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CREDIT CHAINS AND THE PROPAGATION OF FINANCIAL DISTRESS

by Frederic Boissay

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CONTENTS

Abstract

The purpose of this paper is to analyze how shocks propagate through a network of firms that borrow from, and lend to, each other in a trade credit chain, and to quantify the effects of financial contagion across firms. I develop a theoretical model of financial contagion, in which the default of one firm may cause a chain reaction such that its creditors also get into financial difficulties, even though they are sound in the first place. I calibrate and simulate the model using US annual data over the period 1986-2004. At the microeconomic level, I find that, when customers of a sound firm are financially distressed, then this firm gets into financial difficulties with probability that ranges from 4.1% to 12.8% (depending on the business cycle and the underlying economic scenario). Looking at the macroeconomic level, I find that defaults on trade debts lower aggregate GDP by at least 0.4%. During the second half of the 90's, these deadweight losses doubled and reached a high of 0.9% to 2.3% of GDP (depending on the underlying economic scenario) before the recession of 2001. The results of the simulations also suggest that financial contagion across businesses had been 25% higher during the last recession than during the recession of the early 90's.

JEL classification: E32, G29,G33

Key words: Financial contagion, trade credit, business fluctuations

Non-technical Summary

The purpose of this paper is to discuss and quantify the impact of the use of trade credit on firms' financial distress at the firm and aggregate levels. Trade credit is the single most important source of external finance for firms. It appears on every balance sheet and represented more than one half of US businesses' short term liabilities in 2004.[1] It is also is a very peculiar source of finance. First, firms simultaneously grant and receive trade credits, which therefore appear on both sides of their balance sheet. Second, trade credit is in general not well diversified at the firm level, as firms' customers tend to belong to a specific sector. It is indeed not rare for a company to have one large trade credit vis-a-vis one main client on its books, which may represent the entire profit of the year.

These two features led some authors, like Kiyotaki and Moore (1997), to argue that the use of trade credit causes shocks to propagate in the economy. For instance, if the main client of a company has no money, then this company might not get paid prior to the end of the year, meaning that it will have no profit for the year. In the worst case, the company will default on its debts as well; in the best case, its credit rating will be downgraded, making it more expensive to raise external funds and to invest. In Kiyotaki and Moore's words, "a small, negative, specific shock to one firm's technology or net worth may cause a chain reaction in which other firms get into financial difficulties". The recent crisis in the Airline sector provides typical examples of such chain reactions. In September 2004, for instance, the "exposure of Electronic Data Systems (EDS) to US Airways was such that it could slice almost a third from its third-quarter earnings as it [EDS] joined the list of creditors and suppliers to suffer following the airline's bankruptcy filing" (*Financial Times*, 17/09/04).[2] Non-payments of trade credit do not affect only large companies. Collecting the answers from 131 US small businesses that filed for bankruptcy to a questionnaire about the causes for their failures in 2002, Bradley and Rubach (2002) indeed find that entrepreneurs consider non-payments of trade credit as the most important cause for their bankruptcy (31% of the answers), before poor sales (28%).

The widespread use of trade credit implies that trade credit chains have probably harmful consequences at the macroeconomic level too. Although there is, to my knowledge, no evidence on the link between trade credit and macroeconomic crises, it is worth noting that the most rapid growth in trade credit took place prior to the Great Depression. McElvaine (1984) thus reports that "by the end of the 1920's 60% of cars and 80% of radios were bought on installment credit" and that "between 1925 and 1929 the total amount of outstanding installment credit more than doubled" in the US. Trade credit chains probably still have some impact on the business cycles nowadays.

[1] See Figure 1a.

[2] In this press article, titled "*EDS joins US Airways casualties*", it was also mentioned that EDS had Dollars 27 millions in receivables and work in progress due from US Airways, as well as Dollars 16 millions in other assets. While EDS was one of US Airways's smaller creditors, the potential charge came at a sensitive time for the group, which was already struggling with flat revenue and was to cut 20,000 staff. This announcement came after Snecma, the French aero-engine maker, said its commitments with US Airways would force it to take a Euros 35 million charge if the airline remained in Chapter 11 bankruptcy protection.

How harmful are chain reactions at the firm and aggregate levels? Can sound businesses default because of their trade debtors? If so, do these defaults concern a large proportion of businesses? There has been no thorough answer to these questions so far, and common wisdoms on this matter are not clear cut. The purpose of the present paper is to answer these questions.

The novelty of the paper is to develop a theoretical macroeconomic model that describes trade credit chains and makes it possible to identify the impact of the latter, both at the firm and aggregate levels. In the model firms are identical ex ante, live one period, invest at the beginning of the period, and have random returns on their investment at the end of the period. Projects are financed thanks to trade credit: firms both borrow from and (by symmetry) lend to each other at the beginning of the period. At the end of the period, firms have two types of assets: the return on investment, and the repayment of their accounts receivable from their customers. By assuming a continuous distribution of returns, it is possible to identify three groups of firms ex post. Firms whose returns are above a certain threshold have enough assets to clear their debts without even being repaid one cent from their customers; these are "liquid" firms. By contrast, firms whose returns are below another, lower threshold default on their debts even if they are fully repaid by their customers; these are "unsound" firms. And there are intermediate firms, whose returns are in-between the two thresholds. Although these firms are sound, they may default if they have bad customers. This third group is of particular interest because it is the one exposed to financial contagion.

Another innovation of the paper is to provide a quantitative insight into the consequences of chain reactions, both at the firm and aggregate levels. To do so, I calibrate and simulate the model using recent US annual data. At the microeconomic level, I find that, when customers of a sound firm are financially distressed, then this firm gets into financial difficulties with a probability that ranges from 4.1% to 12.8% (depending on the underlying economic scenario). Looking at the macroeconomic level, I find that defaults on trade debts lower GDP by at least 0.4%. During the second half of the 90's, these deadweight losses doubled and reached a high of 0.9% to 2.3% of GDP (depending on the underlying economic scenario) before the recession of 2001. The results of the simulations also suggest that financial contagion across businesses had been 25% higher during the last recession than during the recession of the early 90's.

1 Introduction

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Table 1: Major Causes for Bankruptcy (in %)

Non-payment of Trade Credit	31
Poor sales	28
Lawsuits	8
No major cause identified	33

The widespread use of trade credit implies that trade credit chains have probably harmful consequences at the macroeconomic level too. Although there is, to my knowledge, no evidence on the link between trade credit and macroeconomic crises, it is worth noting that the most rapid growth in trade credit took place prior to the Great Depression. McElvaine (1984) thus reports that "by the end of the 1920's 60% of cars and 80% of radios were bought on installment credit" and that "between 1925 and 1929 the total amount of outstanding installment credit more than doubled" in the US. Trade credit chains probably still have some impact on the business cycles nowadays, as suggested by the fact that both the weight of trade debts in the economy and the share of bad trade debts in total trade debts increased steadily (by more than 60% for the latter) over the 6 years *preceding* the last US recession (see Figures 1a-b) [5]

How harmful are chain reactions at the firm and aggregate levels? Can sound businesses default because of their trade debtors? If so, do these defaults concern a large proportion of businesses? There has been no thorough answer to these questions so far, and common wisdoms on this matter are not clear cut. On the one hand, business managers typically

[5] Sources: IMF, Flows of Funds Accounts (FFA) and Compustat. In figure 1a, I have reported two trade debt to output ratios. One is the trade debt, as reported in the FFA, divided by GDP. The other is the total trade debt for the S&P500 companies, as reported in Compustat, divided by total sales for the same firms. (This latter ratio is consistent with the bad debt-to-trade debt ratio showed in Figure 1b.) As the maturity of trade debt is in general lower than a quarter, the end-of-the-year stock of trade debts corresponds to a quarterly flow. In order to have meaningful trade debt-to-output ratios, I therefore reported in Figure 1a the trade debt observed at the end of the year divided by the average quarterly output the same year. In Figure 1b, I reported for each year the average of the "bad trade debts" to "total accounts receivable" ratio for S&P500 companies. In compustat, the item "bad trade debts " corresponds to the receivables estimated doubtful and represents the provisions for bad debts, i.e. the amount of receivables estimated to be uncollectable. The GDP Gap is the annual average of the quarterly HP-filtered cycle, in percentage deviation from the trend.

answer that customers' defaults are detrimental to their financial health. On the other hand, however, macroeconomists usually argue that such defaults have only a minor effect on the macroeconomy because the amount of net trade credit is negligible and, therefore, defaults should cancel out across firms at the aggregate level. One major difficulty in answering the above questions in a rigorous manner is to disentangle the various causes for default. In particular, one needs to distinguish the effect of a chain reaction that causes financial difficulties to otherwise sound and healthy firms from that of a chain reaction that aggravates financial distress of intrinsically unsound, faltering firms. Another difficulty is the paucity of data, as available databases like Compustat do not have the details needed to identify chains of trade debtors/creditors.

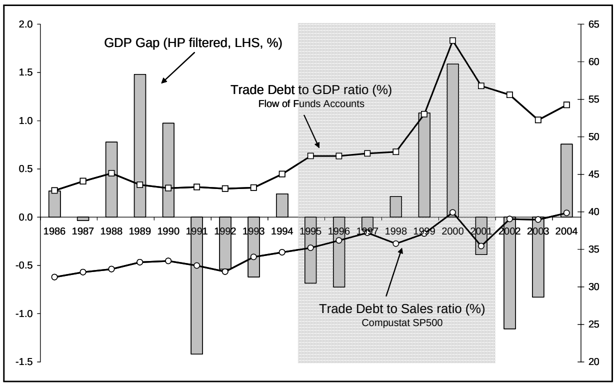

Figure 1a: Trade debt ratio

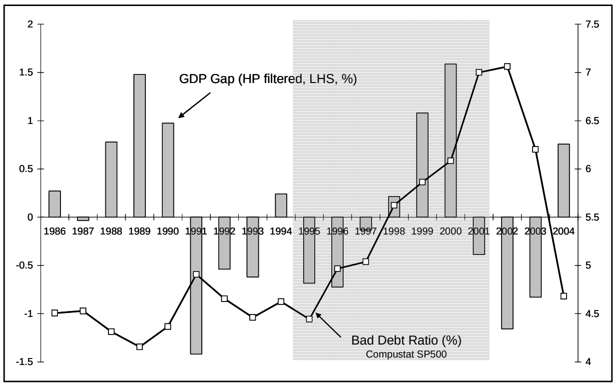

Figure 1b: Bad trade debt ratio

The novelty of the present paper is to develop a theoretical macroeconomic model that describes trade credit chains and makes it possible to identify the impact of the latter, both at the firm and aggregate levels. In the model firms are identical ex ante, live one period, invest at the beginning of the period, and have random returns on their investment at the end of the period. Projects are financed thanks to trade credit: firms both borrow from and (by symmetry) lend to each other at the beginning of the period. At the end of the period, firms have two types of assets: the return on investment, and the repayment of their accounts receivable from their customers. By assuming a continuous distribution of returns, it is possible to identify three groups of firms ex post. Firms whose returns are above a certain threshold have enough assets to clear their debts without even being repaid one cent from their customers; these are "liquid" firms. By contrast, firms whose returns are below another, lower threshold default on their debts even if they are fully repaid by their customers; these are "unsound" firms. And there are intermediate firms, whose returns are in-between the two thresholds. Although these firms are sound, they may default if they have bad customers. This third group is of particular interest because it is the one exposed to financial contagion.

Another innovation of the paper is to provide a quantitative insight into the consequences of chain reactions, both at the firm and aggregate levels. To do so, I calibrate and simulate the model using recent US annual data. At the microeconomic level, I find that, when customers of a sound firm are financially distressed, then this firm gets into financial difficulties with a probability that ranges from 4.1% to 12.8% (depending on the underlying economic scenario). Looking at the macroeconomic level, I find that defaults on trade debts lower GDP by at least 0.4%. During the second half of the 90's, these deadweight losses doubled and reached a high of 0.9% to 2.3% of GDP (depending on the underlying economic scenario) before the recession of 2001. The results of the simulations also suggest that financial contagion across businesses had been 25% higher during the last recession than during the recession of the early 90's.

The papers closest to mine are Kiyotaki & Moore (1997) and Allen & Gale (2000). The common feature with Kiyotaki and Moore's paper is the focus on contagion across firms. One difference is that their contagion mechanism occurs on the real side of interfirm trade and relies on the assumption that firms produce "customized" goods. In a nutshell, financial distress leads firms to cut back orders to their suppliers, who are then left with unsold goods. The fact that goods are customized makes them hard to trade and recycle, implying that suppliers get into financial difficulties as well. The present paper also departs from Allen and Gale's paper, as the latter focuses on financial contagion across banks and consider a liquidity preference shock. Finally, none of these two related papers provide quantitative insights into the size of contagion and its effects on the macroeconomy.

The rest of the paper is organized as follows. I present the basic setup of the model in Section 2, describe financial contagion in Section 3. I discuss the results of the model in Section 4, and present simulations in Section 5. Section 6 concludes.

2 The Model

I consider a one period economy with N small risk-neutral entrepreneurs (or firms) of mass $\frac{1}{N}$ (with N large) who are perfectly competitive. Each entrepreneur is endowed with entrepreneur-specific labour and has access to a technology that transforms L units of labour during the period into q units of final goods at the end of the period.[6] The returns q are random, identically and independently distributed across entrepreneurs according to a return distribution $f(q)$ with mean

$$\overline{q} = AL^{\alpha} \tag{2.1}$$

where $\alpha \in (0, 1)$. These returns are publicly and perfectly observable. I will denote by $F(q)$ the cumulative of f, defined over $\mathbb{R}$. A negative value of q means that the entrepreneur incurred an adverse liquidity shock at the end of the period. (This assumption is a simple and convenient way to have firms default ex post.) Entrepreneurs supply and demand labour at the beginning of the period and consume final goods at the end of the period. They draw a utility equal to one at the end of the period from every final good they consume, and a disutility equal to one at the beginning of the period from every unit of labour they supply. I assume that the unit wage is equal to one (so that the net disutility from supplying labour is equal to zero) and that entrepreneurs discount future at a factor R.

A prerequisite for trade credit to exist is to have interfirm trade exist. With this purpose, I assume that entrepreneurs cannot use their own specific labour in their own project, and have to hire another entrepreneur (say "supplier") to run it. By symmetry, each entrepreneur supplies his own specific labour to another entrepreneur ("customers"). To prevent barters, I also assume that it is never the case that a supplier needs his customer's specific labour (i.e. there is no "double-coincidence of wants"). Labour is hired at the beginning of the period. As in Kiyotaki and Moore (1997), entrepreneurs do not raise finance from outside investors and borrow from their suppliers only (see next paragraph). Every entrepreneur/customer thus purchases labour on credit, i.e. pays the wage L to his supplier at the end of the period. It follows that every entrepreneur both borrows L (accounts payable) from his supplier and

[6] What I will refer to as "labour" throughout the paper could also be understood in a broad sense –rather than literally– as business-to-business services or intermediate goods.

simultaneously (by symmetry) lends L to his customer (accounts receivable) at the beginning of the period. This dual nature is a well-known feature of trade credit.

There are many reasons why firms use trade credit (see Petersen and Rajan (1997) for a review of the traditional literature). Burkart and Ellingsen (2004), for instance, propose an explanation based on the idea that for firms it is easier to divert cash than to divert inputs, implying that, unlike suppliers, banks may refuse to lend for fear that entrepreneurs run away with the cash. In the present paper I will justify the use of trade credit by another theory based on input quality and asymmetric information (see Lee and Stowe (1993)). I assume that suppliers have the choice to deliver high or low quality labour, and that they draw no disutility from supplying low quality labour. I also assume that customers have a negative return on their project when they use low quality labour. Labour quality is observable by suppliers and customers, but it is not verifiable by third parties (e.g. by a Court), notably because negative returns may also occur with high quality labour. In this environment, it is easy to see that if entrepreneurs/customers borrow from banks and pay their suppliers cash at the beginning of the period, then they will be supplied with low quality labour. In contrast, if they ask for late payment, then they will be supplied with high quality labour. The reason is that suppliers want their customers' projects to have positive returns so that customers are able to pay their trade debt at the end of the period. As a consequence, it is optimal for entrepreneurs to finance entirely through trade credit.[7]

At the beginning of the period, entrepreneurs sign debt contracts with their suppliers and their customers. These contracts specify the gross interest rate factor on trade credit, denoted by $\overline{R}$, as well as the amount of the trade debt, L. Figure 2 represents the trade credit chain the N entrepreneurs are involved into. The typical entrepreneur chooses his demand for labour L and the lending rate $\overline{R}$ that maximize his expected profit. To solve for the equilibrium I will proceed in three steps. First, I describe how debt repayments take place at the end of the period. Second, I derive the equilibrium lending rate. Third, I derive the equilibrium demand for labour (or trade debt)

[7] Suppliers would deliver low quality labour even if they were partially paid cash because the net utility they draw from supplying high quality labour at the unit wage is equal to zero.

2.1 Repayment Distribution

The assumption that projects have random returns implies that debt repayments are uncertain. In particular, firms with low returns will not be able to honor their debts. As a result, repayments depend crucially on how well diversified their credit portfolio is.

Diversification. Observationally, trade credit is imperfectly diversified. (This is another well known of its features.) As firms usually belong to a specific geographical area or sector, they are likely to be sensitive to the idiosyncratic shocks that hit their customers. To capture this idea and simplify the exposure of the model, I make temporarily the assumption that every entrepreneur has only one customer (I will relax this assumption in Section 4.2). Obviously, firms do trade with a pool of customers in practice and this assumption is a simple way to model the imperfect diversification of such pool. By symmetry, every firm also has only one supplier. In this context, a shock to the liquidity of one firm may cause a chain reaction along which other firms get into financial difficulties.

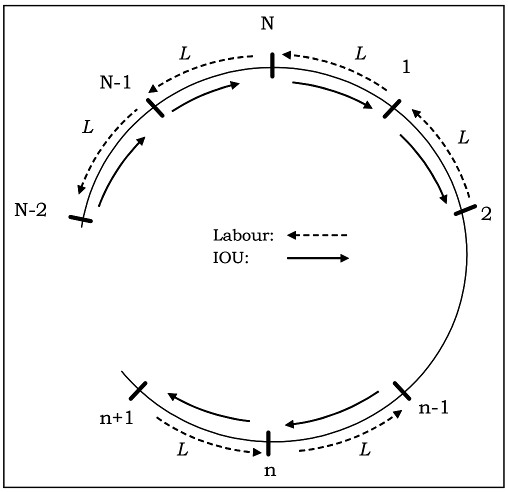

Figure 2: The trade credit chain

Given the uncertainty surrounding debt repayments, an entrepreneur, say $n-1$, may repay only a fraction $x \leqslant 1$ of his due payables to his supplier, i.e. entrepreneur n, at the end of the period (see Figure 2). This happens when his total assets are lower than his total debt. By symmetry, entrepreneur n may be able to repay only a fraction $u \leqslant 1$ of his due payables, as shown in the balance sheets below.

	Firm $n-1$: A	Firm $n-1$: L		Firm n: A	Firm n: L	
	q			q		
$\longrightarrow$	$v\overline{R}L-$ collection cost	$x\overline{R}L$	$\longrightarrow$	$x\overline{R}L-$ collection cost	$u\overline{R}L$	$\longrightarrow$

Table 2: End-of-period Balance Sheets

The total ex post assets of entrepreneur n at the end of the period is equal to its ex post net return q plus the reimbursement of trade credit from entrepreneur $n-1$, $x\overline{R}L$, net of the cost of collecting the money (see next paragraph). How large x is depends on the assets of entrepreneur $n-1$, and in particular on his project's returns. As a result, the fractions of repayment x and u are also random variables, whose distributions are related to the distribution of q. I will denote by g the distribution of repayments, and by G the corresponding cumulative. (Note that distribution of assets at the end of the period is fully characterized by f and g.)

Collection Cost. It is not rare that trade creditors hire the services of collection agencies to help them collect their money at the due date of payment. The implied costs are of various natures. They can be transaction costs: insolvent customers may shuffle to repay their debt and thereby force trade creditors to visit them several times to get some money back. Costs can also arise from a moral hazard problem (e.g. costly state verification): solvent customers may invoke bad financial health in order not to repay their debt, and force suppliers to monitor, at a cost, all defaulting customers. In the rest of the paper I will assume that the collection cost associated with a repayment $x\overline{R}L$ is equal to $\phi(x)\,x\overline{R}L$. The function ϕ is such that $\phi(1) = 0$, which means that the cost is null when the customer is solvent. I also assume that $\phi(x) \in [0,1]$, which means that the repayment always covers the collection cost, and that $\phi'(x) < 0$, which means that the lower the repayment, the higher the fraction of the repayment devoted to paying the collection cost. To fix ideas, I set $\phi(x) = 1 - x^{\eta}$ with $\eta \geqslant 0$, where η captures the size of the cost. In net terms, suppliers therefore get $x^{1+\eta}\overline{R}L$ when customers repay a fraction x of their trade debt. While they collect the money at no cost if $\eta = 0$, they do not get anything in net terms as $\eta \to +\infty$.

As Figure 3 shows, collection costs do not increase monotonously with the amount of the default or payment. This captures the idea that it does not cost anything to the supplier to collect payments either if there is nothing to collect or if the customer is solvent, while it is costly to collect payments in intermediate cases. Collection costs will play an important role in the model as they imply that defaults generate deadweight losses, increase the cost of external finance, and eventually have a negative impact on labour demand and aggregate output (see Section 2.3).

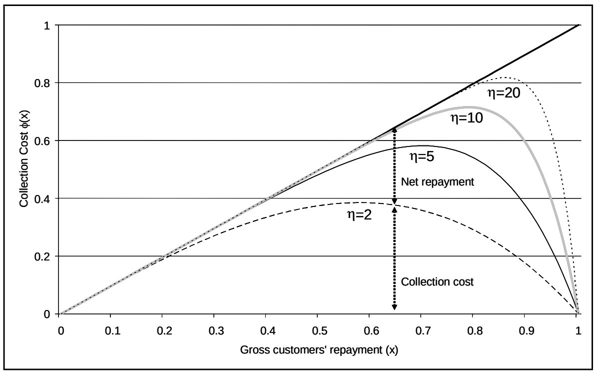

Figure 3: Collection cost

Repayment Distribution. If entrepreneur n is repaid a fraction x of his accounts receivable by entrepreneur $n-1$, then his total asset amounts to $q+x^{1+\eta}\overline{R}L$ (see Table 2). Therefore, he will not be able to repay more than a fraction u of his accounts payable to entrepreneur $n+1$ if his total assets are lower than a fraction u of his total debt: $q + x^{1+\eta}\overline{R}L \leqslant u\overline{R}L$. It follows that the probability $G(u)$ that entrepreneur n repays less than a fraction u of his trade debt writes:

$$\begin{cases} G(u) = \int_0^1 F\left((u - x^{1+\eta})\overline{R}L\right) g(x)dx \ \forall u \in [0,1) \\ \\ \text{with } g \text{ is such that } G(u) = \int_0^u g(x)dx \ \forall u \in [0,1] \text{ and } G(1) = 1 \end{cases} \tag{2.2}$$

Note that the distribution of entrepreneur n's repayments depends not only on the distribution of his own return, but also on the distribution of entrepreneur $n-1$'s repayments. As a consequence, the repayment distribution $g(.)$ is the fixed point solution to equation (2.2). (I show in Appendix 8 that a solution exists and is unique.) The probability $G(u)$ increases with the entrepreneur's total debt. It is also closely related to the distribution of returns f, to the extent that the riskier the distribution of returns, the less likely are entrepreneurs to repay their debts.

In the rest of the paper, I will say that a firm is in financial distress (or, equivalently, that it is insolvent or defaults on its debt) when its total assets are below its total debt at the end of the period: $q + x^{1+\eta}\overline{R}L < \overline{R}L$. I will also make a distinction between liquid and "sound" firms. A firm will be said liquid when it can repay its whole debt cash on the nail, i.e. without being repaid by its customer: $q \geqslant \overline{R}L$. In contrast, I will say that a firm is sound if it can repay its debt as soon as it is fully repaid by its customer, i.e. if $q \geqslant 0$. Hence, a sound firm may be in financial distress if its customer does not repay enough of his due payables.

2.2 Equilibrium Lending Rates

The objective of the typical entrepreneur/supplier is to fix the lending interest rate on trade credit that maximizes his total expected profit. However, the assumption of perfect competition implies they break-even in the equilibrium. Hence, the equilibrium lending interest rate on trade credit is defined by the following relation:

$$\int_0^1 x^{1+\eta}\overline{R}^* g(x)dx = R \tag{2.3}$$

which means that the average net repayment to suppliers (left hand side) is equal to the opportunity cost of lending (right hand side). Relation (2.3) implies that the lending rate is higher than the riskless rate $\overline{R}^* > R$. On the entrepreneur/customer's side, the expected cost of the project is equal to:

$$\int_0^1 x\overline{R}^* g(x)dx = (1+\theta)R \text{ with } \theta \equiv \frac{\int_0^1 xg(x)dx}{\int_0^1 x^{1+\eta}g(x)dx} - 1 \geqslant 0 \tag{2.4}$$

where the term θ is the external finance premium imputable to collection costs. Relation (2.4) implies $R \leqslant (1+\theta)R < \overline{R}^*$. When collection costs are null, $\theta = 0$ and entrepreneurs/customers repay on average the safe interest rate. In Figure 4 below I have represented the relationship between the lending rate and the external finance premium due to collection costs on the one hand, and risk on the other hand. I measured the latter by the variance of the distribution f, the mean of f staying constant. In this numerical application, I set $\alpha = 0.5$, $A = 1$, $L = 0.2$, $R = 1.02$, and $\eta = 10$. Unlike the lending rate, the external finance premium due to collection costs does not increase monotonously with risk. The reason is that collection costs do not increase monotonously with the amount collected: they are low when repayments are either low or large, that is to say when the variance of returns is high, while they are large when repayments are intermediate, i.e. when the variance of returns is low (see Figure 3). It follows that the total deadweight loss due to debt collection first increases with risk, as more firms default, and then decreases as the amount collected per defaulter diminishes.

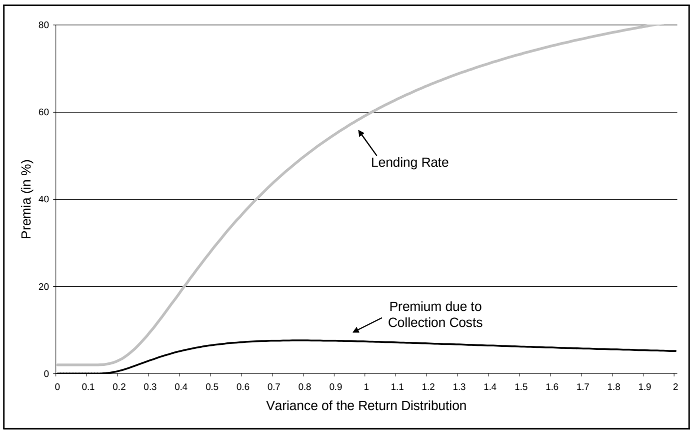

Figure 4: External Finance Premium and Risk

2.3 Equilibrium Labour Demand and Trade Debt

The objective of the typical entrepreneur/customer consists in choosing the quantity of labour L that maximizes his total expected profit. Labour demand determines the amount of trade debt contracted by the entrepreneur. Since the average return of the project is equal to $\overline{q} = AL^{\alpha}$ and trade debt repayments are on average equal to $(1+\theta)RL$, the expected profit of the entrepreneur is equal to $AL^{\alpha} - (1+\theta)RL$. The first order condition determines the optimal labour demand and trade debt:

$$L^{*} = \left(\frac{A\alpha}{(1+\theta)R}\right)^{\frac{1}{1-\alpha}} \tag{2.5}$$

as well as the optimal labour/trade debt to GDP ratio:

$$\frac{L^*}{\overline{q}^*} = \frac{\alpha}{(1+\theta)R} \tag{2.6}$$

Labour demand depends negatively on the riskless interest rate factor R (since entrepreneurs pay labour on credit) as well as on the external finance premium due to collection costs θ and thus on the number of defaults in the economy. It therefore also depends on the return and repayment distributions f and g (see relations (2.2) and (2.4)). A shift of the return distribution to its left (following, for instance, a negative technology shock) entails a shift of repayment distribution to its left, which raises the cost of external finance and thereby magnifies the initial effects of the shock.

Aggregate Cost of Defaults. In absence of collection costs, defaults only affect the distribution of wealth across entrepreneurs at the end of the period. As there is then no deadweight loss in the economy and entrepreneurs are risk-neutral, entrepreneurs do not account for these distributional effects when they choose the size of their project. As a consequence, defaults do not affect aggregate output. Things are different when debt collection is costly. In this case, defaults have a negative effect on output because entrepreneurs/suppliers ask for a positive interest rate premium on trade credit ($\theta > 0$) to cover the collection cost they expect to incur at the end of the period. The higher this premium, the lower the size of the projects. The effect of defaults on aggregate output can be measured by the gap between the outputs that would prevail without ($\theta = 0$) and with ($\theta > 0$) collection costs. Denoting by ρ this gap and using relations (2.5) and (2.6), one gets $\rho = (1+\theta)^{\frac{\alpha}{1-\alpha}} - 1 > 0$.[8]

3 Chain Reactions

It is now possible to derive the probability to get into financial distress, which I denote by P:

$$P = \lim_{u \nearrow 1} G(u) = \int_0^1 F\left((1 - x^{1+\eta})\overline{R}L\right) g(x)dx \tag{3.1}$$

[8] More precisely: $\rho = \left(\frac{A\alpha}{R}\right)^{\frac{\alpha}{1-\alpha}} / \left(\frac{A\alpha}{(1+\theta)R}\right)^{\frac{\alpha}{1-\alpha}} - 1$.

The reason why an entrepreneur defaults is twofold: either he is unsound, or he is sound but illiquid and his client did not repay enough of his trade debt. In the first case, the entrepreneur defaults, irrespective of the quality of his customer. This happens with probability $P_{unsound}$:

$$P_{unsound} = F(0) \tag{3.2}$$

The assumption that entrepreneurs may experience negative liquidity shock (i.e. $F(0) > 0$) is crucial to have financial contagion (see relation (3.4) below), to the extent that the repayments made to unsound firms are absorbed by the liquidity shock, which thus prevents unsound firms from repaying there own debt. In the second case, a so-called domino effect (or financial contagion) is present in the sense that the entrepreneur defaults because his customer primarily defaulted on him. This happens with probability P_{contag}, which can be derived easily by decomposing the unconditional probability of default as follows:

$$P = (1 - P^{*}).P_{unsound} + P.(P_{unsound} + P_{contag}) = P_{unsound} + P.P_{contag} \tag{3.3}$$

where $P_{unsound}+P_{contag}$ is the probability that an entrepreneur defaults given that his customer defaulted and $P.P_{contag}$ is the probability to be sound and yet insolvent. Hence, P_{contag} can also be viewed as the proportion of sound among insolvent firms. Relation (3.3) implicitly defines the probability of default due to financial contagion:

$$P_{contag} = \frac{P - P_{unsound}}{P} \tag{3.4}$$

Relations (3.1) and (3.2) show that financial contagion arises even in absence of collection costs ($P > P_{unsound}$ even when $\eta = 0$). This illustrates the idea that financial contagion is due to distributional effects in the first place. I have represented in Figure 5 the relationship between the default probabilities and risk. (I consider the same numerical example as in Section 2.2.)

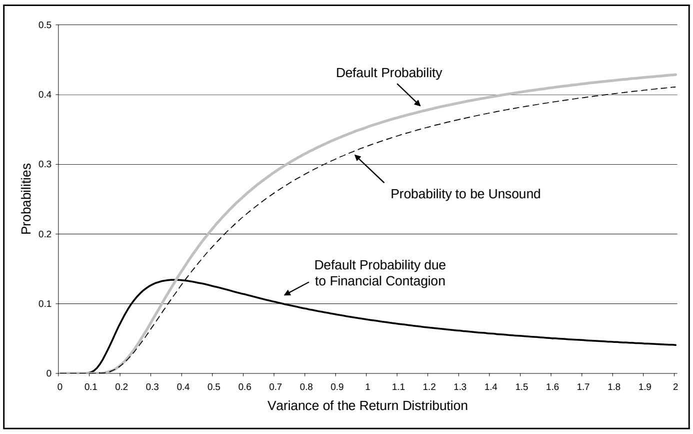

Figure 5: Default Probabilities and Risk

Unlike the probabilities to default and to be unsound, the default probability due to the domino effect does not increase monotonously with risk. It first increases and then decreases as the variance of returns goes up. To understand this point, consider three economies, with low, intermediate and high risks, in which the expected return and the total debt are both equal to one ($\overline{q} = 1$), and the interest rates are such that $\overline{R}L = 1/2$. In the low risk economy, all firms produce $q = 1$ with certainty and are therefore liquid. There is no default. In the intermediate risk economy, returns are distributed according to a Uniform law over the interval $[0, 2]$. Since there is a continuum of returns, some firms are illiquid, sound, and not repaid by their customer. As a result, a domino effect is present. In the high risk economy, firms produce either $q = 0$ or $q = 2$ with probability $1/2$, so that they are either unsound or liquid. Since no firm relies on its customer to repay its own debt (i.e. no firm is both sound and illiquid), there is no domino effect. It follows that the domino effect only exists in intermediate risk economies where some firms rely on their receivables to repay their own debt.

4 Discussion

4.1 Contagion and Containment

In the economy, an initial adverse shock to the financial wealth of an entrepreneur, say n, might cause a chain reaction in which entrepreneurs $n + 1$, $n + 2$,... get into financial difficulties. Obviously, the amplitude of the chain reaction depends on the financial wealth of entrepreneurs $n+1$, $n+2$,... at the end of the period. If entrepreneur $n+1$ is liquid, then he is able to repay his whole debt despite the losses he incurs on his receivables. In this case, the chain reaction is "contained". (See Allen and Gale (2000) for a definition of the notion of "containment".) If, by contrast, entrepreneur $n+1$ is not liquid, then the initial shock to his customer has an impact on his repayment capacity, which affects entrepreneur $n+2$'s financial wealth, etc. In this case, there is a chain reaction: the probability that entrepreneur $n+1$ defaults owing to the default of entrepreneur n is P_{contag} (direct link in the trade chain), the probability that entrepreneur $n+2$ defaults because entrepreneur n defaulted is P_{contag}^2 (two links ahead in the trade chain), and the probability that entrepreneur $n+j$ defaults because of n is P_{contag}^j (j links ahead). Obviously, the financial difficulties of entrepreneur n are more contagious for entrepreneur $n+1$ (his direct supplier) than for entrepreneurs $n+2$, $n+3$,... and the probability that an entrepreneur "contaminates" another entrepreneur decreases geometrically with the number of links that separate them in the trade chain. Eventually, the average number of defaults generated exclusively by the default of one particular entrepreneur in the economy is equal to $\frac{P_{contag}}{1-P_{contag}}$.

4.2 Diversification

In the basic model, the assumption that every entrepreneur has only one customer made his financial health particularly dependent of his customer's and thereby gave strength to the domino effect. Here I generalize the basic model by assuming that each entrepreneur trades with a pool of K suppliers and a pool of K customers indexed by $k = 1, ..., K$, with $1 \leqslant K < N$. In this new setup, every entrepreneur holds a fraction $\frac{1}{K}$ of its customers' trade credit claims. For example, entrepreneur n holds a fraction $\frac{1}{K}$ of entrepreneur k's trade credit claims and gets $\frac{1}{K}x_k^{1+\eta}\overline{R}L$ from entrepreneur k at the end of the period, where x_k is the fraction of trade credit that k repays. I will denote the total repayment to entrepreneur n by $\overline{x}\overline{R}L$, where $\overline{x} \equiv \frac{1}{K}\sum\limits_{k=1,...,K} x_k^{1+\eta}$. Entrepreneur n will not be able to repay more than a fraction u of its debts if his ex post return q is such that $q + \overline{x}\overline{R}L \leqslant u\overline{R}L$. By analogy with the case $K = 1$, it is possible to derive the repayment distribution g, defined by:

$$G(u) = \int_0^1 ... \int_0^1 F\left((u - \overline{x})\,\overline{R}L\right) g(x_1)...g(x_K)dx_1...dx_K \;\forall u \in [0, 1)$$

as well as the equilibrium lending rate, which is still given by relation (2.3).

In the special case of perfect diversification, when $K \longrightarrow +\infty$ (and $N > K + 1 \longrightarrow +\infty$), the law of large numbers applies, implying that entrepreneurs are repaid on average:

$$\lim_{K \longrightarrow +\infty} \frac{1}{K} \sum_{k=1,...,K} x_k^{1+\eta} \overline{R}L = \int_0^1 x^{1+\eta} \overline{R}Lg(x)dx = RL$$

As entrepreneurs are certain to receive RL from their customers at the end of the period, balance sheets are not interdependent anymore. The repayment distribution is then defined by $G(u) = F\left((u\overline{R} - R)L\right) \;\forall u \in [0, 1)$ and the probability to default is equal to $P = F\left((\overline{R} - R)L\right)$. Note that perfect diversification does not eliminate financial contagion ($P > F(0)$) because it does not enable entrepreneurs to close the gap between what they get from their customers (RL) and what they owe ($\overline{R}L$).

5 Quantification of the Domino Effect

In this section I simulate the model in order to evaluate the domino effect and, more generally, the consequences of defaults at the firm and aggregate levels. To do so, I calibrate the model using annual data from US National Accounts and Compustat (see footnote 5). Assuming that returns are distributed Normally, I have to calibrate the parameters A, α, R, K, η, and the standard deviation σ of the return distribution. I take the economy with perfect diversification ($K = +\infty$) and without collection cost ($\eta = 0$) as a benchmark, and set the technology parameters $A = 1.44$, $\alpha = 0.4$, and $\sigma = 0.6$ in order to have, in this benchmark economy, aggregate output normalized to one ($\overline{q} = 1$), the trade debt to output ratio equal to $L/\overline{q} = 39.8\%$, which corresponds to the ratio observed in 2004 (see Figure 1a), and the probability of default equal to $P = 4.7\%$, which corresponds to the bad trade debt to total trade debt ratio observed in 2004 (see Figure 1b).[9] I also set $R = 0.988$, which is equal to one plus the real interbank loan rate in 2004. The results of the simulations for various degrees of diversification ($K = 1, 2, +\infty$) and collection costs ($\eta = 0, 2, +\infty$) are reported in Table 3.

When the customers of a sound firm are financially distressed, then this firm gets into financial difficulties with a strictly positive probability. This probability ranges from 3.4% to 11.3%, depending on the level of collection costs and the degree of diversification. Although the existence of collection costs is not a condition for financial contagion to arise, the latter is more than 2 times larger when collection costs are high ($\eta = +\infty$) than when they are null ($\eta = 0$).

(in %)	$L/\overline{q}$	P	$P_{unsound}$	P_{contag}	$\overline{R} - R$	θ	ρ
Observed	39.8	4.7	-	-	-	-	-
			Benchmark: $\eta = 0$				
$K = +\infty$	39.8	4.7	4.5	**3.4**	2.4	0	**0**
$K = 2$	39.8	4.7	4.5	**4.1**	2.4	0	**0**
$K = 1$	39.8	4.8	4.5	**5**	2.5	0	**0**
			"Best case" scenarios: $\eta = 2$				
$K = +\infty$	39.4	4.9	4.6	**4.8**	3.6	1.0	**0.7**
$K = 2$	39.4	4.9	4.6	**6.1**	3.6	1.0	**0.7**
$K = 1$	39.4	5.0	4.6	**7.5**	3.7	1.0	**0.7**
			"Worst case" scenarios: $\eta = +\infty$				
$K = +\infty$	38.8	5.2	4.8	**6.9**	5.3	2.6	**1.7**
$K = 2$	38.8	5.3	4.8	**8.9**	5.5	2.6	**1.7**
$K = 1$	38.8	5.4	4.8	**11.3**	5.6	2.7	**1.7**

Table 3: Numerical application

The magnitude of financial contagion also depends on how well diversified entrepreneurs are (see discussion in Section 4.2). I thus find that perfect diversification reduces on average

[9] Note that the normalization of the scale of output does not affect the results of the simulations to the extent that I fix the trade debt to output ratio and the probability of default.

financial contagion by 60% (compare cases $K = +\infty$ with cases $K = 1$). At the aggregate level, non-repayments have an effect on GDP ($\rho > 0$) only if one assumes strictly positive collection costs, i.e. if $\eta > 0$. Factoring companies usually advertise fees for managing and collecting receivables from 1% to 3%. In the model, the premium due to collection costs θ is compatible with these figures if $\eta > 2$. Beside the benchmark $\eta = 0$, I therefore reported the results of the simulations in two additional scenarios. In the "best case scenario" ($\eta = 2$), I find that GDP losses due to the defaults on trade debt (ρ) amount to 0.7% of GDP, while in the "worst case scenario" ($\eta = +\infty$), I find 1.7%. Unlike collection costs, diversification has little effect on aggregate output, for the reason that it mostly affects the distribution of wealth across entrepreneurs but not wealth itself. The model also predicts the interest rate spread on trade debt ranges from 3.6% to 5.6%, which is above the observed 2.1% spread on short term bank loans to businesses in 2004[10], which is therefore consistent with the widespread view that trade credits are more expensive than bank loans.

Financial Contagion in the Business Cycle. The aim of this paragraph is to evaluate financial contagion (P_{contag}) and the aggregate cost of defaults (ρ) over the last US business cycle. To obtain a range of plausible values for P_{contag} and ρ, I simulate the model in the worst and best case scenarios described above, with $K = 1$ and $K = +\infty$ respectively. To have meaningful results, I re-calibrate the model so that it faithfully replicates the average trade debt to sales ($L/\overline{q}$) and bad debt to trade debt (P) ratios observed for the S&P500 companies, as well as the US GDP cycle ($\overline{q}$). I normalize the GDP trend to one, and define the GDP cycle as equal to one plus the observed GDP gap ($\overline{q} = 1+$ GDP gap[11]). As riskless interest rate, I use the observed real interbank loan rate. Given the observations of the endogenous variables $\overline{q}_t$, $L_t/\overline{q}_t$, P_t and the interest rate factor R_t in year t, I invert the model by solving the three equations (2.1), (2.6), and (3.1) for the three parameters A_t, σ_t, α_t.[12] I do this calibration exercise for each year over the period 1986-2004 and each scenario. The parameters thus

[10] This spread is the difference between the prime rate on bank loans to businesses and the interbank loan rate. The prime rate is one of several base rates used by banks to price short-term Business loans (Source: Federal Reserve Bank).

[11] As for Figures 1a-b, I define the GDP Gap as the annual average of the quarterly HP-filtered cycle, in percentage deviation from the trend.

[12] To solve this system of equations one needs to solve simulataneously for the repayment distribution g (equation (2.2)), the lending rate (equation (2.3)), and θ (equation (2.4)).

calibrated are reported in Figures 6a-c below. The comparison with Figures 1a-b shows that the scale parameter A_t, the dispersion parameter σ_t, and the elasticity α_t respectively help the model replicate the observed level of output, trade debt to output ratio, and probability of default. In particular, one has to assume an increase in the variance of the return distribution in the second half of the 90's in order the model to reproduce the observed rise in the bad debt ratio (see Figures 1b and 6b, as well as the discussion in Section 3).

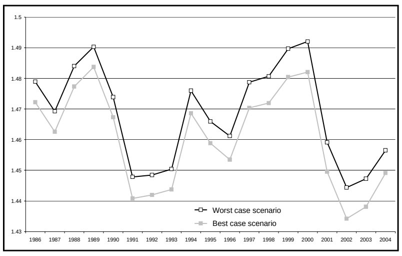

Fig. 6a: Parameter A_t

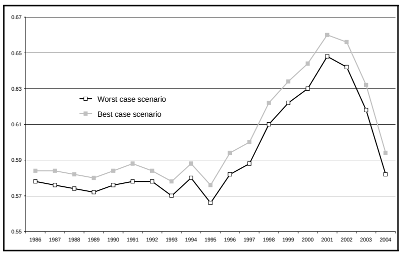

Fig. 6b: Parameter σ_t

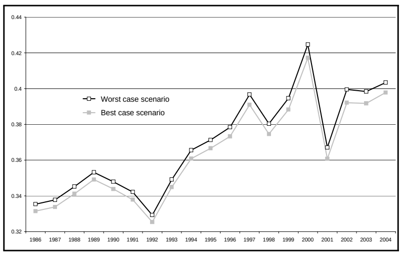

Fig. 6c: Parameter α_t

The results of the simulations are reported in Figures 7a-d. I find that the rises in the probability of default on trade debt and the amount of trade debt in the economy observed between 1995 and 2000 (Figure 1a-b) worked to increase collection costs. In terms of GDP, the implied deadweight losses went up to 2.2% of GDP in 2000 in the worst case scenario (against 0.9% in the best case scenario), before decreasing in 2001. Overall, these GDP losses were above 0.4% of GDP over the whole period. At the firm level, the default probability due to financial contagion rose from 8.4% in 1992 in the worst case scenario (against 4.1% in the best case scenario) up to 12.8% in 2000 (against 5.8% in the best case scenario).

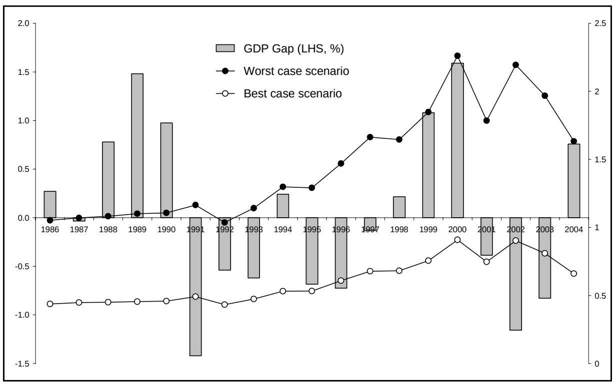

Fig. 7a: Cost of defaults (ρ, in % of GDP)

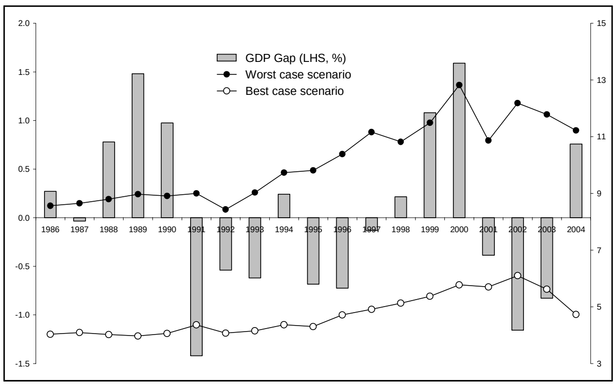

Fig. 7b: Financial Contagion (P_{contag}, in %)

The implied interest rate spread on trade debt ($\overline{R} - R$), followed the same pattern as the probability of default, with a surge in 2000-2001 and a drop in 2003-2004. It stayed above the observed interest rate spread on bank loans and was positively correlated with the latter, with a correlation of 0.55 in the two scenarios, over the period.

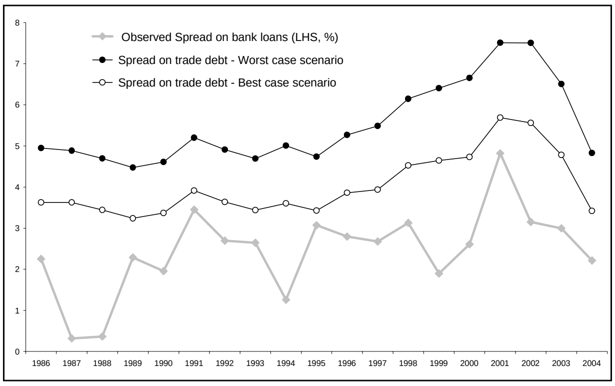

Fig. 7c: Spreads, in %

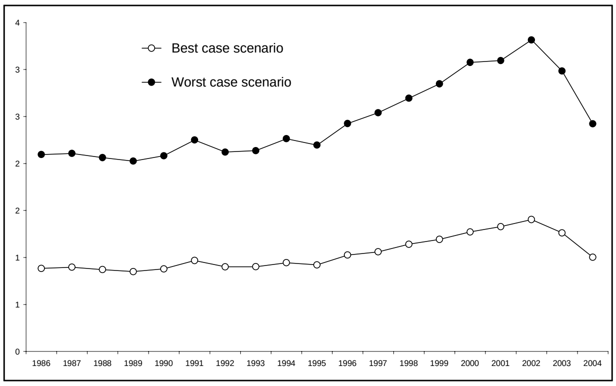

Fig. 7d: Ext. finance premia (θ, in %)

The interest rate spread on trade debt is driven by two forces. The first force is the change in the distribution of returns. For instance, Figures 6a-b show that expected returns increased in the late 90's but that, at the same time, the variance of these returns increased as well. As a result, risks increased and lenders charged a higher risk-premium. The second force is the change in the external finance premium due to collection costs (θ). At the end of the 90's, firms increased their lending rates not only because the risk of not being paid back rose, but also because they expected higher collection costs due to non- repayments (Figure 7d).

The results of the simulations also suggest that financial contagion among businesses had been about 25% higher during the last recession than in the early 90's. The reason is that the economy was overall riskier at the end of the 90's (Figure 6b) while, at the same time, firms were more subject to financial contagion, as the weight of trade debt in the economy was more that 20% higher at the end of the 90's than in the 80's (Figure 1a).

6 Conclusion

This paper is the first to model financial contagion across firms through chains of trade credit. I show how, in presence of idiosyncratic shocks, some firms may not be able to repay their trade payables and thereby have a negative wealth effect on their suppliers. The model makes it possible to identify liquid, sound, and insolvent firms. When firms use trade credit, even sound firms may turn out to be insolvent, for the reason that they are not paid back by their customers. I calibrate the model on US data and simulate it in order to quantitatively assess the impact of trade credit chains on the economy. Consistent with the conventional wisdom that defaults on trade credit are a major cause for firms' financial distress, I find that the proportion of sound firms among insolvent firms goes from 4.1% up to 12.8%, depending on the business cycle and the economic scenario (high or low diversification, high or low collection costs). At the macroeconomic level, the results of the simulations contrast with macroeconomists' common view that financial contagion across firms and defaults on trade debts are innocuous. Under a reasonable calibration of the model, I indeed find that defaults on trade debts lower aggregate GDP by at least 0.4%. These deadweight losses doubled during the second half of the 90's and reached a high of 0.9% to 2.3% of GDP (depending on the underlying economic scenario) before the recession of 2001. The results of the simulations also suggest that financial contagion across businesses had been 25% higher during the last recession than during the recession of the early 90's.

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8 Appendix: A unique repayment distribution g exists

I will give here the sketch of the proof that a unique density function g is solution to equation (2.2) in the simple case where $\eta = 0$ (the generalization to $\eta > 0$ is straightforward). Consider the continuous function Γ derivable almost everywhere over interval $[0,1]$ and such that $\Gamma(x) = G(x)\ \forall x \in [0,1)$ and $\Gamma(1) = \lim_{x \nearrow 1} G(x)$. I look for the function $\Gamma(.)$ defined by (from relation (2.2)):

$$\begin{aligned}\Gamma(u) = \textstyle\int_0^1 F\left((u-x)\overline{R}L\right)\gamma(x)dx + (1-\Gamma(1)).F\left((u-1)\overline{R}L\right) \\ +\Gamma(0).F\left(u\overline{R}L\right)\end{aligned} \tag{8.1}$$

where $\gamma(.)$ satisfies $\Gamma(u) = \int_0^u \gamma(x)dx\ \forall u \in [0,1]$ and $\gamma(x) = g(x)\ \forall x \in (0,1)$. Taking the first derivative of (8.1), I get:

$$\begin{aligned}\gamma(u) = \overline{R}L.\textstyle\int_0^1 f\left((u-x)\overline{R}L\right)\gamma(x)dx + \overline{R}L.(1-\Gamma(1)).f\left((u-1)\overline{R}L\right) \\ +\overline{R}L.\Gamma(0).f\left(u\overline{R}L\right)\end{aligned} \tag{8.2}$$

Using the rescaling function φ of f defined by $\varphi(u) \equiv f\left(u\overline{R}L\right)\ \forall u$, I get:

$$\gamma(u) = \overline{R}L.\left[\int_0^1 \varphi(u-x)\gamma(x)dx + (1-\Gamma(1)).\varphi(u-1) + \Gamma(0).\varphi(u)\right] \tag{8.3}$$

where the first term into brackets is the convolution product of φ and γ. Taking the Fourier transformations $\widehat{\varphi}$ and $\widehat{\gamma}$ of φ and γ, and using the property that the Fourier transformation of the convolution product of two functions is equal to the ordinary product of the Fourier transformations of these functions, I get:

$$\widehat{\gamma}(v) = \overline{R}L.\left[\widehat{\gamma}(v).\widehat{\varphi}(v) + (1-\Gamma(1))e^{-iv}\widehat{\varphi}(v) + \Gamma(0)\widehat{\varphi}(v)\right]$$

$$\Rightarrow \widehat{\gamma}(v) = \frac{\left[(1-\Gamma(1))e^{-iv} + \Gamma(0)\right]}{\frac{1}{\overline{R}L\widehat{\varphi}(v)} - 1} \text{ almost everywhere}$$

Using the inversion theorem of Fourier functions, I know that the density function $\gamma(u)$ exists and is uniquely defined by relation

$$\gamma(u) = \int \widehat{\gamma}(v).e^{ivu}dv < \int \left|\widehat{\gamma}(v).e^{ivu}\right| dv \leqslant \int \left|\widehat{\gamma}(v)\right| dv \tag{8.4}$$

Since $\widehat{\varphi}(v)$ is continuous, bounded, and $\lim_{|v|\to\infty}\widehat{\varphi}(v) = 0$, the function under the integral sign on the right-hand side of the inequality in (8.4) is also continuous, bounded, and tends to zero as $|v| \to \infty$. Hence, the integral on the left-hand side of the inequality exists, as well as $\gamma(u)$. Therefore, $\Gamma(.)$ exists.

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