)$$

where d_{-i} are decisions made by other investors.

This payoff structure implies that investors will choose to roll over the debt if and only if they believe that

$$\text{Prob}(D \geq 1 - \theta)R \geq 1. \qquad (2)$$

Suppose that at the time of the investors' roll–over decisions θ is not known. Assume however, that it is public knowledge that the realization will be drawn from a normal distribution with mean τ and variance σ. The *apriori* probability $1 - p$ of being repayed is

$$1 - p = \text{Prob}(D \geq 1 - \theta) = \Phi(\frac{1}{\sqrt{\sigma}}(D - 1 + \tau)), \qquad (3)$$

where Φ is standard normal CDF, and p is the probability of default.

Our assumptions imply that if $\theta < 0$, there will be a default unless "new money" is injected. In our model, we do not allow for this possibility, therefore there will always be default if $\theta < 0$, and therefore not rolling over the debt ($d = 0$) is a dominant strategy for every investor in this case.[10] Likewise, if $\theta > 1$, the default will not occur even if $D = 0$, because the government can repay all the debt from its own resources, thus rolling over the debt ($d = 1$) is a dominant strategy in this case. We focus our attention on the equilibria with $\theta \in (0; 1)$, which will depend on the information structure regarding θ; however, as shown in Morris and Shin (2003), the dominance regions described are necessary for the existence of the unique equilibrium.

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[9]These assumptions, while unrealistic, are not driving our results but simplify the model presentation.

[10]To make the model work, all we really need is that there is some amount of borrowing beyond which the government is unable to repay the debt and will default. This notion seems quite realistic. Specifying that there is no new money simplifies notation.

2.1 Full information

If the share of debt that the government can repay, θ, is common knowledge to all investors, then there are two symmetric equilibria in which either $D = 0$ or $D = 1$. Both are self–fulfilling and can occur at any level of $\theta \in (0; 1)$. There are also infinitely many non–symmetric equilibria (see Morris and Shin (2004)). This multiplicity will disappear as we introduce imperfect heterogeneous signals and strategic behavior.[11]

This perfect information set–up is frequently a benchmark model for thinking about the investor coordination. With perfect information on can think of the role of credit rating agencies being to help investors coordinate on one of the two equilibria, which then implies that credit rating agencies could be responsible for swings in international capital flows. We will see from the model below that some of this coordination effect is present when we allow for imperfect information and heterogeneous beliefs. We now turn to the model with a unique equilibrium.

2.2 Private signals

Suppose now that in addition to their proper common prior about $\theta \sim N(\tau, \sigma)$, investors get private noisy signals about θ. Denote investors i's signal $\tilde{\theta}_i$:

$$\tilde{\theta}_i = \theta + \varepsilon_i, \quad \varepsilon_i \ \sim \ N\left(0; \frac{1}{\beta}\right),$$

where β is the precision of private signal. This gives us investor i's posterior belief about θ, θ_i:

$$\theta_i \ \sim \ N\left(\frac{\gamma\tau + \beta\tilde{\theta}_i}{\gamma + \beta}, \frac{1}{\gamma + \beta}\right),$$

where we denoted the precision of the prior $1/\sigma$ as γ. We will assume that only investor i observes her signal $\tilde{\theta}_i$, while β is common knowledge.

This posterior distribution of beliefs about θ gives us a unique equilibrium if private signals are sufficiently informative as compared to the prior, $\beta \geq \frac{\gamma^2}{2\pi}$.[12] In what follows, we assume that this

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[11]The same logic applies if all investors get the same noisy signal about θ and this signal is common knowledge.
[12]See Morris and Shin (2004) for the proof in a similar setting.

condition holds. We return to the question of uniqueness later in the paper.

The unique equilibrium can then be described by the threshold level of θ below which there will be a default in equilibrium, because fewer than $1 - \theta$ investors will choose to roll over their debt. Above that level, a sufficient number of investors will choose to roll over the debt and there will be no default. This unique level of θ should be consistent with the belief of a pivotal investor who is indifferent between rolling over debt and not doing so. We denote this equilibrium threshold of θ as θ^*. The equilibrium in this model, θ^*, is a switching point such that the fundamental $\theta \geq \theta^*$ results in a successful roll–over, while $\theta < \theta^*$ results in default.[13]

Proposition 1 *(Morris and Shin 2004) Given the information structure above, and if $\beta \geq \frac{\gamma^2}{2\pi}$, the equilibrium is unique and θ^* is implicitly determined by*

$$\theta^* = \Phi \left(\frac{1}{\sqrt{\beta}} \left[\gamma(\theta^* - \tau) + \sqrt{\gamma + \beta} \ \Phi^{-1} \left(\frac{1}{R} \right) \right] \right) \tag{4}$$

and is $\in (0; 1)$. In addition, $\partial \theta^* / \partial \tau < 0$, $\partial \theta^* / \partial R < 0$.

Proof. Follows from Morris and Shin (2004). Signs of the derivatives are obvious.

For convenience, we will denote $\Phi^{-1} \left(\frac{1}{R} \right)$ as ρ in what follows. It will be useful to keep in mind that $\rho > 0 \ \Leftrightarrow \ R \in (1; 2)$ and $\rho < 0 \ \Leftrightarrow \ R > 2$.[14] For the rest of the paper we typically start with the assumption that $\rho > 0$ $(R < 2)$, but, where results depend on ρ, we discuss the case where $\rho < 0$.

We are most interested in the actual probability of default, p^*. This is equal to the probability that actual θ will be below the threshold θ^*.

$$p^* = \mathrm{Prob} \left(\theta < \theta^* \right) = F_\theta(\theta^*) = \Phi(\gamma(\theta^* - \tau)),$$

which is monotonically increasing in θ^*. Therefore for any variable $\bullet$, sign $(\partial \theta^* / \partial \bullet) = $ sign $(\partial p^* / \partial \bullet)$.

We can see from Proposition 1 that the probability of default is lower if τ is higher and if R is higher. Both better fundamentals and a higher spread $(R - 1)$ on sovereign debt increase the incentives

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[13]Since for any $0 < \theta < \theta^*$ there will be default in equilibrium, and this default would not occur if all investors could coordinate on rolling over the debt, θ^* could be interpreted as a measure of inefficiency due to coordination failure.

[14]Note that $R = 2$ corresponds to 100 percentage points spread over the risk–free rate which we normalized to be equal to 1.

to rolling over the debt and therefore more investors choose to do so, which in turn lowers the probability of default. In addition, better fundamentals lower the probability of default for a given number of investors willing to roll over the debt.

3 The rating agency

In this section we introduce the rating agency and then show how it affects the equilibrium: through its effect on the probability of default, volatility of capital flows, and the uniqueness of the equilibrium.

3.1 Model with a rating agency

Suppose now that there is a rating agency that has the same prior about fundamentals as all the investors and receives a signal $\tilde{\theta}^A$ with noise ν that has mean 0 and precision α.[15]

$$\tilde{\theta}^A = \theta + \nu, \quad \nu \sim N\left(0, \frac{1}{\alpha}\right).$$

We will assume that only the rating agency observes $\tilde{\theta}^A$ but α is common knowledge. Suppose that the agency directly announces its signal $\tilde{\theta}^A$ and investors then update their common prior accordingly. The new prior has mean

$$\theta^A \equiv \frac{\gamma\,\tau + \alpha\,\tilde{\theta}^A}{\gamma + \alpha}$$

and variance $1/(\gamma + \alpha)$. We will assume that investors get the same private signals as before, and therefore the new posterior beliefs θ_i^A are distributed

$$\theta_i^A \sim N\left(\frac{\gamma\,\tau + \alpha\,\tilde{\theta}^A + \beta\,\tilde{\theta}_i}{\gamma + \alpha + \beta}, \frac{1}{\gamma + \alpha + \beta}\right). \tag{5}$$

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[15]The three rating agencies report that they base their sovereign ratings largely on economic fundamentals (Fitch 1998, Moody's 1999, Standard and Poor's 2002).

Now for the equilibrium to be unique, we have to impose a stricter condition on β. While before it was necessary and sufficient that $\beta \geq \frac{\gamma^2}{2\pi}$, now it is necessary that $\beta \geq \frac{(\gamma+\alpha)^2}{2\pi} > \frac{\gamma^2}{2\pi}$ if $\alpha > 0$. This gives us the first effect that introducing rating agency can have:

Proposition 2 *For $\beta \in \left[\frac{\gamma^2}{2\pi}, \frac{(\gamma+\alpha)^2}{2\pi} \right)$, there is a unique equilibrium in the absence of a rating agency and multiple equilibria if a rating agency is introduced.*

Proof. Follows from Morris and Shin (2004) with α replaced by $\gamma + \alpha$.

In other words, if private signals are precise enough to ensure uniqueness of equilibrium in the absence of a rating agency, but not precise enough relative to the precision of the agency's signal, introducing a rating agency will lead to multiplicity of equilibria. We will first focus on the case where a unique equilibrium exists both with and without the rating agency, i.e. $\beta \geq \frac{(\gamma+\alpha)^2}{2\pi}$. We will return to the issue of multiplicity in section 3.4.

Proposition 3 *(Morris and Shin 2004) Given the information structure above and $\beta \geq \frac{(\gamma+\alpha)^2}{2\pi}$, the equilibrium is unique and θ^{A*} is implicitly determined by*

$$\theta^{A*} = \Phi \left(\frac{1}{\sqrt{\beta}} \left((\gamma + \alpha)\theta^{A*} - \gamma\tau - \alpha\tilde{\theta}^A + \sqrt{\gamma + \alpha + \beta}\,\rho \right) \right) \tag{6}$$

and is $\in (0; 1)$. In addition, $\partial\theta^{A*}/\partial\tilde{\theta}^A < 0$, $\partial\theta^{A*}/\partial\tau < 0$, $\partial\theta^{A*}/\partial R < 0$.

Proof. Follows from Morris and Shin (2004). Signs of derivatives are obvious.

The probability of default is now

$$p^* = \mathrm{Prob}\left(\theta < \theta^*\right) = F_\theta(\theta^{A*}) = \Phi(\gamma(\theta^{A*} - \tau)),$$

as before, the probability of default is lower if the prior mean of θ, τ, is higher or the spread $R - 1$ is higher. It is worth noting that θ^{A*}, and thus the probability of default, depend on the rating agency's signal, which depends in turn on ν (the error in the agencies signal). As one might expect, a positive ν (a high assessment by the agency) lowers the probability of default. This additional sensitivity, however, is a notable effect of the introduction of the rating agency and, as will be shown below, has important implications for the equilibrium reached in the model.

We now turn to our analysis of the effect the introduction of the rating agency has on the equilibrium, assuming the equilibrium is unique both with and without the rating agency.

3.2 Effects of rating agency on the probability of default

We first examine how the addition of the rating agency affects the probability of default. Since, as noted above, the probability of default with the rating agency depends in a non–linear way on two random variables (θ and ν), and we only have an implicit solution for θ^*, we have to resort to numeric methods in this analysis.

We can analyze the effects of the rating agency on the *ex-post* probability of default, i.e. for a given τ we analyze the effects for each particular realization of θ while taking the expectation over the distribution of ν in the case with a rating agency. We will refer to this as a short–run effect, interpreting random realizations of θ as short–run changes in fundamentals. We can also analyze the effect of the addition of the rating agency on the *ex–ante* probability of default (i.e the expectation over the distributions of θ and ν), where we interpret changes in τ itself as long–run changes in fundamentals.

The effects of introducing a rating agency on the probability of default *ex–ante* and *ex-post* are illustrated in Figure 1. The left–hand panel represents the effects of the rating agency on the probability of default as a function of long–run fundamentals. We observe that the addition of the rating agency increases the likelihood of default for borrowers with high τ (good long–run fundamentals) and reduces the likelihood of default for borrowers with lower τ. The point at which the switch occurs moves to the left as R increases, as illustrated in the Appendix. The results are otherwise robust to a variety of parameter values, although for some sets of parameters the differences in the probability of default with and without a rating agency are quite small.

The intuition for this result is that for the high τ borrowers, the increase in the probability of default from a low rating is not fully offset by the benefit of getting a high rating. To see this, assume that if the rating agency's announcement is 0, D_0 investors will roll over the debt. With high τ, this $D_0 > 0.5$, and the pivotal investor is located on the upward sloping part of the PDF of the posterior
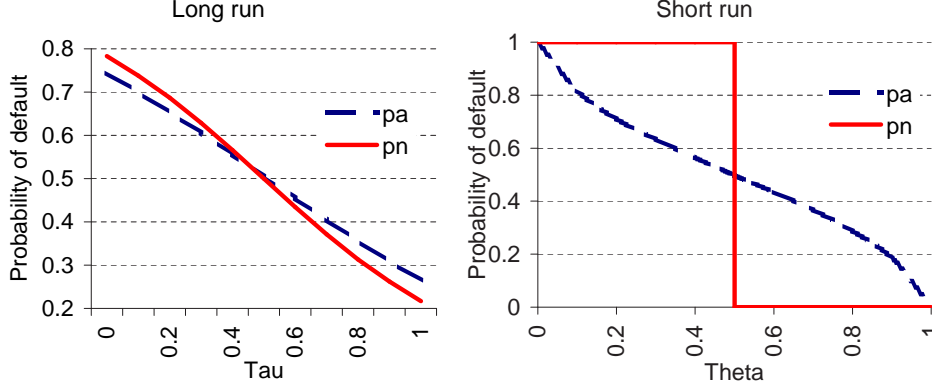
Figure 1: Probability of default with (pa) and without (pn) a rating agency as a function of τ and as a function of θ (with $\tau = 0.5$) for $\alpha = \beta = \gamma = 1$, $R = 2$

beliefs.[16] If the rating announcement is positive, this PDF will shift to the right and additional D_g of investors will now choose to roll over the debt. If the rating announcement is negative (but the same in absolute value), the posterior distribution will shift (by the same amount) to the left. Now, D_b of investors that would roll over the debt with zero rating, will no longer choose to do so. Since D_0 was on the upward part of the posterior distribution PDF, $D_b > D_g$. Thus, the change in investors' posterior beliefs is larger in absolute value when the rating agency's announcement is unfavorable than when it is favorable. Thus, on average, for the high level of τ, a credit rating lowers D and therefore, increases the probability of default.

The reverse intuition holds for borrowers with low τ. For $R = 2$, the switch occurs at $\tau = 0.5$. For very low levels of R the probability of default is always lower with the rating agency than without, while for very high levels of R, the probability of default is always higher with the rating agency than without. The *ex–ante* (long run) effect of a credit rating on the probability of default as a function of R for $\tau = 0.5$ is represented in Figure 2.

Thus, borrowers with poor initial fundamentals are likely to benefit, in terms of default probability, from obtaining a credit rating. However, if the spreads are high, the set of the borrowers that benefit is small. On the other hand, if spreads are low, most borrowers, including those with rather good fundamentals, will benefit from obtaining a credit rating.

In the short run, represented on the right–hand panel of Figure 1, the effect of introducing the

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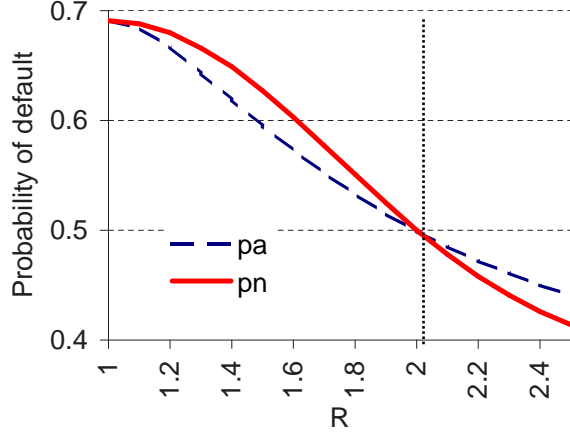[16]All investors with better beliefs choose to roll over their debt.

Figure 2: Probability of default with (pa) and without (pn) a rating agency as a function of R for $\alpha = \beta = \gamma = 1$, $\tau = 0.5$

rating agency is qualitatively the same as in the long run, only with a much larger quantitative effect. Without the rating agency, θ^* is completely determined by the parameters of the model and thus the probability of default is 0 for $\theta > \theta^*$ and is 1 for $\theta < \theta^*$. With the rating agency, the probability of default depends also on the rating announcement, which itself depends on the realizations of θ and ν (over which we take an expectation), thus, the probability of default is a smooth function of θ for a given τ. This effect of the rating agency can be interpreted as stabilizing, since small changes in the realization of fundamentals do not have large impacts on the outcome, as happens in the case without the rating agency. On the other hand, while the probability of default is 0 for some θ without the rating agency, it is always positive in a presence of a credit rating.

3.3 Effects of the rating agency on the response to changes in fundamentals

While our model is not a dynamic one, comparative statics do offer some insight into how the addition of the rating agency affects investors' response to a change in economic fundamentals. It has been argued in the empirical literature that rating agencies affect the volatility of capital flows. One way to examine this in our model is to analyze how introducing a rating agency affects the response of the share of investors who choose to roll over the debt to changes in fundamentals: $\partial D/\partial \tau$ in the long run, or $\partial D/\partial \theta$ for a given τ in the short run.

This share, D, depends on the equilibrium threshold θ^*, on the realization of θ, and, if there is a rating agency, on the realization of rating agency's signal, i.e. realization of ν for each θ. It can

be derived explicitly once we know θ^*.[17] A pivotal investor is indifferent between rolling over the debt and withdrawing his money, i.e. for her

$$1 - \tilde{p} = \frac{1}{R},$$

where $\tilde{p}$ is the estimate of default probability that is based on this investor's posterior belief about the realization of θ.

$$\tilde{p} = \Phi \left(\sqrt{\alpha + \beta + \gamma} \left(\theta^* - \frac{\alpha \tilde{\theta}^A + \beta \tilde{\theta} + \gamma \tau}{\alpha + \beta + \gamma} \right) \right),$$

which we can solve to recover the signal $\tilde{\theta}$ that this pivotal investor received.

$$\frac{\alpha \tilde{\theta}^A + \beta \tilde{\theta} + \gamma \tau}{\alpha + \beta + \gamma} = \theta^* + \frac{\rho}{\sqrt{\alpha + \beta + \gamma}},$$

$$\tilde{\theta} = \frac{\alpha + \beta + \gamma}{\beta} \left(\theta^* + \frac{\rho}{\sqrt{\alpha + \beta + \gamma}} \right) - \frac{\alpha \tilde{\theta}^A + \gamma \tau}{\beta}.$$

Once we know the signal that the pivotal investor received, we know that all the investors with a better signal will roll over the debt, while all the investors with a lower signal will withdraw their money. Thus, D is just the CDF over the distribution of investors' signals, which has mean θ and variance $1/\beta$.

$$D = 1 - \Phi \left(\sqrt{\beta}(\tilde{\theta} - \theta) \right).$$

We will first analyze the ex–ante effect of changes in τ on the capital flows, i.e. we again will be taking the expectation of D over the distribution of θ and ν. The share of investors that roll over their debt in the absence of a rating agency can be obtained by setting $\alpha = 0$ in the derivations above.

The results of our simulations are presented in Figure 3.[18] Again, qualitative predictions are robust with respect to changes in parameter values as can be seen in the Appendix. One can immediately see that in the long run (in response to the changes in τ) the share of investors that choose to roll over the debt mirrors the behavior of the default probability. This is not a coincidence: if the share

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[17] In fact, it is derived in the process of finding θ^*.

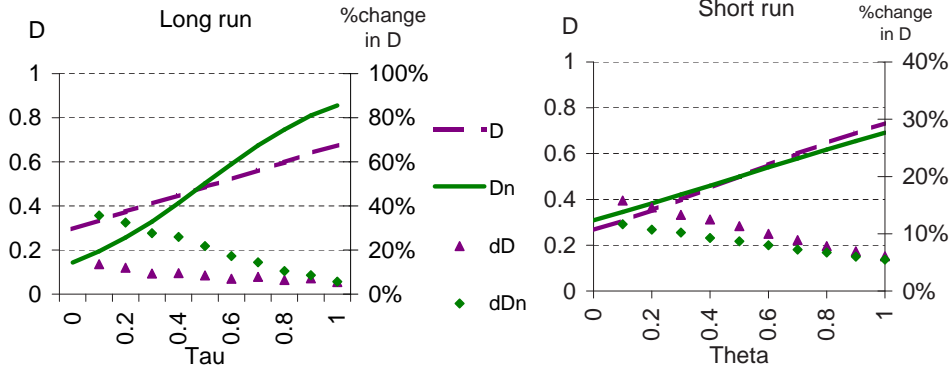[18] We still have to use numeric methods to calculate θ^*.

Figure 3: Debt roll–over with (D) and without (Dn) a rating agency, and the change in debt roll–over with (dD) and without (dDn) a rating agency in response to changes in τ and in θ (for $\tau = 0.5$), with $\alpha = \beta = \gamma = 1$, $R = 2$

of investors that choose to roll over their debt is higher, default is less likely to occur.

Here we are more interested in the magnitude of the response of capital flows to changes in fundamentals, thus, the right–hand axis shows the percentage changes in D as a result of an incremental changes in τ (by 0.1). We find that in the long run, the response of capital flows to changes in fundamentals is always lower with the rating agency, but that the difference is smaller when the fundamentals (τ) are better, as depicted on the left–hand panel of the Figure 3.

The reason for this result is that posterior beliefs are more dispersed in the absence of a credit rating, i.e. the distribution of posterior beliefs in the absence of the rating agency has fatter tails. For poor fundamentals only the investors in the upper tail of the posterior distribution decide to roll over the debt, since the tails are thinner with the rating agency, fewer investors change their decision in response to a change in τ.[19] As τ improves, however, investors closer to the middle of distribution start rolling over their debt and at some point further improvement in fundamentals would have a larger effect with the rating agency than without. This point, however, is not reached for $\tau \in [0; 1]$ for the sets of parameters we analyzed. Presumably, if R increases further, this effect will eventually be reversed for the very high τ.

Thus, we find that the introduction of a rating agency can have a stabilizing effect on capital flows by reducing their response to long–run changes in fundamentals, and that this is especially true for the borrowers with worse long–run fundamentals. In addition, this stabilizing effect is more

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[19]Note that a change in τ shifts the posterior distribution by the same amount with and without the rating agency.

pronounced when the spreads are relatively low.

The right–hand panel of Figure 3 presents the effects of the rating agency on the response of capital flows to the short–run volatility in fundamentals. Here, the effect is reversed: the introduction of the rating agency actually makes capital flows more responsive to the short run changes in θ. This reversal occurs because the threshold θ^* is not affected by the changes in the realization of θ in the absence of a rating agency but is affected when rating agency is present, through the rating announcement. This amplifies the impact of the changes in fundamentals on the equilibrium. However, there is also a direct effect of the changes in θ on D which is larger without the rating agency, due to the fact that the posterior distribution is more dispersed. This direct effect can more than compensate for the additional effect of the changes in the rating announcement on θ^*. Consequently, as Appendix 2 shows, the result that capital flows are more responsive in the short run to changes in θ, when the agency is present, depends on the levels of R and τ. In particular, it holds for low and high levels of R, while for the intermediate levels of R, it depends on τ.

3.4 Multiplicity versus uniqueness of equilibrium

Until now, we have focused on the situation where the equilibrium has been unique. In order to ensure uniqueness, we had to assume that the condition on the precision of private signals relative to that of public information held: in particular, $\beta \geq (\gamma + \alpha)^2/2\pi$. This condition requires that private information be rather precise relative to the rating agency's information. In fact, as shown on Figure 4, if $\alpha \geq 4$, and $\gamma = 1$, uniqueness requires that $\beta > \alpha$, i.e. private signals that carry information in addition to that present in the common prior and the rating agency's announcement, are more precise than the rating agency's announcement. This is not usually the way we think about the quality of information, given that rating agencies are in the business of gathering, analyzing and supplying information.

If we think of the original common prior as easily available information, such as news media, it is not unrealistic to think that the information obtained by the analysts of rating agencies is much more precise. Indeed, the reason that investors make use of credit ratings is because they believe that
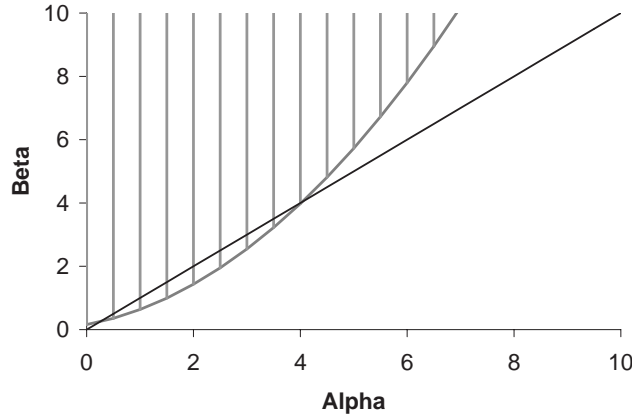
Figure 4: α, β and uniqueness condition (shaded area) for $\gamma = 1$

rating agencies have better access to information and therefore their information is more accurate.[20]

While a unique equilibrium may be easily achieved in the absence of a rating agency, it is much less likely to be achieved when a rating agency is introduced. If uniqueness is not achieved, instead of the single equilibrium, there will be three: two stable equilibria and one unstable equilibrium. With multiple equilibria, the volatility of capital flows may be increased further as shifts in credit ratings may trigger jumps from one equilibrium to another.

The switch from a situation with a unique equilibrium to multiple equilibria due to the introduction of a rating agency is illustrated in Figure 5. Equilibrium E is the unique equilibrium in the absence of the rating agency. When the rating agency is introduced, the equilibrium will shift to either of the stable equilibria (A or C) unless equilibrium E is exactly equal to the unstable equilibrium B. In the case illustrated in Figure 5, θ^* is high enough that the equilibrium converges to C, with a higher probability of default. This need not be the case in general. For some initial θ^*, the addition of the rating agency will shift the equilibrium to A, in which case the addition of the rating agency lowers the probability of default in equilibrium. In this latter scenario, moving from E to A, a credit rating indeed gives "courage" to capital by increasing capital flows and thus, lowering the

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[20]What really matters for the model is not whether the rating agency's information is actually more accurate, but instead that investors believe that the rating agency's information is more accurate.
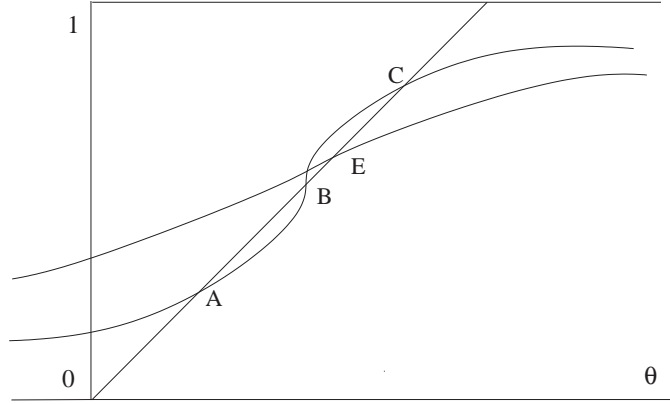
Figure 5: Equilibria (equations (4) and (6)) with and without a rating agency

probability of default. However, this improvement comes at a cost — changes in ratings may shift the equilibrium to the one with higher probability of default.[21]

In our setup, rating agencies announce their beliefs about economic fundamentals. This is somewhat unrealistic, since rating agencies typically announce a rating that corresponds to the likelihood that the sovereign (or some other borrower) will default. If the rating agency announces its expectation of the equilibrium default probability and there is a unique equilibrium, this equilibrium will be the same as when the agency announces its beliefs about fundamentals, provided the investors know what the agency is doing, because investors can always back out the information on which the rating agency is basing its announcement.

In case of multiple equilibria, however, the rating agency would have to choose whether to announce the default probability that corresponds to a good equilibrium or the one that corresponds to a bad equilibrium. It can then serve as a self–fulfilling equilibrium selection mechanism — investors will coordinate on the equilibrium that corresponds to the announced credit rating. This ability to select a self–fulfilling equilibrium arguably gives the rating agency a lot of influence on the markets; and it could give rise to the disciplinary ability of the rating agency described by Boot, Milbourn, and Schmeits (2004), or to an opportunity to "blackmail" the borrower being rated.[22] Our model

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[21]Furthermore, Angeletos, Hellwig, and Pavan (2003) show that allowing the rating agency to act strategically (while limiting its action space) might lead to multiple equilibria even if the condition on β is satisfied. This gives us an additional reason to believe that introducing a rating agency into a coordination game might lead to multiple equilibria.

[22]As was insinuated in a series of articles published in The Washington Post between November 22 and November 24, 2004.

suggests that with multiple equilibria the ability to leverage the impact of their announcement is a possibility for rating agencies, even if the investors are quite sophisticated.

4   Conclusion

We use a global game model of sovereign debt roll–over to analyze the effects of introducing a rating agency to financial markets. The credit rating agency affects the equilibrium by affecting investors individual assessment of the condition of the borrower and the investors' beliefs about the actions of other borrowers. While the model is formulated with sovereign debt in mind, it can be directly applied to corporate debt roll–over with a rating agency. The difference will be in the interpretation of "fundamentals".

We find that the addition of the rating agency increases the probability of default for the borrowers with good fundamentals and lowers it for the borrowers with poor fundamentals in the long and short run, reduces the magnitude of the response in capital flows to long–run changes in fundamentals, and can increase or reduce the magnitude of the response in capital flows to short–run changes in fundamentals. In addition, the introduction of a rating agency may decrease market stability by giving rise to situations in which multiple equilibria exist so that markets respond sharply to changes in beliefs.

It is important to emphasize that this paper is by no means a complete cost–benefit analysis of the role rating agencies play in financial markets. We merely point out some potential costs and benefits that can arise from introducing a rating agency in a coordination game by heterogeneous investors. One additional channel through which rating agencies affect markets is by reducing the cost of collecting information, which could also have important implications for how rating agencies affect financial markets. We attempt such analysis in a related project, still in progress.
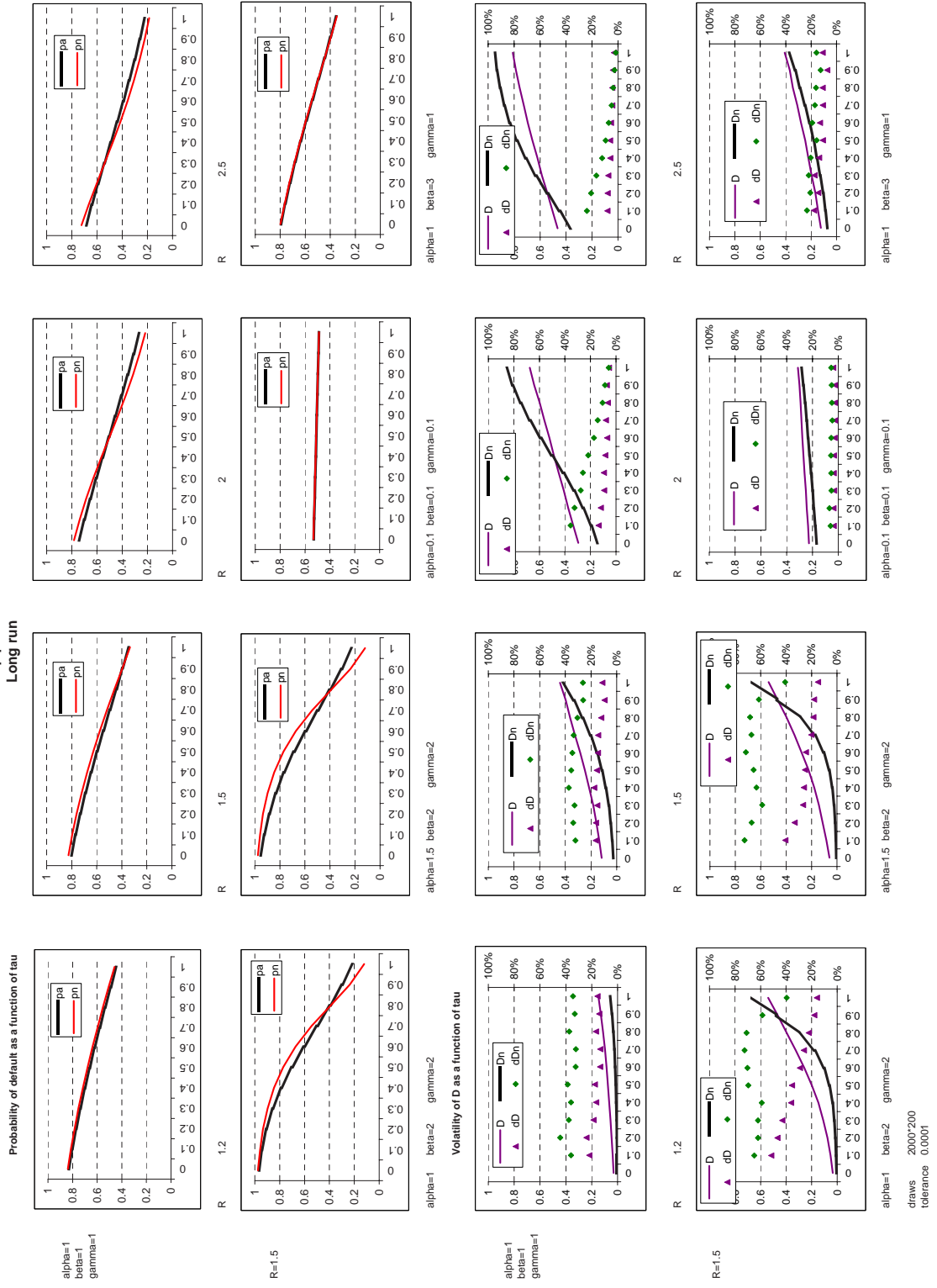
The model can also serve as a stepping stone for many other potentially instructive extensions. One venue is to introduce the dynamics that would endogenize the cost of debt or risk–premium, the size of the debt stock, and potentially even the cost of default.

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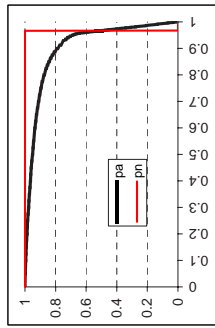GOLDSTEIN, I., AND A. PAUZNER (2003): "Demand Deposit Contracts and the Probability of Bank Runs," Available from http://www.tau.ac.il/ pauzner/papers/.

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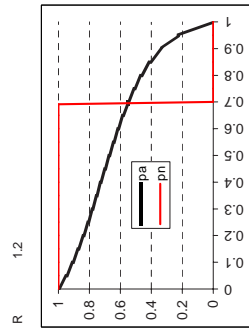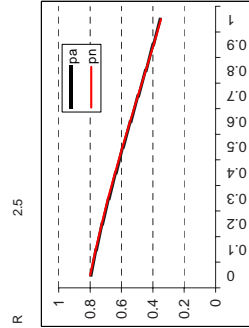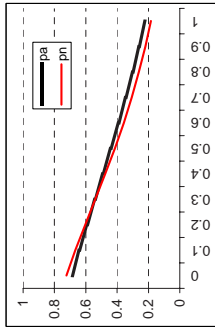
Appendix
Long run

Probability of default as a function of tau

alpha=1
beta=1
gamma=1

alpha=1   beta=2   gamma=2

alpha=1.5   beta=2   gamma=2

alpha=0.1   beta=0.1   gamma=0.1

alpha=1   beta=3   gamma=1

R=1.5

Volatility of D as a function of tau

alpha=1
beta=1
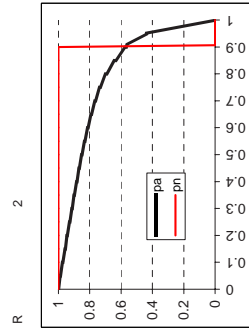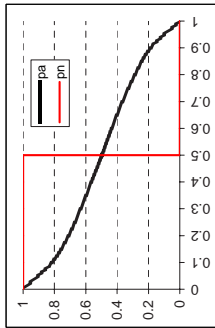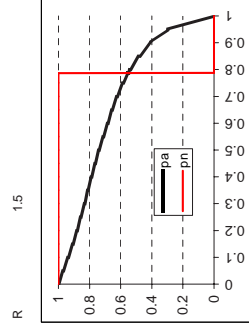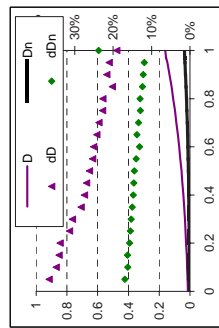gamma=1

R=1.5

draws   2000*200
tolerance   0.0001

23

Short run

Probability of default as a function of theta

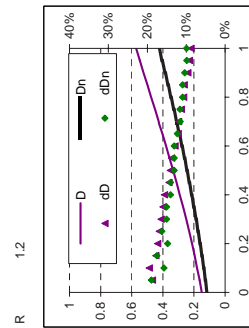**Volatility of D as a function of theta**

alpha=1
beta=1
gamma=1
tau=0.5

R=1.5
alpha=1
beta=1
gamma=1

alpha=1 beta=3 gamma=1

draws   10000
tolerance   0.0001