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Abstract

We show that in a large production economy, the cost of collecting the information required by a planner to set nearly optimal prices is negligible relative to the total output of the economy. The cost of collecting the information required to set a nearly optimal production plan for each firm in the economy is not negligible. This conclusion stands in contrast to common opinion that determining optimal prices requires as much information as determining an optimal plan.

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ORGANIZING PRODUCTION IN A LARGE ECONOMY WITH COSTLY COMMUNICATION

Paul R. Milgrom and Robert J. Weber[1]

Economists have long been interested in the problem of coordinating the production activities of the many diverse but interdependent firms in a large economy. The market system, which coordinates these activities indirectly using prices, works well in neoclassical production environments if the proper prices can be determined. In principle, a socialist "command" system could also work well if the planner were fully and perfectly informed about the technological capabilities of the firms and if there were no motivational or monitoring problems to overcome.[2] Under such idealized conditions, command systems and price-oriented systems are equally effective; both are capable of producing optimal outcomes according to whatever criterion is deemed appropriate. Any comparison of a command system with a price-oriented system must therefore be based on how the two systems perform when the planner lacks complete information or on how well the systems motivate firms and managers to advance social objectives.

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[2]We use "planner" to refer to the central authority in a planned economy, "firm" to refer to a productive unit, and "optimal" to describe a plan or allocation that maximizes the planner's objective function in a socialist economy or that is Pareto optimal in a market economy. In a "command" system, the planner instructs each firm concerning what to produce and which inputs to use.

In the economic planning literature, it is usually assumed that the planner lacks information about the productive capabilities of any individual firm; that information is directly available only to the firm's manager. The planning process itself proceeds in stages. At the first stages, the planner communicates with the firms to gather the information that will be used in determining the plan. At the final stage, he communicates instructions to the firms concerning what is to be produced and by what means. These instructions can take a great variety of forms. In a command economy, the planner specifies detailed goals, quotas, or production plans for each firm separately. An alternative approach is to set prices that guide firms' choices. The planning model in which the planner sets prices differs from standard market models in its focus on the precise process by which prices are determined.

There is some confusion in the economics literature concerning whether a price-oriented planning system has any advantages over a command system in terms of economizing on information. In a study of iterative planning algorithms, Marglin "expose(s) as a myth the conventional idea that price systems economize greatly on calculation and flows of information relative to command systems in searching for an optimal allocation of resources."[3] Weitzman put the matter this way:

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[3]S.A. Marglin [1969], "Information in Price and Command Systems of Planning," in J. Margolis and H. Guitton (eds.), Public Economics: An Analysis of Public Production and Consumption and their Relations to the Private Sectors.

> A reason often cited for the theoretical superiority of prices as planning instruments is that their use allegedly economizes on information. The main thing to note here is that generally speaking it is neither easier nor harder to name the right prices than the right quantities because in principle exactly the <u>same</u> information is needed to correctly specify either.[4]

The final sentence in the Weitzman quotation is not quite correct. In general, setting the right prices requires only information about the average production opportunities available in the economy. Such information is <u>aggregate</u> information; knowledge of the average production set does not require detailed knowledge of the production set of each individual firm. Moreover, in practice a planner can never hope to name the right prices or quantities precisely. To specify approximately correct quantities in a command system still requires information concerning the production sets of essentially all of the individual firms. However, estimation of the aggregate information required to operate a price-oriented planning system can be accomplished through sampling.

In an economy with many small firms, if the firms' production sets are subject to independent perturbations between the time the plan is formed and the time production is completed, or if identifying and communicating the production possibilities of individual firms to a planner is costly, then a system of decentralization using prices can prove superior to a command system. In the first case, the information needed to operate a command system efficiently is unavailable. In the second, it is more expensive than

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[4] Martin L. Weitzman, "Prices vs. Quantities," <u>Review of Economic Studies</u>, 41, page 478. (477-491)

the information needed to determine optimal prices. The first of these points is obvious and the second is developed in the formal model that follows.

The Formal Model

Consider an economy with J firms and ℓ commodities. There are k possible technologies, represented by k strictly convex, compact subsets of R^{ℓ}: $T_1,\ldots,T_k$. The production possibilities can also be described by supply functions $S^1,\ldots,S^k$ which map the price simplex Δ^{ℓ} into R^{ℓ}, where $S^i(p)$ maximizes profit at the prices p over the set T_i. In view of the compactness and strict convexity of the production sets, these supply functions are well-defined, bounded, and continuous.

The supply function for a firm j is denoted by S_j. Thus, $S_j(p)$ is the profit maximizing production plan for firm j when the prices are p. Firm j's supply function coincides with one of the S^i's and is assumed to be unknown to the planner.

Let $c \in R^{\ell}_+$ represent the cost (in terms of resources expended) of gathering and transmitting detailed information about any firm to the planner. The cost might arise from the need to monitor operations at the firm with unusual precision, to plan operations ahead of time that are usually coordinated at the last moment, to gather information earlier than it usually becomes available in the production process, to evaluate a wider range of alternatives than is ordinarily necessary, to verify, collate, summarize and communicate the information, or from any of many related sources.

The planner's von Neumann-Morgenstern utility when he receives information from n firms, chooses the price vector p, and the actual supply functions are $S_1,\ldots,S_J$, is

$$U\{[-nc + S_1(p) + \ldots + S_J(p)]/J\}.$$

Three things are noteworthy about this expression. First, the planner's utility depends on average output net of communication costs. Intuitively, U depends on per capita consumption. The size of firms is held fixed in our model but larger economies contain larger numbers of firms. Second, our model is not a Bayesian one, so we treat the supply functions as unknown parameters rather than as random variables. The n firms with whom communication takes place are assumed to be selected at random, and the planner's choice of p depends on the information communicated to him, so p is a random variable whose distribution depends on the actual supply functions of the firms in the economy. Third, the model formulated here makes no explicit allowance for the possible inconsistency of the firms' plans. This can be handled adequately in the specification of U is the frasible set is "nice." For example if plans are feasible whenever $\Sigma S_i(p) \epsilon A$ where A is the closure of its interior the planner's actual objective function U can, if continuous, be approximated by a continuous function which is a large negative constant off A and which coincides with U on A except near the boundary of A.

Proposition. Let U be a continuous function on a compact convex set containing the sets $T_1, \ldots, T_k$ in its interior. Then there exists a price-oriented planning procedure such that the maximum difference over all k^J possible technological environments between the planner's expected utility from the procedure and the full information optimum utility goes to 0 as J goes to infinity.

Proof. It suffices to display one such procedure. Let $n = O(J^{\frac{1}{2}})$, so that n goes to infinity with J but n/J goes to zero.

Let the planner sample randomly with replacement and let $N_1, \ldots, N_k$ be the (random) numbers of firms with technologies $T_1, \ldots, T_k$ found in the sample. Define $\hat{S} = (N_1 S^1 + \ldots + N_k S^k)/n$. $\hat{S}(p)$ is an estimate of the output per firm that would result if prices p are announced.

If the actual proportions of firms with each of the k technologies are $q_1,\ldots,q_k$, then the actual production per firm if the price vector p is announced will be $S(p) \equiv q_1 S^1(p) + \ldots + q_k S^k(p)$. Both S and $\hat{S}$ lie in $\mathcal{S}$, the convex hull of $\{S^1,\ldots,S^k\}$. The set $\mathcal{S}$ is a family of equicontinuous, uniformly bounded functions.

Consider, for fixed p, the properties of the random vector $\hat{S}(p) - S(p)$. For any $q = (q_1 \ldots, q_k)$ and p, this random vector has expectation zero and $E[||\hat{S}(p) - S(p)||^2] = \sum_{j=1}^{k} q_j ||S^j(p) - S(p)||^2/n$. Since the numerator is a continuous function (of (q,p)) on a compact set, it has an upper bound B, so $E[||\hat{S}(p) - S(p)||^2] \le B/n$, for all p and q. It then follows from the equicontinuity and boundedness of the functions in $\mathcal{S}$, the compactness of the price simplex Δ, and the uniform continuity of U on the compact set $\{S(p)|S\in\mathcal{S},\ p\in\Delta\}$ that, uniformly in q, $E[\sup||\hat{S}(p) - S(p)||]$ and $E[\sup_p|U(\hat{S}(p)) - U(S(p))|]$ converge to zero as n tends to infinity.

Let the planner choose p to maximize $U(S(p))$, and let $\sim$ denote asymptotic equality as J, and therefore n, become large. Then the realized expected utility for any production environment will be

$$
\begin{aligned}
E[U(S(\hat{p}) - nc/J)] &\sim E[U(S(\hat{p}))] \\
&\sim E[U(\hat{S}(\hat{p}))] \\
&= E[\max_p U(\hat{S}(p))] \\
&\sim \max_p U(S(p))
\end{aligned}
$$

uniformly in q, and so uniformly in the technological environments, as was to be proved.

Since the proof proceeds by studying the quality of the approximation of the actual per capita supply function S by the "estimated per capita supply function," $\hat{S}$, where distance is measured in terms of the sup-norm, it is clear that the proposition can be extended to cover the case where the set of possible supply functions is compact in the sup-norm topology. Thus, instead of assuming a finite set of supply functions, we could have assumed that the set of possible supply functions was equicontinuous and uniformly bounded. The proof would then proceed by approximating that set of functions by a finite set and then applying the proposition given above.

Discussion

It is interesting to compare our results with those obtained by Weitzman in the paper cited above. Weitzman's analysis, which takes the planner's information as a given, concludes that decentralization by prices is advantageous when the marginal benefit curve is flatter than the marginal cost curve, so that the planner can better estimate the marginal cost of output at the optimum than the quantity to be produced at the optimum. The command system is relatively advantageous when the reverse is true. For example, one calls a single ambulance to retrieve a heart attack victim, without regard to marginal cost. One does not call two, even if ambulances are very cheap. Weitzman argues that, with fixed coefficients and some fixed resources in the short run, command systems will usually be preferred to a price-decentralized system.

Our analysis allows the planner to gather information at a cost, and finds that when there are many small firms, optimal prices can be estimated cheaply. When the social value function and the supply functions are continuous, these estimated prices lead to nearly optimal outcomes. And, as we argued earlier, when the conditions of production change between the time the plan is determined and the time that production is completed, the price system guides the substitutions among inputs that firm managers must make.

It seems clear that the importance of the effect we have described depends on the timing and availability of information for the planner, the possibilities for substitution in short and medium run production, and the number of firms producing each output. The problem of optimally organizing production is a complicated one, and remains fertile ground for future research.