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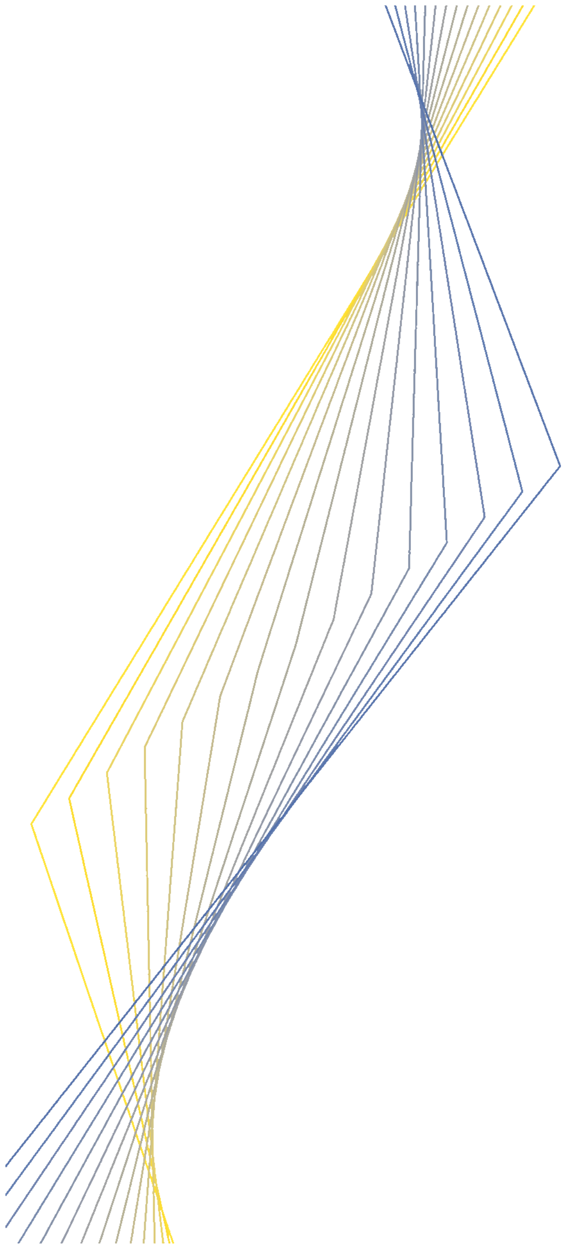

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BY THOMAS WERNER[2]
AND CHRISTIAN UPPER[2]

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Abstract

The present paper focuses on three questions: (i) Are heavy tails a relevant feature of the distribution of BUND futures returns? (ii) Is the tail behaviour constant over time? (iii) If it is not, can we use the tail index as an indicator for financial market risk and does it add value in addition to classical indicators? The answers to these questions are (i) yes, (ii) no, and (iii) yes. The tail index is on average around 3, implying the nonexistence of the fourth moments. A recently developed test for changes in the tail behaviour indicated several breaks in the degree of heaviness of the return tails. Interestingly, the tails of the return distribution do not move in parallel to realised volatility. This suggests that the tails of futures returns contain information for risk management that complements that gained from more standard statistical measures.

Keywords: Tail index, futures returns, extreme value theory, risk management
JEL: C14, G13

Non-Technical summery

For the purpose of estimating market risks, it is important to know how probable extreme price fluctuations in the financial markets are. Essentially, account needs to be taken of price movements in both directions as a sharp increase in prices can imply large losses to holders of short positions just as much as falling prices among investors with long positions. If the probability of extreme values occurring is greater than implied by the normal distribution, the term “fat tails” is used. Recent developments in extreme value theory enable this phenomenon to be analysed without explicit assumptions having to be made about the distribution of returns. A tail index can be used to measure the fatness of the tails.

The literature on extremes of the distribution of returns has so far dealt primarily with exchange rates and stock prices. By contrast, hardly any consideration has yet been given to the prices of bonds and notes or futures contracts on bonds. This is surprising as banks, in particular, often hold fairly large open positions in futures contracts during a trading day. This discussion paper endeavours to fill this gap. To this end, high frequency data for BUND futures - especially five-minute returns - are analysed. Three questions are at the forefront of the analysis: (i) Does the distribution of BUND futures returns have ”fat tails”?; (ii) Is the probability of extreme price movements constant over time?; (iii) Can a tail index provide information about the degree of market uncertainty which cannot be gained using classical indicators such as volatility?

We were able to show that the ”fat tails” phenomenon does occur in the distribution of BUND futures returns. A tail index of approximately 3 implies that the fourth and all higher moments of the distribution do not exist. This shows analyses based on the sample kurtosis to be problematic and justifies the use of the extreme value theory. Recently developed tests have revealed breaks in the fatness of the tails in the distribution of returns. The tail index does not always move in the same direction as volatility. The two indicators may well even send different signals at times. For instance, a decline in volatility might be offset by an increase in the fatness of the tail. For this reason, risk estimates based solely on volatility are to be viewed with extreme caution. Observing the tails of the distribution of returns thus provides information relevant to risk management which cannot be derived using conventional methods.

The results of this research project may be viewed as the first step towards modelling the fatness of tails. Future work could identify factors affecting the fatness of the tails in order to gain a better understanding of its development over time and, where appropriate, to enable measures to be taken to influence it.

1 Introduction

It has been well known at least since the contribution of Mandelbrot (1963) that the distribution of asset returns is not well approximated by the Gaussian normal. Above all, the return distribution seems to have fatter tails than the normal distribution. The literature on the tail behaviour of returns has grown rapidly over recent years for a number of reasons. First of all, long series of asset prices and higher frequencies have become available. Together with the concurrent increase in computing power, this permitted the use of data intensive methods that previously had been difficult if not impossible to implement. Secondly, several new methods based on extreme value theory have been developed. An important example used in this paper is the bootstrap Hill estimator. Finally, the turbulences in the international financial markets in the summer and autumn of 1998 have questioned many of the assumptions of quantitative trading and risk management models, in particular the use of normal distributions.

The literature on the tail behaviour of asset prices has focussed mainly on the foreign exchange (for example Müller, Dacorogna, and Pictet (1998)), and stock markets (for example Lux (2001) for the spot and Cotter (2001) for the futures market). Only few papers deal with bonds or bond futures. Perhaps this is because bond returns are less volatile than stock or forex returns and are therefore believed to pose less risk than other assets. We believe that this argument is wrong and that the omission is not justified. What matters is not the volatility of an asset price per se but the volatility of a position in that asset relative to capital. Even the safest asset becomes risky if leverage is sufficiently high.

Our paper addresses this omission. Rather than working with data on the bond market directly, we estimate the tail behaviour of the BUND future. The BUND future contract traded on Eurex has become the main instrument for hedging long term interest rate risk in the Euro area. Trading is much heavier in the futures than in the spot market, and transactions data is available for a longer time span. Furthermore, since futures are traded electronically on a centralized exchange, the data is also of higher quality than that on the underlying bonds, where trading is more fragmented. Nevertheless, we show in a companion paper (Upper and Werner (2002)) that prices in the futures and spot market move together very closely, so our main findings should apply to the bond market as well.

We focus on three questions: (i) Are heavy tails a relevant feature of the distribution of BUND futures returns? (ii) Is the tail behaviour constant over time? (iii) If it is not, then can we use the tail index as an indicator for financial market risk and does it add value in addition to classical indicators? The answers to questions (i) and (ii) have important implications for the design of trading and risk management models. If the return distribution has fat tails, then the assumption of normality in many of such models would lead one to seriously underestimate the likelihood of sharp falls and gains. For example, the Value at Risk (VaR) measure often used in risk management corresponds to the maximum loss that can occur with a given probability. In mathematical terms, it refers to a quantile, which depends crucially on the shape of the distribution. Another aspect is the liquidation risk of an asset. In a recent paper Duffie and Ziegler (2001) analyse the riskiness of different liquidation strategies and show that the riskiness of different liquidation strategies depends significantly on the fatness of the tails. But it is not only the fatness of tails alone that is important, also the variation of the tail behaviour over time is of interest. If the fatness of a tail is changing over time it is necessary to recognize this for risk evaluation and modelling.

We find that the distribution of high-frequency returns of the BUND future is indeed characterized by heavy tails. At five-minute intervals, the tail index estimated over our complete sample is around three. This implies that the distribution has infinite kurtosis but finite variance. The tail index increases as one reduces the frequency, although the tails remain significantly heavy even for daily data.

The tail behaviour is not stable over time. What is interesting is that the tail index does not move in parallel with more standard measures for volatility such as the variance of returns. This suggests that the tail index does indeed provide information not contained in more commonly used volatility measures.

The paper is organized as follows. In section 2 we present the theoretical foundations for the tail index estimation discussed in section 3. This is followed by sections presenting the data and the empirical results. A final section concludes.

2 What are heavy tails?

Until now we have used a loose definition of heavy tails. It is not easy to define heavy tails precisely. Even the name is not used uniformly. Sometimes heavy-tailed distributions are called fat-tailed, thick-tailed or long-tailed. In the following we use all of these terms as synonyms.

An often used definition of heavy-tailness is based on the 4th central moment. If X is a random variable and μ_X and σ_X are the mean and the standard deviation of X, then X is called heavy-tailed if

$$E\left[\frac{(X-\mu_X)^4}{\sigma_X}\right] > 3.$$

This property is called excess kurtosis because the 4th central moment (the kurtosis) of the normal distribution is 3. However, this definition can only be applied in a sensible way if the 4th moment of a random variable actually exists. If two variables have infinite 4th moments, then no discrimination between their distributions is possible on the basis of the kurtosis.

Unfortunately there is no general accepted definition of tail-heavyness under which a tail ranking is possible. We obtain such a ranking only for particular classes of distributions. In the following we briefly discuss five classes.[1]

E: nonexistence of exponential moments

[1]This classification is borrowed from Bamberg and Dorfleitner (2001).

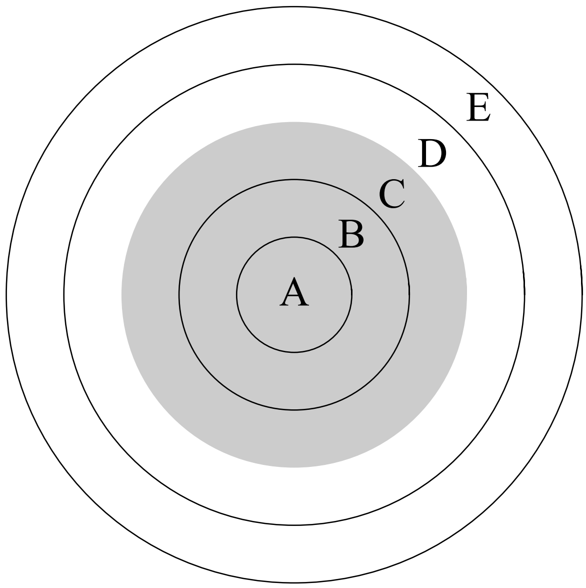

Figure 1: Different classes of heavy-tailed distributions

D: subexponential distributions

C: regular variation with tail index $\alpha > 0$

B: Pareto tails with $\alpha > 0$

A: stable (non-normal) distributions

These classes of distributions are nested as demonstrated in figure 1. The broadest class E encompasses all distributions with

$$E\left(e^{X}\right) = \infty.$$

It is important to note that the normal distribution is not contained in this class as its tail probability $P(X > x) = \overline{F}(x) = 1 - F(x)$ declines faster than exponentially.[2] In this sense all distributions of class E are heavy-tailed with respect to the normal distribution.

[2] $F(x)$ is the cumulative distribution function of X with $F(x) = P(X \leq x)$.

Since the normal distribution has comparatively thin tails, stronger assumptions are possible for heavy tails. The class D contains the subexponential distributions.[3] A distribution is subexponential if

$$\lim_{x \to \infty} \frac{P(X_1 + \cdots + X_n) > x)}{P(\max(X_1, \ldots, X_n) > x))} = 1. \tag{1}$$

This condition has a nice interpretation: the sum of n iid subexponential random variables is likely to be large if and only if their maximum is likely to be large. It is possible to show that equation 1 implies

$$\lim_{x \to \infty} \frac{\overline{F}(x)}{e^{-\epsilon x}} \to \infty \qquad \forall \epsilon > 0.$$

As the name suggests, the tails of a subexponential distribution decrease more slowly than any exponential distribution.

In this paper we focus on the class C of distributions, which are characterized by regular variation in the tails. They form a subclass of the subexponential distributions[4] and satisfy the condition[5]

$$\lim_{t \to \infty} \frac{\overline{F}(tx)}{\overline{F}(t)} = x^{-\alpha}. \tag{2}$$

This condition states that far out in the tail ($t \to \infty$) the distribution behaves like a Pareto distribution. As a consequence, the tail probabilities $P(X > x)$ decline according to a power function. The parameter α is called "tail index" and can be used as a measure of tail-heavyness. An important member of class C is the Student-t distribution.

In contrast, distributions in class B have exact Pareto tails. The cumulative distribution function of the Pareto distribution is

$$F(x) = 1 - u^{\alpha} x^{-\alpha} \qquad \text{where } x \geq u \text{ and } u > 0. \tag{3}$$

[3] A short survey about subexponential distributions is given by Goldie and Klüppelberg (1998).

[4] This is shown, for example, by Goldie and Klüppelberg (1998) pp. 442-443.

[5] See for example Embrechts, Klüppelberg, and Mikosch (1997) p. 335.

The tail probability $P(X > x) = 1 - F(x) = \overline{F}$ of a class B distribution is therefore $u^\alpha x^{-\alpha}$. The tail index α can be related to the moments of a distribution with Pareto tails. From[6]

$$E[X^k] = \alpha u^\alpha \int_u^\infty x^{k-\alpha-1} dx$$

it follows that only the first k-moments with $k < \alpha$ are bounded. This property is important to understand the class A, the class of stable or, as they are also called, α-stable distributions. The distributions of this class have Pareto tails with $\alpha < 2$, which implies infinite variance and, as a consequence, very fat tails. In spite of this restriction, class A is of great importance because asymptotic theory similar to central limit laws is possible.[7] Unfortunately, it is only possible to represent the stable distributions in a analytical way by the characteristic function (spectral representation). The density function can only be computed by numerical approximation.[8] In this paper we use a semi-parametric approach based on the Hill estimator that encompasses α-stable distributions. However, if the variance of a distribution is known to be infinite, then a parametric approach based on α-stable distributions would be more appropriate.[9]

Let us consider class C in more detail. There is a nice connection between this class and classical extreme value theory. A main topic of extreme value theory is the modelling of the fluctuation of sample maxima. If $X_1, X_2, \ldots$ is a sequence of iid random variables then the sample maximum M_n is defined as

$$M_1 = X_1, \qquad M_n = \max(X_1, X_2, \ldots, X_n), \qquad n \geq 2.$$

One of the most famous theorems of extreme value theory is the Fisher-

[6]The density of the Pareto law is $\alpha u^\alpha x^{-\alpha-1}$. See for example Dacorogna, Müller, Pictet, and de Vries (1998).

[7]Note that the central limit theorems are not applicable to stable distributions because the variance is infinite.

[8]See McCulloch (1998).

[9]For a recent application of α-stable distributions to asset pricing models see Kim (2002)

Tippett theorem.[10] It states that if the properly normalised sample maximum converges to a non-degenerate distribution, then this distribution belongs to one of the following three distributions (density functions):

$$\text{Fréchet: } \Phi_\alpha(x) = \begin{cases} 0, & x \leq 0 \\ \exp(-x^{-\alpha}), & x > 0, \end{cases} \qquad \alpha > 0$$

$$\text{Weibull: } \Psi_\alpha(x) = \begin{cases} \exp(-(-x^{-\alpha})), & x \leq 0 \\ 0, & x > 0, \end{cases} \qquad \alpha > 0$$

$$\text{Gumbel: } \Lambda_\alpha(x) = \exp(-e^{-x}), \quad x \in \mathbb{R},$$

If the distribution of the sample maximum of a give distribution converges to one of the three, then it belongs to the maximum domain of attraction of $\Phi_\alpha(x)$, $\Psi_\alpha(x)$ or $\Lambda_\alpha(x)$. It is possible to characterize distributions that belong to the maximum domain of attraction of the Gumbel distribution as having thin or moderately heavy tails. Distributions that belongs to the domain of attraction of the Weilbull distribution have a fixed upper end point. Important for our discussion of heavy tails is the class of distributions that belongs to the domain of attraction of the Fréchet distributions. A distribution belongs to this class if and only if its tails are regularly varying.[11] This exactly corresponds to our class C. The heaviness of the tails depends negatively on the tail index α. We turn to its estimation in the next section.

3 Estimating and testing the tail index

3.1 The Hill estimator

The tail-index α of equation 2 can be estimated with the Hill estimator. If the distribution under consideration is exactly Pareto, then the Hill estimator can

[10] See for example Embrechts, Klüppelberg, and Mikosch (1997) pp. 121-125.

[11] This is proven for example in Embrechts, Klüppelberg, and Mikosch (1997) pp. 131-132.

easily be constructed as a maximum likelihood estimation. The likelihood for the observation of k values $x_1, \ldots, x_k$ is

$$L(x_1, \ldots, x_k; \alpha) = \prod_{i=1}^{k} \alpha u^{\alpha} x_i^{-\alpha-1}$$

and the log-likelihood function

$$\log L = \sum_{i=1}^{k} \left(\log(\alpha) + \alpha \log(u) - (\alpha+1)\log(x_i)\right). \tag{4}$$

Maximizing equation 4 with respect to α gives the Hill estimator

$$\frac{1}{\hat{\alpha}} = \sum_{i=1}^{k} \left(\log(x_i) - \log(u)\right).$$

The distributions of class C are not exactly Pareto but their tails behave like the tails of a Pareto distribution. Therefore the Hill estimator can be used for the outer parts of the distribution. Let x_i be the ith order statistic such that $x_i \geq x_{i-1}$ for all $i = 2, \ldots, n$. If we choose to include k observations from the right tail in our estimate of α, the Hill estimator becomes

$$\hat{\gamma}(k) = \frac{1}{\hat{\alpha}(k)} = \sum_{i=1}^{k} \left(\log(x_{n-i+1}) - \log(x_{n-k})\right). \tag{5}$$

Whereas the concept and the calculation of the Hill estimator are straightforward, the choice of k is not. On the one hand, the approximation of the tails by the Pareto distribution improves as one moves further out into the tails. On the other hand, this leads to a reduction in the number of data points available, which drives up the variance. No general solution for this trade-off exists and many competing methods are available. An often used heuristical method is the Hill plot.[12] The Hill estimates are plotted for all possible values of k and an optimal k is selected by eye-ball search for a range that is robust with respect to k. Unfortunately, the Hill plot is sometimes erratic and may thus not be very useful.

[12]See for example Reiss and Thomas (2001) chapter 5.

Regression based estimator

A recently developed alternative to the Hill plot by Huisman, Koedijk, Kool, and Palm (2001) is especially useful for small samples. Their regression-based approach is based on an approximation of the asymptotic expected value of the Hill estimator as a linear function of k

$$E(\gamma(k)) \approx \frac{1}{\alpha} - ck. \tag{6}$$

Here c is a constant depending on parameters of the distribution and the sample size. If k becomes small, the bias goes down and the expectation goes to the true value $\gamma = \frac{1}{\alpha}$. The variance of the estimator increases with small k

$$var(\gamma(k)) \approx \frac{1}{k\alpha^2} \tag{7}$$

The idea of Huisman, Koedijk, Kool, and Palm (2001) is to use equation 6 in a regression analysis and regress the $\gamma(k)$ values (computed with an ordinary Hill estimator) against k as follows:

$$\gamma(k) = \beta_0 + \beta_1 k + \epsilon(k), \qquad k = 1, \ldots, \kappa.$$

The estimated $\hat{\beta}_0$ is an estimator of $\gamma = \frac{1}{\alpha}$. The authors propose to choose $\kappa = n/2$ where n is the sample size[13]. Furthermore they propose a weighted least squares method to improve the efficiency of the estimator by use of equation 7 and show that the resulting estimator has good small sample properties.

We are especially interested in α because it can be related directly to the existence of moments. One can show, by monte-carlo simulation, that the distribution of α is asymmetric. To construct confidence intervals for the point estimation of this parameter we use therefore a nonparametric percentile bootstrap proposed and tested in a similar context by Caers, Beirlant, and Vynckier (1998).

[13] The authors show that the results are robust with respect to the choice of κ.

Bootstrap based estimator

Another method to determine the optimal value of k has been developed by Danielsson, de Haan, Peng, and de Vries (1998). It is based on an evaluation of the mean squared error of $\hat{\gamma}$ defined as

$$MSE(k) = E\left((\hat{\gamma}(k) - \gamma)^2\right). \tag{8}$$

To evaluate this value the authors have proposed a bootstrap approach. The idea is to randomly draw with replacement from the original data set and to compute $\hat{\gamma}$ from this artificial sample. If this procedure is repeated for a large number of bootstrap samples the MSE can be calculated in principle. The optimal value of k is then found be minimizing equation 8. The problem is the value of γ in this equation. It is unknown and more problematic, and estimation is only possible if k is know in advance. Danielsson, de Haan, Peng, and de Vries (1998) have solved this problem by a method based on a combination of subsample bootstraps and the use of asymptotic theory. The details are summarized by Matthys and Beirlant (2000).[14]

3.2 Testing for structural breaks in the tail behaviour

Although it is well known that the volatility of asset returns varies over time, not much is known about the stability of the tail behaviour. To deal with this question, Quintos, Fan, and Phillips (2001) have recently developed a test for structural change in tail behaviour. If a priori a change from thinner to thicker tails is suspected then a recursive version of the Hill estimator (equation 5) is proposed by the authors. The test is based on

$$Y_T(t) = \left(\frac{tm_t}{T}\right)^{1/2}\left(\frac{\hat{\alpha}_t}{\hat{\alpha}_T} - 1\right). \tag{9}$$

[14] This method should not be confused with the bootstrapping of confidence intervals. We use it only to asses the robustness of the tail index estimations and do not provide confidence intervals because bootstrapping a bootstrap method is computational infeasible in our case.

The test is performed by the computation of equation 9 for a recursively increasing sample size t. In equation 9, $\hat{\alpha}_T$ is the Hill estimator for the whole sample size T and $\hat{\alpha}_t$ is the same estimator for the sample up to time t. m_t is equal to k in the Hill estimator. In addition, Quintos, Fan, and Phillips (2001) have proposed a modification to this test to deal with GARCH type dependency. It is known[15] that the Hill estimator is a consistent estimator for a large class of dependent processes but the variance is effected by dependency. This problem is solved by the authors by a variance correction. In this paper we use the more general version allowing for GARCH type dependency.

4 Data

Our analysis is based on tick data for the Bund future obtained from Deutsche Börse AG. It covers all transactions from January 1997 to December 2001, a total of 12.7 million trades. For each transaction, we have a time stamp (up to the centisecond), volume and price as well as the expiry date of the contract. Our data does not, however, include quotes.[16]

Contracts expire in March, June, September and December of each year. Trading is concentrated on the nearby maturity and switches to the next contract within days just before expiry. Since contracts with different expiry dates tend to differ in price, we look only at the most actively traded maturity on each trading day. Since the changeover occurs very rapidly, we loose only about 5 % of the observations. We then link the individual contracts to a long series covering the whole sample period.

Trades occur at irregular intervals whereas the statistical methods used in this paper require equally spaced data. We create such a series by recording

[15]See for example Resnick and Stărică (1996).

[16]Until the December 1998 contract, each Bund future refered to a notional German government bond with a face value of 250,000 DM and a coupon of 6 %. The Euro Bund future, which replaced the Bund future in the transition to EMU, has a contract value of 100,000 Euro.

Year	Method	5 minute		1 hour		1 day	
		Left-tail	Right-tail	Left-tail	Right-tail	Left-tail	Right-tail
1997-2001	Regression	3.01	3.33	3.36	3.90	4.55	6.28
		[2.91, 3.09]	[3.20, 3.46]	[3.06, 3.73]	[3.52, 4.35]	[3.35, 6.98]	[4.61, 10.16]
	Bootstrap	3.16	3.44				
1997	Regression	2.46	2.88	2.91	3.40		
		[2.34, 2.61]	[2.72, 3.09]	[2.43, 3.58]	[2.74, 4.34]		
	Bootstrap	3.16	3.75				
1998	Regression	2.46	2.88	3.18	3.77		
		[2.33, 2.62]	[2.70, 3.09]	[2.66, 4.12]	[3.18, 5.02]		
	Bootstrap	2.66	2.77				
1999	Regression	3.75	4.41	3.51	3.91		
		[3.49, 4.04]	[4.06, 4.81]	[2.94, 4.55]	[3.23, 5.07]		
	Bootstrap	3.46	3.86				
2000	Regression	3.22	3.19	4.09	4.59		
		[2.98, 3.46]	[2.96, 3.45]	[3.29, 5.37]	[3.67, 6.15]		
	Bootstrap	2.72	3.12				
2001	Regression	2.78	2.91	3.38	4.37		
		[2.60, 2.97]	[2.74, 3.11]	[2.79, 4.57]	[3.62, 5.87]		
	Bootstrap	2.66	3.17				

Table 1: The tail index of Bund futures returns

price of the last transaction in each time bracket. Intervals where no transactions take place are treated as missing data. This means that we do not fill in with the last available price, as is often done in the literature. Overnight returns are discarded for frequencies higher than a day. We also construct a series with daily data. In this case, we take the last trade on or before 5 p.m., when trading activity is at its peak.

5 Empirical results

5.1 Tail estimates

The results of the tail index estimations are summarized in table 1. Remember from section 2 that α declines as the tails of the distribution become thicker and that a tail index of less than 4 implies infinite forth moments and therefore an infinitely high kurtosis. Let us first discuss the results for the complete sample (1997-2001) shown at the top of table 1. For the five-minute returns, both methods estimate α to be greater than 3 but smaller than 4. The values in square brackets are the upper and lower bounds of 95% bootstrap confidence intervals for the regression-based tail index.[17] High-frequency returns thus appear to have fat tails with infinite kurtosis, possibly infinite third moments but definitely finite variance. At lower frequencies, the tail index increases, and the tails become thinner.[18] For daily returns, the estimates for α obtained by the regression-based method are above 4, suggesting a bounded fourth moment. But it is important to note that even in this case the tails remain heavier than those of the normal distribution, which has a very large, in theory infinite, tail index. Therefore our results suggest that the first of our questions can be answered in the affirmative: Yes, the returns on the Bund future do have thick tails. Moreover, the left tails, corresponding to negative returns, tend to be slightly thicker than the right tails irrespective of the frequency.

Let us now turn to the estimates for the different years of our sample. Unfortunately, only results for frequencies higher than a day are available as there are not sufficient data points at the daily level. We find that the tails seem to be particularly fat during 1998 and 2001, and less so in 1999 and 2000. This is not surprising, given that 1998 saw some of the worst turbulence in the international financial markets in living memory, and 2001 was marred by the September 11 shock. In contrast, 1999 and 2000 were rather tranquil years. The confidence intervals of the 5 minute losses (left-tail) are non-overlapping for 1998 to 2001. This finding is a first sign of time variation in the tail behaviour, which we shall explore in the following subsection.

[17] Computing confidence intervals for the bootstrap method would involve bootstrapping the bootstrap. In our case, it has turned out to be unfeasible

[18] We use the bootstrap method only for the 5 minute returns because it is known from the literature that it requires a very large sample size (>5000), see Matthys and Beirlant (2000).

5.2 Structural break in the tail behaviour during 1998 and 2001

The estimation of the tail index α for fixed subperiods provides only rough-and-ready evidence for time variation in the tail behaviour. To answer our second question in a more rigorous manner, we apply the test on structural change in the tail behaviour that has been described in section 3.2. In particular, we are interested whether events such as the turbulences in international financial markets in 1998 or the attacks on September 11, 2001 have affected the tail behaviour of the return distribution. In order to limit the computational burden, we do the estimations for each year separately. Here we present the results for the years 1998 and 2001, when the tails of the distribution seemed to have been particularly heavy.

The recursive test statistic of equation 9 for the year 1998 is plotted in figure 2. The critical value at the 1 % confidence level is 2.54. We find that the maximum of the test statistic is well above 20 and the null hypothesis of constant tail behaviour is therefore soundly rejected. The test statistic has two peaks: one at the end of July 1998 and another at the beginning of September 1998. These dates roughly match the Russian devaluation on August 17th and the LTCM crisis.[19]

The corresponding test statistic for the year 2001 is plotted in figure 3. Again the maximum is above twenty and the null hypothesis of stability is clearly rejected. But the shape of the plot is very different from that of the year 1998. During most of 2001, the test statistic is low but it rises sharply towards the end of the year, possibly in response to the September 11 attacks. If one performs a change test for the year up to the end of August only, the highest value of the test statistic is less than 1.8 and the null hypothesis cannot be rejected.[20]

[19]LTCM was recapitalized on September 23rd, but there had been massive disruptions in the markets during the previous weeks.

[20]We have also found breaks during the other years of our sample, but the test statistics were much lower and not associated with any identifiable events. For the sake of brevity,

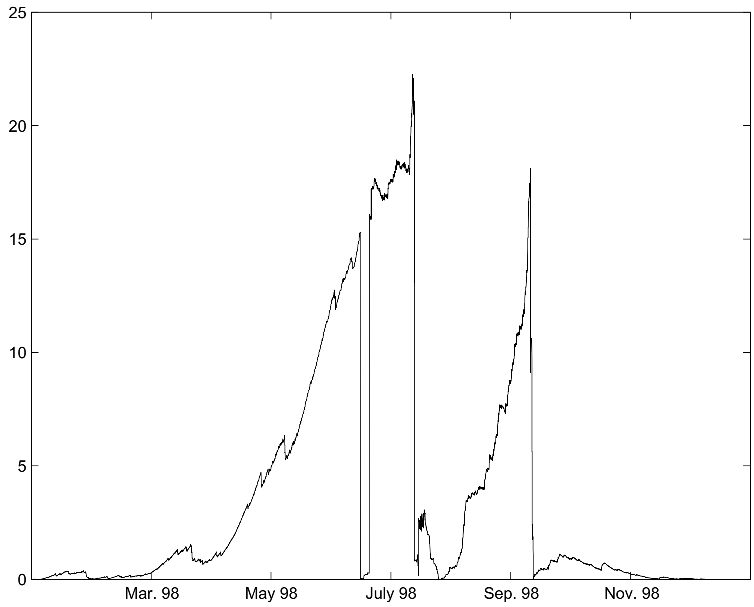

Figure 2: Change test for 1998

The time variation of the tails seems to be limited to high-frequencies, as we could not find any breaks in the tail behaviour for the one hour and one day returns.

5.3 The tail index as a risk indicator

Let us now focus on the third and final of the questions posed in the introduction: Can we use the tail index as an indicator for financial market risk and does it add value in addition to classical indicators? Although there is a wide variety of indicators for financial risk, we limit our analysis to realized volatility and use this measure to compare it with the tail index.

Realized volatility

Unusually strong price fluctuations are an important characteristic of financial risk. It is therefore natural to use volatility measures as indicators for

they are not presented here.

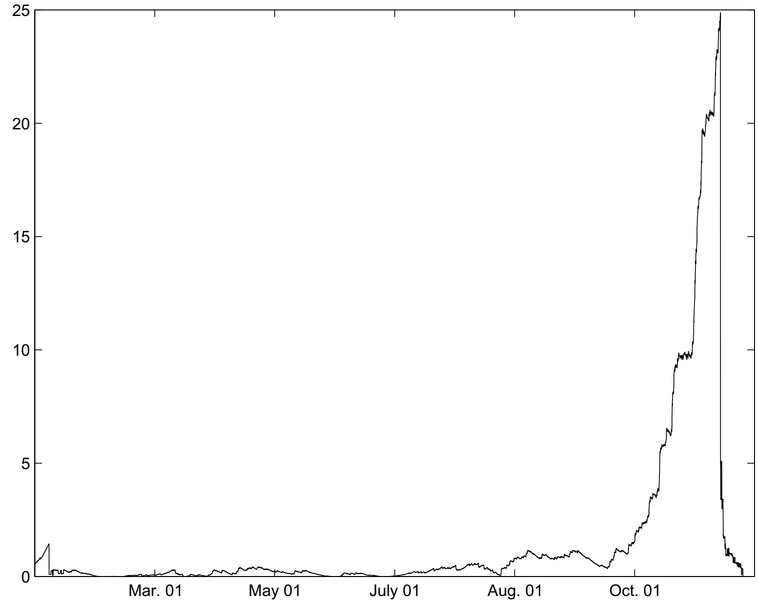

Figure 3: Change test for 2001

financial turbulence. In contrast to tail indices from extreme value theory, they are computed using the complete support of the distribution of returns. We compute the realized volatility for each trading day by summing up the squared five-minute returns

$$\hat{\sigma}_t = \sum_i (\Delta log P_i)^2. \tag{10}$$

Anderson, Bollerslev, Diebold, and Labys (2001) show that this approaches the price volatility of a continuous process as the intervals between the observations goes to zero.

Realized volatility versus tail index

We asses whether the tail index is a useful indicator for financial market uncertainty by means of a recursive estimation of the tail index over a rolling 20-day window using equally-spaced date with 5-minute intervals. On average, around 1,000 data points are included in the estimation of the tail

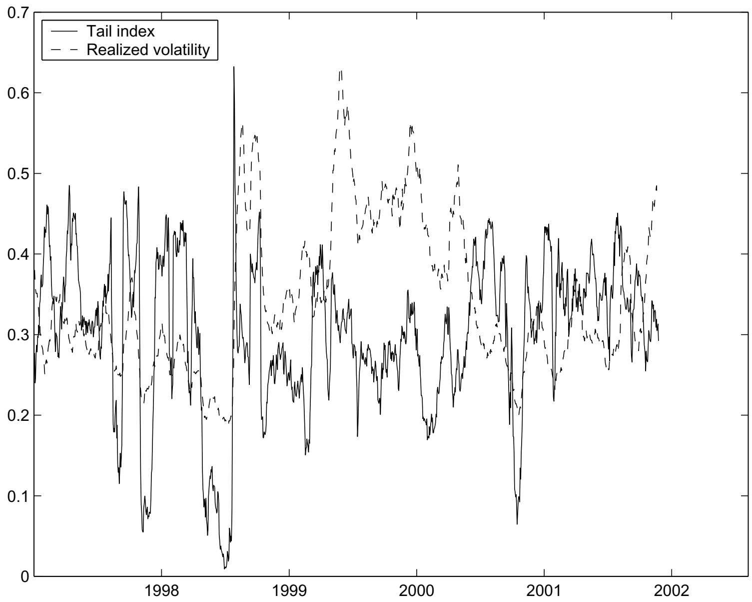

Figure 4: Tail index and realized volatility

index.[21] To compare this index with a classical measure of financial market uncertainty, we have computed the average of the realized volatility for the last 20 days. The two indicators are plotted in figure 4. The dark line is $\hat{\gamma}$ which is the inverse of the tail index $\hat{\alpha}$. A high value of $\hat{\gamma}$ thus implies thicker tails. The dotted line is the rescaled realized volatility.

The tail index and realized volatility move in parallel during most of our sample, suggesting that the tail index added no additional information beyond that contained in our volatility measure. A closer inspection of the graph reveals that this view is not correct. For example, during 1999 realized volatility was relatively high but the tail index relative low. Consequently, this period was a good one for classical risk management. The volatility is captured by standard methods and the relative thin tails might be a justification for the normal approximation that is commonly used. The development during the following year was less benign for risk managers. Volatility declined but the tail index rose for more than half a year. Although the decline

[21] The precise number varies because of missing values.

Year	Correlation between tail index and realized volatility
1997	0.51
1998	0.49
1999	0.14
2000	-0.06
2001	-0.32

Table 2: Correlation between tail index and realized volatility

in volatility suggested lower risk, the actual probability of extreme events went up rather than down. As a consequence, the risk of extreme events could easily have been underestimated and caught market participants on the wrong foot. It is therefore fair to say that the tail index does indeed add information beyond that contained in realized volatility. The information contend of the tail index relativ to realised volatility can be analysed further by the correlation between the two indexes. Table 2 shows the correlation coefficients for the years 1997-2001. In the years 1997-1999 the correlation between the tail index and the realized volatility is positive but in 2000 and 2001 the correlation is even negative. This means that the two indexes move on average in the opposite direction during this years. The tail index contribute strongly with information in addition to the volatility.

5.4 Implications of the tail behaviour on Value-at-Risk

To emphasize the importance of the variation of the tail behaviour through time, we compare value-at-risk measures based on the tail index and on the normal distribution, respectively. Value-at-Risk (VaR) is generally defined as the "possible maximum loss over a given holding period with a fixed confidence level". That is VaR at the $100(1-\alpha)$ percent confidence level is defined as the lower 100α percentile of the return distribution.[22] Our VaR measures are based on 5 minute returns and quoted in basis points.

Year	Volatility	Tail-Index ($1/\alpha$)	VaR 99% (normal)	VaR 99% (tail-index) (left)	VaR 99% (tail-index) (right)
1997	0.244	0.406	5.7	10.1	8.8
1998	0.263	0.406	6.1	11.2	9.7
1999	0.343	0.267	8.0	10.2	9.2
2000	0.293	0.311	6.9	9.7	9.8
2001	0.262	0.359	6.1	9.7	8.3

Table 3: Value-at-Risk for the years 1997-2001

The results are collected in table 3. The second column contains the volatility of the futures returns measured by the standard deviation times 1,000. Realised volatiltiy increases between 1997 to 1999, and falls during the following two years. The VaR calculated under the assumption of normally distributed returns is given in column four. Clearly this VaR measure is driven by the volatility because the distribution is fixed.[23] The values imply a significant decline in the market risk in the years 2000 and 2001 in comparison to 1999. In column five the VaR based on the tail-index for the left-hand of the distribution is given.[24] It is a measure for the riskiness of a long trading position. Two points are important. Firstly the VaR based on the tail-index is always higher than the VaR based on normal distribution. This is a direct consequence of the fat tails. Secondly, and more important, the VaR based

[22]It is very common to use α as the probability in a VaR measure and we do not like to change this habit. Please do not mix up this α with the tail-index.

[23]The mean of the return distribution is virtually zero at high frequencies. Any changes in the distribution are therefore driven by the higher moments

[24]How VaRs can be calculated based on tail-index and Hill estimation is described in Gourieroux and Jasiak (2001). The basic idea is to use an empirical quantile (in our case the 90% quantile) to compute an extreme quantile based on the tail index and the pareto like behaviour of the tails.

on the tail-index does not decline much in 2000 and 2001. The reason is the increase of the fatness of the tails in 2000 and 2001 in comparison to 1999. This partially compensates the decline in volatility. It is therefore dangerous to asses the market risk exclusively on the basis of volatility measures. In column six we have added the VaR based on the tail-index for the right-hand of the distribution. According to the tail indexes in 1 the right-hand of the distribution is less fat tailed than the left-hand. This is the reason for the lower associated VaR measures and implies a lower risk of a short position in the BUND future than a long position. Nevertheless our main argument about the importance of time variation in the tail is not effected. A risk assessment based on the normal distribution implies a symmetric VaR and the risk of a short position in the BUND future should lower in 2000 than in 1999 because the variance is lower in 1999. Contrary to this argument the VaR values in table 3 show, because of the increase in the tail fatness, an even higher risk in 2000. Neglecting the time variation of the tail behaviour results in an absolut misleading assessment of the market risk.

6 Conclusion

In this paper we have focused on three questions. (i) Are the bond futures returns heavy-tailed? (ii) Is the tail behaviour constant during time? (iii) Does the tail index add further information with respect to classical indicators of financial market uncertainty?

We have found a significant heaviness of the tails of the Bund futures log-returns. The tail index is on average around 3, implying the nonexistence of the fourth moments. The tails of the 1-hour and 1-day returns are slightly thinner than the 5-minute return tails but remain thicker than those of the normal distribution.

With the aid of a recently developed test for changes in tail behaviour we have identified several breaks in the degree of heaviness of the log-return tails. Such breaks were particularly pronounced during 1998 and 2001, probably in relation with the Russia and LTCM crises in the former, and the September 11 attacks in the latter year.

Another finding is that the behaviour of the tails of a distribution is not necessarily captured by measures for volatility. For example, in 2000 volatility declined, suggesting a reduction in risks, whereas the probability of extreme price changes, as measured by the tail index, actually increased. This shows that the tail index contains important information for financial market risk assessment beyond that captured in standard volatility measures.

In some sense our paper is a first step in modelling time variation of tail behaviour. The results we have presented show the need for such an investigation. Unfortunately we can not forecast the tail behaviour because the used method is only able to extract information for the tail behaviour out of a given data set. There is no explicit modelling of the time variation. There is some very recent research about autoregressive conditonal kurtosis by Brooks, Burke, and Persand (2002), which seems promising. The idea is to model time variation in the fourth moment similar to the well knows GARCH type modelling. Nevertheless, our results show that the fourth moment may not exist. More research in this area is indicated for the future.

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