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**DISCRETION RATHER  
THAN RULES?**

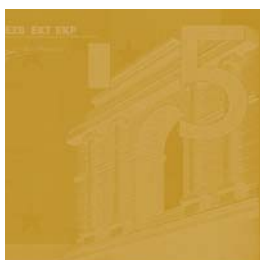
**WHEN IS DISCRETIONARY  
POLICY-MAKING BETTER  
THAN THE TIMELESS  
PERSPECTIVE?**

by Stephan Sauer



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### WHEN IS DISCRETIONARY POLICY-MAKING BETTER THAN THE TIMELESS PERSPECTIVE? <sup>1</sup>

by Stephan Sauer <sup>2</sup>



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## Abstract

Discretionary monetary policy produces a dynamic loss in the New Keynesian model in the presence of cost-push shocks. The possibility to commit to a specific policy rule can increase welfare. A number of authors since Woodford (1999) have argued in favour of a timeless perspective rule as an optimal policy. The short-run costs associated with the timeless perspective are neglected in general, however. Rigid prices, relatively impatient households, a high preference of policy makers for output stabilisation and a deviation from the steady state all worsen the performance of the timeless perspective rule and can make it inferior to discretion.

*Keywords:* Optimality; Timeless perspective; Policy rules.

*JEL classification:* E5

## Non-technical summary

Kydland and Prescott (1977) showed that rule-based policy-making can increase welfare. The timeless perspective proposed by Woodford (1999) represents a prominent modern form of such a rule in monetary policy analysis. It helps to overcome not only the traditional inflation bias in the sense of Barro and Gordon (1983), but also the stabilisation bias, a dynamic loss stemming from cost-push shocks in the New Keynesian model as described in Clarida, Galí, and Gertler (1999). These represent the long-run gains from rule-based policy-making in the New Keynesian model.

The timeless perspective, however, is associated with short-run costs because the monetary authority demonstrates its commitment to the timeless perspective by not exploiting given inflation expectations in the initial period. Instead, it follows a policy “to which it would have been optimal to commit to at a date far in the past”, i.e. it behaves as if the policy rule had been in place already for a long time. This policy is strategically coherent because it avoids any initial period effects that are one reason for the time inconsistency of standard commitment solutions, but it is initially suboptimal. These short-run costs from the timeless perspective are the price to pay to make the commitment to it arguably more credible than an overall optimal commitment solution that exploits given inflation expectations. This paper analyses under which circumstances these short-run costs exceed the long-run gains from commitment.

After deriving a formal condition for the superiority of discretion over the timeless perspective rule, we investigate the influence of structural and preference parameters on the performance of monetary policy both under discretion and the timeless perspective. Discretion gains relatively to the timeless perspective rule, i.e. the short-run losses become relatively more important, if the private sector behaves less forward-looking or if the monetary authority puts a greater weight on output gap stabilisation. For empirically reasonable values of price stickiness, the relative gain from discretion rises with stickier prices. A fourth parameter which influences the relative gains is the persistence of shocks: Introducing serial correlation into the model only strengthens the respective relative performance of policies in the situation without serial correlation in shocks. In particular, we show conditions for each parameter under which discretion performs strictly better than the timeless perspective rule.

Furthermore, the framework of short-run losses and long-run gains also allows explaining why an economy that is sufficiently far away from its steady-state suffers rather than gains from implementing the timeless perspective rule. In general, this paper uses unconditional expectations of the loss function as welfare criterion, in line with most of the literature. The analysis of initial conditions, however, requires reverting to expected losses conditional on the initial state of

the economy because unconditional expectations of the loss function implicitly treat the economy's initial conditions as stochastic. Altogether, in the normal New Keynesian model all conditions for the superiority of discretion need not be as adverse as one might suspect.

Finally, we introduce an “optimal” timeless policy rule based on Blake (2001), Jensen and McCallum (2002) and Jensen (2003). While the general influence of structural and preferences parameters on the performance of monetary policy under this rule is not affected, discretion is never better than this rule when evaluated with unconditional expectations as it is common in the literature on monetary policy rules. The reason is that this allegedly optimal rule optimally accounts for the use of unconditional expectations as the welfare criterion. For *any* timeless rule, however, initial conditions can be sufficiently adverse to make the rule inferior to discretion.

As a policy conclusion, the timeless perspective in its standard formulation is not optimal for all economies at all times. In particular, if an economy is characterised by rigid prices, a low discount factor, a high preference for output stabilisation or a sufficiently large deviation from its steady state, it should prefer discretionary monetary policy over the timeless perspective. The critical parameter values obtained in this paper with the simplest version of the New Keynesian model suggest that – for a number of empirically reasonable combinations of parameters – the long-run losses from discretion may be less relevant than previously thought. Furthermore, the short-run costs in this paper can be interpreted as a lower bound for the actual costs because they are derived under the assumption of full credibility of the monetary authority. Incomplete credibility would raise the costs from commitment even further, since it takes some time until the central bank can reap the full gains from commitment.

# 1 Introduction

Kydland and Prescott (1977) showed that rule-based policy-making can increase welfare. The timeless perspective proposed by Woodford (1999) represents a prominent modern form of such a rule in monetary policy analysis. It helps to overcome not only the traditional inflation bias in the sense of Barro and Gordon (1983), but also the stabilisation bias, a dynamic loss stemming from cost-push shocks in the New Keynesian model as described in Clarida et al. (1999). It is, however, associated with short-run costs that may be larger than the long-run gains from commitment.

After deriving a formal condition for the superiority of discretion over the timeless perspective rule, this paper investigates the influence of structural and preference parameters on the performance of monetary policy both under discretion and the timeless perspective in the sense of Woodford (1999). Discretion gains relatively to the timeless perspective rule, i.e. the short-run losses become relatively more important, if the private sector behaves less forward-looking or if the monetary authority puts a greater weight on output gap stabilisation. For empirically reasonable values of price stickiness, the relative gain from discretion rises with stickier prices. A fourth parameter which influences the relative gains is the persistence of shocks: Introducing serial correlation into the model only strengthens the respective relative performance of policies in the situation without serial correlation in shocks. In particular, we show conditions for each parameter, under which discretion performs strictly better than the timeless perspective rule.

Furthermore, the framework of short-run losses and long-run gains also allows explaining why an economy that is sufficiently far away from its steady-state suffers rather than gains from implementing the timeless perspective rule. In general, this paper uses unconditional expectations of the loss function as welfare criterion, in line with most of the literature. The analysis of initial conditions, however, requires reverting to expected losses conditional on the initial state of the economy because unconditional expectations of the loss function implicitly treat the economy's initial conditions as stochastic. Altogether, in the normal New Keynesian model all conditions for the superiority of discretion need not be as adverse as one might suspect.

We also introduce an “optimal” timeless policy rule based on Blake (2001), Jensen and McCallum (2002) and Jensen (2003). While the general influence of structural and preferences parameters on the performance of monetary policy under this rule is not affected, discretion is never better than this rule when evaluated with unconditional expectations as it is common in the literature on monetary policy rules. The reason is that this allegedly optimal rule optimally accounts for the use of unconditional expectations as the welfare criterion. For *any* timeless rule, however, initial conditions can be sufficiently adverse to make the rule inferior to discretion.



The following section 2 presents the canonical New Keynesian Model. Section 3.1 explains the relevant welfare criteria. The analytical solution in section 3.2 is followed by simulation results and a thorough economic interpretation of the performance of policies under discretion and the timeless perspective, while section 3.4 concludes the discussion of Woodford's timeless perspective by looking at the effects of initial conditions. Section 4 introduces the optimal timeless policy rule and repeats the analysis from section 3.3, whereas section 5 concludes.

## 2 New Keynesian Model

The New Keynesian or New Neoclassical Synthesis model has become the standard toolbox for modern macroeconomics. While there is some debate about the exact functional forms, the standard setup consists of a forward-looking Phillips curve, an intertemporal IS-curve and a welfare function.<sup>1</sup> Following, e.g., Walsh (2003), the New Keynesian Phillips curve based on Calvo (1983) pricing is given by

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + u_t \quad (1)$$

with

$$\alpha \equiv \frac{(1 - \zeta)(1 - \beta\zeta)}{\zeta}. \quad (2)$$

$\pi_t$  denotes inflation,  $E_t$  the expectations operator conditional on information in period  $t$ ,  $y_t$  the output gap, and  $u_t$  a stochastic shock term that is assumed to follow a stationary AR(1) process with AR-parameter  $\rho$  and innovation variance  $\sigma^2$ . While the output gap refers to the deviation of actual output from natural or flexible-price output,  $u_t$  is often interpreted as a cost-push shock term that captures time-varying distortions from consumption or wage taxation or mark-ups in firms' prices or wages. It is the source of the stabilisation bias.  $0 < \beta < 1$  denotes the (private sector's) discount factor and  $0 \leq \zeta < 1$  is the constant probability that a firm is not able to reset its price in period  $t$ . A firm's optimal price depends on current and (for  $\zeta > 0$ ) future real marginal costs, which are assumed to be proportional to the respective output gap.<sup>2</sup> Hence,  $\zeta$  and  $\alpha$  reflect the degree of price rigidity in this model which is increasing in  $\zeta$  and decreasing in  $\alpha$ .

The policy-maker's objective at an arbitrary time  $t = 0$  is to minimise

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t L_t \quad \text{with} \quad L_t = \pi_t^2 + \omega y_t^2, \quad (3)$$

<sup>1</sup>Depending on the purpose of their paper, some authors directly use an instrument rule or a targeting rule without explicitly maximising some welfare function.

<sup>2</sup>In (1), the proportionality factor is set equal to 1.

where  $\omega \geq 0$  reflects the relative importance of output-gap variability in policymaker preferences. We assume zero to be the target values of inflation and the output gap, respectively. While the former assumption is included only for notational simplicity and without loss of generality, the latter is crucial for the absence of a traditional inflation bias in the sense of Barro and Gordon (1983).

The New Keynesian model also includes an aggregate demand relationship based on consumers' intertemporal optimisation in the form of

$$y_t = E_t y_{t+1} - b(R_t - E_t \pi_{t+1}) + v_t, \quad (4)$$

where  $R_t$  is the central bank's interest rate instrument and  $v_t$  is a shock to preferences, government spending or the exogenous natural-rate value of output, for example.<sup>3</sup> The parameter  $b > 0$  captures the output gap elasticity with respect to the real interest rate. Yet, for distinguishing between the timeless-perspective and the discretionary solution, it is sufficient to assume that the central bank can directly control  $\pi_t$  as an instrument. Hence, the aggregate demand relationship can be neglected below.<sup>4</sup>

## 2.1 Model Solutions

If the monetary authority neglects the impact of its policies on inflation expectations and reoptimises in each period, it conducts monetary policy under *discretion*. This creates both the Barro and Gordon (1983) inflation bias for positive output gap targets and the Clarida et al. (1999) stabilisation bias caused by cost-push shocks. To concentrate on the second source of dynamic losses in this model, a positive inflation bias is ruled out by assuming an output gap target of zero in the loss function (3). Minimising (3) subject to (1) and to given inflation expectations  $E_t \pi_{t+1}$  results in the Lagrangian

$$\Lambda_t = \pi_t^2 + \omega y_t^2 - \lambda_t (\pi_t - \beta E_t \pi_{t+1} - \alpha y_t - u_t) \quad \forall t = 0, 1, 2, \dots \quad (5)$$

The first order conditions

$$\begin{aligned} \frac{\partial \Lambda_t}{\partial y_t} &= 2\omega y_t + \alpha \lambda_t = 0 \\ \frac{\partial \Lambda_t}{\partial \pi_t} &= 2\pi_t - \lambda_t = 0 \end{aligned}$$

imply

$$\pi_t = -\frac{\omega}{\alpha} y_t. \quad (6)$$

<sup>3</sup> $v_t$  is generally referred to as a *demand* shock. But in this model,  $y_t$  reflects the output *gap* and not output alone. Hence, shocks to the flexible-price level of output are also included in  $v_t$ . See, e.g., Woodford (2003, p. 246).

<sup>4</sup>Formally, adding (4) as a constraint to the optimisation problems below gives a value of zero to the respective Lagrangian multiplier.

If instead the monetary authority takes the impact of its actions on expectations into account and possesses an exogenous possibility to credibly commit itself to some future policy, it can minimise the loss function (3) over an enhanced opportunity set. Hence, the resulting *commitment solution* must be at least as good as the one under discretion. The single-period Lagrangian (5) changes to

$$\Lambda = E_0 \sum_{t=0}^{\infty} \beta^t [(\pi_t^2 + \omega y_t^2) - \lambda_t(\pi_t - \beta\pi_{t+1} - \alpha y_t - u_t)]. \quad (7)$$

This yields as first order conditions

$$\begin{aligned} \frac{\partial \Lambda}{\partial y_t} &= 2\omega y_t + \alpha \lambda_t = 0, & t = 0, 1, 2, \dots, \\ \frac{\partial \Lambda}{\partial \pi_t} &= 2\pi_t - \lambda_t = 0, & t = 0, \\ \frac{\partial \Lambda}{\partial \pi_t} &= 2\pi_t - \lambda_t + \lambda_{t-1} = 0, & t = 1, 2, \dots, \end{aligned}$$

implying

$$\pi_t = -\frac{\omega}{\alpha} y_t, \quad t = 0 \quad \text{and} \quad (8)$$

$$\pi_t = -\frac{\omega}{\alpha} y_t + \frac{\omega}{\alpha} y_{t-1}, \quad t = 1, 2, \dots \quad (9)$$

The commitment solution improves the short-run output/inflation trade-off faced by the monetary authority because short-run price dynamics depend on expectations about the future. Since the authority commits to a history-dependent policy in the future, it is able to optimally spread the effects of shocks over several periods. The commitment solution also enables the policy maker to reap the benefits of discretionary policy in the initial period without paying the price in terms of higher inflation expectations, since these are assumed to depend on the future commitment to (9). Indeed, optimal policy is identical under commitment and discretion in the initial period. In a recent paper, Dennis and Söderström (2006) compare the welfare gains from commitment over discretion under different scenarios.

However, the commitment solution suffers from time inconsistency in two ways: First, by switching from (9) to (6) in any future period, the monetary authority can exploit given inflationary expectations and gain in the respective period. Second, the monetary authority knows at  $t = 0$  that applying the same optimisation procedure (7) in the future implies a departure from today's optimal plan, a feature McCallum (2003, p. 4) calls "strategic incoherence".

To overcome the second form of time inconsistency and thus gain true credibility, many authors since Woodford (1999) have proposed the concept of policy-making under the *timeless perspective*: The optimal policy in the initial period should

be chosen such that it would have been optimal to commit to this policy at a date far in the past, not exploiting given inflationary expectations in the initial period.<sup>5</sup> This implies neglecting (8) and applying (9) in all periods, not just in  $t = 1, 2, \dots$ :

$$\pi_t = -\frac{\omega}{\alpha}y_t + \frac{\omega}{\alpha}y_{t-1}, \quad t = 0, 1, \dots \quad (10)$$

Hence, the only difference to the commitment solution lies in the different policy in the initial period, unless the economy starts from its steady-state with  $y_{-1} = 0$ .<sup>6</sup> But since the commitment solution is by definition optimal for (7), this difference causes a loss of the timeless perspective policy compared to the commitment solution. If this loss is greater than the gain from the commitment solution (COM) over discretion, rule-based policy making under the timeless perspective (TP) causes larger losses than policy under discretion (DIS):

$$L_{TP} - L_{COM} > L_{DIS} - L_{COM} \quad \Leftrightarrow \quad L_{TP} > L_{DIS}. \quad (11)$$

The central aim of the rest of this paper is to compare the losses from TP and DIS.

## 2.2 Minimal state variable (MSV) solutions

Before we are able to calculate the losses under the different policy rules, we need to determine the particular equilibrium behaviour of the economy, which is given by the New Keynesian Phillips curve (1)<sup>7</sup> and the respective policy rule, i.e. DIS (6) or TP (10). Following McCallum (1999), the minimal state variable (MSV) solution to each model represents the rational expectations solution that excludes bubbles and sunspots.

Under *discretion*,  $u_t$  is the only relevant state variable in (1) and (6)

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \alpha y_t + u_t \\ \pi_t &= -\frac{\omega}{\alpha} y_t, \end{aligned}$$

so the conjectured solution is of the form

$$\begin{aligned} \pi_{t,DIS} &= \phi_1 u_t \\ y_{t,DIS} &= \phi_2 u_t. \end{aligned}$$

<sup>5</sup>Woodford (1999) compares this “commitment” to the “contract” under John Rawls’ veil of uncertainty.

<sup>6</sup>Due to the history-dependence of (10), the different initial policy has some influence on the losses in subsequent periods, too.

<sup>7</sup>Without loss of generality but to simplify the notation, the MSV solutions are derived based on (1) without reference to (2). The definition of  $\alpha$  in (2) is substituted into the MSV solutions for the simulation results in section 3.3.



Since  $E_t\pi_{t+1} = \phi_1\rho u_t$  in this case, the MSV solution is given by

$$\pi_{t,DIS} = \frac{\omega}{\omega(1 - \beta\rho) + \alpha^2}u_t \quad (12)$$

$$y_{t,DIS} = \frac{-\alpha}{\omega(1 - \beta\rho) + \alpha^2}u_t. \quad (13)$$

Under the *timeless perspective*,  $y_{t-1}$  and  $u_t$  are the relevant state variables from (1) and (10):

$$\begin{aligned} \pi_t &= \beta E_t\pi_{t+1} + \alpha y_t + u_t \\ \pi_t &= -\frac{\omega}{\alpha}y_t + \frac{\omega}{\alpha}y_{t-1}. \end{aligned}$$

Hence, the conjectured solution becomes

$$\pi_{t,TP} = \phi_{11}y_{t-1} + \phi_{12}u_t \quad (14)$$

$$y_{t,TP} = \phi_{21}y_{t-1} + \phi_{22}u_t. \quad (15)$$

After some calculations,<sup>8</sup> the resulting MSV solution is described by

$$\pi_{t,TP} = \frac{\omega(1 - \delta)}{\alpha}y_{t-1} + \frac{1}{\gamma - \beta(\rho + \delta)}u_t \quad (16)$$

$$y_{t,TP} = \delta y_{t-1} - \frac{\alpha}{\omega(\gamma - \beta(\rho + \delta))}u_t. \quad (17)$$

with  $\gamma \equiv 1 + \beta + \frac{\alpha^2}{\omega}$  and  $\delta \equiv \frac{\gamma - \sqrt{\gamma^2 - 4\beta}}{2\beta}$ . Given these MSV solutions, we are now able to evaluate the relative performance of monetary policy under discretion and the timeless perspective rule.

## 3 Policy Evaluation

### 3.1 Welfare criteria

**Unconditional expectations:** The standard approach to evaluate monetary policy performance is to compare average values for the period loss function, i.e. values of the unconditional expectations of the period loss function in (3), denoted as  $E[L]$ .<sup>9</sup> We follow this approach for the analysis of the influence of preference and structural parameters mainly because it is very common in the

<sup>8</sup> These calculations include a quadratic equation in  $\phi_{21}$ , of which only one root,  $0 < \delta < 1$ , is relevant according to both the stability and MSV criteria.

<sup>9</sup>The unconditional expectations of the period loss function  $L_t$  are equal to the unconditional expectations of the total loss function  $\mathcal{L}$  in (3), scaled down by the factor  $(1 - \beta)$ .

literature<sup>10</sup> and allows an analytical solution. However, it includes several implicit assumptions.

First,  $\pi_t$  and  $y_t$  need to be covariance-stationary. This is not a problem in our setup since  $u_t$  is stationary by assumption and  $0 < \delta < 1$  is chosen according to the stability criterion, see footnote 8. Second, using unconditional expectations of (3) implies treating the initial conditions as stochastic (see, e.g., King and Wolman, 1999, p.377) and thus averages over all possible initial conditions. Third, Rudebusch and Svensson (1999) and Dennis (2004, Appendix A) show that the standard approach is formally correct only for  $\lim \beta \rightarrow 1$ , the central bank's discount factor being close to 1. This may influence the precise parameter values for which DIS performs better than TP in section 3.3, but it only strengthens the general argument with respect to the influence of  $\beta$  as will be shown below.

**Conditional expectations:** At the same time, using unconditional expectations impedes an investigation of the effects of specific initial conditions and transitional dynamics to the steady state on the relative performance of policy rules. For this reason and to be consistent with the microfoundations of the New Keynesian model, Kim and Levin (2005), Kim, Kim, Schaumburg, and Sims (2005) and Schmitt-Grohé and Uribe (2004) argue in favour of conditional expectations as the relevant welfare criterion. If future outcomes are discounted, i.e.  $\beta < 1$ , the use of conditional expectations, i.e.  $\mathcal{L}$  in (3) as welfare criterion, implies that short-run losses from TP become relatively more important to the long-run gains compared to the evaluation with unconditional expectations.

Both concepts can be used to evaluate the performance of monetary policy under varying parameter values and the results are qualitatively equivalent. Besides its popularity and analytical tractability, the choice of unconditional expectations as the general welfare measure has a third advantage: by implicitly averaging over all possible initial conditions and treating all periods the same, we can evaluate policies for all current and future periods and thus consider the policy problem from a “truly timeless” perspective in the sense of Jensen (2003), that does not bias our results in favour of discretionary policy-making. Only the analysis of the effects of different initial conditions requires reverting to conditional expectations.

## 3.2 Analytical solution

In principle, the relative performance of DIS and TP can be solved analytically if closed form solutions for the unconditional expectations of the period loss function are available. This is possible, since

$$L_i = E[L_{t,i}] = E[\pi_{t,i}^2] + \omega E[y_{t,i}^2], \quad i \in \{DIS, TP\} \quad (18)$$

<sup>10</sup>See, e.g., various articles in the conference volume by Taylor (1999) and Clarida et al. (1999), Woodford (1999), Jensen and McCallum (2002).

from (3) and the MSV solutions in section 2.2 determine the unconditional variances  $E[\pi_{t,i}^2]$  and  $E[y_{t,i}^2]$ . The MSV solution under *discretion*, (12) and (13) with  $u_t$  as the only state variable and  $E[u_t^2] = \frac{1}{1-\rho^2}\sigma^2$ , give the relevant welfare criterion

$$\begin{aligned} L_{DIS} &= \left[ \frac{\omega}{\omega(1-\beta\rho) + \alpha^2} \right]^2 \frac{1}{1-\rho^2} \sigma^2 + \omega \left[ \frac{-\alpha}{\omega(1-\beta\rho) + \alpha^2} \right]^2 \frac{1}{1-\rho^2} \sigma^2 \\ &= \frac{\omega(\omega + \alpha^2)}{[\omega(1-\beta\rho) + \alpha^2]^2} \cdot \frac{1}{1-\rho^2} \sigma^2. \end{aligned} \quad (19)$$

For the *timeless perspective*, the MSV solution (16) and (17) depends on two state variables,  $y_{t-1}$  and  $u_t$ . From the conjectured solution in (14) and (15), we have

$$\begin{aligned} E[\pi_{t,TP}^2] &= \phi_{11}^2 E[y_{t-1}^2] + \phi_{12}^2 E[u_t^2] + 2\phi_{11}\phi_{12} E[y_{t-1}u_t] \\ E[y_{t,TP}^2] &= \phi_{21}^2 E[y_{t-1}^2] + \phi_{22}^2 E[u_t^2] + 2\phi_{21}\phi_{22} E[y_{t-1}u_t]. \end{aligned} \quad (20)$$

These two equations are solved and plugged into (18) in Appendix A. The result is

$$L_{TP} = \frac{2\omega(1-\delta)(1-\rho) + \alpha^2(1+\delta\rho)}{\omega(1-\delta^2)(1-\delta\rho)[\gamma - \beta(\delta + \rho)]^2} \cdot \frac{1}{1-\rho^2} \sigma^2. \quad (21)$$

Hence, discretion is superior to the timeless perspective rule, if

$$\begin{aligned} L_{DIS} < L_{TP} &\Leftrightarrow \frac{\omega(\omega + \alpha^2)}{[\omega(1-\beta\rho) + \alpha^2]^2} < \frac{2\omega(1-\delta)(1-\rho) + \alpha^2(1+\delta\rho)}{\omega(1-\delta^2)(1-\delta\rho)[\gamma - \beta(\delta + \rho)]^2} \\ &\Leftrightarrow RL \equiv L_{TP}/L_{DIS} - 1 > 0. \end{aligned} \quad (22)$$

(22) allows analytical proofs of several intuitive arguments: First, the variance of cost-push shocks  $\frac{1}{1-\rho^2}\sigma^2$  affects the magnitude of absolute losses in (19) and (21), but has no effect on the relative loss  $RL$  because it cancels out in (22). Second, economic theory states that with perfectly flexible prices, i.e.  $\zeta = 0$  and  $\alpha \rightarrow \infty$ , respectively, the short-run Phillips curve is vertical at  $y_t = 0$ . In this case, the short-run output/inflation trade-off and hence the source of the stabilisation bias disappears completely and no difference between DIS, COM and TP can exist.

Third, if the society behaves as an “inflation nutter” (King, 1997) and only cares about inflation stabilisation, i.e.  $\omega = 0$ , inflation deviates from the target value neither under discretion nor under rule-based policy-making. This behaviour eliminates the stabilisation bias because the effect of shocks cannot be spread over several periods. Shocks always enter the contemporaneous output gap completely. Furthermore, the initial conditions do not matter, since  $y_{-1}$  receives a weight of 0 in (10) and no short-run loss arises. The last two statements are summarised in the following proposition.

**Proposition 1** *Discretion and Woodford's timeless perspective are equivalent for*

1. *perfectly flexible prices or*
2. *inflation nutter - preferences.*

**Proof:**

1.  $\lim_{\alpha \rightarrow \infty} RL = 0.$
2.  $\lim_{\omega \rightarrow 0} RL = 0.$

Finally, proposition 2 states that discretion is not always inferior to Woodford's timeless perspective. If the private sector discounts future developments at a larger rate, i.e.  $\beta$  decreases, firms care less about optimal prices in the future, when they set their optimal price today. Hence, the potential to use future policies to spread the effects of a current shock via the expectations channel decreases. Therefore, the loss from the stabilisation bias under DIS, where this potential is not exploited, i.e. the long-run gains  $L_{DIS} - L_{COM}$ , also decreases with smaller  $\beta$ , while the short-run costs from TP,  $L_{TP} - L_{COM}$ , remain unaffected under rule (10). In the extreme case of  $\beta = 0$ , expectations are irrelevant in the Phillips curve (1) and the source of the stabilisation bias disappears. If the reduction in the long-run gain is sufficiently large, conditions (11) and (22) are fulfilled.

**Proposition 2** *There exists a discount factor  $\beta$  small enough such that discretion is superior to Woodford's timeless perspective as long as some weight is given to output stabilisation and prices are not perfectly flexible.*

**Proof:**  $RL$  is continuous in  $\beta$  because stability requires  $0 \leq \delta, \rho < 1$ . Furthermore,  $\lim_{\beta \rightarrow 0} RL = \frac{[\alpha^2 + 2(1-\rho)\omega + (1+\rho)\omega](\alpha^2 + \omega)}{(\alpha^2 + 2\omega)[\alpha^2 + (1-\rho)\omega]} - 1 > 0$  for  $\omega > 0 \wedge \alpha < \infty$ .

In principle, (22) could be used to look at the influence of structural ( $\zeta, \rho$ ) and preference ( $\beta, \omega$ ) parameters on the relative performance of monetary policy under discretion and the timeless perspective rule more generally.<sup>11</sup> Unfortunately, (22) is too complex to be analytically tractable. Hence, we have to turn to results from simulations.

### 3.3 Simulation results

Preference ( $\beta, \omega$ ) and structural ( $\zeta, \rho$ ) parameters influence the relative performance of monetary policy under discretion and the timeless perspective rule. To evaluate each effect separately, we start from a benchmark model with parameter values presented in table 1 and then vary each parameter successively.

<sup>11</sup>Please note that it would be conceptually nonsense to compare one policy over several values of a preference parameter. Here, however, we always compare two policies (DIS and TP) holding all preference and structural parameters constant.



Parameter	$\beta$	$\omega$	$\zeta$	$\alpha$	$\rho$
Value	0.99	0.0625	0.8722	0.02	0

Table 1: Parameter values for benchmark model.

If one period in the model reflects one quarter, the discount factor of  $\beta = 0.99$  corresponds to an annual real interest rate of 4%. Setting  $\omega = 1/16$  implies an equal weight on the quarterly variances of annualised inflation and the output gap. For  $\beta = 0.99$ ,  $\zeta = 0.8722$  corresponds to  $\alpha = 0.02$ , the value used in Jensen and McCallum (2002) based on empirical estimates in Galí and Gertler (1999).<sup>12</sup>

**Discount factor  $\beta$ :** Figure 1 presents the results for the variation of the discount factor  $\beta$  as the loss from the timeless perspective relative to discretionary policy,  $RL$ . A positive (negative) value of  $RL$  means that the loss from the timeless perspective rule is greater (smaller) than the loss under discretion, while an increase (decrease) in  $RL$  implies a relative gain (loss) from discretion.

[Insert figure 1 about here]

The simulation shows that  $RL$  increases with decreasing  $\beta$ , i.e. DIS gains relative to TP, if the private sector puts less weight on the future. This pattern reflects proposition 2 in the previous section. Since the expectations channel becomes less relevant with smaller  $\beta$ , the stabilisation bias and thus the long-run gains from commitment also decrease in  $\beta$ , whereas short-run losses remain unaffected.

In particular, DIS becomes superior to TP in the benchmark model for  $\beta < 0.839$ , but with  $\omega = 1$  already for  $\beta < 0.975$ . Differentiating between the central bank's and the private sector's discount factor  $\beta$  as in section 4, when the optimal timeless policy rule is derived analytically, shows that the latter drives  $RL$  because it enters the Phillips curve, while the former is irrelevant due to the use of unconditional expectations as the welfare criterion as discussed in section 3.1. But since using the unconditional expectations of the loss function gives equal weight to all periods and hence greater weight to future periods than actually valid for  $\beta < 1$ , this effect only strengthens the general argument.

This can be shown with the value of the loss function (3),  $\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t L_t$ , conditional on expectations at  $t = 0$  instead of the unconditional expectations  $E[L]$ . As figure 2 demonstrates, the general impact of  $\beta$  on  $RL$  is similar to figure 1.<sup>13</sup> The notable difference is the absolute superiority of DIS over TP in our benchmark model, independently of  $\beta$ . In order to get a critical value of  $\beta$

<sup>12</sup> $\zeta$  and  $\alpha$  are linked through the definition of  $\alpha$  in (2).

<sup>13</sup>The use of conditional expectations requires setting the initial conditions, i.e.  $y_{-1}$  and  $u_0$ , to specific values. In figure 2,  $y_{-1} = -0.01$  and  $u_0 = 0$ .

for which DIS and TP produce equal losses, other parameters of the benchmark model have to be adjusted such that they favour TP, e.g. by reducing  $\omega$  as explained below. Hence, figure 2 provides evidence that the use of unconditional expectations does not bias the results towards lower losses for discretionary policy. For reasons presented in section 3.1, we focus only on unconditional expectations from now on.

[Insert figure 2 about here]

**Output gap weight  $\omega$ :** In Barro and Gordon (1983), the traditional inflation bias increases in the weight on the output gap, while the optimal stabilisation policies are identical both under discretion and under commitment.<sup>14</sup> In our intertemporal model without structural inefficiencies, however, the optimal stabilisation policies are different under DIS and COM/TP. The history-dependence of TP in (10) improves the monetary authority's short-run output/inflation trade-off in each period because it makes today's output gap enter tomorrow's optimal policy with the opposite sign, but the same weight  $\omega/\alpha$  in both periods. Hence, optimal current inflation depends on the *change* in the output gap under TP, but only on the contemporaneous output gap under DIS. This way, rule-based policy-making eliminates the stabilisation bias and reduces the relative variance of inflation and output gap, which is a prominent result in the literature.<sup>15</sup>

The short-run costs from TP arise because the monetary authority must be tough on inflation already in the initial period. These short-run costs increase with the weight on the output gap  $\omega$ .<sup>16</sup> The long-run gains from TP are caused by the size of the stabilisation bias and the importance of its elimination given by the preferences in the loss function. Equation (10) shows that increasing  $\omega$  implies a softer policy on inflation today, but is followed by a tougher policy tomorrow. Although the effect of tomorrow's policy is discounted by the private sector with  $\beta$ , the size of the stabilisation bias, i.e. the neglect of the possibility to spread shocks over several periods, appears to be largely independent from  $\omega$ . However, the reduction in the relative variance of inflation due to TP becomes less important the larger the weight on the variance of the output gap in the loss function, i.e. the long-run gains from TP decrease in  $\omega$ . Since short-run costs increase and long-run gains decrease in the weight on the output gap ( $\omega \uparrow$ ), a larger preference for output gap stabilisation favours DIS relative to TP for reasonable ranges of parameters.

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<sup>14</sup>In Barro and Gordon (1983), a larger  $\omega$  increases the marginal utility of higher inflation. Under discretion, the marginal utility of higher inflation must equal its marginal cost such that the ex ante expected policy is also ex post optimal *on average*, which leaves the optimal stabilisation policy unaffected.

<sup>15</sup>See, e.g., Woodford (1999) and Dennis and Söderström (2006).

<sup>16</sup>The optimal output gap  $y_t$  under DIS is decreasing in  $\omega$ , see equation (6).

In the benchmark model of figure 3,  $RL$  initially decreases from 0 for  $\omega = 0$  with  $\omega \uparrow$ .<sup>17</sup> But for reasonable values of  $\omega$ , i.e.  $\omega > 0.0009$  in the benchmark model,  $RL$  increases in the preference for output stabilisation and becomes even positive for  $\omega > 5.28$ .<sup>18</sup>

[Insert figure 3 about here]

**Price rigidity  $\zeta$ :** Proposition 1 states that DIS and TP are equivalent for perfectly flexible prices, i.e.  $\zeta = 0$  or  $\alpha \rightarrow \infty$ , respectively. Increasing price rigidity, i.e. increasing  $\zeta$ , has two effects: First, firms' price-setting becomes more forward-looking because they have less opportunities to adjust their prices. This effect favours TP over DIS for  $\zeta \uparrow$  because TP optimally incorporates forward-looking expectations. Second, more rigid prices imply a flatter Phillips curve and thus the requirement of TP to be tough on inflation already in the initial period becomes more costly. Hence, the left-handside of (11), the short-run losses from TP over DIS, increases. Figure 4 demonstrates that for  $\zeta > 0.436$ , the second effect becomes more important, and for  $\zeta > 0.915$ , the second effect even dominates the first.<sup>19</sup>

[Insert figure 4 about here]

Galí and Gertler (1999) provide evidence that empirically reasonable estimates for price rigidity lie within  $\alpha \in [0.01; 0.05]$ , i.e.  $\zeta \in [0.909; 0.804]$ . In this range, figure 5 shows that  $RL$  increases with the firms' probability of not being able to reset their price,  $\zeta$ , and exceeds 0 for  $\zeta > 0.915$  or  $\alpha < 0.009$ .

[Insert figure 5 about here]

**Correlation of shocks  $\rho$ :** The analysis of the influence of serial correlation in cost push shocks,  $\rho$ , is more complex.  $L_{DIS}$  exceeds  $L_{TP}$  in the benchmark model with  $\rho = 0$  and raising  $\rho$  ceteris paribus strengthens the advantage of TP as demonstrated in figure 6. If shocks become more persistent, their impact on future outcomes increases and thus TP gains relative to DIS because it accounts for these effects in a superior way. The long-run gains from TP dominate its short-run losses and  $RL$  decreases with  $\rho$ .

<sup>17</sup>Note the magnifying glass in figure 3.

<sup>18</sup> $RL$  may approach 0 again for  $\omega \rightarrow \infty$ , the (unreasonable) case of an "employment nutter".

<sup>19</sup>Since the relationship between  $\zeta$  and  $\alpha$  given by equation (2) also depends on  $\beta$ , there is a qualitatively irrelevant and quantitatively negligible difference between varying the probability of no change in a firm's price,  $\zeta$ , and directly varying the output gap coefficient in the Phillips curve,  $\alpha$ .

[Insert figure 6 about here]

However, the relationship between  $\rho$  and  $RL$  is not independent of the other parameters in the model, while the relationships between  $RL$  and  $\beta, \zeta$  and  $\omega$ , respectively, appear to be robust to alternative specifications of other parameters. Broadly speaking, as long as  $L_{DIS} > L_{TP}$  for  $\rho = 0$ , varying  $\rho$  results in a diagram similar to figure 6, i.e.  $L_{DIS} > L_{TP}$  for all  $\rho \in [0; 1)$  and  $RL$  decreases in  $\rho$ . If, however, due to an appropriate combination of  $\beta, \zeta$  and  $\omega$ ,  $L_{DIS} \leq L_{TP}$  for  $\rho = 0$ , a picture symmetric to the horizontal axis in figure 6 emerges, as shown in figure 7.<sup>20</sup> That means that a higher degree of serial correlation only strengthens the dominance of either TP or DIS already present without serial correlation. Hence, serial correlation on its own seems not to be able to overcome the result of the trade-off between short-run losses and long-run gains from TP implied by the other parameter values.<sup>21</sup>

[Insert figure 7 about here]

### 3.4 Effects of initial conditions

As argued in section 3.1, we have to use conditional expectations of  $\mathcal{L}$  in 3 in order to investigate the effects of the initial conditions, i.e. the previous output gap  $y_{-1}$  and the current cost-push shock  $u_0$  on the relative performance of policy rules. Figure 8 presents the relative loss  $\widehat{RL} = \mathcal{L}_{TP}/\mathcal{L}_{DIS} - 1$  conditional on  $y_{-1}$  and  $u_0$ .

[Insert figure 8 about here]

Starting from the steady state with  $y_{-1} = u_0 = 0$  where  $\widehat{RL} = -0.0666$  in the benchmark model,<sup>22</sup> increasing the absolute value of the initial lagged output gap  $|y_{-1}|$  increases the short-run cost from following TP instead of DIS and leaves long-run gains unaffected: While  $\pi_{0,DIS} = y_{0,DIS} = 0$  from (12) and (13),  $\pi_{0,TP}$  and  $y_{0,TP}$  deviate from their target values as can be seen from the history-dependence of (10) or the MSV solution (16) and (17). Hence, TP becomes suboptimal under conditional expectations for sufficiently large  $|y_{-1}|$ . Note also that this short-run cost is of course symmetric to the steady-state value  $y_{-1} = 0$ .

<sup>20</sup>For parameter combinations that result in  $L_{DIS}$  in the neighbourhood of  $L_{TP}$  for  $\rho = 0$ , increasing  $\rho$  has hardly any influence on  $RL$ , but for high degrees of serial correlation from about  $\rho > 0.8$ ,  $RL$  increases rapidly.

<sup>21</sup>This shows that the results in McCallum and Nelson (2004, p. 48), who only report the relationship visible in figure 6, do not hold in general.

<sup>22</sup>Here, I have set  $\omega = 1/16$ , implying an equal weight on the quarterly variance of the output gap and the annualised inflation rate.

If in addition to  $|y_{-1}| > 0$  a cost-push shock  $|u_0| > 0$  hits the economy, the absolute losses both under DIS and TP increase. Since TP allows an optimal combination of the short-run cost from TP, the inclusion of  $|y_{-1}| > 0$  in (10), with the possibility to spread the impact of the initial shock  $|u_0| > 0$  over several periods, a larger shock  $u_0$  alleviates the short-run cost from TP. Hence, the relative loss  $\widehat{RL}$  from TP decreases in  $|u_0|$  for any given  $|y_{-1}| > 0$ .

However, this effect is the weaker the closer  $|y_{-1}|$  is to 0, as can be seen from the less bent contour lines in figure 8. If  $y_{-1} = 0$ , the size of  $|u_0|$  has no influence on  $\widehat{RL}$  any more since DIS and TP do not differ in  $t = 0$ .<sup>23</sup> In this case,  $\widehat{RL}$  is parallel to the  $u_0$ -axis. While  $u_0$  still influences the *absolute* loss-values  $\mathcal{L}$  under both policies and how these losses are spread over time under TP, it has no influence on the *relative* gain from TP as measured by  $\widehat{RL}$ , which is solely determined by the long-run gains from TP for  $y_{-1} = 0$ .

Note that  $RL$  is symmetric both to  $y_{-1} = 0$  for any given  $u_0$  and to  $u_0 = 0$  for any given  $y_{-1}$ . Under DIS,  $y_{-1}$  has no impact because (6) is not history-dependent and  $u_0$  only influences the respective period loss  $L_0$ , which is the weighted sum of the *variances*  $\pi_0^2$  and  $y_0^2$ . Hence,  $\mathcal{L}_{DIS}$  is independent of  $y_{-1}$  and symmetric to  $u_0 = 0$ .

Under TP, however, the history-dependence of (9) makes  $y_{-1}$  and  $u_0$  influence current and future losses. While the transitional dynamics differ with the relative sign of  $u_0$  and  $y_{-1}$ , the total absolute loss  $\mathcal{L}_{TP}$  does not for any given combination of  $|y_{-1}|$  and  $|u_0|$ . If the economy was in a recession ( $y_{-1} < 0$ ), for example,<sup>24</sup> the price to pay under TP is to decrease  $\pi_0$  through dampening  $y_0$ . In figure 9, the shift of the steady-state aggregate demand curve  $AD^*$  to  $AD_0$  reflects this policy response.

[Insert figure 9 about here]

**Scenario 1:** If additionally a negative cost-push shock  $u_0 < 0$  hits the economy, i.e. with the *same* sign as  $y_{-1} < 0$ , this shock lowers  $\pi_0$  further as the Phillips curve (1) is shifted downwards from its steady-state locus  $AS^*$  to  $AS'_0$  in figure 9. At the same time,  $u_0 < 0$  increases  $y_0$  ceteris paribus,<sup>25</sup> brings  $y_0$  closer to the target of 0 and thus reduces the price to pay for TP in the next periods

<sup>23</sup>To be precise, the policy “rules” (6) and (10) do not differ in  $t = 0$ , but the losses differ because of the more favourable output-inflation trade-off through the impact of TP on  $E_0\pi_1$  in (1). This benefit of TP is part of the long-run gains, however, because it is also present under COM.

<sup>24</sup>The following arguments run in a completely analogous manner for  $y_{-1} > 0$ .

<sup>25</sup>Formally, partial derivatives of (16) and (17) with respect to both state variables  $(y_{t-1}, u_t)$  show that both have the same qualitative effect on  $\pi_t$  and an opposing effect on  $y_t$ :  $\partial\pi_t/\partial y_{t-1} = \frac{\omega(1-\delta)}{\alpha} > 0$  and  $\partial\pi_t/\partial u_t = \frac{1}{\gamma-\beta(\rho+\delta)} > 0$  while  $\partial y_t/\partial y_{t-1} = \delta > 0$  and  $\partial y_t/\partial u_t = \frac{-\alpha}{\omega(\gamma-\beta(\rho+\delta))} < 0$ .

$t = 1, \dots$ . The anticipation of this policy in turn lowers inflation expectations  $E_0\pi_1$  compared to the steady-state and thus shifts  $AS'_0$  even further down.  $B$  denotes the resulting equilibrium in figure 9 and is always closer to the  $\pi_0$ -axis than  $A$ .

**Scenario 2:** If, however, the initial cost-push shock  $u_0$  is positive, i.e. of *opposite* sign to  $y_{-1} < 0$ , the transitional dynamics are reversed. The Phillips curve (1) is shifted upwards to  $AS''_0$  in figure 9. In contrast to scenario 1 with  $u_0 < 0$ , this reduces the negative impact of  $y_{-1}$  on  $\pi_0$  but increases  $y_0$  to point  $C$ . Hence, the price to pay under TP in  $t = 1$  is larger than in scenario 1, which in turn also lowers inflation expectations  $E_0\pi_1$  by more. The additional shift of  $AS''_0$  downwards is thus larger than for  $u_0 < 0$  and the new equilibrium is at point  $D$ .

Figure 10 presents the discounted period losses under TP for both cases in the benchmark model. The behaviour of the economy as described above causes a larger loss in the initial period for the first scenario with  $\text{sign}(y_{-1}) = \text{sign}(u_0)$  compared to the case with  $\text{sign}(y_{-1}) = -\text{sign}(u_0)$  because the expectations channel has a smaller impact, but a reversal of the magnitude of losses for  $t \geq 1$  because the price to pay for TP then is larger until the period loss converges to its unconditional value. Since the sum of the discounted losses, however, is equal in both scenarios,  $\mathcal{L}_{TP}$  is symmetric to  $u_0 = 0$  given  $y_{-1}$  and to  $y_{-1} = 0$  given  $u_0$ .

[Insert figure 10 about here]

To summarise, Figure 8 presents the influences of the initial conditions on the relative performance of TP and DIS and the rest of this section provides intuitive explanations of the effects present in the model.  $\widehat{RL}$  becomes positive, i.e. DIS performs better than TP, in the benchmark model for quite realistic values of the initial conditions, e.g.  $\widehat{RL} > 0$  for  $|y_{-1}| = 0.015$  and  $|u_0| = 0.01$ . Hence, it may not be welfare increasing for an economy to switch from DIS to TP if it is not close to its steady state.

## 4 Optimal timeless policy rule

So far, we have compared policy under discretion and under the timeless perspective rule in the sense of Woodford (1999). The latter appears to be the most common “optimal” rule in the recent literature on monetary policy. However, as noted in the introduction, several authors have already mentioned that TP is not always an optimal rule - without providing an analysis of the influence of different parameters on the performance of TP and without an intuitive interpretation of their result, the main objectives of our paper. In particular, Blake (2001) and Jensen (2003) derive the optimal timeless policy (OP) based on the unconditional

expectations of the timeless perspective's MSV solution, i.e. equations (16) and (17) in section 2.2, as

$$\pi_t = -\frac{\omega}{\alpha}y_t + \beta\frac{\omega}{\alpha}y_{t-1} \quad \forall t. \quad (23)$$

Starting from the root of the problem, however, and differentiating between the monetary authority's discount factor  $\beta_{MA}$ , at which the intertemporal losses in (3) are discounted, and the private sector's discount factor  $\beta_{PS}$ , that enters the New Keynesian Phillips curve (1), allow further insights. The intertemporal Lagrangian (7) changes to

$$\Lambda = E_0 \sum_{t=0}^{\infty} \beta_{MA}^t [(\pi_t^2 + \omega y_t^2) - \lambda_t(\pi_t - \beta_{PS}\pi_{t+1} - \alpha y_t - u_t)]. \quad (24)$$

This yields as first order conditions

$$\begin{aligned} \frac{\partial \Lambda}{\partial y_t} &= 2\omega y_t + \alpha \lambda_t = 0, & t = 0, 1, 2, \dots, \\ \frac{\partial \Lambda}{\partial \pi_t} &= 2\pi_t - \lambda_t = 0, & t = 0, \\ \frac{\partial \Lambda}{\partial \pi_t} &= 2\beta_{MA}\pi_t - \beta_{MA}\lambda_t + \beta_{PS}\lambda_{t-1} = 0, & t = 1, 2, \dots, \end{aligned}$$

implying

$$\pi_t = -\frac{\omega}{\alpha}y_t, \quad t = 0 \quad \text{and} \quad (25)$$

$$\pi_t = -\frac{\omega}{\alpha}y_t + \frac{\beta_{PS}}{\beta_{MA}}\frac{\omega}{\alpha}y_{t-1}, \quad t = 1, 2, \dots \quad (26)$$

Again, the timeless perspective requires neglecting (25) and applying (26) in all periods. We know from the discussion in section 3.1 and Dennis (2004, Appendix A) that using the “truly timeless” perspective with unconditional expectations implicitly sets  $\beta_{MA} = 1$ . Hence, the optimal timeless rule by Blake (2001) and Jensen (2003) is in fact<sup>26</sup>

$$\pi_t = -\frac{\omega}{\alpha}y_t + \beta_{PS}\frac{\omega}{\alpha}y_{t-1}, \quad \forall t. \quad (27)$$

This rule causes a loss under unconditional expectations of

$$L_{OP} = \frac{\omega[1 - \eta^2 + (\beta_{PS} - \eta)^2] + \alpha^2}{\omega(1 - \eta^2)(\xi - \beta_{PS}\eta)^2} \cdot \sigma^2, \quad (28)$$

<sup>26</sup>Recall that Blake (2001) and Jensen (2003) cannot account for the difference between  $\beta_{PS}$  and  $\beta_{MA}$  because they optimise over unconditional expectations of the loss function.

where  $\xi \equiv 1 + \beta_{PS}^2 + \frac{\alpha^2}{\omega}$ ,  $\eta \equiv \frac{\xi - \sqrt{\xi^2 - 4\beta_{PS}^2}}{2\beta_{PS}}$  and  $\rho = 0$  for simplicity.

Performing simulations analogous to the ones in section 3.3, but with the optimal timeless rule (27) instead of (10) and  $\widetilde{RL} \equiv L_{OP}/L_{DIS} - 1$ , gives graphs with similar patterns to the respective figures in section 3.3. The critical difference is that  $\widetilde{RL}$  never becomes positive for *any* parameter combinations (see figures 11 to 15), even for figure 7, where  $RL$  is positive, but  $\widetilde{RL}$  negative for all  $\rho$ .<sup>27</sup> This suggests that as long as the private sector is not completely myopic<sup>28</sup> and some weight is given to output stabilisation and prices are not perfectly flexible, the inclusion of  $\beta_{PS}$  in the optimal policy rule (27) is superior to DIS from a truly timeless perspective.<sup>29</sup> The optimal policy rule reduces its reaction to the lagged output gap in all periods and thus optimally accounts for the decreasing potential to use future policies to spread the effects of a current shock both in the initial and future periods, given that the future is not discounted in the welfare function ( $\beta_{MA} = 1$ ). The reason is that (27) reduces the weight on  $y_{t-1}$  by  $\beta_{PS}$  whereby today's output gap receives exactly the same weight in tomorrow's policy with which the private sector discounts tomorrow's policy today.<sup>30</sup>

[Insert figures 11 to 15 about here]

Hence, the inclusion of  $\beta_{PS}$  optimally accounts for the use of unconditional expectations as the welfare criterion. But the general argument, that the relative performance of policy-making under the timeless perspective and discretion reflects the trade-off between short-run losses and long-run gains in (11), remains valid for two reasons: First, the general pattern of the parameter influences is not affected by OP. Second, the influence of initial conditions on the relative performance is alleviated, but still present in the benchmark model with  $\beta = 0.99$  as can be seen in figure 16, which plots  $\widetilde{RL}$  as in figure 8, but with OP instead of TP compared to DIS.

[Insert figure 16 about here]

<sup>27</sup>Here,  $L_{DIS} \geq L_{OP}$  for  $\rho = 0$  with any combination of parameters, and increasing  $\rho$  only aggravates this situation.

<sup>28</sup>For a completely myopic private sector, i.e.  $\beta_{PS} = 0$ , the optimal timeless rule causes a loss equivalent to the one under discretion because equations (6) and (23) are identical for  $\beta_{PS} = 0$ . Hence, there is no equivalent to Proposition 2 for OP.

<sup>29</sup>An analytical proof of this result could be given as follows: Since  $\lim_{\beta_{PS} \rightarrow 0} L_{OP} = L_{DIS}$  and  $\frac{dL_{OP}}{d\beta} < 0$  for  $0 < \beta \leq 1$ , while  $\frac{dL_{DIS}}{d\beta} = 0$ ,  $L_{OP} < L_{DIS}$  for  $0 < \beta \leq 1$ . But  $\frac{dL_{OP}}{d\beta}$  is too complex to allow an analytical determination of sign  $\left(\frac{dL_{OP}}{d\beta}\right)$ .

<sup>30</sup>Recall also the discussion of the influence of  $\beta$  and  $\omega$  in sections 3.2 and 3.3.



## 5 Conclusion

This paper explores the theoretical implications of different policy rules and discretionary policy under varying parameters in the New Keynesian model. With the comparison of short-run gains from discretion over rule-based policy and long-run losses from discretion, we have provided a framework in which to think about the impact of different parameters on monetary policy rules versus discretion. This framework allows intuitive economic explanations of the effects at work.

Already Blake (2001), Jensen and McCallum (2002) and Jensen (2003) provide evidence that a policy rule following the timeless perspective can cause larger losses than purely discretionary modes of monetary policy making in special circumstances. But none of these contributions considers an economic explanation for this rather unfamiliar result let alone analyses the relevant parameters as rigorously as this paper.

What recommendations for economic policy making can be derived? Most importantly, the timeless perspective in its standard formulation is not optimal for all economies at all times. In particular, if an economy is characterised by rigid prices, a low discount factor, a high preference for output stabilisation or a sufficiently large deviation from its steady state, it should prefer discretionary monetary policy over the timeless perspective. The critical parameter values obtained in this paper suggest that – for a number of empirically reasonable combinations of parameters – the long-run losses from discretion may be less relevant than previously thought.

In an overall laudatory review of Woodford (2003), Walsh (2005) argues that Woodford's book "will be widely recognized as the definitive treatise on the new Keynesian approach to monetary policy". He criticises the book, however, for its lack of an analysis of the potential short-run costs of adopting the timeless perspective rule. Walsh (2005) sees these short-run costs arising from incomplete credibility of the central bank. Our analysis has completely abstracted from such credibility effects and still found potentially significant short-run costs from the timeless perspective. Obviously, if the private sector does not fully believe in the monetary authority's commitment, the losses from sticking to a rule relative to discretionary policy are even greater than in the model used in this paper. One way to incorporate such issues is to assume that the private sector has to learn the monetary policy rule. Evans and Honkapohja (2001) provide a convenient framework to analyse this question in more detail.

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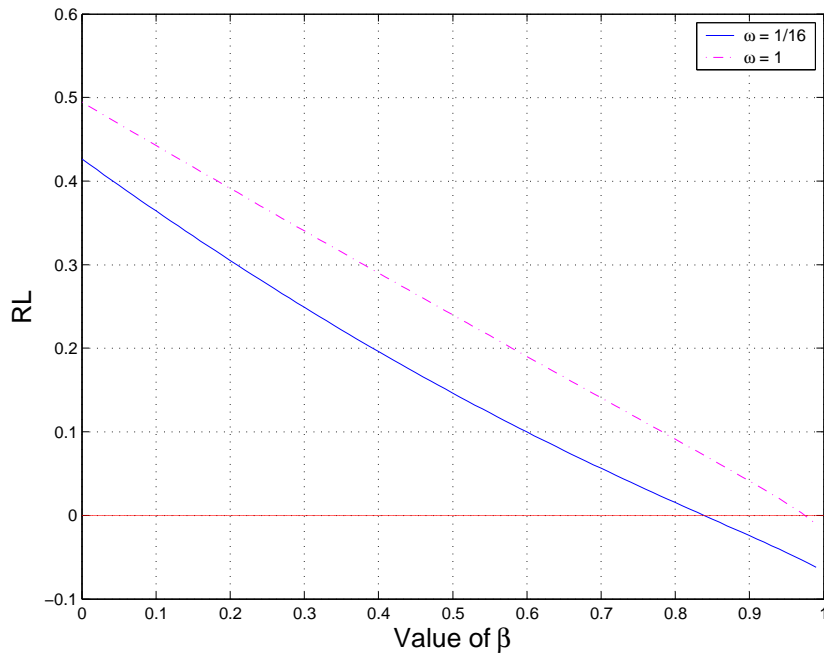


Figure 1: Variation of discount factor  $\beta$ , TP vs. DIS.

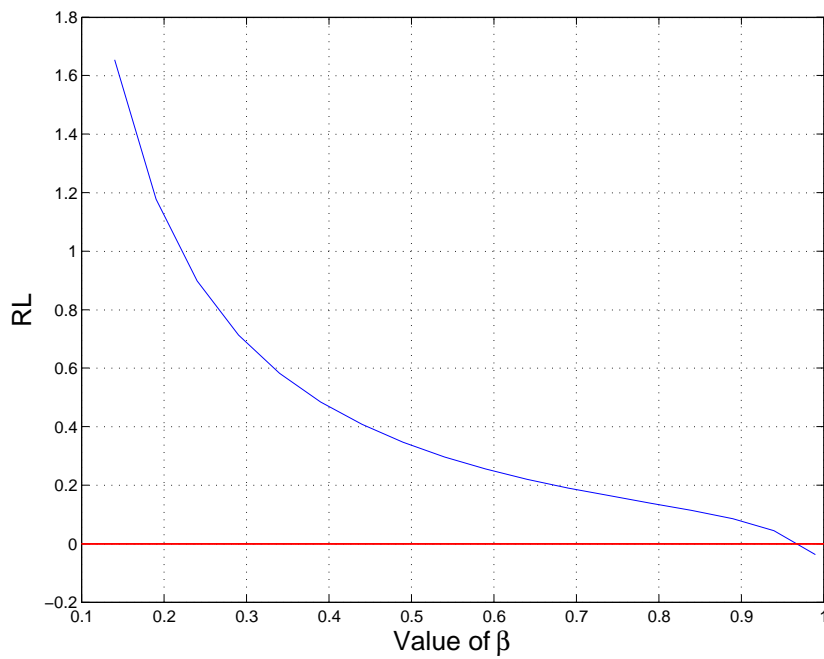


Figure 2: Variation of discount factor  $\beta$  using conditional expectations of loss function, TP vs. DIS.

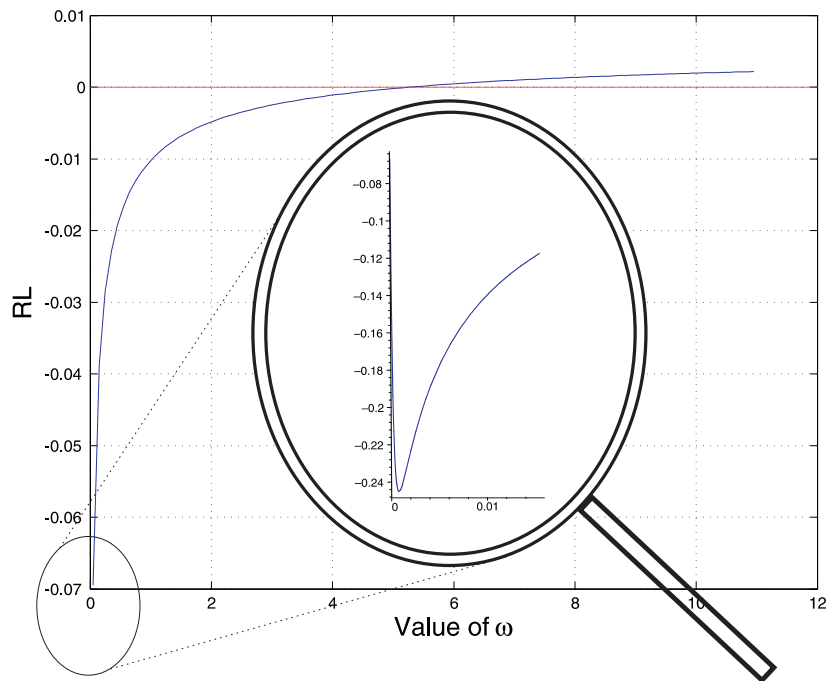


Figure 3: Variation of weight on the output gap  $\omega$ , TP vs. DIS.

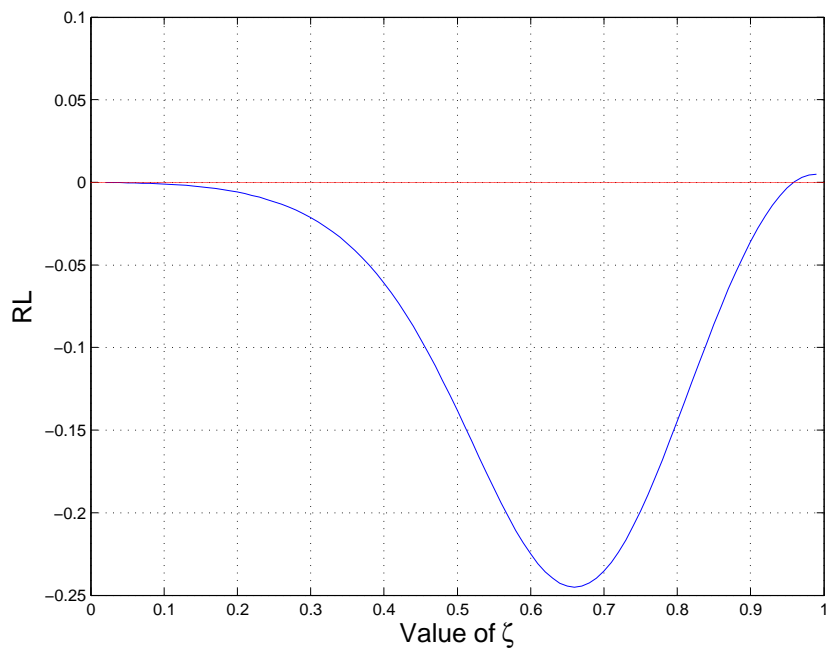


Figure 4: Variation of degree of price rigidity  $\zeta$ , TP vs. DIS.

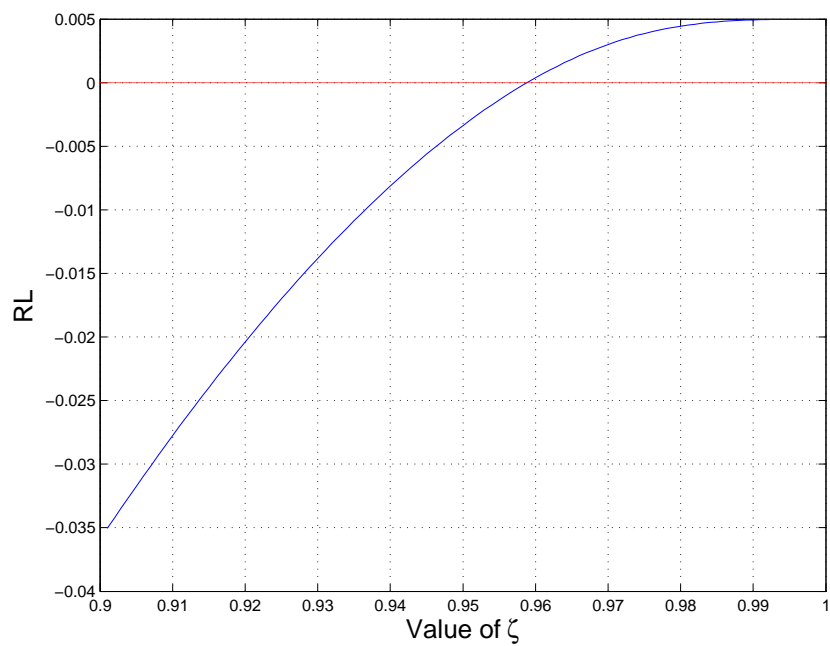


Figure 5: Variation of degree of price rigidity  $\zeta$ , TP vs. DIS.

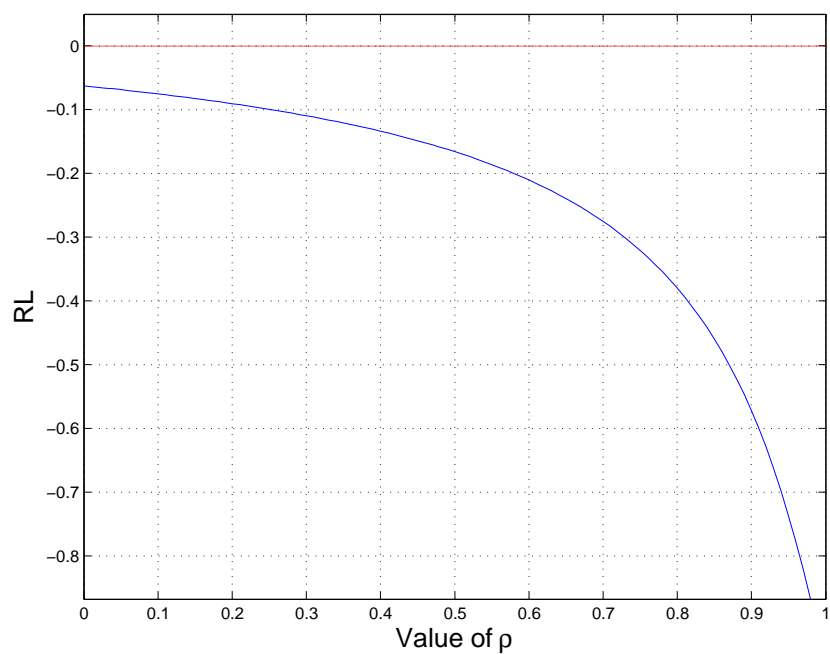


Figure 6: Variation of degree of serial correlation  $\rho$  in benchmark model, TP vs. DIS.

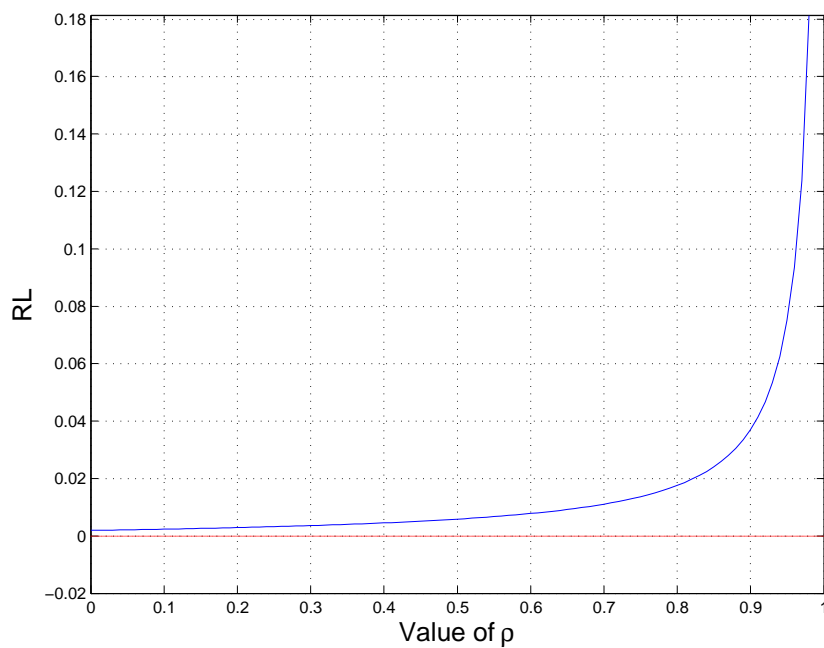


Figure 7: Variation of degree of serial correlation  $\rho$  with  $\omega = 10$ , TP vs. DIS.

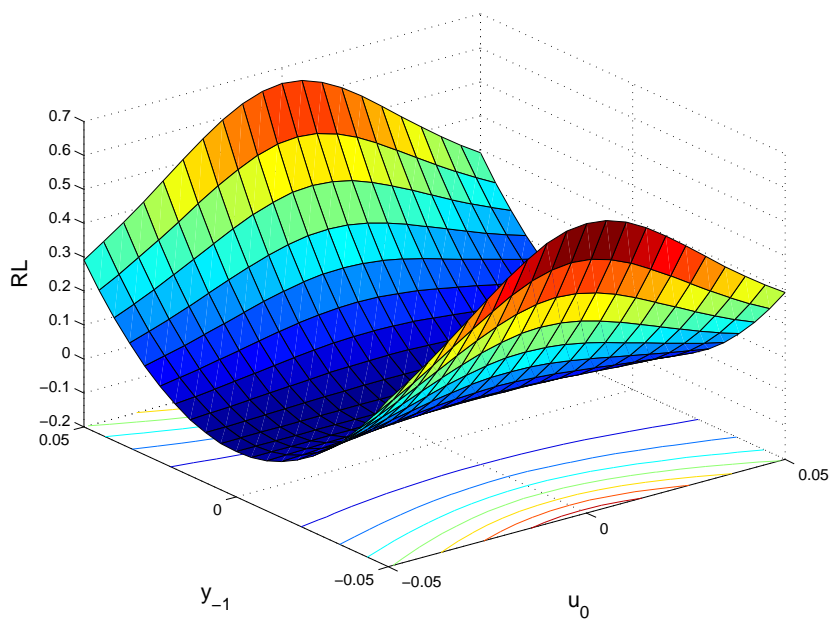


Figure 8:  $\widehat{RL}$  depending on  $y_{-1}$  and  $u_0$ .

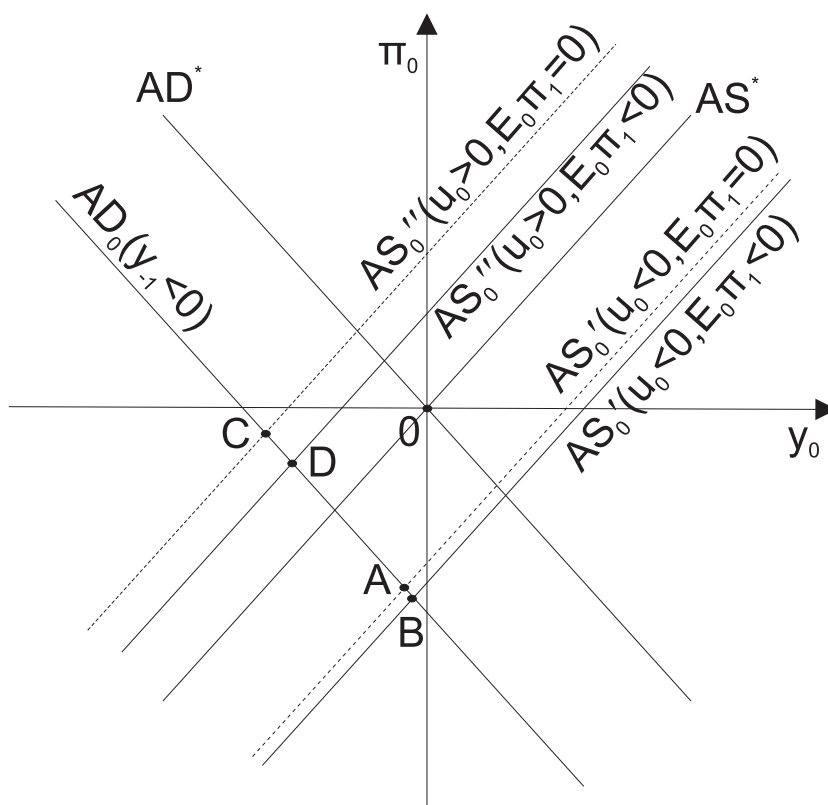


Figure 9: AS-AD-Diagram in  $t = 0$  for two symmetric cost-push shocks  $u_0$ .



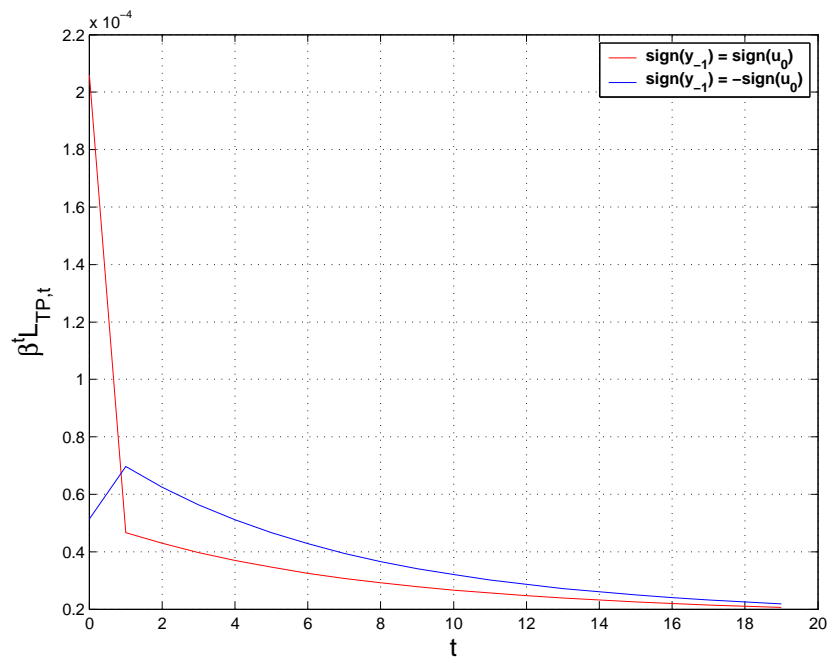


Figure 10: Discounted per-period loss values  $L_{TP,t}$  for  $|y_{-1}| = 0.02$  and  $|u_0| = 0.01$ .

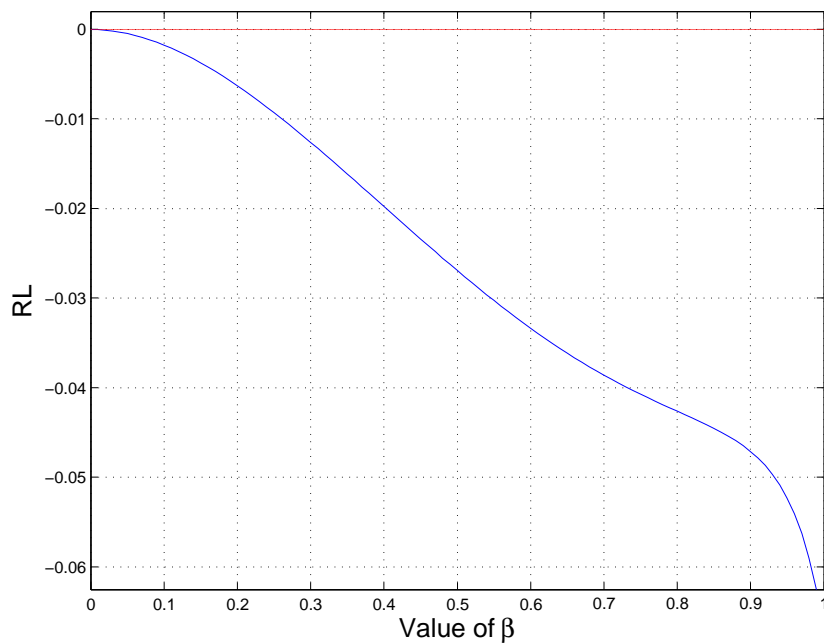


Figure 11: Variation of discount factor  $\beta$ , OP vs. DIS.

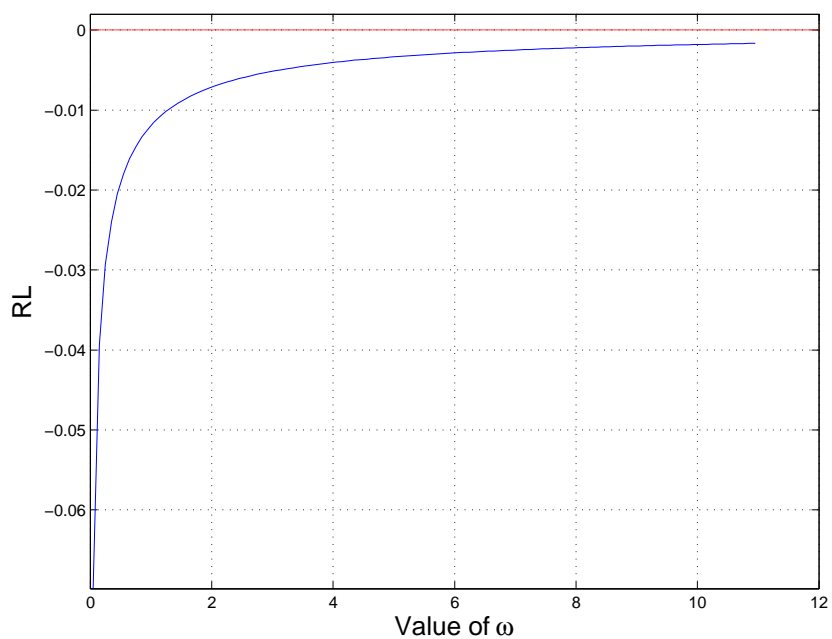


Figure 12: Variation of weight on the output gap  $\omega$ , OP vs. DIS.

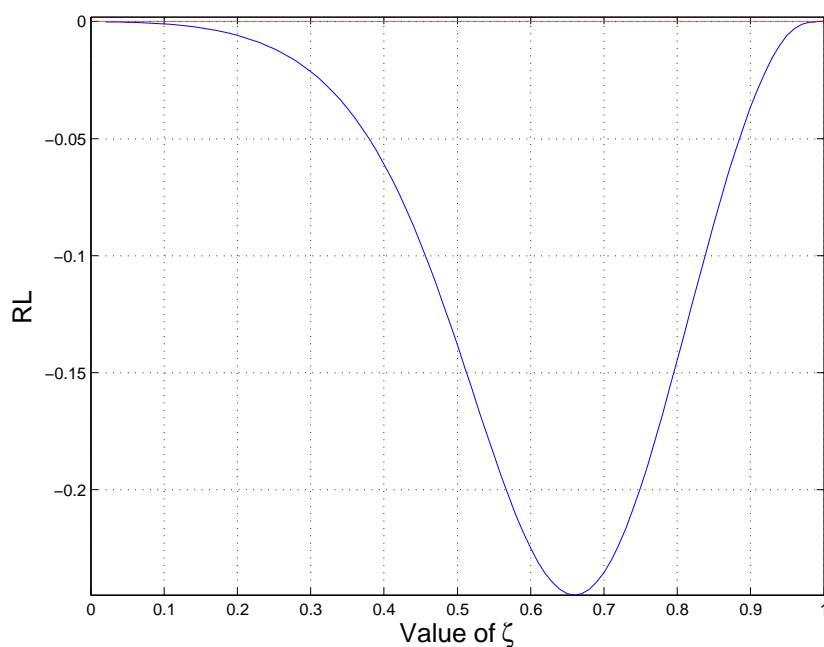


Figure 13: Variation of degree of price rigidity  $\zeta$ , OP vs. DIS.

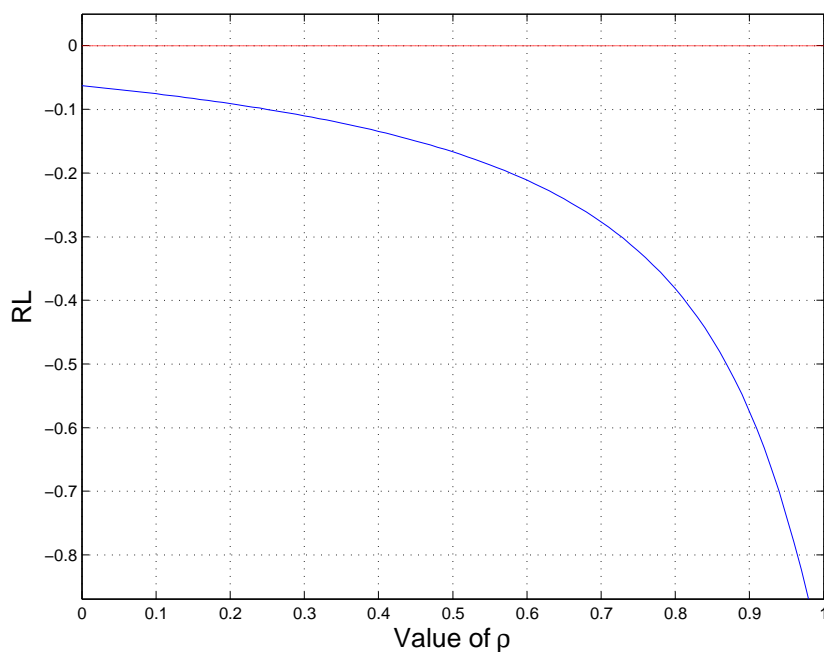


Figure 14: Variation of degree of serial correlation  $\rho$ , OP vs. DIS.

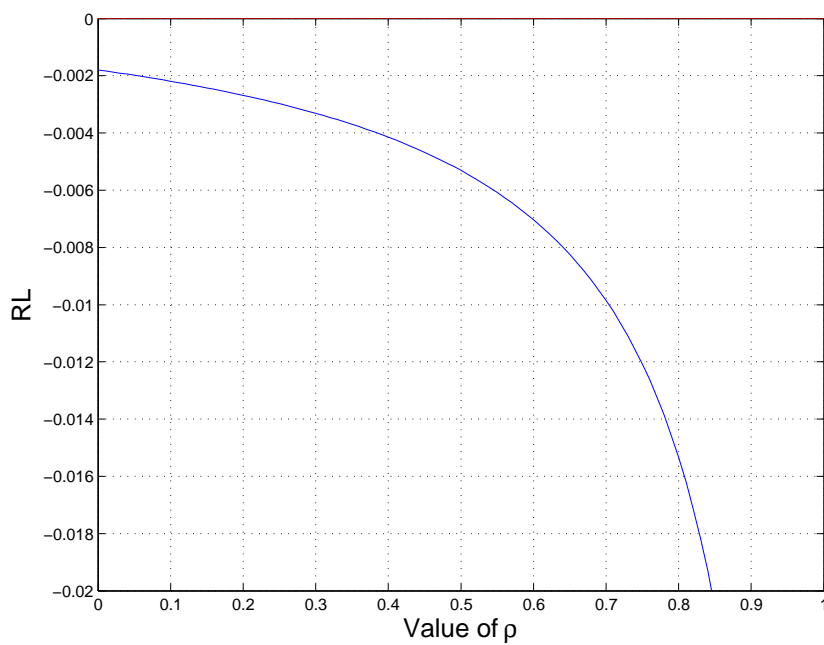


Figure 15: Variation of degree of serial correlation  $\rho$  with  $\omega = 10$ , OP vs. DIS.

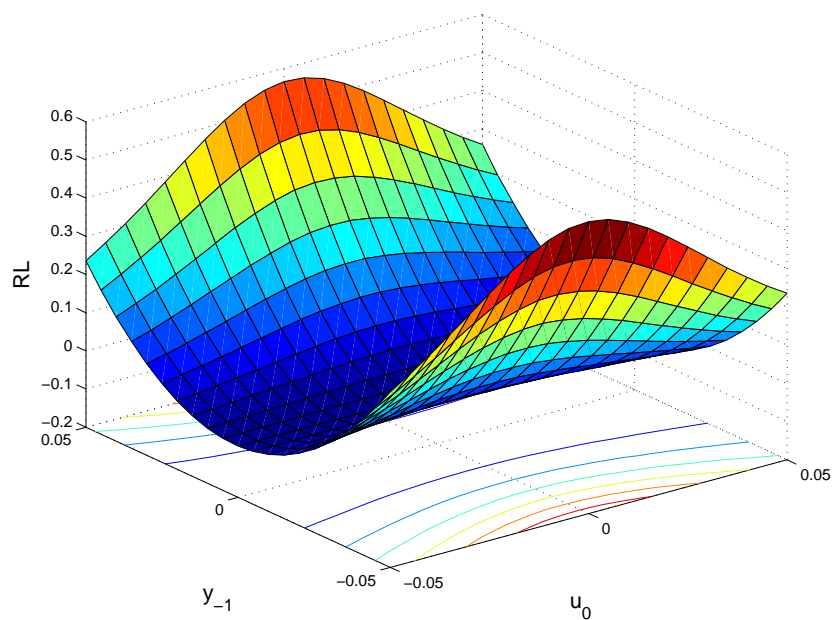


Figure 16:  $\widehat{RL} = \frac{\mathcal{L}_{OP}}{\mathcal{L}_{DIS}}$  depending on  $y_{-1}$  and  $u_0$ .

## A Derivation of $L_{TP}$

The unconditional loss for the timeless perspective, equation (21), can be derived in several steps. The MSV solution (16) and (17) depends on two state variables,  $y_{t-1}$  and  $u_t$ . From the conjectured solution in (15), we have

$$E[y_t^2] = \phi_{21}^2 E[y_{t-1}^2] + \phi_{22}^2 E[u_t^2] + 2\phi_{21}\phi_{22} E[y_{t-1}u_t]. \quad (29)$$

$E[y_{t-1}u_t]$  can be calculated from (15) with  $u_t = \rho u_{t-1} + \epsilon_t$  as

$$\begin{aligned} E[y_{t-1}u_t] &= E[(\phi_{21}y_{t-2} + \phi_{22}(\rho u_{t-2} + \epsilon_{t-1}))(\rho u_{t-1} + \epsilon_t)] \\ &= E[\underbrace{\phi_{21}\rho y_{t-2}u_{t-1}}_{=E[y_{t-1}u_t]} + \phi_{22}(\underbrace{\rho^2 u_{t-1}u_{t-2}}_{=\rho\sigma_u^2} + \underbrace{\rho u_{t-1}\epsilon_{t-1}}_{=\sigma^2})] + 3 \cdot 0, \end{aligned} \quad (30)$$

since the white noise shock  $\epsilon_t$  is uncorrelated with anything from the past. Solving for  $E[y_{t-1}u_t]$  with  $\sigma_u^2 = \frac{1}{1-\rho^2} \sigma^2$  gives

$$E[y_{t-1}u_t] = \frac{\phi_{22}\rho}{1 - \phi_{21}\rho} \cdot \frac{1}{1 - \rho^2} \sigma^2. \quad (31)$$

Plugging this into (29), using  $E[y_t^2] = E[y_{t-1}^2] = E[y^2]$  and  $\phi_{21}, \phi_{22}$  from the MSV solution (17) leaves

$$\begin{aligned} E[y^2] &= \frac{1}{1 - \phi_{21}^2} \left( \phi_{22}^2 + \frac{2\phi_{21}\phi_{22}^2\rho}{1 - \phi_{21}\rho} \right) \frac{1}{1 - \rho^2} \sigma^2 \\ &= \frac{\alpha^2(1 + \delta\rho)}{\omega^2(1 - \delta^2)(1 - \delta\rho)[\gamma - \beta(\delta + \rho)]^2} \cdot \frac{1}{1 - \rho^2} \sigma^2. \end{aligned} \quad (32)$$

From the conjectured solution in (14), we have

$$E[\pi_t^2] = \phi_{11}^2 E[y_{t-1}^2] + \phi_{12}^2 E[u_t^2] + 2\phi_{11}\phi_{12} E[y_{t-1}u_t]. \quad (33)$$

Combining this with the previous results and the MSV solution (16) results in

$$E[\pi^2] = \frac{2(1 - \rho)}{(1 + \delta)(1 - \delta\rho)[\gamma - \beta(\delta + \rho)]^2} \cdot \frac{1}{1 - \rho^2} \sigma^2. \quad (34)$$

Hence,  $L_{TP}$  as the weighted sum of  $E[\pi^2]$  and  $E[y^2]$  is given by

$$L_{TP} = \frac{2\omega(1 - \delta)(1 - \rho) + \alpha^2(1 + \delta\rho)}{\omega(1 - \delta^2)(1 - \delta\rho)[\gamma - \beta(\delta + \rho)]^2} \cdot \frac{1}{1 - \rho^2} \sigma^2. \quad (35)$$

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