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ENTRY AND INNOVATION IN VERTICALLY DIFFERENTIATED MARKETS

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Abstract

This paper analyzes the optimal entry into experience goods markets with vertically differentiated buyers. We consider the case where the value of the new product is imperfectly known, but common to all buyers (common values) as well as the case where the quality is different across buyers (private values).

We distinguish between new products that are improvements to existing products and new products that are substitutes. Different types of products have qualitatively distinct diffusion paths. Improvements are introduced slowly relative to the full information case, while substitutes are introduced more aggressively. The slow entry strategy is associated with increasing supply and decreasing prices over time. The reverse patters holds for an aggressive entry strategy.

The incentives to innovate display a similar distinction. A firm with a currently inferior product opts for a large but risky innovation, whereas a currently superior producer chooses a smaller but certain innovation.

Keywords: Experience Goods, Vertical Differentiation, Entry, Innovation.

JEL Classification: D81, D83

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1 Introduction

The standard model of experience goods and repeated purchases predicts that new products are launched in the market with a low initial price. Sales in the periods following entry are made at a higher price and only the goodwill customers from the initial period make subsequent purchases. At the same time, casual observation suggests that experience goods are launched in the market using a number of alternative strategies. When a new pizzeria locates in a residential neighborhood, it is common that it creates a pool of goodwill customers in the initial weeks by setting its prices lower than in the subsequent weeks. In different types of markets, however, we would not necessarily expect to see such deep initial price cuts. If an expensive French restaurant decided to locate in the same neighborhood, its prices would probably be at a consistently high level and it would rely on favorable word-of-mouth communication.

In this paper, we analyze the optimal entry strategies for different types of experience goods in a number of different markets. Our main goal is to obtain a characterization of the features of the new product that lead to qualitatively different entry strategies. In a two period model of competition, we derive results connecting the incentives to give initial discounts to the size of the improvement embodied in the new product. If the new product represents a definitive improvement on the existing products, then the monopolist prefers a slow entry strategy. In particular, the first period equilibrium quantities of the new product fall short of the equilibrium quantities in a single period myopic model and the second period sales are extended to buyers that did not buy in the first period. When the new product is less clearly an improvement, or when it is perhaps a lower quality substitute for an existing product, the first period equilibrium quantities of the new product may exceed the single period quantities. The second period clientele in this case is a subset of the first period buyers.

We treat the case where an objective measure for the uncertain quality of the new product is available as well as the case where the buyers' initial uncertainty is only about their idiosyncratic taste parameters. We refer to them as the *common values* and the *private values* model respectively. The key modeling feature in both of these cases is that the buyers are assumed to be vertically differentiated. In other words, even when there is a commonly agreed upon measure of quality for the new product, different buyers have a different willingness to pay for the product, for example due to differences in disposable income.

We consider first the common values model. A new product of initially uncertain quality enters the market and new information about the product quality is generated only through purchases. We assume that the experiences of those buyers that decide to buy the new product in the first period are publicly observable. The exact mechanism of information transmission from first to second period is left unmodeled, but are motivated by considerations such as word of mouth communication between the buyers and consumer reports services. As a consequence, all buyers have identical beliefs about the new product. We also assume that the amount of information that becomes available is increasing in the sales of the new product in the first period. Markets of this type include among others new airlines carriers and new providers of communications services.

We analyze two distinct market structures within this model. In the first, the substitute product is provided by a competitive industry, in the second, the substitute is supplied by a single competitor. In the monopoly model, there are no differences between price and quantity competition. We show that the usual option value considerations may lead the producer of the new product to sell larger than myopic quantities in the first period if the product is not a certain improvement. If the good is an improvement with probability 1, then the option value vanishes and sales take place at the myopic quantity.

The situation is much more interesting in the case of a quality differentiated duopoly. In this case, we focus mainly on the quantity competition case. By doing this, we extend the scope of viable new products. In particular, quantity competition allows for the possibility that an innovation is launched which brings the two competitors closer to each other without change in the leadership. We show that the stage game equilibrium profit of the superior producer is concave in the parameter of differentiation while the equilibrium profit of the inferior producer is convex. Since sales by the new firm generate additional information about the product, we can use an argument based on Jensen's inequality to show that whenever the new product is myopically superior, its sales in the first period of the dynamic game are lower than the myopic sales. By a similar argument, we show that if the new product is myopically inferior, then the first period sales exceed the myopic sales. For completeness, we also sketch the price competition model. While the stage game equilibrium profits of the two firms are no longer monotone in the quality parameter, and only innovations that guarantee sufficient distance between the competitors are viable, we show that our main conclusions on entry strategies remain valid.

We also use the result on the curvature of the value functions to characterize the optimal innovations for the leader and the follower. The leader prefers relatively safe product innovations, whereas the follower prefers risky innovations that have a positive probability of leapfrogging the leader.

The private values model makes the polar opposite assumption on the form of uncertainty relating to the new product. We assume that there is no aggregate uncertainty about the performance of the new good, but individual tastes for the product are independent across the potential buyers, and not known a priori. A given buyer learns how well the new product suits her needs by trying it. For simplicity, we assume that a single trial is sufficient to determine the preferences. In this model, the buyers have different beliefs on the quality of the new product in the second period depending on their purchases and experiences in the first period. The main driving force for our results in this case is the interplay between the vertical differentiation, and the idiosyncratic differentiation. First period sales lead naturally to market segmentation, and the entrant chooses the entry strategy to maximally extract surplus along these two dimensions. Examples of products of this type include pharmaceuticals where extensive tests prior to entry are sufficient to determine the aggregate effectiveness of the treatment in the population. How well the drug works for an individual patient is, however, a random variable.

Our main qualitative finding in the monopoly version of this model is again that the aggressiveness of first period sales is determined by the quality of the new product relative to the existing products. Depending on the size of the improvement, the price path takes one of two possible shapes. If the improvement is large enough for most buyers, then prices are initially high and declining. If the improvement is negligible for a large part of the clientele, then the prices are initially low and increasing. These two strategies differ mostly in the implied intertemporal surplus extraction. In the first case, the monopolist is cream skimming the top of the vertical differentiation distribution in the first period. In the second case, the monopolist extracts the top of the idiosyncratic quality distribution in the second period. We also show that these qualitative results extend to the duopoly case as well.

1.1 Related Literature

Our model is related to a number of branches in the literature on imperfect competition. The model of vertical differentiation was first developed in the context of a duopoly model by Gabszewicz & Thisse (1979), (1980), and Shaked & Sutton (1982), (1983). The emphasis in those models was on the optimal choices of product qualities for competing producers. The product characteristics were commonly known to all the participants in the market, and the quality choices by the firms were followed by a second stage price competition. Gal-Or (1983) and Bonnano (1986) first considered quantity competition in a model of vertical differentiation. Our primary interest in this paper is in explaining observed differences in the qualitative features of initial pricing. To allow for a wide range of possibilities, we want to have the flexibility in the demand structure afforded by vertical differentiation.

The recent literature on experimentation and strategic experimentation has considered models closely related to the common values case of the current paper. Early models such as Rothschild (1974) and McLennan (1984) consider the learning problem of a monopolist facing a fixed demand curve with unknown parameters.[1] Aghion, Espinosa & Jullien (1993), Harrington

[1] The monopoly learning problem is further analyzed, among others, in Prescott (1972),

(1995) and Keller & Rady (1998) analyze a duopolistic market where two competitors learn bout the substitutability between their products. In these models, useful information becomes available whenever either of the firms makes a sale. The main difference between these papers and the current paper is that here the actual demand curve, and not only the beliefs about the demand, depends on past sales. Bergemann & Välimäki (1997) considers the entry problem of a new product in a situation where the buyers are horizontally differentiated. To our knowledge, the model of entry with vertical differentiation and common values is new to this paper. In the absence of the vertical differentiation, the previous models of entry cannot generate qualitatively different predictions for the speed of entry for different types of new products. The public observability of utility signals is central to some recent models of word-of-mouth communication such as McFadden & Train (1996).

Monopoly models similar to our private values case were treated in Milgrom & Roberts (1986), Farrell (1986) and Tirole (1988). All of these models make the assumption that the perceived quality is either high or otherwise of value zero. We view this restriction as unnecessary and unrealistic in many situations. In a vertically differentiated model, there ought to be enough flexibility to incorporate the possibility that the idiosyncratic perception of quality and the willingness to pay for quality interact in a non-trivial manner. Our model allows for the possibility that the monopolist makes some sales to buyers with a high willingness to pay and a moderate perception of the quality of the new product. Buyers with a lower willingness to pay for quality must have more optimistic beliefs in order to make the purchases. This generalization allows for richer characterization of the optimal policy of the monopolist. In our model, it is possible that the marginal buyer in the second period might have a lower willingness to pay for quality than the marginal buyer in the first period and as a result, buyers have an incentive for experimental consumption. Cremer (1984) considers a model with initially identical buyers and idiosyncratic experience to explain coupons and

Kihlstrom, Mirman & Postlewaite (1984), Easley & Kiefer (1988), Aghion, Bolton, Harris & Jullien (1991), Mirman, Samuelson & Urbano (1993), and Treffler (1993).

entry fees for shopping clubs.

Finally, conditions for initially high prices have been obtained in asymmetric information models of entry. In those papers, the monopolist is assumed to know the true value of the product, and the prices chosen serve as signals of the true quality. A prominent example of such models is Bagwell & Riordan (1991) where high and declining prices serve as signals of high product quality. Judd & Riordan (1994) consider a model with initially symmetric information where private signals are received by the monopolist and the buyers after first period choices. The firm then faces a signalling problem in the second period. The results in these models depend on the details of the information revelation mechanism and the cost structure. In our model, the results depend only on the quality difference between the products which can in principle be inferred directly from the realized prices.

The paper is organized as follows. Section 2 contains the model used in the paper. Section 3 presents the analysis in the model with common values. Section 4 considers the private values model. Section 5 concludes.

2 The Model

An entrant launches a new product in a two period model with $t \in \{0, 1\}$. The buyers have unit demand for the goods. They are vertically differentiated and distributed on the unit square. The horizontal coordinate indexes their willingness to pay for quality, $v \in [0, 1]$, and the vertical coordinate indexes their taste for the new product, θ. We assume throughout that the taste θ is independent of the vertical differentiation v. The marginal distribution on v is assumed to be uniform. The taste θ^i of buyer i for the new product is initially unknown to buyer i (and everybody else) and θ^i is a random variable in period 0. Conditionally on knowing θ^i, buyer i is willing to pay up to $\theta^i v^i$ for the new product. At the beginning of period 0, all players share the common prior beliefs:

$$\tilde{\theta}^i \sim F_0(\theta),$$

where $\theta \in \left[\underline{\theta}, \bar{\theta}\right] \subset [0,1]$. Notice that this allows for the possibility of *common values*, where $\theta^i = \theta^j$ for all i and j as well as the *private values* case where each θ^i is independent of θ^j for all $j \neq i$. The taste θ^i may therefore describe an objective unknown quality such as reliability in travel or communication services or an idiosyncratic quality which represent the value of the match between the product and a specific buyer i, as in a new pharmaceutical product. The expected quality is denoted by

$$\theta_t^i = \mathbb{E}\left[\tilde{\theta} \left| F_t^i (\theta)\right.\right],$$

where we have allowed for the possibility of different beliefs on the quality in period 1 based on period 0 experiences.

The buyers have access to a substitute product which is known and has a safe quality s. Buyers located at v, are willing to pay up to sv for the established product.

We consider two alternative market structures. In the first, we assume that the safe product is produced competitively, and as a result its equilibrium price is always at marginal cost normalized to 0. In the second market structure, the established product is produced by a single competitor. The equilibrium prices are determined by the quantities supplied which are denoted by q_t^N and q_t^S for the new and the safe firm(s), respectively. In any case, the firms choose quantities in each period simultaneously and the market clearing conditions determine the equilibrium prices.

The new product is an experience good and sales in period 0 generate valuable information for purchases in the subsequent period. In the common values case, we assume that all information is publicly observed and the amount of information depends on the sales q_0^N in period 0. The common posterior probability on the product quality is denoted by $F_1(\theta)$. The (ex ante) distribution of the posterior beliefs about the quality, denoted by $F_1\left(\theta_1 \left| F_0, q_0^N\right.\right)$, depends on the prior beliefs $F_0(\theta)$ and on the volume of the sales q_0^N. In the common values case, larger quantities are assumed to yield more precise information. Formally, $F_1\left(\theta_1 \mid F_0, q_0^N\right)$ is assumed to be second order stochastically decreasing in q_0^N.

In the case of private values, we assume for simplicity that buyers learn their tastes in a single trial with the new product. If buyers at location v purchases the new product in period 0, they learn their perceived quality parameters with certainty.[2] As a result, ex ante identical buyers may have different tastes ex-post. We denote the belief of a given buyer, i again by $F_1^i(\theta)$. The difference to the common values case is that $F_1(\theta)$ is no longer constant across i. For all i that did not purchase the product in period 0, $F_1^i(\theta) = F_0(\theta)$, but for all those i that purchased the product, $F_1^i(\theta) \in \{0,1\}$. Even though each $F_1^i(\theta)$ is random, the aggregate distribution of $F_1^i(\theta)$ is deterministic by the law of large numbers as long as all buyers with the same v make the same purchasing decisions. We denote the vector of second period expected qualities by θ_1.

Given θ_1, we can write the period 1 inverse demand functions as

$$p_1^j = d_1^j\left(q_1^S, q_1^N, \theta_1\right),$$

and find the Nash equilibria of the period 1 stage game between the firms. For every vector of posterior probabilities θ_1, the quantities chosen by the firms must be optimal given what other firms are doing. Denoting the equilibrium choices by $q_1^j(\theta_1)$ for $j \in \{N, S\}$, period 1 equilibrium profits are given by:

$$\pi_1^j(\theta_1) = q_1^j(\theta_1)\, d_1^j\left(q_1^S(\theta_1), q_1^N(\theta_1), \theta_1\right).$$

In the initial period, players take into account their effect on F_1. Denote the inverse demand functions by:

$$p_0^j = d_0^j\left(q_0^N, q_0^S, F_0\right).$$

The overall payoff functions for the firms are:

$$\Pi^j\left(q_0^N, q_0^S, F_0\right) = q_0^j d_0^j\left(q_0^N, q_0^S, F_0\right) + \int \pi_1^j(\theta_1)\, dF_1\left(\theta_1 \left| F_0, q_0^N\right.\right).$$

The subgame perfect equilibrium (SPE) is given by a vector of quantities, $\left(q_0^N, q_0^S\right)$ and $\left(q_1^S(\theta_1), q_1^N(\theta_1)\right)$ for each possible realization of θ_1 such that they form a Nash equilibrium for every subgame.

[2] We are assuming that the law of large numbers holds in this case.

3 Common Values

With common values, all buyers agree about the true quality of the new product, or $\theta^i = \theta^j$ for all i, j. The monopoly with a competitive fringe is considered as a benchmark model in Subsection 3.1. The duopoly is analyzed in Subsection 3.2 and 3.3. The incentives to innovate in the strategic environment are discussed in 3.4. The robustness of the quantity competition results are considered in Subsection 3.5, where they are contrasted to a model with price competition.

3.1 Monopoly

We analyze first the optimal entry strategy in the case where a substitute product is supplied at zero price by a competitive fringe. The new firm decides the optimal sales quantities in a two-period model. The buyers of type v value the existing product at sv, whereas their (expected) valuation for the new product in period $t \in \{0, 1\}$ is $\theta_t v$.[3] The (expected) quality difference between the two products is denoted by α_t :

$$\alpha_t \equiv \theta_t - s.$$

We solve for the optimal quantities by backwards induction. The period 1 market clearing price for the new firm when selling quantity q_1^N is given by the indifference of the buyers with $v = 1 - q_1^N$:

$$p_1^N = \left(1 - q_1^N\right)(\theta_1 - s).$$

This price is positive only if $\theta_1 > s$.[4] In that case, the optimal sales quantity is independent of θ_1 and identical to the myopic quantity, denoted by m_t^N, with $m_t^N = \frac{1}{2}$. We are mostly interested in two specifications for the values of $\bar{\theta}$ and $\underline{\theta}$. In the first, the new product is certainly an improvement on the existing products, or $\underline{\theta} > s$, and it follows that $\theta_1 > s$ with probability 1.

[3] Recall that we are assuming that the second period beliefs of the buyers are identical since all first period experiences are publicly known.

[4] Notice that s can also be interpreted as the constant marginal cost of production. A negative price corresponds then to a negative markup.

As a result, the optimal profit of the new seller is linear in θ_1. If $\bar{\theta} \leq s$, then the new product is at best a perfect substitute for the existing products. In this case, $\theta_1 < s$ with probability 1, and the optimal action is to sell $q_1^N = 0$, which coincides with the myopically optimal action $m_1^N = 0$.

To link the two periods of the game, we need to specify the mechanism of information revelation. The sales in period 0 generate information through the experience the buyers make with new product. The new information becomes available in period 1 either through word-of-mouth communication or consumer report services. We make the following assumption in this section.

Assumption 1. $F_1\left(\theta_1 \left| F_0, q_0^N\right.\right)$ *is second order stochastically decreasing in* q_0^N.

In other words, the quantity q_0^N generates a mean-preserving spread in the sense that the expected quality in period 1 is independent of the volume of sales q_0^N in period 0, or

$$\int \theta_1 dF_1\left(\theta_1 \left| F_0, q_0^N\right.\right) = \theta_0,$$

for all F_0 and q_0^N. However, the variability in the beliefs tomorrow is increasing in q_0^N. This implies that a larger volume of sales carries more information and leads to larger (expected) changes in the posterior belief θ_1. In order to be able to make use of calculus, we also make the following technical assumption.

Assumption 2. $F_1\left(\theta_1 \left| F_0, q_0^N\right.\right)$ *is twice continuously differentiable in* q_0^N.

Notice also that as long as $F_1\left(\theta_1 \left| F_0, q_0^N\right.\right)$ is continuous in q_0^N all buyers are informationally small. A buyer would therefore not have a strict incentive to lie when asked about her experience in period 0. In other words, the assumption of word-of-mouth communication is incentive compatible under a weaker version of Assumption 2.

The improvement case where $\underline{\theta} > s$ and the substitute case where $\bar{\theta} < s$ display a linearity of period 1 profit in θ_1. Under the assumption of second order stochastic dominance $\mathbb{E}\left[\theta_1 \left| F_0, q_0^N\right.\right] = \theta_0$ regardless of q_0^N and hence period 1 profits are independent of the equilibrium quantities in period 0.

In the intermediate case where $\underline{\theta} < s < \bar{\theta}$, it is possible that both $\{\theta_1 > s\}$ and $\{\theta_1 < s\}$ are events with positive probability ex ante. In this case, the profit function of the monopolist in period 1 is convex in θ_1, and information is valuable. By Assumption 1, more information is generated through higher initial sales, and it is then optimal for the monopolist to make sales beyond the myopically optimal quantity $m_0^N = \frac{1}{2}$. We can summarize the findings below in

Proposition 1 (Monopoly Sales)

1. *For $\underline{\theta} \geq s$, the equilibrium quantities are $q_t^N = m_t^N = \frac{1}{2}$;*
2. *For $\bar{\theta} \leq s$, $q_t^N = m_t^N = 0$;*
3. *For $\underline{\theta} < s < \bar{\theta}$, $q_0^N \geq m_0^N$, and $q_1^N = m_1^N$.*

Hence we observe that new products that are certain improvements on the existing product are introduced at quantities equal to the myopically optimal quantities, but products that have real uncertainty surrounding their viability may be introduced at quantities above the myopically optimal ones. This observation connecting the initial quantities to the product quality will also be the key feature in the model of strategic competition which is considered next. The inequality $q_0^M \geq m_0^N$ is not necessarily strict as the spread in the distribution generated by q_0^N may be restricted to occur only inside the events $\{\theta_1 > s\}$ and $\{\theta_1 < s\}$ while leaving the probability of each event unchanged. In this case, the problem remains linear and the inequality is only a weak one.

3.2 Static Duopoly

In this subsection, we assume that the substitute product is produced by a single competitor rather than a competitive industry. This allows us to focus on the strategic effects of entry on the two firms depending on the features of the new product. We start the analysis with the competition in period 1.

For the remainder of this section, we assume without loss of generality that $\alpha_1 > 0$. For $\alpha_1 < 0$, the roles of the two firms are simply reversed. To calculate the Nash equilibrium, we need to determine the equilibrium prices that satisfy three properties: (i) buyers with valuations $v \in \left[1 - q_1^S - q_1^N, 1 - q_1^N\right]$ prefer the safe to the new firm, (ii) buyers with $v \in \left[1 - q_1^N, 1\right]$ prefer the new firm and (iii) all buyers get a nonnegative expected utility from their purchases. Let p_1^S, p_1^N be the equilibrium prices. Then

$$p_1^S = s\left(1 - q_1^S - q_1^N\right)$$

and

$$p_1^N = s\left(1 - q_1^S - q_1^N\right) + \alpha_1\left(1 - q_1^N\right).$$

The second period payoffs can be written as functions of the quantities $\left(q_1^N, q_1^S\right)$:

$$\pi_1^S\left(q_1^S, q_1^N\right) = q_1^S s\left(1 - q_1^S - q_1^N\right)$$

and

$$\pi_1^N\left(q_1^S, q_1^N\right) = q_1^N\left(s\left(1 - q_1^S - q_1^N\right) + \alpha_1\left(1 - q_1^N\right)\right).$$

Notice that at $\alpha_1 = 0$, the payoffs coincide with those in a homogenous goods Cournot model. The Nash equilibrium of the duopoly is obtained by solving simultaneously for profit maximizing $\left(q_1^N, q_1^S\right)$.[5] Observe first that the established firm's reaction function is independent of the level of differentiation. The quantity set by the new firm, q_1^N determines the size of the market for the established firm, and the price is determined from the zero surplus condition of the marginal buyer. Neither involves the level of differentiation once the output level of the new firm is given:

$$q_1^S\left(q_1^N, \alpha_1\right) = \frac{1}{2}\left(1 - q_1^N\right).$$

The reaction function of the new firm is given by:

$$q_1^N\left(q_1^S, \alpha_1\right) = \frac{1}{2} - \frac{s}{2\left(s + \alpha_1\right)} q_1^S.$$

[5] With the linear demand specification, the profit function of each firm is concave in its own quantity, and therefore first order conditions are also sufficient for optimality.

For all levels of α_1, the optimal reaction to $q_1^S = 0$ is given by the monopoly quantity $q_1^M = \frac{1}{2}$. At $\alpha_1 = 0$, the effect of the established firm's decisions on the optimal reactions of the new firm is at its strongest. As α_1 increases, the reaction curve of the new firm becomes flatter, eventually converging to a constant on the monopoly quantity. In a sense, the new firm becomes strategically independent of the established firm as α_1 grows.

The reaction function of the new firm can be written as:

$$q_1^N\left(q_1^S, \alpha_1\right) = \frac{\alpha_1}{s + \alpha_1} q_1^M + \frac{s}{s + \alpha_1} q_1^N\left(q_1^S, 0\right).$$

From this equation, we see that the reaction of the new firm is a weighted average of the monopoly best response and the Cournot reaction function at zero differentiation.

With the reaction functions, it is easy to solve for the equilibrium quantities:

$$q_1^N(\alpha_1) = \frac{s + 2\alpha_1}{3s + 4\alpha_1},$$

and

$$q_1^S(\alpha_1) = \frac{s + \alpha_1}{3s + 4\alpha_1}.$$

Notice that these quantities converge to the Cournot quantities as α_1 converges to zero. When α_1 is large, both firms behave as if they were monopolists on their parts of the demand curve. Equilibrium prices for the firms are:

$$p_1^N(\alpha_1) = \frac{(s + \alpha_1)(s + 2\alpha_1)}{3s + 4\alpha_1}$$

and

$$p_1^S(\alpha_1) = \frac{s(s + \alpha_1)}{3s + 4\alpha_1}.$$

The equilibrium second period profits, $\pi_1^j(\alpha_1)$ are then given by:

$$\pi_1^N(\alpha_1) = (s + \alpha_1)\left(q_1^N(\alpha_1)\right)^2$$

and

$$\pi_1^S(\alpha_1) = s\left(q_1^S(\alpha_1)\right)^2. \tag{1}$$

A direct calculation yields the following result on the curvatures of the equilibrium profit functions.

Proposition 2 (Curvatures)

1. *For* $\alpha_1 > 0$, $\pi_1^N(\alpha_1)$ *and* $q_1^N(\alpha_1)$ *are concave and* $\pi_1^S(\alpha_1)$ *and* $q_1^S(\alpha_1)$ *are convex in* α_1.

2. *For* $\alpha_1 < 0$, $\pi_1^N(\alpha_1)$ *and* $q_1^N(\alpha_1)$ *are convex, and* $\pi_1^S(\alpha_1)$ *and* $q_1^S(\alpha_1)$ *are concave in* α_1.

An intuition for the concavity of the value function of the new firm runs as follows. At low α_1 and $\alpha_1 > 0$, a marginal increase in α_1 increases the profit of the new firm through two channels. First, it increases the price for fixed quantities. This direct effect is the same at all levels of α_1 as long as the quantities supplied are unchanged. There is also the indirect effect from a stronger competitive position of the new firm and the corresponding reduction in the quantity of the established firm. This effect is strongest when α_1 is close to 0, and vanishes as α_1 increases. The combination of these two effects leads to a concave overall profit function. The argument for the convexity in the profit function of the established firm is established similarly. This general argument also indicates that the curvature properties continue to hold for a more general class of densities over the space of the vertical differentiation parameter v than the uniform density assumed here. Further use of the curvature properties is made when we consider the incentives to innovate.

3.3 Dynamic Duopoly

Next, we relate the intertemporal competition game to the learning from experience. It is convenient to define, with slight abuse of notation,

$$\pi_1^j\left(q_0^N\right) \equiv \int \pi_1^j\left(\theta_1\right) dF_1\left(\theta_1 \left| F_0, q_0^N\right.\right) \quad \text{for } j \in \{S, N\}.$$

The intertemporal profit function of firm j is given by backwards induction:

$$\Pi^j\left(q_0^N, q_0^S, F_0\right) = \pi_0^j\left(q_0^N, q_0^S, F_0\right) + \pi_1^j\left(q_0^N\right). \tag{2}$$

For each firm the objective function in period 0 is given by $\Pi^j\left(q_0^N, q_0^S, F_0\right)$. The indifference condition for the buyers in period 0 is identical to the static indifference condition as the purchase decision of any single buyer is without influence on the posterior distribution of the common beliefs in period 1. The following lemma is an immediate consequence of the results on the curvatures of period 1 profit functions.

Lemma 1

1. *Suppose that* $\Pr\{\alpha_1 \geq 0\} = 1$. *Then* $\pi_1^S\left(q_0^N\right)$ *is increasing in* q_0^N *and* $\pi_1^N\left(q_0^N\right)$ *is decreasing in* q_0^N.
2. *Suppose that* $\Pr\{\alpha_1 \leq 0\} = 1$. *Then* $\pi_1^S\left(q_0^N\right)$ *is decreasing in* q_0^N *and* $\pi_1^N\left(q_0^N\right)$ *is increasing in* q_0^N.

P roof. See Appendix. ■

The next proposition proves the existence of an equilibrium, and derives the qualitative implications on equilibrium quantities for the firms in the two cases of the Lemma 1. Informally, the proposition states that a certain improvement on an existing product is introduced relatively slowly while a lower quality substitute is launched aggressively in the market.

Proposition 3 *An SPE in pure strategies exists. If*

1. $\Pr\{\alpha_1 \geq 0\} = 1$, *then* $q_0^N < m_0^N$, *and* $q_0^S > m_0^S$ *in any equilibrium of the dynamic game.*
2. $\Pr\{\alpha_1 \leq 0\} = 1$, *then* $q_0^N > m_0^N$, *and* $q_0^S < m_0^S$ *in any equilibrium of the dynamic game.*

P roof. See Appendix. ■

Figure 1 illustrates the proposition by showing the shifts in period 0 reaction functions induced by the intertemporal considerations.

[Insert Figure 1 here]

For $\alpha_1 \geq 0$, the new firm is slow to introduce the new product as the profit at the expected value of the product is exceeds the expected profit at the true quality. For $\alpha_1 \leq 0$, the marginal value of an increase in the quality is increasing and hence the firm wishes to accelerate the diffusion of information, leading to a more aggressive stance. As only the sales by the new product carry information, the best response function of the established firm is constant across α_1. The inequalities in Proposition 3 are strict as the payoff functions have a non-zero curvature in contrast to the linearity in the monopoly model. The equilibrium in period 0, however, need not to be unique since the information revelation represented by $F_1\left(\theta_1\left|F_0, q_0^N\right.\right)$ may induce non-concavities into the payoff functions. The qualitative result is however robust to the multiplicity of equilibria.

The characterization result in Proposition 3 can be extended to the intermediate case when the new product is neither a certain improvement nor a certain substitute with additional structure on the uncertainty resolution. To this end consider the following example where the resolution of uncertainty has the following linear form. With probability q_0^N, the signal is informative and distributed uniformly $\tilde{\theta} \sim \mathcal{U}\left[\theta_0 - \varepsilon, \theta_0 + \varepsilon\right]$ for an arbitrary $\varepsilon > 0$, with the complementary probability $1 - q_0^N$ the signal is uninformative. In this case, there is a θ such that for all $\theta_0 \geq \theta$, we have $q_0^N \leq m_0^N$ and $q_0^S \geq m_0^S$, while for all $\theta_0 \leq \theta$,we observe $q_0^N \geq m_0^N$ and $q_0^S \leq m_0^S$. Furthermore, the equilibrium is unique for all θ_0.

We conclude this subsection by considering an alternative assumption on the market structure. Suppose there are two firms with safe products in the market, and one of them comes up with a product innovation. If the innovator is the currently superior firm, a natural question to ask is whether the innovator would delay information revelation in the market by selling the improved product as well as the safe product. It is not hard to see that the losses resulting from selling the myopically inferior old product outweigh the gains from having less uncertainty resolution. Hence the innovator would sell only the myopically better new product in period 0, and the analysis would be essentially unchanged.[6]

[6] The only qualitatively new effect would arise if the product innovation is potentially inferior to the existing product. In this case, the innovator would have an option value component in the new product that is not present here.

3.4 Incentives to Innovate

So far we have emphasized the implications that uncertain product quality has on equilibrium prices and quantities. The first set of results showed that the entry strategy depends on the ranking of the new product in terms of its value relative to the competing product. In this section, we shift the focus to the incentives of the firms to introduce new products into the market. In particular, we ask whether firms differ in their innovation choices depending on their current position in the market. More precisely, consider the new and the established firm with initially arbitrary positions in the quality spectrum. The new firm wishes to introduce a new product and has to make a decision about the size and the riskiness of the new product. While we consider exclusively the innovation choice by the new firm and maintain the current position of the established firm, it is immediate that the new product could also be thought of as being generated by an established firm who is simply at the end of its current product cycle.

The first result concerns the optimal riskiness of the innovation when the expected innovation size is fixed to be θ_1. If the cost of an innovation were only a function of its expected return, this would represent the case of a fixed $R\&D$ budget. For simplicity we evaluate the payoff resulting from the uncertain innovation directly at the post-entry payoff. As before we call the new firm the leader if $\alpha_0 = \theta_0 - s > 0$ and follower if $\alpha_0 < 0$. The following result points to a difference in the optimal choices of the leader and the follower. Denote by $\delta_{\theta'}(\theta)$ the Dirac distribution with $G(\theta) = 0$ for all $\theta < \theta'$ and $G(\theta) = 1$ for all $\theta \geq \theta'$.

Proposition 4 (Optimal Innovation)
For a given expected improvement θ_1,

1. *the optimal strategy of a leader is $\delta_{\theta_1}(\theta)$;*
2. *the optimal strategy of a follower is:*

$$\frac{\theta' - \theta_1}{\theta' - \theta_0}\delta_{\theta'}(\theta) + \frac{\theta_1 - \theta_0}{\theta' - \theta_0}\delta_{\theta_0}(\theta),$$

for a unique θ' with $s < \theta' < \infty$ if $\theta_1 < \theta'$, otherwise it is $\delta_{\theta_1}(\theta)$.

P roof. See Appendix. ■

The different attitudes of the follower and the leader towards uncertain innovations follow from the curvature properties of the post-entry payoffs derived in Proposition 2. A leader always chooses a certain improvement. In contrast, a follower selects an uncertain project unless the improvement is sufficiently large, or $\theta_1 > \theta' > s$, in which case he overtakes the leader with certainty and with a sizable quality difference. The value θ' is determined by a tangential line to the equilibrium profit function of the follower in the quality-profit space:$\left(\theta, \pi^N(\theta)\right)$. More precisely, it is the line starting at $(\theta_0, \pi(\theta_0))$ and being tangential to some point $(\theta', \pi^N\left(\theta'\right)$. The uniqueness of θ' results again from Proposition 2, which states that the equilibrium profit function is first convex and turns concave as the current follower overtakes the current leader. The optimal allocation policy for the follower is then to increase the probability of success at θ' until θ' is reached with probability one, after which his marginal behavior is naturally identical to the one of a leader. The characterization results rely on the fact that the firm can choose an arbitrary risk profile. As the attitude towards the uncertain innovation depends exclusively on the distinct curvature properties of the follower and the leader, similar results would obtain for any restricted class of distribution functions.

With a fixed $R\&D$ budget the leader undertakes a small but certain improvement, whereas a follower attempts to overtake the leader but at the cost of suffering from failure. These results are suggestive for an extension of the current model into one with an infinite time horizon. Since the innovation of the follower is risky it suggests that on average the distance between follower and leader increases, which points to some stability in the leadership. On the other hand, the strategy of the follower is designed to overtake the leader and hence there will be turnover in the leadership as well. However such an extension is beyond the scope of the current paper.[7]

[7] Interestingly, the current literature on dynamic innovation and competition considers mostly fixed innovation choices such as step-by-stop or leapfrogging innovations, but not the optimal risk profile of an innovation, see e.g. Grossman & Helpman (1991), Aghion & Howitt (1992) and**?**.

The previous results considered innovations with a fixed expected size. Next, we consider the optimal size of the innovation where we assume that the cost of the innovation depends only on its expected value. A consequence of Proposition 4 is that all optimal innovations belong to a simple parametric family which are parametrized by the probability of success, p, and the value conditional on success, θ'. The expected innovation is then $\theta_1 = p\theta'$. The relative size of the optimal investment of leader and follower depend on the curvature of the marginal cost of innovations. This dependence is best illustrated with the following parametrization of the cost function. For every γ, assume that $c(\Delta\theta, \gamma)$ is strictly increasing and convex in the size of the expected improvement $\Delta\theta \equiv \theta_1 - \theta_0$. Assume also that $\partial c(\Delta\theta, \gamma) / \partial(\Delta\theta)$ is increasing in γ. Suppose further $\partial c(0, \gamma) / \partial(\Delta\theta) = 0$ for all γ. Denote the expected size of the innovation by θ_1^L and θ_1^F for leader and follower respectively. The value of the competing product is fixed at s.

Proposition 5 (Size) *There is a $\hat{\gamma}$ with $0 < \hat{\gamma} < \infty$, such that*

1. *for all $\gamma < \hat{\gamma}$, $\theta_1^F > \theta_1^L$;*
2. *for all $\gamma \geq \hat{\gamma}$, $\theta_1^F \leq \theta_1^L$.*

P roof. See Appendix. ■

The marginal gains from increasing the quality are maximal at a symmetric position with the competing product: $\theta_0 = s$. If the marginal costs rise quickly in the size of the improvement, the leader will always have higher marginal returns. But due to the concavity, the marginal returns are strictly decreasing for a leader but constant for a follower as long as increases in the expected size of the innovation results only in putting more probability on the outcome θ'. This explains why neither the leader nor the follower have uniformly stronger incentives to produce innovations.

3.5 Price Competition

The results on the entry and innovation behavior presented in the previous sections were derived in a model of quantity competition. In this section, we show that the qualitative features of the results are valid in a model of price competition as well. To this end it will be sufficient to examine the post-entry behavior and then link entry behavior to its impact on post-entry payoffs. It is again convenient to distinguish between the strategy of follower and leader and we start with the leader, or $\alpha_1 > 0$. The indifference conditions of the marginal buyers remains as before:

$$\left(1-q_1^N\right)(s+\alpha_1)-p_1^N=\left(1-q_1^N\right)s-p_1^S$$

and

$$\left(1-q_1^N-q_1^S\right)s-p_1^S=0.$$

The pricing game by the sellers has a unique equilibrium and as the derivation is standard, we simply state the equilibrium prices

$$p_1^N(\alpha_1)=2\alpha_1\frac{s+\alpha_1}{3s+4\alpha_1},$$

and

$$p_1^S(\alpha_1)=\frac{s\alpha_1}{3s+4\alpha_1}.$$

The equilibrium value functions are given by

$$\pi_1^N(\alpha_1)=4\alpha_1\frac{(s+\alpha_1)^2}{(3s+4\alpha_1)^2},$$

and

$$\pi_1^S(\alpha_1)=s\alpha_1\frac{s+\alpha_1}{(3s+4\alpha_1)^2}.$$

It can be verified that both $\pi_1^S(\alpha_1)$ and $\pi_1^N(\alpha_1)$ are increasing and concave in α_1. It then follows immediately that both firms would prefer less information about the new product to be generated in period 0. The immediate implications for period 0 equilibrium prices and quantities are that the new firm will offer higher prices relative to the myopic price to reduce the quantity sold and thereby reduce the information flow. Notice that the established seller can also affect the period 0 sales through his prices. As his period 1 value function is concave, he has an incentive to offer prices below the myopically optimal one given the price of the new firm. The shifts in the reaction curves of the two firms are depicted in Figure 2. Notice that the effect on period 0 quantities relative to the myopic quantities is unambiguous whereas the impact on period 0 prices depends on the magnitudes by which the reaction curves shift.

[Insert Figure 2 here]

Since the price of the new firm is concave in α_1, it follows by the martingale property of the beliefs that the expected equilibrium price in period 1 is lower than the equilibrium price in period 0.

The case of the follower can be analyzed in a similar fashion with one important exception. Due to the price competition, the follower's incentive to increase his quality at the margin are not monotone. As the value θ_1 approaches s from below, the competition between the two firms becomes more severe and yields zero profit in the limit as $\theta_1 = s$, or $\alpha_1 = 0$. The corresponding profit functions are for $\alpha_1 < 0$:

$$\pi_1^N(\alpha_1) = -\frac{\alpha_1 s (s + \alpha_1)}{(3s - \alpha_1)^2}$$

and

$$\pi_1^S(\alpha_1) = -\frac{4s^2 \alpha_1}{(3s - \alpha_1)^2}.$$

In fact, the profit function of the new firm is now concave (except for very small values of θ_1). However since it is first decreasing and then increasing as α_1 passes through zero, the intertemporal implications for the optimal innovation and entry strategy remain as before. The follower introduces an innovation only if it results in a positive probability of overtaking the current leader by a wide enough margin since it is otherwise preferable to stay at a distance from the leader in the quality space. The kink in the profit function at $\alpha_1 = 0$, then implies that current follower and leader both prefer a resolution of uncertainty which leads to an increased distance between them. The resulting period 0 equilibrium quantities are then biased to support more sales of the new firm to generate information which in turn leads to the same intertemporal pattern of sales as in the quantity competition model.

4 Private Values

This section considers market entry when the experience of each consumer is purely idiosyncratic. The buyers have identical ex-ante (expected) taste for the product but differ in their ex-post taste due to different experiences with the new product. For simplicity, we assume that the aggregate distribution of tastes is common knowledge at the outset, but individual buyers do not know if the product suits their idiosyncratic needs. As examples of this, one may think of new pharmaceuticals that have been tested prior to approval by the FDA, and their performance in the population is accurately known. How well the new drug suits a particular patient is, however, uncertain at the outset. As the individual experience carries no information for the other buyers in the market, we refer to this situation as the private value model. The idiosyncratic aspect of the experience leads to segmentation in the post entry market. The segmentation occurs independently of the market structure. The problem of the monopolist is considered first in Subsection 4.1. The strategic aspects arising with the duopoly are analyzed in Subsection 4.2.

4.1 Monopoly

Consider a monopolist who introduces a new product in the market. The true value of the new product for buyer i is distributed uniformly on the unit interval:

$$\tilde{\theta}^i \sim \mathcal{U}[0,1],$$

where $\tilde{\theta}^i$ is independent of v^i and of $\tilde{\theta}^j$ for all $i \neq j$.[8] For simplicity we assume that each buyer learns his true preference for the new product upon trying it once and then values it at $\theta^i v^i$. The expected value of the product is:

$$\mathbb{E}\left[\tilde{\theta}^i\right] v^i = \frac{1}{2} v^i.$$

[8] As a robustness check, we also analyzed the model with an arbitrary two-point distribution for the possible private values. The results remain qualitatively the same, as in the uniform case. The details of the computations are available from the authors upon request.

To interpret the idiosyncratic variance in tastes, we can think of the new product as containing new features that add utility to the buyers in different degrees. There is also a competitively supplied substitute product that has a value sv_i to the buyer v_i.[9]

Initially, all consumers are uncertain about the utility they would derive from the new product. After period 0, the market is segmented between informed and uninformed consumers. The size of each segment depends on the volume of sales, q_0^N, in period 0. The set of informed consumers display heterogeneity along two dimensions: (i) the willingness to pay, v^i, and (ii) the private value for the new product, θ^i.

We begin by analyzing the sales decision in the post-entry phase. The essential question to be addressed by the monopolist is whether or not he extends his market in period 1 to new and uninformed customers. Denote by i_t^j and u_t^j the mass of informed and uninformed buyers, respectively, which buy from firm j in period t. Consider first the situation where q_0^N was chosen sufficiently large, so that it is optimal to sell only to informed buyers or $u_1^N = 0$. The marginal buyer in the second period is then indifferent between buying from the monopolist and from the competitive fringe:

$$v^i\theta^i - p_1^N = v^i s.$$

Each informed consumer is identified by two characteristics v^i and θ^i, and the marginal buyer is described by a hyperbola in the (v, θ) space:

$$v^i\left(\theta^i - s\right) = p_1^N$$

as described in Figure 3.

[INSERT FIGURE 3 HERE]

All buyers above the hyperbola strictly prefer to acquire the new product at price p_1. To the left of the vertical line intersecting at $1 - q_0^N$, the buyers

[9] The symmetry in the valuations of the current product is an assumption made merely for analytical convenience. It prevents the segmentation to exist already in the entry phase. All subsequent results continue to hold qualitatively with ex-ante segmented markets.

are homogenous along the dimension θ and simply maintain the expected value $\mathbb{E}\left[\widetilde{\theta}^{i}\right]$. For a given price p_1, an increase in the value of the alternative s lifts the hyperbola upwards, though faster for small values of v and slower for high values of v. The hyperbola intersects with the vertical line above $\frac{1}{2}$, indicating that the new product is too expensive to attract new buyers.

To describe the implications of the post-entry behavior for the initial period, consider the marginal buyer at $v = 1 - q_0$. By buying in period 0, she expects to purchase the new product again if her experience is sufficiently positive. The option value of buying today is therefore given by

$$w(q_0) \equiv \int_{s+\frac{p_1^N}{1-q_0^N}}^{1} \left(\left(1-q_0^N\right)\theta - p_1^N\right) d\theta.$$

The lower bound in the integral identifies the lowest experience θ in period 0 such that the marginal buyer $v = 1 - q_0^N$ in period 0 is the marginal buyer in period 1, or

$$\left(1-q_0^N\right)\theta - p_1^N = \left(1-q_0^N\right)s.$$

While the option value is realized in period 1, the monopolist is able to extract the entire value from the marginal buyer in period 0 through higher prices. The indifference condition for the marginal buyer is given by

$$\left(1-q_0^N\right)\mathbb{E}\left[\tilde{\theta}_i\right] + w\left(q_0^N\right) = p_0^N$$

The assumption of private values changes the behavior of the marginal buyer with respect to her decision to select the new product. With common values, the buyer is informationally small as her decision doesn't influence her of for that matter anybody's posterior belief. In contrast, with private values, each buyer is informationally large with respect to her own posterior beliefs, as only her own private experience can lead to a change in her posterior belief. This leads to the appearance of option value arguments in the calculus of the buyer as well. Notice that for a given p_1^N, a decrease in the quantity q_0^N increases the option value as a higher v leads to a larger set of experiences for which the buyer will return to the new product. While the increase in the option value would give the monopolist an incentive to decrease the supply in the initial period, a countervailing incentive is his interest to establish a sufficiently large franchise in the post-entry phase. As all buyers with types (v, θ) below the hyperbola drop out in period 1, he seeks to extend q_0^N sufficiently to generate a large base of goodwill clients for the future. More precisely, conditional on not selling to new buyers in period 1, his continuation value is increasing in current sales q_0^N. As the goodwill customers are all acquired in the initial period, we refer to this scenario as the fast entry strategy.

We contrast this with a slow entry strategy in which new customers are acquired in both periods. The intertemporal incentives to deviate from the myopic optimum are pointing in the reverse direction in this case. Conditional on selling to new customers in period 1, it is now optimal (for period 1 revenues) to leave as many customers as possible without prior experience as the idiosyncratic realizations will necessarily leave some consumers disappointed and hence induce them to drop out as potential clients.

[Insert Figure 4 here]

This will therefore induce the seller to restrict his quantity relative to the myopically optimal strategy in period 0. Notice that there is again an option value associated with purchases in period 0, but the option value is now derived from the negative outcomes in the experiment. Since the marginal buyer today would certainly buy tomorrow if she doesn't today, by making the purchase today she avoids future purchases when the product doesn't suit her needs:

$$-\int_0^{s+\frac{p_1^N}{1-q_0^N}} \left(\left(1-q_0^N\right)\theta - p_1^N\right) d\theta.$$

Again, the option value and future sales present countervailing incentives as they are the intertemporal equivalent to marginal vs. inframarginal revenues.

The arguments just presented indicate that slow and fast entry strategy introduce distinct biases for the supply in the initial period. Slow entry tailors the surplus extraction of the high valuation buyers to occur in the entry phase. Fast entry reverses the timing of the surplus extraction but also the composition of the market. The arrival of information in the entry phase leads to a narrower market in which high joint realizations of θ^i and v^i dominate. Based on these considerations on the timing and the composition of sales, we can deduce the optimal form of entry. Denote by u_1^N the mass of new and hence uninformed buyers in period 1.

Proposition 6 (Optimal Entry Strategy) *There is an $\widehat{s} > 0$ such that:*

1. *for $s < \widehat{s}$, $u_1^N > 0$ and for $s > \widehat{s}$, $u_1^N = 0$;*
2. *for $s < \widehat{s}$, $q_0^N < \frac{1}{2}$, and for $s > \widehat{s}$, $q_0^N > \frac{1}{2}$.*

P roof. See Appendix. ■

The proposition suggest that for a new and largely superior product (small s), the optimal entry strategy is to cream-skim the buyers with high willingness-to-pay and then extend the market in the subsequent period to new buyers with lower valuations. If the margin between the new product and the established product is small (large s), then it is optimal to quickly build up the goodwill market and focus exclusively on buyers with a positive experience. The switch in the strategy is accompanied by a discrete upwards jump in the initial quantity, testimony of the change in the regime. The jump occurs from a value below to one above the myopic quantity, which is given by $\frac{1}{2}$ and is independent of s. In contrast, the quantity offered in the fast entry regime is increasing in s This monotonicity is entirely due to the creation of heterogeneity by the idiosyncratic experiences. In a completely informed market the segment of agents with a positive experience but only moderate willingness-to-pay who buy the new product in equilibrium disappear quickly as s increases. This leads the monopolist to increase his quantity in the initial period so as to create a sufficiently large franchise for his product tomorrow. Notice that the marginal cost of deviating from the myopic optimum is also decreasing in s as the price the monopolist can charge today is decreasing in s. The shape of the intertemporal price path is determined by the entry rate.

Corollary 1 *The entry rate determines the intertemporal price path:*

1. $u_1^N = 0 \Leftrightarrow p_0^N < p_1^N$;
2. $u_1^N > 0 \Leftrightarrow p_0^N > p_1^N$.

The value of the outside option s can alternatively be interpreted as the marginal cost of production. In this case, the results state that lower margin products are introduced rapidly into the market whereas high margin products are introduced slowly and with substantial cream-skimming.

We conclude this section by briefly considering the incentives to innovate and the choice in the riskiness of the innovation. Consider again a mean-preserving spread of the initial distribution of experiences. The mean-preserving spread has no influence on the expected valuations in the initial period. Consider therefore its effect on the intertemporal values and policies. Higher dispersion increases the option value of the initial purchase, independently of whether the option value stems from the upside or downside of the experiment. The essential impact is on the sales in period 1. The sales pattern associated with a slow entry strategy implies that the monopolist sells more aggressively to informed customers in period 1. In other words, he will sell to buyers with experiences below the average in period 1. This contrasts with a fast entry strategy where the monopolist sells only to high experience customers. The comparison is illustrated in Fig.3 and Fig. 4. A mean-preserving spread then implies that the slow entry strategy tends to loose more customers due to the spread downwards, whereas a fast entry strategy never sold any substantial quantity to buyers with average experiences. In fact, in balance the spread tends to increase the number of customers as it adds new buyers with very positive experiences. This leads to the conclusion that when the new product is only marginally better than the current product, the monopolist prefers an innovation with more variance in the individual experiences. For a clearly superior product, the conclusion is the opposite. The optimal innovation has a small variance to minimize the heterogeneity in the experience.

4.2 Duopoly

The duopoly problem shares many similarities with the problem of the monopolist, and here we focus mostly on the differences. The setting is identical to the previous one except for the fact that the competing product of value sv^i for buyer i is now supplied by a single firm. The central question is whether the entrant attempts to acquire all goodwill buyers in period 0, or whether he proceeds slowly in the introduction of the new product. The results are formally presented for the case that the new product is in expectations superior to the safe product, or $s < \mathbb{E}\left[\tilde{\theta}^i\right]$. We briefly discuss the introduction of an (expectationally) inferior product towards the end of this section.

The essential modifications to the earlier arguments are due to strategic considerations. In the entry phase, the buyers are only differentiated along their willingness to pay. The post-entry phase on the other hand introduces a second dimension through the idiosyncratic experience. The outcomes of the individual experiences suggest a natural sorting in the market across different values of v. As individuals with a positive experience continue to adopt the new product, but individuals with a negative experience reject it, this sorting reduces the competition in the post-entry phase. The idiosyncratic experience endogenously generates an element of horizontal differentiation among all buyers with the same willingness-to-pay v. The structure of the competition in the period 1 is perhaps best illustrated by Figure 5.:

Insert Figure 5 here

For a given supply of q_1^N and q_1^S, each firm makes sales in the informed and uninformed segment. Denote by i_1^j and u_1^j the mass of informed and uniformed customers associated with firm j, where $q_1^j = i_1^j + u_1^j$. The equilibrium prices satisfy the following indifference conditions in the segment of the uninformed buyers:

$$\left(1 - q_0^N - u_1^N\right) \mathbb{E}\left[\tilde{\theta}^i\right] - p_1^N = \left(1 - q_0^N - u_1^N\right) s - p_1^S$$

and

$$\left(1 - q_0^N - u_1^N - u_1^S\right) s - p_1^S = 0.$$

The indifference condition among the informed customers is given by

$$\theta^i v^i - p_1^N = sv^i - p_1^S$$

which forms the hyperbola. The quantity of informed consumers who purchase form the safe firm is then given by the area below the hyperbola:

$$i_1^S = \int_{1-q_0^N}^{1} \left(s + \frac{p_1^N - p_1^S}{v} \right) dv,$$

and for the entrant by the complement:

$$i_1^N = q_0 - \int_{1-q_0^N}^{1} \left(s + \frac{p_1^N - p_1^S}{v} \right) dv.$$

As both firms attempt to maintain their inframarginal profits on the informed buyers, the competition for the marginal buyer with no prior experience of the new good will be less severe The cost of acquiring a new marginal buyer is therefore smaller in the post entry phase for the new firm. This change in the acquisition cost across periods leads the new firm to adopt a slow entry strategy for all s.

Proposition 7 *For all $s < \mathbb{E}\left[\tilde{\theta}^i\right]$, there exists a unique SPE such that:*

1. *$q_0^N < q_1^N$, $p_0^N > p_1^N$ and $u_1^N > 0$;*
2. *$q_0^S > q_1^S$ and $p_0^S < p_1^S$.*

P roof. See Appendix. ■

The fact that the competition becomes softer with the idiosyncratic differentiation is also documented by the fact that the average willingness to pay of the buyer associated with the safe firm is increasing and thus makes firm S less aggressive towards the marginal buyer.

It should finally be noted that the duopoly model allows for a possibility that was not present in the monopoly case. With a competitive supply of substitute products, it is never optimal to enter the market with a product whose quality is no higher than the substitute product quality for any buyer. In the duopoly case this is a possibility. Since the producer of the existing product never supplies the entire market, there is a residual demand on which even an (expectationally) inferior product may enter.

The emergence of the horizontal differentiation in the post-entry phase then suggests a reversal of the strategic incentives. The established firm will restrict its quantity in the initial period to allow the new firm to enter the market rapidly. The differential experience leads then to a sorting in the market and a reduction in the competition for the marginal buyer. The corresponding representation in the (v, θ) space is given in Figure 6:

INSERT FIGURE 6 HERE

The informed consumers in period 1 now consist of an intermediate segment in the unit interval of v. The lower and upper end don't experiment with the new product in period 0. The indifference hyperbola of the informed segment now has two parts: (i) a decreasing part and (ii) an increasing part. The increasing part is the equivalent of the indifference earlier, with the exception that

$$s + \frac{p_1^N - p_1^S}{v}$$

is now increasing in v as $p_1^N - p_1^S < 0$, as the safe firm offers the superior product or $\mathbb{E}\left[\tilde{\theta}^i\right] < s$. The decreasing part arises in the lowest segment of v where firm S doesn't sell as $vs - p_1^S < 0$, but the new firm still sells to buyers provided that their initial experience was sufficiently positive, or:

$$v^i \theta^i - p_1^N \geq 0.$$

The firms then compete in the post entry market in such a way that the safe firm makes sales to consumers who were disappointed with the new product, and the new firm sells to satisfied old customers as well as some new buyers.

The analysis in this case is complicated by the fact that the new firm sells its product to an intermediate range of buyers in the first period. This leads to two discontinuities in the (v, θ) space in the post-entry period, one at the upper end and one at the lower end. In particular, the discontinuity at the lower end creates a kink in the demand curve at the equilibrium quantities. The kink implies that there are a continuum of equilibria in period

1 differing in the market share that each firm receives in equilibrium.[10] The continuum of equilibria in period 1 creates a continuum of period 0 equilibria as well. It is therefore very hard to get tight predictions on intertemporal equilibrium behavior as well as on the comparative statics and we omit the formal analysis here.

5 Conclusion

This paper addressed the (strategic) problem of entry with experience goods in vertically differentiated markets. In a number of different market settings, we showed that the incentives for fast versus slow entry depend on the characteristics of the new product. In particular, we demonstrated that large innovations are launched slowly in the market whereas marginal ones are introduced aggressively. In the common values model, we showed that the strategic effects between leader and follower are strongest when the firms close to each other in the quality spectrum. As a result, leader and follower display opposite curvatures in their equilibrium value functions with respect to the distance between them. With aggregate uncertainty about the true quality this leads to different preferences in the speed of market entry. In consequence the equilibrium entry of an improvement is qualitatively different from the one of a substitute product. In the private values model, with idiosyncratic but no aggregate uncertainty leader and follower adopt similarly distinct strategies. In contrast to the common values, the difference in the optimal entry strategies was mostly driven by the optimal intertemporal extractions of consumer surplus by the new firm. In both of these cases, the

[10] The kink in the demand curve is due to the differential response of the price and the composition of sales relative to a marginal increase or decrease in q_1^j. To see this, consider an equilibrium $\left(q_1^N, q_1^S\right)$ and suppose that the new firm ponders a marginal change in the supply. An increase in q_1^N would leave the differential $p_1^N - p_1^S$ unchanged as the safe firm still supplies q_1^S, and so the additional quantity would have to be absorbed in its entirety by the lower end of the informed segment, leading to a downwards shift in the decreasing part of the hyperbola. In contrast a decrease in q_1^N would remove buyers from the informed as well as uniformed segment and would lead to an upward shift of the entire hyperbola.

optimal entry strategies for improvements and substitutes are empirically distinguishable from each other in terms of the implied prices as well as the quantities sold.

We assumed throughout that the sellers cannot price discriminate. In many industries, the buyers are hooked up to the suppliers either physically as in cable television or telephone services or through contracts. In this case sellers may give differential treatment to buyers depending on their past choices and experiences. An interesting extension of the current work would allow for price discrimination based on the buyers' past history, or 'behavior based' price discrimination in the terms of Fudenberg & Tirole (1997).

The analysis in this paper was simplified considerably by the assumption of a single innovation. However, the results on the incentives to innovate suggest a rich set of issues for future research in the dynamic evolution of oligopolies in the presence of product innovation. The different attitudes of follower and leader in the adoption of new products suggested an interesting behavior of the ergodic distribution of firms. We hope to report more on these issues in future research.

6 Appendix

The appendix contains the proofs to all propositions in main body of the text.

Proof of Lemma 1. (1). If $\alpha_1 \geq 0$, the equilibrium profit function $\pi_1^S(\alpha_1)$ is convex in α_1 and $\pi_1^N(\alpha_1)$ is concave in α_1 by Proposition 2. Since the distribution of θ_1 and hence the linear translation α_1 is second order stochastically decreasing in q_0^N, the claim follows from the results in Rothschild & Stiglitz (1970).

(2). The proof is analogous.■

Proof of Proposition 3. Let $m_0^j\left(q_0^k, \alpha_0\right)$ be the myopic reaction functions of firm j. Observe first that the dynamic reaction function of the safe firm is identical to the myopic reaction function: $q_0^S\left(q_0^N, \alpha_0\right) = m_0^S\left(q_0^N, \alpha_0\right)$ for all q_0^N and α_0. It is therefore continuous and downward sloping. For the existence of the equilibrium, it is sufficient to show that the reaction function of the new firm is downward sloping in the following sense: if q_0^N is a best response to q_0^S in the dynamic game, and $\hat{q}_0^S < q_0^S$, then there is a $\hat{q}_0^N \geq q_0^N$ such that $\hat{q}_0^N$ is a best response to $\hat{q}_0^S$. But this is an immediate consequence of

$$\frac{\partial^2 \Pi^N\left(q_0^S, q_0^N\right)}{\partial q_0^S \partial q_0^N} < 0.$$

(1.) By Assumption 2, $\pi_1^N\left(q_0^N\right)$ is continuously differentiable. From (2), we derive the first order necessary condition for the new firm:

$$2\left(s+\alpha_0\right) q_0^N - \pi_1^{N\prime}\left(q_0^N\right) = s\left(1-q_0^S\right) + \alpha_0.$$

Since $\pi_0^{N\prime}\left(q_0^N\right)$ is negative by the Lemma 1, any q_0^N satisfying the first order condition must satisfy:

$$q_0^N\left(q_0^S, \alpha_0\right) < m_0^N\left(q_0^S, \alpha_0\right) = \frac{s\left(1-q_0^S\right)+\alpha_0}{2\left(s+\alpha_0\right)}.$$

Also since $q_0^S\left(q_0^N, \alpha_0\right) = m_0^S\left(q_0^N, \alpha_0\right)$ for all q_0^N and α_0,all the equilibria lie on $m_0^S\left(q_0^N, \alpha_0\right)$ with $q_0^N < m_0^N$, and the claim follows.

(2.) The proof is analogous.■

Proof of Proposition 4. The equilibrium value function of the new firm before the introduction of the innovation is

$$\pi_{-}^{N}(\theta_0) = \frac{\theta_0 s^2}{(4s - \theta_0)^2}$$

if the new firm is currently a follower ($\alpha_0 < 0$) and

$$\pi_{+}^{N}(\theta_0) = \frac{\theta_0 (2\theta_0 - s)^2}{(4\theta_0 - s)^2}. \tag{3}$$

if the new firm is currently a leader ($\alpha_0 > 0$).

(1) This follows immediately from the concavity of the leader's value function (3) in his own quality.

(2) If the new firm is a follower, its payoff is increasing and convex as long as $\alpha_0 < 0$. It is continuous at $\alpha_0 = 0$ and is concave and increasing for $\alpha_0 > 0$. We first show that any arbitrary distribution $F(\theta)$ is always dominated by a two-point distribution where the lower point is θ_0. Consider the probability measure, μ_F, induced by an arbitrary distribution F on $[\theta_0, \infty)$. Define

$$\mu_F^- \equiv \mu_F I_{[\theta_0, s)}, \text{ and } \mu_F^+ \equiv \mu_F I_{[s,\infty)},$$

where I_A is the indicator function of set A. Also define

$$m^- \equiv \frac{1}{\mu_F([\theta_0, s))} \int \theta d\mu_F^- \text{ and } m^+ \equiv \frac{1}{\mu_F([s,\infty))} \int \theta d\mu_F^+.$$

Since $\pi^N(\theta)$ is convex in θ on $[\theta_0, s)$,

$$\begin{aligned} & \int \pi^N(\theta)\, d\left[\frac{\mu_F([\theta_0, s))(s - m^-)}{s - \theta_0}\delta_{\theta_0} + \frac{\mu_F([\theta_0, s))(m^- - \theta_0)}{s - \theta_0}\delta_s + \mu_F^+\right] \\ \geq & \int \pi^N(\theta)\, d\mu_F. \end{aligned}$$

Since $\pi^N(\theta)$ is concave in θ on $[s, \infty)$,

$$\begin{aligned} & \int \pi^N(\theta)\, d\left[\frac{\mu_F([\theta_0, s))(s - m^-)}{s - \theta_0}\delta_{\theta_0} + \left(1 - \frac{\mu_F([\theta_0, s))(s - m^-)}{s - \theta_0}\right)\delta_{\bar{m}}\right] \\ \geq & \int \pi^N(\theta)\, d\left[\frac{\mu_F([\theta_0, s))(s - m^-)}{s - \theta_0}\delta_{\theta_0} + \frac{\mu_F([\theta_0, s))(m^- - \theta_0)}{s - \theta_0}\delta_s + \mu_F^+\right], \end{aligned}$$

where

$$\bar{m} = \frac{1}{\mu_F([s,\infty)) + \frac{\mu_F([\theta_0,s))(m^- - \theta_0)}{s-\theta_0}} \int \theta d \left(\frac{\mu_F([\theta_0,s))(m^- - \theta_0)}{s-\theta_0} \delta_s + \mu_F^+ \right).$$

Since at least one of the inequalities is strict if the support of μ_F consists of more than two points, the claim is proved. For a given mean $\theta_1 = p\theta'$, the follower's problem is then solved by finding the unique θ' such that

$$\frac{\pi^N(\theta') - \pi^N(\theta_0)}{\theta' - \theta_0} = \pi^{N\prime}(\theta'), \tag{4}$$

which determines θ' by the curvature properties of $\pi^N(\theta)$. If the uniquely implied p displays $p > 1$, it follows that $\theta' = \theta_1$ and the follower is with probability one the leader in the next period.■

Proof of Proposition 5. Consider first the decision problem of the follower. The marginal benefit of increasing θ_1 for $p < 1$ is given by $\pi^{N\prime}(\theta')$ determined earlier by (4). The marginal benefit is therefore constant at $\pi^{N\prime}(\theta')$ until $p = 1$ beyond which $p = 1$ and all further marginal improvements are marginal and certain innovations. Consider a leader with $\hat{\theta}_0$ such that $s < \hat{\theta}_0 < \theta'$, and a follower with $\theta_0 < s$. Then by the concavity of profit function

$$\pi^{N\prime}(\theta) > \pi^{N\prime}(\theta')$$

for all θ with $s < \theta < \theta'$. Since the marginal benefit of increasing expenditure is exactly $\pi^{N\prime}(\theta')$ for the follower this together with the supermodularity of the cost function establishes the desired result.■

Proof of Proposition 6. The problem is analyzed recursively. Since the quantity of the competitive fringe is simply determined by $q_1^S = 1 - q_1^N$, we omit the superscript in the following with the understanding that $q_t^N = q_t$ as well as $p_t^N = p_t$. Consider first a given q_0. Due to the discontinuity in the (v,θ) space in period 1, it is convenient to distinguish between $u_1 = 0$ and $u_1 > 0$. We index the optimal policies and value function for $u_1 = 0$ with a lower bar and for $u_1 > 0$ with an upper bar. Consider first $u_1 = 0$. For a given price p_1 the demand is given by:

$$q_1 = q_0 - \int_{1-q_0}^{1} \left(s + \frac{p_1}{v} \right) dv$$

as the lower bound of θ for every v is determined by the equality $\theta v - p_1 = sv$. Inverting the price-quantity relation yields:

$$p_1 = \frac{q_1 - q_0 (1 - s)}{\ln (1 - q_0)}.$$

The optimal quantity is determined by maximizing

$$\pi_1 (q_1) = q_1 \frac{q_1 - q_0 (1 - s)}{\ln (1 - q_0)}$$

which leads to

$$\underline{q}_1 = \frac{1}{2} (1 - s) q_0$$

and

$$\underline{p}_1 = -\frac{1}{2} \frac{(1 - s) q_0}{\ln (1 - q_0)}. \tag{5}$$

The equilibrium profits $\underline{\pi}_1 (q_0) = \underline{p}_1 \underline{q}_1$ then is increasing and concave in q_0, and decreasing and concave in s.

Suppose next that $u_1 > 0$. For a given price p_1, the sold quantity is then composed of informed buyers:

$$i_1 = q_0 - \int_{1-q_0}^{1} \left(s + \frac{p_1}{v} \right) dv \tag{6}$$

and uninformed buyers u_1 which are determined by

$$\left(\frac{1}{2} - s \right) (1 - q_0 - u_1) = p_1. \tag{7}$$

Inverting the price-quantity relation in (6) and (7) we obtain the objective function for the monopolist:

$$\pi_1(q_1) = q_1 \left(\frac{(1 - 2s) (1 - q_1 - s q_0)}{2 - (1 - 2s) \ln (1 - q_0)} \right)$$

which yields:

$$\bar{q}_1 = \frac{1}{2} (1 - s q_0)$$

and

$$\bar{p}_1 = \frac{1}{2} \frac{(1 - 2s) (1 - s q_0)}{2 - (1 - 2s) \ln (1 - q_0)}.$$

The equilibrium profit function $\bar{\pi}_1(q_0) = \bar{p}_1 \bar{q}_1$ is decreasing and concave in q_0, and decreasing and convex in s. Thus for any given s, there exists a unique $\widetilde{q}$ such that

$$\bar{\pi}_1(\widetilde{q}) = \underline{\pi}_1(\widetilde{q}). \tag{8}$$

It follows that $\bar{q}_1$ is optimal for all $q_0 \leq \widetilde{q}$, and $\underline{q}_1$ is optimal for all $q_0 \geq \widetilde{q}$, where $\widetilde{q}$ is strictly decreasing in s. (Conversely for any given q_0, there exists a unique $\widetilde{s}$ such that $\bar{q}_1$ is optimal for all $s \leq \tilde{s}$, and $\underline{q}_1$ is optimal for all $s > \tilde{s}$.)

Consider now the optimal policy in period 0 contingent on continuation policies $\underline{q}_1$ or $\bar{q}_1$ respectively. Again we start with the aggressive policy $\underline{q}_1$. The option value of the marginal buyer in period 0 is given by:

$$\underline{w}(q_0) = \int_{s+\frac{\underline{p}_1}{1-q_0}}^{1} \left((1-q_0)(\theta - s) - \underline{p}_1\right) d\theta$$

where the lower bound is determined by

$$(1-q_0)\,\theta - \underline{p}_1 = (1-q_0)\,s$$

and $\underline{p}_1$ is as given in (5). The option value is then

$$\underline{w}(q_0) = \frac{1}{8}\frac{(q_0 + 2(1-q_0)\ln(1-q_0))^2 (1-s)^2}{\left(\ln^2(1-q_0)\right)(1-q_0)}.$$

which is decreasing in q_0. The market clearing condition in the initial period is given by:

$$(1-q_0)\frac{1}{2} + \underline{w}(q_0) - p_0 = (1-q_0)\,s$$

and the monopolist seeks to maximize:

$$\underline{\Pi}(q_0) = q_0\left((1-q_0)\left(\frac{1}{2} - s\right) + \underline{w}(q_0)\right) + \underline{\pi}_1(q_0). \tag{9}$$

Using the curvature properties of $\underline{w}(q_0)$ and $\underline{\pi}_1(q_0)$, it can be shown that (9) has a unique solution $\underline{q}_0$which is monotone increasing in s and for $s = 0$ displays $\underline{q}_0 > \frac{1}{2}$. The basic argument for this result is that as s increases the marginal cost of deviating from the myopic optimal becomes smaller and decreases more rapidly than the intertemporal benefits. It can further be verified that $\underline{q}_0 > \widetilde{q}$, so that $\underline{q}_1$ is the optimal continuation policy for $\underline{q}_0$.

Consider next the slow entry strategy. The option value of the marginal type is given by

$$\bar{w}(q_0) = -\int_0^{s+\frac{\bar{p}_1}{1-q_0}} \left((1-q_0)(\theta - s) - \bar{p}_1\right) d\theta$$

or

$$\bar{w}(q_0) = \frac{1}{8} \frac{\left(1 + 2q_0 s^2 + 2s - 5q_0 s - 2s(1-2s)(1-q_0)\ln(1-q_0)\right)^2}{\left(2 - (1-2s)\ln(1-q_0)\right)^2 (1-q_0)}$$

With slow entry, the option value for the marginal buyer is generated by early information about low experiences which allow her to switch to the safe firm in period 1. The option value is decreasing and convex in q_0 and increasing in s. The market clearing condition is as before and the monopolist seeks to maximize

$$\overline{\Pi}(q_0) = q_0 \left((1-q_0)\left(\frac{1}{2} - s\right) + \bar{w}(q_0) \right) + \bar{\pi}_1(q_0).$$

Again, using the curvature properties of $\bar{w}(q_0)$ and $\bar{\pi}_1(q_0)$, it can be shown that this problem has a unique solution which is first decreasing and then increasing in s. For all values of s, it can be shown that the unconstrained solution $\bar{q}_0$ satisfies

$$\bar{\Pi}(\bar{q}_0) > \underline{\Pi}\left(\underline{q}_0\right).$$

However the solution $\bar{q}_0$ becomes eventually constrained by the requirement that $u_1 > 0$ is the optimal continuation policy for $\bar{q}_0$, or that

$$\bar{\pi}_1(\bar{q}_0) \geq \underline{\pi}_1(\bar{q}_0).$$

More precisely there is an $\widetilde{s}$ such that $\bar{q}_0 = \widetilde{q}$ for all $s > \widetilde{s}$ where $\bar{q}_0$ is determined as the solution to the equation (8). As $s \to \frac{1}{2}$, $\widetilde{q} \to 0$ and it follows that there is an s such that $\underline{q}_0$ becomes optimal. It can further be shown that there is at most one such switch as an application of the envelope theorem shows that for $s > \widetilde{s}$:

$$\frac{\partial \bar{\Pi}(\widetilde{q}(s), s)}{\partial s} + \frac{\partial \bar{\Pi}(\widetilde{q}(s), s)}{\partial \widetilde{q}} \frac{\partial \widetilde{q}(s)}{\partial s} < \frac{\partial \underline{\Pi}\left(\underline{q}_0(s), s\right)}{\partial s},$$

where the second term in the inequality is due to the fact $\bar{q}_0$ is restricted by the equality (8) to $\bar{q}_0 = \widetilde{q}$, which concludes the proof.■

Proof of Proposition 7. The set-up and the proof strategy is as before. We briefly describe the equilibrium conditions, analyze the properties of the equilibrium, but omit the entirely standard but tedious calculus arguments. Again, it is convenient to distinguish between $u_1^N = 0$ and $u_1^N > 0$, and we start with $u_1^N = 0$. The equilibrium quantity q_1^N for any given price differential $p_1^N - p_1^S$ is given by:

$$q_1^N = q_0^N - \int_{1-q_0^N}^{1} \left(s + \frac{p_1^N - p_1^S}{v} \right) dv.$$

Inverting the price-quantity relation:

$$p_1^N - p_1^S = \frac{q_1^N - q_0^N (1-s)}{\ln \left(1 - q_0^N\right)} \tag{10}$$

and the equilibrium prices are determined by (10) and

$$\left(1 - q_1^N - q_1^S\right) s = p_1^S. \tag{11}$$

The firms $j \in \{N, S\}$ then maximize:

$$\pi_1^j \left(q_1^j\right) = p_1^j q_1^j.$$

subject to (10) and (11). The Nash equilibrium in quantities conditional on $n_1^N = 0$ is unique and the equilibrium prices are

$$\underline{p}_1^N = \frac{\left(s \ln \left(1 - q_0^N\right) - 1\right) \left(s \ln \left(1 - q_0^N\right) - 2q_0^N (1-s)\right)}{\left(3s \ln \left(1 - q_0^N\right) - 4\right) \ln \left(1 - q_0^N\right)} \tag{12}$$

and

$$\underline{p}_1^S = \frac{\left(s \ln \left(1 - q_0^N\right) + q_0^N (1-s) - 2\right) s}{3s \ln \left(1 - q_0^N\right) - 4}. \tag{13}$$

The resulting equilibrium value function $\underline{\pi}_1^N \left(q_0^N\right)$of N is increasing and concave in q_0^N.

The equilibrium conditions associated with $u_1^N > 0$ are likewise derived as in the monopoly case with the obvious exception that now $p_1^S > 0$. Again there is an equilibrium, which is unique conditional on $u_1^N > 0$. The equilibrium prices are

$$\bar{p}_1^N = \frac{\left(1 - s(1-2s)\ln\left(1-q_0^N\right)\right)\left(2(1-s) - s(1-2s)\left(2q_0^N + \ln\left(1-q_0^N\right)\right)\right)}{\left(2(2-s) - 3s(1-2s)\ln\left(1-q_0^N\right)\right)\left(2 - (1-2s)\ln\left(1-q_0^N\right)\right)}. \quad (14)$$

and

$$\bar{p}_1^S = \frac{\left(s(1-2s)\ln\left(1-q_0^N\right) - (1-2s)q_0^N s - 1\right)s}{\left(2(2-s) - 3s(1-2s)\ln\left(1-q_0^N\right)\right)}. \quad (15)$$

With the period 1 continuation values established, consider now period 0 equilibrium conditions. For $u_1^N = 0$, the option value for the marginal buyer is as in the monopoly case (with the exception of $p_1^S > 0$ instead of $p_1^S = 0$) given by:

$$\underline{w}\left(q_0^N\right) = \int_{s+\frac{\underline{p}_1^N - \underline{p}_1^S}{1-q_0^N}}^{1} \left(\left(1-q_0^N\right)(\theta - s) - \underline{p}_1^N + \underline{p}_1^S\right) d\theta,$$

with $\underline{p}_1^N$ and $\underline{p}_1^S$ as in (12) and (13), or

$$\underline{w}\left(q_0^N\right) = \frac{\left((1-s)\left(1+q_0^N\right) - \underline{p}_1^N + \underline{p}_1^S\right)^2}{2\left(1-q_0^N\right)}.$$

The indifference conditions in period 0 are

$$\left(1-q_0^N\right)\mathbb{E}\left[\tilde{\theta}^i\right] + \underline{w}\left(q_0^N\right) - p_0^N = \left(1-q_0^N\right)s - p_0^S,$$

and

$$\left(1 - q_0^N - q_0^S\right)s - p_0^S = 0.$$

The firms $j \in \{N, S\}$ are maximizing the intertemporal objective function:

$$\underline{\Pi}^j\left(q_0^j\right) = p_0^j q_0^j + \underline{\pi}_1^j\left(q_0^N\right) \quad (16)$$

with $\underline{\pi}_1^j\left(q_0^N\right)$ resulting from (12) and (13). The first order conditions resulting from (16) are:

$$\frac{1}{2} - \frac{1}{2}q_0^S - q_0^N + \underline{w}\left(q_0^N\right) + q_0^N \underline{w}'\left(q_0^N\right) + \underline{\pi}_1^{N\prime}\left(q_0^N\right) = 0$$

and

$$s - sq_0^N - 2sq_0^S = 0,$$

We observe that the intertemporal payoff elements $\underline{w}\left(q_0^N\right)$ and $\underline{\pi}_1^N\left(q_0^N\right)$ only appear in the first order conditions of the new firm. Moreover $\underline{q}_0^N \underline{w}\left(q_0^N\right) + \underline{\pi}_1^N\left(q_0^N\right)$ is increasing and concave in q_0^N. The derivation of the slow entry equilibrium conditions is identical with the exception of the option value which is given by:

$$\bar{w}\left(q_0^N\right) = \frac{\left(s\left(1 - q_0^N\right) + \bar{p}_1^N - \bar{p}_1^S\right)^2}{2\left(1 - q_0^N\right)}.$$

The equilibrium is otherwise identical to before.

We now argue for the uniqueness of the slow entry equilibrium. Again we denote the continuation payoff associated with the slow and fast entry equilibrium by upper and lower case. We observe first that for all q_0^N,

$$q_0^N \bar{w}\left(q_0^N\right) + \bar{\pi}_1^N\left(q_0^N\right) > q_0^N \underline{w}\left(q_0^N\right) + \underline{\pi}_1^N\left(q_0^N\right), \tag{17}$$

and

$$\bar{w}\left(q_0^N\right) + q_0^N \bar{w}'\left(q_0^N\right) + \bar{\pi}_1'^N\left(q_0^N\right) < \underline{w}\left(q_0^N\right) + q_0^N \underline{w}'\left(q_0^N\right) + \underline{\pi}_1^{N\prime}\left(q_0^N\right), \tag{18}$$

We argue by way of contradiction. Suppose that there exists an equilibrium such that $q_0^N = \underline{q}_0^N$. The inequality (18) implies that $\underline{q}_0^N > m_0^N$ and $\underline{q}_0^S < m_0^S$. But then by (17) there exists another q_0^N such that $q_0^N = \bar{q}_0^N$ and $q_0^N < \underline{q}_0^N$ such that the intertemporal benefits as collected in (17) are at least as high as at $\underline{q}_0^N$. Since the new q_0^N is smaller by (18), it also increases the net payoff in period 0 which is less biased away from the myopic best response, which leads to the contradiction.■

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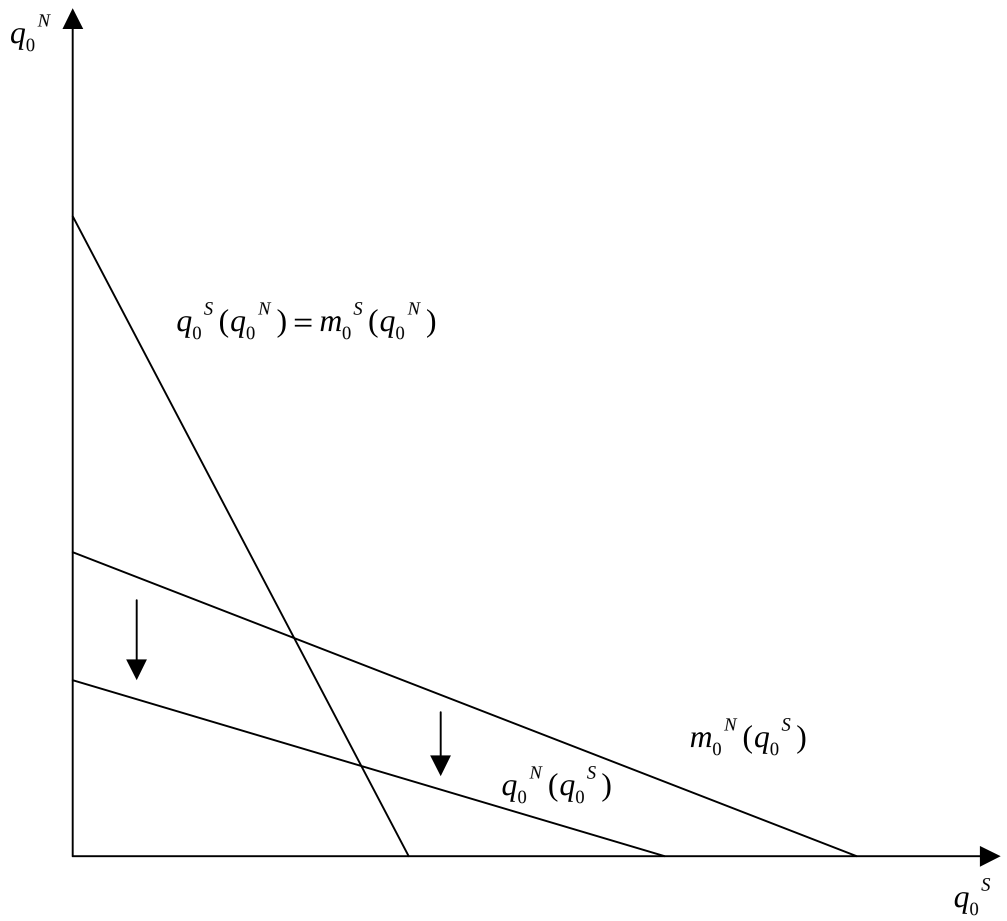

Figure 1.: Myopic vs. Dynamic Best Response (in Quantities)

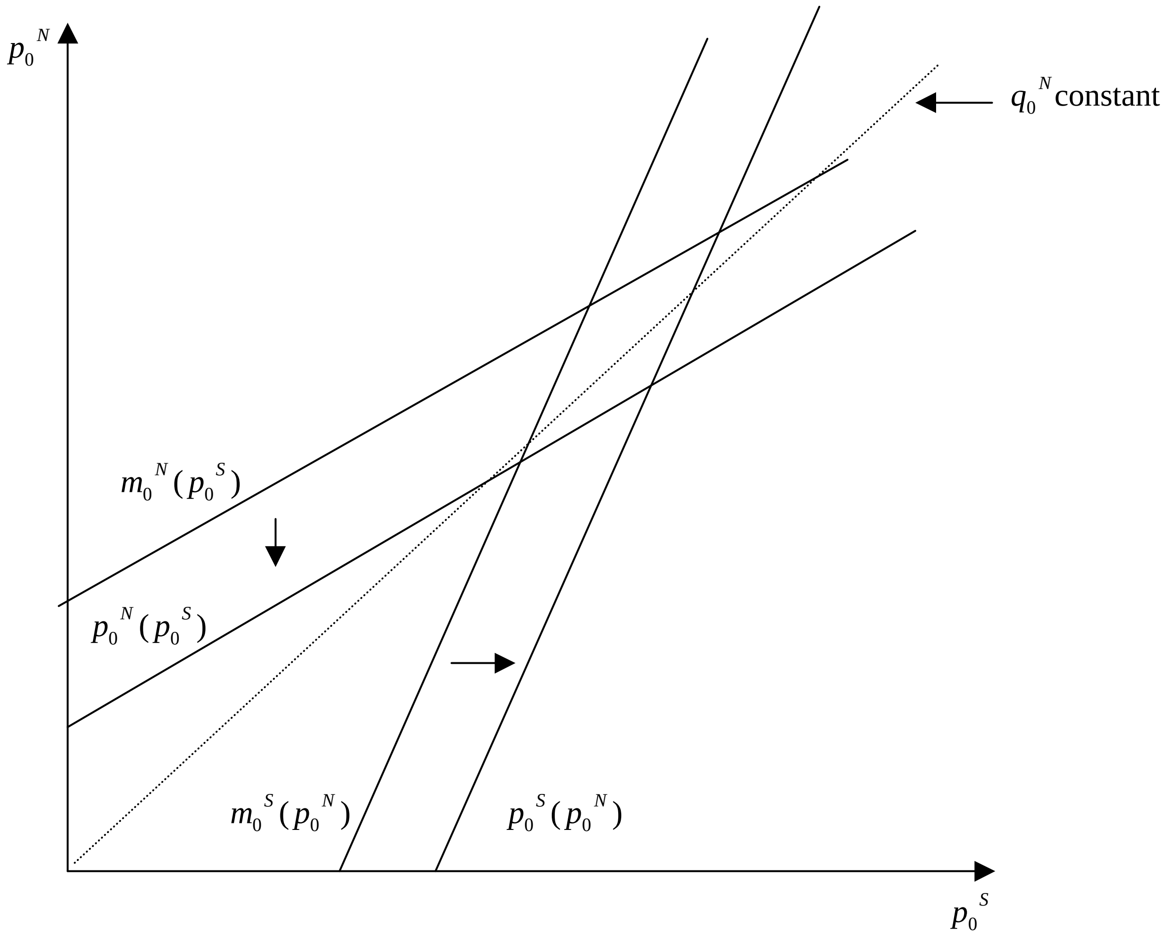

Figure 2.: Myopic vs Dynamic Best Response (in Prices)

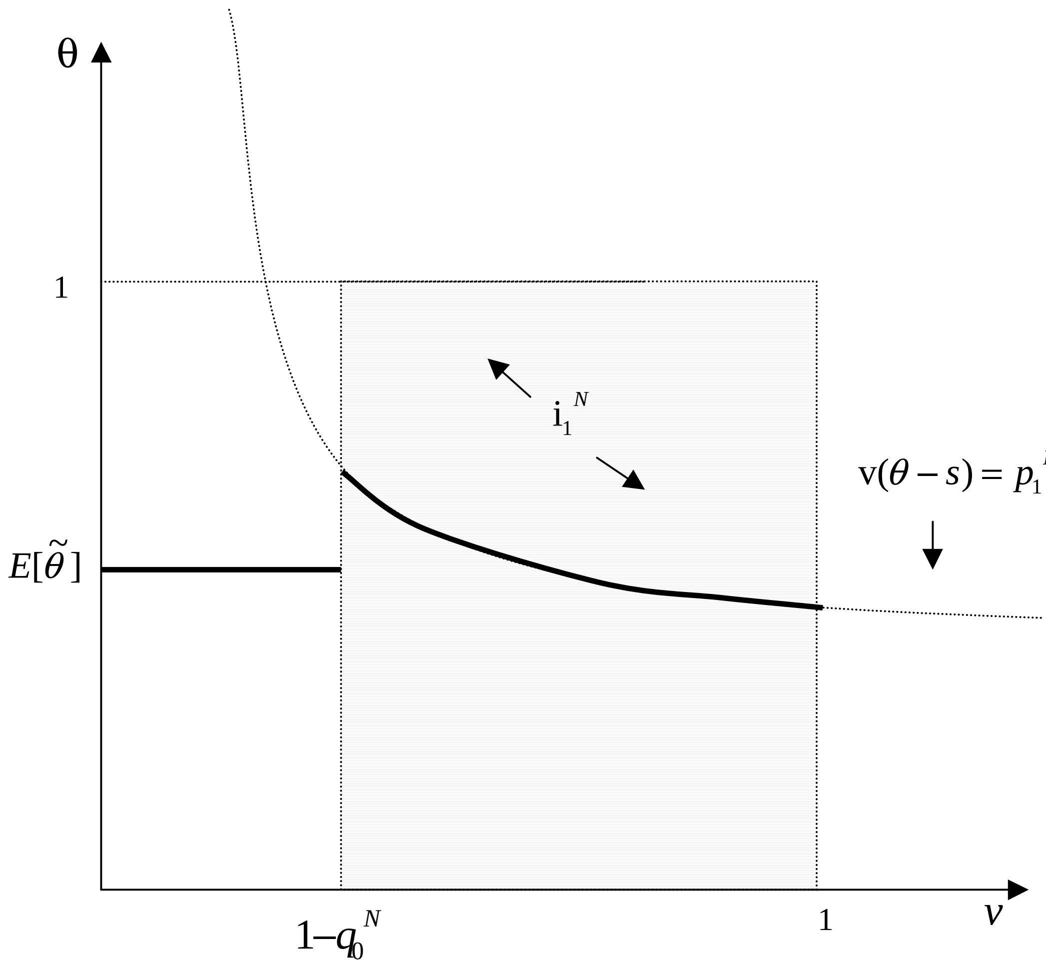

Figure 3: Fast Entry ($u_1^N = 0$).

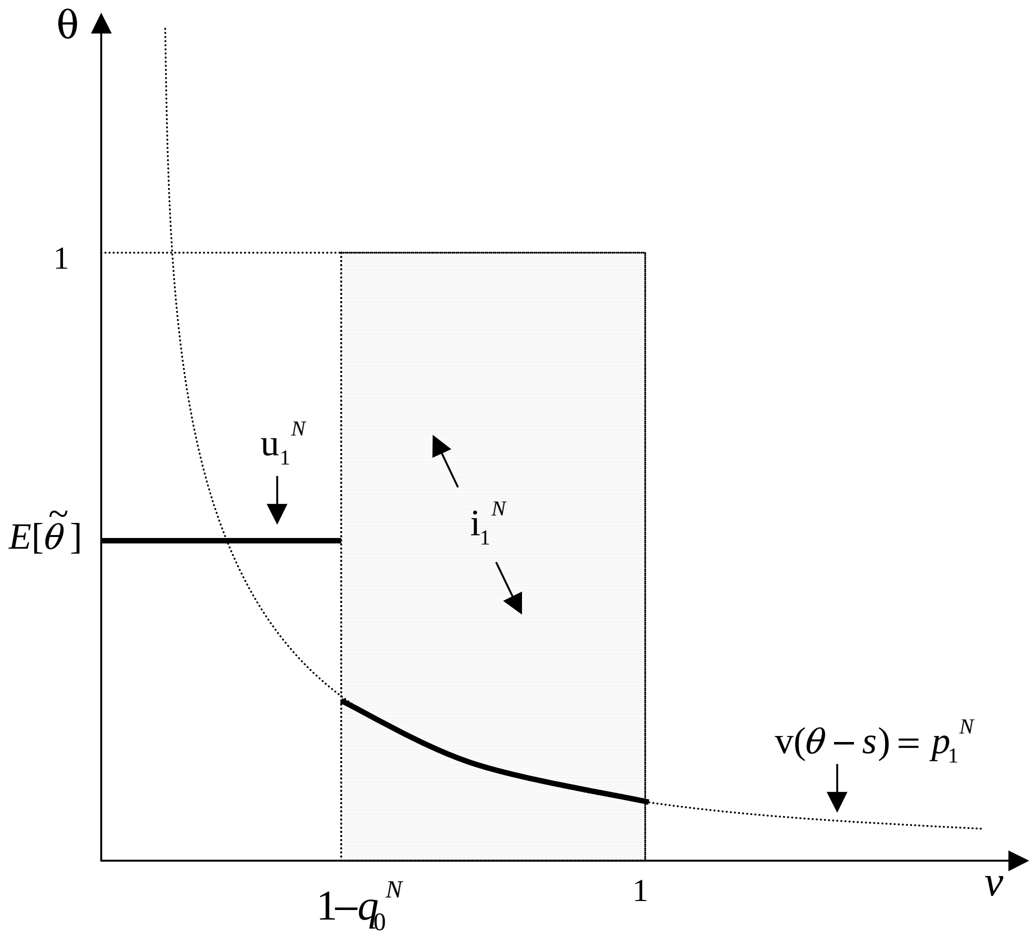

Figure 4: Slow Entry $(u_1^N > 0)$.

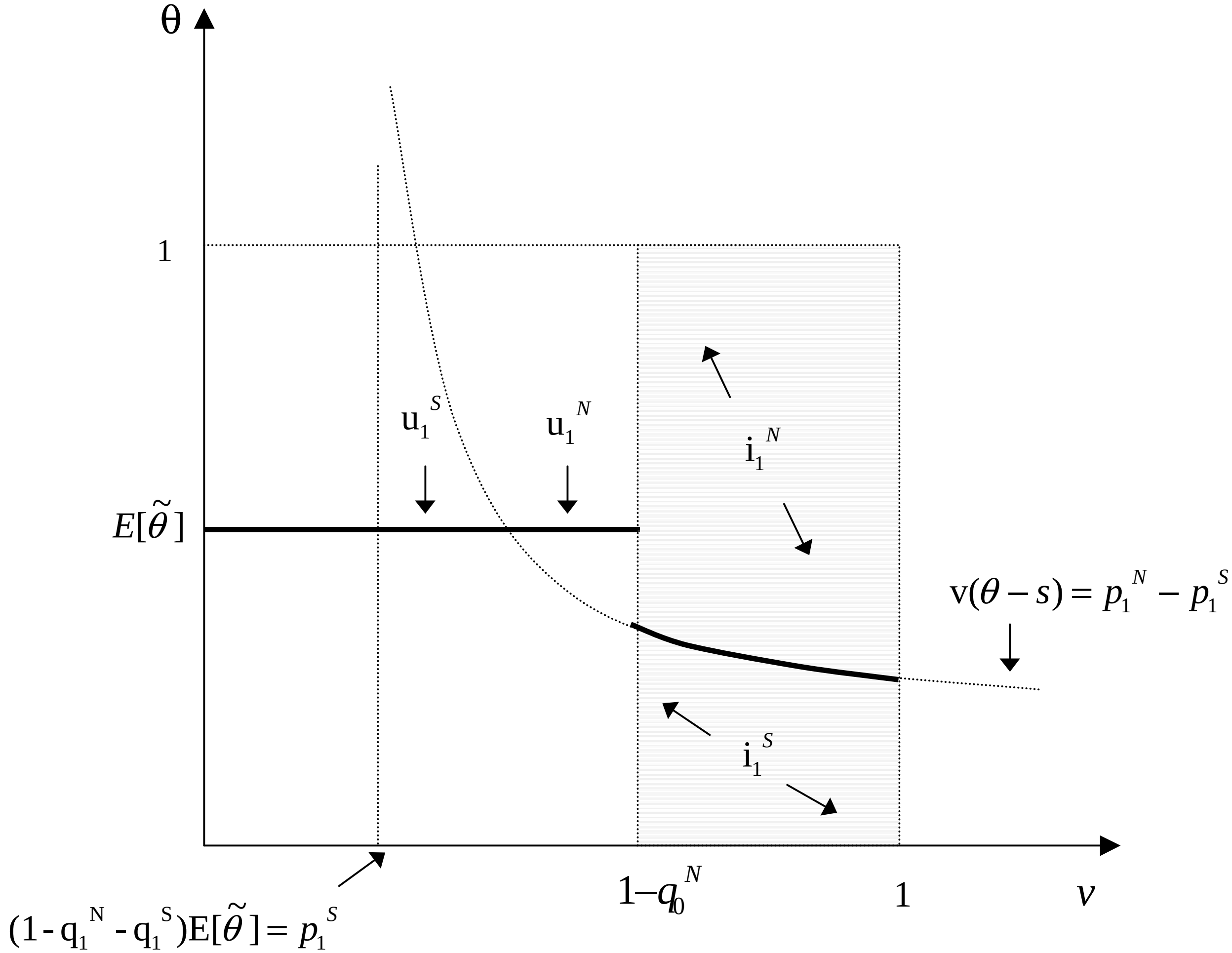

Figure 5: Slow Entry $(u_1^N > 0)$ in Duopoly.

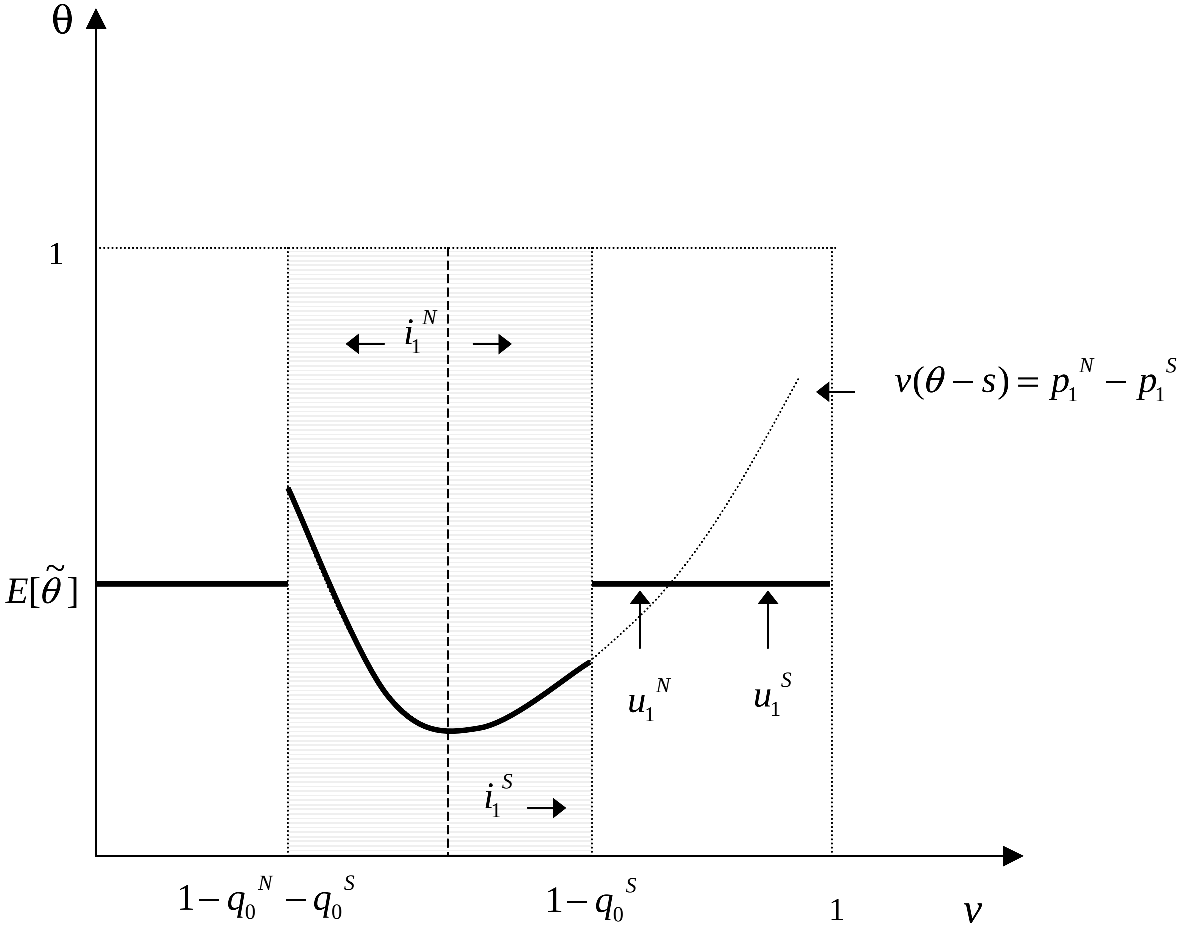

Figure 6: Intermediate Entry in Duopoly