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May 1999

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Abstract

This paper proposes a forward-looking indicator of risk in the foreign exchange markets calculated from the implied volatilities of currency options according to the Garman-Kohlhagen model. We discuss the properties of such indicator and stress that it is related to a notion of risk that does not coincide with that of Gaussian risk underlying most mainstream models. We postulate that it is associated with a broader definition of risk, which we call hazard in order to avoid confusion. The properties of the Global Hazard Indicator (GHI) are assessed against the background of the market turbulence in 1998. This period has been characterized by abnormal fluctuations in the exchange rate markets spurred by a sequence of shocks in some emerging economies and in South East Asia, which have raised fear of contagion in developed countries.

JEL classification codes: F01;F31

Keywords: Currency markets; exchange rates; wild risk; currency options; implied volatility.

Introduction

As the euro will become one of the dominant international currencies, its exchange rate dynamics will have wider repercussions, to be monitored and examined in the lights of the global economic conditions, rather than merely in relation to the trade balance, or the domestic cyclical phase.

The world foreign exchange market is structured around a small number of currency pairs, broadly traded internationally. It often happens that one of the currency pairs exhibits strong volatility, owing to asymmetric shocks affecting one particular country. Nevertheless wide swings (and large volumes of trade) in one major exchange rate market have repercussions around the world, so that the currency markets tends to be either in an "excited" or in a "quiescent" state. Stated somewhat differently, movements in one major spot exchange rate often affect other spot exchange rates[1] beyond what could be expected if they followed a simple stochastic process or were exclusively influenced by the fundamentals. This phenomenon is highlighted by the co-movements in currency volatilities, which are not subject to a tight no-arbitrage condition unlike spot rates.[2]

The literature on exchange rate volatility has focused almost exclusively on the volatility of currency pairs. Several methods have been proposed for example to model empirical volatilities, while a vast literature has examined whether volatility is excessive compared to the predictions of some model or if the historical volatility exceeds the boundaries consistent with the efficient market hypothesis.[3]

However these studies by focusing on particular bilateral exchange rates have often neglected the tensions in the global exchange rates market, which are an essential source of information to policy makers for a twofold reason.

1) The simultaneous fluctuation of exchange rate volatilities around the world implies that exchange rates between currency pairs do not necessarily reflect current macroeconomic fundamentals in two countries, but are affected also by the broader conditions in the world economy over which national policy makers might have little control.
2) Given this sensitivity to global conditions, simultaneous exchange rate fluctuations provide useful information on the markets' sentiment and perception of the risks that threaten the world economic stability.

This latter point leads to some important considerations about risk and its measurement. Volatility, risk and variance are roughly considered synonyms in most research in finance and exchange rates. While this association is justified in a Gaussian world, ruled by the central limit theorem, the reality is more variegated.

There exist various forms of risk (not necessarily measurable through statistical analyses performed on a set of data) whose nature deserves careful attention. In the 1960's Mandelbrot was the first to point out by a rigorous approach that the notion of risk has qualitative connotations that are difficult to pin down because "the variety of natural and social phenomena is infinite, while the mathematical techniques capable of capturing them are very few".[4] In this context Mandelbrot

1 A glance through articles in the financial press shows how often commentators and market analysts use this notion of spill over to justify swings in exchange rate markets that are unrelated to fundamentals.
2 A kind of no arbitrage condition on volatilities holds in the following sense. Given three liquid currency pair markets, the volatility in one of the markets cannot exceed the sum of the volatilities in the other two.
3 The interested reader might find a very useful survey in MacDonald and Taylor (1992), or can refer to many textbooks, for example Isard (1995).
4 Mandelbrot (1997b) part II.

makes a distinction between benign risk and wild risk. Loosely speaking the first is the kind of risk associated with the Gaussian paradigm; the second is the risk deriving from large, rare and erratic jumps and/or the persistence of deviations from the mean.[5] As this second kind of risk cannot be treated within the realm of standard statistical models it has been largely ignored in the literature. Exceptions are represented by studies based on Pareto-stable distributions, long memory processes, chaotic dynamics, fractal structures and so on.

However this paper does not follow these avenues: our attention focuses on the notion of wild risk, which in order to avoid confusion we prefer to call **hazard**, as measured by the currency options implied volatilities derived from the Garman and Kohlhagen (1983) (henceforth GK) model. Paradoxically, as it will be explained in section 3.1 below, this interpretation of implied volatilities as a measure of hazard is only possible because the normality assumption underlying the GK model does not hold in reality (if it did the actual implied volatilities would be constant). In heuristic terms we interpret the extension of the GK model to a multicurrency setting as providing a yardstick to measure the global wild risk (hazard) in foreign exchange markets.

Therefore we construct an index of the global currency markets hazard from actual data on currency options implied volatilities. This Global Hazard Index (GHI) is a real time, forward-looking market-based indicator, which provides a composite measure of hazard in the major spot exchange rate markets across the world, rather than the risk from trade in two particular currencies.[6] Furthermore we show that from the currency option data one can derive a term structure of hazard whose slope is a powerful indicator of market tensions.

It is reasonable to postulate that a measure of aggregate hazard in the currency markets can be related to expectations on key economic developments as well as on policy variables. Therefore the GHI constitutes a synthetic measure that can be used for two main purposes:

1) Comparing currency market hazard across time and different global economic conditions;
2) Monitoring the effects of policy actions on market behavior and expectations.

This paper is divided into five additional sections. The first one focuses on co-movements of exchange rates and presents an unconventional measure of these co-movements in the currency markets. By arguing that there is clear evidence of a common process driving exchange rate fluctuations it sets the stage for the second section which presents the mathematical underpinning of the GHI based on the GK currency option valuation model. The third section discusses some statistical properties of the GHI and the term structure of hazard that can be inferred from the data. The fourth analyzes the behavior of this indicator in the lights of the recent episodes of currency turmoil. The fifth and final section summarizes the results.

Throughout the paper, we adopt as benchmark the GK option pricing model, which assumes that exchange rates follow a stochastic process in continuous time, but for the sake of brevity we will omit to provide the details of the underlying probability and measure theory. The interested reader is referred to the original article or to the many textbooks on continuous time finance, for example Duffie (1992), part II.

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5 Mandelbrot, with reference to the Biblical characters, called these two effects the Noah and Joseph effects.

6 On the contrary measures of volatility calculated from the actual data are essentially backward looking as they reflect past behavior and neglect the markets' view about future dynamics.

1 The quadratic variation and the co-movements of foreign exchange spot rates

The first point we wish to emphasize is that the volatilities of major exchange rates have a common behavior that has rarely been examined in the literature. Exceptions are represented by certain studies on market efficiency tests or the validation of exchange rate models (see for example Bartolini and Bodnar (1995)). This section shows how these co-movements can be investigated by comparing the sequence of exchange rate quadratic variations.

The quadratic variation of the (logarithm of the) spot is a natural measure of the volatility of the foreign exchange spot rate in a continuous time setting.

Definition 1: **Quadratic variation associated with the partition Δ.** (Kunita (1990))

Let X_t be a continuous stochastic process $t \in [0, T]$, let Δ be a partition of the interval [0,T] such that $\Delta = \{0 = t_0 < t_1 < \ldots\ldots < t_n = T\}$ and let $|\Delta| = \max_k (t_{k+1} - t_k)$ be the largest interval associated with the partition Δ. Then the continuous process

$$< X >_t^{\Delta} = \sum_{k=0}^{t-1} (X_{t_{k+1}} - X_{t_k})^2 \tag{1}$$

is called the quadratic variation of X_t associated with the partition Δ.

Definition 2: **Quadratic variation.** (Kunita (1990))

Given a sequence of partitions $\{\Delta_n\}$ whose maximum interval tends to zero, i.e. $\lim_{n\to\infty} |\Delta_n| \to 0$, the quadratic variation is defined as the following probability limit (if it exists and is independent almost surely of the choice of the partition sequence $\{\Delta_n\}$)

$$< X_t > \equiv \text{p}\lim\{< X >_t^{\Delta_n}\} \tag{2}$$

Fig. 1 depicts the behavior over time of the estimated quadratic variations of some selected monthly bilateral exchange rates during the last twenty years (from January 1979 to August 1998) recorded by the Bank of International Settlements at 2:15 GMT. It displays three main features.

1) It is an irregularly increasing function, meaning that periods during which risk is predominantly high, alternate randomly with periods of relatively lower risk.
2) Market risk tends to persist in the sense that higher risk months tend to cluster together and the same is true for months when the risk is lower; however the size of these clusters, i.e. the length of high or low risk periods, is erratic. In general high-risk periods tend to be shorter.
3) The periods of high and low risk in different currency markets tend to coincide, which is the phenomenon we call co-movement of volatilities.

This latter visual impression can be corroborated by comparing the correlation of the increments of the quadratic variations, (i.e. the squares of the increments of the log spot exchange rate) for 5000 daily observations over the last 20 years and the same correlation obtained from simulated realizations of Brownian motion.[7] The results are shown in table 1.[8]

7 *Notice that the empirical quadratic variations obtained for the Brownian motion are much more rectilinear, than those appearing on the graph above.*

8 *The complete array of the comparison is provided in the Annex.*

Table 1. Correlations of daily quadratic variation increments over the period January 1979 – August 1998 compared to simulated realizations of Brownian motion

SPOT 1	SPOT 2	**Actual**	**Simulated Bm**
USD/DEM	USD/JPY	**47%**	**25%**
USD/DEM	DEM/JPY	**40%**	**27%**
USD/JPY	DEM/JPY	**44%**	**23%**

Data Source: Bank of International Settlements

Two points are noteworthy:

1) The correlation of increments of the quadratic variation calculated over the true data consistently exceeds that obtained from the simulated Brownian motion.
2) The correlations in the first column are within a narrow range, which supports the hypothesis of a common global behavior of exchange rate volatilities.

The comparison between increments in the quadratic variations puts in evidence the existence of a common stochastic process (likely of non Gaussian nature) driving the fluctuations in exchange rates. However an analysis based on quadratic variation cannot be used to build a rigorous real-time indicator of global hazard in the currency markets because it would be a backward looking analysis, while, in our view, a useful indicator of market volatility should be forward looking.

The next section will focus on an indicator that can better capture the agents' expectations, while providing a measure of hazard quickly reacting to changes in market conditions and policy actions.

2 An indicator of global volatility in the foreign exchange markets

An indicator of global currency markets hazard to be used for real-time monitoring of market conditions and the evaluation of policy actions should have four critical characteristics.

1) It should be forward looking in the sense that it should capture the market assessment of future developments rather than merely track the evolution of past behavior.
2) It should be unique in the sense that its calculation should not involve any parameters whose value is determined by subjective criteria.
3) It should allow a comparison with the volatilities of the bilateral exchange rates.
4) It should be easy to calculate and its construction be as straightforward as possible from available market data

In this section we present an extension of the GK model demonstrating how such an indicator can be calculated from currency option data.

In a world with only two currencies a meaningful indicator would be the at-the-money forward implicit volatility between the two currencies (colloquially referred to as "implied vol" in the jargon of traders).[9] This volatility can be translated into an option price by using the GK formula, which is essentially a variant of the Black-Scholes (1973) stock option pricing formula.

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9 Two points are worth mentioning about "implied vols": 1) They are the standard way of quotation of currencies options (just like French BTAN or Finnish government bonds are quoted in yields instead of prices); in other words the option contract specifies that the Garman-Kohlhagen option pricing formula must be used to translate the implied volatility into the price of the option. 2) The most liquid contracts are the one-month and the three-month options.

In the real world currency pairs have liquid option markets to the extent that they are traded in a liquid spot market. In practice this is the case only for a limited number of currency pairs: dollar-mark (now euro-dollar), dollar-yen and mark-yen (euro-yen), and to a lesser extent for the British pound and the Swiss franc against both dollar and mark.

How can we find an analogous notion of the at-the-money forward implicit volatility in a multi-currency case? Given that the properties of the implied volatilities are based on a specific option pricing model and that *such model has a precise mathematical underpinning*, the question is eminently mathematical.

In the next subsections we show how to answer this question using two theorems. Theorem 1, the central one, demonstrates that there exists one and only one measure extending the concept of bilateral implied volatility to the multi-currency case. Theorem 2 provides an explicit expression for this measure in the 3-currency case. The proofs require a detailed presentation of the multi-currency extension of GK model, which is given in subsection 2.1. Subsection 2.2 introduces a relevant geometrical notion and presents the central mathematical results of the paper. Subsection 2.3 utilizes those results to build a global indicator of volatility on currencies markets. The reader not interested in the mathematical details might skip to equation (18) without loss of continuity.

2.1 Presentation of the model

2.1.1 Basic features

As a matter of notation we denote currencies with capital letters. The set of currencies is assumed to be finite and is denoted by the symbol Д.

Let us denote by S_{MN} the price of currency M expressed in currency N (the number of units of currency N necessary to buy one unit of currency M), i.e. the spot of the currency pair MN. We denote by L_{MN} the natural logarithm of this spot (the "log-spot"), in symbols

$$L_{MN} \equiv \ln(S_{MN}) \tag{3}$$

If more than two currencies are involved, then the exchange rates are constrained by an obvious no arbitrage relationship (the "cross-currency" equality), which in terms of log-spots is expressed by

$$L_{AB} = L_{AC} + L_{CB} \tag{4}$$

Equality (4) has the standard form of the of vector sum, sometimes called the Chasles relation.

2.1.2 Dynamic features

The multi-currency GK model assumes that the vector of log-spots follows an Itô process with constant diffusion term (this term being a matrix). Let us denote by $\boldsymbol{\sigma}_{MN}$ the column of this matrix that corresponds to L_{MN}. It follows then from equality (4) that those $\boldsymbol{\sigma}$ vectors fulfil:

$$\sigma_{AB} = \sigma_{AC} + \sigma_{CB} \tag{5}$$

This formula, too, has the form of the Chasles relation. Each σ is a parameter of the model, which according to equality (5) has the following properties:

- Each currency is identified by a point in an Euclidean space E, (hence Д$\subset$ E.)
- The $\boldsymbol{\sigma}_{MN}$ are then simply the vectors joining point M to point N (see Graph 1 below) in the space Д.

Thus thanks to equality (4) we can consider a vector variable $\mathbf{x}$ such that

$$\forall M,N \qquad L_{MN} = \mathbf{MN} \cdot \mathbf{x} \tag{6}$$

(where bold fonts indicate vectors.) The vector variable $\mathbf{x}$ is uniquely defined by (6).

The dynamics of the exchange rate postulated by the model can be written in the following form

$$dL_{MN} = \mathbf{MN} \cdot \mathbf{dW}_t + (...) \cdot dt \tag{7}$$

where $\mathbf{W}_t$ is a (multivariate) Wiener process taking values in E, and (…) is the drift (whose precise specification is immaterial). Equation (7) can be equivalently written as:

$$\mathbf{dx} = \mathbf{dW}_t + (\mathbf{...}) \cdot dt \tag{8}$$

2.1.3 Riskless interest rates

The only other parameters of the multivariate GK model are the riskless interest rates associated with each currency. We will denote by r_X the rate corresponding to currency X. Generally, the currency-points are linearly independent. In this case, two features are noteworthy:

- The riskless rates do not have to fulfil any particular constraint.
- There exists a unique affine[10] function mapping E into the real axis $\Re$, which associates each currency to its own riskless rate. (We will denote this function by R).

(Note that the more general case, where currencies are *not* linearly independent, is of no practical interest for exchange rate markets studies. It would have some theoretical interest, as it provides a link between the GK model and the Heath Jarrow Morton (1992) models of the term structure, but this is beyond the scope of the present paper. Hence we will always assume that currencies are linearly independent.)

10 Affine functions are at the same time concave and convex, hence of the form constant plus linear function.

2.1.4 Pricing argument and methodology

Our extension of the GK model belongs to the class of Black and Scholes (1973) models and relies on the same no-arbitrage pricing argument. The pricing algorithm follows as well the same methodology.

Definition 3: **N-neutral probability**.

By Girsanov's Theorem (see Duffie 1992)) for any currency $N \in Д$, there exists one unique martingale equivalent probability measure i.e. one having the same null sets as the probability measure defined as the weak solution of (7), and such that for any currency M in Д, the ((r_N-r_M)-discounted) spot S_{MN} follows a martingale. We call this unique probability the N-neutral probability. We denote it by Π_N. We also denote by E_N the expectation operator under probability Π_N.

The next step consists of finding an explicit expression for these probabilities.

2.2 Main results

This subsection presents the fundamental theorem, which, in a nutshell proves that the mathematical invariant of the multi-currency extension of the GK model involves a well-known geometrical concept, the orthocenter of a set.

Definition 4: **Orthocenter**.

Without loss of generality we can assume that E is linearly spanned by Д. Since we have assumed that points of Д are linearly independent, a standard result of geometry applies and ensures that there is a unique point equidistant from each point of Д. We call this point the *orthocenter* of the points in Д. We denote it by Z. We call the distance between Z and a point in Д the *radius of the currency system*. We denote it by Ω.

We start by proving the following auxiliary result.

Lemma 1: Under the P-neutral probability, one obtains

$$\begin{aligned} dL_{MN} &= \mathbf{MN} \cdot \mathbf{dW_t} + \left(r_N - r_M - \frac{\mathbf{MP}^2 - \mathbf{NP}^2}{2} \right) dt \\ \frac{dS_{MN}}{S_{MN}} &= \mathbf{MN} \cdot \mathbf{dW_t} + \left(r_N - r_M - \mathbf{MN} \cdot \mathbf{PN} \right) dt \end{aligned} \tag{9}$$

Proof: In the special case where N = P, one can write

$$\frac{dS_{MP}}{S_{MP}} = \mathbf{MP} \cdot \mathbf{dW_t} + \left(r_P - r_M \right) dt \tag{10}$$

thanks to definition 3. By applying Ito's lemma, one gets

$$dL_{MP} = \mathbf{MP} \cdot \mathbf{dW_t} + \left(r_P - r_M - \frac{\mathbf{MP}^2}{2} \right) dt \tag{11}$$

Applying (11) to two distinct currency pairs, and using Chasles relation (4), one can eliminate the restrictive condition N = P, and obtain the first equation in (9). By applying Ito's lemma to this first equation one obtains the second. This completes the proof of the lemma.

We state now the main result.

Theorem 1: Under the P-neutral probability, the motion of **x** is given by

$$\mathbf{dx} = \mathbf{dW_t} + (\nabla R - \mathbf{ZP})\,dt \tag{12}$$

where ∇ indicates the gradient.

Proof: Let us denote by H the middle point of the MN segment. A straightforward application of the identity $(x+y)(x-y) = x^2 - y^2$ allows us to

$$\frac{\mathbf{MP}^2 - \mathbf{NP}^2}{2} = \mathbf{HP} \cdot \mathbf{MN} \tag{13}$$

But since Z is equidistant from N and M, it follows that **MN** is orthogonal to **ZH**. Hence (13) leads to

$$\frac{\mathbf{MP}^2 - \mathbf{NP}^2}{2} = \mathbf{ZP} \cdot \mathbf{MN} \tag{14}$$

Therefore the first expression in (9) can be rewritten as

$$dL_{MN} = \mathbf{MN} \cdot \mathbf{dW_t} + (r_N - r_M - \mathbf{MN} \cdot \mathbf{ZP})\,dt \tag{15}$$

Since R is the affine function such that $r_X = R(X)$, (15) leads to

$$dL_{MN} = \mathbf{MN} \cdot \mathbf{dW_t} + (\mathbf{MN} \cdot \nabla R - \mathbf{MN} \cdot \mathbf{ZP})\,dt \tag{16}$$

Then (12) follows from (6), and the proof of Theorem 1 is complete.

The central point of the paper is that formula (12) appears as the simplest possible expression of the general P-neutral probability. Any invariant of the model should appear in (12). Therefore, the only geometric invariants of the model are R and Z, and the only numerical invariant is Ω.

We now give the expression of Ω in the particular, but important, 3-currency case.

Theorem 2: If Д = {A, B, C}, then Ω is given by

$$\frac{\|\mathbf{AB}\|\,\|\mathbf{BC}\|\,\|\mathbf{AC}\|}{\sqrt{(\|\mathbf{AB}\|+\|\mathbf{BC}\|+\|\mathbf{AC}\|)(-\|\mathbf{AB}\|+\|\mathbf{BC}\|+\|\mathbf{AC}\|)(\|\mathbf{AB}\|-\|\mathbf{BC}\|+\|\mathbf{AC}\|)(\|\mathbf{AB}\|+\|\mathbf{BC}\|-\|\mathbf{AC}\|)}} \tag{17}$$

Proof: In the case of three currencies, the representative points describe a triangle. This triangle follows the famous *law of sines*, according to which the ratio between a side and the sinus of its opposite angle is the same for the three sides. This ratio is then a characteristic of the triangle with respect to the particular Euclidean metric. Elementary calculations show that it is equal to twice the distance between a currency-point and the orthocenter, and provide expression (17) for this distance.

2.3 Empirical construction of the indicator.

Since the radius of the currency system is the only mathematical invariant of the model, one is led to use it in the construction of a global volatility indicator.

It should be noted that other mathematical constructions, such as averages or weighted averages of volatilities, do not generalize the notion of implied volatility to the case of a multi-currency GK model. In fact none of these constructions plays any role in the generalized expression of the risk-neutral probabilities with respect to the various currencies, while a quantity generalizing the concept of volatility must appear in such an expression.

This radius is a function of the volatility of all the currency pair involved: if there are n currencies, the number of volatilities is n(n-1)/2. In the case of two currencies, there exists only one volatility and Ω is half of this volatility. The quantity extending the role of volatility to the multi-currency case is then twice the distance between any currency-point and the orthocenter. (Given the definition of orthocenter this distance does not depend on the choice of the particular currency point).

The indicator, in order to encompass the world currency market, should in principle include all currencies, but on the other hand, it should have a simple expression. These conflicting objectives can be reconciled considering the relatively minor importance of the British pound (GBP) and Swiss franc (CHF), and the fact that the market for exchange rate options between those currencies and the yen is rather illiquid. Hence, in practice, the relevant indicator can be formed using only the implied volatilities of the exchange rates among the dollar, the mark and the yen.

In other words by theorem 2 the indicator will be given by the law of sines resulting in a ratio of the dollar-mark, dollar-yen and mark-yen implied volatilities.[11] It is important to notice that this number is invariant under permutations of the 3 currencies, and is not affected by the conversion of marks into euro. Hence the indicator is expressed through a formula where the 3 currencies play symmetrical roles. In symbols

$$\mathrm{GHI} = \frac{2\sigma_{\$M}\,\sigma_{\$¥}\,\sigma_{M¥}}{\sqrt{(\sigma_{\$M}+\sigma_{\$¥}+\sigma_{M¥})(-\sigma_{\$M}+\sigma_{\$¥}+\sigma_{M¥})(\sigma_{\$M}-\sigma_{\$¥}+\sigma_{M¥})(\sigma_{\$M}+\sigma_{\$¥}-\sigma_{M¥})}} \quad (18)$$

Expression (18) highlights that the indicator, which we call the Global Hazard Indicator (GHI), has the same dimension and the same magnitude order as any (bilateral) volatility. It can therefore be interpreted as a form of composite implied volatility and readily compared to the implied volatilities of currency pairs. Stated in different terms, the comparison of a currency pair implied volatility to this global indicator is meaningful because their ratio is a dimensionless number.

The final point regards the choice among the maturity of the option contracts whose data are used to build the GHI. There exist liquid contract whose maturity range between one week to one year, the one-month and the three-month being the most liquid. We concentrate our attention on the three-month option contracts because their price movements are relatively smoother than the one-month options. As a matter of notation, when confusion might arise, we will indicate by GHI-3M the orthocenter constructed from data on 3-month option contracts, by GHI-6M the

11 Therefore, it should be computed as $\sigma/\sqrt{(1-\rho^2)}$, where σ is the implied dollar-mark volatility and ρ is the implied correlation between dollar-yen and mark-yen ortocenter relative to 6-month options, GHI-1Y the one relative to 1-year options, and so on. Later we will devote some attention to the term structure of the GHI, which turns out to be an important measure of global market conditions.

3 The nature of the GHI, its statistical properties and the term structure of hazard

In the introduction we mentioned that the GHI could be considered as a measure of wild risk. In this section we try to explain the rationale supporting such an assertion and clarify the notion of wild risk in the particular context we analyze. Subsequently we will briefly consider several related issues. Are increments of the GHI approximately Gaussian or do they significantly deviate from normality? Does the GHI exhibit a strong autocorrelation? What does the GHI imply for the implied volatilities of currency pairs other than the dollar the mark (euro) and the yen? How does the GHI compare to other indicators based on implied volatilities for example a simple average of implied volatilities? Subsequently we turn the attention to the term structure of implied volatilities as reflected in the term structure of the GHI.

The data used in the analysis were taken from Reuters (TS**1**). They are the OTC implied volatilities of at the money (forward) currency options with different maturities.

3.1 The GHI, market turbulence and the notion of hazard

There is a subtle point to grasp when employing the implied volatilities or the GHI as indicators of market conditions. The implied volatilities are derived from the GK model. However we know that the GK model is not an accurate description of reality because if it were, the implied volatility would be constant over time. So why do we focus on the GK model and why do market participants use it so extensively in their trading activity?

The answer is that although the GK model does not provide a precise description of reality it provides a useful yardstick to measure what we call hazard, i.e. encompassing those forms of risk whose nature is too complex to be analyzed by standard mathematical tools.

In essence the GK formula merely translates into a price the level of risk (as expressed by the implied volatility) that the market perceives. In fact one should keep in mind that the GK formula does not include relevant factors that in reality do affect option prices, in particular the randomness of the quadratic variation of the log spot exchange rate (see section 1), which we interpret as being associated with wild risk (in fact the quadratic variation of a Brownian motion would be constant) . Therefore the fluctuations in the implied volatilities capture the effects of those factors neglected by the GK model. More to the point, we conjecture that they reflect to a large extent the market assessment of future realizations of the quadratic variation (and hence of wild risk).

Paradoxically, even if the true model existed and were known, it could still be meaningful to use the GK formula in order to get a price for the options and hence a synthetic measureof hazard. On the contrary if the GK model were a precise description of reality then the implied volatilities (which would be constant) could not convey any information on market conditions.[12]

We can explain this apparent paradox by an analogy. Let us consider a bond. One can define its internal yield by assuming unrealistically that the yield curve is flat. This erroneous model contains

12 This would mean that the risk faced by the market participants is (mostly) Gaussian, or benign in the jargon of Mandelbrot.

as parameters the values taken by the yield curve at different maturities. These parameters provide a measure (a numeraire) to price the bond. In fact the yield curve is the standard way of quoting some bonds (French BTAN, PO and coupon strips), just as GK implied volatilities are the standard way of quoting currency options.[13] Moreover yields are used worldwide as financial indicators, just as currencies implied volatilities are used in this paper. In other words the role of the GK model is to provide a unit of measure for the hazard in the exchange rate markets.

To summarize the argument, we can say that the suitability of implied volatilities and the GHI as indicators of wild risk (as perceived by the market) is totally independent of the veridicality of the assumptions underpinning the GK model, which merely provides a metric for the analysis of hazard.

3.2 Normality

Increments of the logarithm of the GHI strongly deviate from normality (kurtosis of 128 on the available sample) and do not seem to exhibit a great deal of asymmetry, if one relies upon the empirical histogram in Fig. 3. This confirms the tendency visible from Fig. 2 for the GHI-3M to remain stable (the median of the distribution is in fact zero), but subject to infrequent large jumps. These characteristics are exactly among those one would expect from a measure of wild risk in the sense of Mandelbrot. We interpret this result as further evidence that the traditional notion of risk (which is essentially linked to the Gaussian paradigm) might be inadequate for describing the kind of uncertainty actually faced by participants in currency markets. We wish to underline that this result is indeed not surprising given that the currency pair implied volatilities do exhibit the same leptocurtic increments.

3.3 The autocorrelation function

Fig. 5 exhibits the autocorrelation function of the GHI. Its shape has a distinctive pattern of negative dependence for a time lag of 3-4 days. Such a feature is clearly unusual because (logarithms of) liquid prices generally display an autocorrelation function, which is 1 at lag 1, and zero otherwise. In principle any time dependence in volatilities represents an arbitrage opportunity that traders would immediately exploit.

Since implied volatilities are in principle tradable values, the efficient market hypothesis predicts that increments must be uncorrelated thereby ruling out such systematic time dependence. There are two possible explanations for this phenomenon.

1) The trading strategy (i.e. the portfolio) taking advantage of the negative autocorrelation would meet too many transaction costs (this is likely since bid-ask spreads on volatilities are far from negligible).
2) Reliable time series on foreign exchange implied volatilities are presently publicly available only for a limited sample period (from Reuters) so the estimate of the autocorrelation function is biased and unreliable or perceived as such by market agents.

If the first explanation is the correct one, this autocorrelation structure will not vanish with time; in case the second one is correct, with time the information contained in the autocorrelation function will be exploited to make profits and hence disappear.

13 The paradox can also be explained by a simpler analogy. Let's assume that we want to determine how fast a car travels from point A to point B. In order to determine what we mean by fast or slow we need a metric. One can provide this metric by assuming (erroneously) that the car travels at uniform speed. Then by dividing the actual time of the trip by the "theoretical" time predicted by the model one obtains a measure of the car speed. The role of the GK implied volatility is exactly to provide an analogous metric to measure wild risk (hazard) in exchange rate markets.

It is important to remark that this autocorrelation pattern cannot be attributed to herd behavior or noise trading because in such cases one would observe a positive autocorrelation.

3.4 Other currencies

Another important finding is that the GHIs contain information about other currency pairs volatilities. Available data allow a test for the exchange rates between the Swiss franc and the dollar, the Swiss franc and the mark, the British pound and the dollar, the British pound and the mark. We will call volatilities on those currency pairs "external volatilities". We calculate (using daily data covering 1997 and 1998) the correlation of those external volatilities, at various maturities, and the GHI at maturities of 1-month, 2-month, 3-month and 6-month. We find two interesting regularities (see Table 2)

- The correlation is mostly significant and higher at shorter maturities.
- The Swiss franc displays higher correlations with the GHI than the British pound

As one would expect, the hazard of other widely internationally traded currencies is not immune to the effects of global hazard. We interpret this result as indicating that the GHI, although not covering the entire world currency market, offers a good measure of the tensions that have important repercussions on all currency pairs.

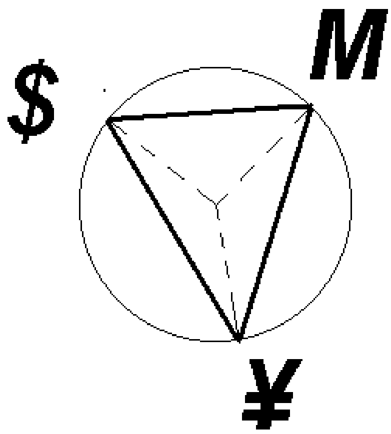
Table 2: Correlations of the Swiss franc and the British pound with the GHI at various maturities

Pair	Term	GHI-1M	GHI-2M	GHI-3M	GHI-6M
USD/CHF	1Month	29%	25%	19%	6%
USD/CHF	2Month	22%	18%	12%	0%
USD/CHF	3Month	14%	11%	6%	-6%
DEM/CHF	1week	42%	39%	35%	25%
DEM/CHF	1Month	43%	40%	37%	30%
DEM/CHF	1Year	22%	22%	21%	18%
DEM/CHF	2Month	37%	35%	32%	26%
DEM/CHF	3Month	32%	30%	28%	23%
DEM/CHF	6Month	26%	26%	24%	20%
GBP/USD	1Month	24%	21%	17%	8%
GBP/USD	2Month	19%	17%	14%	7%
GBP/USD	3Month	15%	14%	11%	6%
GBP/USD	6Month	8%	8%	6%	3%
GBP/USD	1Year	4%	4%	3%	1%
GBP/DEM	1week	16%	12%	7%	-4%
GBP/DEM	1Month	12%	9%	4%	-6%
GBP/DEM	2Month	11%	8%	4%	-5%
GBP/DEM	3Month	11%	9%	5%	-3%
GBP/DEM	6Month	6%	4%	2%	-5%
GBP/DEM	1Year	-2%	-3%	-5%	-10%

3.5 The GHI versus alternative indicators

As explained earlier, other combinations of implied volatilities could be considered as alternative forward looking indicators, but they would have a serious drawback: the weights would be chosen by an ad hoc criterion. Therefore the indicator would not be fully consistent with the way individual options are actually priced in the market. Why is it important that the indicator is consistent with the way market price options? Or, equivalently, what feature does in practice the GHI display that makes it a better indicator than, for example, the simple average of the individual implied volatilities? An intuitive explanation can be offered by looking at the geometrical representation of the ortocenter in Graph 1.

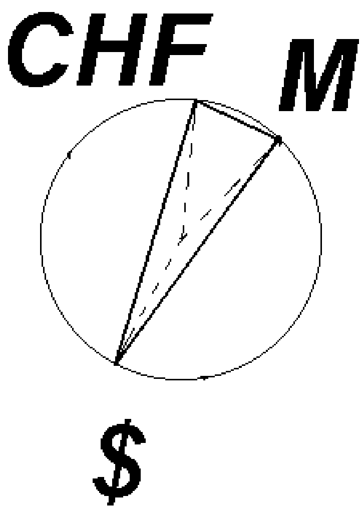
Graph 1 The orthocenter



Thick line lengths represent volatilities. The radius of the circle (dotted lines) is Ω. Hence, the indicator is the diameter of the circle.

The orthocenter is the diameter of the circle. The simple average would be one third of the perimeter of the triangle inscribed in the circle formed by the bilateral implied volatilities. Now let us compare Graph 1 and 2. In the former the volatilities are relatively large, while the latter depicts an extreme case in which one of the volatilities is extremely tiny. In this second case (where the circle would have a different diameter) it is intuitive that the natural measure of global volatility almost coincides with the size of the bilateral volatility dominating the market. In such a case the GHI correctly registers this fact, while the simple average of individual volatilities obviously does not. In other words the simple average would be a meaningful indicator when the sizes of the individual volatilities are roughly similar. Analogous arguments can be made for any weighted average of the individual volatilities and be easily extended to include any function however complicated of the individual volatilities.

Graph 2 The orthocenter in the case of one currency pair displaying low implied volatility



The consequences of this difference can be grasped by contrasting as in Fig. 4 the GHI-3M, the simple average and the difference between the two (whose scale is on the left axis). Because of the co-movements of currencies that we already observed, they are likely to exhibit similar behavior. That is in fact what happens over the 27 months for which data are available. But the difference widens when the hazard is highest as one would guess from the analytical definition of the GHI in equation (18). So in periods of turbulence when the size of volatilities diverges the simple average understates the "excitement" of the market. Equivalently, it can be shown on a scatter graph that there exist no functional relationship between the average of volatilities and the GHI.

The geometric representation in graphs 1 and 2 provides also a natural way of comparing the GHI with another measure involving the currency options implied volatilities: the implied correlation (see for example Butler and Cooper (1997) who interpret it as an indicator of the market sentiment towards EMU). This measure can be defined starting from the relationship

$$\sigma^2_{\$M} = \sigma^2_{¥M} + \sigma^2_{\$¥} - 2\sigma_{\$¥}\sigma_{¥M}\rho \tag{19}$$

where ρ is again (see footnote 10) the implied correlation between the dollar-yen and yen-mark exchange rates. By a simple transformation one obtains

$$\rho = \frac{-\sigma^2_{\$M} + \sigma^2_{\$¥} + \sigma^2_{M¥}}{2\sigma_{\$¥}\sigma_{M¥}} \tag{20}$$

As Butler and Cooper (1997) remark this indicator can be characterized as expressing the "degree of co-movement between two currencies using a third as numeraire".

In terms of the representations in graphs 1 and 2, the implied correlation is actually the cosine of the angle formed by the two implied volatilities. As a matter of fact, the GHI and the implied correlation are based on two well-known relationships between sides and angles of a triangle. Indicating by a b and c the sides of triangle and by A B and C the angles lying opposite to them the law of sines states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \mathrm{diam}(\Theta) \tag{21}$$

where Θ is the circle that inscribes the triangle. The law of cosines states that

$$c^2 = a^2 + b^2 - 2ab\cos C \tag{22}$$

with analogous relationships holding for the other sides. The form of relationship (22) is the same as (19).

In practice the two indicators emphasize different characteristics of a three-currency world: the implied correlation concentrates on the pairwise relationships, while the GHI condenses the three-way relationship. For this reason we can claim that it provides a meaningful measure of global foreign exchange market conditions.

3.6 The term structure

So far we have paid attention mostly to the GHI-3M. Other liquid contracts can be used to calculate the GHIs for different maturities, so to get a term structure of global hazard. Fig. 6 depicts this term structure for the daily data in 1998. The most peculiar trait is the widening of the term structure in conjunction with periods of high volatility. This phenomenon is analogous to the one observed in the term structure of the interest rate. We will notice in the next section that the term structure of risk is particularly sensitive to policy actions to the point that it can be taken as an indicator of market tension and of the effects of policy measures on market dynamics.

4 The behaviour of the GHI during the market turbulence of 1998 and the birth of the euro

Fig. 2 shows the daily GHI-3M between September 13 1996 and February 3 1999. This section focuses on the main features one can detect in the time series of the GHI-3M over the sample period and an overview of the euro's first few weeks of life.

4.1 The movements in the GHI-3M

The first feature is the increasing trend over the sample period. However this increase is not monotonic. Rather it proceeds in steps until the last part of the period when the GHI-3M oscillates widely and flares up in rapid succession, most notably at the beginning of October 1998 when the GHI-3M reaches its peak of about 42,[14] against a "normal" range between 9 and 18.

The second feature regards the troughs in the GHI-3M over the last two years, which precede the sudden increase related to the crises in South East Asia. After each of these crises the GHI-3M rose to higher plateaus.

14 The scale of Fig. 2 actually does not include the peak value, in order not to compress the graph too much.

The third and most important feature is connected to the most severe episode of jump in hazard that took place on October 8 1998 and was not related to any particular episode of crisis or the disclosure of news on economic fundamentals in the large developed economies. Rather this episode highlights how hazardous conditions can be the result of tensions over which the national authorities have little control.

The graph of the GHI term structure over 1998 and the first weeks of 1999, shown in Fig. 6, stresses the point. The Russian crisis in August triggered a substantial increase in the GHI associated with a widening of the term structure that persisted throughout September and was only briefly dampened when, in early October, the US Congress approved the increase of the of IMF quota equivalent to USD 18 billion. Then on October 8th the world financial markets were suddenly shaken by the extraordinarily volatility in the dollar exchange rate which led to a loss of about 18 percent over the yen in 48 hours. According to market participants this episode was caused by a large build up of speculative positions by hedge funds that had been arbitraging on the interest rate differential between Japan and the US. The unexpected increase of the yen (statements by the Japanese authorities had been widely interpreted as pressing for a weaker yen to boost the export) had forced them to cover precipitously their short positions. However the scale of the operations had a severe impact on the market liquidity which, already reduced by fears of a global credit crunch, quickly dried up.

Hazardous conditions persisted for a few days until the U.S. Federal Reserve decided to lower the federal funds rate. This measure calmed the markets, as the investors become convinced that the American monetary institutions were ready to inject liquidity if necessary, thereby producing two effects. The GHIs dropped as one would expect and, more interestingly, the term structure of hazard became suddenly flat.

4.2 The birth and early life of the euro

It is widely accepted that the euro was de facto born on December 3rd 1998 with the concerted action by central banks in the EU-11 countries to lower their reference rate to 3.0 percent.[15] The GHIs actually increased to different degrees, an effect that we believes highlights how the move was largely unexpected. Nevertheless the move by the euro area central banks coincided with the rejection by the Brazilian Parliament of the fiscal package agreed in the program supported by the IMF financial support. As the effects of these two events cannot be disentangled a conclusive assessment is not possible.

The launch of the euro at the beginning of 1999 was also followed by a temporary widening of the term structure of hazard, and a noticeable increase in the hazard, which we interpret as ensuing from a general adjustment of global markets to the introduction of a new major currency. Nevertheless early January was characterized also by two other events totally unrelated to EMU.

The USD on January 5th 1999 depreciated and reached a level of 110.50 to the yen, its lowest level since October 1998. The main reason behind this movement was the continuing repatriation of funds by Japanese companies, due to the "credit crunch" in Japan. Furthermore senior Japanese government officials, expressed pessimism about the USD/JPY rate over the next year while at the same time rumors of expiring options in the USD/JPY with knockout levels of 110 added, albeit temporarily, some uncertainty.

Subsequently on January 6 and 7 1999 the USD fell against the Japanese yen on concern that a worsening Brazilian economy would in turn slow the US economic growth (the United States conduct one-fifth of its trade with Brazil). These concerns were initiated by information that

15 Only the Bank of Italy, although reducing its discount rate, maintained the rate at 3.5 percent.

Brazil's second-largest state declared a 3-month moratorium on interest payments on a USD 15 billion loan owed to the federal government. In addition, the auction of the new 10-year Japanese government bonds went very well, which had a positive impact on Japanese bond yields and provided a further base for the appreciation of the yen. The political convulsions in Brazil at a difficult economic juncture prompted a few days later the devaluation of the real and the abandonment of the exchange rate mechanism, which exacerbated the volatility in the global currency markets.

However as a prima facie evidence we can observe that since the introduction of the euro even such a traumatic event has failed to trigger an outburst of volatility, which although high by historical comparison, appears to have been so far confined to a relatively narrow range.

5 Conclusion

After the launch of Stage 3 of EMU the foreign exchange market will most likely strengthen its triangular structure of which the euro will become a pivotal currency. Thus the policy followed by the European Central Bank will have wide repercussions in the foreign currency markets that need to be closely monitored through some synthetic measures of the global market's behavior.

This paper has proposed a forward-looking indicator of market wild risk, (here referred to as hazard) that takes into account markets expectations of future currency movements. We have argued that such a measure, constructed from data on currency option implied volatilities, can be interpreted as gauging the level of uncertainty and hence perceived global hazard prevailing in the foreign exchange markets. We have shown how it differs from other measures of volatility, which are essentially backward looking (and biased by the low signal to noise ratio typical of high frequency data).

The GHI constitutes a useful instrument that could help to assess systemic risks, pinpoint some of the factors that affect market perception of risk and evaluate the effects of policy decisions. In this regard we have shown how the turbulence that has persisted in the currency markets in the second half of 1998 has affected the GHI and the kind of implications one can draw from the daily analysis of the GHI at various maturities.

Further research in this area should focus on one important element, which was highlighted in section 3.6, namely the term structure of implied volatilities. A noteworthy feature of fig. 6 is the inversion of the term structure during periods of crisis. In normal times, intuitively, the GHI-1M would be lower than the GHI-2M, which in turn would be lower than GHI-3M and so on. But at the outset of crises notably in Thailand, Indonesia, Korea, and in the aftermath of the debt default in Russia we observe that the term structure of the GHI is suddenly inverted (see Fig. 7) and the discrepancy widens.

This stylized fact suggests a conjecture. One of the most difficult judgement calls for policy makers and investors is to distinguish short term outbursts of volatility spurred by lack of information or uncertainty over future course of policy from volatility induced by exchange rate adjustments reflecting changes in fundamentals. A clear cut solution to this dilemma is unlikely to be devised, but one can convincingly argue that the inverted GHI term structure is a sign that short term uncertainty predominates in the market. When this happens it would be a symptom that either agents have insufficient information or that they are unable to process the available information efficiently and unambiguously. On the contrary when the term structure of the GHI displays synchronous movements one could argue that the market is adjusting in response to perceived changes in the fundamentals.

This conjecture needs certainly to be put on sounder theoretical grounds and be corroborated by econometric evidence, but it would be extremely useful if it could provide an interpretation of the foreign exchange movements to guide policy decisions.

Annex. Comparison between the actual and simulated increments of quadratic variation

Spot 1	Spot 2	Actual	Simulated Bm
USDDEM	USDJPY	47%	25%
USDDEM	USDCHF	89%	91%
USDDEM	USDGBP	71%	50%
USDDEM	USDCAD	17%	0%
USDDEM	DEMJPY	40%	27%
USDDEM	DEMCHF	26%	0%
USDDEM	GBPDEM	20%	54%
USDDEM	CADDEM	89%	94%
USDJPY	USDCHF	51%	25%
USDJPY	USDGBP	32%	13%
USDJPY	USDCAD	10%	1%
USDJPY	DEMJPY	44%	23%
USDJPY	DEMCHF	9%	2%
USDJPY	GBPDEM	9%	12%
USDJPY	CADDEM	44%	25%
USDCHF	USDGBP	66%	46%
USDCHF	USDCAD	17%	1%
USDCHF	DEMJPY	31%	25%
USDCHF	DEMCHF	32%	8%
USDCHF	GBPDEM	18%	50%
USDCHF	CADDEM	82%	87%
USDGBP	USDCAD	15%	0%
USDGBP	DEMJPY	29%	13%
USDGBP	DEMCHF	16%	0%
USDGBP	GBPDEM	34%	1%
USDGBP	CADDEM	57%	48%
USDCAD	DEMJPY	8%	0%
USDCAD	DEMCHF	3%	0%
USDCAD	GBPDEM	2%	1%
USDCAD	CADDEM	25%	6%
DEMJPY	DEMCHF	26%	1%
DEMJPY	GBPDEM	10%	14%
DEMJPY	CADDEM	37%	24%
DEMCHF	GBPDEM	10%	-1%
DEMCHF	CADDEM	26%	0%
GBPDEM	CADDEM	22%	51%

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Fig. 1. Empirical quadratic variations of selected currency pairs

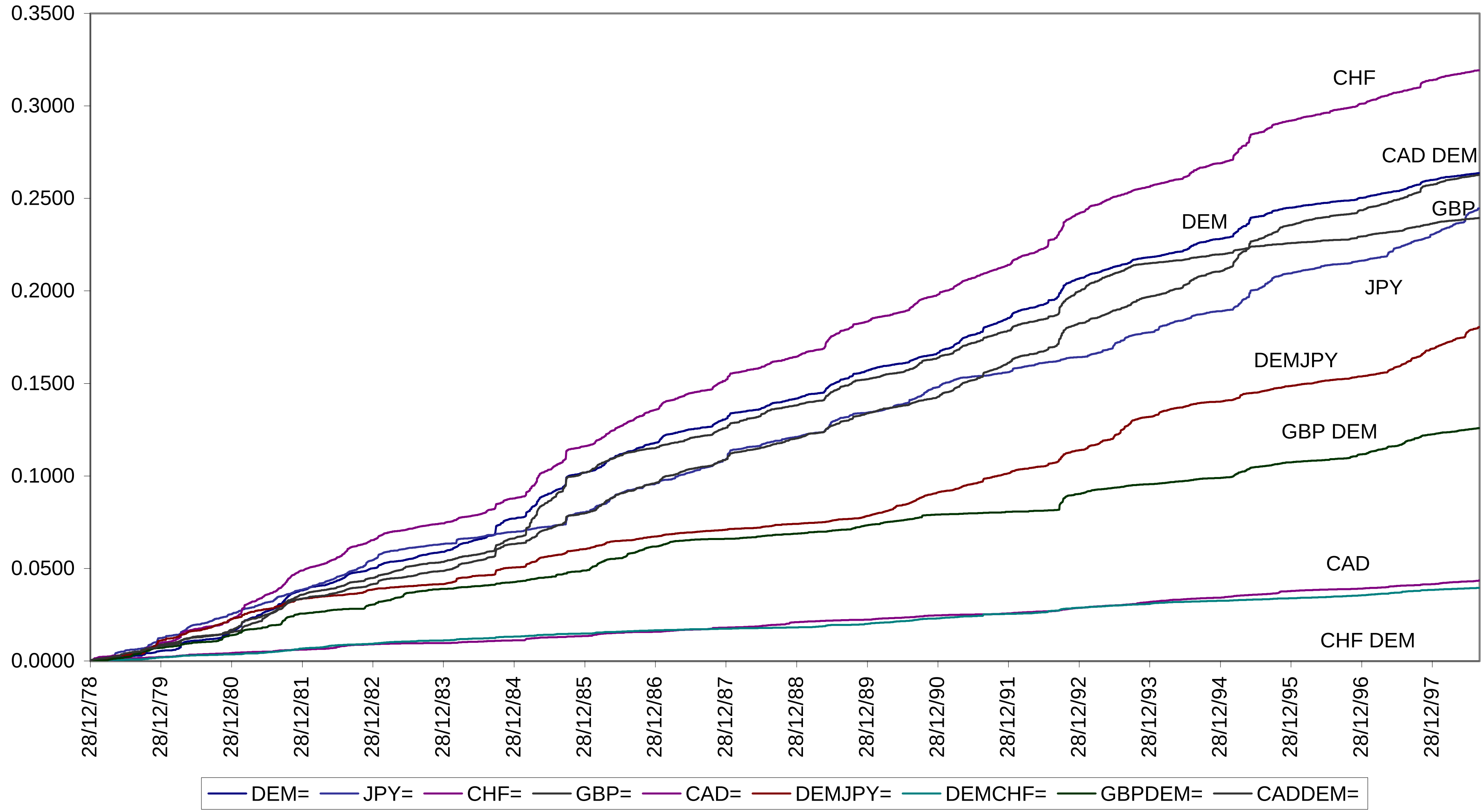


Fig. 2. Global Hazard Indicator (3-Month)

(daily data Sept. 13 1996-Feb. 3 1999)

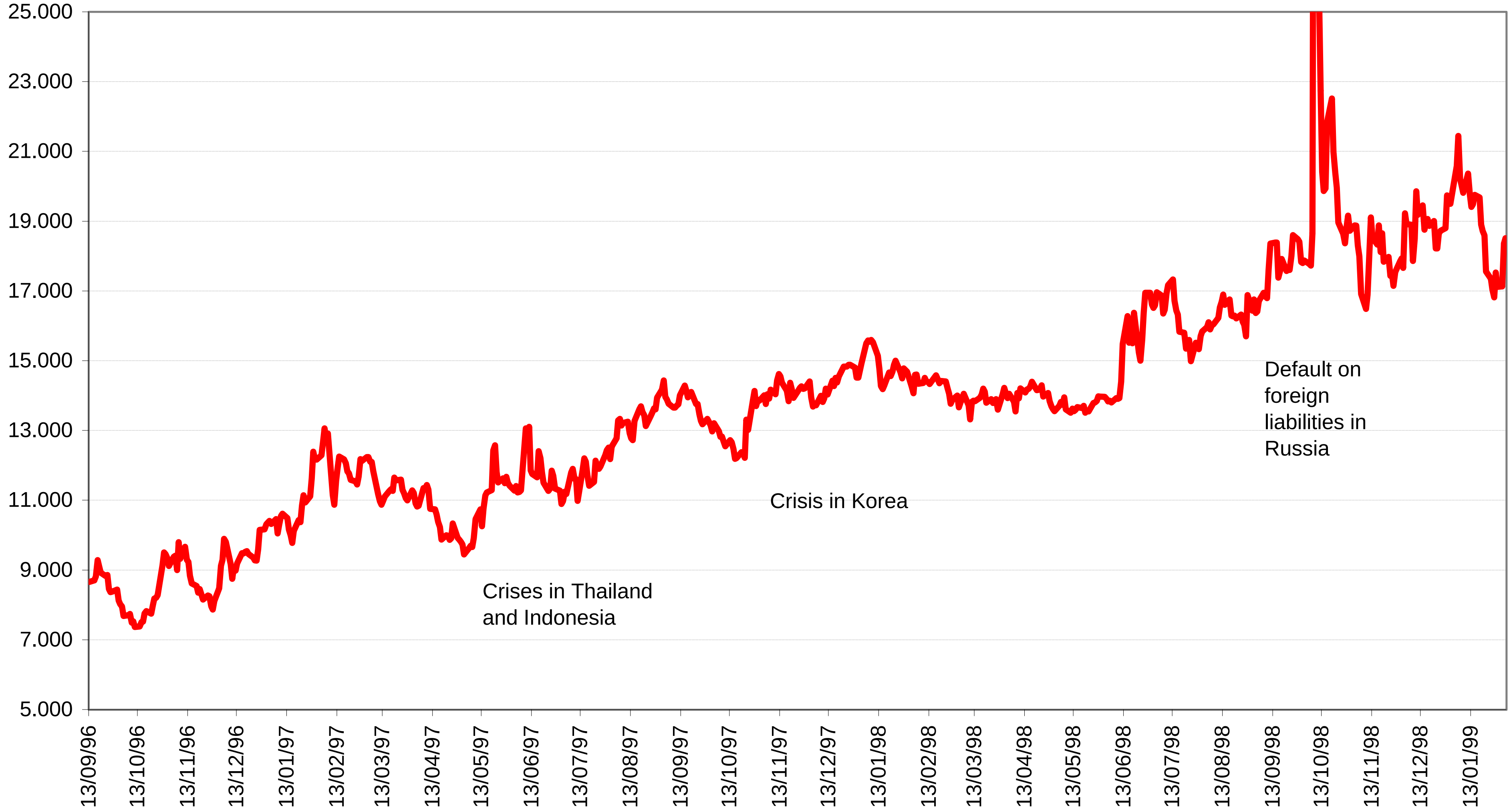


Fig. 3. Histogram of first differences of GHI-3M
(daily data from Sept. 13 1996 to Feb. 3 1999)

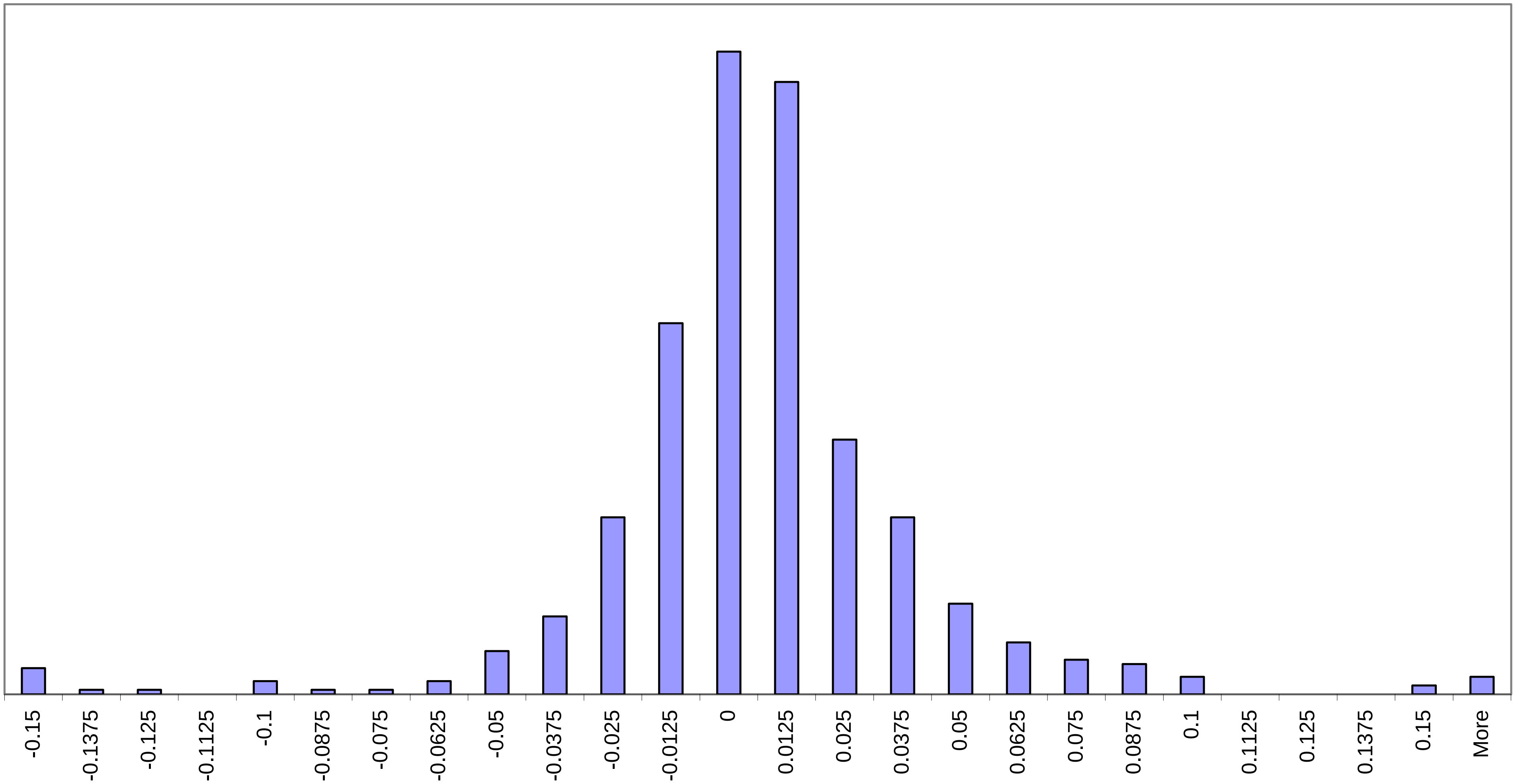


Fig. 4. GHI-3M versus the simple average of implied volatilites

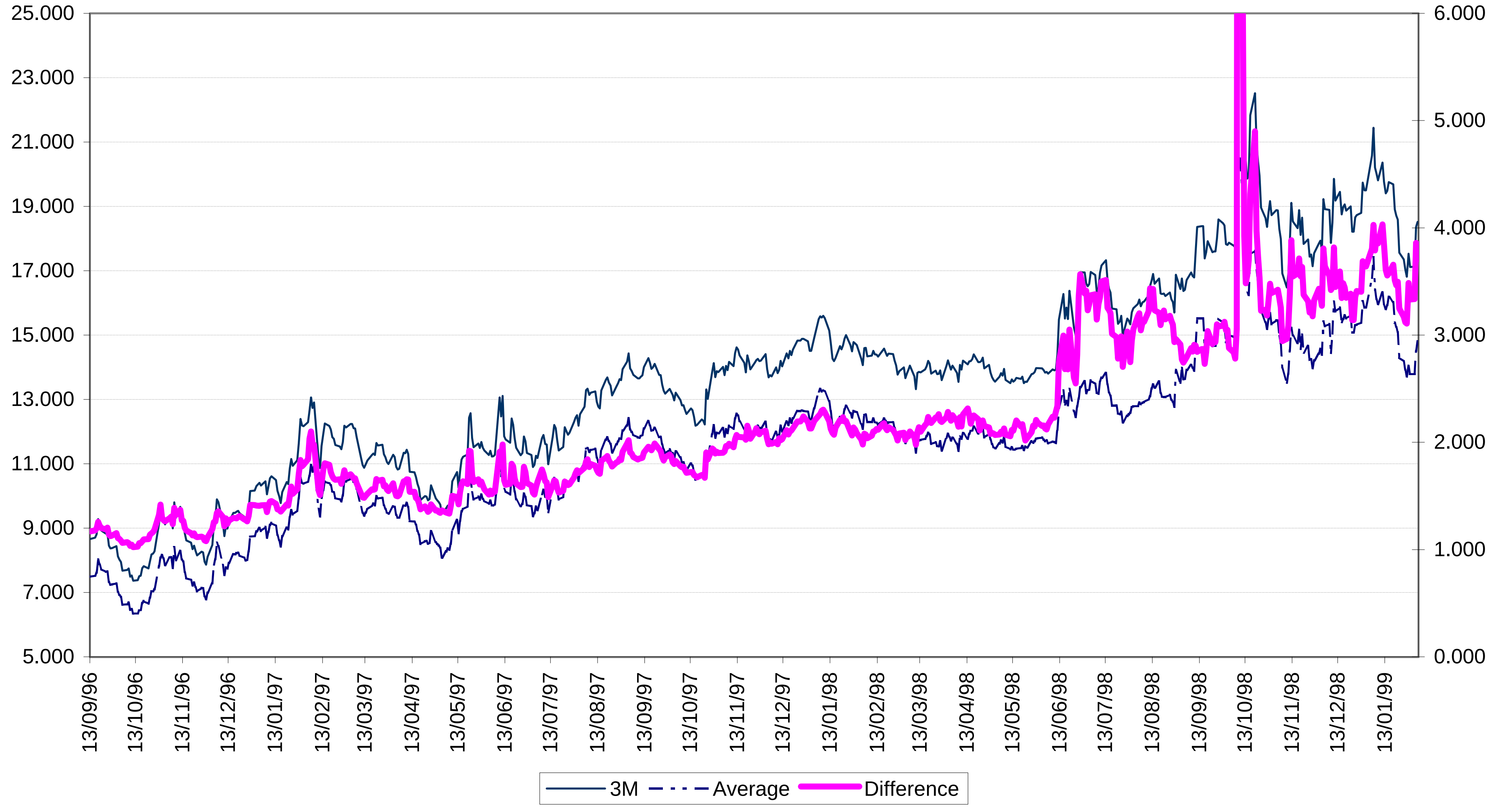


Fig. 5. Autocorrelation function of GHI-3M

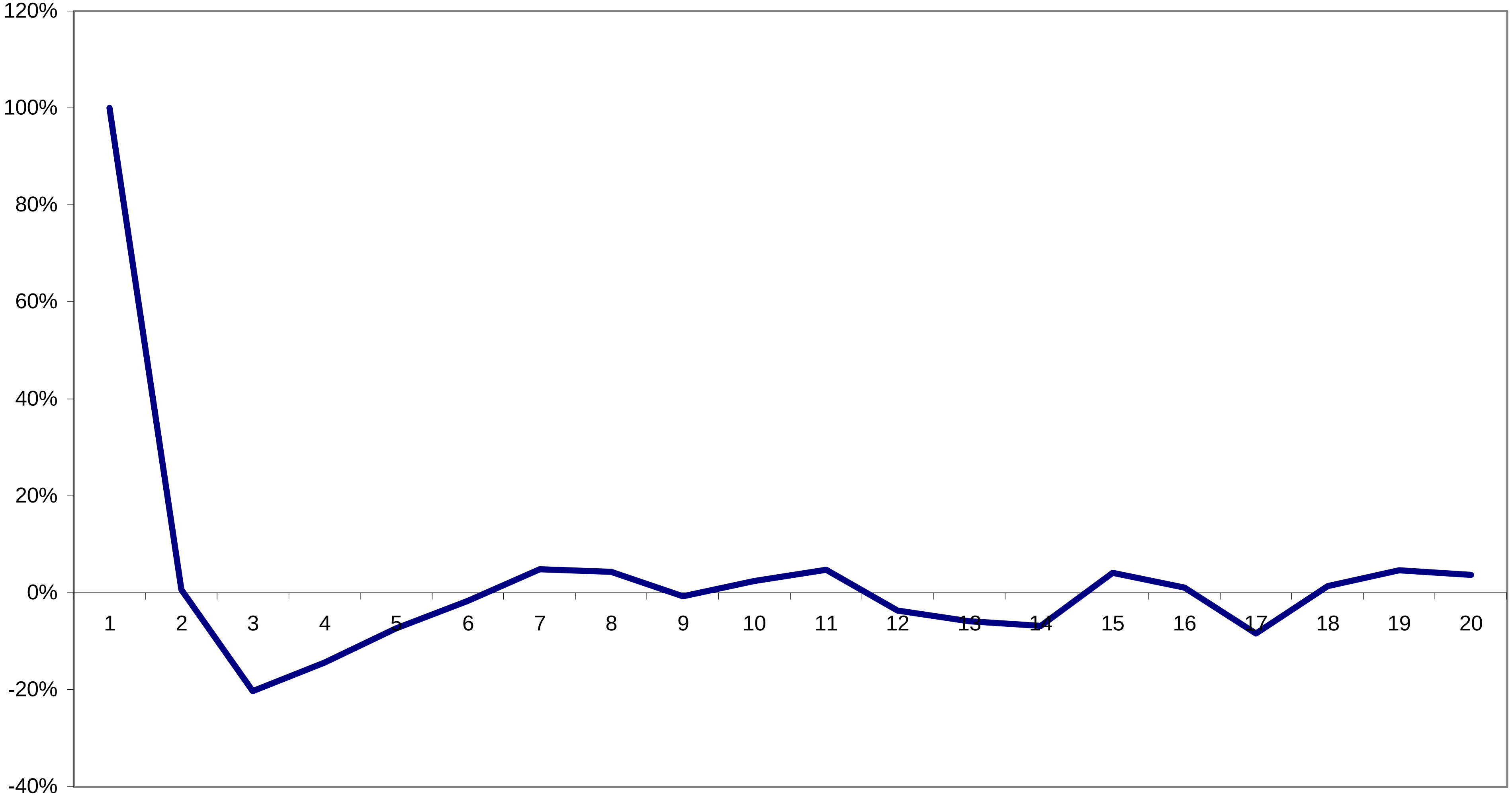
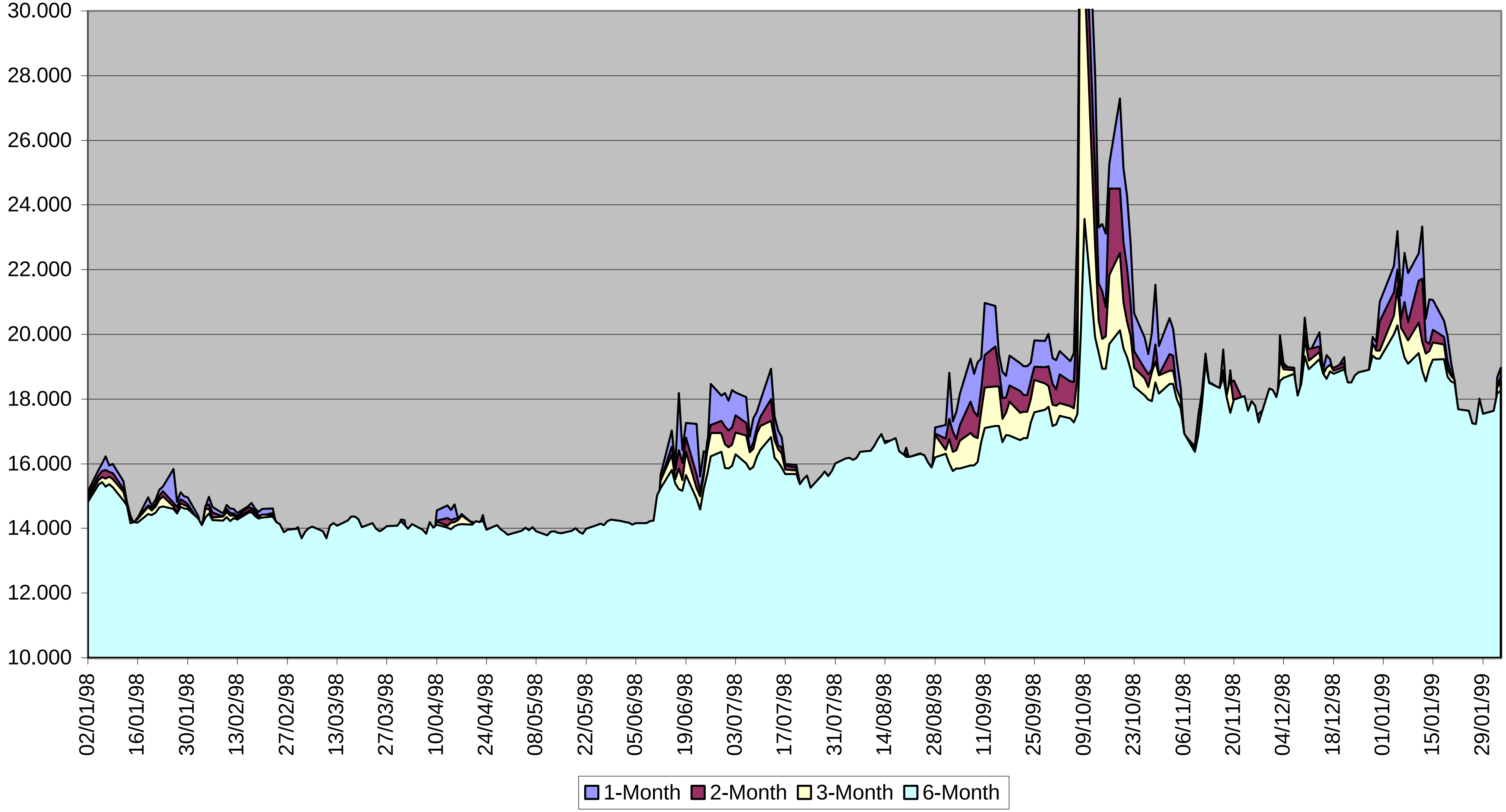


Fig. 6. Term structure of the daily implied volatility orthocenters from Jan. 1998



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