COWLES FOUNDATION FOR RESEARCH IN ECONOMICS

AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut 06520

COWLES FOUNDATION DISCUSSION PAPER NO. 361

PRODUCTION THEORY AND THE STOCK MARKET

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June 4, 1973

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*This work was supported by National Science Foundation Grant GS-2874-A1 to the Institute for Mathematical Studies in the Social Sciences, Stanford University and by grants from the National Science Foundation and the Ford Foundation at Yale University. The author expresses his gratitude to Karl Shell, Robert Wilson, William Sharpe, and Joe Stiglitz, who not only made valuable suggestions, but listened with remarkable patience to earlier versions of this work. Peter Diamond, William Brainard, and John Lintner also provided valuable insights.

An earlier version of this paper appeared as "Stockholders' Interests, Value Maximization, and the Optimality of Stock Markets," Working Paper No. 20, Institute for Mathematical Studies in the Social Sciences, Stanford University.

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ABSTRACT

Traditional economic models separate firms' production decisions from equilibrium in stock markets. In this paper, we develop an integrated model of production in the presence of capital asset market equilibrium. Our theory indicates that, in a stochastic environment, production and financial variables are inextricably interrelated.

Following the financial equilibrium models of Sharpe [13], Lintner [10], and Mossin [11], we assume that profits and therefore portfolio returns are random. But stockholders can alter their distributions of returns by altering firms' production decisions as well as by altering their portfolios. The key to the analysis is a "unanimity theorem," which shows that in many environments stockholders will agree on optimal output decisions, despite their different expectations and attitudes towards risk.

We develop equilibrium conditions which must be satisfied by production decisions. Profit maximization is indeed optimal for a firm whose profits are riskless. But risky firms' outputs depend on financial as well as cost variables, and the equilibrium conditions lead to a theory of production under uncertainty which replaces the now-vacuous notion of profit maximization. We further show that the output decisions will be Pareto optimal for stockholders, and that these decisions maximize market value only in a "purely competitive" world. Our results provide a synthesis of the conflicting conclusions of Diamond [4], Stiglitz [14], and Wilson [17], [18] on the optimality of stock markets.

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Theories of production and of finance have developed along remarkably independent paths. "The Stock Market" is not an essential element in neoclassical production models. And until recently, production has not appeared in models of capital asset market equilibrium.[1]

Neoclassical production models have not needed stock markets because profit maximization provides a complete description of firm behavior under certainty. Stockholders are invoked to justify profit maximization, but thereupon are hastily retired from the scene. Indeed, the existence of the stock market has been somewhat embarrassing, since traditional economic models are complete without it.

The neoclassical approach breaks down, however, in the presence of uncertainty. The assumption of profit maximization no longer ranks alternative decisions, since profits are not uniquely determined by firms' actions. Production theory, and the welfare propositions constructed thereupon, must be reconstructed when the economic environment is stochastic.[2]

Extensions of traditional theory have been suggested to explain production decisions under uncertainty. The models of Sandmo [12], Lintner [9], and Leland [8] suggest that firms facing random prices act to maximize expected utility of profit. But in an environment in which owners possess differing attitudes towards risk as well as differing expectations, whose utility function and subjective probabilities are to be used?

Models of financial equilibrium, in contrast, have explicitly recognized the random character of profits, and the ownership of firms by diverse

stockholders. But while the pioneering capital asset models of Sharpe [13], Lintner [10], and Mossin [11] have helped explain optimal portfolios and relative values of risky assets, they have shed no light on production, since these decisions are presumed fixed.

Not until Diamond's notable paper [4] were production decisions examined in the context of stock market equilibrium. Subsequent papers by Stiglitz [14], Jensen and Long [7], and Fama [6] have questioned the Pareto optimality of stock markets in the allocation of investment. All those studies have assumed firms maximize market value. Yet Wilson [17], using a variant of Stiglitz's model, shows that every stockholder would recommend production decisions which do not maximize market value. Wilson concludes that what is to be questioned is not the optimality of stock markets, but rather the presumption of value maximization by firms.[3]

We shall build on an element underlying both finance and production theories: firms should operate "in the stockholders' interests." Production theories have interpreted this principle to imply profit maximization. Financial studies have interpreted it to imply value maximization. But neither approach has rigorously justified its interpretation, and Wilson's work seriously questions the validity of value maximization.

We develop below a model with production and with a stock market. Investors have diverse expectations and attitudes towards risk, and choose optimal portfolios given (equilibrium) market values. Firms face random prices for their products and must select outputs. These outputs are assumed to be selected in the stockholders' interests. A key to our analysis is the proof that, in competitive stochastic environments, outputs selected by firms in equilibrium will be in all stockholders' interests--regardless

of their expectations or willingness to bear risk. The chosen outputs are Pareto optimal for stockholders, but in general will not maximize market value. We can show, however, that in certain "purely competitive" environments, firms acting as if they maximized market value will make choices which coincide with stockholders' interests. The contrasting results of Diamond, Stiglitz, and Wilson are readily explicable by our theory.

Outputs chosen by firms will depend upon both production (cost) and financial (market value) variables. Thus, our theory indicates the stock market and production are inextricably connected under uncertainty. For competitive firms facing nonrandom prices, stockholders will choose profit maximizing policies ($p = MC$). For firms facing random prices, stockholders select outputs which behave like certainty outputs in some ways, but differently in others.

Our results, hopefully, point the way to further integration of the theories of production and finance, and to the reconstruction of value and welfare theory in stochastic environments. While we do not undertake empirical testing of the theory of this paper, it might be noted that our results do not require estimates of subjective probabilities for testing. In contrast with most other economic models under uncertainty, we need know only equilibrium market values, rather than the subjective probabilities which they reflect.

I. Firms Facing Random Prices

We consider an economy with firms indexed $j = 0, \ldots, N$. A firm has a profit function relating profit π^j to the vector of decisions of the firm d^j and the unknown state of nature θ:

$$\pi^j = \pi^j(d^j, \theta) \,.^4 \tag{1}$$

Randomness could enter from any or all of several factors, including uncertain demand or uncertain technology. In the present section, we limit ourselves to firms similar to the competitive models of Sandmo [12] and Baron [2], where selling price is random but cost functions are known. In this environment,

$$\pi^j = p^j(\theta)q^j - C^j(q^j) \,, \quad j = 0, \ldots, N \,, \tag{2}$$

where q^j is the (scalar) output decision of the j^{th} firm. Each firm owns certain resources essential to production, such as capital, "goodwill," or technical expertise. $C^j(q^j)$ reflects the costs incurred by the hiring of further factors of production, and as such could be viewed as a "variable cost" function. We assume there exists a firm which faces a certain price:[5]

$$\pi^0 = \bar{p}^0 q^0 - C^0(q^0) \,. \tag{3}$$

Subsequent sections consider noncompetitive environments, multiple decision variables, and technological uncertainty.

Firms issue shares, which represent a claim to profits, such that the ownership of a fraction s_i^j of the shares of firm j entitles the i^{th} stockholder to a return $s_i^j\pi^j$. We denote the current total market value of the j^{th} firm's shares by $\bar{V}^j$, and define $\bar{V} = (\bar{V}^0, \ldots, \bar{V}^N)$.[6]

II. Investors

We assume there are M investors, indexed $i = 1, \ldots, M$. Investors are described by three attributes:

(a) $\bar{s}_i = (\bar{s}_i^0, \ldots, \bar{s}_i^N)$, the current portfolio measured in fractions of each firm's stock, owned by investor i. We assume $\sum_i \bar{s}_i^j = 1$, $j = 0, \ldots, N$.

(b) $U_i(R_i, \theta)$, i's utility function, unique to a linear transformation, which gives the utility of return R_i in state θ. We assume $U_i'(R_i, \theta) \equiv \partial U_i(R_i, \theta)/\partial R_i > 0$ (nonsatiation); $U_i''(R_i, \theta) \equiv \partial^2 U_i(R_i, \theta)/\partial R_i^2 < 0$ (concavity in R_i).

(c) μ_i, the i^{th} investor's subjective probability measure over the states of nature.[7] E_i is the expectation with respect to μ_i.

Investors are assumed to rank alternatives by expected utility.[8] Given arbitrary market values $V = (V^0, \ldots, V^N)$ of firms' stocks, we can define budget sets $B(V, \bar{s}_i)$ for each investor:

$$B(V, \bar{s}_i) = \{s_i \in R^{n+1} \mid \sum_{j=0}^{N} V^j(s_i^j - \bar{s}_i^j) \leq 0\} . \tag{4}$$

Clearly $B(V, \bar{s}_i)$ gives the set of portfolios which an investor could afford when market values are V and his current portfolio is $\bar{s}_i$. Nonsatiation implies that <u>optimal</u> portfolios will be restricted to the subset of $B(V, \bar{s}_i)$ such that

$$\sum_{j=0}^{N} V^j(s_i^j - \bar{s}_i^j) = 0 , \quad \text{for all } i . \tag{5}$$

Following Diamond [4] and Stiglitz [14], we define R_i , the return to a portfolio s_i , as

$$R_i = \sum_{j=0}^{N} \pi^j s_i^j , \tag{6}$$

or substituting for s_i^0 from (5),

$$R_i = r \sum_{j=0}^{N} V^j \bar{s}_i^j + \sum_{j=1}^{N} (\pi^j - rV^j) s_i^j , \tag{7}$$

where $r \equiv \pi^0(q^0)/V^0$, the return on the riskless asset.[9]

III. Financial Equilibrium

Portfolios $\hat{s}_i = (\hat{s}_i^0, \ldots, \hat{s}_i^N)$, $i = 1, \ldots, M$, and market values $\hat{V} = (\hat{V}^0, \ldots, \hat{V}^N)$ constitute a financial equilibrium relative to q if

(a) For each investor i , $\hat{s}_i$ is optimal in $B(\hat{V}, \bar{s}_i)$, given q . That is, $\hat{s}_i$ maximizes $E_i[U_i(R_i, \theta)]$ for $s_i \in B(\hat{V}, \bar{s}_i)$, given q .

(b) $\hat{V}$ equates supply and demand in each security's market, given q . That is, $\sum_i \hat{s}_i^j = 1$, $j = 0, \ldots, N$.

Financial equilibrium is a familiar concept. Condition (a) states that each consumer is in equilibrium given market values $\hat{V}$. Condition (b) states that values $\hat{V}$ are equilibrium market values. Given fixed q and further assumptions about subjective probability functions and preferences, Sharpe [13] and others have examined properties of equilibrium portfolios and market values.

In our model, it is clear that equilibrium portfolios and market values depend upon q. This is because a change in q will change the distributions of profits perceived by each investor. Let us denote the financial equilibrium correspondence by

$$[\hat{s}_1(q), \ldots, \hat{s}_M(q); \hat{V}(q)] .^{10}$$

We assume that, for each q in a relevant domain, the image of the equilibrium correspondence is non-empty, implying a financial equilibrium exists. Later, we shall assume $\hat{s}_i(q)$ and $\hat{V}(q)$ are differentiable functions (as they will be in a mean-variance framework).

Conditions (a) of financial equilibrium imply $\hat{s}_i$ is optimal, given q, for all i. $(\hat{s}_i^1, \ldots, \hat{s}_i^N)$ will maximize (with no constraints) $E_i[U_i(R_i, \theta)]$ when s_i^0 is eliminated by (5) and R_i is given by (7). Necessary first order maximizing conditions are

$$(8) \qquad E_i\{U_i'(R_i, \theta)[\pi^j - r\hat{V}^j]\} = 0 ; \; j = 1, \ldots, N , \quad i = 1, \ldots, M .$$

When profits can be written in the competitive form (2), we may rewrite (8) as

$$(9) \qquad E_i\{U_i'(R_i, \theta)[p^j(\theta)q^j - C^j(q^j) - r\hat{V}^j]\} = 0 .$$

Conditions (b) of financial equilibrium imply

$$(10) \qquad \sum_i \hat{s}_i^j = 1 , \quad j = 0, \ldots, N .$$

(5), (8) or (9), and (10) form $(N+1)(M+1)$ equations in the $(N+1)(M+1)$ unknowns $\hat{s}_i^j$ and $\hat{V}^j$, $i = 1, \ldots, M$; $j = 0, \ldots, N$. Because these equations are homogeneous of degree zero in $\hat{V}$, only relative prices are determined, and we normalize prices by choosing

$$\hat{V}^0 = 1 \text{ , for all } q \text{ .}^{11} \tag{11}$$

This implies that the equilibrium correspondence of market values takes the form $\hat{V}(q) = [1, \hat{V}^1(q), \ldots, \hat{V}^N(q)]$, and that

$$r = \pi^0(q^0)/\hat{V}^0(q) = \pi^0(q^0) \text{ .} \tag{12}$$

IV. Equilibrium Production Decisions

A vector of outputs $\hat{q} = (\hat{q}^0, \ldots, \hat{q}^N)$ is termed a production equilibrium if outputs $\hat{q}$ are "in the stockholders' interests," when current portfolios and market values are a financial equilibrium relative to $\hat{q}$.

The requirement that current portfolios and market values be a financial equilibrium given $\hat{q}$, i.e. that $[\bar{s}_i = \hat{s}_i(\hat{q});\ \bar{V} = \hat{V}(\hat{q})]$, implies there is no impetus for portfolio or market value changes if $\hat{q}$ remains fixed. And the requirement that $\hat{q}$ be "in the stockholders' interests" implies that $\hat{q}$ won't change, given current portfolios and market values.

We have purposefully left imprecise the meaning of outputs being "in the stockholders' interests." Any number of voting rules might actually be used, as long as they lead to some set of outputs being chosen. But whatever the rule for determining output, we presume there cannot be some $\bar{q}^k \neq \hat{q}^k$, and associated (different) financial equilibrium, such that all stockholders of k are better off at $\bar{q}^k$ than at $\hat{q}^k$. Clearly,

if such a $\bar{q}^k$ exists, and stockholders influence the firm (or the manager is a stockholder), then the firm will change output from $\hat{q}^k$, and $\hat{q}$ cannot represent a production equilibrium.

To examine the implications of production equilibrium, we first prove

Theorem I: Unanimity Theorem. In the competitive random environment (2), assume current portfolios and market values are in financial equilibrium given current production decisions $q = (q^0, \ldots, q^N)$. Then $\partial E_i[U_i(R_i, \theta)]/\partial q^k$ has the same sign for all stockholders. That is, stockholders will unanimously vote for (or against) small changes in output.

Proof

(a) For the riskless firm:

$$\frac{\partial E_i[U_i(R_i, \theta)]}{\partial q^0} = E_i\left[U_i'(R_i, \theta)\left(\frac{\partial R_i}{\partial q^0}\right)\right] \tag{13}$$

where R_i is given by (7), r by (12), and $\bar{s}_i = \hat{s}_i(q)$. Since $\hat{V} = \hat{V}(q)$, r, and $\hat{s}_i$ will change as q^0 changes,

$$(14)\quad \frac{\partial E_i[U_i(R_i,\ \theta)]}{\partial q^0} = E_i\Bigg\{ U_i'(R_i,\ \theta)\Bigg\{\Bigg[\bar{s}_i^0 + \sum_{j=1}^{N}[\hat{V}^j(\bar{s}_i^j - \hat{s}_i^j)]\Bigg]\frac{dr}{dq^0} + r\sum_{j=1}^{N}\Bigg[(\bar{s}_i^j - \hat{s}_i^j)\frac{\partial \hat{V}^j}{\partial q^0}\Bigg] + \sum_{j=1}^{N}\Bigg[(\pi^j - r\hat{V}^j)\frac{\partial \hat{s}_i^j}{\partial q^0}\Bigg]\Bigg\}\Bigg\}$$

$$= E_i\Bigg[U_i'(R_i,\ \theta)\left(\frac{dr}{dq^0}\right)\bar{s}_i^0\Bigg]\ ,\quad \text{using (8) and } \bar{s}_i = \hat{s}_i\ ;$$

$$= E_i\Bigg[U_i'(R_i,\ \theta)\frac{d\pi^0}{dq^0}\Bigg]\bar{s}_i^0\ ,\quad \text{using (12);}$$

$$= E_i[U_i'(R_i,\ \theta)]\bar{s}_i^0[p^0 - MC(q^0)]\ ,\quad \text{using (3),}$$

where $MC^j(q^j) \equiv dC^j(q^j)/dq^j$, $j = 0, \ldots, N$. Now $E_i[U_i'(R_i,\ \theta)]\bar{s}_i^0$ is positive (by nonsatiation) for all investors with $\hat{s}_i^0 = \bar{s}_i^0 > 0$; that is, <u>for all stockholders</u>. Therefore (14) has the sign of $[p^0 - MC^0(q^0)]$ for every stockholder.

(b) <u>For Risky Firms</u>:

For arbitrary k , $1 \leq k \leq N$,

$$(15)\quad \frac{\partial E_i[U_i(R_i,\ \theta)]}{\partial q^k} = E_i\Bigg[U_i'(R_i,\ \theta)\left(\frac{\partial R_i}{\partial q^k}\right)\Bigg]$$

$$= E_i\Bigg\{U_i'(R_i,\ \theta)\Bigg[r\sum_{j=1}^{N}\Bigg[(\bar{s}_i^j - \hat{s}_i^j)\frac{\partial \hat{V}^j}{\partial q^k}\Bigg] + \sum_{j=1}^{N}\Bigg[(\pi^j - r\hat{V}^j)\frac{\partial \hat{s}_i^j}{\partial q^k}\Bigg] + \frac{d\pi^k}{dq^k}\hat{s}_i^k\Bigg]\Bigg\}$$

$$= E_i\Bigg\{U_i'(R_i,\ \theta)\Bigg[\frac{d\pi^k}{dq^k}\bar{s}_i^k\Bigg]\Bigg\}\ ,\quad \text{using (8) and } \bar{s}_i = \hat{s}_i$$

$$= E_i\{U_i'(R_i,\ \theta)[p^k(\theta) - MC^k(q^k)]\bar{s}_i^k\}\ ,\quad \text{using (2).}^{12}$$

Now from conditions (9), since $\hat{s}_i = \bar{s}_i$ is a financial equilibrium,

$$(16) \qquad E_i\{U_i'(R_i, \theta)[p^k(\theta)]\} = E_i\{U_i'(R_i, \theta)\}\left[\frac{C^k(q^k) + r\hat{V}^k}{q^k}\right] .$$

Substituting (16) into (15) gives

$$(17) \qquad \frac{\partial E_i[U_i(R_i, \theta)]}{\partial q^k} = E_i[U_i'(R_i, \theta)]\bar{s}_i^k\left[\frac{C^k(q^k) + r\hat{V}^k}{q^k} - MC^k(q^k)\right] .$$

But as before, since $E_i[U_i'(R_i, \theta)]\bar{s}_i^k > 0$ for all stockholders of k, the sign of (17) will be <u>identical</u> for all stockholders, and will have the same sign as $\frac{C^k(q^k) + r\hat{V}^k}{q^k} - MC^k(q^k)$. This last expression, of course, depends only on observable variables: cost functions and current market values.

Theorem I leads immediately to conditions which will be satisfied by equilibrium outputs in a competitive environment.

<u>Theorem II</u>: <u>Fundamental Equilibrium Conditions</u>. If $\hat{q} = (\hat{q}^0, \ldots, \hat{q}^N)$ is a <u>production equilibrium</u>, then

(a) For the riskless firm:

$$(18) \qquad p^0 = MC^0(\hat{q}^0) ;$$

(b) For risky firms:

$$(19) \qquad \frac{C^j(\hat{q}^j) + r\hat{V}^j(\hat{q}^j)}{\hat{q}^j} = MC^j(\hat{q}^j) , \quad j = 1, \ldots, N .$$

Proof: If conditions (18) and/or (19) are not satisfied at $\hat{q}$, then a small change in some q^k can occur after which all stockholders in firm k are better off, since $\partial E_i[U_i(R_i, \theta)]/\partial q^k > 0$ (or < 0) for all stockholders of k by Theorem I. But this contradicts the assumption that $\hat{q}$ is a production equilibrium.[13]

V. Theory of Firm Behavior under Uncertainty

Equilibrium condition (18) confirms what every schoolboy knows: the competitive firm under certainty sets price equal to marginal cost, and therewith maximizes profits. Thus, our theory confirms that profit maximization is optimal from the point of view of stockholders if profits are nonstochastic.

Under uncertainty, we showed earlier that profit maximization and "p = MC" are meaningless concepts. Conditions (19) replace the familiar relation (18). (19) can be interpreted as follows: a risky firm j should choose a $\hat{q}^j$ such that the "total" average cost equals marginal cost, where the total average cost is the sum of average variable costs $C^j(\hat{q}^j)/\hat{q}^j$ plus an imputed average capital cost $r\hat{V}^j(\hat{q}^j)/\hat{q}^j$.[14] Thus stock market value of the firm becomes crucial in the firm's output decision--our theory indicates the inextricable relationship between production (cost) and financial (market value) variables.

If the firm's physical assets are transferrable and marketed, the analysis can be extended to consider entry and exit of firms. If $\hat{V}^j$, the market value of the j^{th} firm, exceeds the market value of its assets, entry will take place since any entrepreneur could make a sure profit by purchasing physical assets and then selling stock (whose market value will exceed the cost of the physical assets). In long run equilibrium, the value of firms will equal the value of their physical assets.[15]

Conditions (19) can be used to generate useful theorems on firm behavior.

Theorem III. $\hat{V}^j \geqq 0$ if and only if $MC^j(\hat{q}^j) \geqq AC^j(\hat{q}^j)$, where $AC^j(\hat{q}^j) = C^j(\hat{q}^j)/\hat{q}^j$.

Proof: From (19),

$$\hat{V}^j(\hat{q}^j) = \frac{\hat{q}^j}{r}\left[MC^j(\hat{q}^j) - AC^j(\hat{q}^j)\right], \tag{20}$$

which is nonnegative if and only if $MC^j(\hat{q}^j) \geqq AC^j(\hat{q}^j)$.

In long run equilibrium, we will expect $\hat{V}^j \geqq 0$.[16] This implies that optimal output $\hat{q}^j$ will be chosen so $MC^j(\hat{q}^j) \geqq AC^j(\hat{q}^j)$, or in the region where marginal cost lies above average (variable) costs. Note the similarity with the traditional theory of the competitive firm under certainty, which says $p = MC(\hat{q})$ for $p \geqq AVC(\hat{q})$, where AVC is average variable cost. Combining these gives the familiar condition $MC(\hat{q}) \geqq AVC(\hat{q})$, which is precisely the condition we derive for the uncertainty case! Thus market value being nonnegative is the substitute for long run profit being nonnegative under certainty.

Theorem IV. Any change in expectations, attitudes towards risk, etc., which leads to a rise in the firm's market value, but does not affect costs, will lead to greater output by the firm.

Proof: If the $\hat{V}^j(q)$ function shifts upwards to $V^j_*(q)$, all investors will find

$$\frac{C^j(\hat{q}^j) + rV_*^j(\hat{q}^j)}{\hat{q}^j} - MC^j(\hat{q}^j) > 0 ,$$

which by (17) implies all stockholders will urge an increase in q^j until a new equilibrium (19) is reached at a greater $\hat{q}^j$.[17]

Theorem IV is a replacement for the certainty statement that "a rise in price leads to a rise in output." Since price is not defined under uncertainty, we see that market value takes its place in this context.

Theorem V. In general, an increase in fixed costs will lead to a change in output.

Proof: If there is no change in optimal output, then

$$\frac{C(\hat{q}) + \Delta F + r(\hat{V} + \Delta\hat{V})}{\hat{q}} = MC(\hat{q}) = \frac{C(\hat{q}) + r\hat{V}(\hat{q})}{\hat{q}}$$

or $\Delta\hat{V}/\Delta F = -1/r$. In general, it can be shown that $\Delta\hat{V}/\Delta F = -1/r$ if and only if each investor exhibits constant absolute risk aversion.[18]

VI. Stockholders Directives and Value Maximization

Will the $\hat{q}^j$ chosen by stockholders also maximize the market value of the firm? In general, the answer is no.[19] But under conditions which we argue are truly "competitive," the stockholders' policies will maximize perceived market value.

First, we demonstrate that equilibrium outputs $q^j = \hat{q}^j$ will not in general maximize market value. For simplification, we use the special case of a single risky firm in addition to the riskless firm, although we still suppose firms act as if prices are independent of their decisions. Differentiating (8) with respect to q^1, the risky

firm's output, and evaluating at $q^1 = \hat{q}^1$ gives

$$(21) \qquad E_i\left\{U_i'(R_i,\ \theta)\left[\frac{d\pi^1}{dq^1} - r\,\frac{\partial \hat{V}^1}{\partial q^1}\right]\right\}$$

$$+ E_i\left\{U_i''(R_i,\ \theta)(\pi^1 - r\hat{V}^1)(\pi^1 - r\hat{V}^1)\,\frac{\partial \hat{s}_i^1}{\partial q^1} + \frac{d\pi^1}{dq^1}\,\hat{s}_i^1\right]\right\} = 0$$

or, using (15) and the fact that in equilibrium $\partial E_i\left[U_i'(R_i,\ \theta)\frac{d\pi^1}{dq^1}\right] = 0$,

$$-D_i^{-1}E_i\ [U_i'(R_i,\ \theta)]\left(r\,\frac{\partial \hat{V}^1}{\partial q^1}\right) + \frac{\partial \hat{s}_i^1}{\partial q^1} + D_i^{-1}\left[E_i\left\{U_i''(R_i,\ \theta)(\pi^1 - r\hat{V}^1)\,\frac{d\pi^1}{dq^1}\,s_i^1\right\}\right] = 0\ ,$$

where $D_i \equiv E_i\{U_i''(R_i,\ \theta)[\pi^1 - r\hat{V}^1]^2\} < 0$. Now summing (21) over individuals $i = 1, \ldots, M$ and noting $\sum_i \partial \hat{s}_i^1/\partial q^1 = 0$ gives

$$(22) \qquad \left.\frac{\partial \hat{V}^1}{\partial q^1}\right|_{q^1=\hat{q}^1} = \frac{\sum_i D_i^{-1}E_i\left[U_i''(R_i,\ \theta)(\pi^1 - r\hat{V}^1)\,\frac{d\pi^1}{dq^1}\,\hat{s}_i^1\right]}{r\sum_i D_i^{-1}E_i[U_i'(R_i,\ \theta)]}\ .$$

By nonsatiation, the denominator is always negative. In general, the numerator will be nonzero, which explains why Stiglitz and others can show that value maximization does not lead to Pareto optimal choices of outputs (or "techniques"). If the firm chooses a $q_V^j \neq \hat{q}^j$, our model indicates <u>everyone</u> could be made better off by a small change in q^j away from q_V^j . We support Wilson's contention that what Stiglitz brings into question is not the Pareto optimality of the stockmarket, but rather the value maximization criterion.

There is, however, an environment in which firms, <u>thinking</u> they are

maximizing market value, make decisions which coincide with the $\hat{q}^k$ that stockholders wish.

Theorem VI. (Value Maximization in a Competitive Environment). Assume that a firm operates in an industry in which firms face the same random price and possess identical convex cost functions. Then the firm will maximize its market value relative to other firms in the industry by setting $q^k = \hat{q}^k$, the output which is unanimously supported by stockholders.

Proof: Consider firms k and ℓ in industry A, facing common price $p^A(\theta)$ and having identical cost functions $C^A(\cdot)$. Assume $q^k = \hat{q}^k$, where $\hat{q}^k$ is the shareholders' desired policy which satisfies (19). Further assume $q^\ell \neq \hat{q}^k$. We show that in the financial equilibrium associated with those outputs,

$$\hat{V}^\ell < \hat{V}^k .$$

From (9), for all i we know

$$E_i\{U_i'(R_i, \theta)[p^A(\theta)\hat{q}^k - C^A(\hat{q}^k) - r\hat{V}^k]\} = 0 \tag{23}$$

$$E_i\{U_i'(R_i, \theta)[p^A(\theta)q^\ell - C^A(q^\ell) - r\hat{V}^\ell]\} = 0$$

which imply

$$\hat{V}^k > \hat{V}^\ell \text{ iff } E_i\{U_i'(R_i, \theta)[p^A(\theta)\hat{q}^k - C^A(\hat{q}^k)]\} \tag{24}$$

$$> E_i\{U_i'(R_i, \theta)[p^A(\theta)q^\ell - C^A(q^\ell)]\} .$$

From (15) and the requirement $\partial E_i[U_i(R_i, \theta)]/\partial q^k = 0$,

$$E_i[U_i'(R_i,\ \theta)p^A(\theta)] = E_i\{U_i'(R_i,\ \theta)[MC^A(\hat{q}^k)]\} .$$

Substituting this into (24) and noting $E_i[U_i'(R_i,\ \theta)] > 0$ gives

$$(25) \qquad \hat{V}^k > \hat{V}^\ell \quad \text{iff} \quad MC^A(\hat{q}^k)\hat{q}^k - C^A(\hat{q}^k) > MC^A(\hat{q}^k)q^\ell - C^A(q^\ell) .$$

The inequality clearly will hold if

$$(26) \qquad \max_q [MC^A(\hat{q}^k)q - C^A(q)] = MC^A(\hat{q}^k)\hat{q}^k - C^A(\hat{q}^k) ,$$

and the maximum will be unique when $C^A(\cdot)$ is strictly convex. Now a necessary and sufficient condition to maximize the LHS of (26) is that

$$(27) \qquad \frac{d}{dq}[MC^A(\hat{q}^k)q - C^A(q)] = MC^A(\hat{q}^k) - MC^A(q) = 0 ,$$

or $q = \hat{q}^k$. The theorem then follows immediately from (25).

Theorem VI is definitely in the spirit of "perfect competition." If a firm maximizes its relative value but considers others' values independent of its decisions, it thinks it is maximizing its own market value. Thus competitive firms, attempting to maximize value by maximizing relative value, will make output choices which coincide with stockholders' interests. This result justifies Diamond's assumption that firms should maximize a market valuation function of the form he chooses. Note that Stiglitz, in excluding perfectly correlated firms, excludes our notion of a "perfectly competitive" environment.

A final question to be asked is whether a firm acts "as if" it maximized expected utility of profit. In general, the answer is "no," but if all shareholders are alike (e.g., like "the manager"), and prices of different

firms are independently distributed, then it can be shown that $\hat{q}^j$ is "as if" firm j were maximizing expected utility of profit, and the theorems of Baron, Sandmo, and Leland are valid in this environment.

VII. Pareto Optimality of Production Equilibrium

We now examine the optimality properties, from the point of view of stockholders, of a production equilibrium $\hat{q}$ and its associated financial equilibrium.[20] Pareto optimality requires that if every person ℓ except an arbitrary individual m has fixed expected utility $E_\ell[U_\ell(R_\ell, \theta)] = \bar{U}_\ell$, then the output and portfolio decisions must maximize $E_m[U_m(R_m, \theta)]$. Feasibility requires $\sum_{i=1}^{M} s_i^j = 1$ for all j, so we can express the portfolio of the m^{th} individual as $s_m = (1 - \sum_{\ell \neq m} s_\ell^0, \ldots, 1 - \sum_{\ell \neq m} s_\ell^N)$. The returns of portfolios are $R_i = \sum_{j=0}^{N} \pi^j s_i^j$, $i = 1, \ldots, M$. If a production equilibrium is to be Pareto optimal, $\hat{q}$ and $\hat{s}_i(\hat{q})$, $i = 1, \ldots, M$, must yield a stationary point to the Lagrangean

$$L = E_m[U_m(R_m, \theta)] + \sum_{\ell \neq m} \lambda_\ell \{E_\ell[U_\ell(R_\ell, \theta)] - \bar{U}_\ell\},$$

implying

$$(28) \quad \frac{\partial L}{\partial q^k} = E_m\left[U_m'(R_m, \theta)\frac{\partial \pi^k}{\partial q^k}\right]\hat{s}_m^k + \sum_{\ell \neq m} \lambda_\ell E_\ell\left[U_\ell'(R_\ell, \theta)\frac{\partial \pi^k}{\partial q^k}\right]\hat{s}_\ell^k = 0,$$

$$k = 0, \ldots, N;$$

$$(29) \quad \frac{\partial L}{\partial s_\ell^j} = -E_m[U_m'(R_m, \theta)\pi^j] + \lambda_\ell E_\ell[U_\ell'(R_\ell, \theta)]\pi^j = 0, \quad j = 0, \ldots, N; \ \ell \neq m.$$

Now (14), (15), and the equilibrium requirements (18) and (19) imply (28) will be satisfied by the production equilibrium $\hat{q}$ and its associated financial equilibrium. Using conditions (8), we see (29) will be satisfied when we choose $\lambda_\ell = \dfrac{E_m[U'_m(R_m, \theta)\pi^j]}{E_\ell[U'_\ell(R_\ell, \theta)\pi^j]}$, $\ell \neq m$. We are free to choose the λ's in such a manner, since (28) will be satisfied for arbitrary λ's at the production equilibrium. Thus, we conclude that production equilibrium satisfies the first order necessary conditions for Pareto optimality. If $\hat{s}_i^j(\hat{q})$ is positive for all i and j, then $E_m[U_m(R_m, \theta)]$ will indeed be maximized for arbitrary m, and we conclude that production equilibrium is Pareto optimal from the point of view of the stockholders.[21]

VIII. Some Generalizations: A Preliminary Look

A. Imperfect Competition and Random Technologies

The interested reader can verify that a "unanimity theorem" can be proved whenever

$$(30) \qquad \pi^j(d^j, \theta) = f^j(d^j) + g^j(d^j)h^j(\theta), \quad j = 0, \ldots, N,$$

where d^j may be a vector of decision variables.[22] Note Diamond's model with technological uncertainty fulfills this condition, as does the competitive firm facing random selling price. But if output is the only decision variable and costs remain nonstochastic, (30) is satisfied if random demand functions take the (imperfectly competitive) form

$$(31) \qquad p^j(q^j, \theta) = f^j(q^j) + g^j(q^j)h^j(\theta), \quad j = 0, \ldots, N,$$

which includes both multiplicative random demand curves $[f^j(q^j) \equiv 0]$ and additive random demand curves $[g^j(q^j) \equiv 1]$. A value theory similar to "MR = MC" can be constructed, and Pareto optimality (for the stockholders) will follow. If $\pi^j(d^j, \theta)$ cannot be put in the form (30), then in the absence of further assumptions about mean-variance preferences or special types of securities, our "unanimity" theorem breaks down, and with it all other results in the preceding sections.[23]

B. A Different Objective Function

Earlier results assumed investors ranked alternative portfolios on the basis of expected utility of returns. While ours is consistent with the studies of Diamond and Stiglitz, it could be argued that final portfolio value is the appropriate argument in a world which does not self-destruct at the end of the current period. We consider now a model in which end-period wealth is the appropriate argument of the utility function.

If $\hat{V}^j$ is the equilibrium market value of firm j, $(\hat{V}^j + \pi^j)\hat{s}_i^j$ will be the assumed end-period wealth resulting from investor i holding a fraction $\hat{s}_i^j$ of firm j.[24] We can then express

$$W_i = \sum_{j=0}^{N} (\hat{V}^j + \pi^j)\hat{s}_i^j ,$$

or substitution from the budget constraint (5),

$$W_i = (1+r) \sum_{j=0}^{N} \hat{V}^j \bar{s}^j + \sum_{j=1}^{N} (\pi^j - r\hat{V}^j)\hat{s}_i^j .$$

The investor seeks to

$$\underset{s_i \in B_i}{\text{Maximize}}\ E_i[U_i(W_i,\ \theta)]\ ,$$

which gives the portfolio optimizing conditions

$$E_i\left[\ U_i'(W_i,\ \theta)\ \frac{\partial W_i}{\partial s_i^j}\ \right] = E_i\{U_i'(W_i,\ \theta)[\pi^j - r\hat{V}^j]\} = 0\ ,\quad j = 1,\ \ldots,\ N\ .$$

These conditions are identical to (8) except W_i replaces R_i. But, assuming $\hat{s}_i^j = \bar{s}_i^j$ (current portfolios are optimal),

$$\frac{\partial E_i[U_i(W_i,\ \theta)]}{\partial \hat{q}^k} = E_i\left\{U_i'(W_i,\theta)\left[\frac{d\pi^k}{dq^k}\hat{s}_i^k + \sum_{j=1}^{N}\frac{\partial \hat{V}^j}{\partial q^k}\,\hat{s}_i^j\right]\right\}\ .$$

In general, we cannot expect a unanimity theorem. However, in the purely competitive environment, we can assume investors <u>perceive</u> (a) $\partial\hat{V}^j/\partial q^k = 0$, $j \neq k$; (b) maximizing relative value maximizes market value.

The argument which under competitive markets showed $\hat{q}^k$ maximized relative value V^k (Theorem VI) also will hold when W_i replaces R_i. Thus, q_*^k, the q^k which sets

$$E_i\left[\ U_i'(W_i,\ \theta)\ \frac{d\pi^k}{dq^k}\ \right] = 0\ ,$$

also sets $\partial\hat{V}^k/\partial q^k = 0$ as perceived by the investor. Together, conditions (a) and (b) imply

$$E_i\left\{U_i'(W_i,\ \theta)\left[\frac{d\pi^k}{dq^k} + \sum_{j=1}^{N}\frac{\partial V^j}{\partial q^k}\,\hat{s}_i^j\right]\right\} = 0\ ,\quad k = 1,\ \ldots,\ N$$

when $q^k = q_*^k$, for all investors $i = 1, \ldots, M$. Under these conditions, then, the value theory for competitive markets developed in Section VI continues to hold, as will the Pareto optimality results of Section VII (with W_i replacing R_i and q_*^j replacing $\hat{q}^j$, for all i and j). The firm facing certain price p^0 will continue to choose the profit maximizing $q^0 = \hat{q}^0$.

IX. A Mean-Variance Example

Let us briefly descend to a world in which

(i) all investors rank portfolios on the basis of mean and variance of return;

(ii) all investors share identical expectations for prices, and therefore for profits.

This is, of course, the world of Sharpe, Jensen and Long, Stiglitz, and Wilson, where relative market values are given by the formula[25]

$$\hat{V}^j(q) = \frac{E\pi^j - k \sum_{\ell=0}^{N} E(\pi^j - \bar{\pi}^j)(\pi^\ell - \bar{\pi}^\ell)}{r} \tag{32}$$

where k is the "price of risk," and $\pi^j = p^j(\theta)q^j - C^j(q^j)$. For the present, let us further assume that $E(\pi^j - \bar{\pi}^j)(\pi^\ell - \bar{\pi}^\ell) = 0$ for all $\ell \neq j$. Then (32) becomes

$$\hat{V}^j(q) = \frac{E\pi^j - k\mathrm{Var}\ \pi^j}{r} . \tag{33}$$

Now substitute (33) into the fundamental equilibrium relation (19):

$$(34) \qquad \frac{C^j(\hat{q}^j) + E(p^j)\hat{q}^j - C^j(\hat{q}^j) - k\mathrm{Var}[\pi^j(\hat{q}^j)]}{\hat{q}^j} = MC^j(\hat{q}^j) ,$$

or

$$E(p^j) - \frac{k\mathrm{Var}\ \pi^j(\hat{q}^j)}{\hat{q}^j} = MC^j(\hat{q}^j) .$$

Clearly (34) implies $E(p^j) > MC^j(\hat{q}^j)$, which if MC is increasing in q implies output of the competitive firm under uncertainty is less than output under uncertainty.[26] However, as the variance of profit shrinks to zero, or as the risk adjustment factor k becomes small, the firm moves closer to maximizing expected profit.

If a firm has a substantial negative covariance with other firms, it is possible that $\sum_{\ell=0}^{N} (\pi^j - \bar{\pi}^j)(\pi^\ell - \bar{\pi}^\ell) < 0$, in which case the firm would have a larger output under uncertainty than it would if price equaled its expected value with probability one.

X. Conclusion

Examining production in the context of a stock market, we have shown

(1) Despite differences in expectations and attitudes towards risk of stockholders, outputs selected by firms will be (locally) optimal for all stockholders;

(2) firms whose profits are not random will choose output to maximize profit;

(3) the outputs selected with be Pareto optimal for stockholders;[27]

(4) the outputs selected will not in general maximize the market value of the firm. But they will maximize market value in a "perfectly competitive" environment, in which firms in the same industry have perfectly correlated returns.

The contrasting results of Diamond, Stiglitz, and Wilson on the optimality of stock markets are readily interpretable in the context of our model. Diamond assumes the "perfectly competitive" environment which implies value maximization coincides with stockholders' interests. Stiglitz assumes non-perfectly correlated firms but asserts firms maximize market value, an assertion which Wilson shows will not coincide with stockholders' interests.

Our theory indicates the essential role of the stock market in determining production decisions under uncertainty. Equilibrium output conditions involve both production (cost) and financial (market value) variables. The equilibrium conditions provide a basis for deriving comparative static results, and seem to offer promise for the next task: the construction of a model which includes stochastic equilibrium in the markets for outputs, as well as equilibrium in capital asset markets.

Finally, we note that our theory generates testable conclusions which do not require estimates of subjective probabilities. In this respect, it is different from the capital asset market equilibrium models, which have required approximations of subjective probabilities of stockholders for testing. In our model, subjective probabilities help to determine relative market values, but we need only measure these market values and cost functions of firms to implement empirical tests.

FOOTNOTES

[1]Recent contributions which examine production (or investment) include Diamond [4], Stiglitz [14], Jensen and Long [7], and Fama [6]. These studies are discussed below. Stigum [16] considers existence of general equilibria under uncertainty, and Dreze [5] develops a decentralized "tatonnement" process in a model quite similar to ours.

[2]An important exception to this is the case where separate contracts can be made for each state of nature: see Arrow [1]. (It seems clear that the actual economic environment does not provide such a complete set of markets.) See also Debreu [3].

[3]Although Stiglitz's published paper [14] treats only firms which maximize market value, his unpublished manuscript [15] clearly indicates his reservations about the appropriateness of value maximization. In the manuscript, he develops an analysis which parallels several of the ideas in Section IV, and indicates that different notions of equilibrium can give rise to different optimal strategies for stockholders.

[4]To formalize our notion of "states of nature," we presume that θ is an element in a measurable space (Θ, Σ), where Σ is a σ-field of subsets of Θ ("events"). If D is a feasible set of decisions, then π^j is a measurable function from $D \times \Theta$ into R^1, for all j. The specification of a probability measure over Σ will then generate a joint probability distribution of profits of firms, conditional on d.

[5]Alternatively, we could have assumed the existence of a riskless bond paying a fixed coupon rate.

[6]Note our total market value is the market value of equities. For simplicity, we ignore possible debt financing. But one can show that, if debt is not issued beyond the point of possible bankruptcy (as perceived by any investor), the Modigliani-Miller theorem survives: see footnote 18.

[7]More exactly, μ_i is a probability measure on the measurable space (Θ, Σ). Thus the specification of different probability measures for different investors implies each investor has his own subjective probability distribution for the profits of firms, conditional upon their decisions.

[8]Since the utility function in (b) is state dependent, we are dealing with a more general form of expected utility than is typical. Peter Diamond has pointed out that all theorems are valid in the more general case where (assuming there are a finite number of states of nature) choice between decisions can be ranked by a nonsatiated, quasi-concave function

$$U[R_i^1(d), \ldots, R_i^S(d)] ,$$

where $R_i^s(d)$ is the return to investor i in state of nature s, given decisions d.

[9]The assumption that returns to holding shares depend only on firms' profits is adopted by Diamond [4], Stiglitz [14], and others. This assumption is reasonable in a stationary world with intertemporal independence, or in a world in which the movement of stock prices is closely dependent upon profits, as seems reasonable in the long run. In the short run, however, speculators may have (probabilitistic) expectations of returns which bear little or no connection to firms' decisions. If speculators perceive no connection, they can be included in our model without altering the results which follow. The diligent reader also can ascertain that a two-period consumption model will not alter the nature of our results. An alternative formulation to (6), in which investors rank portfolios according to end-period wealth, is considered in Section VIIIB.

[10]Of course the financial equilibrium also depends on the current portfolios $\bar{s}_i$, but this dependence is suppressed since we shall not consider variations in $\bar{s}_i$. Also note that the financial equilibrium correspondence gives optimal portfolios and equilibrium market values if q is regarded as fixed--i.e. in equilibrium. If all investors know q will change to some $\bar{q}$, and values will change to $\hat{V}(\bar{q})$, then $\hat{V}(q)$ may no longer describe the actual market values at q.

[11]If we start with a two period consumption model, this normalization will not be possible. Nonetheless, all our results will follow.

[12]Note that our conclusion follows for any possible $\partial\hat{V}^j/\partial q^k$. Indeed, different investors can have different opinions regarding $\partial\hat{V}^j/\partial q^k$ and our results will still follow, as long as all investors face the equilibrium price $\hat{V}^j(\hat{q})$.

[13]At the chosen outputs, (18) and (19) and appropriate second order conditions imply that all stockholders find $\hat{q}$ a local optimum. I have not been able to show that $\hat{q}$ represents a global optimum for all stockholders, although it will be in the mean-variance framework of Section IX, when cost functions are convex.

An alternative proof of the equilibrium condition (19) can be based on an examination of arbitrage. Assume stockholder i (with $\bar{s}_i^k > 0$) could change the output of firm k by Δq^k. The resulting change in his return will be $[p^k(\theta) - MC^k(q^k)]\bar{s}_i^k\Delta q^k$. At the same time, assume he exchanges a fraction of his shares $\Delta s_i^k = \bar{s}_i^k\Delta q/q$ for $V^k\Delta s_i^k$ shares of the riskless asset, producing a change $\left[-p^k(\theta) + \frac{C^k(q^k) + rV^k}{q^k}\right]\bar{s}_i^k\Delta q^k$. The net return of the simultaneous changes is

$$\left[\frac{C^k(q^k) + rV^k}{q^k} - MC^k(q^k)\right]\bar{s}_i^k\Delta q^k,$$

whose sign is independent of θ and of i. Thus, all stockholders would urge changes in q^k until $\hat{q}^k$ is achieved and (19) is satisfied. Even if no riskless asset is present, the arbitrage argument can be used when the distributions of marginal returns are spanned by the distributions of total returns: see Ekern and Wilson [18].

[14]Note that $r\hat{V}(\hat{q})/\hat{q}$ is the average opportunity cost of the market value of the firm. If the firm is a monopoly this market value may be considerably higher than the true cost of the capital in that industry.

(Clearly, readers should resist the temptation to conclude that "average cost equals marginal cost implies zero profits." First, profits are random so "zero profits" is meaningless. Second, we showed above that our "average cost" is dependent on the stock market value of the firm, which may not be closely related to the true value of the firm's capital stock.)

[15]In our model, the signal for entry is not when "excess" profits are present, but rather when market value exceeds the cost of duplicating a firm. Clearly this is a much more satisfactory situation in the presence of uncertainty. Even if <u>expected</u> profit could be determined under uncertainty, it would still have to be "risk adjusted" in some manner to make the old criterion for entry operable.

Note how, in long run equilibrium with entry, the firm will be producing <u>at the bottom of its</u> (<u>long run</u>) <u>average cost curve</u>. This is because in long run equilibrium $r\hat{V}^j$ is the true opportunity cost of the firm's capital stock, since $\hat{V}^j$ will equal the actual value of the firm's capital. Conditions (19) now imply true average cost equals marginal cost at the equilibrium output, implying the firm under uncertainty will produce at minimum average cost. If expectations are identical, the results in Section IX imply that expected price will exceed marginal cost in equilibrium. Thus, while with free entry firms are producing at minimum average cost, their expected profits will be positive, and total output of the industry will be less than if p^j equalled its expected value with certainty. Greater uncertainty, then leads to fewer firms and higher expected prices in long run equilibrium, though all firms will be producing at the minimum average cost.

[16]Free disposal of securities will guarantee $\hat{V}^j \geqq 0$.

[17]For the theorem to hold when changes in V are large and when stockholders are permitted to suggest non-marginal changes, we would further require $E_i[U_i(R_i, \theta)]$ to be globally concave in q. Local concavity is assured by the fact that $\hat{q}$ provides a local maximum in $E_i[U_i(R_i, \theta)]$, for all stockholders.

[18]The proof relies on the fact that there are no wealth effects if an investor possesses a utility function exhibiting constant absolute risk aversion. Thus a rise in fixed costs with an offsetting change in market cost (by a factor $1/r$) will lead all investors to demand precisely the same percentage of the firm's stock as before. Whereas Theorem V might seem to indicate that optimal output depends on the debt-equity ratio, such is not the case when a firm's debt is risk free. Each investor can always generate the same distribution of returns, independent of the debt-equity ratio, by an appropriate holding of the firm's debt and equity. (This is not possible when there is simply an increase in fixed cost.) There is good reason to think, however, that firms will be motivated to issue debt beyond the "zero probability of bankruptcy" level. We hope to explore this topic in a future paper.

[19]That is, there will in general exist a q_v^k such that $\hat{V}^k(q_v^k)$ will exceed $\hat{V}^k(\hat{q}^k)$, given that other $\hat{q}^i$s remain fixed. But $\hat{V}^k(q_v^k)$ is the value of the firm's stock if q_v^k is perceived to be an <u>equilibrium</u> output. If q_v^k is chosen temporarily and investors know that q^k will move to $\hat{q}^k$, then the current $V(q_v^k)$ will not equal $\hat{V}(q_v^k)$ and indeed (if adjustment is rapid) will not exceed $\hat{V}(\hat{q}^k)$. (See footnote 10.) This explains why "raiders" may not be able to buy a firm, temporarily change from $\hat{q}^k$ to q_v^k, and make a sure profit, since the stockholders who purchase the the firm from the "raider" would vote to change the output back to $\hat{q}^k$, and the move to q_v^k would be temporary.

[20]The optimality of outputs in our model is with respect to the distribution of profits they generate. Since our model is not closed (prices are exogenous, given θ), we cannot examine optimality of outputs from the point of view of their consumption.

[21]If investor m holds a short position in firm k in equilibrium, his utility will be <u>minimized</u> with respect to q^k at $\hat{q}^k$. It is not clear that we have Pareto optimality in this situation: perhaps investor m could bribe the stockholders of firm k to change output in exchange for some of the riskless asset (for example), leaving all better off.

[22]Ekern and Wilson [18] show that unanimity will hold under somewhat more general conditions: when the distributions of marginal profits are spanned by the distributions of current profits. (See also footnote 13 above). When profits take the form (30), conditions (18) are replaced by

$$V^j(\hat{d}^j) = \frac{f^j(\hat{d}^j) - (f^j_k/g^j_k)g^j(\hat{d}^j)}{r}$$

where f^j_k is the partial derivative of f^j with respect to the k^{th} component of the vector d^j, evaluated at $\hat{d}^j$. This condition collapses to (19) when $f^j(d^j) = -C^j(q^j)$ and $g^j(d^j) = q^j$. A further paper will examine implications of equilibrium in noncompetitive environments.

[23]In a working paper, I have shown that if condition (30) is not satisfied, stockholders in many situations will unanimously vote to split the firm into separate firms, each of whose profit functions will take the form (30). Thus, the condition for unanimity may not be as restrictive as it first appears.

[24]See footnote 9 for a discussion of when "capital gains" are included in this formulation.

[25]See Stiglitz [14], p. 36.

[26]Note $\hat{q}^j$ does <u>not</u> coincide with the value maximizing output q^j_v, which would satisfy the condition

$$Ep^j - \frac{2k \text{ Var } \pi^j(q^j_v)}{q^j_v} = MC^j(q^j_v) .$$

[27]See our caveat in footnote 21 regarding Pareto optimality with equilibrium short positions.

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