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A MICRO-FOUNDED OPEN-ECONOMY MODEL FOR FORECASTING AND POLICY ANALYSIS

by Kai Christoffel, Günter Coenen and Anders Warne

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In 2008 all ECB publications feature a motif taken from the €10 banknote.

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Abstract

In this paper, we outline a version of the New Area-Wide Model (NAWM) of the euro area designed for use in the (Broad) Macroeconomic Projection Exercises regularly undertaken by ECB/Eurosystem staff. We present estimation results for the NAWM that are obtained by employing Bayesian inference methods and document the properties of the estimated model by reporting its impulse-response functions and forecast-error-variance decompositions, by inspecting the model-based sample moments, and by examining the model's forecasting performance relative to a number of benchmarks, including a Bayesian VAR. We finally consider several applications to illustrate the potential contributions the NAWM can make to forecasting and policy analysis.

JEL Classification: C11, C32, E32, E37

Keywords: DSGE modelling, open-economy macroeconomics, Bayesian inference, forecasting, policy analysis, euro area

Non-Technical Summary

This paper reports on the progress made in estimating a version of the New Area-Wide Model (NAWM)—a micro-founded open-economy model of the euro area under development at the European Central Bank (ECB)—that is designed for use in the (Broad) Macroeconomic Projection Exercises regularly undertaken by ECB/Eurosystem staff and for policy analysis. The NAWM is neo-classical in nature and centred around intertemporal decisions of households and firms which are maximising expected life-time utility and the expected stream of profits, respectively. As a result, forward-looking expectations play a key role in influencing the adjustment dynamics of both quantities and prices, and changes in supply-side factors have a pronounced impact already in the short run. At the same time, the NAWM includes a number of nominal and real frictions that have been identified as empirically important, such as sticky prices and wages (so that some Keynesian features prevail in the short run), habit persistence in consumption and adjustment costs in investment. Moreover, it incorporates analogous frictions relevant in an open-economy setting, including local-currency pricing (giving rise to imperfect exchange-rate pass-through in the short run), and costs of adjusting trade flows.

The design of the NAWM for use in the ECB/Eurosystem staff projections has been guided by two important considerations, namely (i) to provide a comprehensive set of core projection variables and (ii) to allow conditioning on monetary, fiscal and external developments which, in the form of technical assumptions, are an important element of the projections. As a consequence, the scale of the model—compared with a typical DSGE model—is relatively large. Employing Bayesian methods, it is estimated on 18 key macroeconomic variables, including real GDP, private consumption, total investment, government consumption, exports and imports, a number of deflators, employment and wages, and the short-term nominal interest rate. In addition, data for the nominal effective exchange rate, euro area foreign demand, euro area competitors' export prices as well as oil prices are used, which are deemed important variables in the projections capturing the influence of external developments. Conformable with the number of variables, 18 structural shocks are considered in the estimation. These shocks are latent factors with an economic interpretation that help typifying the sources of the observed fluctuations in the data.

The use of Bayesian methods for estimating the NAWM enables us to assess the data coherence of the model as a whole and to compare alternative, not necessarily nested specifications of the model. Another advantage of using Bayesian methods is that it allows us to construct probability distributions for the model's parameters and for model-based fore-

casts, and to thereby quantify the uncertainty associated with model-based analysis and forecasting. This probabilistic approach also extends to validating the properties of the model economically by assessing the propagation of its structural shocks (on the basis of the model's impulse-response functions), the contribution of the structural shocks to the observed fluctuations in the variables used in the estimation (forecast-error-variance decompositions), and the implied sample moments for these variables (unconditional means, standard deviations and correlations). It also permits comparing the model's out-of sample forecast performance (as measured by its root-mean-squared forecast errors) with that of alternative benchmarks, including Bayesian VARs.

All in all, this paper shows that the NAWM fares quite well along these dimensions, and therefore the model seems well suited for forecasting and policy analysis. This is illustrated by a number of applications that, inter alia, exemplify how the NAWM can aid in understanding the evolution of the euro area economy by interpreting historical developments in terms of its structural shocks and by conducting scenarios that are concerned with counterfactual economic events. It is also demonstrated how to use the NAWM for constructing interval forecasts and to analyse the influence of conditioning information.

1 Introduction

Recent years have witnessed the development of a new generation of dynamic stochastic general equilibrium (DSGE) models that build on explicit micro-foundations with optimising agents. Major advances in estimation methodology allowed estimating variants of these models that are able to compete, in terms of data coherence, with more standard time-series models, such as vector autoregressions.[1] Accordingly, the new generation of micro-founded DSGE models provides a framework that appears particularly suited for evaluating the consequences of alternative macroeconomic policies. More recently, increasing efforts have been undertaken to use these models also for forecasting.[2]

Against this background, this paper reports on the progress made in estimating a version of the New Area-Wide Model (NAWM)—a micro-founded open-economy model of the euro area under development at the European Central Bank (ECB)—that is designed for use in the (Broad) Macroeconomic Projection Exercises regularly undertaken by ECB/Eurosystem staff.[3] The estimated version of the NAWM complements a calibrated version—richer in detail and open for topic-driven extensions—that has been developed for analysing a broader range of policy issues in a flexible manner (cf. Coenen, McAdam and Straub, 2007).[4] In pursuing this parallel development strategy, the NAWM builds extensively on the earlier work by Smets and Wouters (2003) and Adolfson, Laséen, Lindé and Villani (2007), who estimated, respectively, a closed and a small-open economy model of the euro area using Bayesian techniques. Moreover, the NAWM also draws on the advances made in developing the International Monetary Fund's calibrated Global Economy Model (GEM; cf. Bayoumi, Laxton and Pesenti, 2004) and the Federal Reserve Board's calibrated open-economy model named SIGMA (cf. Erceg, Guerrieri and Gust, 2006).

[1] See, among others, Smets and Wouters (2003, 2007), del Negro, Schorfheide, Smets and Wouters (2007), and Adolfson, Lindé, Laséen and Villani (2007), who estimate, using Bayesian methods, variants of the model by Christiano, Eichenbaum and Evans (2005).

[2] See Smets and Wouters (2004), Adolfson, Andersson, Lindé, Villani and Vredin (2007), Adolfson, Lindé and Villani (2007), and Edge, Kiley and Laforte (2008).

[3] In connection with the NAWM project, a Matlab program for Bayesian estimation and evaluation of DSGE and vector autoregressive models has been developed (cf. Warne, 2008). This program is named YADA (Yet Another DSGE Application) and available upon request.

[4] For applications that focus on fiscal policy issues see Coenen and McAdam (2006), Coenen, McAdam and Straub (2007), and Coenen, Mohr and Straub (2008).

The NAWM succeeds the Area-Wide Model (AWM; cf. Fagan, Henry and Mestre, 2001), which is a traditional macroeconomic model of the euro area that has been extensively used at the ECB over the past ten years. There are notable differences between the two models. The NAWM is neo-classical in nature and centred around intertemporal decisions of households and firms which are maximising expected life-time utility and the expected stream of profits, respectively. As a result, forward-looking expectations play a key role in influencing the adjustment dynamics of both quantities and prices, and changes in supply-side factors have a pronounced impact already in the short run. At the same time, the NAWM includes a number of nominal and real frictions that have been identified as empirically important, such as sticky prices and wages (so that some Keynesian features prevail in the short run), habit persistence in consumption and adjustment costs in investment. Moreover, it incorporates analogous frictions relevant in an open-economy setting, including local-currency pricing (giving rise to imperfect exchange-rate pass-through in the short run), and costs of adjusting trade flows. In contrast, the AWM features predominantly Keynesian behaviour in the short run, even though it has a basic neo-classical steady state. Thus, supply-side effects consistent with profit maximisation materialise only in the long run. The short-run dynamics of aggregate supply and aggregate demand are not explicitly derived from an optimisation framework, but instead specified and estimated in an ad hoc error-correction form. For this reason, and because of the absence of an explicit treatment of expectations, quantities and prices tend to adjust only very sluggishly.

The development of the estimated version of the NAWM has been guided by two important considerations, namely (i) to provide a comprehensive set of core projection variables and (ii) to allow conditioning on monetary, fiscal and external developments which, in the form of technical assumptions, are an important element of the ECB/Eurosystem staff projections.[5] As a consequence, the scale of the model—compared with a typical DSGE model—is relatively large. Employing Bayesian methods, it is estimated on 18 key macroeconomic variables, including real GDP, private consumption, total investment, government consumption, exports and imports, a number of deflators, employment and wages, and the short-term nominal interest rate. In addition, we utilise data on the nominal effective

[5] For an overview of the projection exercises and a description of the techniques and tools used therein, see ECB (2001).

exchange rate, euro area foreign demand, euro area competitors' export prices as well as oil prices, which are deemed important variables in the projections capturing the influence of external developments.[6] As a novel feature, we use the recently compiled extra-euro area trade data (both volumes and prices) which appears economically more compelling than the use of total trade data. Conformable with the number of variables, 18 structural shocks are considered in the estimation. These shocks are latent factors with an economic interpretation that help typifying the sources of the observed fluctuations in the data.

Compared with the calibrated version of the NAWM, which consists of two symmetric countries representing the euro area and the rest of the industrialised world, the estimated version maintains the simplifying assumption that the euro area is a small open economy, motivated by the aforementioned fact that the ECB/Eurosystem staff projections are made conditional on assumptions regarding external developments. Moreover, to ease the estimation of the model, the fiscal sector has been simplified by assuming that Ricardian equivalence holds. In this context, the assumption that a fraction of households is limited in their ability to participate in asset markets has been abandoned, along with the usage of money as a means of facilitating transactions. In contrast, the estimated version has been augmented by including a unit-root technology to account for a common stochastic trend in real quantities. We also explore the importance of allowing for a possibly time-varying inflation objective to capture the nominal convergence process in the run up to the formation of the European Economic and Monetary Union (EMU).

The use of Bayesian inference methods for estimating the NAWM enables us to assess the data coherence of the model as a whole and to compare alternative, not necessarily nested specifications of the model. Another advantage of using Bayesian methods is that it allows us to construct probability distributions for the model's parameters and for model-based forecasts, and to thereby quantify the uncertainty associated with model-based analysis and forecasting. This probabilistic approach also extends to validating the properties of the model economically by assessing the propagation of its structural shocks (on the basis of the model's impulse-response functions), the contribution of the structural shocks to

[6]The version of the NAWM actually used in the projections has been extended by bridge equations for HICP excluding energy and HICP energy, the latter measuring the direct impact of changes in oil prices on the HICP, as well as a number of other macroeconomic variables, including unemployment.

the observed fluctuations in the variables used in the estimation (forecast-error-variance decompositions), and the implied sample moments for these variables (unconditional means, standard deviations and correlations). It also permits comparing the model's out-of sample forecast performance (as measured by its root-mean-squared forecast errors) with that of alternative benchmarks, including Bayesian VARs.

All in all, this paper shows that the NAWM fares quite well along these dimensions, and therefore the model seems well suited for forecasting and policy analysis. This is illustrated by a number of applications that, inter alia, exemplify how the NAWM can aid in understanding the evolution of the euro area economy by interpreting historical developments in terms of its structural shocks and by conducting scenarios that are concerned with counterfactual economic events. It is also demonstrated how to use the NAWM for constructing interval forecasts and to analyse the influence of conditioning information.

The remainder of the paper is organised as follows. Section 2 outlines the NAWM for forecasting and policy analysis, while Section 3 presents our estimation results, along with some sensitivity analysis. Section 4 documents the properties of the estimated model, whereas Section 5 considers several applications. Section 6 concludes and discusses directions for future extensions. Two appendices provide details on the log-linear version of the model and the computation of its steady state.

2 Model Specification

Within the euro area—henceforth, domestic—economy, there are four types of economic agents: households, firms, a fiscal authority, and a monetary authority. As regards firms, we distinguish between producers of tradable differentiated intermediate goods and producers of three non-tradable final goods: a private consumption good, a private investment good, and a public consumption good. In addition, there are foreign intermediate-good producers that sell their differentiated goods in domestic markets, and a foreign retail firm that combines the exported domestic intermediate goods. International linkages arise from the trade of intermediate goods and international assets, allowing for limited exchange-rate pass-through on the import side and imperfect risk sharing.

In the following, we describe the behaviour of the different types of agents, formulate the market-clearing conditions and the aggregate resource constraint, and state the law of motion for the domestic (net) holdings of foreign assets. In this context, we also define expressions for the trade balance and the terms of trade. To the extent needed, foreign variables and parameters are indexed with an asterisk, '*'.

2.1 Households

There is a continuum of households indexed by $h \in [0, 1]$, the instantaneous utility of which depends on the level of consumption as well as hours worked. Each household accumulates physical capital, the services of which it rents out to firms, and buys and sells domestic government bonds as well as internationally traded foreign bonds. This enables households to smooth their consumption profile in response to shocks. The households supply differentiated labour services to firms and act as wage setters in monopolistically competitive markets. As a consequence, each household is committed to supply sufficient labour services to satisfy firms' labour demand.

Preferences and Constraints

Each household h maximises its lifetime utility in a given period t by choosing purchases of the consumption good, $C_{h,t}$, purchases of the investment good, $I_{h,t}$, which determines next period's physical capital stock, $K_{h,t+1}$, the intensity with which the existing capital stock is utilised in production, $u_{h,t}$, and next period's (net) holdings of domestic government bonds and internationally traded foreign bonds, $B_{h,t+1}$ and $B^*_{h,t+1}$, respectively, given the following lifetime utility function:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} \beta^k \left(\epsilon^C_{t+k} \ln \left(C_{h,t+k} - \kappa \, C_{t+k-1} \right) - \frac{\epsilon^N_{t+k}}{1+\zeta} \left(N_{h,t+k} \right)^{1+\zeta} \right) \right], \tag{1}$$

where β denotes the discount factor and ζ is the inverse of the Frisch elasticity of labour supply.[7] The parameter κ measures the degree of external habit formation in consumption. Thus, the utility of household h depends positively on the difference between the current level of individual consumption, $C_{h,t}$, and the lagged economy-wide consumption level,

[7] In contrast to the calibrated version of the NAWM, the utility function is assumed to be logarithmic in consumption in accordance with the balanced-growth property of the model.

C_{t-1}, and negatively on the number of hours worked, $N_{h,t}$. We will refer to ϵ_t^C and ϵ_t^N as consumption preference and labour-supply shocks, respectively.

Household h faces the following period-by-period budget constraint:

$$
\begin{aligned}
&(1+\tau_t^C)\, P_{C,t}\, C_{h,t} + P_{I,t}\, I_{h,t} \\
&\qquad + (\epsilon_t^{RP} R_t)^{-1} B_{h,t+1} + ((1-\Gamma_{B^*}(s_{B^*,t+1};\epsilon_t^{RP^*}))\, R_t^*)^{-1} S_t\, B_{h,t+1}^* + \Upsilon_{h,t} + \Xi_t \\
&= (1-\tau_t^N-\tau_t^{W_h})\, W_{h,t}\, N_{h,t} + (1-\tau_t^K)\,(R_{K,t}\, u_{h,t} - \Gamma_u(u_{h,t})\, P_{I,t})\, K_{h,t} \\
&\qquad + \tau_t^K\, \delta\, P_{I,t}\, K_{h,t} + (1-\tau_t^D)\, D_{h,t} - T_t + B_{h,t} + S_t\, B_{h,t}^*, \qquad (2)
\end{aligned}
$$

where $P_{C,t}$ and $P_{I,t}$ are the prices of a unit of the private consumption good and the investment good, respectively. $N_{h,t}$ denotes the labour services provided to firms at wage rate $W_{h,t}$; $R_{K,t}$ indicates the rental rate for the effective capital services rented to firms, $u_{h,t}\, K_{h,t}$, and $D_{h,t}$ are the dividends paid by the household-owned firms. R_t and R_t^* denote the respective risk-less returns on domestic government bonds and internationally traded foreign bonds. The latter are denominated in foreign currency and, thus, their domestic value depends on the nominal exchange rate S_t (expressed in terms of units of the domestic currency per unit of the foreign currency).

The fiscal authority absorbs part of the household's gross income to finance its expenditure. In this context, τ_t^C denotes the consumption tax rate levied on the household's consumption purchases; and τ_t^N, τ_t^K and τ_t^D are the tax rates levied on the different sources of the household's income: wage income, $W_{h,t}\, N_{h,t}$, capital income, $R_{K,t}\, K_{h,t}$, and dividend income, $D_{h,t}$.[8] Here, for simplicity, we assume that the utilisation cost of physical capital as well as physical capital depreciation, $\delta\, P_{I,t}\, K_{h,t}$, are exempted from taxation. $\tau_t^{W_h}$ is the additional payroll tax rate levied on wage income (representing the household's contribution to social security). The term T_t denotes lump-sum taxes.

The effective return on the risk-less domestic bonds depends on a financial intermediation premium, represented by the exogenous "risk premium" shock ϵ_t^{RP}, which drives a wedge between the interest rate controlled by the monetary authority and the re-

[8]For simplicity, it is assumed that dividends are taxed at the household level.

turn required by the household.[9] Similarly, when taking a position in the international bond market, the household encounters an external financial intermediation premium $\Gamma_{B^*}(s_{B^*,t+1}; \epsilon_t^{RP^*})$ which depends on the economy-wide (net) holdings of internationally traded foreign bonds expressed in domestic currency relative to domestic nominal output, $s_{B^*,t+1} = S_t B^*_{t+1}/P_{Y,t}Y_t$, and takes the form:

$$\Gamma_{B^*}(s_{B^*,t+1}; \epsilon_t^{RP^*}) = \gamma_{B^*} \left(\left(\epsilon_t^{RP^*}\right)^{\frac{1}{\gamma_{B^*}}} \exp\left(\frac{S_t B^*_{t+1}}{P_{Y,t} Y_t}\right) - 1 \right) \tag{3}$$

with $\gamma_{B^*} > 0$.[10] That is, if the domestic economy is a net debtor, households have to pay an increasing external intermediation premium on their international debt. The shock $\epsilon_t^{RP^*}$ represents the exogenous component of the external intermediation premium and will be referred to as external risk premium shock.[11] This specification implies that, in the non-stochastic steady state, households have no incentive to hold foreign bonds and the economy's net foreign asset position is zero. The incurred intermediation premium is rebated in a lump-sum manner, being indicated by Ξ_t.

For analytical convenience, each household h is assumed to hold state-contingent securities, $\Upsilon_{h,t}$. These securities are traded amongst households and provide insurance against household-specific wage-income risk. This guarantees that the marginal utility of consumption out of wage income is identical across households.[12] As a result, all households will choose identical allocations in equilibrium.[13]

The physical capital stock owned by household h evolves according to the following capital accumulation equation:

$$K_{h,t+1} = (1-\delta) K_{h,t} + \epsilon_t^I \left(1 - \Gamma_I(I_{h,t}/I_{h,t-1})\right) I_{h,t}, \tag{4}$$

where δ is the depreciation rate, $\Gamma_I(I_{h,t}/I_{h,t-1})$ represents a generalised adjustment cost function formulated in terms of the (gross) rate of change in investment, $I_{h,t}/I_{h,t-1}$, and ϵ_t^I

[9] See Smets and Wouters (2007) for further discussion.

[10] Note that we have used current nominal output and the current exchange rate to scale B^*_{t+1}, because the net foreign asset position is a predetermined variable.

[11] The exponential weighing of the external risk premium shock implies that the shock enters the log-linearised version of the intermediation premium with a unit coefficient.

[12] The existence of state-contingent securities renders the model tractable under staggered wage setting when households are supplying differentiated labour services.

[13] This in turn guarantees that $C_{h,t} = C_t$ in equilibrium.

denotes an investment-specific technology shock affecting the efficiency of the newly installed investment good. The adjustment cost function is assumed to take the following form:

$$\Gamma_I(I_{h,t}/I_{h,t-1}) \quad = \quad \frac{\gamma_I}{2}\left(\frac{I_{h,t}}{I_{h,t-1}} - g_z\right)^2 \tag{5}$$

with $\gamma_I > 0$. The term g_z denotes the economy's trend growth rate in the non-stochastic steady state.

Finally, as regards the provision of effective capital services, varying the intensity of utilising the physical capital stock, $u_{h,t}$, is subject to a proportional cost $\Gamma_u(u_{h,t})$ which is assumed to take the following form:

$$\Gamma_u(u_{h,t}) \quad = \quad \gamma_{u,1}\,(u_{h,t}-1) + \frac{\gamma_{u,2}}{2}\,(u_{h,t}-1)^2 \tag{6}$$

with $\gamma_{u,1}, \gamma_{u,2} > 0$.

Choice of Allocations

Defining as $\Lambda_{h,t}/P_{C,t}$ and $\Lambda_{h,t}\,Q_{h,t}$ the Lagrange multipliers associated with the budget constraint (2) and the capital accumulation equation (4), respectively, the first-order conditions for maximising the household's lifetime utility function (1) with respect to $C_{h,t}$, $I_{h,t}$, $K_{h,t+1}$, $u_{h,t}$, $B_{h,t+1}$ and $B^*_{h,t+1}$ are given by:

$$\Lambda_{h,t} \quad = \quad \epsilon_t^C\,\frac{(C_{h,t}-\kappa\,C_{t-1})^{-1}}{1+\tau_t^C}, \tag{7}$$

$$\begin{aligned}\frac{P_{I,t}}{P_{C,t}} \quad = \quad & Q_{h,t}\,\epsilon_t^I\left(1-\Gamma_I(I_{h,t}/I_{h,t-1})-\Gamma'_I(I_{h,t}/I_{h,t-1})\,\frac{I_{h,t}}{I_{h,t-1}}\right) \\ & +\beta\,\mathrm{E}_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\,Q_{h,t+1}\,\epsilon_{t+1}^I\,\Gamma'_I(I_{h,t+1}/I_{h,t})\,\frac{I_{h,t+1}^2}{I_{h,t}^2}\right],\end{aligned} \tag{8}$$

$$\begin{aligned}Q_{h,t} \quad = \quad & \beta\,\mathrm{E}_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\Big((1-\delta)\,Q_{h,t+1}\right. \\ & \left.+(1-\tau_{t+1}^K)\,\frac{R_{K,t+1}}{P_{C,t+1}}\,u_{h,t+1}+\Big(\tau_{t+1}^K\,\delta-(1-\tau_{t+1}^K)\,\Gamma_u(u_{h,t+1})\Big)\,\frac{P_{I,t+1}}{P_{C,t+1}}\Big)\right],\end{aligned} \tag{9}$$

$$R_{K,t} \quad = \quad \Gamma'_u(u_{h,t})\,P_{I,t}, \tag{10}$$

$$\beta\,\epsilon_t^{RP}\,R_t\,\mathrm{E}_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\,\frac{P_{C,t}}{P_{C,t+1}}\right] \quad = \quad 1, \tag{11}$$

$$\beta\,(1-\Gamma_{B^*}(s_{B^*,t+1};\epsilon_t^{RP^*}))\,R_t^*\,\mathrm{E}_t\left[\frac{\Lambda_{h,t+1}}{\Lambda_{h,t}}\,\frac{P_{C,t}}{P_{C,t+1}}\,\frac{S_{t+1}}{S_t}\right] \;=\; 1. \qquad (12)$$

Here, $\Lambda_{h,t}$ represents the shadow price of a unit of the consumption good; that is, the marginal utility of consumption out of income. Similarly, $Q_{h,t}$ measures the shadow price of a unit of the investment good; that is, Tobin's Q.[14]

In equilibrium, with all households choosing identical allocations, the combination of the first-order conditions with respect to the holdings of domestic and internationally traded bonds, (11) and (12), yields a risk-adjusted uncovered interest parity (UIP) condition, reflecting the assumption that the return on internationally traded bonds is subject to an external financial intermediation premium.

Wage Setting

Each household h supplies its differentiated labour services $N_{h,t}$ in monopolistically competitive markets. There is sluggish wage adjustment due to staggered wage contracts à la Calvo (1983). Accordingly, household h receives permission to optimally reset its nominal wage contract $W_{h,t}$ in a given period t with probability $1-\xi_W$.

All households that receive permission to reset their wage contracts in a given period t choose the same wage rate $\tilde{W}_t = \tilde{W}_{h,t}$. Those households which do not receive permission are allowed to adjust their wage contracts to productivity developments and inflation according to the following scheme:

$$W_{h,t} \;=\; g_{z,t}\,\Pi_{C,t}^{\dagger}\,W_{h,t-1}, \qquad (13)$$

where $g_{z,t}$ represents the (gross) rate of underlying labour productivity growth and $\Pi_{C,t}^{\dagger} = \Pi_{C,t-1}^{\chi_W}\bar{\Pi}_t^{1-\chi_W}$ is a geometric average of past (gross) consumer price inflation, $\Pi_{C,t-1} = P_{C,t-1}/P_{C,t-2}$, and the monetary authority's possibly time-varying (gross) inflation objective, $\bar{\Pi}_t$. The weight of past inflation is determined by the indexation parameter χ_W.

Each household h receiving permission to reset its wage contract in period t maximises its lifetime utility function (1) subject to its budget constraint (2), the demand for its differentiated labour services (the formal derivation of which we postpone until we consider

[14]We note that the domestic risk premium shock, ϵ_t^{RP}, affects investment via Tobin's Q and helps to explain the co-movement of consumption and investment observed in the data. In contrast, the consumption preference shock, ϵ_t^{C}, moves consumption and investment in opposite directions.

the decision problem of the domestic intermediate-good firms in Section 2.2.1) and the wage-indexation scheme (13).

Hence, we obtain the following first-order condition characterising the households' optimal wage-setting decision:

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}(\xi_W\beta)^k\left(\Lambda_{t+k}\,(1-\tau^N_{t+k}-\tau^{W_h}_{t+k})\,g_{z;t,t+k}\,\frac{\Pi^{\dagger}_{C;t,t+k}}{\Pi_{C;t,t+k}}\,\frac{\tilde{W}_t}{P_{C,t}}\right.\right. \quad (14)$$

$$\left.\left.-\,\varphi^W_{t+k}\,\epsilon^N_{t+k}\,(N_{h,t+k})^{\zeta}\right)N_{h,t+k}\right] = 0,$$

where Λ_{t+k} denotes the marginal utility out of income (equal across all households), $g_{z;t,t+k} = \prod_{s=1}^{k} g_{z,t+s}$, $\Pi^{\dagger}_{C;t,t+k} = \prod_{s=1}^{k}\Pi^{\chi_W}_{C,t+s-1}\bar{\Pi}^{1-\chi_W}_{t+s}$ and $\Pi_{C;t,t+k} = \prod_{s=1}^{k}\Pi_{C,t+s-1}$.

This expression states that in those labour markets in which wage contracts are re-optimised, the latter are set so as to equate the households' discounted sum of expected after-tax marginal revenues, expressed in consumption-based utility terms, Λ_{t+k}, to the discounted sum of expected marginal cost, expressed in terms of marginal disutility of labour, $\Delta_{h,t+k} = -N^{\zeta}_{h,t+k}$. In the absence of wage staggering ($\xi_W = 0$), the factor φ^W_t represents a possibly time-varying markup of the real after-tax wage charged over the households' marginal rate of substitution between consumption and leisure,

$$(1-\tau^N_t-\tau^{W_h}_t)\,\frac{\tilde{W}_t}{P_{C,t}} \;=\; -\,\varphi^W_t\,\epsilon^N_t\,\frac{\Delta_t}{\Lambda_t}, \quad (15)$$

reflecting the existence of monopoly power on the part of the households.[15]

Aggregate Wage Dynamics

With the continuum of households setting the wage contracts for their differentiated labour services on the basis of equation (13) and equation (14), respectively, the aggregate wage index W_t (see Section 2.2.1 for details) evolves according to

$$W_t \;=\; \left(\xi_W\left(g_{z,t}\,\Pi^{\dagger}_{C,t}W_{t-1}\right)^{\frac{1}{1-\varphi^W_t}}+(1-\xi_W)\left(\tilde{W}_t\right)^{\frac{1}{1-\varphi^W_t}}\right)^{1-\varphi^W_t}. \quad (16)$$

2.2 Firms

There are two types of monopolistically competitive intermediate-good firms: A continuum of domestic intermediate-good firms indexed by $f\in[0,1]$ which produce differentiated

[15] Here we have used the fact that, in equilibrium, the marginal disutility is equal across households; that is $\Delta_{h,t} = \Delta_t$.

outputs that are sold domestically or abroad, and a continuum of foreign intermediate-good firms indexed by $f^* \in [0,1]$ which produce differentiated outputs that are sold in domestic markets. In addition there is a set of three representative domestic final-good firms which combine the purchases of domestically-produced intermediate goods with purchases of imported intermediate goods into three distinct non-tradable goods, namely a private consumption good, a private investment good and a public consumption good. Finally, there is a representative foreign retail firm that combines the exported domestic intermediate goods.

2.2.1 Domestic Intermediate-Good Firms

Technology

Each domestic intermediate-good firm f produces a differentiated intermediate good $Y_{f,t}$ with an increasing-returns-to-scale Cobb-Douglas technology that is subject to fixed costs of production, $z_t\,\psi$,

$$Y_{f,t} = \max\left[\varepsilon_t\,(K^s_{f,t})^{\alpha}\,(z_t\,N_{f,t})^{1-\alpha} - z_t\,\psi,\,0\right], \tag{17}$$

utilising as inputs homogenous capital services, $K^s_{f,t}$, that are rented from households in fully competitive markets, and an index of differentiated labour services, $N_{f,t}$, which combines household-specific varieties of labour that are supplied in monopolistically competitive markets,

$$N_{f,t} = \left(\int_0^1 \left(N^h_{f,t}\right)^{\frac{1}{\varphi^W_t}} dh\right)^{\varphi^W_t}. \tag{18}$$

Here, the possibly time-varying parameter $\varphi^W_t > 1$ is inversely related to the intratemporal elasticity of substitution between the differentiated labour services supplied by the households, $\varphi^W_t/(\varphi^W_t - 1) > 1$. As shown above, the parameter φ^W_t has a natural interpretation as a markup in the household-specific labour market.[16]

The variable ε_t represents a serially correlated, but *transitory* technology shock that affects total-factor productivity, while the variable z_t denotes a *permanent* technology shock shifting the productivity of labour lastingly. The permanent technology shock introduces

[16]Note that the exposition of the calibrated version of the NAWM is centred around the intratemporal elasticity of substitution which is assumed to be time invariant.

a unit root in the firms' output and evolves according to the following serially correlated process,

$$g_{z,t} = (1-\rho_{g_z})\, g_z + \rho_{g_z}\, g_{z,t-1} + \eta_t^{g_z}, \qquad (19)$$

where $g_{z,t} = z_t/z_{t-1}$ represents the (gross) rate of labour-augmenting productivity growth with steady-state value g_z.

Both technology shocks, and the fixed costs of production, are assumed to be identical across firms. The fixed costs are scaled by the permanent technology shock to guarantee that fixed costs as a fraction of output do not vanish as output grows.[17]

Capital and Labour Inputs

Taking the rental cost of capital $R_{K,t}$ and the aggregate wage index W_t as given, the intermediate-good firm's optimal demand for capital and labour services solves the problem of minimising total input cost $R_{K,t}\, K_{f,t} + (1+\tau_t^{W_f})\, W_t\, N_{f,t}$ subject to the technology constraint (17). Here, $\tau_t^{W_f}$ denotes the payroll tax rate levied on wage payments (representing the firm's contribution to social security).

Defining as $MC_{f,t}$ the Lagrange multiplier associated with the technology constraint (17), the first-order conditions of the firm's cost minimisation problem with respect to capital and labour inputs are given, respectively, by

$$\alpha\, \frac{Y_{f,t} + z_t\,\psi}{K_{f,t}^s}\, MC_{f,t} = R_{K,t}, \qquad (20)$$

$$(1-\alpha)\, \frac{Y_{f,t} + z_t\psi}{N_{f,t}}\, MC_{f,t} = (1+\tau_t^{W_f})\, W_t, \qquad (21)$$

or, more compactly,

$$\frac{\alpha}{1-\alpha}\, \frac{N_{f,t}}{K_{f,t}^s} = \frac{R_{K,t}}{(1+\tau_t^{W_f})\, W_t}. \qquad (22)$$

Here, the Lagrange multiplier $MC_{f,t}$ measures the shadow price of varying the use of capital and labour services; that is, nominal marginal cost. We note that, since all firms f face the same input prices and since they all have access to the same production technology, nominal marginal cost $MC_{f,t}$ are identical across firms; that is, $MC_{f,t} = MC_t$ with

$$MC_t = \frac{1}{\varepsilon_t\, z_t^{1-\alpha} \alpha^\alpha (1-\alpha)^{1-\alpha}}\, (R_{K,t})^\alpha ((1+\tau_t^{W_f})\, W_t)^{1-\alpha}. \qquad (23)$$

[17]The parameter ψ will be chosen to ensure zero profits in steady state. This in turn guarantees that there is no incentive for other firms to enter the market in the long run.

With the nominal wage contract $W_{h,t}$ for the differentiated labour services of household h being set in monopolistically competitive markets, firm f takes $W_{h,t}$ as given and chooses the optimal input of each labour variety h by minimising the total wage-related labour cost $\int_0^1 W_{h,t}\, N_{f,t}^h dh$, subject to the aggregation constraint (18).

The resulting demand for labour variety h is a function of the household-specific wage rate $W_{h,t}$ relative to the aggregate wage index W_t:

$$N_{f,t}^h \;=\; \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W-1}} N_{f,t} \tag{24}$$

with $\varphi_t^W/(\varphi_t^W - 1)$ representing the wage elasticity of labour demand.

The wage index W_t can be obtained by substituting the labour index (18) into the labour demand schedule (24) and then integrating over the continuum of households:

$$W_t \;=\; \left(\int_0^1 W_{h,t}^{\frac{1}{1-\varphi_t^W}}\, dh\right)^{1-\varphi_t^W}. \tag{25}$$

Aggregating over the continuum of firms, we obtain the following aggregate demand for the labour services of a given household h:

$$N_t^h \;=\; \int_0^1 N_{f,t}^h\, df \;=\; \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W-1}} N_t. \tag{26}$$

Price Setting

Each firm f sells its differentiated output $Y_{f,t}$ in both domestic and foreign markets under monopolistic competition. We assume that the firm charges different prices at home and abroad, setting prices in producer (that is, domestic) currency regardless of the market of destination.[18] In both markets, there is sluggish price adjustment due to staggered price contracts à la Calvo (1983). Accordingly, firm f receives permission to optimally reset prices in a given period t either with probability $1 - \xi_H$ or with probability $1 - \xi_X$, depending on whether the firm sells its differentiated output in the domestic or the foreign market.

Defining as $P_{H,f,t}$ the domestic price of good f and as $P_{X,f,t}$ its foreign price, all firms that receive permission to reset their price contracts in a given period t choose the same

[18] This contrasts with the assumption of local-currency pricing that is maintained in the calibrated version of the NAWM for goods that are sold abroad. As shown in the sensitivity analysis in Section 3.5, producer-currency pricing is strongly preferred by the data.

price, $\tilde{P}_{H,t} = \tilde{P}_{H,f,t}$ and $\tilde{P}_{X,t} = \tilde{P}_{X,f,t}$, depending on the market of destination. Those firms which do not receive permission are allowed to adjust their prices according to the following schemes:

$$P_{H,f,t} = \Pi_{H,t-1}^{\chi_H} \bar{\Pi}_t^{1-\chi_H} P_{H,f,t-1}, \tag{27}$$

$$P_{X,f,t} = \Pi_{X,t-1}^{\chi_X} \bar{\Pi}_t^{1-\chi_X} P_{X,f,t-1}. \tag{28}$$

That is, the price contracts are indexed to a geometric average of past (gross) intermediate-good inflation, $\Pi_{H,t-1} = P_{H,t-1}/P_{H,t-2}$ and $\Pi_{X,t-1} = P_{X,t-1}/P_{X,t-2}$, and the possibly time-varying (gross) inflation objective of the domestic monetary authority, $\bar{\Pi}_t$, where χ_H and χ_X are indexation parameters determining the weight on past inflation.

Each firm f receiving permission to optimally reset its domestic and/or foreign price in period t maximises the discounted sum of its expected nominal profits,

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(\xi_H^k \, D_{H,f,t+k} + \xi_X^k \, D_{X,f,t+k} \right) \right], \tag{29}$$

subject to the price-indexation schemes (27) and (28) and taking as given domestic and foreign demand for its differentiated output, $H_{f,t}$ and $X_{f,t}$ (to be derived in Sections 2.2.3 and 2.2.4). The stochastic discount factor $\Lambda_{t,t+k}$ can be obtained from the consumption Euler equation of the households, and

$$D_{H,f,t} = P_{H,f,t} \, H_{f,t} - MC_t \, H_{f,t}, \tag{30}$$

$$D_{X,f,t} = P_{X,f,t} \, X_{f,t} - MC_t \, X_{f,t} \tag{31}$$

are period-t nominal profits (net of fixed costs) yielded in the domestic and foreign markets, respectively, which are distributed as dividends to the households.[19] Hence, we obtain the following first-order condition characterising the firm's optimal pricing decision for its output sold in the domestic market:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} \xi_H^k \, \Lambda_{t,t+k} \left(\Pi_{H,t,t+k}^{\dagger} \, \tilde{P}_{H,t} - \varphi_{t+k}^H \, MC_{t+k} \right) \, H_{f,t+k} \right] = 0, \tag{32}$$

where we have substituted the indexation scheme (27), noting that $P_{H,f,t+k} = \Pi_{H,t,t+k}^{\dagger} \tilde{P}_{H,t}$ with $\Pi_{H,t,t+k}^{\dagger} = \prod_{s=1}^{k} \Pi_{H,t+s-1}^{\chi_H} \bar{\Pi}_{t+s}^{1-\chi_H}$.

[19]Note that we have made use of the first-order conditions (9) and (21) to derive the expressions for nominal profits.

This expression states that in those intermediate-good markets in which price contracts are re-optimised, the latter are set so as to equate the firm's discounted sum of expected revenues to the discounted sum of expected marginal cost. In the absence of price staggering ($\xi_H = 0$), the factor φ_t^H represents a possibly time-varying markup of the price charged in domestic markets over nominal marginal cost, reflecting the degree of monopoly power on the part of the intermediate-good firm.

Similarly, we obtain the following first-order condition characterising the firm's optimal pricing decision for its output sold in the foreign market:

$$\mathrm{E}_t \left[\sum_{k=0}^{\infty} \xi_X^k \, \Lambda_{t,t+k} \left(\Pi_{X,t,t+k}^{\dagger} \, \tilde{P}_{X,t} - \varphi_{t+k}^X \; MC_{t+k} \right) \; X_{f,t+k} \right] \quad = \quad 0, \qquad (33)$$

where we have substituted the indexation scheme (28), noting that $P_{X,f,t+k} = \Pi_{X,t,t+k}^{\dagger} \tilde{P}_{X,t}$ with $\Pi_{X,t,t+k}^{\dagger} = \prod_{s=1}^{k} \Pi_{X,t+s-1}^{\chi_X} \bar{\Pi}_{t+s}^{1-\chi_X}$.

Aggregate Price Dynamics

With the continuum of intermediate-good firms setting the price contracts for their differentiated products sold domestically on the basis of equation (27) and equation (32), respectively, the aggregate price index $P_{H,t}$ (see Section 2.2.3) evolves according to

$$P_{H,t} \quad = \quad \left((1-\xi_H)(\tilde{P}_{H,t})^{\frac{1}{1-\varphi_t^H}} + \xi_H \left(\Pi_{H,t-1}^{\chi_H} \bar{\Pi}_t^{1-\chi_H} P_{H,t-1} \right)^{\frac{1}{1-\varphi_t^H}} \right)^{1-\varphi_t^H} . \qquad (34)$$

A similar relationship holds for the aggregate index of price contracts set for the differentiated products sold abroad (see Section 2.2.4) with

$$P_{X,t} \quad = \quad \left((1-\xi_X)(\tilde{P}_{X,t})^{\frac{1}{1-\varphi_t^X}} + \xi_X \left(\Pi_{X,t-1}^{\chi_X} \bar{\Pi}_t^{1-\chi_X} P_{X,t-1} \right)^{\frac{1}{1-\varphi_t^X}} \right)^{1-\varphi_t^X} . \qquad (35)$$

2.2.2 Foreign Intermediate-Good Firms

Each foreign intermediate-good firm f^* sells its differentiated good $Y_{f^*,t}^*$ in domestic market under monopolistic competition, setting the price in local (that is, domestic) currency, as in Betts and Devereux (1996). Again, there is sluggish price adjustment due to staggered price contracts à la Calvo. Accordingly, the foreign intermediate-good firm, or exporter, receives permission to optimally reset its price in a given period t with probability $1 - \xi^*$

and has access to the following indexation scheme with parameter χ^*:

$$P_{IM,f^*,t} \;=\; \Pi_{IM,t-1}^{\chi^*} \bar{\Pi}_t^{1-\chi^*} P_{IM,f^*,t-1}, \tag{36}$$

where $P_{IM,f^*,t} = P^*_{X,f^*,t}$ and $\Pi_{IM,t-1} = P_{IM,t-1}/P_{IM,t-2}$ with $P_{IM,t} = P^*_{X,t}$. Here, we have utilised the fact that, with foreign exporters setting prices in domestic currency, the price of the intermediate good imported from abroad (the import price index of the home country) is equal to the price charged by the foreign exporter in the home country (the export price index of the foreign country).

Each foreign exporter f^* receiving permission to optimally reset its price in period t maximises the discounted sum of its expected nominal profits,

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty} (\xi^*)^k \, \Lambda^*_{t,t+k} \, D^*_{f^*,t+k}/S_{t+k}\right], \tag{37}$$

subject to the price-indexation scheme and the domestic (import) demand for its differentiated output, $IM_{f^*,t} = X^*_{f^*}$ (to be derived in Sections 2.2.3), where

$$D^*_{f^*,t} \;=\; P_{IM,f^*,t}\, IM_{f^*,t} - MC^*_t \, IM_{f^*,t} \tag{38}$$

with $MC^*_t = S_t \,(P_{O,t})^{\omega^*} \,(P^*_{Y,t})^{1-\omega^*}$ representing the foreign exporter's nominal marginal cost. The latter is defined as a simple geometric average of the price of oil, $P_{O,t}$, and foreign prices, $P^*_{Y,t}$, with ω^* measuring the share of oil in imports.

Hence, we obtain the following first-order condition characterising the foreign exporter's optimal pricing decision for its output sold in the domestic market:

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty} (\xi^*)^k \, \Lambda^*_{t,t+k} \left(\Pi^{\dagger}_{IM,t,t+k} \, \tilde{P}_{IM,t} - \varphi^*_{t+k} \, MC^*_{t+k}\right) \; IM_{f^*,t+k}/S_{t+k}\right] \;=\; 0, \tag{39}$$

where we have substituted the indexation scheme (36), noting that $P_{IM,f^*,t+k} = \Pi^{\dagger}_{IM,t,t+k}$ $\tilde{P}_{IM,t}$ with $\Pi^{\dagger}_{IM,t,t+k} = \prod_{s=1}^{k} \Pi_{IM,t+s-1}^{\chi^*} \bar{\Pi}_{t+s}^{1-\chi^*}$.

The associated aggregate index of price contracts for the differentiated products sold in domestic markets (see Section 2.2.3) evolves according to

$$P_{IM,t} \;=\; \left((1-\xi^*)(\tilde{P}_{IM,t})^{\frac{1}{1-\varphi^*_t}} + \xi^* \left(\Pi_{IM,t-1}^{\chi^*} \bar{\Pi}_t^{1-\chi^*} P_{IM,t-1}\right)^{\frac{1}{1-\varphi^*_t}}\right)^{1-\varphi^*_t}. \tag{40}$$

2.2.3 Domestic Final-Good Firms

There are three different types of final-good firms which combine the purchases of the domestically-produced intermediate goods with purchases of the imported intermediate goods into three distinct non-tradable final goods, namely a private consumption good, Q_t^C, a private investment good, Q_t^I, and a public consumption good, Q_t^G.

The representative firm producing the non-tradable final private consumption good, Q_t^C, combines purchases of a bundle of domestically-produced intermediate goods, H_t^C, with purchases of a bundle of imported foreign intermediate goods, IM_t^C, using a constant-returns-to-scale CES technology,

$$Q_t^C = \left(\nu_{C,t}^{\frac{1}{\mu_C}} \left(H_t^C\right)^{1-\frac{1}{\mu_C}} + (1-\nu_{C,t})^{\frac{1}{\mu_C}} \left((1-\Gamma_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM}))\, IM_t^C \right)^{1-\frac{1}{\mu_C}} \right)^{\frac{\mu_C}{\mu_C-1}} \quad (41)$$

where μ_C denotes the intratemporal elasticity of substitution between the distinct bundles of domestic and foreign intermediate goods, while the possibly time-varying parameter $\nu_{C,t}$ measures the home bias in the production of the consumption good.

The final-good firm incurs a cost $\Gamma_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM})$ when varying the use of the bundle of imported goods in producing the consumption good,

$$\Gamma_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM}) = \frac{\gamma_{IM^C}}{2} \left(\left(\epsilon_t^{IM}\right)^{-\frac{1}{\gamma_{IM^C}}} \frac{IM_t^C/Q_t^C}{IM_{t-1}^C/Q_{t-1}^C} - 1 \right)^2 \quad (42)$$

with $\gamma_{IM^C} > 0$. As a result, the import share is relatively unresponsive in the short run to changes in the relative price of the bundle of imported goods, while the level of imports is permitted to jump in response to changes in overall demand.[20] We will refer to ϵ_t^{IM} as an import demand shock.

Defining as $H_{f,t}^C$ and $IM_{f^*,t}^C$ the use of the differentiated output produced by the domestic intermediate-good firm f and the differentiated output supplied by the foreign exporter f^*, respectively, we have

$$H_t^C = \left(\int_0^1 \left(H_{f,t}^C\right)^{\frac{1}{\varphi_t^H}} df \right)^{\varphi_t^H}, \quad (43)$$

$$IM_t^C = \left(\int_0^1 \left(IM_{f^*,t}^C\right)^{\frac{1}{\varphi_t^*}} df^* \right)^{\varphi_t^*}, \quad (44)$$

[20] While our treatment of the adjustment cost as being external to the firm would formally involve assuming the existence of a large number of firms with appropriate changes in notation (see, e.g., Bayoumi, Laxton and Pesenti, 2004), we abstract from these changes for ease of exposition.

where the possibly time-varying parameters $\varphi_t^H, \varphi_t^* > 1$ are inversely related to the intratemporal elasticities of substitution between the differentiated outputs supplied by the domestic firms and the foreign exporters, respectively, with $\varphi_t^H/(\varphi_t^H - 1) > 1$ and $\varphi_t^*/(\varphi_t^* - 1) > 1$. As shown above, the parameters φ_t^H and φ_t^* have a natural interpretation as markups in the markets for domestic and imported intermediate goods.[21]

With nominal prices for the differentiated goods f and f^* being set in monopolistically competitive markets, the final-good firm takes their prices $P_{H,f,t}$ and $P_{IM,f^*,t}$ as given and chooses the optimal use of the differentiated goods f and f^* by minimising the expenditure for the bundles of differentiated goods, $\int_0^1 P_{H,f,t}\, H_{f,t}^C\, df$ and $\int_0^1 P_{IM,f^*,t}\, IM_{f^*,t}^C\, df^*$, subject to the aggregation constraints (43) and (44). This yields the following demand equations for the differentiated goods f and f^*:

$$H_{f,t}^C = \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{-\frac{\varphi_t^H}{\varphi_t^H - 1}} H_t^C, \tag{45}$$

$$IM_{f^*,t}^C = \left(\frac{P_{IM,f^*,t}}{P_{IM,t}}\right)^{-\frac{\varphi_t^*}{\varphi_t^* - 1}} IM_t^C, \tag{46}$$

where

$$P_{H,t} = \left(\int_0^1 (P_{H,f,t})^{\frac{1}{1-\varphi_t^H}}\, df\right)^{1-\varphi_t^H}, \tag{47}$$

$$P_{IM,t} = \left(\int_0^1 (P_{IM,f^*,t})^{\frac{1}{1-\varphi_t^*}}\, df^*\right)^{1-\varphi_t^*} \tag{48}$$

are the aggregate price indexes for the bundles of domestic and imported intermediate goods, respectively.

Next, taking the price indexes $P_{H,t}$ and $P_{IM,t}$ as given, the consumption-good firm chooses the combination of the domestic and foreign intermediate-good bundles H_t^C and IM_t^C that minimises $P_{H,t}\, H_t^C + P_{IM,t}\, IM_t^C$ subject to aggregation constraint (41). This yields the following demand equations for the intermediate-good bundles:

$$H_t^C = \nu_{C,t} \left(\frac{P_{H,t}}{P_{C,t}}\right)^{-\mu_C} Q_t^C, \tag{49}$$

$$IM_t^C = (1 - \nu_{C,t}) \left(\frac{P_{IM,t}}{P_{C,t}\, \Gamma_{IM^C}^{\dagger}(IM_t^C/Q_t^C; \epsilon_t^{IM})}\right)^{-\mu_C} \frac{Q_t^C}{1 - \Gamma_{IM^C}(IM_t^C/Q_t^C; \epsilon_t^{IM})}, \tag{50}$$

[21] Note that the exposition of the calibrated version of the NAWM is centred around the intratemporal elasticities of substitution which are assumed to be time invariant.

where

$$P_{C,t} = \left(\nu_{C,t} \left(P_{H,t} \right)^{1-\mu_C} + (1-\nu_{C,t}) \left(\frac{P_{IM,t}}{\Gamma^{\dagger}_{IM^C}(IM^C_t/Q^C_t;\epsilon^{IM}_t)} \right)^{1-\mu_C} \right)^{\frac{1}{1-\mu_C}} \tag{51}$$

is the price of a unit of the private consumption good and

$$\Gamma^{\dagger}_{IM^C}(IM^C_t/Q^C_t;\epsilon^{IM}_t) = 1 - \Gamma_{IM^C}(IM^C_t/Q^C_t;\epsilon^{IM}_t) - \Gamma'_{IM^C}(IM^C_t/Q^C_t;\epsilon^{IM}_t)\, IM^C_t. \tag{52}$$

The representative firm producing the non-tradable final private investment good, Q^I_t, is modelled in an analogous manner. Specifically, the firm combines its purchase of a bundle of domestically-produced intermediate goods, H^I_t, with the purchase of a bundle of imported foreign intermediate goods, IM^I_t, using a constant-returns-to-scale CES technology,

$$Q^I_t = \left(\nu_{I,t}^{\frac{1}{\mu_I}} \left(H^I_t \right)^{1-\frac{1}{\mu_I}} + (1-\nu_{I,t})^{\frac{1}{\mu_I}} \left((1-\Gamma_{IM^I}(IM^I_t/Q^I_t;\epsilon^{IM}_t))\, IM^I_t \right)^{1-\frac{1}{\mu_I}} \right)^{\frac{\mu_I}{\mu_I-1}}, \tag{53}$$

where μ_I denotes the intratemporal elasticity of substitution between the distinct bundles of domestic and foreign intermediate inputs, while the possibly time-varying parameter $\nu_{I,t}$ measures the home bias in the production of the investment good.

All other variables related to the production of the investment good—import adjustment cost, $\Gamma_{IM^I}(IM^I_t/Q^I_t;\epsilon^{IM}_t)$; the optimal demand for firm-specific and bundled domestic and foreign intermediate goods, $H^I_{f,t}$, H^I_t and $IM^I_{f^*,t}$, IM^I_t, respectively; as well as the price of a unit of the investment good, $P_{I,t}$—are defined or derived in a manner analogous to that for the consumption good.[22]

In contrast, the non-tradable final public consumption good, Q^G_t, is assumed to be a composite made only of domestic intermediate goods; that is, $Q^G_t = H^G_t$ with

$$H^G_t = \left(\int_0^1 \left(H^G_{f,t} \right)^{\frac{1}{\varphi^H_t}} df \right)^{\varphi^H_t}. \tag{54}$$

Hence, the optimal demand for the differentiated intermediate good f is given by

$$H^G_{f,t} = \left(\frac{P_{H,f,t}}{P_{H,t}} \right)^{-\frac{\varphi^H_t}{\varphi^H_t - 1}} H^G_t, \tag{55}$$

and the price of a unit of the public consumption good is $P_{G,t} = P_{H,t}$.

[22]We note that, even in the absence of import adjustment cost, the prices of the consumption and investment goods may differ due to differences in the home bias parameters.

Aggregating across the three final-good firms, we obtain the following aggregate demand equations for domestic and foreign intermediate goods f and f^*:

$$H_{f,t} = H^C_{f,t} + H^I_{f,t} + H^G_{f,t} = \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{-\frac{\varphi^H_t}{\varphi^H_t - 1}} H_t, \tag{56}$$

$$IM_{f^*,t} = IM^C_{f^*,t} + IM^I_{f^*,t} = \left(\frac{P_{IM,f^*,t}}{P_{IM,t}}\right)^{-\frac{\varphi^*_t}{\varphi^*_t - 1}} IM_t, \tag{57}$$

where $H_t = H^C_t + H^I_t + H^G_t$ and $IM_t = IM^C_t + IM^I_t$.

2.2.4 Foreign Retail Firm

We assume that a representative foreign retail firm combines the purchases of the differentiated goods, $X_{f,t}$, that are produced by the domestic intermediate-good firms f and sold abroad, using a CES technology,

$$X_t = \left(\int_0^1 (X_{f,t})^{\frac{1}{\varphi^X_t}}\, df\right)^{\varphi^X_t}, \tag{58}$$

where the possibly time-varying parameter $\varphi^X_t > 1$ is inversely related to the intratemporal elasticity of substitution between the differentiated goods supplied by the domestic firms, with $\varphi^X_t/(\varphi^X_t - 1) > 1$.

With nominal prices for the exported intermediate goods f being set in producer currency under monopolistic competition, the foreign retailer takes its input prices $P_{X,f,t}/S_t$ as given and decides on the optimal use of the differentiated inputs by minimising the expenditure for the bundle of differentiated goods, $\int_0^1 P_{X,f,t}/S_t\, X_{f,t}\, df$, subject to the aggregation constraint (58).

This yields the following demand equation for the differentiated good f:

$$X_{f,t} = \left(\frac{P_{X,f,t}}{P_{X,t}}\right)^{-\frac{\varphi^X_t}{\varphi^X_t - 1}} X_t, \tag{59}$$

where

$$P_{X,t} = \left(\int_0^1 (P_{X,f,t})^{\frac{1}{1-\varphi^X_t}}\, df\right)^{1-\varphi^X_t} \tag{60}$$

is the aggregate price index for the bundle of exported domestic intermediate goods in producer currency.

The retailer takes the aggregate price index $P_{X,t}$ as given and supplies the quantity of the export bundle, X_t, that satisfies foreign demand. The latter is given by an equation similar in structure to the domestic import equation,

$$X_t = \nu_t^* \left(\frac{P_{X,t}/S_t}{P_{X,t}^{c,*}\, \Gamma_X^\dagger(X_t/Y_t^{d,*};\epsilon_t^X)} \right)^{-\mu^*} \frac{Y_t^{d,*}}{1-\Gamma_X(X_t/Y_t^{d,*};\epsilon_t^X)} \tag{61}$$

with μ^* denoting the price elasticity of exports. Here ν_t^* represents the possibly time-varying export share of the domestic intermediate-good firms, which captures the foreign non-price related preferences for domestic goods. The variable $P_{X,t}^{c,*}$ denotes the price of foreign firms that are competing with the domestic firms in their export markets, $Y_t^{d,*}$ is a measure of overall foreign demand, and $\Gamma_X(X_t/Y_t^{d,*};\epsilon_t^X)$ is an adjustment cost function given by

$$\Gamma_X(X_t/Y_t^{d,*};\epsilon_t^X) = \frac{\gamma^*}{2} \left(\left(\epsilon_t^X\right)^{-\frac{1}{\gamma^*}} \frac{X_t/Y_t^{d,*}}{X_{t-1}/Y_{t-1}^{d,*}} - 1 \right)^2 \tag{62}$$

and

$$\Gamma_X^\dagger(X_t/Y_t^{d,*};\epsilon_t^X) = 1 - \Gamma_X(X_t/Y_t^{d,*};\epsilon_t^X) - \Gamma_X'(X_t/Y_t^{d,*};\epsilon_t^X)\, X_t. \tag{63}$$

2.3 Fiscal and Monetary Authorities

2.3.1 Fiscal Authority

The fiscal authority purchases the final public consumption good, G_t, issues bonds to refinance its outstanding debt, B_t, and raises both distortionary and lump-sum taxes. The fiscal authority's period-by-period budget constraint then has the following form:

$$\begin{aligned} P_{G,t}\, G_t + B_t \;=\; & \tau_t^C\, P_{C,t}\, C_t + (\tau_t^N + \tau_t^{W_h}) \int_0^1 W_{h,t}\, N_{h,t}\, dh + \tau_t^{W_f}\, W_t\, N_t \\ & + \tau_t^K \left(R_{K,t}\, u_t - (\Gamma_u(u_t) + \delta)\, P_{I,t} \right) K_t + \tau_t^D\, D_t + T_t + R_t^{-1}\, B_{t+1}, \end{aligned} \tag{64}$$

where all variables are expressed in economy-wide terms, except for the households' labour services and wages, $N_{h,t}$ and $W_{h,t}$, which are differentiated across households.

The purchases of the public consumption good G_t are determined exogenously. As regards the effects of changes in public consumption, we note that "Ricardian equivalence" is assumed to hold. Hence, the particular time path of debt is irrelevant for the households' choice of allocations. For this reason and without loss of generality, we assume that lump-sum taxes close the fiscal authority's budget constraint each period. Finally, all distortionary tax rates τ_t^j with $j = C, D, K, N, W_h$ and W_f are assumed to be set exogenously.

2.3.2 Monetary Authority

The monetary authority sets the nominal interest rate according to a simple log-linear interest-rate rule,

$$\begin{aligned} \widehat{r}_t &= \phi_R \, \widehat{r}_{t-1} + (1-\phi_R) \left(\widehat{\bar{\pi}}_t + \phi_\Pi \left(\widehat{\pi}_{C,t-1} - \widehat{\bar{\pi}}_t \right) + \phi_Y \, \widehat{y}_t \right) \\ & \quad + \phi_{\Delta\Pi} \left(\widehat{\pi}_{C,t} - \widehat{\pi}_{C,t-1} \right) + \phi_{\Delta Y} \left(\widehat{y}_t - \widehat{y}_{t-1} \right) + \widehat{\eta}_t^R, \end{aligned} \tag{65}$$

where $\widehat{r}_t = \log(R_t/R)$ is the logarithmic deviation of the (gross) nominal interest rate from its steady-state value. Similarly, $\widehat{\pi}_{C,t} = \log(\Pi_{C,t}/\bar{\Pi})$ denotes the logarithmic deviation of (gross) quarter-on-quarter consumer price inflation $\Pi_{C,t} = P_{C,t}/P_{C,t-1}$ from the monetary authority's long-run inflation objective $\bar{\Pi}$, while $\widehat{\bar{\pi}}_t = \log(\bar{\Pi}_t/\bar{\Pi})$ represents the logarithmic deviation of the monetary authority's possibly time-varying inflation objective from its long-run value, which is assumed to follow a serially correlated process with mean zero,

$$\widehat{\bar{\pi}}_t = \rho_{\bar{\Pi}} \, \widehat{\bar{\pi}}_{t-1} + \widehat{\eta}_t^{\bar{\Pi}}. \tag{66}$$

Finally, $\widehat{y}_t = \widehat{Y_t/z_t}$ is the logarithmic deviation of aggregate output from the trend output level implied by the permanent technology shock, which will be referred to as output gap, and $\widehat{\eta}_t^R$ is a serially uncorrelated shock to the nominal interest rate.[23]

2.4 Market Clearing and Aggregate Resource Constraint

In this sub-section, we formulate the various market-clearing conditions which need to hold in equilibrium, along with the aggregate resource constraint.

Market Clearing in the Labour Markets

Each household h acts as wage setter in a monopolistically competitive market. Hence, in

[23] In using a trend-based output gap measure we follow Adolfson, Laséen, Lindé and Villani (2007), whereas Smets and Wouters (2003) employ an output gap concept which is based on the assumption of full price and wage flexibility (cf. Woodford, 2003).

equilibrium the supply of its differentiated labour service needs to equal the intermediate-good firms' demand,

$$N_{h,t} \;=\; \int_0^1 N_{f,t}^h \, df \;=\; N_t^h. \tag{67}$$

Aggregating over the continuum of households h, we have

$$\begin{aligned}\int_0^1 N_{h,t}\,dh &= \int_0^1 N_t^h\,dh \\ &= \int_0^1 \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W-1}} N_t\,dh \\ &= \check{s}_{W,t}\,N_t, \end{aligned} \tag{68}$$

where the variable

$$\check{s}_{W,t} \;=\; \int_0^1 \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W-1}} dh \tag{69}$$

measures the degree of wage dispersion across the differentiated labour services h.

Given the optimal wage-setting strategies for the households, the measure of wage dispersion evolves according to

$$\check{s}_{W,t} \;=\; (1-\xi_W)\left(\frac{\tilde{W}_t}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W-1}} + \xi_W\left(\frac{W_t}{W_{t-1}}\,\frac{\Pi_{C,t}}{\Pi_{C,t-1}^{\chi_W}\,\bar{\Pi}_t^{1-\chi_W}}\right)^{-\frac{\varphi_t^W}{\varphi_t^W-1}} \check{s}_{W,t-1}, \tag{70}$$

where $\tilde{W}_t$ denotes the optimal wage contract chosen by those households that have received permission to reset their wages in period t, and $\Pi_{C,t} = P_{C,t}/P_{C,t-1}$.[24]

As regards the total wage sum paid by firms to the households, we have

$$\int_0^1 W_{h,t}\,N_{h,t}\,dh \;=\; N_t\int_0^1 W_{h,t}\left(\frac{W_{h,t}}{W_t}\right)^{-\frac{\varphi_t^W}{\varphi_t^W-1}} dh \;=\; W_t\,N_t, \tag{71}$$

where the first equality has been obtained using the aggregate demand for labour services of variety h, while the last equality uses the properties of the wage index W_t.

Market Clearing in the Capital Market

Market clearing in the rental market for capital services implies that the effective utilisation of capital by households satisfies

$$u_t\,K_t \;=\; u_t\int_0^1 K_{h,t}\,dh \;=\; \int_0^1 K_{f,t}^s\,df \;=\; K_t^s. \tag{72}$$

[24] The dispersion term $\check{s}_{W,t}$ is equal to one in steady state, and fluctuations in $\check{s}_{W,t}$ do vanish in the log-linearised version of the model.

Market Clearing in the Markets for Domestic Intermediate Goods

Each intermediate-good producing firm f acts as price setter in domestic and foreign monopolistically competitive markets. Hence, in equilibrium the supply of its differentiated output needs to equal domestic and foreign demand,

$$Y_{f,t} = H_{f,t} + X_{f,t}. \tag{73}$$

Aggregating over the continuum of firms f, we obtain the following aggregate resource constraint:

$$\begin{aligned} Y_t = \int_0^1 Y_{f,t}\, df &= \int_0^1 H_{f,t}\, df + \int_0^1 X_{f,t}\, df \\ &= \int_0^1 \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{-\frac{\varphi_t^H}{\varphi_t^H-1}} H_t\, df + \int_0^1 \left(\frac{P_{X,f,t}}{P_{X,t}}\right)^{-\frac{\varphi_t^X}{\varphi_t^X-1}} X_t\, df \\ &= \check{s}_{H,t}\, H_t + \check{s}_{X,t}\, X_t, \end{aligned} \tag{74}$$

where the variables

$$\check{s}_{H,t} = \int_0^1 \left(\frac{P_{H,f,t}}{P_{H,t}}\right)^{-\frac{\varphi_t^H}{\varphi_t^H-1}} df, \tag{75}$$

$$\check{s}_{X,t} = \int_0^1 \left(\frac{P_{X,f,t}}{P_{X,t}}\right)^{-\frac{\varphi_t^X}{\varphi_t^X-1}} df \tag{76}$$

measure the degree of price dispersion across the differentiated goods f sold either domestically or abroad.

Given the optimal price-setting strategies for intermediate-good firms, the two measures of price dispersion evolve according to

$$\check{s}_{H,t} = (1-\xi_H)\left(\frac{\tilde{P}_{H,t}}{P_{H,t}}\right)^{-\frac{\varphi_t^H}{\varphi_t^H-1}} + \xi_H \left(\frac{\Pi_{H,t}}{\Pi_{H,t-1}^{\chi_H}\bar{\Pi}_t^{1-\chi_H}}\right)^{-\frac{\varphi_t^H}{\varphi_t^H-1}} \check{s}_{H,t-1}, \tag{77}$$

$$\check{s}_{X,t} = (1-\xi_X)\left(\frac{\tilde{P}_{X,t}}{P_{X,t}}\right)^{-\frac{\varphi_t^X}{\varphi_t^X-1}} + \xi_X \left(\frac{\Pi_{X,t}}{\Pi_{X,t-1}^{\chi_X}\bar{\Pi}_t^{1-\chi_X}}\right)^{-\frac{\varphi_t^X}{\varphi_t^X-1}} \check{s}_{X,t-1}, \tag{78}$$

where $\tilde{P}_{H,t}$ and $\tilde{P}_{X,t}$ denote the optimal price contracts chosen by those firms that have received permission to reset their prices in their home and foreign markets in period t, and

$\Pi_{H,t} = P_{H,t}/P_{H,t-1}$ and $\Pi_{X,t} = P_{X,t}/P_{X,t-1}$.[25]

Similarly, in nominal terms we have

$$\begin{aligned} P_{Y,t}\, Y_t &= \int_0^1 P_{H,f,t}\, H_{f,t}\, df + \int_0^1 P_{X,f,t}\, X_{f,t}\, df \\ &= H_t \int_0^1 P_{H,f,t} \left(\frac{P_{H,f,t}}{P_{H,t}} \right)^{-\frac{\varphi_t^H}{\varphi_t^H - 1}} df + X_t \int_0^1 P_{X,f,t} \left(\frac{P_{X,f,t}}{P_{X,t}} \right)^{-\frac{\varphi_t^X}{\varphi_t^X - 1}} df \\ &= P_{H,t}\, H_t + P_{X,t}\, X_t, \end{aligned} \tag{79}$$

where the second-to-last equality has been obtained using the aggregate demand relationships for the domestic intermediate goods sold in home and foreign markets, $H_{f,t}$ and $X_{f,t}$, while the last equality has been obtained using the properties of the aggregate price indexes $P_{H,t}$ and $P_{X,t}$.

Finally, as regards aggregate intermediate-good firms' profits, we have

$$\begin{aligned} D_t &= \int_0^1 D_{H,f,t}\, df + \int_0^1 D_{X,f,t}\, df \\ &= P_{H,t}\, H_t + P_{X,t}\, X_t - MC_t \left(\check{s}_{H,t}\, H_t + \check{s}_{X,t}\, X_t + \psi\, z_t \right) \end{aligned} \tag{80}$$

or, written as profit share,

$$s_{D,t} = \frac{D_t}{P_{Y,t}\, Y_t} = 1 - \frac{MC_t}{P_{Y,t}} \frac{\check{s}_{H,t}\, H_t + \check{s}_{X,t}\, X_t + \psi\, z_t}{Y_t}. \tag{81}$$

Market Clearing in the Markets for Imported Intermediate Goods

Each foreign exporter f^* acts as price setter for its differentiated output in domestic monopolistically competitive markets. Hence, in equilibrium the supply of its differentiated output needs to equal demand, $IM_{f^*,t}$.

Aggregating over the continuum of firms f^*, we have

$$\begin{aligned} \int_0^1 IM_{f^*,t}\, df^* &= \int_0^1 \left(\frac{P_{IM,f^*,t}}{P_{IM,t}} \right)^{-\frac{\varphi_t^*}{\varphi_t^* - 1}} IM_t\, df^* \\ &= \check{s}_{IM,t}\, IM_t, \end{aligned} \tag{82}$$

[25] As for wage dispersion, the terms $\check{s}_{H,t}$ and $\check{s}_{X,t}$ are equal to one in steady state, and fluctuations in $\check{s}_{H,t}$ and $\check{s}_{X,t}$ do vanish in the log-linearised version of the model.

where the variable

$$\check{s}_{IM,t} = \int_0^1 \left(\frac{P_{IM,f^*,t}}{P_{IM,t}} \right)^{-\frac{\varphi_t^*}{\varphi_t^*-1}} df^* \tag{83}$$

measures the degree of price dispersion across the differentiated goods f^*.

Given the optimal price-setting strategies for foreign exporters, the measure of price dispersion evolves according to

$$\check{s}_{IM,t} = (1-\xi^*) \left(\frac{\tilde{P}_{IM,t}}{P_{IM,t}} \right)^{-\frac{\varphi_t^*}{\varphi_t^*-1}} + \xi^* \left(\frac{\Pi_{IM,t}}{\Pi_{IM,t-1}^{\chi^*} \bar{\Pi}_t^{1-\chi^*}} \right)^{-\frac{\varphi_t^*}{\varphi_t^*-1}} \check{s}_{IM,t-1}, \tag{84}$$

where $\tilde{P}_{IM,t}$ denotes the optimal price contracts chosen by those exporters that have received permission to reset their prices in period t, and $\Pi_{IM,t} = P_{IM,t}/P_{IM,t-1}$.[26]

Market Clearing in the Final-Good Markets

Market clearing in the fully competitive final-good markets implies:

$$Q_t^C = C_t, \tag{85}$$

$$Q_t^I = I_t + \Gamma_u(u_t)\, K_t, \tag{86}$$

$$Q_t^G = G_t. \tag{87}$$

Subsequently, combining the market-clearing conditions for domestic intermediate-good and final-good markets results in the following representation of the nominal aggregate resource constraint (79):

$$\begin{aligned} P_{Y,t}\, Y_t &= P_{H,t}\, H_t + P_{X,t}\, X_t \\ &= P_{C,t}\, C_t + P_{I,t}\, (I_t + \Gamma_u(u_t)\, K_t) + P_{G,t}\, G_t + P_{X,t}\, X_t \\ &\quad - P_{IM,t} \left(IM_t^C \frac{1-\Gamma_{IM^C}(IM_t^C/Q_t^C;\epsilon_t^{IM})}{\Gamma_{IM^C}^{\dagger}(IM_t^C/Q_t^C;\epsilon_t^{IM})} + IM_t^I \frac{1-\Gamma_{IM^I}(IM_t^I/Q_t^I;\epsilon_t^{IM})}{\Gamma_{IM^I}^{\dagger}(IM_t^I/Q_t^I;\epsilon_t^{IM})} \right), \end{aligned} \tag{88}$$

where the last equality has been obtained using the demand functions for the bundles of the domestic and foreign intermediate goods utilised in the production of the final consumption and investment goods, H_t^C and H_t^I as well as IM_t^C and IM_t^I, along with the prices of the two types of final goods, $P_{C,t}$ and $P_{I,t}$.

[26] Yet again, the dispersion term $\check{s}_{IM,t}$ is equal to one in steady state, and fluctuations in $\check{s}_{IM,t}$ do vanish in the log-linearised version of the model.

Market Clearing in the Domestic Government Bond Market

The equilibrium holdings of domestic government bonds evolve over time according to the fiscal authority's budget constraint, reflecting the fiscal authority's need to issue debt in order to finance its deficit. However, given the simplifying assumption that the budget is closed by lump-sum taxes in each period, the outstanding debt is zero in equilibrium,

$$B_t \;=\; \int_0^1 B_{h,t}\, dh \;=\; 0. \tag{89}$$

Market Clearing in the Market for Foreign Bonds

At a given point in time t, the supply of internationally traded foreign bonds is fully elastic and matches the (net) holdings of foreign bonds accumulated by domestic households,

$$B_t^* \;=\; \int_0^1 B_{h,t}^*\, dh. \tag{90}$$

2.5 Net Foreign Assets, Trade Balance and Terms of Trade

The domestic economy's net foreign assets equal the economy-wide net holdings of foreign bonds (denominated in foreign currency) and evolve according to

$$(R_t^*)^{-1} B_{t+1}^* \;=\; B_t^* + \frac{TB_t}{S_t}, \tag{91}$$

where

$$TB_t \;=\; P_{X,t}\, X_t - P_{IM,t}\, IM_t \tag{92}$$

is the domestic economy's trade balance.[27]

For reporting purposes, the net foreign assets, as well as the trade balance, are conveniently expressed as a share of domestic output, with $s_{B^*,t+1} = S_t\, B_{t+1}^*/P_{Y,t}Y_t$ and $s_{TB,t} = TB_t/P_{Y,t}Y_t$, respectively.

Finally, the terms of trade (defined as the domestic price of imports relative to the price of exports) are given by:

$$ToT_t \;=\; \frac{P_{IM,t}}{P_{X,t}}. \tag{93}$$

[27] As discussed in Section 2.1, the existence of a financial intermediation premium guarantees that, in the non-stochastic steady state, domestic holdings of foreign bonds are zero.

3 Bayesian Estimation

We adopt the empirical approach outlined in Schorfheide (2000) and An and Schorfheide (2007) and estimate the NAWM employing Bayesian inference methods. This involves obtaining the joint posterior distribution of the model's structural parameters based on its log-linear state-space representation.[28, 29]

In the following we briefly sketch the adopted approach and describe the data and the prior distributions used in its implementation. In this context, we also provide information on the structural shocks that we consider in the estimation and describe the calibration of those parameters that we keep fixed. We then present our estimation results.

3.1 Methodology

Employing Bayesian inference methods allows formalising the use of prior information from earlier studies at both the micro and macro level in estimating the parameters of a possibly complex DSGE model. This seems particularly appealing in situations where the sample period of the data is relatively short, as is the case for the euro area. From a practical perspective, Bayesian inference may also help to alleviate the inherent numerical difficulties associated with solving the highly non-linear estimation problem.

Formally, let $p(\theta|m)$ denote the prior distribution of the parameter vector $\theta \in \Theta$ for some model $m \in \mathcal{M}$, and let $\mathcal{L}(\mathcal{Y}_T|\theta, m)$ denote the likelihood function for the observed data, $\mathcal{Y}_T = \{\, y_t \,\}_{t=1}^T$, conditional on parameter vector θ and model m. The joint posterior distribution of the parameter vector θ for model m is then obtained by combining the likelihood function for $\mathcal{Y}_T$ and the prior distribution of θ,

$$p(\theta|\mathcal{Y}_T, m) \quad \propto \quad \mathcal{L}(\mathcal{Y}_T|\theta, m)\, p(\theta|m),$$

where "$\propto$" indicates proportionality.

As the posterior distribution cannot be determined analytically, we adopt a Monte-Carlo Markov-Chain (MCMC) sampling algorithm to simulate the distribution of the parameter

[28] For details on the derivation of the log-linear version of the NAWM, see Appendix A.

[29] For all computations, we use YADA, a Matlab program for Bayesian estimation and evaluation of DSGE models (cf. Warne, 2008).

vector θ.[30] The resulting posterior distribution is typically characterised by measures of central location, such as the mode or the mean, measures of dispersion, such as the standard deviation, or selected percentiles.

As discussed in Geweke (1999), Bayesian inference also provides a framework for comparing alternative, not necessarily nested, and potentially misspecified models on the basis of their marginal likelihood. For a given model m the latter is computed by integrating out the parameter vector θ,

$$\mathcal{L}(\mathcal{Y}_T|m) \quad = \quad \int_{\theta \in \Theta} \mathcal{L}(\mathcal{Y}_T|\theta, m)\, p(\theta|m)\, d\theta.$$

Thus, the marginal likelihood gives an indication of the overall likelihood of the observed data conditional on a model.

3.2 Data and Shocks

3.2.1 Data

In estimating the NAWM, we use times series for 18 macroeconomic variables which feature prominently in the ECB/Eurosystem staff projections:

- real GDP (Y)
- private consumption (C)
- total investment (I)
- government consumption (G)
- extra-euro area exports (X)
- extra-euro area imports (IM)
- GDP deflator (P_Y)
- consumption deflator (P_C)
- extra-euro area import deflator (P_{IM})
- total employment (E)
- compensation per head (W)
- nominal interest rate (R)
- nominal effective exchange rate (S)
- foreign demand ($Y^{d,*}$)†
- foreign prices (P_Y^*)†
- foreign interest rate (R^*)†
- competitors' export prices ($P_X^{c,*}$)†
- oil prices (P_O)†

All time series are taken from an updated version of the AWM database (see Fagan, Henry and Mestre, 2001), except for the time series of extra-euro area trade data the construction of which is detailed in Dieppe and Warmedinger (2007). The sample period

[30] More specifically, we rely on the Random-Walk Metropolis (RWM) algorithm to obtain a large number of random draws from the posterior distribution of θ. The mode and a modified Hessian of the posterior distribution, the latter evaluated at the mode, are used to determine the initial proposal density for the RWM algorithm. The posterior mode and the Hessian matrix are computed by standard numerical optimisation routines, namely Christopher Sims' optimiser csminwel.

ranges from 1985Q1 to 2006Q4 (using the period 1980Q2 to 1984Q4 as training sample). The times series marked with a dagger ('†') are modelled using a structural vector-autoregressive (SVAR) model, the estimated parameters of which are kept fixed throughout the estimation of the NAWM.[31] Similarly, government consumption is specified by means of a univariate autoregressive (AR) model with fixed estimated parameters.[32]

Prior to estimation, we transform real GDP, private consumption, total investment, extra-euro area exports and imports, the associated deflators, compensation per head (henceforth, wages), as well as foreign demand and foreign prices into quarter-on-quarter growth rates, approximated by the first difference of their logarithm. Furthermore, a number of additional transformations are made to ensure that variable measurement is consistent with the properties of the NAWM's balanced-growth path and in line with the underlying assumption that all relative prices are stationary:

- We match the sample growth rates of extra-euro area exports and imports as well as foreign demand with the sample growth rate of real GDP by removing the sample growth rate differentials, reflecting the fact that trade volumes and foreign demand tend to grow at a significantly higher rate than real GDP.
- We take the logarithm of government consumption and remove a linear trend consistent with the NAWM's steady-state growth rate of 2.0 percent per annum which is assumed to have two components: labour productivity growth of roughly 1.2 percent and labour force growth of approximately 0.8 percent (see Section 3.3.1 for details).
- We take the logarithm of employment and remove a linear trend consistent with a steady-state labour force growth rate of 0.8 percent, noting that, in the absence of a reliable measure of hours worked, we use data on employment in the estimation.[33]

[31] Identification within the SVAR model is achieved by a Choleski decomposition of its estimated variance-covariance matrix, the ordering of variables being: foreign prices, foreign demand, foreign interest rate, oil prices and competitors' export prices.

[32] In this version of the NAWM, the distortionary tax rates are assumed to be time-invariant. Details on their calibration are provided in Section 3.3.1.

[33] We relate the employment variable, E, to the unobserved hours-worked variable, N, by an auxiliary equation following Smets and Wouters (2003),

$$\widehat{E}_t = \frac{\beta}{1+\beta}\,\mathrm{E}_t[\widehat{E}_{t+1}] + \frac{1}{1+\beta}\,\widehat{E}_{t-1} + \frac{(1-\beta\xi_E)\,(1-\xi_E)}{(1+\beta)\,\xi_E}\left(\widehat{N}_t - \widehat{E}_t\right),$$

where a hat ('$\widehat{\ }$') denotes the logarithmic deviation from trend in the case of employment and from the steady-state value in the case of hours worked. This is consistent with our assumption that the model can be expressed in per-capita terms. The parameter ξ_E determines the sensitivity of employment with respect

- We construct a measure of the real effective exchange rate from the nominal effective exchange rate, the domestic GDP deflator and foreign prices (defined as a weighted average of foreign GDP deflators) and then remove the mean.
- We deflate competitors' export prices and oil prices (both expressed in the currency basket underlying the construction of the nominal effective exchange rate) with foreign prices and then remove unrestricted linear trends.

Figure 1 shows the time series of the transformed variables for the sample period 1985Q1 to 2006Q4.

3.2.2 Shocks

Out of the total of 19 structural shocks incorporated in the NAWM, we employ a subset of 12 shocks, plus the 6 shocks in the AR and SVAR models for government consumption and the foreign variables, respectively:

- domestic risk premium shock (ϵ^{RP})
- external risk premium shock (ϵ^{RP^*})
- permanent technology shock (g_z)
- transitory technology shock (ε)
- investment-specific techn. shock (ϵ^{I})
- wage markup shock (φ^{W})
- price markup shock: domestic (φ^{H})
- price markup shock: exports (φ^{X})
- price markup shock: imports (φ^{*})
- import demand shock (ϵ^{IM})
- export preference shock (ν^{*})
- government consumption shock (η^{G})
- interest rate shock (η^{R})
- foreign demand shock ($\eta^{Y^{d,*}}$)
- foreign price shock ($\eta^{\Pi^*_Y}$)
- foreign interest rate shock (η^{R^*})
- competitors' export price shock ($\eta^{P_X^{c,*}}$)
- oil price shock (η^{P_0})

All shocks are assumed to follow first-order autoregressive processes, except for the interest rate shock and the shocks in the AR and SVAR models, which are assumed to be serially uncorrelated. In addition, we account for measurement error in extra-euro area trade data (both volumes and prices) in view of the fact that they are prone to revisions. We also allow for small errors in the measurement of real GDP and the GDP deflator to alleviate discrepancies between the national accounts framework underlying the construction of official GDP data and the NAWM's aggregate resource constraint.

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to hours worked.

3.3 Calibration and Prior Distributions

3.3.1 Calibration

Our calibration strategy follows common practice and assigns values to those parameters that affect the NAWM's non-stochastic steady state.[34] Regarding the latter, all real variables are assumed to evolve along a balanced-growth path with a trend growth rate of 2.0 percent per annum, which roughly matches average real GDP growth in our estimation sample. The steady-state growth rate of 2.0 percent in turn is assumed to consist of two components: a steady-state growth rate of 1.2 percent for labour productivity, g_z, broadly in line with average labour productivity growth over the sample period, and a steady-state growth rate of 0.8 percent for the labour force, g_E, as a proxy for steady-state population growth.[35] Hence, once labour force growth is accounted for, all quantities within the NAWM can be interpreted in (approximate) per-capita terms. Consistent with the balanced-growth assumption, we then calibrate key steady-state ratios of the model by matching their empirical counterparts over the sample period. Specifically, the expenditure shares of private consumption, total investment and government consumption are set to, respectively, 57.5, 21.0 and 21.5 percent of nominal GDP, while the export and import shares are set to 16.0 percent, ensuring balanced trade in steady state.

Conditional on the steady-state rate of productivity growth, the discount factor β is chosen to be consistent with an annualised equilibrium real interest rate of 2.5 percent, while the monetary authority's long-run (net) inflation objective $\bar{\Pi} - 1$ is assumed to equal 1.9 percent at an annualised rate, consistent with the ECB's quantitative definition of price stability of inflation being below, but close to 2 percent. In line with the literature, we set the depreciation rate δ in the capital accumulation equation equal to 0.025 and fix the capital share α in the intermediate-good firms' production technology at 0.36. Our calibration of the wage and price markups with $\varphi^W = 1.30$ and $\varphi^H = \varphi^X = 1.35$ is based on studies conducted at the OECD (cf. Martins, Scarpetta and Pilat, 1996, and Jean and Nicoletti, 2002).[36]

[34] For details on the computation of the steady state, see Appendix B.

[35] Using labour force growth as a proxy for population growth is motivated by the fact that the latter variable does not play a role in the ECB/Eurosystem staff projections, whereas the former is considered to be an important determinant of trend output growth.

[36] See Bayoumi, Laxton and Pesenti (2004) for further discussion. We note that the foreign price markup

The parameter ψ, determining the fixed costs of the intermediate-good firms' production technology, is calibrated such that profits are zero in steady state. Regarding final-good production, we choose steady-state values for the home-bias parameters ν_C and ν_I that allow the model to replicate the import content of consumption and investment spending—roughly 10 and 6 percent, expressed as shares of nominal GDP—utilising information from input-output tables for the euro area (cf. Statistics Netherlands, 2006). On the fiscal side, we use information provided in OECD (2004) and Eurostat (2006) and calibrate the tax rates on consumption purchases and labour income and the contribution rates to social security with $\tau^C = 0.183$, $\tau^N = 0.122$, $\tau^{W_h} = 0.118$ and $\tau^{W_f} = 0.219$. In the absence of reliable data, the tax rate on dividend income τ^D is set to zero, while the tax rate on capital income τ^K is determined as a free parameter in the steady-state computation.

Finally, we calibrate a small number of additional parameters that are inherently difficult to identify. This concerns the inverse of the labour supply elasticity ζ, which we set equal to 2 in line with the range of available estimates, and the sensitivity of the external intermediation premium γ_{B^*}, which we fix at 0.01 so that the evolution of net foreign assets has only a small impact on the exchange rate and trade in the short run, while guaranteeing that the net foreign asset position is stabilised at zero in the long run.

3.3.2 Prior Distributions

The vertical panel in the middle of **Table 1** summarises our assumptions regarding the prior distributions for those parameters that affect the dynamics, but not the steady state of the NAWM, and that we wish to estimate using Bayesian methods.[37] As the closed-economy model of the euro area by Smets and Wouters (2003) is essentially nested within the NAWM, we follow Smets and Wouters and make broadly similar prior assumptions for those structural parameters that are common to both models. Our choice of priors for the parameters concerning the NAWM's open-economy dimension is informed by the priors

φ^* neither affects the steady state nor does it enter the log-linearised version of the model.

[37] Note that we are only concerned with those steady-state properties that are relevant for estimating the log-linear version of the NAWM. While some of the parameters influence the steady-state *level* of real variables such as hours worked and capital, their steady-state *ratios*, which matter for the log-linear model, are invariant to changes in those parameters.

used in Adolfson, Laséen, Lindé and Villani (2007). However, our preferred specification of the NAWM incorporates neither variable capital utilisation (as in Smets and Wouters) nor a time-varying inflation objective (as in Adolfson et al.).[38]

The (marginal) prior distributions for the parameters are chosen in conformity with the constraints on the parameter space implied by theory. That is, for those parameters that are bounded between 0 and 1, we choose a standardised beta distribution. This group of parameters comprises, inter alia, the habit formation parameter, the Calvo and indexation parameters constraining the wage and price-setting decisions of households and firms, the parameter determining the degree of interest-rate smoothing in the interest-rate rule, and the autoregressive coefficients of the shock processes.

Highlighting the priors for just a few of these parameters, we note that a common prior mean of 0.75 has been chosen for both the Calvo and the indexation parameters, while the standard deviation for the latter is assumed to be twice as large, reflecting the high degree of uncertainty about the persistence of price and wage inflation. As to the prior distributions for the autoregressive coefficients of the shock processes, we set the prior means uniformly to 0.75, except for the wage and price markup shocks, for which the means are set to 0.5, and the interest rate shock, which is assumed to be serially uncorrelated. The lower prior mean for the autoregressive coefficients of the markup shocks is motivated by the fact that some studies assume that they are actually serially uncorrelated.

For parameters that are bounded from below at zero, we have chosen either a gamma or an inverse gamma distribution to model the prior distributions. Specifically, the priors for the intratemporal elasticities of substitution between domestic and imported intermediate goods in final-good production and the elasticity of export demand are specified as gamma distributions with a common mean of 1.5 and a standard deviation equal to 0.25. Similarly, the adjustment-cost parameters are assumed to follow a gamma distribution a priori, albeit with different means and standard deviations. In contrast, the prior distributions for the standard deviations of the structural shocks are modelled as inverse gamma distributions with 2 degrees of freedom and a common mode of 0.10, reflecting the fact that there is little prior information on these parameters.

[38] The sensitivity of our estimation results to alternative specifications is studied in Section 3.5.

Finally, the prior distributions for the parameters of the interest-rate rule are modelled as normal distributions, with the exception of the parameter determining the degree of interest-rate smoothing. The precise parametrisation of the prior distributions follows Smets and Wouters (2003), except for the prior regarding the response coefficients on the level of the output gap. Specifically, the means of the prior distributions equal 1.7 for the inflation response coefficient, 0.3 for the response coefficient on the change in inflation, and 0.0625 for the response coefficient on the *change* in the output gap, while the response coefficient on the output gap *level* is set to zero.

Our decision to restrict the interest-rate response to the output gap level to zero is motivated by the empirical finding that it is inherently difficult to pin down the precise level of the output gap, as most data have been transformed into growth rates prior to estimation. At the same time, changes in the output gap are deemed a useful, and relatively robust indicator of demand pressures. Yet we acknowledge that the employed output gap measure—the deviation of actual output from the stochastic trend implied by the permanent technology shock—has no strong theoretical basis (cf. Galí and Gertler, 2007). Estimating the NAWM with a theory-consistent measure of the output gap—that is, the deviation of actual output from the output level that can be attained in an environment of full nominal flexibility (cf. Woodford, 2003)—is left for the future.

3.4 Estimation Results

The right-hand columns in **Table 1** present estimation results for our preferred specification of the NAWM. The entries in the posterior-mode column refer to the values of the structural parameters that are obtained by maximising the model's posterior distribution. The remaining four columns report the mean, the median as well as the 5 and 95 percent percentiles of the (marginal) posterior distributions which are computed using a posterior sampling algorithm based on a Markov chain with 550,000 draws, with 50,000 draws being discarded as burn-in draws.

One commonly used approach for monitoring convergence of the posterior sampling algorithm is to use multivariate analysis of variance (MANOVA). Based on the method discussed by Brooks and Gelman (1998) we have calculated the so-called multivariate potential scale reduction factor (MPSRF) that summarises the information about within-chain and between-chain variation for a number of parallel Markov chains. This factor is greater than unity for a finite number of posterior draws per chain and a finite number of chains. As the simulation converges, this factor declines to 1, where values less than 1.1 are often regarded as signs of convergence. Accordingly, in **Figure 2** we have plotted sequential estimates of the MPSRF based on 4 chains, each having 550,000 posterior draws, with the first 50,000 draws being discarded as burn-in draws. The sequential estimates start with 10,000 post burn-in posterior draws and are updated every 10,000 draws. After around 150,000 post burn-in draws we find that the MPSRF has dropped below 1.1, and at 500,000 draws the MPSRF is very close to unity.

Comparing the plots of the prior and posterior distributions in **Figure 3** gives an indication of how informative the observed data are about the structural parameters. That is, for those parameters where the posterior distribution turns out to be close to the prior distribution, the data are likely to be rather uninformative. Hence, the figure suggests that the observed data provide additional information for most parameters. Exceptions are the Calvo parameter constraining the price-setting decisions of domestic intermediate-good firms that are selling their goods abroad, ξ_X, and the adjustment cost parameter dampening the short-run price elasticity of exports, γ^*. This finding may reflect the fact that no data on export prices are used in the estimation.

A number of estimation results are noteworthy. First, the estimates of the key parameters shaping the dynamics of domestic demand in response to the model's structural shocks—the degree of habit formation in consumption, κ, and the investment adjustment cost parameter, γ_I—are in line with those reported in Smets and Wouters (2003). Regarding the investment adjustment cost parameter, the somewhat lower estimate is likely to arise from the fact that households in the NAWM can more effectively smooth consumption due to their ability to borrow from abroad. As a result, investment will be reduced by a lesser extent in response to adverse shocks, and hence observed investment fluctuations are consistent with a smaller adjustment cost parameter.

Second, the estimated elasticities of substitution between domestic and imported intermediate goods in final-good production, μ_C and μ_I, are found to be rather similar across the consumption and the investment good, once parameter uncertainty is taken into account. In contrast, the estimates of the adjustment-cost parameters associated with changing the import content differ substantially across the two types of final goods. In particular, the estimated cost of changing the import content of the consumption good, γ_{IM^C}, is substantially higher than the cost associated with changing the import content of the investment good, γ_{IM^I}. Comparing the posterior and prior distributions of these parameters in Figure 3, we see that this result is strongly informed by the data. Apparently, the relative smoothness of the consumption data requires that shocks affecting import quantities are mainly propagated via adjustments in the import content of investment.

Third, on the nominal side, we observe that the estimate of the Calvo parameter constraining the price-setting decisions of domestic firms that are selling in home markets, ξ_H, is rather high. Yet our posterior mode estimate of about 0.92 is broadly comparable with a point estimate of about 0.90 for the Calvo parameter in the model by Smets and Wouters (2003). The estimate for ξ_H implies that the domestic Phillips curve within the NAWM is rather flat or, in other words, that the sensitivity of domestic inflation with respect to movements in aggregate marginal cost is low.[39] Interestingly, as shown in **Figure 4**, recursively estimating the NAWM over the years 1998 to 2006 results in a sequence of estimates of ξ_H that are gradually rising, in line with the view that the slope of the euro area Phillips curve has been declining over recent years.[40] In contrast, the estimate of the Calvo parameter constraining the price-setting decisions of foreign exporters, ξ^*, is rather low, resulting in a relatively high degree of exchange-rate pass-through to import prices, already in the short run. The estimates of the Calvo parameters constraining the adjustment of export prices

[39] Our estimate for ξ_H does not necessarily imply a high degree of nominal rigidity, as its interpretation in terms of the implied mean duration of price contracts with $1/(1-\xi_H)$ would suggest. The reason is that the Calvo-style Phillips curve, in general, does not permit to separately identify nominal and real rigidities which jointly influence the price-setting behaviour of firms. Indeed, the inclusion of alternative sources of real rigidities, such as a "kinked demand curve" of firm-specific inputs, would allow re-interpreting the estimate of the Calvo parameter without affecting the slope coefficient of the Phillips curve. See Eichenbaum and Fisher (2007) and Coenen, Levin and Christoffel (2007) for further discussion.

[40] We note that Smets and Wouters (2003) estimate their model on data from 1980 to 1998, which may thus explain, why they obtain a lower estimate for the Calvo parameter

and wages, ξ_X and ξ_W, remain close to their prior means. As to the indexation parameters, the estimation results suggest that the persistence in wage inflation is noticeably higher than that in price inflation. However, the posterior distributions of the indexation parameters are rather wide, indicating that there is substantial parameter uncertainty.

Fourth, regarding the interest-rate rule, we find that the estimated response coefficients are broadly similar to the estimates reported in Smets and Wouters (2003), despite the differences in the underlying output gap concept. The estimated inflation response is safely above unity, ensuring determinacy of the model solution, while the response to the change in the output gap is positive, albeit small. We also find supportive evidence for a relatively high degree of interest-rate smoothing.

And fifth, regarding the properties of the structural shocks, we observe that none of the estimated shock processes appears excessively persistent. Indeed, the 95 percent percentile of the posterior distribution for the autoregressive coefficient does not exceed 0.95 for any of the structural shocks. It is also notable that the estimated degree of persistence for the markup shocks is at the lower end of the range of estimates.

Finally, **Figure 5** and **Figure 6** display the smoothed estimates of the structural shocks, along with their innovation component. A selective interpretation of the estimated shocks will be given in Section 5 in the context of a model-based decomposition of historical developments in the observed data.

3.5 Sensitivity Analysis

In **Table 2** we provide information on the sensitivity of our estimation results with respect to several changes in the specification of the NAWM. A formal comparison is made on the basis of the marginal likelihood which gives an indication of the data coherence of each, not necessarily nested specification.

First, we observe that allowing for variable capital utilisation is not supported by the data, as indicated by a fall in the marginal likelihood by more than 20 units.[41] Regarding the parameter estimates, we see a substantial fall in the standard deviation of the transitory

[41] The prior distribution of the utilisation cost parameter $\gamma_{u,2}$ is given by a gamma distribution with mean 0.01 and standard deviation 0.01, while the parameter $\gamma_{u,1}$ is pinned down by the model's steady-state calibration (see Appendix B).

technology shock. Second, the inclusion of a time-varying inflation objective to capture the nominal convergence process in the run up to the formation of EMU is not supported by the data either.[42] In fact, the variation in the smoothed estimate of the inflation objective shock (not shown) is found to be quantitatively unimportant, but surrounded by a high degree of uncertainty. Starting at a relatively low level in the mid eighties, the inflation objective creeps up by only a small amount following the surge in inflation in the early nineties, before gradually falling again in the run up to EMU. Third, we restrict all markup shocks to be serially uncorrelated, as occasionally assumed in other studies. Not surprisingly, the estimated degree of indexation characterising the wage and price-setting behaviour of households and firms rises noticeably. However, on the basis of the marginal likelihood, restricting the markup shocks is found to be at odds with the data. Finally, estimating the NAWM under the assumption that domestic exporters set their prices in local rather than producer currency results in a dramatic deterioration of the marginal likelihood by almost 60 units.[43] This deterioration is accompanied by marked changes in the parameters characterising the price-setting behaviour of domestic exporters, the elasticity of export demand and the parameters of the export price markup shock.

4 Model Properties

In this section, we validate the empirical properties of our preferred specification for the NAWM by reporting the model's impulse-response functions and forecast-error-variance decompositions, by inspecting the sample moments implied by the model and by examining the model's forecasting performance. In this context, we use the adopted Bayesian approach to estimation to quantify, depending on the type of validation exercise, different sources of uncertainty, notably parameter uncertainty and the uncertainty associated with the structural shock processes.[44]

[42] The inflation objective is assumed to follow a highly persistent autoregressive process. The autoregressive coefficient has been fixed at 0.975, while the prior for the standard deviation follows an inverse gamma distribution with 2 degrees of freedom and a mode of 0.1.

[43] Local-currency pricing on the part of both domestic and foreign exporters is the maintained assumption in the calibrated version of the NAWM.

[44] Additional sources of uncertainty are the measurement errors allowed for in the estimation and the uncertainty about the model's unobserved state variables.

4.1 Impulse-Response Functions

Figure 7 to **Figure 10** show the NAWM's impulse-response functions to four distinct structural shocks: an interest rate shock, a transitory technology shock, a domestic price markup shock and an export preference shock.[45] While the interest rate shock informs us about the transmission of monetary policy within the NAWM, the three other shocks provide examples of a supply, a cost-push and a demand shock, respectively. Focusing on the observed variables that are endogenously determined within the NAWM, the figures show the mean and the 70 and 90 percent equal-tail uncertainty bands for the impulse responses to shocks equal to one standard deviation. These uncertainty bands reflect the uncertainty about the model's structural parameters, as described by their posterior distribution. All impulse responses are reported as percentage deviations from the model's non-stochastic steady state, except for those of the inflation and interest rates which are reported as annualised percentage-point deviations.

Interest Rate Shock

The impulse responses to an interest rate shock are in line with common wisdom regarding the transmission of monetary policy. Domestic demand is temporarily curtailed, with investment falling more strongly than consumption. Moreover, due to the induced appreciation of the domestic currency, import prices drop noticeably, causing an improvement of the terms of trade. The improved terms of trade lead to expenditure-switching from domestic towards foreign goods. As a consequence, exports are retrenched, whereas imports fall by less than domestic demand. Following the broad-based decline in aggregate demand, firms cut back their demand for labour and, therefore, employment falls. Via its impact on firms' marginal cost, the resulting decline in wages puts downward pressure on domestic prices.[46] The fall in domestic prices translates into a decline in consumer prices, which is further strengthened by the drop in import prices due to the appreciation of the domestic currency. In sum, the peak absolute effect on economic activity and *inflation* is reached after about one year, whereas the implied *price level* effects are gradual and long-lasting.

[45] Additional impulse-response functions are shown in Appendix Figures A.1 to A.9.

[46] Similarly (but not shown), firms utilise fewer capital services. The ensuing decline in the rental rate of capital amplifies the negative impact on marginal cost.

Transitory Technology Shock

A transitory technology shock triggers a pronounced decline in real marginal cost. This decline in marginal cost causes domestic prices to fall, as the prices of intermediate goods are set as a markup on marginal cost. With domestic demand adjusting only sluggishly to the increase in supply, both employment and nominal wages go down, whereas real wages move very little. Monetary policy, as prescribed by the estimated interest-rate rule, aims at counteracting the induced deflationary pressures by gradually lowering the nominal interest rate. However, the real interest rate actually rises in the short run and restrains the increase in domestic demand. At the same time, the deterioration of the terms of trade going along with the depreciation of the domestic currency leads to expenditure switching away from foreign towards domestic goods, thereby boosting exports, while dampening imports. The depreciation, in nominal terms, gives rise to a noticeable increase in import prices. As a consequence, the drop in consumer prices is smaller than the decline in domestic prices, or the GDP deflator.

Domestic Price Markup Shock

A shock to the price markup that domestic intermediate-good firms charge over their marginal cost when selling their outputs domestically causes a surge in domestic inflation which translates into a sharp rise in consumer price inflation. The monetary authority counteracts the inflationary push by raising the nominal interest rate. However, its attempt to deal with the trade-off between stabilising inflation and mitigating the adverse effects on economic activity results in an initial fall of the real interest rate. Nevertheless, with forward-looking households and firms, the interest rate response induces a noticeable decline in consumption and investment alike, which in turn leads to a fall in imports. At the same time, the decline in exports is subdued as the improvement in the terms of trade, which accompanies the induced real appreciation of the domestic currency, is rather short-lived and contained. The decline in aggregate demand causes employment to fall and exerts downward pressure on real wages, while nominal wages are sharply raised with a one-quarter delay, to compensate for the upward shift in consumer prices in line with the indexation scheme embodied in the model.

Export Preference Shock

The export preference shock boosts domestic exports and gives rise to an contemporaneous increase in real GDP. In an attempt to counteract the implied demand pressures, monetary policy is tightened by raising the nominal interest rate. As a result, the external impulse spills over only slowly to investment and, with some further delay, to consumption—akin to the notion of an export-led expansion. The broad-based increase in aggregate demand translates into a heightened demand for labour on the part of firms, which causes employment to strengthen. Both nominal and real wages pick up and, via their impact on firms' marginal cost, domestic inflation accelerates. However, the pass-through of the increase in domestic prices to consumer prices is dampened by a fall in import prices. The latter is triggered by the appreciation of the domestic currency following the tightening of monetary policy. The implied improvement in the terms of trade in turn bolsters imports.

4.2 Forecast-Error-Variance Decompositions

In **Table 3** we provide details on the contributions of the NAWM's structural shocks to the forecast error variances of a selected set of observed variables. In particular, we report mean estimates and 90 percent equal-tail uncertainty bands for the contributions to the forecast error variances over short (1 and 4-quarter) and medium-term (20 and 40-quarter) horizons, noting that the uncertainty is relative large, as revealed by the width of the 90 percent uncertainty bands. This reflects the fact that, in contrast to the computation of the impulse-response functions, not only parameter, but also shock uncertainty is taken into account.In the following, we concentrate on the decompositions for real GDP (in levels), GDP deflator inflation, wage inflation and the nominal interest rate.[47]

In the short run, the observed fluctuations in real GDP are primarily driven by domestic risk premium shocks, investment-specific technology shocks and export preference shocks. In the very short run, the former two shocks explain, respectively, 29 and 14 percent of GDP fluctuations, while the latter accounts for 24 percent. Over the medium term, the contributions of domestic risk premium shocks and export preference shocks are gradually

[47] Additional forecast-error-variance decompositions are reported in Appendix Table A.

reduced and permanent technology shocks become more important. At the 40-quarter horizon, transitory, investment-specific and permanent technology shocks together explain 60 percent of real GDP fluctuations, with permanent shocks accounting for about 30 percent. The contribution of domestic risk premium shocks falls to 11 percent. Throughout, the contribution from interest rate shocks is relatively small, diminishing quickly from 10 percent in the very short run to 3 percent after 40 quarters.

It is striking that only 30 percent of the forecast-error-variance of real GDP is explained by the permanent technology shock at the 40-quarter horizon. In the long run, the share for this shock is, by assumption, 100 percent. To assess convergence to the long-run we may relate the 40-quarter horizon variance to an estimate of the long-run variance. A value close to unity for this ratio indicates convergence, while a very large or a small value indicates a lack thereof. For real GDP we obtain a value around 2.2 for this ratio (200 quarters is used when estimating the long-run variance).[48] For the other variables in Table 3 this long run convergence statistic is below but very close to unity after 40 quarters.

Turning to GDP deflator inflation, we observe that domestic and export price markup shocks account for most of the variability in inflation at short horizons.[49] The remaining variability in inflation is explained by transitory demand shocks, wage markup shocks and domestic risk premium shocks. Apparently, the relatively small contribution of transitory technology shocks and wage markup shocks—affecting domestic and export price inflation through the marginal cost channel—reflects the rather low sensitivity of domestic inflation to real marginal cost. Over the medium term, the joint contribution of the price markup shocks declines from 85 to 53 percent, while transitory technology and wage markup shocks become more important. Similarly, the contribution of domestic risk premium shocks gradually increases, reflecting the delayed and indirect influence of the induced demand pressures on prices via marginal cost. At the 40-quarter horizon, domestic risk premium shocks explain

[48] Since the forecast-error-variance of real GDP is based on an accumulation of forecast errors for real GDP growth, the ratio can only be greater than unity when negative cross-period products of impulse responses for real GDP growth dominate the same-period products. For a variable that is not an accumulation of a growth-rate variable, the ratio cannot be greater than unity and very small values occur when the impulse responses converge slowly to zero. For details, see Warne (2008).

[49] In this context, we recall that the NAWM's nominal aggregate resource constraint implies that nominal output equals the sum of the intermediate-good firms' nominal revenues from selling their goods domestically and abroad (see equation (79) in Section 2).

16 percent of the variability in GDP deflator inflation, while transitory technology shocks and wage markup shocks account for 13 and 6 percent, respectively.

Regardless of the horizon, the forecast error variance of wage inflation is dominated by wage markup shocks. In the short run, wage markup shocks account for more than three-fourth of the variability of wage inflation, while explaining still more than 50 percent after 40 quarters. Other important sources of wage variability are domestic risk premium and permanent technology shocks, which explain a fraction of, respectively, 14 and 10 percent of wage fluctuations at the 40-quarter horizon. This result points to the limitations of the labour-market setup in the current version of the NAWM which, like the large majority of the new-generation DSGE models, rests on the assumption of monopolistically competitive labour markets with rigid nominal wages.

Finally, fluctuations in the interest rate are mostly attributable to interest rate shocks in the very short run. However their contribution is diminishing quickly, falling from about 40 percent in the very short run to below 5 percent in the medium run. Conversely, the contributions of domestic risk premium are becoming increasingly important. However, investment-specific technology shocks, price markup shocks, export preference shocks and the group of foreign shocks are also making noticeable contributions.

4.3 Implied Sample Moments

In **Table 4** we report the sample means and standard deviations implied by the NAWM, along with the sample moments based on the observed data over the estimation period 1985Q1 to 2006Q4. The model-based sample moments have been estimated through simulation. Specifically, for each one of 500 parameter values from the posterior distribution 500 samples of all the observed variables have been simulated by drawing from the estimated shock processes. For each such simulated sample, the sample moments have been calculated. The averages of the model-based sample means and standard deviations and their 5 and 95 percent percentiles are shown in the table. Furthermore, in **Figure 11** we provide model and data-based sample autocorrelations between real GDP and the observed variables that are endogenously determined within the NAWM. The figure displays the averages of the model-based sample autocorrelations and the 70 and 90 percent equal-tail uncertainty bands, along with the sample autocorrelations based on the observed data.

Regarding the sample means, we conclude from Table 4 that the NAWM is broadly in line with the data. The data means for the GDP and consumption deflator inflation series are somewhat greater than the 95 percent percentile, while the data-based mean for import deflator inflation is smaller than the 5 percent percentile. Moreover, the data-based sample mean of the nominal interest rate is somewhat greater than the model-based 95 percent percentile.[50] With the exception of employment and the real effective exchange rate, which are demeaned, all the other variables in the table have a data-based sample mean that is higher than the model-based mean, but also well below the 95 percent percentile.

Turning to the standard deviations, we find that real GDP, consumption, investment, exports and imports have a data-based sample volatility measure that is below the 5 percent percentile generated by the NAWM. For all other variables the model-based sample standard deviation is roughly consistent with the data-based estimate. Moreover, studying the dynamics between real GDP and the endogenous variables through the implied autocorrelations, it can be seen from Figure 11 that the pattern is similar. The point estimates do not match exactly, but the data-based correlations are generally within standard bounds from the distribution of the model-based correlations. Overall, these findings suggest that the NAWM is able to explain a fair share of the fluctuations in the observations of the endogenous variables over the sample period.

4.4 Forecasting Performance

Finally, **Figure 12** gives an indication of the relative forecasting performance of the NAWM. To this end, the figure shows the root-mean-squared errors of unconditional one to eight quarters-ahead forecasts for year-on-year real GDP growth, year-on-year GDP inflation and the annual short-term nominal interest rate. The NAWM-based forecasts are compared to

[50] Ultimately, these discrepancies are rooted in our calibration of the monetary authority's long-run inflation objective within the NAWM which is set to 1.9 percent at an annualised rate, in line with the ECB's quantitative definition of price stability. This calibration compares with a situation in which a large number of countries forming the euro area experienced a rather protracted period of higher inflation rates prior to joining EMU. Hence, for inflation and interest rates, the data-based sample means are quite a bit higher than their model-based means.

forecasts that are generated by a Bayesian vector-autoregressive (BVAR) model with a steady-state prior and two naïve forecasts. The naïve forecasts are given by the random-walk assumption (the last pre-forecast sample observation representing the forecast over the full forecast horizon) and the pre-forecast sample mean (corresponding to a random walk with drift for the accumulated variable; e.g., the level of real GDP).[51] The forecasts are made out-of-sample with the end of the estimation sample being gradually extended from 1998Q4 to 2005Q4. For the NAWM and the BVAR, the point forecasts for computing the RMSEs are given by the means of the predictive densities that are based on 500 draws from the posterior distributions of the models' parameters using 500 prediction paths per parameter draw. To ease the computational burden for this Bayesian approach to forecasting, both the NAWM and the BVAR are re-estimated in the fourth quarter of each calendar year, rather than in each individual quarter.

Regarding real GDP growth, we observe that the NAWM has smaller RMSEs for all prediction horizons when compared to the BVAR and the naïve forecasts, except for the 6 to 8 quarter-ahead horizons, over which the sample mean has slightly smaller RMSEs. As regards GDP deflator inflation, the NAWM again outperforms the BVAR at all horizons, but the random walk-based forecasts are found to have smaller RMSEs, except for the two-year horizon. The sample mean largely over-predicts inflation over the forecast evaluation period, and therefore we have dropped the respective RMSEs from the figure.[52] Finally, in the case of the nominal interest rate, the NAWM, the BVAR and the random walk generate broadly similar RMSEs. As in the case of GDP deflator inflation, the RMSEs from the sample mean are too large to fit into the figure.

In summary, on the basis of our limited forecast comparison exercise, the NAWM seems to fare quite well relative to the BVAR model and the naïve benchmarks. A more comprehensive analysis of the NAWM's forecasting properties is ongoing.

[51] The variables in the BVAR model are real GDP, the GDP deflator, the nominal short-term interest rate, the real effective exchange rate, foreign demand, foreign prices and the foreign interest rate. All variables are measured in exactly the same way as the variables used in the estimation of the NAWM. The BVAR has a prior similar to the BVAR model in Adolfson, Andersson, Lindé, Villani and Vredin (2007) and is described in detail in Villani (2005).

[52] The good performance of the random walk-based forecasts can be explained by the fact that inflation has been relatively stable during the EMU period. In contrast, the bad performance of the sample mean reflects the protracted period of higher average inflation rates in the pre-EMU period.

5 Applications

We now consider a number of applications to illustrate the potential contributions that the NAWM can make to forecasting and policy analysis at the ECB. The first application shows how the NAWM can aid in understanding the evolution of the euro area economy by interpreting historical developments through the lens of a structural model. In the second application we give an example of how the NAWM can be used to conduct scenarios that are concerned with counterfactual developments in its observed variables. The third application demonstrates how a Bayesian approach to forecasting can be used to construct interval, as opposed to point forecasts. In this context, we also demonstrate that incorporating conditioning information can be useful in forecasting. Finally, the fourth application exemplifies the calculation of probabilities for specific prediction events, such as the probability of entering a recession.

5.1 Historical Decompositions

What have been the driving forces behind the historical fluctuations in economic activity, inflation and interest rates since the formation of EMU? The NAWM can be utilised to answer this question by decomposing the observed variables used in the estimation into the contributions of its structural shocks (see Figures 5 and 6). As an illustration, we focus on the decomposition of year-on-year real GDP growth (in deviation from its steady-state growth rate of 2 percent per annum). Moreover, to facilitate the presentation, we group the structural shocks in five categories: technology shocks, demand shocks, markup shocks, the monetary policy shock and the foreign shocks.[53]

Figure 13 shows that the acceleration of real GDP growth in the first two years of EMU can be largely attributed to favourable markup and demand shocks, which offset the overall negative contribution of technology shocks. Interestingly, declining markups, possibly as

[53] The technology shock group comprises the permanent technology shock, the transitory technology shock and the investment-specific technology shock. The demand shock group includes the shocks to the domestic risk premium, government consumption and import demand, while the markup shock group consists of the wage markup, the domestic price markup and the export price markup shocks. The monetary policy shock is represented by the innovation in the model's interest rate rule. Finally, the foreign shock group comprises the external risk premium shock, the export preference shock, the import price markup shock, and the shocks to the foreign variables, including foreign demand.

a result of enhanced competition in goods and labour markets in the run up to EMU, are leading the expansion in economic activity, with demand factors gaining importance only later on. The NAWM suggests that the subsequent downturn starting in the second half of 2000 was triggered by adverse influences from abroad. For instance, the emergence of new competitors in euro area export markets eventually led to losses in export market shares. Moreover, the sharp deceleration of economic activity in the United States and its spillovers to the rest of the world caused a pronounced fall in euro area foreign demand from 2001 onwards. Within the NAWM, these developments are captured by negative export preference and negative foreign demand shocks, respectively.

The subdued growth of real GDP over the period 2002-2005 is largely explained by negative demand shocks, notably domestic risk premium shocks, that entailed a protracted slump in domestic spending. Throughout this period, monetary policy shocks (i.e., unanticipated deviations of the short-term nominal interest rate from the prescriptions of the estimated interest-rate rule) supported domestic demand and prevented a stronger slowing of real GDP growth. Since 2003 the overall contribution of the foreign shocks has been rather modest. This masks the fact that in 2003 the adverse impact of external risk premium shocks (accounting for the marked appreciation of the euro) was largely offset by the unwinding of the previous shocks to export preferences. In contrast, the favourable developments in euro area foreign demand during the 2004-2006 period have been largely compensated by the continued appreciation of the euro and a renewed deterioration of foreign preferences for euro area exports. Interestingly, compared to the expansion in the initial years of EMU, the pickup in 2006 seems at least partly demand-led.

Decomposing the contribution of the technology shock group reveals that positive investment-specific technology shocks, which account for the period of strong investment growth from 2002 onwards, have gradually offset, by means of capital deepening, the adverse effects on total factor productivity of negative transitory technology shocks. The contribution of the permanent technology shocks has been positive on average, albeit small, which is in line with the subdued developments in trend labour productivity over recent years. Similarly, decomposing the contribution of the markup shock group shows that negative wage markup shocks, which capture the protracted period of moderate wage settlements since the mid 1990s, have had a favourable impact on real GDP growth throughout the EMU period. However, from 2002 onwards this impact has been gradually overturned by the adverse effects of a sequence of positive price markup shocks. These shocks account for idiosyncratic factors pushing up firms' production costs and hence prices, but they may also reflect the strengthening of firms' market power over recent years that allowed passing on cost increases to a larger extent.

5.2 Counterfactual Scenarios

The NAWM permits conducting counterfactual scenarios in order to assess the consequences of assuming alternative paths for selected structural shocks or, more directly, for a subset of its observed variables. Here, we concentrate on the second type of scenario and examine the economic impact of higher nominal wage growth as has been repeatedly called for in the public debate in view of the moderate wage settlements over recent years.

As shown in the upper left-hand panel in **Figure 14**, year-on-year nominal wage growth in the euro area has been subdued throughout the EMU period and well below the NAWM's steady-state growth rate for wages of 3.1 percent per annum (corresponding to 1.2 percent trend labour productivity growth plus 1.9 percent steady-state inflation). Moreover, the average rate of wage growth of 2.7 percent recorded over the 1999-2002 period has decelerated further to 2.1 percent in the period 2003-2006. Against this background, we consider a counterfactual scenario according to which average nominal wage growth in the latter period is shifted upward to the average rate of wage growth registered for the former period, while maintaining the dynamic profile of the year-on-year growth rates. Of course, the impact of higher wage growth on economic activity and inflation depends crucially on the underlying sources, such as increases in labour productivity, which would leave unit labour costs unaffected, or autonomous increases in wage settlements, which would push up labour costs. Here, we are concerned with the second case and assume that the rise in wage growth is caused by a sequence of exogenous wage markup shocks.[54]

[54] See Appendix Figure A.5 for details on the propagation of wage markup shocks within the NAWM.

The remaining panels of Figure 14 show the outcomes of the counterfactual wage scenario, focusing on employment, year-on-year real GDP growth and year-on-year consumer price inflation. As the higher level of wage growth implies a sustained increase in labour costs, employment falls cumulatively below its historical level. At the same time, since the wage rate is the main determinant of firms' marginal cost and as firms are setting their prices as a markup on the latter, the sustained increase in labour costs causes domestic prices to drift upward. This in turn translates into a noticeable and protracted increase in consumer price inflation. With nominal interest rates being set according to the estimated policy rule, monetary policy is gradually tightened in an attempt to counteract the surge in inflation, thereby curbing domestic demand. Moreover, the induced appreciation of the euro reduces the euro area's external price competitiveness and, thus, export demand. Overall, the fall in domestic and export demand results in a prolonged slowing of real GDP growth. In this respect, the wage scenario complements the historical decomposition of real GDP growth in the previous subsection which showed that negative wage markup shocks have contributed positively to real GDP growth throughout the EMU period.

Figure 14 also depicts 90 percent uncertainty bands for the scenario outcomes that reflect the uncertainty associated with the estimation of the NAWM's structural parameters.[55] As can be seen, parameter uncertainty has only a limited bearing on the scenario outcomes, corroborating the model-based finding that higher wage settlements, unrelated to productivity, would have a negative impact on real GDP growth, while pushing up inflation.

5.3 Mean and Interval Predictions

One important advantage of a Bayesian approach to forecasting is that it permits computing predictive densities for the observed variables used in the estimation of the NAWM. From these densities, it is straightforward to calculate both point (e.g., mean) and interval predictions, with the latter providing an indication of the overall degree of forecast uncertainty.

[55] By construction, the scenario analysis disregards the uncertainty originating in the structural shocks, the measurement errors and the unobserved states of the NAWM. These additional sources of uncertainty would need to be taken into consideration in a model-based forecast exercise.

In our application we concentrate on year-on-year real GDP growth, year-on-year consumer price inflation and the annual short-term nominal interest rate. In **Figure 15** we have plotted the unconditional mean predictions and the unconditional equal-tail 70 percent prediction intervals for the prediction sample beginning in 2001Q1.[56] The CEPR Business Cycle Dating Committee (2003) concluded that the euro area had essentially stagnated since 2001Q1, even though the downturn thereafter was not regarded as a recession. However, it can be seen that euro area real GDP growth had already started to decline prior to early 2001 (the peak is located in 2000Q2). Based on our data, the NAWM predicts that real GDP growth will continue to decelerate, albeit gradually, over the full prediction sample until the end of 2002. While the ex-post realisation of real GDP growth is well covered by the prediction interval, the mean prediction overstates GDP growth throughout the prediction sample and misses its trough in 2002Q1 by a noticeable margin.[57] As regards consumer price inflation, the mean prediction tracks actual inflation developments fairly well. In contrast, the interest rate is overpredicted throughout the sample.

Figure 15 also shows conditional predictions that are computed under the assumption that the actual paths for the NAWM's foreign variables, including foreign demand, are known. While this assumption is unrealistic in the sense that the *ex post* realisations of the foreign variables would not be available in a real-time forecasting exercise, it conforms with the practice of the ECB's macroeconomic projections, which are conditioned on technical assumptions regarding external developments. In our application, we assume that the conditioning assumption satisfies *hard conditions* (a particular path) rather than *soft conditions* (a range for the path). Following Leeper and Zha (2003), it is also assumed that particular shocks—in our case, the shocks of the model's SVAR block—are manipulated to ensure that the conditioning information is met by the predictions.[58] For real GDP growth it is noteworthy that the mean prediction, along with the prediction interval, shifts downward relative to the unconditional prediction over the remainder of 2001, before slightly picking up. This is

[56]The predictions have been made out of sample, using the 2000Q4 vintage of the model.

[57]It is worth noting that the prediction intervals are relatively wide. This is a common finding for prediction intervals based on DSGE models and relates mainly to the uncertainty associated with the structural shocks (and, to a lesser extent, measurement errors), as opposed to parameter or state uncertainty.

[58]For alternative approaches to conditional forecasting, see Waggoner and Zha (1999), and Robertson, Tallman and Whiteman (2005).

line with the findings of the historical decomposition of real GDP growth in Subsection 5.1, which identified a decline in foreign demand as an important factor contributing to the downturn in 2001.[59] The conditioning information implies also a downward shift of the predictions for consumer price inflation and the nominal interest rate.

5.4 Prediction Events

Another advantage of a Bayesian approach to forecasting is that we can calculate probabilities of certain events over the prediction sample. For example, we may want to know the probability that the euro area economy is going into a recession. Similarly, we may want to learn about the probability that inflation at some point in time is greater than, say, 2 percent. The predictive densities that are obtained from the NAWM make it straightforward to calculate the probabilities of such prediction events.

In our application we are concerned with 3 different prediction events for 2 observed variables. First, we define a recession as the case when year-on-year real GDP growth is negative for at least 3 consecutive quarters. Second, we consider the event that year-on-year consumer price inflation lies between 0 and 2 percent; and third, that inflation falls below 0 percent. The empirical results for these events are summarised in **Figure 16**. The dates on the horizontal axis refer to the first forecast period of rolling out-of-sample predictions that extend up to 8 quarters into the future, with the NAWM being re-estimated in the fourth quarter of each calendar year.

It is noteworthy that the prediction event probabilities for the economy going into a recession (see the left-hand panel of Figure 16) are elevated over the period 2000Q2-2002Q1, in line with the previous observation that real GDP growth had decelerated over this period. At the same time, the probabilities that year-on-year consumer price inflation lies between 0 and 2 percent are roughly 40 percent during this period, while the probabilities of deflation are no more than 20 percent (see, respectively, the blue and the red solid line in the right-hand panel of the figure). Hence, the probability of consumer price inflation rising above 2 percent is yet regarded as quite high.

[59] The decomposition also revealed that, from 2002Q1 onwards, the continued weakness of real GDP growth is caused by a slump in domestic demand, which is accounted for by domestic risk premium shocks.

In connection with the protracted period of negative inflation rates in Japan, there was heightened concern for deflationary risks also in the United States and in Europe. From an ex post perspective, however, this concern seems not warranted for the euro area. Figure 16 shows that, during the 2003-2004 period, the deflation probabilities for consumer prices are hardly rising. In contrast, the probability of deflation exceeds 40 percent in the first half of 1999, reflecting the rather low rates of inflation in the run up to EMU.

6 Conclusion

In this paper, we have outlined the specification of an estimated version of the NAWM which is designed for regular use in the ECB/Eurosystem staff projections. This version of the model incorporates a relatively high degree of detail and comes close to meeting our objective of providing a comprehensive set of core projection variables. At the same time, it allows conditioning forecasts on monetary, fiscal and external assumptions which form a key element of the projections exercises.

We have presented estimation results obtained by employing Bayesian methods and examined the empirical properties of the model by studying its impulse-response functions and forecast-error-variance decompositions, by inspecting the implied sample moments, and by evaluating its relative forecasting performance. Overall, the estimated NAWM is found to have economically plausible properties, especially with regard to the propagation of key economic shocks and the identification of the main sources of economic fluctuations. Furthermore, in terms of forecasting ability, the NAWM seems to fare quite well compared to a BVAR model as well as naïve benchmarks. Finally, several applications illustrated that the NAWM can make potentially useful contributions to forecasting and policy analysis, including the assessment of uncertainties and risks.

Yet, it is important to note that the estimated version of the NAWM will be subject to further refinements in the light of the practical experience that will be gained over time regarding its use in the projections. Moreover, since the current version of the model maintains a number of simplifying assumptions, notably that the euro area does not influence its external environment and that Ricardian equivalence holds, we will explore the possibility of extending the estimated model along these dimensions in the future. Other possible extensions concern the addition of financial frictions that go beyond the existence of a fraction of households with limited ability to participate in asset markets, as incorporated in the calibrated version of the NAWM, and the specification of a more realistic labour market with an explicit role for unemployment.[60]

[60]Regarding first steps towards extending the NAWM's labour market set up, see Christoffel, Kuester and Linzert (2008), who estimate a model of the euro area with search and matching frictions. Possible directions for incorporating financial frictions are explored in Lombardo and McAdam (2008).

References

Adolfson, M., M. K. Andersson, J. Lindé, M. Villani and A. Vredin, 2007, "Modern Forecasting Models in Action: Improving Macroeconomic Analysis at Central Banks", *International Journal of Central Banking*, 3, 111-144.

Adolfson, M., S. Laséen, J. Lindé and M. Villani, 2007, "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through", *Journal of International Economics*, 72, 481-511.

Adolfson, M., J. Lindé and M. Villani, 2007, "Forecasting Performance of an Open Economy DSGE Model", *Econometric Reviews*, 26, 289-328.

An, S. and F. Schorfheide, 2007, "Bayesian Analysis of DSGE Models", *Econometric Reviews*, 26, 113-172.

Bayoumi, T., D. Laxton and P. Pesenti, 2004, "Benefits and Spillovers of Greater Competition in Europe: A Macroeconomic Assessment", ECB Working Paper No. 341, European Central Bank, April.

Betts, C. and M. B. Devereux, 1996, "The Exchange Rate in a Model of Pricing-to-Market", *European Economic Review*, 40, 1007-1021.

Brooks, S. P. and A. Gelman, 1998, "General Methods for Monitoring Convergence of Iterative Simulations", *Journal of Computational and Graphical Statistics*, 7, 434-455.

Calvo, G. A., 1983, "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, 12, 383-398.

CEPR, 2003, *Findings of the Business Cycle Dating Committee of the Centre for Economic Policy Research*, September, available at http://cepr.org/data/dating.

Christiano, L. J., M. Eichenbaum and C. L. Evans, 2005, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, 113, 1-45.

Christoffel, K., K. Kuester and T. Linzert, 2008, "The Role of Labor Markets for Euro Area Monetary Policy", mimeo, European Central Bank, June.

Coenen, G., A. T. Levin and K. Christoffel, 2007, "Identifying the Influences of Nominal and Real Rigidities in Aggregate Price-Setting Behavior", *Journal of Monetary Economics*, 54, 2439-2466.

Coenen, G. and P. McAdam, 2006, "How Do VAT Changes Affect the Economy? An Illustration Using the New Area-Wide Model", Research Bulletin No. 4, DG Research, European Central Bank, April.

Coenen, G., P. McAdam and R. Straub, 2007, "Tax Reform and Labour-Market Performance in the Euro Area: A Simulation-Based Analysis Using the New Area-Wide Model", ECB Working Paper No. 747, April, forthcoming in the *Journal of Economic Dynamics and Control*.

Coenen, G., M. Mohr and R. Straub, 2008, "Fiscal Consolidation in the Euro Area: Long-Run Benefits and Short-Run Costs", ECB Working Paper No. 902, May, forthcoming in *Economic Modelling*.

Del Negro, M., F. Schorfheide, F. Smets and R. Wouters, 2007, "On the Fit and Forecasting Performance of New-Keynesian Models", *Journal of Business and Economics Statistics*, 25, 123-143.

Dieppe, A. and T. Warmedinger, 2007, "Modelling Intra- and Extra-Area Trade Substitution and Exchange Rate Pass-Through in the Euro Area", ECB Working Paper No. 760, European Central Bank, June.

ECB, 2001, "A Guide to Eurosystem Staff Macroeconomic Projection Exercises", European Central Bank, June.

Edge, R., K. Kiley and J. P. Laforte, 2008, "A Comparison of Forecast Performance between Federal Reserve Staff Forecasts, Simple Reduced-Form Models, and a DSGE Model", presented at the NBER Summer Institute, Boston, 10-11 July.

Eichenbaum, M. and J. D. M. Fisher, 2007, "Estimating the Frequency of Price Re-Optimization in Calvo-Style Models", *Journal of Monetary Economics*, 54, 2032-2047.

Erceg, C. J., L. Guerrieri and C. Gust, 2006, "SIGMA: A New Open Economy Model for Policy Analysis", *International Journal of Central Banking*, 2, 1-50.

Eurostat, 2006, "Structures of the Taxation Systems in the European Union, 1995-2004", European Commission, Brussels.

Fagan, G., J. Henry, and R. Mestre, 2001, "An Area-Wide Model (AWM) for the Euro Area", ECB Working Paper No. 42, European Central Bank, January.

Galí, J., and M. Gertler, 2007, "Macroeconomic Modeling for Monetary Policy Evaluation", *Journal of Economic Perspectives*, 21, 25-45.

Geweke, J. F., 1999, "Using Simulation Methods for Bayesian Econometric Models: Inference, Developments and Communication", *Econometric Reviews*, 18, 1-126.

Jean, S. and G. Nicoletti, 2002, "Product Market Regulation and Wage Premia in Europe and North America", OECD Economic Department Working Paper ECO/WKP(2002)4, Organisation for Economic Co-operation and Development.

Leeper, E. M. and T. Zha, 2003, "Modest Policy Interventions", *Journal of Monetary Economics*, 50, 1673-1700.

Lombardo, G. and P. McAdam, 2008, "Adding Financial Market Frictions to the NAWM", mimeo, European Central Bank, June.

Martins, J. O., S. Scarpetta and D. Pilat, 1996, "Mark-up Pricing, Market Structure and the Business Cycle", *OECD Economic Studies*, 27, 71-106.

OECD, 2004, "Taxing Wages 2003-2004", OECD Statistics, Organisation for Economic Co-operation and Development.

Robertson, J. C., E. W. Tallman and C. H. Whiteman, 2005, "Forecasting Using Relative Entropy", *Journal of Money, Credit, and Banking*, 37, 383-401.

Schorfheide, F., 2000, "Loss-Function-Based Evaluation of DSGE Models", *Journal of Applied Econometrics*, 15, 645-670.

Smets, F. and R. Wouters, 2003, "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area", *Journal of the European Economic Association*, 1, 1123-1175.

Smets, F. and R. Wouters, 2004, "Forecasting with a Bayesian DSGE Model: An Application to the Euro Area", *Journal of Common Market Studies*, 42, 841-867.

Smets, F. and R. Wouters, 2007, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach", *American Economic Review*, 97, 586-607.

Statistics Netherlands, 2006, "Attributing the Euro Area GDP Growth Rate to Final Demand Components", Report, July.

Villani, M., 2005, "Inference in Vector Autoregressive Models with an Informative Prior on the Steady State", Sveriges Riksbank Working Paper Series, No. 181, March.

Waggoner, D. F. and T. Zha, 1999, "Conditional Forecasts in Dynamic Multivariate Models", *Review of Economics and Statistics*, 81, 639-651.

Warne, A., 2008, "YADA Manual — Computational Details", mimeo, European Central Bank, June.

Woodford, M., 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton, Princeton University Press.

Appendix A: The Log-Linearised Model

In this appendix we provide details on the derivation of the log-linear version of the NAWM. The exposition follows closely the structure of Section 2 in the main text, which outlines the model specification in non-linear form.

Transformation of Variables

We start by casting the NAWM's structural relationships into stationary form. The reason is twofold: First, because of the assumed unit-root process underlying the evolution of productivity, and consistent with the balanced-growth property of the model, all real variables, with the exception of hours worked, have a *real* stochastic trend in common. And second, as the monetary authority aims at stabilising inflation, rather than the price level, all nominal variables share a *nominal* stochastic trend. To render the model stationary, we therefore scale all variables that share the common real trend with the level of productivity, z_t, while we divide all nominal variables by the price of the consumption good, $P_{C,t}$. In order to simplify the notation, we introduce the convention that all transformed variables are represented by lower-case letters, instead of the upper-case letters employed for the original variables. For example, we use $y_t = Y_t/z_t$ to denote the stationary level of aggregate output, while we use $p_{I,t} = P_{I,t}/P_{C,t}$ to represent the price of the investment good relative to the price of the consumption good.

There are, however, a few exceptions from these conventions that are noteworthy. First, since the nominal wage rate is assumed to grow in line with productivity, it not only needs to be transformed with the price of the consumption good, $P_{C,t}$, but also with the productivity level, z_t, in order to become stationary; and accordingly we define $w_t = W_t/(z_t\, P_{C,t})$. Second, as the model's endogenous state variables, such as the capital stock, are predetermined in a given period t, they will be scaled with the lagged value of productivity; that is, $k_t = K_t/z_{t-1}$. Third, the marginal utility of consumption needs to be *scaled up* with the level of productivity to become stationary; and hence we define $\lambda_t = z_t \Lambda_t$. Finally, we scale the foreign real variables with the productivity trend prevailing abroad, z_t^*, while maintaining the assumption that z_t and z_t^* share the same stochastic trend. Thus, we treat z_t/z_t^* as a stationary process that captures the degree of asymmetry in productivity developments in the domestic versus the foreign economy.

Log-Linearisation around the Non-Stochastic Steady State

After having made the necessary stationary-inducing transformations, we proceed with the log-linearisation of the NAWM around its non-stochastic steady state.[61] We indicate the logarithmic deviation of a variable from its steady-state value by a hat ('$\widehat{\ }$') and define the latter implicitly by dropping the time subscript t. For example, the log-deviation from steady state for the scaled output variable is $\widehat{y}_t = \log(y_t/y)$.

[61]See Appendix B for the derivation of the NAWM's non-stochastic steady state.

A.1 Households

In the following we present the log-linearised first-order conditions characterising the households' optimal choice of allocations and state the log-linear wage Phillips curve which is derived from the first-order condition describing the households' optimal wage-setting decision and the law of motion for the aggregate wage index. Since all households make identical decisions in equilibrium, the household-specific index h can be dropped.

The Households' Choice of Allocations

Applying the before-mentioned stationarity-inducing transformations to the households' first-order conditions (7) to (12) yields:

$$\lambda_t = \epsilon_t^C \frac{(c_t - \kappa\, g_{z,t}^{-1}\, c_{t-1})^{-1}}{1+\tau_t^C}, \tag{A.1}$$

$$\begin{aligned} p_{I,t} &= Q_t\, \epsilon_t^I \left(1 - \Gamma_I(g_{z,t}\, i_t/i_{t-1}) - \Gamma_I'(g_{z,t}\, i_t/i_{t-1})\, g_{z,t}\, \frac{i_t}{i_{t-1}}\right) \\ &\quad + \beta\, \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}\, Q_{t+1}\, \epsilon_{t+1}^I\, \Gamma_I'(g_{z,t+1}\, i_{t+1}/i_t)\, g_{z,t+1}\, \frac{i_{t+1}^2}{i_t^2}\right], \end{aligned} \tag{A.2}$$

$$\begin{aligned} Q_t &= \beta\, \mathrm{E}_t \Bigg[\frac{\lambda_{t+1}}{\lambda_t}\, g_{z,t+1}^{-1} \Big((1-\delta)\, Q_{t+1} \\ &\quad + (1-\tau_{t+1}^K)\, r_{K,t+1}\, u_{t+1} + \left(\tau_{t+1}^K\, \delta - (1-\tau_{t+1}^K)\, \Gamma_u(u_{t+1})\right) p_{I,t+1}\Big)\Bigg], \end{aligned} \tag{A.3}$$

$$r_{K,t} = \Gamma_u'(u_t)\, p_{I,t}, \tag{A.4}$$

$$\beta\, \epsilon_t^{RP} R_t\, \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}\, g_{z,t+1}^{-1}\, \Pi_{C,t+1}^{-1}\right] = 1, \tag{A.5}$$

$$\beta\, (1 - \Gamma_{B^*}(s_{B^*,t+1}; \epsilon_t^{RP^*}))\, R_t^*\, \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}\, g_{z,t+1}^{-1}\, \Pi_{C,t+1}^{-1}\, \frac{s_{t+1}}{s_t}\, \frac{\Pi_{Y,t+1}}{\Pi_{Y,t+1}^*}\right] = 1, \tag{A.6}$$

where $p_{I,t} = P_{I,t}/P_{C,t}$ is the relative price of the investment good, $r_{K,t} = R_{K,t}/P_{C,t}$ is the real rental rate of capital, $s_t = S_t\, P_{Y,t}^*/P_{Y,t}$ denotes the real exchange rate defined in terms of the domestic and foreign output deflators, $P_{Y,t}$ and $P_{Y,t}^*$, whereas $\Pi_{Y,t} = P_{Y,t}/P_{Y,t-1}$ and $\Pi_{Y,t}^* = P_{Y,t}^*/P_{Y,t-1}^*$ represent the respective (gross) inflation rates.

Similarly, the transformation of the capital accumulation equation (4) yields:

$$k_{t+1} = (1-\delta)\, g_{z,t}^{-1}\, k_t + \epsilon_t^I\, (1 - \Gamma_I(g_{z,t}\, i_t/i_{t-1}))\, i_t, \tag{A.7}$$

where we recall that $k_t = K_t/z_{t-1}$, as the physical capital stock is a predetermined state variable.

Noting that the financial intermediation premium (3) and the capital utilisation cost function (6) are already expressed in stationary form, the transformation of the adjustment cost function (5) yields:

$$\Gamma_I(g_{z,t}\, i_t/i_{t-1}) \;=\; \frac{\gamma_I}{2}\left(g_{z,t}\,\frac{i_t}{i_{t-1}} - g_z\right)^2 . \tag{A.8}$$

After some straightforward algebra and noting that $\Pi_C = \Pi_Y = \bar{\Pi}$, we obtain the following log-linearised expressions for the transformed first-order conditions (A.1) to (A.6):

$$\widehat{\lambda}_t \;=\; -\frac{1}{1-\kappa\, g_z^{-1}}\,\widehat{c}_t + \frac{\kappa\, g_z^{-1}}{1-\kappa\, g_z^{-1}}\,\widehat{c}_{t-1} - \frac{\kappa\, g_z^{-1}}{1-\kappa\, g_z^{-1}}\,\widehat{g}_{z,t} - \frac{1}{1+\tau^C}\,\widehat{\tau}_t^c + \widehat{\epsilon}_t^C, \tag{A.9}$$

$$\begin{aligned}\widehat{p}_{I,t} \;=\; \widehat{Q}_t + \widehat{\epsilon}_t^I + \gamma_I\, g_z^2\Big(\beta\left(\mathrm{E}_t\left[\widehat{i}_{t+1}\right] - \widehat{i}_t\right) \\ - \left(\widehat{i}_t - \widehat{i}_{t-1}\right) + \beta\,\mathrm{E}_t\left[\widehat{g}_{z,t+1}\right] - \widehat{g}_{z,t}\Big),\end{aligned} \tag{A.10}$$

$$\begin{aligned}\widehat{Q}_t \;=\; &\frac{\beta\,(1-\delta)}{g_z}\,\mathrm{E}_t\left[\widehat{Q}_{t+1}\right] + \mathrm{E}_t\left[\widehat{\lambda}_{t+1}\right] - \widehat{\lambda}_t - \mathrm{E}_t\left[\widehat{g}_{z,t+1}\right] \\ &- \frac{\beta\,(1-\tau^K)\,r_K}{g_z}\,\mathrm{E}_t\left[\frac{1}{1-\tau^K}\,\widehat{\tau}_{t+1}^K - \widehat{r}_{K,t+1}\right] + \frac{\beta\,\delta\,p_I}{g_z}\,\mathrm{E}_t\left[\tau^K\widehat{p}_{I,t+1} + \widehat{\tau}_{t+1}^K\right],\end{aligned} \tag{A.11}$$

$$\widehat{r}_{K,t} \;=\; \frac{\gamma_{u,2}}{\gamma_{u,1}}\,\widehat{u}_t + \widehat{p}_{I,t}, \tag{A.12}$$

$$\mathrm{E}_t\left[\widehat{\lambda}_{t+1}\right] - \widehat{\lambda}_t - \mathrm{E}_t\left[\widehat{g}_{z,t+1}\right] + \widehat{r}_t - \mathrm{E}_t\left[\widehat{\pi}_{C,t+1}\right] + \widehat{\epsilon}_t^{RP} \;=\; 0, \tag{A.13}$$

$$\begin{aligned}&\mathrm{E}_t\left[\widehat{\lambda}_{t+1}\right] - \widehat{\lambda}_t - \mathrm{E}_t\left[\widehat{g}_{z,t+1}\right] + \widehat{r}_t^* - \mathrm{E}_t\left[\widehat{\pi}_{C,t+1}\right] \\ &+ \mathrm{E}_t\left[\widehat{s}_{t+1}\right] - \widehat{s}_t + \mathrm{E}_t\left[\widehat{\pi}_{Y,t+1} - \widehat{\pi}_{Y,t+1}^*\right] - \gamma_{B^*}\,\widehat{s}_{B^*,t+1} - \widehat{\epsilon}_t^{RP^*} \;=\; 0,\end{aligned} \tag{A.14}$$

where we have assumed $\Pi_Y^* = \Pi_Y$ for deriving expression (A.14).

After substitution of the transformed adjustment cost function (A.8), the log-linearisation of the transformed capital accumulation equation (A.7) results in:

$$\begin{aligned}\widehat{k}_{t+1} \;=\; &(1-\delta)\,g_z^{-1}\,\widehat{k}_t - (1-\delta)\,g_z^{-1}\,\widehat{g}_{z,t} \\ &+ (1-(1-\delta)\,g_z^{-1})\,\widehat{\epsilon}_t^I + (1-(1-\delta)\,g_z^{-1})\,\widehat{i}_t.\end{aligned} \tag{A.15}$$

Note that the log-linearised first-order condition for the purchases of the investment good (A.10) can be re-written as an investment equation,

$$\begin{aligned}\widehat{i}_t \;=\; &\frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\widehat{i}_{t+1}\right] + \frac{1}{1+\beta}\,\widehat{i}_{t-1} + \frac{1}{\gamma_I\, g_z^2\,(1+\beta)}\left(\widehat{Q}_t - \widehat{p}_{I,t} + \widehat{\epsilon}_t^I\right) \\ &+ \frac{1}{1+\beta}\left(\beta\,\mathrm{E}_t\left[\widehat{g}_{z,t+1}\right] - \widehat{g}_{z,t}\right),\end{aligned} \tag{A.16}$$

while combining the log-linearised first-order conditions for the purchases of the consumption good (A.9) and the holdings of domestic government bonds (A.13) results in a log-linear consumption equation,

$$\begin{aligned}\widehat{c}_t \;=\;& \frac{1}{1+\kappa\, g_z^{-1}}\,\mathrm{E}_t\left[\widehat{c}_{t+1}\right] + \frac{\kappa\, g_z^{-1}}{1+\kappa\, g_z^{-1}}\,\widehat{c}_{t-1} - \frac{1-\kappa\, g_z^{-1}}{1+\kappa\, g_z^{-1}}\left(\widehat{r}_t - \mathrm{E}_t\left[\widehat{\pi}_{C,t+1}\right] + \widehat{\epsilon}_t^{RP}\right) \qquad \text{(A.17)}\\ & - \frac{1}{1+\kappa\, g_z^{-1}}\left(\mathrm{E}_t\left[\widehat{g}_{z,t+1}\right] - \kappa\, g_z^{-1}\,\widehat{g}_{z,t}\right) + \frac{1-\kappa\, g_z^{-1}}{\left(1+\kappa\, g_z^{-1}\right)\left(1+\tau^C\right)}\left(\mathrm{E}_t\left[\widehat{\tau}_{t+1}^C\right] - \widehat{\tau}_t^C\right)\\ & - \frac{1-\kappa\, g_z^{-1}}{1+\kappa\, g_z^{-1}}\left(\mathrm{E}_t\left[\widehat{\epsilon}_{t+1}^C\right] - \widehat{\epsilon}_t^C\right).\end{aligned}$$

Similarly, combining the log-linearised first-order conditions for the holdings of domestic and internationally traded bonds, (A.13) and (A.14), yields a risk-adjusted uncovered interest parity (UIP) condition,

$$\widehat{r}_t - \widehat{r}_t^* + \widehat{\epsilon}_t^{RP} \;=\; \mathrm{E}_t\left[\widehat{s}_{t+1}\right] - \widehat{s}_t + \mathrm{E}_t\left[\widehat{\pi}_{Y,t+1} - \widehat{\pi}_{Y,t+1}^*\right] - \gamma_{B^*}\,\widehat{s}_{B^*,t+1} - \widehat{\epsilon}_t^{RP^*}. \qquad \text{(A.18)}$$

Wage Setting and Aggregate Wage Dynamics

In the log-linear model, aggregate wage dynamics is determined by a wage Phillips curve. In stating the wage Phillips curve, we let $\widehat{\bar{\pi}}_t = \log(\bar{\Pi}_t/\bar{\Pi})$ denote the logarithmic deviation of the monetary authority's inflation objective from its long-run value and recall that $w_t = W_t/(z_t P_{C,t})$. With these definitions, combining the log-linearised first-order condition characterising the households' optimal wage-setting decision (14) and the log-linearised expression for the aggregate wage index (16) yields the following log-linear expression:[62]

$$\begin{aligned}\widehat{w}_t \;=\;& \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\widehat{w}_{t+1}\right] + \frac{1}{1+\beta}\,\widehat{w}_{t-1} + \frac{\beta}{1+\beta}\,\mathrm{E}_t\left[\widehat{\pi}_{C,t+1}\right] \qquad \text{(A.19)}\\ & - \frac{1+\beta\chi_W}{1+\beta}\,\widehat{\pi}_{C,t} + \frac{\chi_W}{1+\beta}\,\widehat{\pi}_{C,t-1} - \frac{\beta(1-\chi_W)}{1+\beta}\,\mathrm{E}_t\left[\widehat{\bar{\pi}}_{C,t+1}\right]\\ & + \frac{1-\chi_W}{1+\beta}\,\widehat{\bar{\pi}}_{C,t} - \frac{(1-\beta\xi_W)(1-\xi_W)}{(1+\beta)\,\xi_W\,\Psi(\varphi^W,\zeta)}\left(\widehat{w}_t^\tau - \widehat{mrs}_t - \widehat{\varphi}_t^W\right),\end{aligned}$$

where the terms

$$\begin{aligned}\widehat{w}_t^\tau \;&=\; -\frac{\widehat{\tau}_t^N + \widehat{\tau}_t^{W_h}}{1-\tau^N-\tau^{W_h}} + \widehat{w}_t,\\ \widehat{mrs}_t \;&=\; \widehat{\epsilon}_t^N + \zeta\,\widehat{N}_t - \widehat{\lambda}_t\end{aligned}$$

denote, respectively, the log-deviations of the tax and productivity-adjusted real wage and the households' marginal rate of substitution between consumption and leisure, and

$$\Psi(\varphi^W,\zeta) \;=\; 1 + \frac{\varphi^W}{\varphi^W - 1}\,\zeta.$$

[62]Details of the derivations are tedious, but straightforward, and available upon request.

A.2 Firms

In this sub-section, we derive the log-linear expressions summarising the technology constraints and the production and pricing decisions of the various domestic and foreign firms.

A.2.1 Domestic Intermediate-Good Firms

We start with the derivation of the log-linear equations characterising the domestic intermediate-good firms' production technology, the optimal combination of factor inputs and the marginal cost schedule. We then state the log-linear price Phillips curves determining domestic and export price inflation, respectively. The two Phillips curves are derived from the first-order conditions characterising the firms' optimal price-setting decisions and the law of motion for the respective aggregate price indexes. The firm-specific index f can be dropped because all firms make identical decisions in equilibrium.

Technology, Inputs and Marginal Cost

With the above-mentioned conventions, and assuming that production exceeds fixed costs, the stationarity-inducing transformation of the production technology (17) results in:

$$y_t = \varepsilon_t \, (g_{z,t}^{-1} \, k_t^s)^{\alpha} \, N_t^{1-\alpha} - \psi. \tag{A.20}$$

Similarly, the transformation of the combined first-order condition (22) yields:

$$\frac{\alpha}{1-\alpha} \, \frac{g_{z,t} \, N_t}{k_t^s} = \frac{r_{K,t}}{(1+\tau_t^{W_f}) \, w_t}, \tag{A.21}$$

while the transformed version of the marginal cost schedule (23) is:

$$mc_t = \frac{1}{\varepsilon_t \, \alpha^{\alpha} (1-\alpha)^{1-\alpha}} \, (r_{K,t})^{\alpha} ((1+\tau_t^{W_f}) \, w_t)^{1-\alpha}. \tag{A.22}$$

Hence, after some simple algebra, we obtain the following log-linearised expressions for the production technology (A.20), the first-order condition (A.21) and the marginal cost schedule (A.22):

$$\widehat{y}_t = (1+\psi \, y^{-1}) \left(\widehat{\varepsilon}_t + \alpha \, (\widehat{k}_t^s - \widehat{g}_{z,t}) + (1-\alpha) \, \widehat{N}_t \right), \tag{A.23}$$

$$\widehat{r}_{K,t} = \widehat{g}_{z,t} + \widehat{N}_t + (1+\tau^{W_f})^{-1} \, \widehat{\tau}_t^{W_f} + \widehat{w}_t - \widehat{k}_t^s, \tag{A.24}$$

$$\widehat{mc}_t = -\,\widehat{\varepsilon}_t + \alpha \, \widehat{r}_{K,t} + (1-\alpha) \left((1+\tau^{W_f})^{-1} \, \widehat{\tau}_t^{W_f} + \widehat{w}_t \right). \tag{A.25}$$

Price Setting and Aggregate Price Dynamics

For the domestic intermediate-good firms, we obtain two separate Phillips curves which determine domestic and export price inflation, respectively.[63]

[63] Details of the derivations are tedious, but straightforward, and available upon request.

Combining the log-linearised first-order condition (32) characterising the optimal price-setting decision of domestic firms that are selling their outputs in home markets with the log-linearised version of the corresponding aggregate price index (34) results in the following log-linear version of the domestic price Phillips curve:

$$\begin{aligned}(\widehat{\pi}_{H,t}-\widehat{\pi}_t) &= \frac{\beta}{1+\beta\chi_H}\,\mathrm{E}_t\left[\widehat{\pi}_{H,t+1}-\widehat{\pi}_{t+1}\right]+\frac{\chi_H}{1+\beta\chi_H}\,(\widehat{\pi}_{H,t-1}-\widehat{\pi}_t) \\ &\quad +\frac{\beta\chi_H}{1+\beta\chi_H}\,(\mathrm{E}_t\left[\widehat{\pi}_{t+1}-\widehat{\pi}_t\right])+\frac{(1-\beta\xi_H)(1-\xi_H)}{\xi_H(1+\beta\chi_H)}\left(\widehat{mc}_t^H+\widehat{\varphi}_t^H\right).\end{aligned} \tag{A.26}$$

Here, $\widehat{mc}_t^H=\widehat{mc}_t-\widehat{p}_{H,t}$ represents the average real marginal cost of the domestic intermediate-good firms selling in domestic markets (expressed as logarithmic deviation from its steady-state value) with $mc_t=MC_t/P_{C,t}$ and $p_{H,t}=P_{H,t}/P_{C,t}$.

Similarly, combining the log-linearised first-order condition (33) with the log-linearised version of the aggregate export price index (35) results in the following log-linear export price Phillips curve:

$$\begin{aligned}(\widehat{\pi}_{X,t}-\widehat{\pi}_t) &= \frac{\beta}{1+\beta\chi_X}\,\mathrm{E}_t\left[\widehat{\pi}_{X,t+1}-\widehat{\pi}_{t+1}\right]+\frac{\chi_X}{1+\beta\chi_X}\,(\widehat{\pi}_{X,t-1}-\widehat{\pi}_t) \\ &\quad +\frac{\beta\chi_X}{1+\beta\chi_X}\,(\mathrm{E}_t\left[\widehat{\pi}_{t+1}-\widehat{\pi}_t\right])+\frac{(1-\beta\xi_X)(1-\xi_X)}{\xi_X(1+\beta\chi_X)}\left(\widehat{mc}_t^X+\widehat{\varphi}_t^X\right),\end{aligned} \tag{A.27}$$

where $\widehat{mc}_t^X=\widehat{mc}_t-\widehat{p}_{X,t}$ represents the average real marginal cost of the domestic intermediate-good firms selling in foreign markets (expressed as logarithmic deviation from its steady-state value) with $p_{X,t}=P_{X,t}/P_{C,t}$.

A.2.2 Foreign Intermediate-Good Firms

Like in the case of the domestic intermediate-good firms, the price-setting decision of the foreign intermediate-good firms and the dynamics of the aggregate import price index can be summarised by a Phillips-curve relationship. The firm-specific index f^* can be dropped because all firms make identical decisions in equilibrium.

In particular, combining the log-linearised first-order condition for the price-setting decision of the foreign intermediate-good firms (39) with the log-linearised version of the corresponding aggregate import price index (40) results in the following log-linear import price Phillips curve:

$$\begin{aligned}(\widehat{\pi}_{IM,t}-\widehat{\pi}_t) &= \frac{\beta^*}{1+\beta^*\chi^*}\,\mathrm{E}_t\left[\widehat{\pi}_{IM,t+1}-\widehat{\pi}_{t+1}\right]+\frac{\chi^*}{1+\beta^*\chi^*}\,(\widehat{\pi}_{IM,t-1}-\widehat{\pi}_t) \\ &\quad +\frac{\beta^*\chi^*}{1+\beta^*\chi^*}\,(\mathrm{E}_t\left[\widehat{\pi}_{t+1}-\widehat{\pi}_t\right])+\frac{(1-\beta^*\xi^*)(1-\xi^*)}{\xi^*(1+\beta^*\chi^*)}\,(\widehat{mc}_t^*+\widehat{\varphi}_t^*),\end{aligned} \tag{A.28}$$

where $\widehat{mc}_t^*=\widehat{s}_t+\widehat{p}_{Y,t}-\widehat{p}_{IM,t}+\omega^*\,\widehat{p}_{O,t}$ represents the average real marginal cost of the foreign intermediate-good firms (expressed as logarithmic deviation from its steady-state value) with $p_{IM,t}=P_{IM,t}/P_{C,t}$ and $p_{O,t}=P_{O,t}/P_{Y,t}^*$.

A.2.3 Domestic Final-Good Firms

Here we present the log-linearised expressions for the final-good firms' production technologies, along with the optimal demand schedules for the input bundles and the aggregate price index minimising production cost. Without loss of generality, we focus on the consumption good, noting that similar expressions are obtained for the investment good.

The stationarity-inducing transformations of the consumption-good technology (41) and the demand schedules (49) and (50) yield:

$$q_t^C = \left(\nu_{C,t}^{\frac{1}{\mu_C}} \left(h_t^C\right)^{1-\frac{1}{\mu_C}} + (1-\nu_{C,t})^{\frac{1}{\mu}} \left(\left(1-\Gamma_{IM^C}(im_t^C/q_t^C;\epsilon_t^{IM})\right) im_t^C\right)^{1-\frac{1}{\mu_C}}\right)^{\frac{\mu_C}{\mu_C-1}} \tag{A.29}$$

and

$$h_t^C = \nu_{C,t} \left(\frac{p_{H,t}}{p_{C,t}}\right)^{-\mu_C} q_t^C, \tag{A.30}$$

$$im_t^C = (1-\nu_{C,t}) \left(\frac{p_{IM,t}}{p_{C,t}\,\Gamma_{IM^C}^{\dagger}(im_t^C/q_t^C;\epsilon_t^{IM})}\right)^{-\mu_C} \frac{q_t}{1-\Gamma_{IM^C}(im_t^C/q_t^C;\epsilon_t^{IM})}, \tag{A.31}$$

where

$$\Gamma_{IM^C}^{\dagger}(im_t^C/q_t^C;\epsilon_t^{IM}) = 1-\Gamma_{IM^C}(im_t^C/q_t^C;\epsilon_t^{IM}) - \Gamma_{IM^C}'(im_t^C/q_t^C;\epsilon_t^{IM})\, im_t^C. \tag{A.32}$$

Similarly, the transformation of the price index for the final consumption good (51) results in

$$p_{C,t} = \left(\nu_{C,t}\,(p_{H,t})^{1-\mu_C} + (1-\nu_{C,t}) \left(\frac{p_{IM,t}}{\Gamma_{IM^C}^{\dagger}(im_t^C/q_t^C;\epsilon_t^{IM})}\right)^{1-\mu_C}\right)^{\frac{1}{1-\mu_C}}. \tag{A.33}$$

We arrive at the following log-linearised expressions for the transformed final-good technology (A.29) and the transformed demand schedules (A.30) and (A.31):

$$\begin{aligned} \widehat{q}_t^C = {} & \nu_C^{\frac{1}{\mu_C}} \left(\frac{h^C}{q^C}\right)^{1-\frac{1}{\mu_C}} \widehat{h}_t^C + (1-\nu_C)^{\frac{1}{\mu_C}} \left(\frac{im^C}{q^C}\right)^{1-\frac{1}{\mu_C}} \widehat{im}_t^C \\ & + \frac{1}{\mu_C-1} \left(\nu_C^{\frac{1}{\mu_C}} \left(\frac{h^C}{q^C}\right)^{1-\frac{1}{\mu_C}} - \frac{\nu_C}{1-\nu_C} (1-\nu_C)^{\frac{1}{\mu_C}} \left(\frac{im^C}{q^C}\right)^{1-\frac{1}{\mu_C}}\right) \widehat{\nu}_{C,t} \end{aligned} \tag{A.34}$$

and

$$\widehat{h}_t^C = \widehat{\nu}_{C,t} - \mu_C\,(\widehat{p}_{H,t} - \widehat{p}_{C,t}) + \widehat{q}_t^C, \tag{A.35}$$

$$\widehat{im}_t^C = -\frac{\nu_C}{1-\nu_C}\widehat{\nu}_{C,t} - \mu_C \left(\widehat{p}_{IM,t} - \widehat{p}_{C,t} - \widehat{\Gamma}_{IM^C,t}^{\dagger}\right) + \widehat{q}_t^C, \tag{A.36}$$

where

$$\widehat{\Gamma}_{IM^C,t}^{\dagger} = -\gamma_{IM^C} \left(\left(\widehat{im}_t^C - \widehat{q}_t^C\right) - \left(\widehat{im}_{t-1}^C - \widehat{q}_{t-1}^C\right)\right) + \widehat{\epsilon}_t^{IM}. \tag{A.37}$$

Log-linearisation of the transformed price index (A.33) results in

$$\widehat{p}_{C,t} = \nu_C \left(\frac{p_H}{p_C}\right)^{1-\mu_C} \widehat{p}_{H,t} + (1-\nu_C)\left(\frac{p_{IM}}{p_C}\right)^{1-\mu_C} \left(\widehat{p}_{IM,t} - \widehat{\Gamma}^{\dagger}_{IM^C,t}\right) + \frac{\nu_C}{1-\mu_C}\left(\left(\frac{p_H}{p_C}\right)^{1-\mu_C} - \left(\frac{p_{IM}}{p_C}\right)^{1-\mu_C}\right)\widehat{\nu}_{C,t} \tag{A.38}$$

with $p_C = 1$ and $\widehat{p}_{C,t} = 0$ by definition.

With obvious changes in notation, we obtain similar expressions for the investment good—the quantity Q^I_t, the demand for domestic and imported intermediate-good bundles H^I_t and IM^I_t, the transformed adjustment cost $\Gamma^{\dagger}_{IM^I}(IM^I_t/Q^I_t;\epsilon^{IM}_t)$, and the price index $P_{I,t}$. In contrast, for the public consumption good, which only consists of domestic intermediate goods, we yield trivial expressions with $\widehat{q}^G_t = \widehat{h}^G_t$ and $\widehat{p}_{G,t} = \widehat{p}_{H,t}$.

Finally, aggregating over the three final-good firms, we get the following log-linear expressions for the demand for domestic and foreign intermediate-good bundles:

$$\widehat{h}_t = \frac{h^C}{h}\,\widehat{h}^C_t + \frac{h^I}{h}\,\widehat{h}^I_t + \frac{h^G}{h}\,\widehat{h}^G_t, \tag{A.39}$$

$$\widehat{im}_t = \frac{im^C}{im}\,\widehat{im}^C_t + \frac{im^I}{im}\,\widehat{im}^I_t. \tag{A.40}$$

A.2.4 Foreign Retail Firm

Regarding the foreign retail firm, which combines the differentiated domestic intermediate goods that are sold abroad, it is sufficient to derive a log-linearised expression for the export bundle X_t, which is determined by the export demand equation (61).

Scaling aggregate foreign demand Y^*_t by the productivity level prevailing abroad, z^*_t, and denoting the productivity differential between the foreign and the domestic country by $\tilde{z} = z^*_t/z_t$, the export demand equation can be rewritten as

$$x_t = \nu^*_t \left(\frac{p_{X,t}/(p_{Y,t}\,s_t)}{p^c_{X,t}\,\Gamma^{\dagger}_X(x_t/(y^*_t\tilde{z}_t);\epsilon^X_t)}\right)^{-\mu^*} \frac{y^*_t\,\tilde{z}_t}{1-\Gamma_X(x_t/(y^*_t\tilde{z}_t);\epsilon^X_t)} \tag{A.41}$$

with

$$\Gamma^{\dagger}_X(x_t/(y^*_t\tilde{z}_t);\epsilon^X_t) = 1 - \Gamma_X(x_t/(y^*_t\tilde{z}_t);\epsilon^X_t) - \Gamma'_X(x_t/(y^*_t\tilde{z}_t);\epsilon^X_t)\,x_t, \tag{A.42}$$

where $p^c_{X,t} = P^c_{X,t}/P^*_{Y,t}$.

Here, we maintain the assumption that z_t and z^*_t share the same stochastic trend. Thus, we treat $\tilde{z}_t$ as a stationary process capturing the degree of asymmetry in productivity developments in the domestic versus the foreign economy. Further, assuming $z_0 = z^*_0$ implies that the productivity levels are equal across economies in the non-stochastic steady state, with $\tilde{z} = 1$.

Hence, the log-linear expression for exports is given by

$$\widehat{x}_t = \widehat{\nu}_t^* - \mu^* \left(\widehat{p}_{X,t} - \widehat{p}_{Y,t} - \widehat{s}_t - \widehat{p}_{X,t}^c - \widehat{\Gamma}_{X,t}^\dagger\right) + \widehat{y}_t^* + \widehat{\widetilde{z}}_t \tag{A.43}$$

with

$$\widehat{\Gamma}_{X,t}^\dagger = -\gamma^* \left(\left(\widehat{x}_t - \widehat{y}_t^* - \widehat{\widetilde{z}}_t\right) - \left(\widehat{x}_{t-1} - \widehat{y}_{t-1}^* - \widehat{\widetilde{z}}_{t-1}\right)\right) + \widehat{\epsilon}_t^X. \tag{A.44}$$

We note that the log-linear export demand equation features three different types of shocks: the export preference shock, $\widehat{\nu}_t^*$, the export demand shock, $\widehat{\epsilon}_t^X$, and the productivity differential $\widehat{\widetilde{z}}_t$. As their effects are not separately identified, we attribute the joint impact of these shocks to the export preference shock.[64]

A.3 Fiscal and Monetary Authorities

Dividing the fiscal-authority's budget constraint (64) with nominal output and noting that the budget is closed by lump-sum taxes in each period with $B_t = 0$, we obtain

$$\begin{aligned} s_{G,t} &= \tau_t^C \frac{P_{C,t}\, C_t}{P_{Y,t}\, Y_t} + (\tau_t^N + \tau_t^{W_h} + \tau_t^{W_f}) \frac{W_t\, N_t}{P_{Y,t}\, Y_t} \\ &\quad + \tau_t^K\, u_t \frac{R_{K,t}\, K_t}{P_{Y,t}\, Y_t} - \tau_t^K \left(\Gamma_u(u_t) + \delta\right) \frac{P_{I,t}\, K_t}{P_{Y,t}\, Y_t} + \tau_t^D\, s_{D,t} + s_{T,t}, \end{aligned} \tag{A.45}$$

where we have used that $\int_0^1 W_{h,t}\, N_{h,t}\, dh = W_t\, N_t$.

Here, the fiscal authority's purchases of the final public consumption good and lump-sum taxes are specified as a share of nominal output, with $s_{G,t} = P_{G,t}\, G_t / P_{Y,t} Y_t$ and $s_{T,t} = T_t / P_{Y,t} Y_t$, respectively. Similarly, $s_{D,t} = D_t / P_{Y,t}\, Y_t$ denotes the domestic intermediate-good firms' profit share.

The fiscal authority's budget constraint (A.45) can be written in terms of stationary variables as:

$$\begin{aligned} s_{G,t} &= \tau_t^C \frac{p_{C,t}\, c_t}{p_{Y,t}\, y_t} + (\tau_t^N + \tau_t^{W_h} + \tau_t^{W_f}) \frac{w_t\, N_t}{p_{Y,t}\, y_t} \\ &\quad + \tau_t^K\, u_t \frac{r_{K,t}\, k_t\, g_{z,t}^{-1}}{p_{Y,t}\, y_t} - \tau_t^K \left(\Gamma_u(u_t) + \delta\right) \frac{p_{I,t}\, k_t\, g_{z,t}^{-1}}{p_{Y,t}\, y_t} + \tau_t^D\, s_{D,t} + s_{T,t}, \end{aligned} \tag{A.46}$$

and noting that $u = 1$, $\Gamma_u(1) = 0$ and $\Gamma_u'(1) = r_K\, p_I^{-1}$, we obtain the following log-linearised expression for the fiscal authority's transformed budget constraint (A.46):

$$\begin{aligned} \widehat{s}_{G,t} &= \frac{p_C\, c}{p_Y\, y} \left(\widehat{\tau}_t^C + \tau^C \left(\widehat{p}_{C,t} + \widehat{c}_t - \widehat{p}_{Y,t} - \widehat{y}_t\right)\right) \\ &\quad + \frac{w\, N}{p_Y\, y} \left(\widehat{\tau}_t^N + \widehat{\tau}_t^{W_h} + \widehat{\tau}_t^{W_f} + (\tau^N + \tau^{W_h} + \tau^{W_f}) \left(\widehat{w}_t + \widehat{N}_t - \widehat{p}_{Y,t} - \widehat{y}_t\right)\right) \\ &\quad + \frac{r_K\, k\, g_z^{-1}}{p_Y\, y} \left(\widehat{\tau}_t^K + \tau^K \left(\widehat{u}_t + \widehat{r}_{K,t} + \widehat{k}_t - \widehat{g}_{z,t} - \widehat{p}_{Y,t} - \widehat{y}_t\right)\right) \end{aligned} \tag{A.47}$$

[64] The productivity differential also enters the law of motion for the net foreign assets held by domestic households. However, after log-linearisation, the productivity terms cancel out each other (cf. Section A.5).

$$- \frac{p_I\, k\, g_z^{-1}}{p_Y\, y} \left(\delta\, \widehat{\tau}_t^K + \tau^K\, \gamma_{u,1}\, \widehat{u}_t + \tau^K\, \delta\, \left(\widehat{p}_{I,t} + \widehat{k}_t - \widehat{g}_{z,t} - \widehat{p}_{Y,t} - \widehat{y}_t \right) \right)$$

$$+ \tau^D\, \widehat{s}_{D,t} + s_D\, \widehat{\tau}_t^D + \widehat{s}_{T,t}.$$

The log-linearised government consumption share is given by

$$\widehat{s}_{G,t} \quad = \quad s_G\, \left(\widehat{p}_{G,t} + \widehat{g}_t - \widehat{p}_{Y,t} - \widehat{y}_t \right), \tag{A.48}$$

while the log-linearised expression for the profit share $\widehat{s}_{D,t}$ is provided by equation (A.57) below.

The behaviour of the monetary authority is already fully described by the simple log-linear interest-rate rule in the main text (cf. equation (65)).

A.4 Market Clearing and Aggregate Resource Constraint

Appropriate transformations and log-linearisation of the relevant market clearing conditions (72), (85), (86) and (87) yield:

$$\widehat{u}_t + \widehat{k}_t \quad = \quad \widehat{k}_t^s, \tag{A.49}$$

$$\widehat{q}_t^C \quad = \quad \widehat{c}_t, \tag{A.50}$$

$$\widehat{q}_t^I \quad = \quad \widehat{i}_t + r_K\, p_I^{-1}\, g_z^{-1}\, \frac{k}{q^I}\, \widehat{u}_t, \tag{A.51}$$

$$\widehat{q}_t^G \quad = \quad \widehat{g}_t, \tag{A.52}$$

where we recall that $\Gamma_u'(1) = r_K\, p_I^{-1}$.

Similarly, appropriate transformations and subsequent log-linearisation of the real and nominal versions of the aggregate resource constraint, (74) and (89), yield

$$\widehat{y}_t \quad = \quad \frac{h}{y}\, \widehat{h}_t + \frac{x}{y}\, \widehat{x}_t \tag{A.53}$$

and

$$\begin{aligned} \widehat{p}_{Y,t} + \widehat{y}_t \quad = \quad & \frac{p_C\, c}{p_Y\, y} \left(\widehat{p}_{C,t} + \widehat{c}_t \right) + \frac{p_I\, i}{p_Y\, y} \left(\widehat{p}_{I,t} + \widehat{i}_t \right) + \frac{p_I\, k\, g_z^{-1}}{p_Y\, y}\, \gamma_{u,1}\, \widehat{u}_t \\ & + \frac{p_G\, g}{p_Y\, y} \left(\widehat{p}_{G,t} + \widehat{g}_t \right) + \frac{p_X\, x}{p_Y\, y} \left(\widehat{p}_{X,t} + \widehat{x}_t \right) - \left(\frac{p_{IM}\, im^C}{p_Y\, y} \left(\widehat{p}_{IM,t} + \widehat{im}_t^C - \widehat{\Gamma}_{IM^C,t}^{\dagger} \right) \right. \\ & \left. + \frac{p_{IM}\, im^I}{p_Y\, y} \left(\widehat{p}_{IM,t} + \widehat{im}_t^I - \widehat{\Gamma}_{IM^I,t}^{\dagger} \right) \right), \end{aligned} \tag{A.54}$$

recalling that $p_C = 1$ and $\widehat{p}_{C,t} = 0$.

Finally, rewriting the profit share (81) in terms of stationary variables,

$$s_{D,t} \quad = \quad 1 - \frac{mc_t}{p_{Y,t}}\, \frac{\check{s}_{H,t}\, h_t + \check{s}_{X,t}\, x_t + \psi}{y_t}, \tag{A.55}$$

noting that in steady state

$$s_D \;=\; 1 - \frac{1}{\varphi}\,(1 + \psi\, y^{-1}) \tag{A.56}$$

with $\varphi = \varphi^H = \varphi^X = (mc/p_Y)^{-1}$ and recalling that the dispersion terms $\check{s}_{H,t}$ and $\check{s}_{X,t}$ equal zero up to first order (cf. Woodford, 2003), we arrive at the following log-linearised expression for the profit share:

$$\widehat{s}_{D,t} \;=\; -\frac{1}{\varphi}\,(1 + \psi\, y^{-1})\,(\widehat{mc}_t - \widehat{p}_{Y,t}) - \frac{1}{\varphi}\left(\frac{h}{y}\,\widehat{h}_t + \frac{x}{y}\,\widehat{x}_t - \frac{h + x + \psi}{y}\,\widehat{y}_t\right) \tag{A.57}$$

A.5 Net Foreign Assets, Trade Balance and Terms of Trade

For log-linearising the law of motion of the net foreign asset position (91), we substitute the definition of the trade balance (92) and scale the resulting expression by z_t^* and $P_{Y,t}^*$, yielding

$$(R_t^*)^{-1} b_{t+1}^* \;=\; b_t^*\,\tilde{z}_{t-1}\,\tilde{z}_t^{-1} g_{z,t}^{-1} (\Pi_{Y,t}^*)^{-1} + \frac{p_{X,t}}{s_t\, p_{Y,t}}\, x_t\,\tilde{z}_t^{-1} - \frac{p_{IM,t}}{s_t\, p_{Y,t}}\, im_t\,\tilde{z}_t^{-1}, \tag{A.58}$$

where $b_{t+1}^* = B_{t+1}^*/(z_t^*\, P_{Y,t}^*)$, $\Pi_Y^* = P_{Y,t}^*/P_{Y,t-1}^*$ and $s_t = S_t\, P_{Y,t}^*/P_{Y,t}$.

We then obtain the following log-linearised expression:

$$\begin{aligned}(R^*)^{-1}\,\widehat{b}_{t+1}^* \;=\;& g_z^{-1}\,(\bar{\Pi}_Y^*)^{-1}\,\widehat{b}_t^* + \frac{p_X\, x}{s\, p_Y}\left(\widehat{p}_{X,t} + \widehat{x}_t - \widehat{s}_t - \widehat{p}_{Y,t} - \widehat{\tilde{z}}_t\right)\\ &- \frac{p_{IM}\, im}{s\, p_Y}\left(\widehat{p}_{IM,t} + \widehat{im}_t - \widehat{s}_t - \widehat{p}_{Y,t} - \widehat{\tilde{z}}_t\right).\end{aligned} \tag{A.59}$$

Hence, the law of motion of the net foreign asset position, expressed in domestic currency and as a fraction of domestic output, $s_{B^*,t+1} = S_t\, B_{t+1}^*/(P_{Y,t}\, Y_t) = s_t\, b_{t+1}^* \tilde{z}_t/y_t$, can be written in log-linear form as follows:

$$\widehat{s}_{B^*,t+1} \;=\; s\,\tilde{z}\,y^{-1}\,\widehat{b}_{t+1}^* \tag{A.60}$$

with $s = 1$ and $\tilde{z} = 1$ and recalling that the domestic holdings of foreign bonds are zero in the non-stochastic steady state.

The trade balance (92), expressed as a share of nominal output, is conveniently written in terms of the export and the import share,

$$s_{TB,t} \;=\; s_{X,t} - s_{IM,t} \tag{A.61}$$

with $s_{TB,t} = TB_t/(P_{Y,t}\, Y_t)$, $s_{X,t} = P_{X,t}\, X_t/(P_{Y,t}\, Y_t)$ and $s_{IM,t} = P_{IM,t}\, IM_t/(P_{Y,t}\, Y_t)$.

Log-linearisation then yields

$$\widehat{s}_{TB,t} \;=\; \widehat{s}_{X,t} - \widehat{s}_{IM,t}, \tag{A.62}$$

where

$$\widehat{s}_{X,t} \;=\; s_X\,(\widehat{p}_{X,t} + \widehat{x}_t - \widehat{p}_{Y,t} - \widehat{y}_t), \tag{A.63}$$

$$\widehat{s}_{IM,t} \;=\; s_{IM}\,(\widehat{p}_{IM,t} + \widehat{im}_t - \widehat{p}_{Y,t} - \widehat{y}_t). \tag{A.64}$$

Finally, the log-linearised terms of trade are given by

$$\widehat{ToT}_t \;=\; \widehat{p}_{IM,t} - \widehat{p}_{X,t}. \tag{A.65}$$

A.6 Relative Prices

Relative prices within the NAWM are assumed to be stationary, and they are related to the various inflation rates through a set of identities. In particular, using the price of the consumption good as the numeraire, the price indexes for the domestic intermediate goods that are sold in home markets or abroad evolve, after log-linearisation, according to

$$\widehat{p}_{H,t} = \widehat{p}_{H,t-1} + \widehat{\pi}_{H,t} - \widehat{\pi}_{C,t}, \quad \text{(A.66)}$$

$$\widehat{p}_{X,t} = \widehat{p}_{X,t-1} + \widehat{\pi}_{X,t} - \widehat{\pi}_{C,t}, \quad \text{(A.67)}$$

whereas the relative price of aggregate domestic output is given by

$$\widehat{p}_{Y,t} = \widehat{p}_{Y,t-1} + \widehat{\pi}_{Y,t} - \widehat{\pi}_{C,t}. \quad \text{(A.68)}$$

Similarly, the relative prices of the final consumption and investment goods evolve according to

$$\widehat{p}_{C,t} = 0, \quad \text{(A.69)}$$

$$\widehat{p}_{I,t} = \widehat{p}_{I,t-1} + \widehat{\pi}_{I,t} - \widehat{\pi}_{C,t}. \quad \text{(A.70)}$$

Finally, for the price index of the imported intermediate goods we obtain

$$\widehat{p}_{IM,t} = \widehat{p}_{IM,t-1} + \widehat{\pi}_{IM,t} - \widehat{\pi}_{C,t}. \quad \text{(A.71)}$$

Note that both competitors' export prices and oil prices are denominated with foreign prices, $p^{c*}_{X,t} = P^{c*}_{X,t}/P^{*}_{Y,t}$ and $p_{O,t} = P_{O,t}/P^{*}_{Y,t}$. They enter, after suitable transformations, the SVAR model describing the evolution of the foreign variables.[65]

A.7 Normalisations

As in Smets and Wouters (2003, 2007), we normalise some of the structural shocks in the log-linearised version of the NAWM. For example, in the case of the domestic Phillips curve (A.26) characterising the optimal price-setting decision of domestic intermediate-good firms that are selling their outputs in home markets, the normalisation consists of defining a new variable $\widehat{\varphi}^{H,\dagger}_t = const\ \widehat{\varphi}^{H}_t$, where $const = (1-\beta\xi_H)(1-\xi_H)/(\xi_H(1+\beta\chi_H))$, and estimating the standard deviation of the innovation to $\widehat{\varphi}^{H,\dagger}_t$ instead of $\widehat{\varphi}^{H}_t$. That is, the normalisation is chosen such that the price markup shock enters the domestic price Phillips curve with a unit coefficient. We do the same for the the markup shocks $\widehat{\varphi}^{X}_t$, $\widehat{\varphi}^{*}_t$ and $\widehat{\varphi}^{W}_t$ in the export and import price Phillips curves (A.27) and (A.28) and in the wage Phillips curve (A.19). We also normalise the investment-specific technology shock $\widehat{\epsilon}^{I}_t$ so that it enters the investment equation (A.17) with a unit coefficient. These normalisations make it easier to choose reasonable priors for the standard deviations of the innovations to the structural shocks, and often they improve the convergence properties of the MCMC sampling algorithm for simulating the posterior distribution of the model's structural parameters.

[65] See Section 3.2.1 for details.

Appendix B: Computation of the Steady State

In this appendix, we outline the computation of the non-stochastic steady state for the NAWM. Our strategy is to reduce the steady-state version of the model to a system of four equations consisting of an equilibrium relation for the labour market, an equilibrium relation for the capital market and two relations characterising the equilibrium in goods markets and the system of relative prices. These relations allow us to solve simultaneously for the steady-state values of the capital stock, k, consumption, c, hours worked, N, and the price of the investment good relative to that of the consumption good, p_I.[66]

Here, because of the assumed unit-root process for productivity (with trend growth rate g_z) and the non-stationarity of prices, all variables that share the productivity trend are scaled with the level of productivity, while all nominal variables are divided by the price of the consumption good. In order to simplify the notation, the transformed variables are represented by lower-case letters.[67]

B.1 The Households' Allocations

We start by re-stating several first-order conditions characterising the households' optimal allocations in steady state. Using the before-mentioned conventions, the first-order condition characterising the households' optimal purchases of the consumption good yields:

$$\lambda = \frac{1}{(1-\kappa\, g_z^{-1})\,(1+\tau^C)\,c}. \tag{B.1}$$

Similarly, from the first-order condition characterising the optimal purchases of the investment good we obtain:

$$p_I = Q. \tag{B.2}$$

From the first-order condition characterising the optimal holdings of capital we can derive the following steady-state expression for Q:

$$Q = \frac{1-\tau^K}{g_z\,\beta^{-1}+\delta-1-\tau^K\,\delta}\,r_K. \tag{B.3}$$

Evaluating the first-order condition for the capital stock in steady state and making use of the first-order condition for the optimal utilisation of capital,

$$r_K = \gamma_{u,1}\,p_I, \tag{B.4}$$

we can determine the first derivative of the capital adjustment function:

$$\gamma_{u,1} = \frac{r_K}{p_I} = \frac{1}{1-\tau^K}\left(g_z\,\beta^{-1}+\delta-1-\tau^K\,\delta\right). \tag{B.5}$$

Finally, we note that the capital accumulation equation can be written as

$$i = \left(1-\frac{1-\delta}{g_z}\right)k. \tag{B.6}$$

[66] The computation of the steady state follows closely a solution strategy proposed by Paolo Pesenti.

[67] See Appendix A for more details on these stationary-inducing transformations.

B.2 Labour-Market Equilibrium

On the labour-supply side, the first-order condition characterising the households' optimal wage-setting decision yields the following (Lerner) relation:

$$(1-\tau^N-\tau^{W_h})\,w \;=\; \varphi^W\,\frac{N^\zeta}{\lambda}. \tag{B.7}$$

Using the first-order condition (B.1), we can re-write this relation as

$$(1-\tau^N-\tau^{W_h})\,w \;=\; \varphi^W\,N^\zeta\,(1+\tau^C)\,(1-\kappa\,g_z^{-1})\,c \tag{B.8}$$

or, alternatively,

$$w \;=\; \frac{1+\tau^C}{1-\tau^N-\tau^{W_h}}\,(1-\kappa\,g_z^{-1})\,\varphi^W\,N^\zeta\,c. \tag{B.9}$$

Regarding the characterisation of labour demand, we utilise the combined first-order conditions characterising the intermediate-good firms' optimal choice of inputs:

$$\frac{r_K}{(1+\tau^{W_f})\,w} \;=\; \frac{\alpha}{(1-\alpha)}\,\frac{N}{k}\,g_z, \tag{B.10}$$

or, alternatively,

$$w \;=\; \frac{1-\alpha}{\alpha}\,\frac{k\,N^{-1}}{g_z\,(1+\tau^{W_f})}\,r_K. \tag{B.11}$$

Combining (B.9) and (B.11), we obtain the following equilibrium relation for the labour market:

$$\frac{1+\tau^C}{1-\tau^N-\tau^{W_h}}\,(1-\kappa\,g_z^{-1})\,\varphi^W\,N^\zeta\,c \;=\; \frac{1-\alpha}{\alpha}\,\frac{g_z^{-1}k\,N^{-1}}{1+\tau^{W_f}}\,r_K. \tag{B.12}$$

Re-arranging and using (B.5), we obtain

$$g_z\,\frac{1+\tau^C}{1-\tau^N-\tau^{W_h}}\,(1-\kappa\,g_z^{-1})\,\varphi^W\,N^{\zeta+1}\,c\,k^{-1} = \frac{1-\alpha}{\alpha}\,\frac{1}{1+\tau^{W_f}}\,\frac{1}{1-\tau^K}\left(g_z\,\beta^{-1}+\delta-1-\tau^K\,\delta\right)p_I. \tag{B.13}$$

We can re-write this expression as follows:

$$g_z\,\frac{\alpha}{1-\alpha}\,\frac{1+\tau^C}{1-\tau^N-\tau^{W_h}}\,(1-\kappa\,g_z^{-1})\,\frac{1+\tau^{W_f}}{\gamma_{u,1}}\,\varphi^W\,N^{\zeta+1}\,c\,k^{-1}\,p_I^{-1} \;=\; 1. \tag{B.14}$$

Using the definition

$$\Theta \;=\; g_z\,\frac{\alpha}{1-\alpha}\,\frac{1+\tau^C}{1-\tau^N-\tau^{W_h}}\,(1-\kappa\,g_z^{-1})\,\frac{1+\tau^{W_f}}{\gamma_{u,1}}\,\varphi^W, \tag{B.15}$$

we finally obtain:

$$\Theta\,N^\zeta\,c\left(\frac{k}{N}\right)^{-1}p_I^{-1} \;=\; 1. \tag{B.16}$$

B.3 Capital-Market Equilibrium

Combining the first-order condition characterising the intermediate-good firms' optimal demand for capital services,

$$\alpha\, g_z^{1-\alpha} \left(\frac{N}{k}\right)^{1-\alpha} = \frac{r_K}{mc}, \tag{B.17}$$

and the first-order condition characterising the intermediate-good firms' optimal price-setting decision in domestic markets,

$$p_H = \varphi^H\, mc, \tag{B.18}$$

we obtain the following equilibrium condition for the capital market:

$$\alpha = g_z^{-(1-\alpha)} \left(\frac{k}{N}\right)^{1-\alpha} r_K\, \varphi^H\, p_H^{-1}. \tag{B.19}$$

Using (B.5), we can re-write this equilibrium condition as

$$\alpha = g_z^{-(1-\alpha)} \left(\frac{k}{N}\right)^{1-\alpha} \gamma_{u,1}\, \varphi^H\, \frac{p_I}{p_H}. \tag{B.20}$$

B.4 Goods-Markets Equilibrium

As regards the production of intermediate goods, the following real aggregate resource constraint holds in steady state:

$$g_z^{-\alpha} k^{\alpha} N^{1-\alpha} - \psi = h + x. \tag{B.21}$$

Taking into account the identity $h = h^C + h^I + h^G$ and re-calling that the demand for the bundle of domestic intermediate goods used in the production of the consumption good is given by

$$h^C = \nu_C\, (p_H)^{-\mu_C}\, c, \tag{B.22}$$

we can re-write the aggregate resource constraint (B.21) as

$$h^I = g_z^{-\alpha} \left(\frac{k}{N}\right)^{\alpha} N - \psi - x - \nu_C\, (p_H)^{-\mu_C}\, c - h^G. \tag{B.23}$$

Substituting (B.23) into the investment-good technology

$$i^{1-\frac{1}{\mu_I}} = \nu_I^{\frac{1}{\mu_I}} \left(h^I\right)^{1-\frac{1}{\mu_I}} + (1-\nu_I)^{\frac{1}{\mu_I}} \left(im^I\right)^{1-\frac{1}{\mu_I}}, \tag{B.24}$$

re-calling that the demand for the bundle of the imported intermediate goods used in the production of the final investment good is given by

$$im^I = (1-\nu_I) \left(\frac{p_{IM}}{p_I}\right)^{-\mu_I} i, \tag{B.25}$$

and using relation (B.6), we arrive at the following expression for the aggregate resource constraint:

$$\left(\frac{g_z-1+\delta}{g_z}k\right)^{1-\frac{1}{\mu_I}} = \nu_I^{\frac{1}{\nu_I}}\left(g_z^{-\alpha}\left(\frac{k}{N}\right)^{\alpha}N-\psi-x-\nu_C\,(p_H)^{-\mu_C}\,c-h^G\right)^{1-\frac{1}{\mu_I}} + (1-\nu_I)\left(\frac{p_{IM}}{p_I}\right)^{1-\mu_I}\left(\frac{g_z-1+\delta}{g_z}k\right)^{1-\frac{1}{\mu_I}}, \quad \text{(B.26)}$$

or, equivalently,

$$\left(\frac{g_z-1+\delta}{g_z}k\right)^{1-\frac{1}{\mu_I}}\left(1-(1-\nu_I)\left(\frac{p_{IM}}{p_I}\right)^{1-\mu_I}\right) = \nu_I^{\frac{1}{\mu_I}}\left(g_z^{-\alpha}\left(\frac{k}{N}\right)^{\alpha}N-\psi-x-\nu_C\,(p_H)^{-\mu_C}\,c-h^G\right)^{1-\frac{1}{\mu_I}}. \quad \text{(B.27)}$$

B.5 Equilibrium Relative Prices

Using the price of the consumption good as the numeraire, the relative prices of the domestic intermediate good sold at home and the investment good are given by

$$1 = \nu_C\,(p_H)^{1-\mu_C}+(1-\nu_C)\,(p_{IM})^{1-\mu_C} \quad \text{(B.28)}$$

and

$$(p_I)^{1-\mu_I} = \nu_I\,(p_H)^{1-\mu_I}+(1-\nu_I)\,(p_{IM})^{1-\mu_I}\,. \quad \text{(B.29)}$$

Re-arranging equation (B.28), we obtain:

$$p_H = \left(\frac{1-(1-\nu_C)\,(p_{IM})^{1-\mu_C}}{\nu_C}\right)^{\frac{1}{1-\mu_C}}, \quad \text{(B.30)}$$

and, similarly, re-arranging equation (B.29) yields:

$$\frac{p_H}{p_I} = \left(\frac{1-(1-\nu_I)\left(\frac{p_{IM}}{p_I}\right)^{1-\mu_I}}{\nu_I}\right)^{\frac{1}{1-\mu_I}}. \quad \text{(B.31)}$$

We observe that $P_Y = P_H = P_X$, which in turn implies that the relative prices are identical as well; that is, $p_Y = p_H = p_X$.[68]

[68]The equality $P_Y = P_H = P_X$ results from the fact that the prices of the intermediate goods sold domestically or abroad are set as a markup on the same marginal cost measure and assuming that the markups, which are inversely related to the elasticity of substitution between the differentiated intermediate goods, are identical in domestic and foreign markets. The equality of prices also implies that the nominal resource constraint collapses to the real constraint in steady state.

B.6 Computational Issues

Collecting equations (B.16), (B.20), (B.27), (B.30) and (B.31), the steady-state model can now be reduced—by substituting equation (B.30) and conditioning on government consumption, $g = h^G$, exports, x, fixed cost in production, ψ, and the relative price of the bundle of imported goods, p_{IM}—to a set of four equations in the unknown steady-state values of the capital stock, k, consumption purchases, c, hours worked, N, and the relative price of the investment good, p_I:

1. Labour-market equilibrium:

$$\Theta\, N^{\zeta}\, c \left(\frac{k}{N}\right)^{-1} p_I^{-1} \;=\; 1 \tag{B.32}$$

2. Capital-market equilibrium:

$$\alpha \;=\; g_z^{-(1-\alpha)} \left(\frac{k}{N}\right)^{1-\alpha} \gamma_{u,1}\, \varphi^H\, p_I^{-1} \left(\frac{1-(1-\nu_C)\,(p_{IM})^{1-\mu_C}}{\nu_C}\right)^{\frac{-1}{1-\mu_C}} \tag{B.33}$$

3. Goods-markets equilibrium:

$$\left(\frac{g_z-1+\delta}{g_z}\,k\right)^{1-\frac{1}{\mu_I}} \left(1-(1-\nu_I)\left(\frac{p_{IM}}{p_I}\right)^{1-\mu_I}\right) =$$

$$\nu_I^{\frac{1}{\mu_I}} \left(g_z^{-\alpha}\left(\frac{k}{N}\right)^{\alpha} N-\psi-x-\nu_C\left(\frac{1-(1-\nu_C)\,(p_{IM})^{1-\mu_C}}{\nu_C}\right)^{\frac{-\mu_C}{1-\mu_C}} c-h^G\right)^{1-\frac{1}{\mu_I}} \tag{B.34}$$

4. Equilibrium relative prices:

$$\left(\frac{1-(1-\nu_C)\,(p_{IM})^{1-\mu_C}}{\nu_C}\right)^{\frac{1}{1-\mu_C}} p_I^{-1} \;=\; \left(\frac{1-(1-\nu_I)\left(\frac{p_{IM}}{p_I}\right)^{1-\mu_I}}{\nu_I}\right)^{\frac{1}{1-\mu_I}} \tag{B.35}$$

Conditional on the relative price of the bundle of imported goods, p_{IM}, we solve for the unknown steady-state values k, c, N and p_I using numerical methods. In so doing, we simultaneously calibrate some key steady-state ratios of the model in order to pin down $g = h^G$, x and ψ. First, we choose the desired level of the government consumption share $s_G = (p_G\, g)/(p_Y\, y)$. Second, we calibrate the desired import shares $s_{IM^C} = (p_{IM}\, im^C)/(p_Y\, y)$ and $s_{IM^I} = (p_{IM}\, im^I)/(p_Y\, y)$ by appropriately adjusting the quasi-share parameters ν_C and ν_I. Imposing balanced trade in steady state, the export share $s_X = (p_X\, x)/(p_Y\, y)$ is then given by $s_X = s_{IM} = s_{IM^C} + s_{IM^I}$. Finally, we choose the fixed cost in production ψ according to $\psi/y = \varphi = \varphi^H = \varphi^X$ such that firms' profits are zero in steady state. Here, the parameter φ denotes the firms' uniform markup over marginal cost. In addition, we calibrate the desired nominal investment share $s_I = (p_I\, i)/(p_Y\, y)$ by appropriately adjusting the capital income tax rate τ^K.[69] The nominal consumption share $s_C = (p_C\, c)/(p_Y\, y)$ can then be determined as a residual; that is, $s_C = 1 - s_I - s_G - (s_X - s_{IM})$.

As the price of imported goods is set by monopolistically competitive firms abroad, we treat the import price P_{IM} as given. Hence, without loss of generality, we can normalise the relative price of imports to one; that is, we set $p_{IM} = 1$.[70] As a result, all other relative prices are equal to one as well; that is, $p_I = p_Y = p_H = p_G = p_X = 1$. Furthermore, because of $p_{IM} = p_I = 1$, the model's steady state is invariant to changes in the intratemporal substitution elasticities between domestic and imported goods, μ_C and μ_I. Still, the model's steady state depends on additional parameters that are eventually estimated, namely the trend growth rate g_z, the inverse of the labour-supply elasticity ζ and—via Θ—the habit parameter κ. However, while the parameters ζ and κ do influence the steady-state *level* of real variables such as labour, capital and consumption, their steady-state *ratios* are invariant to changes in those parameters. In contrast, the trend growth rate g_z also influences key steady-state ratios, including the capital-to-labour ratio. Hence, as the log-linearised model is parameterised in terms of steady-state ratios rather than steady-state levels, it is only the variation in the estimated trend-growth rate g_z that would require the updating of the steady-state computations at the estimation stage.

[69] Alternatively, the investment share could be calibrated by adjusting the rate of capital depreciation δ or the capital share in production α.

[70] Similarly, treating the foreign price P_Y^* as given, we normalise the real exchange rate to one, $s = 1$.

Table 1: Prior and Posterior Distributions of the Structural Parameters

Parameter		Prior distribution type	mean	std	Posterior distribution mode	mean	median	5%	95%
Preferences									
Habit formation	κ	beta	0.70	0.05	0.564	0.566	0.566	0.511	0.618
Wage and price setting									
Calvo: wages	ξ_W	beta	0.75	0.05	0.765	0.768	0.769	0.703	0.826
Indexation: wages	χ_W	beta	0.75	0.10	0.635	0.638	0.642	0.450	0.815
Calvo: dom. prices	ξ_H	beta	0.75	0.05	0.920	0.921	0.922	0.904	0.935
Indexation: dom. prices	χ_H	beta	0.75	0.10	0.417	0.425	0.424	0.292	0.563
Calvo: exp. prices	ξ_X	beta	0.75	0.05	0.770	0.763	0.767	0.681	0.836
Indexation: exp. prices	χ_X	beta	0.75	0.10	0.489	0.509	0.503	0.333	0.703
Calvo: imp. prices	ξ^*	beta	0.75	0.05	0.528	0.531	0.530	0.465	0.598
Indexation: imp. prices	χ^*	beta	0.75	0.10	0.480	0.498	0.494	0.332	0.680
Oil import share	ω^*	beta	0.15	0.05	0.157	0.156	0.155	0.120	0.192
Final-good production									
Subst. elasticity: cons.	μ_C	gamma	1.50	0.25	1.943	1.966	1.952	1.539	2.437
Subst. elasticity: inv.	μ_I	gamma	1.50	0.25	1.595	1.601	1.587	1.251	1.997
Price elasticity: exp.	μ^*	gamma	1.50	0.25	1.028	1.032	1.023	0.770	1.324
Adjustment costs									
Investment	γ_I	gamma	4.00	0.50	5.169	5.167	5.149	4.294	6.105
Import content: cons.	γ_{IM^C}	gamma	2.50	1.00	5.596	5.729	5.619	3.904	7.937
Import content: inv.	γ_{IM^I}	gamma	2.50	1.00	0.404	0.440	0.410	0.203	0.775
Export market share	γ^*	gamma	2.50	1.00	2.424	2.816	2.618	1.450	4.891
Monetary policy									
Interest-rate smoothing	ϕ_R	beta	0.90	0.05	0.865	0.867	0.867	0.837	0.894
Resp. to inflation	ϕ_Π	normal	1.70	0.10	1.904	1.900	1.899	1.766	2.038
Resp. to change in inflation	$\phi_{\Delta\Pi}$	normal	0.30	0.10	0.185	0.186	0.185	0.113	0.261
Resp. to output growth	$\phi_{\Delta Y}$	normal	0.0625	0.05	0.147	0.153	0.153	0.108	0.197
Employment									
Calvo-style parameter	ξ_E	beta	0.50	0.15	0.851	0.850	0.851	0.822	0.875
Autoregressive coefficients									
Risk premium shock: dom.	ρ_{RP}	beta	0.75	0.10	0.917	0.920	0.921	0.888	0.949
Risk premium shock: ext.	ρ_{RP^*}	beta	0.75	0.10	0.880	0.882	0.883	0.821	0.936
Permanent techn. shock	ρ_{g_z}	beta	0.75	0.10	0.797	0.780	0.791	0.613	0.909
Transitory techn. shock	ρ_ε	beta	0.75	0.10	0.895	0.894	0.897	0.833	0.945
Inv.-spec. techn. shock	ρ_I	beta	0.75	0.10	0.713	0.710	0.716	0.580	0.825
Wage markup shock	ρ_{φ^W}	beta	0.50	0.10	0.670	0.661	0.666	0.545	0.763
Price markup shock: dom.	ρ_{φ^H}	beta	0.50	0.10	0.396	0.397	0.393	0.255	0.554
Price markup shock: exp.	ρ_{φ^X}	beta	0.50	0.10	0.375	0.386	0.381	0.245	0.545
Price markup shock: imp.	ρ_{φ^*}	beta	0.50	0.10	0.548	0.541	0.543	0.376	0.701
Import demand shock	ρ_{IM}	beta	0.75	0.10	0.865	0.859	0.864	0.793	0.909
Export preference shock	ρ_{ν^*}	beta	0.75	0.10	0.807	0.783	0.795	0.639	0.883
Standard deviations									
Risk premium shock: dom.	σ_{RP}	inv. gamma	0.10	2	0.161	0.161	0.158	0.119	0.215
Risk premium shock: ext.	σ_{RP^*}	inv. gamma	0.10	2	0.433	0.439	0.430	0.283	0.624
Permanent techn. shock	σ_{g_z}	inv. gamma	0.10	2	0.116	0.131	0.123	0.072	0.213
Transitory techn. shock	σ_ε	inv. gamma	0.10	2	1.126	1.158	1.133	0.836	1.572
Inv.-spec. techn. shock	σ_I	inv. gamma	0.10	2	0.412	0.422	0.416	0.345	0.521
Wage markup shock	σ_{φ^W}	inv. gamma	0.10	2	0.115	0.118	0.117	0.091	0.151
Price markup shock: dom.	σ_{φ^H}	inv. gamma	0.10	2	0.124	0.126	0.126	0.101	0.152
Price markup shock: exp.	σ_{φ^X}	inv. gamma	0.10	2	1.062	1.080	1.072	0.879	1.309
Price markup shock: imp.	σ_{φ^*}	inv. gamma	0.10	2	0.973	0.994	0.984	0.710	1.313
Import demand shock	σ_{IM}	inv. gamma	0.10	2	4.600	4.780	4.696	3.763	6.086
Export preference shock	σ_{ν^*}	inv. gamma	0.10	2	8.073	8.985	8.563	5.759	13.661
Interest rate shock	σ_R	inv. gamma	0.10	2	0.115	0.118	0.117	0.102	0.136

Note: This table provides information on the marginal prior and posterior distributions of the structural parameters for the preferred specification of the NAWM. For the inverse gamma prior distributions, the mode and the degrees of freedom are reported. The posterior distributions are based on a Markov chain with 550,000 draws, with 50,000 draws being discarded as burn-in draws.

Table 2: Sensitivity of Parameter Estimates and Marginal Likelihood Comparison

Parameter		Preferred specification	Variable capital utilisation	Varying inflation objective	Uncorr. markup shocks	Local-currency pricing
Preferences						
Habit formation	κ	0.564	0.561	0.566	0.555	0.563
Wage and price setting						
Calvo: wages	ξ_W	0.765	0.736	0.771	0.867	0.758
Indexation: wages	χ_W	0.635	0.624	0.651	0.862	0.630
Calvo: dom. prices	ξ_H	0.920	0.918	0.922	0.944	0.920
Indexation: dom. prices	χ_H	0.417	0.362	0.422	0.548	0.409
Calvo: exp. prices	ξ_X	0.770	0.764	0.775	0.817	0.522
Indexation: exp. prices	χ_X	0.489	0.476	0.490	0.623	0.447
Calvo: imp. prices	ξ^*	0.528	0.532	0.531	0.553	0.530
Indexation: imp. prices	χ^*	0.480	0.485	0.477	0.549	0.485
Oil import share	ω^*	0.157	0.156	0.163	0.156	0.155
Final-good production						
Subst. elasticity: cons.	μ_C	1.943	1.884	1.929	1.974	1.823
Subst. elasticity: inv.	μ_I	1.595	1.673	1.600	1.619	1.562
Price elasticity: exp.	μ^*	1.028	0.896	1.065	1.062	0.912
Adjustment costs						
Investment	γ_I	5.169	5.276	5.064	4.932	5.196
Capital utilisation	$\gamma_{u,2}$	—	0.119	—	—	—
Import content: cons.	γ_{IM^C}	5.596	5.637	5.502	5.134	5.132
Import content: inv.	γ_{IM^I}	0.404	0.358	0.400	0.369	0.390
Export market share	γ^*	2.424	1.875	2.476	3.518	2.269
Monetary policy						
Interest-rate smoothing	ϕ_R	0.865	0.861	0.877	0.863	0.861
Resp. to inflation	ϕ_Π	1.904	1.911	1.853	1.906	1.893
Resp. to change in inflation	$\phi_{\Delta\Pi}$	0.185	0.190	0.171	0.190	0.186
Resp. to output growth	$\phi_{\Delta Y}$	0.147	0.143	0.162	0.156	0.137
Employment						
Calvo-style parameter	ξ_E	0.851	0.792	0.858	0.870	0.844
Autoregressive coefficients						
Risk premium shock: dom.	ρ_{RP}	0.917	0.912	0.936	0.931	0.916
Risk premium shock: ext.	ρ_{RP^*}	0.880	0.878	0.890	0.908	0.886
Permanent techn. shock	ρ_{g_z}	0.797	0.806	0.804	0.844	0.798
Transitory techn. shock	ρ_ε	0.895	0.926	0.888	0.867	0.898
Inv.-spec. techn. shock	ρ_I	0.713	0.647	0.750	0.748	0.697
Wage markup shock	ρ_{φ^W}	0.670	0.669	0.674	—	0.672
Price markup shock: dom.	ρ_{φ^H}	0.396	0.365	0.395	—	0.382
Price markup shock: exp.	ρ_{φ^X}	0.375	0.367	0.382	—	0.572
Price markup shock: imp.	ρ_{φ^*}	0.548	0.553	0.553	—	0.546
Import demand shock	ρ_{IM}	0.865	0.863	0.864	0.892	0.868
Export preference shock	ρ_{ν^*}	0.807	0.820	0.813	0.709	0.790
Standard deviations						
Risk premium shock: dom.	σ_{RP}	0.161	0.165	0.142	0.136	0.161
Risk premium shock: ext.	σ_{RP^*}	0.433	0.434	0.412	0.362	0.409
Permanent techn. shock	σ_{g_z}	0.116	0.111	0.119	0.103	0.115
Transitory techn. shock	σ_ε	1.126	0.762	1.220	1.431	1.059
Inv.-spec. techn. shock	σ_I	0.412	0.437	0.394	0.402	0.419
Wage markup shock	σ_{φ^W}	0.115	0.118	0.115	0.285	0.116
Price markup shock: dom.	σ_{φ^H}	0.124	0.128	0.124	0.163	0.127
Price markup shock: exp.	σ_{φ^X}	1.062	1.077	1.054	1.236	2.820
Price markup shock: imp.	σ_{φ^*}	0.973	0.988	0.966	1.377	0.981
Import demand shock	σ_{IM}	4.600	4.438	4.537	4.475	4.530
Export preference shock	σ_{ν^*}	8.073	6.616	8.256	10.304	7.520
Interest rate shock	σ_R	0.115	0.116	0.118	0.121	0.115
Inflation objective shock	$\sigma_{\bar{\Pi}}$	—	—	0.064	—	—
Marginal likelihood	$\ln p(Y_T)$	-1936.198	-1959.190	-1946.023	-1950.217	-1991.542

Note: This table reports the posterior mode estimates of the structural parameters for alternative specifications of the NAWM, and compares their marginal likelihood. The latter is estimated using Geweke's modified harmonic mean estimator.

Table 3: Forecast-Error-Variance Decompositions

	Real GDP			GDP defl. inflation			Wage inflation			Nominal interest rate		
Shock	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%
	Forecast horizon: 1 quarter											
Risk premium: domestic	0.29	0.22	0.36	0.03	0.01	0.05	0.04	0.02	0.07	0.22	0.14	0.30
Risk premium: external	0.01	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.03
Permanent technology	0.06	0.03	0.12	0.00	0.00	0.01	0.08	0.03	0.17	0.01	0.00	0.01
Transitory technology	0.01	0.00	0.03	0.08	0.04	0.13	0.01	0.00	0.03	0.00	0.00	0.00
Inv.-spec. technology	0.14	0.10	0.18	0.00	0.00	0.00	0.01	0.00	0.01	0.07	0.05	0.10
Wage markup	0.00	0.00	0.01	0.02	0.01	0.04	0.83	0.72	0.91	0.00	0.00	0.00
Price markup: domestic	0.07	0.04	0.10	0.36	0.28	0.44	0.01	0.00	0.02	0.03	0.00	0.09
Price markup: exports	0.00	0.00	0.01	0.49	0.39	0.58	0.00	0.00	0.00	0.00	0.00	0.01
Price markup: imports	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Import demand	0.03	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.02	0.09
Export preferences	0.24	0.18	0.31	0.01	0.00	0.01	0.01	0.01	0.03	0.14	0.09	0.20
Government consumption	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01
Interest rate	0.10	0.06	0.14	0.01	0.00	0.01	0.01	0.00	0.02	0.42	0.29	0.56
Foreign	0.04	0.03	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.03	0.02	0.04
	Forecast horizon: 4 quarters											
Risk premium: domestic	0.25	0.18	0.32	0.10	0.06	0.14	0.11	0.06	0.16	0.39	0.30	0.47
Risk premium: external	0.01	0.00	0.02	0.01	0.01	0.01	0.01	0.00	0.01	0.04	0.02	0.06
Permanent technology	0.08	0.03	0.15	0.01	0.00	0.03	0.10	0.04	0.18	0.01	0.00	0.03
Transitory technology	0.04	0.01	0.08	0.15	0.09	0.22	0.03	0.02	0.05	0.01	0.00	0.01
Inv.-spec. technology	0.15	0.11	0.21	0.00	0.00	0.00	0.01	0.00	0.02	0.10	0.06	0.15
Wage markup	0.02	0.00	0.03	0.06	0.04	0.10	0.64	0.54	0.73	0.00	0.00	0.01
Price markup: domestic	0.12	0.07	0.18	0.29	0.22	0.37	0.04	0.01	0.07	0.07	0.02	0.14
Price markup: exports	0.02	0.01	0.04	0.32	0.23	0.42	0.00	0.00	0.01	0.02	0.01	0.04
Price markup: imports	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01
Import demand	0.01	0.00	0.01	0.00	0.00	0.01	0.01	0.00	0.01	0.03	0.01	0.06
Export preferences	0.19	0.14	0.25	0.02	0.01	0.04	0.03	0.01	0.05	0.16	0.10	0.22
Government consumption	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Interest rate	0.08	0.05	0.11	0.02	0.01	0.03	0.02	0.01	0.04	0.10	0.06	0.17
Foreign	0.03	0.02	0.04	0.02	0.01	0.02	0.00	0.00	0.02	0.06	0.05	0.08

Note: This table reports posterior mean estimates and 90 percent equal-tail uncertainty bands for the forecast-error-variance decomposition of selected observed variables at the 1, 4, 20, and 40-quarter horizon. The decomposition is conducted *only* for the structural shock part of the forecast errors, while the shares of the forecast errors due to measurement errors and unobserved state variables are skipped. The results are based on 5,000 draws from the posterior distribution of the NAWM's structural parameters.

Table 3: Forecast-Error-Variance Decompositions (continued)

	Real GDP			GDP defl. inflation			Wage inflation			Nominal interest rate		
Shock	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%
	Forecast horizon: 20 quarters											
Risk premium: domestic	0.15	0.09	0.21	0.16	0.10	0.23	0.14	0.09	0.21	0.56	0.45	0.67
Risk premium: external	0.01	0.00	0.02	0.02	0.01	0.04	0.01	0.01	0.02	0.05	0.03	0.07
Permanent technology	0.17	0.08	0.32	0.02	0.00	0.04	0.10	0.04	0.18	0.02	0.00	0.05
Transitory technology	0.16	0.07	0.29	0.13	0.08	0.19	0.03	0.02	0.06	0.01	0.00	0.02
Inv.-spec. technology	0.16	0.08	0.27	0.00	0.00	0.00	0.01	0.00	0.02	0.07	0.03	0.13
Wage markup	0.08	0.03	0.13	0.06	0.03	0.10	0.58	0.48	0.67	0.00	0.00	0.01
Price markup: domestic	0.08	0.05	0.13	0.22	0.16	0.29	0.04	0.02	0.06	0.03	0.01	0.07
Price markup: exports	0.01	0.00	0.02	0.31	0.22	0.41	0.00	0.00	0.01	0.02	0.01	0.04
Price markup: imports	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
Import demand	0.03	0.01	0.05	0.01	0.00	0.01	0.01	0.00	0.01	0.02	0.00	0.03
Export preferences	0.10	0.06	0.14	0.03	0.01	0.05	0.04	0.02	0.06	0.10	0.06	0.15
Government consumption	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Interest rate	0.04	0.02	0.06	0.02	0.01	0.04	0.02	0.01	0.04	0.04	0.02	0.07
Foreign	0.01	0.01	0.02	0.02	0.02	0.03	0.02	0.01	0.03	0.08	0.05	0.10
	Forecast horizon: 40 quarters											
Risk premium: domestic	0.11	0.06	0.17	0.16	0.10	0.24	0.14	0.09	0.21	0.56	0.44	0.68
Risk premium: external	0.01	0.00	0.02	0.02	0.01	0.04	0.01	0.01	0.02	0.05	0.03	0.07
Permanent technology	0.31	0.15	0.50	0.02	0.00	0.04	0.10	0.04	0.18	0.02	0.00	0.06
Transitory technology	0.16	0.05	0.31	0.13	0.08	0.19	0.03	0.02	0.06	0.01	0.00	0.02
Inv.-spec. technology	0.14	0.06	0.25	0.00	0.00	0.01	0.01	0.00	0.02	0.07	0.03	0.13
Wage markup	0.07	0.03	0.12	0.06	0.03	0.10	0.57	0.48	0.67	0.00	0.00	0.01
Price markup: domestic	0.06	0.03	0.10	0.22	0.16	0.28	0.04	0.02	0.06	0.03	0.01	0.07
Price markup: exports	0.00	0.00	0.01	0.30	0.21	0.41	0.00	0.00	0.01	0.02	0.01	0.04
Price markup: imports	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
Import demand	0.02	0.01	0.04	0.01	0.00	0.01	0.01	0.00	0.01	0.02	0.01	0.03
Export preferences	0.08	0.04	0.12	0.03	0.02	0.00	0.05	0.02	0.06	0.10	0.06	0.16
Government consumption	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Interest rate	0.03	0.01	0.05	0.02	0.01	0.04	0.03	0.01	0.04	0.04	0.02	0.06
Foreign	0.01	0.01	0.02	0.03	0.02	0.04	0.02	0.01	0.03	0.08	0.05	0.11

Note: See above.

Table 4: Sample Means and Standard Deviations

Variable	Sample mean				Sample standard deviation			
	NAWM			Data	NAWM			Data
	Mean	5%	95%		Mean	5%	95%	
Real GDP	0.50	0.33	0.66	0.57	0.84	0.68	1.03	0.48
Consumption	0.50	0.34	0.66	0.55	0.74	0.60	0.90	0.48
Investment	0.50	0.11	0.88	0.71	2.76	2.14	3.50	1.35
Exports	0.50	0.09	0.91	0.57	3.41	2.78	4.15	2.17
Imports	0.50	0.10	0.90	0.57	2.95	2.42	3.55	1.87
GDP defl. inflation	0.48	0.24	0.71	0.75	0.59	0.46	0.75	0.40
Consumption defl. inflation	0.48	0.25	0.70	0.73	0.49	0.36	0.64	0.36
Import defl. inflation	0.48	0.13	0.82	0.01	2.47	2.04	2.94	2.65
Employment	−0.00	−1.51	1.51	0.00	1.72	0.97	2.69	1.79
Wage inflation	0.78	0.48	1.07	0.93	0.84	0.65	1.06	0.57
Nominal interest rate	4.40	2.74	6.06	6.22	1.97	1.25	2.91	3.08
Real effective exchange rate	0.01	−8.70	8.74	0.00	8.55	5.50	12.68	10.38

Note: This table reports posterior mean estimates and 90 percent equal-tail uncertainty bands for the NAWM-based sample means and standard deviations of selected observed variables along with the sample moments based on the data. The results are based on 500 draws from the posterior distribution of the NAWM's structural parameters using 500 sample paths per parameter draw.

Table A: Additional Forecast-Error-Variance Decompositions

Shock	Consumption	Investment	Exports	Imports	Cons. defl. inflation	Imp. defl. inflation	Employment	Real eff. exch. rate
	Forecast horizon: 1 quarter							
Risk premium: domestic	0.54	0.07	0.01	0.02	0.04	0.06	0.16	0.08
Risk premium: external	0.02	0.04	0.08	0.05	0.01	0.25	0.01	0.47
Permanent technology	0.09	0.01	0.00	0.01	0.00	0.00	0.02	0.00
Transitory technology	0.03	0.00	0.00	0.00	0.05	0.00	0.47	0.00
Inv.-spec. technology	0.01	0.69	0.00	0.07	0.00	0.00	0.03	0.00
Wage markup	0.01	0.00	0.00	0.00	0.02	0.00	0.04	0.00
Price markup: domestic	0.09	0.04	0.00	0.00	0.76	0.00	0.08	0.01
Price markup: exports	0.01	0.00	0.03	0.00	0.00	0.00	0.01	0.00
Price markup: imports	0.00	0.01	0.00	0.05	0.01	0.33	0.00	0.00
Import demand	0.00	0.10	0.03	0.75	0.09	0.11	0.00	0.15
Export preferences	0.00	0.01	0.67	0.01	0.00	0.07	0.12	0.12
Government consumption	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Interest rate	0.18	0.02	0.01	0.01	0.01	0.02	0.04	0.03
Foreign	0.02	0.01	0.17	0.03	0.01	0.16	0.02	0.14
	Forecast horizon: 4 quarters							
Risk premium: domestic	0.41	0.08	0.01	0.01	0.13	0.06	0.19	0.06
Risk premium: external	0.04	0.06	0.08	0.14	0.04	0.24	0.01	0.37
Permanent technology	0.10	0.02	0.00	0.02	0.01	0.00	0.02	0.00
Transitory technology	0.06	0.00	0.01	0.00	0.10	0.00	0.34	0.02
Inv.-spec. technology	0.00	0.57	0.00	0.08	0.00	0.00	0.03	0.00
Wage markup	0.02	0.00	0.01	0.00	0.04	0.00	0.07	0.01
Price markup: domestic	0.19	0.06	0.00	0.01	0.54	0.00	0.10	0.02
Price markup: exports	0.01	0.00	0.10	0.00	0.00	0.00	0.02	0.01
Price markup: imports	0.01	0.02	0.00	0.07	0.01	0.27	0.00	0.00
Import demand	0.01	0.12	0.04	0.53	0.07	0.12	0.00	0.19
Export preferences	0.00	0.02	0.58	0.05	0.01	0.08	0.14	0.14
Government consumption	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Interest rate	0.13	0.02	0.00	0.00	0.02	0.02	0.05	0.02
Foreign	0.02	0.03	0.17	0.09	0.03	0.21	0.03	0.16

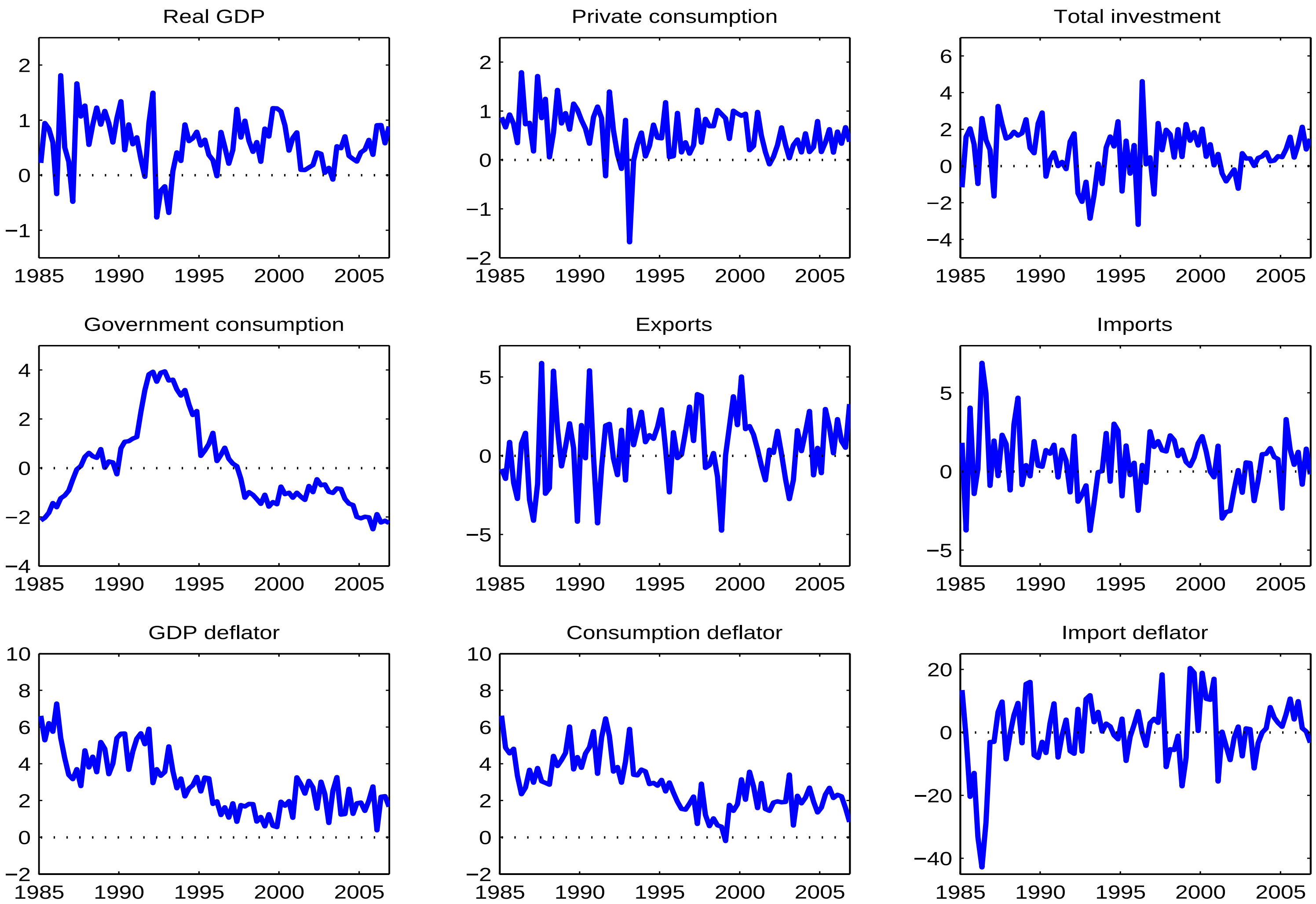
Note: This table reports posterior mean estimates for the forecast-error-variance decompositions of additional observed variables at the 1, 4, 20 and 40-quarter horizon. The decomposition is conducted *only* for the structural shock part of the forecast errors, while the shares of the forecast errors due to measurement errors and unobserved state variables are skipped. The results are based on 5,000 draws from the posterior distribution of the NAWM's structural parameters.

Table A: Additional Forecast-Error-Variance Decompositions (continued)

Shock	Consumption	Investment	Exports	Imports	Cons. defl. inflation	Imp. defl. inflation	Employment	Real eff. exch. rate
	Forecast horizon: 20 quarters							
Risk premium: domestic	0.18	0.09	0.01	0.01	0.22	0.05	0.20	0.04
Risk premium: external	0.04	0.08	0.06	0.16	0.04	0.25	0.01	0.20
Permanent technology	0.15	0.08	0.02	0.06	0.02	0.00	0.03	0.00
Transitory technology	0.16	0.05	0.06	0.00	0.09	0.00	0.15	0.04
Inv.-spec. technology	0.11	0.34	0.02	0.05	0.00	0.00	0.02	0.04
Wage markup	0.07	0.03	0.03	0.00	0.05	0.00	0.26	0.02
Price markup: domestic	0.11	0.07	0.00	0.01	0.42	0.00	0.10	0.02
Price markup: exports	0.01	0.00	0.10	0.00	0.01	0.00	0.02	0.01
Price markup: imports	0.01	0.01	0.00	0.04	0.01	0.26	0.00	0.00
Import demand	0.01	0.10	0.15	0.33	0.06	0.11	0.01	0.29
Export preferences	0.09	0.09	0.37	0.25	0.01	0.08	0.12	0.15
Government consumption	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Interest rate	0.05	0.02	0.00	0.00	0.03	0.01	0.04	0.01
Foreign	0.01	0.04	0.18	0.09	0.04	0.24	0.04	0.18
	Forecast horizon: 40 quarters							
Risk premium: domestic	0.12	0.08	0.01	0.01	0.22	0.05	0.18	0.04
Risk premium: external	0.03	0.08	0.07	0.15	0.04	0.25	0.01	0.19
Permanent technology	0.25	0.12	0.06	0.09	0.02	0.00	0.03	0.00
Transitory technology	0.14	0.06	0.05	0.00	0.09	0.00	0.14	0.04
Inv.-spec. technology	0.13	0.29	0.02	0.05	0.00	0.01	0.03	0.04
Wage markup	0.06	0.03	0.02	0.00	0.05	0.00	0.27	0.02
Price markup: domestic	0.07	0.06	0.00	0.01	0.41	0.00	0.09	0.02
Price markup: exports	0.00	0.00	0.09	0.00	0.01	0.00	0.02	0.00
Price markup: imports	0.00	0.01	0.00	0.03	0.01	0.26	0.00	0.00
Import demand	0.01	0.11	0.17	0.30	0.06	0.11	0.02	0.30
Export preferences	0.13	0.08	0.32	0.25	0.01	0.08	0.13	0.14
Government consumption	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Interest rate	0.03	0.02	0.00	0.00	0.03	0.01	0.04	0.01
Foreign	0.03	0.06	0.19	0.11	0.05	0.23	0.04	0.20

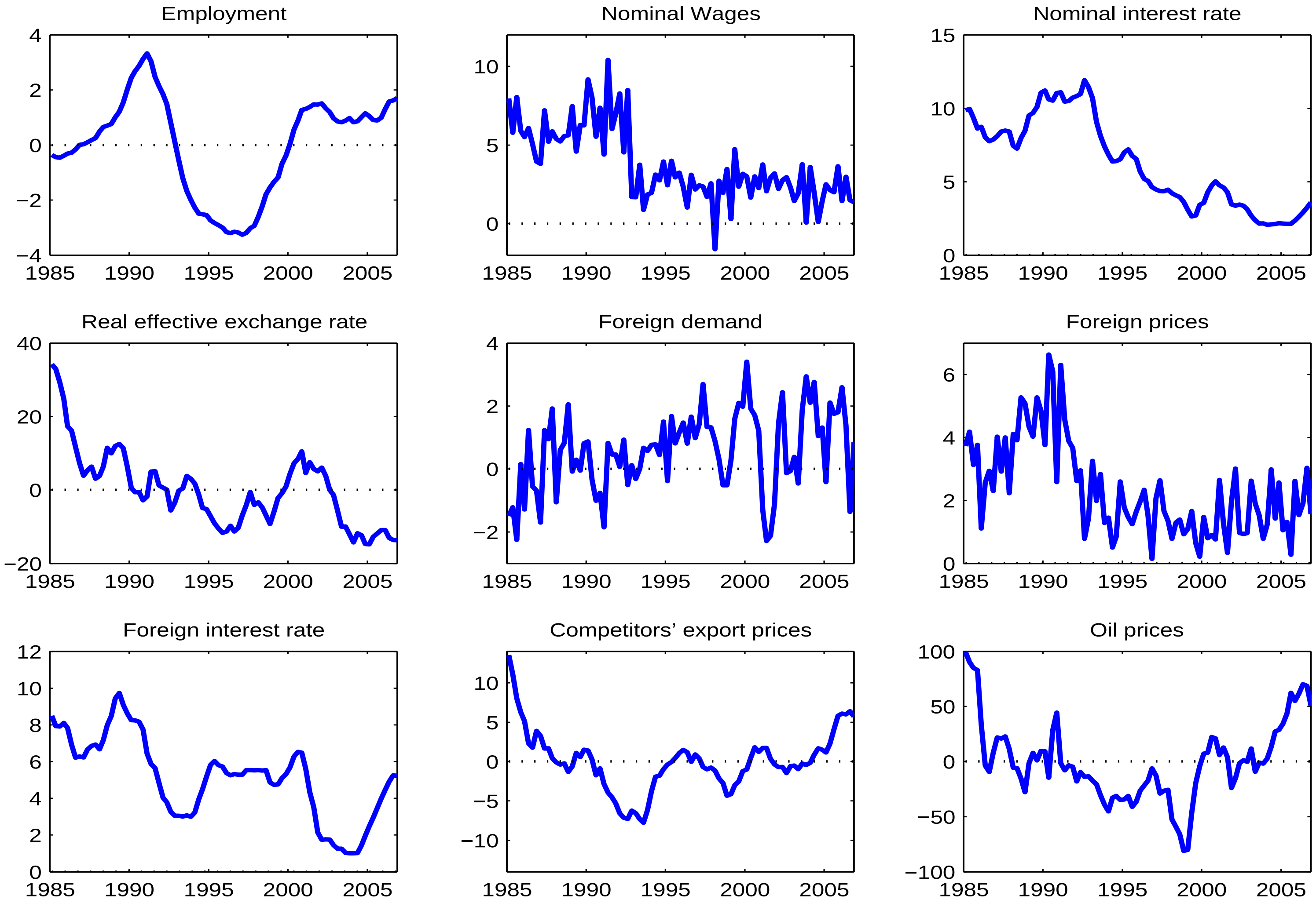
Note: See above.

Figure 1: The Data



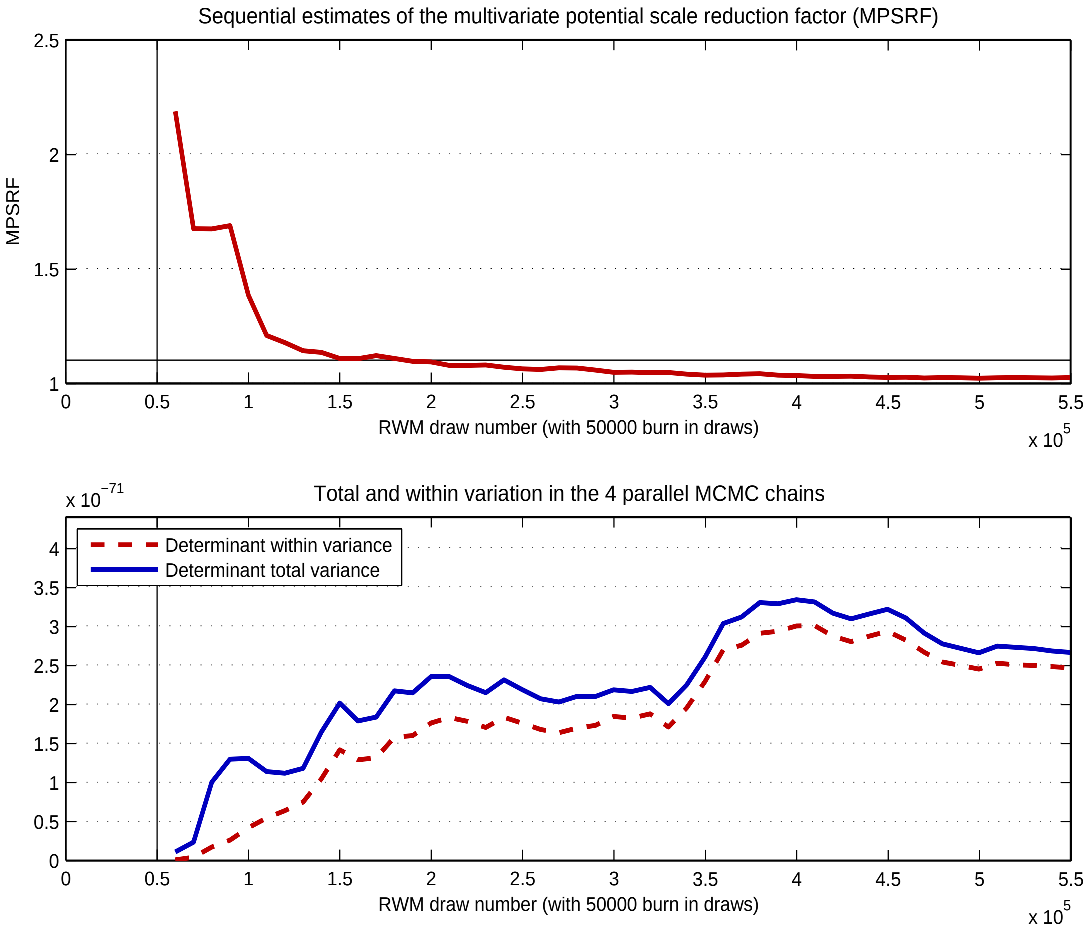
Note: This figure shows the time series of the observed variables used in the estimation of the NAWM. Details on the variable transformations are provided in Section 3.2.1. Inflation and interest rates are reported in annualised percentage terms.

Figure 1: The Data (continued)



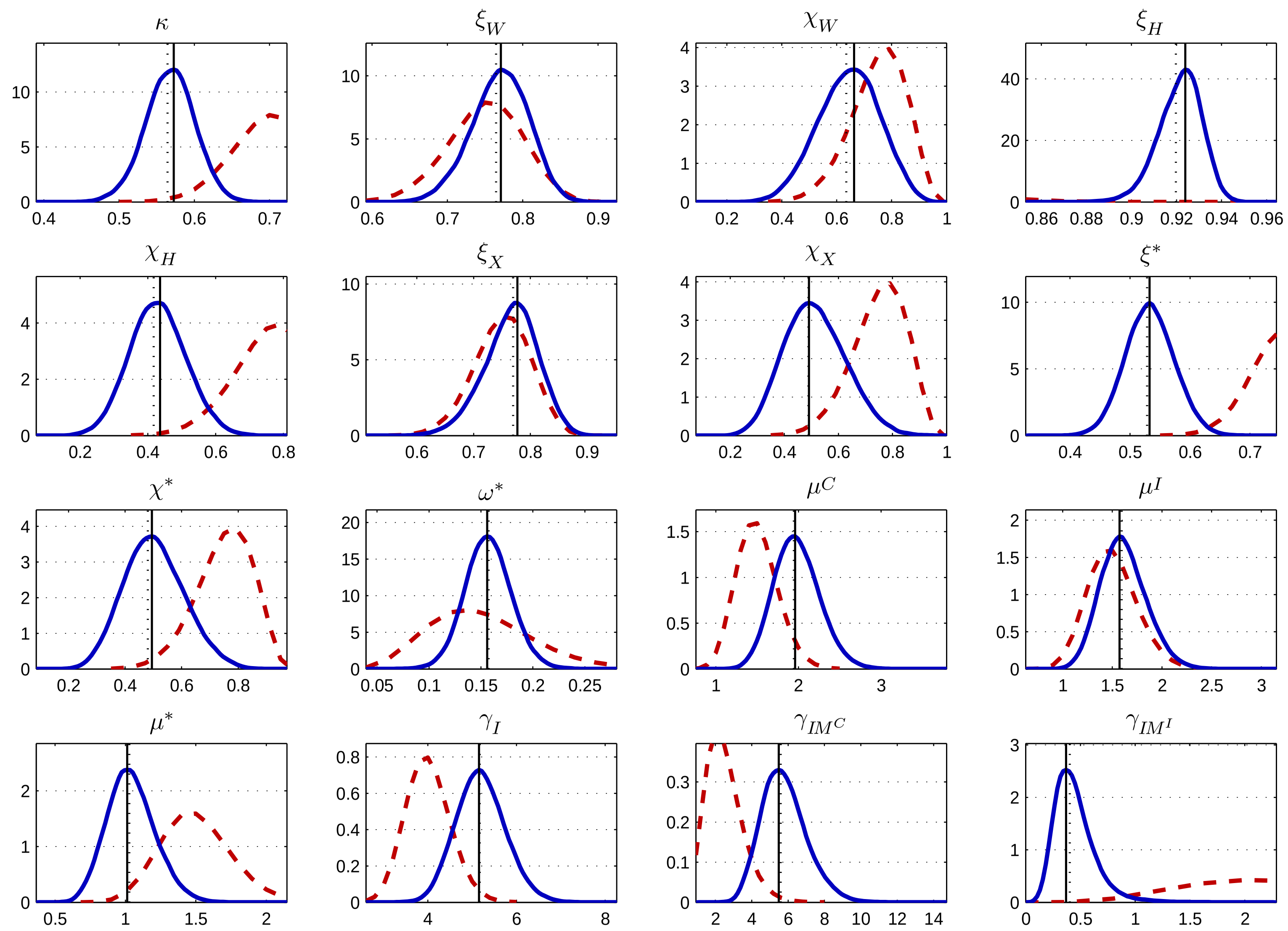
Note: See above.

Figure 2: Convergence of the Posterior Sampling Algorithm



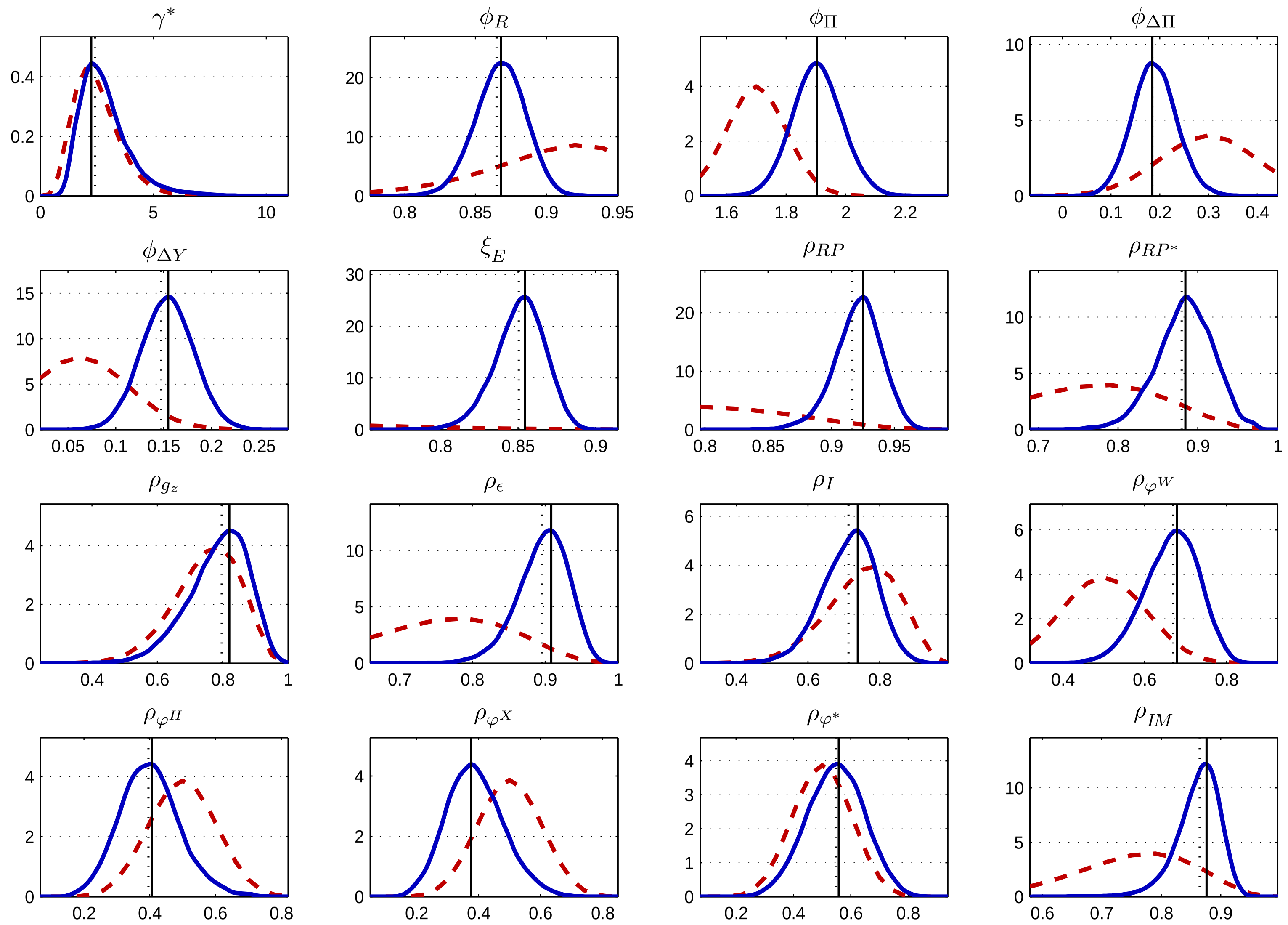
Note: This figure plots sequential estimates of the multivariate potential scale reduction factor (MPSRF) based on 4 Markov chains, each having 550,000 posterior draws, where the first 50,000 are discarded as burn-in draws.

Figure 3: Prior and Posterior Distributions of the Structural Parameters



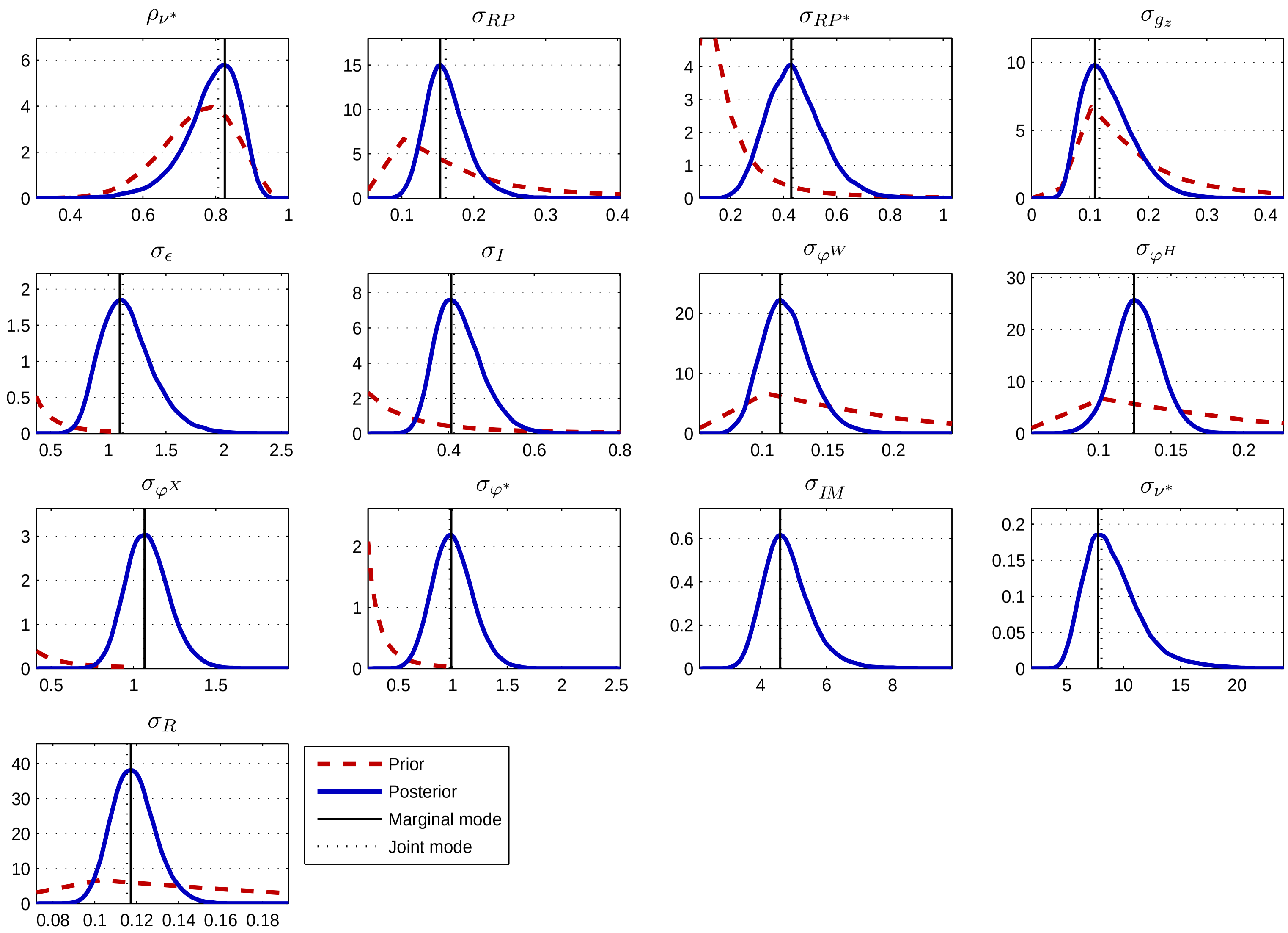
Note: This figure shows the marginal posterior distributions of the NAWM's structural parameters based on a Markov chain with 550,000 draws (blue solid lines) against their marginal prior distributions (red dotted lines), with 50,000 draws being discarded as burn-in draws.

Figure 3: Prior and Posterior Distributions of the Structural Parameters (continued)



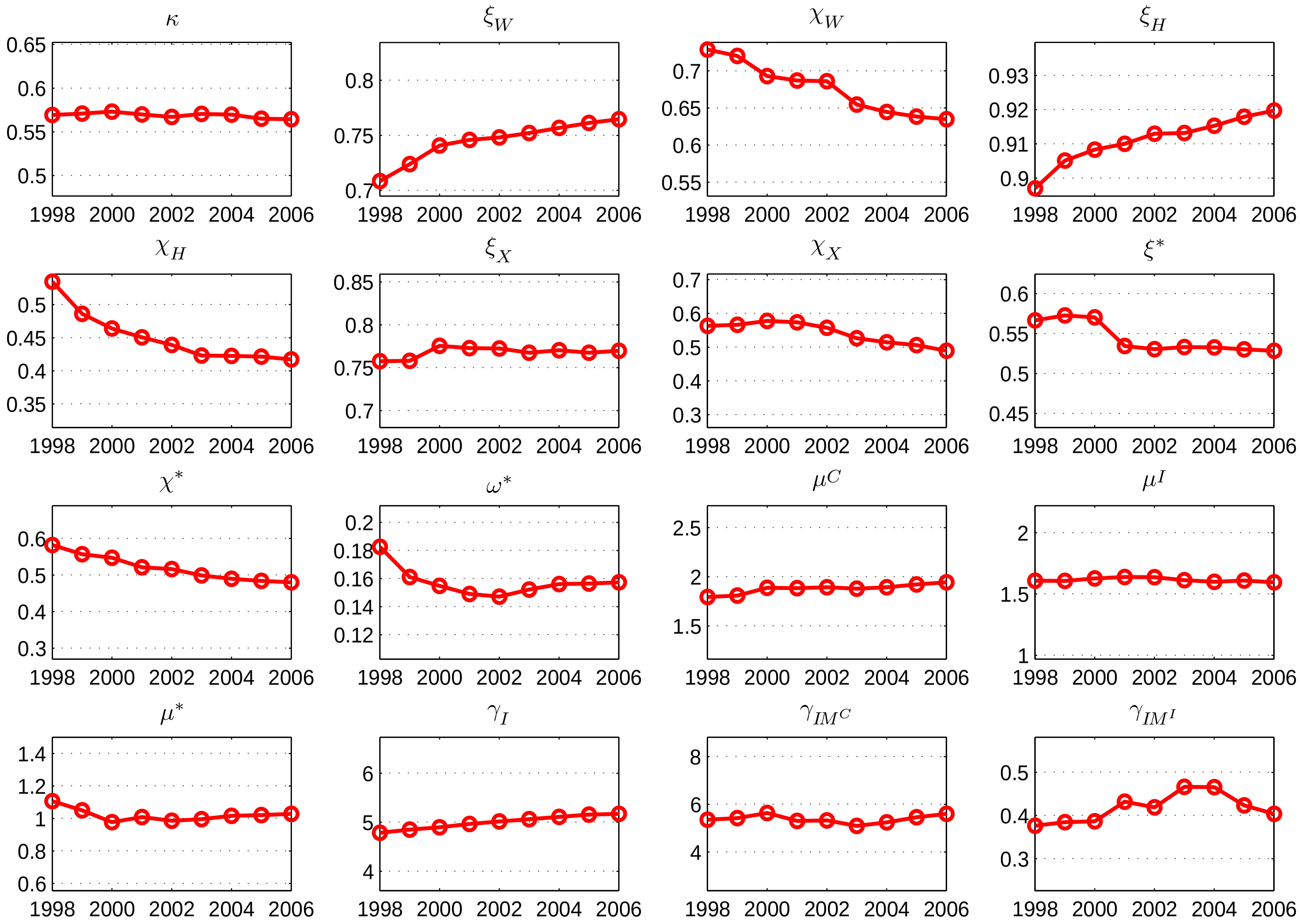
Note: See above.

Figure 3: Prior and Posterior Distributions of the Structural Parameters (continued)



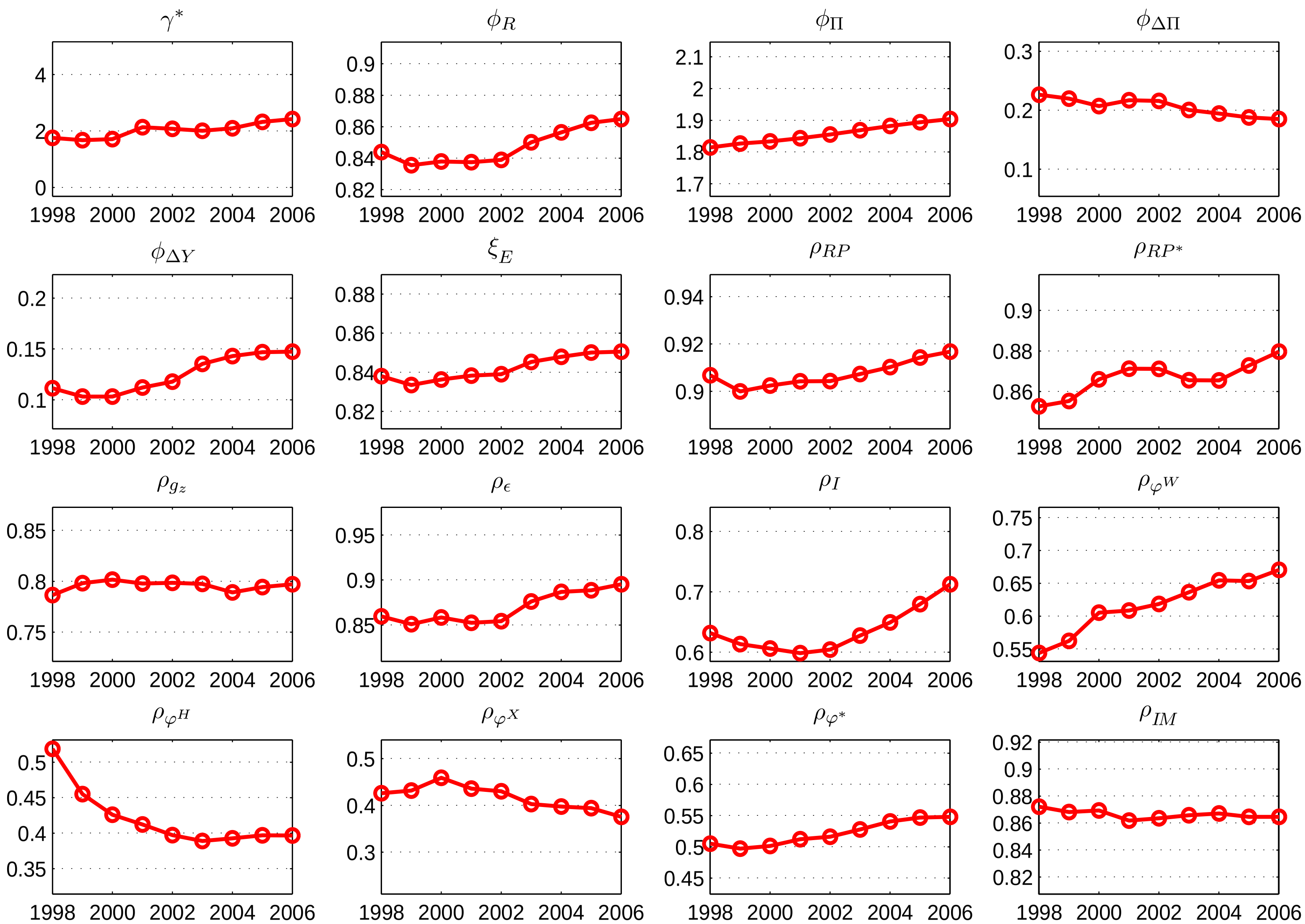
Note: See above.

Figure 4: Recursive Posterior Mode Estimates of the Structural Parameters



Note: This figure shows the posterior mode of the NAWM's structural parameters estimated recursively with the estimation sample being gradually extended by a full year from 1998Q4 to 2006Q4.

Figure 4: Recursive Posterior Mode Estimates of the Structural Parameters (continued)

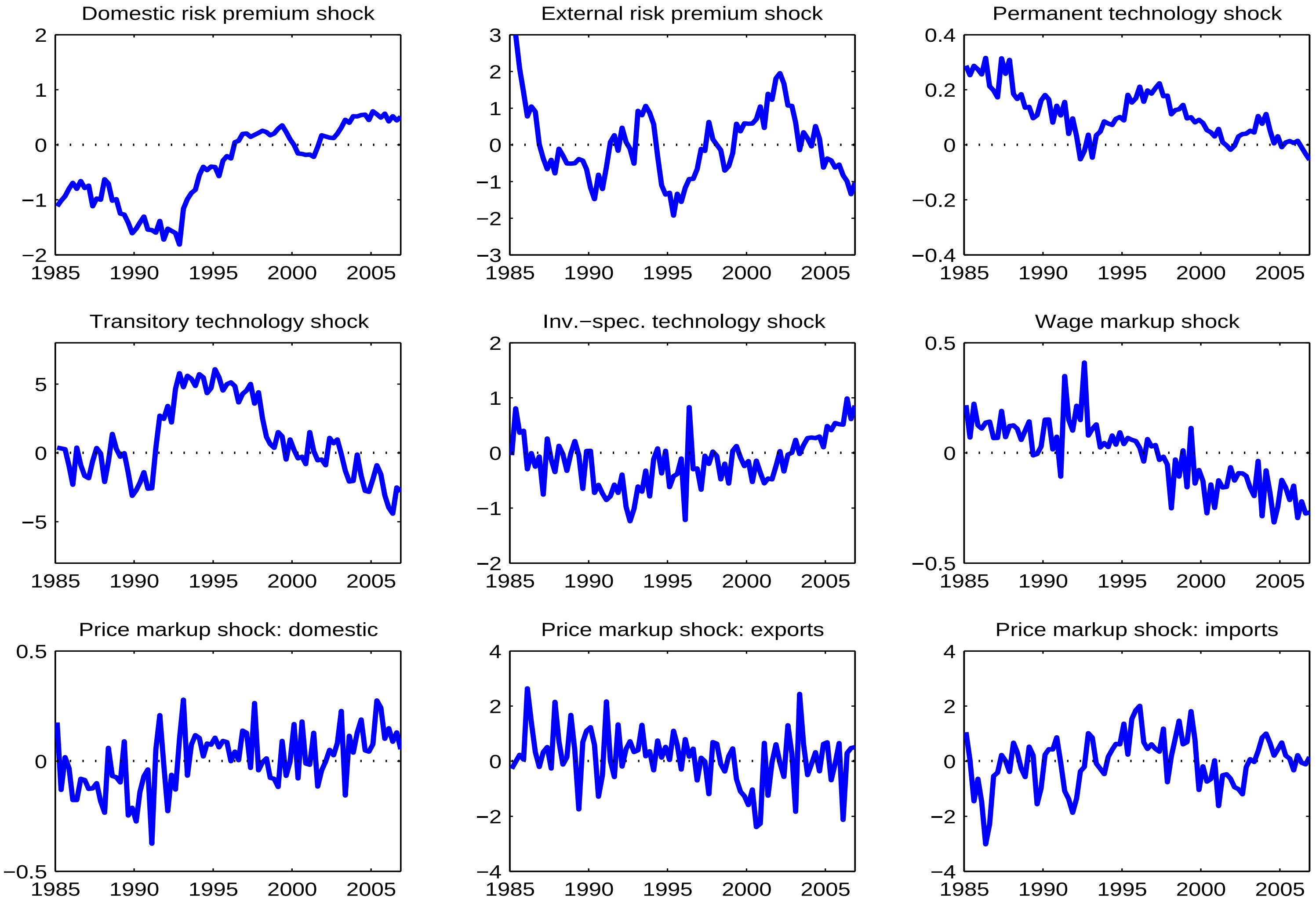


Note: See above.

Figure 4: Recursive Posterior Mode Estimates of the Structural Parameters (continued)

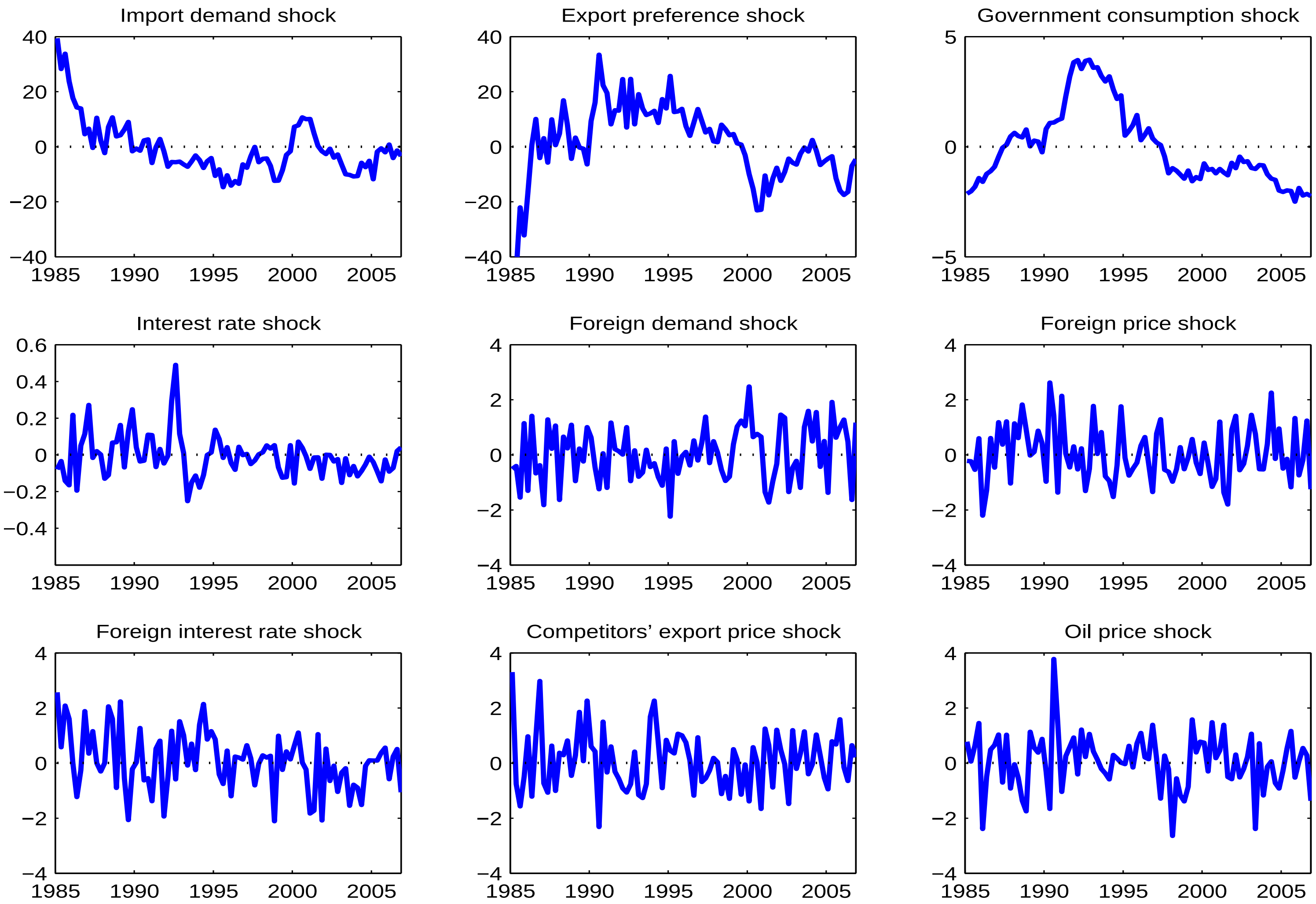
Note: See above.

Figure 5: Smoothed Estimates of the Structural Shocks



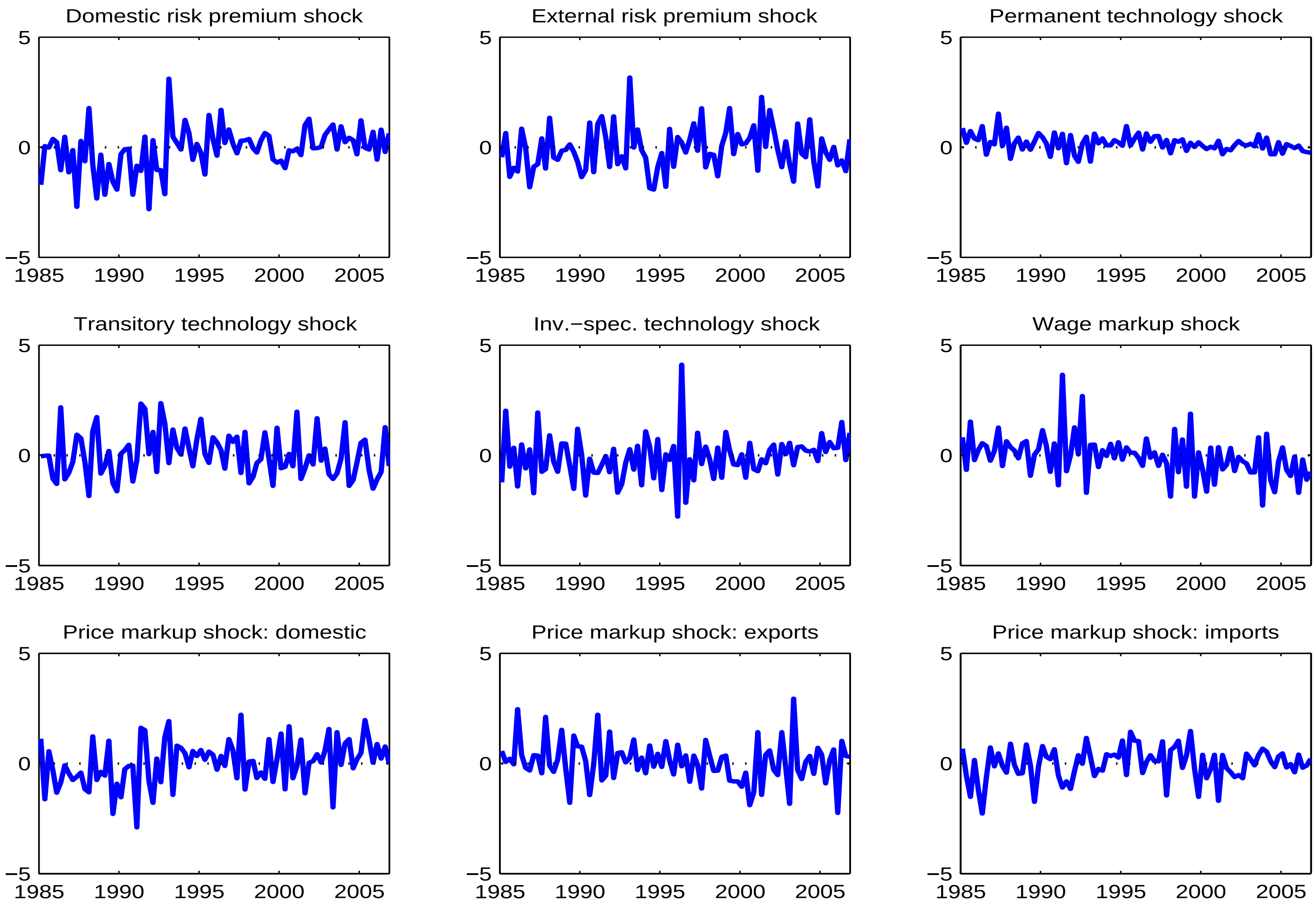
Note: This figure shows the smoothed estimates of the NAWM's structural shocks based on the posterior mode estimates of the model's structural parameters.

Figure 5: Smoothed Estimates of the Structural Shocks (continued)



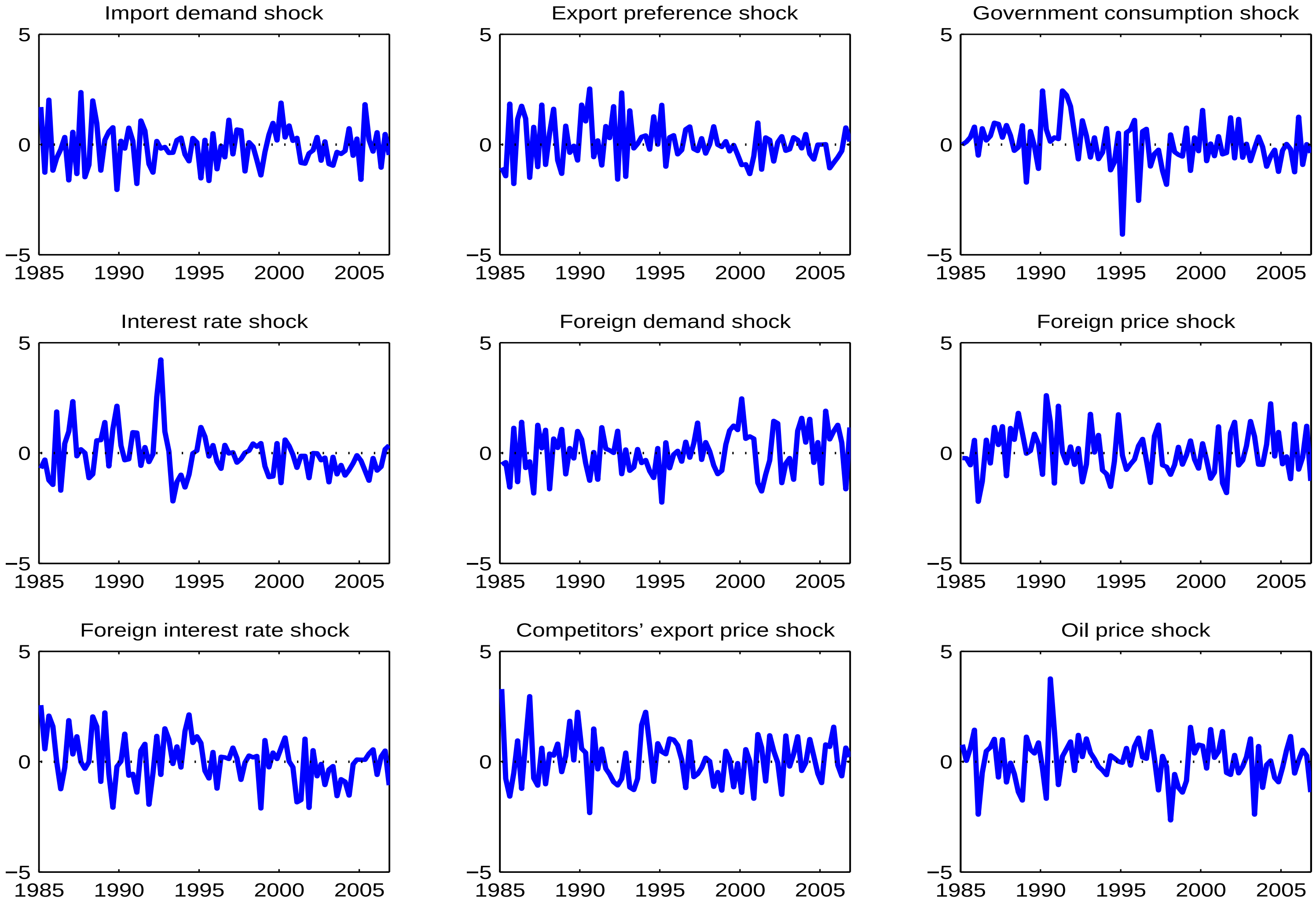
Note: See above.

Figure 6: Smoothed Estimates of the Innovation Component of the Structural Shocks



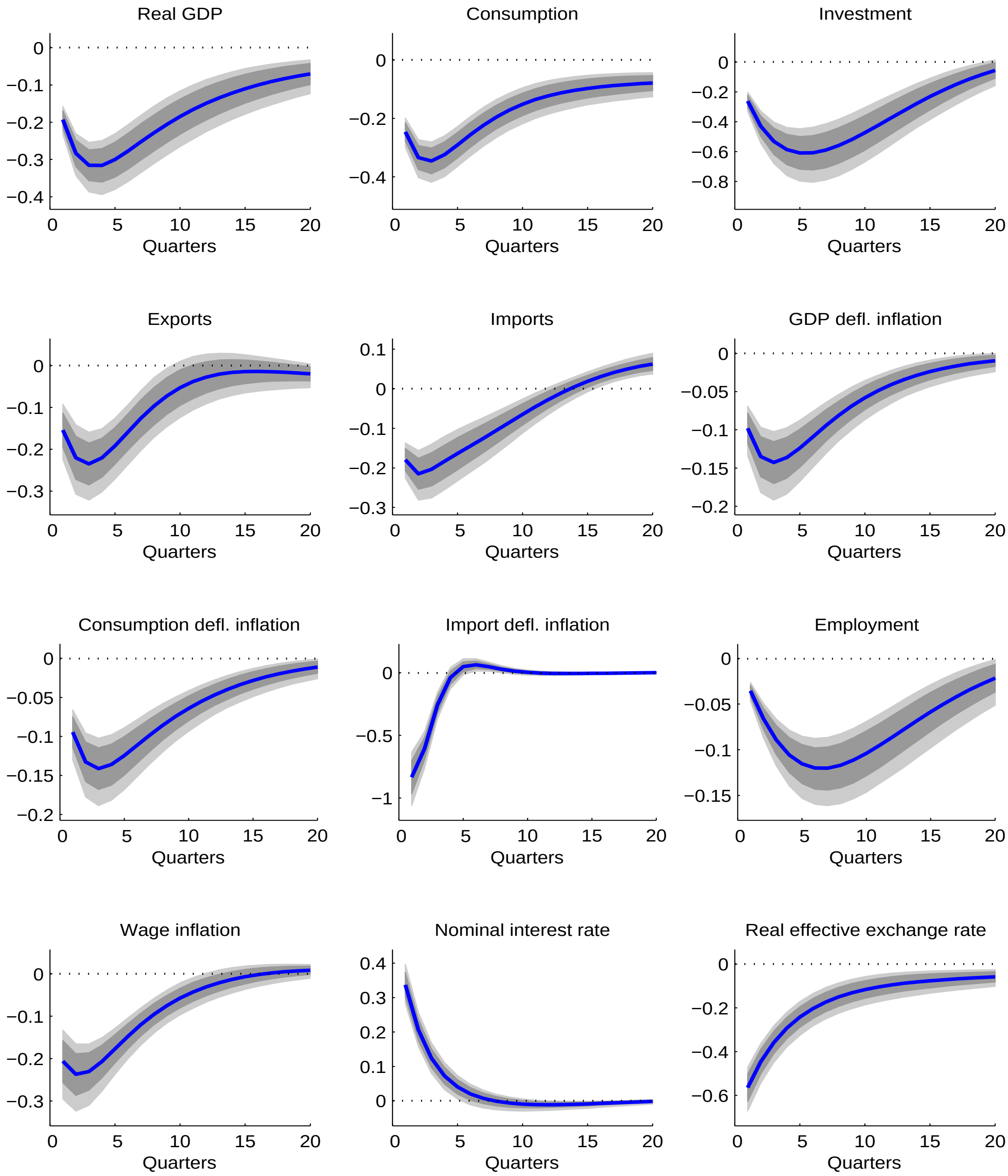
Note: This figure shows the smoothed estimates of the standardised innovation component of the NAWM's structural shocks based on the posterior mode estimates of the model's structural parameters.

Figure 6: Smoothed Estimates of the Innovation Component of the Structural Shocks (continued)



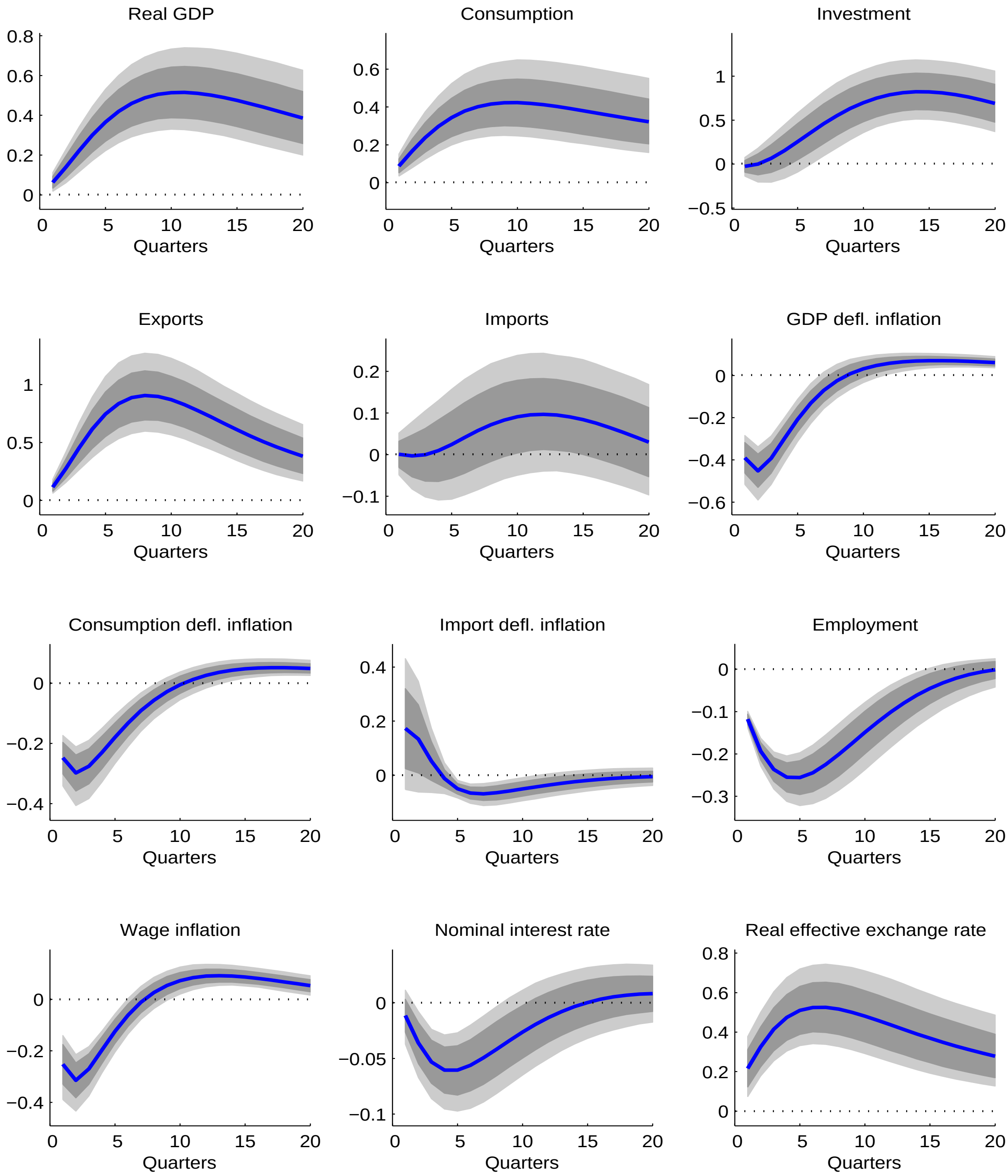
Note: See above.

Figure 7: Impulse Responses to an Interest Rate Shock



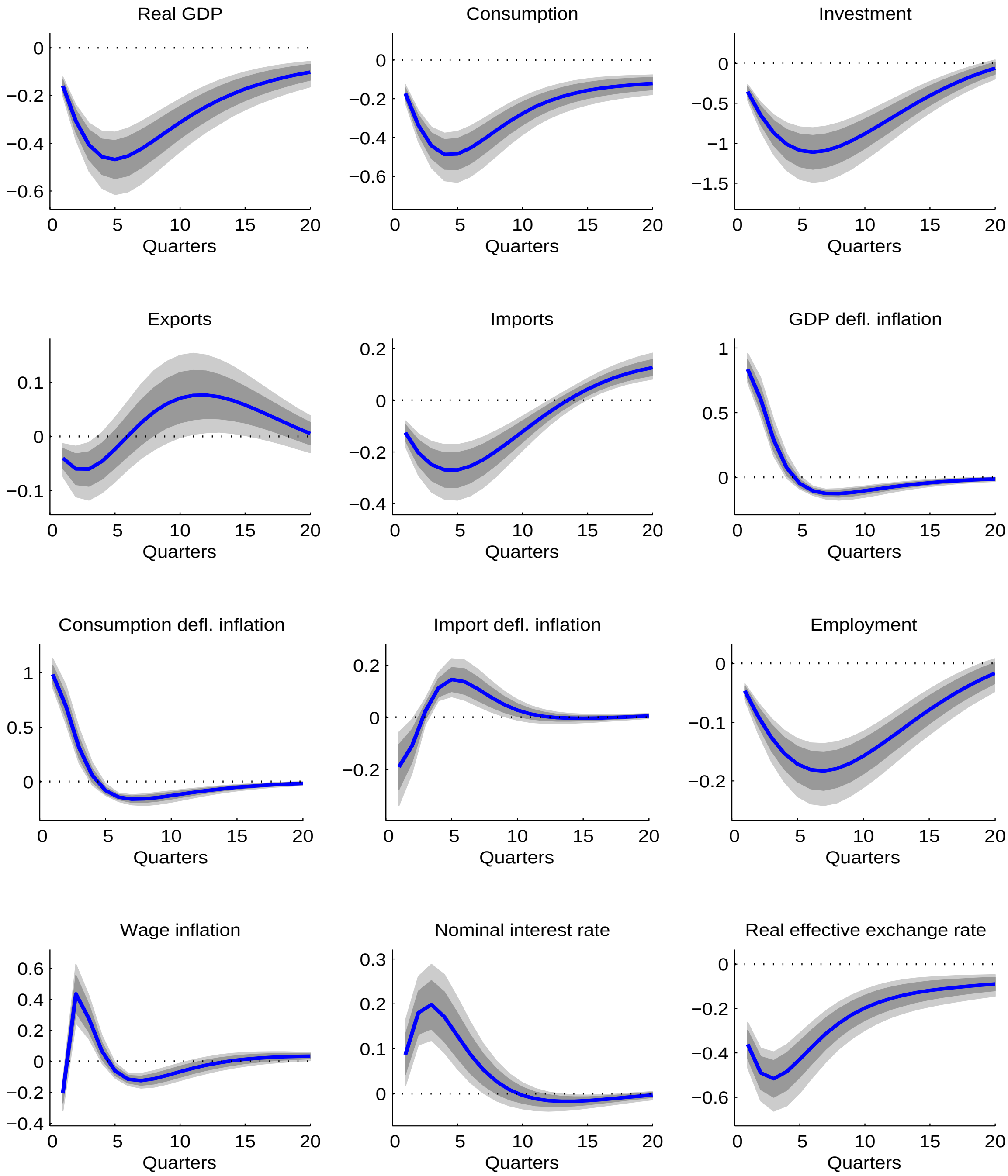
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to an interest rate shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure 8: Impulse Responses to a Transitory Technology Shock



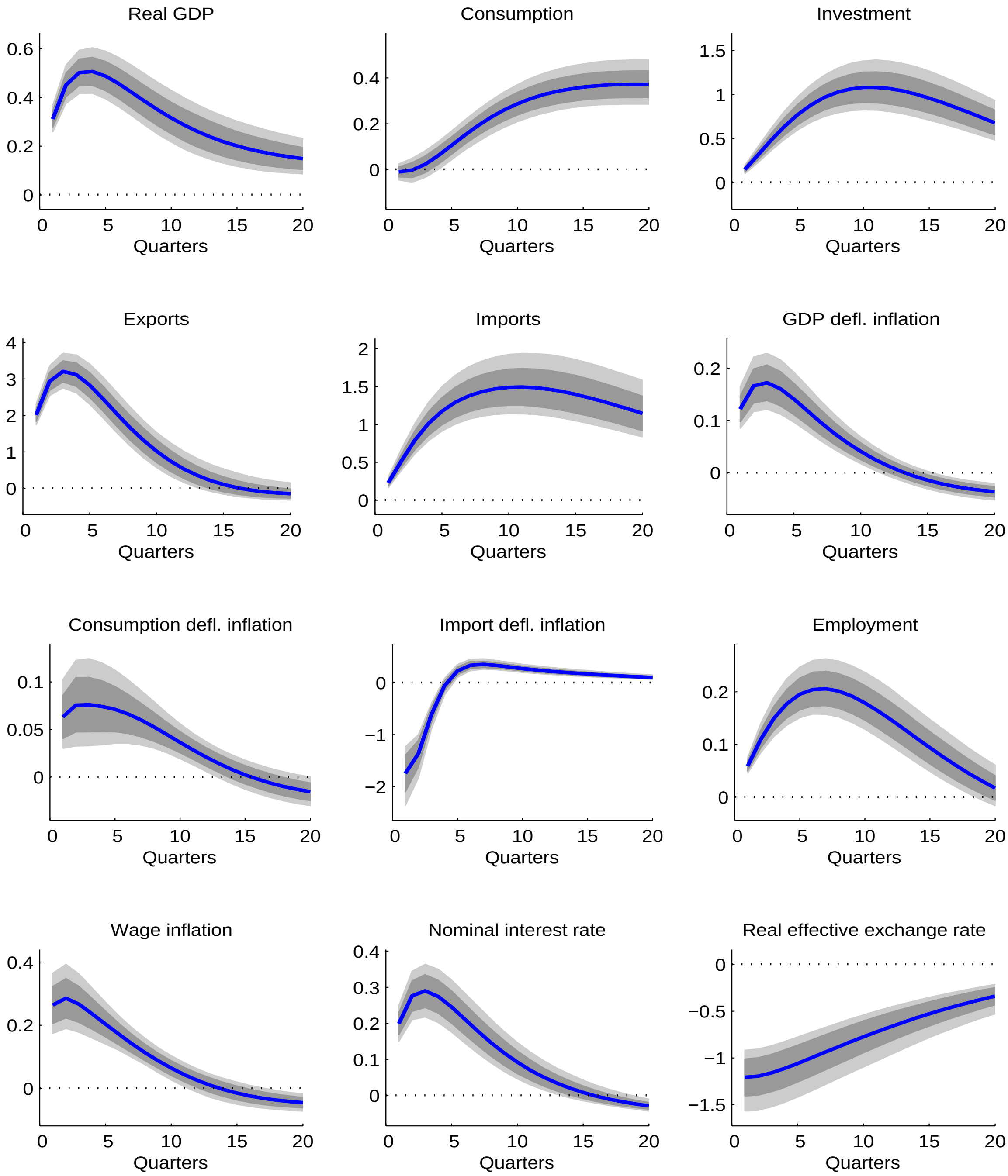
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to a transitory technology shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure 9: Impulse Responses to a Domestic Price Markup Shock



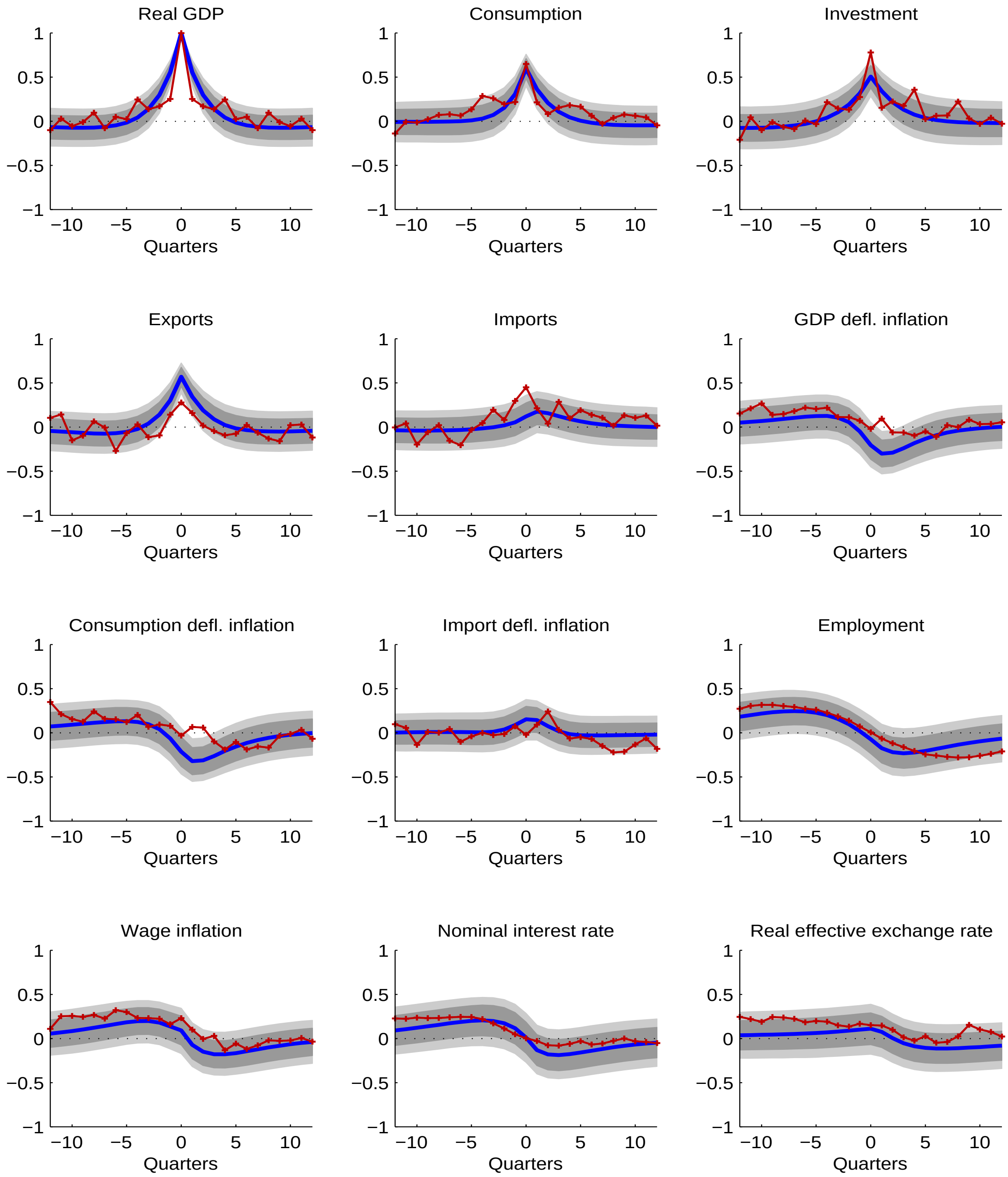
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to a domestic price markup shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure 10: Impulse Responses to an Export Preference Shock



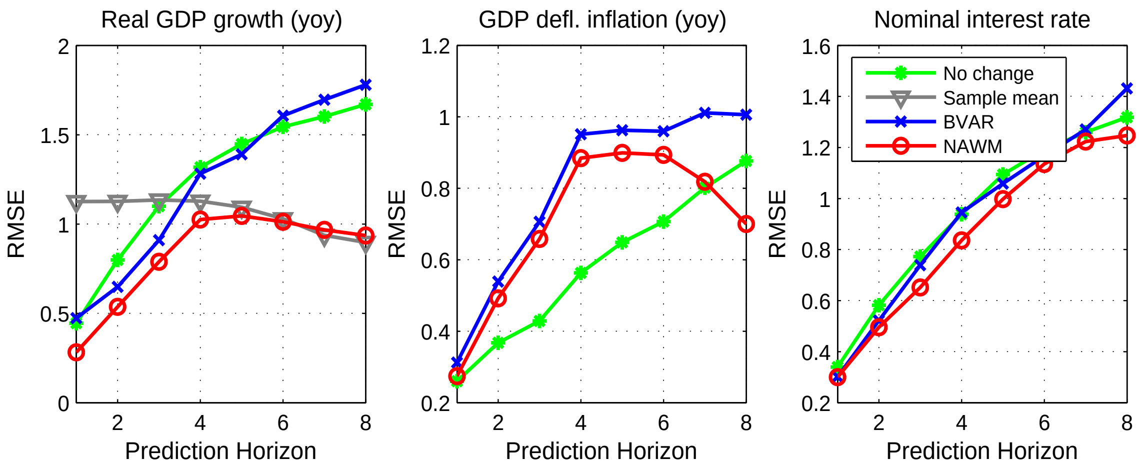
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to an export preference shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure 11: Sample Autocorrelations with respect to Real GDP



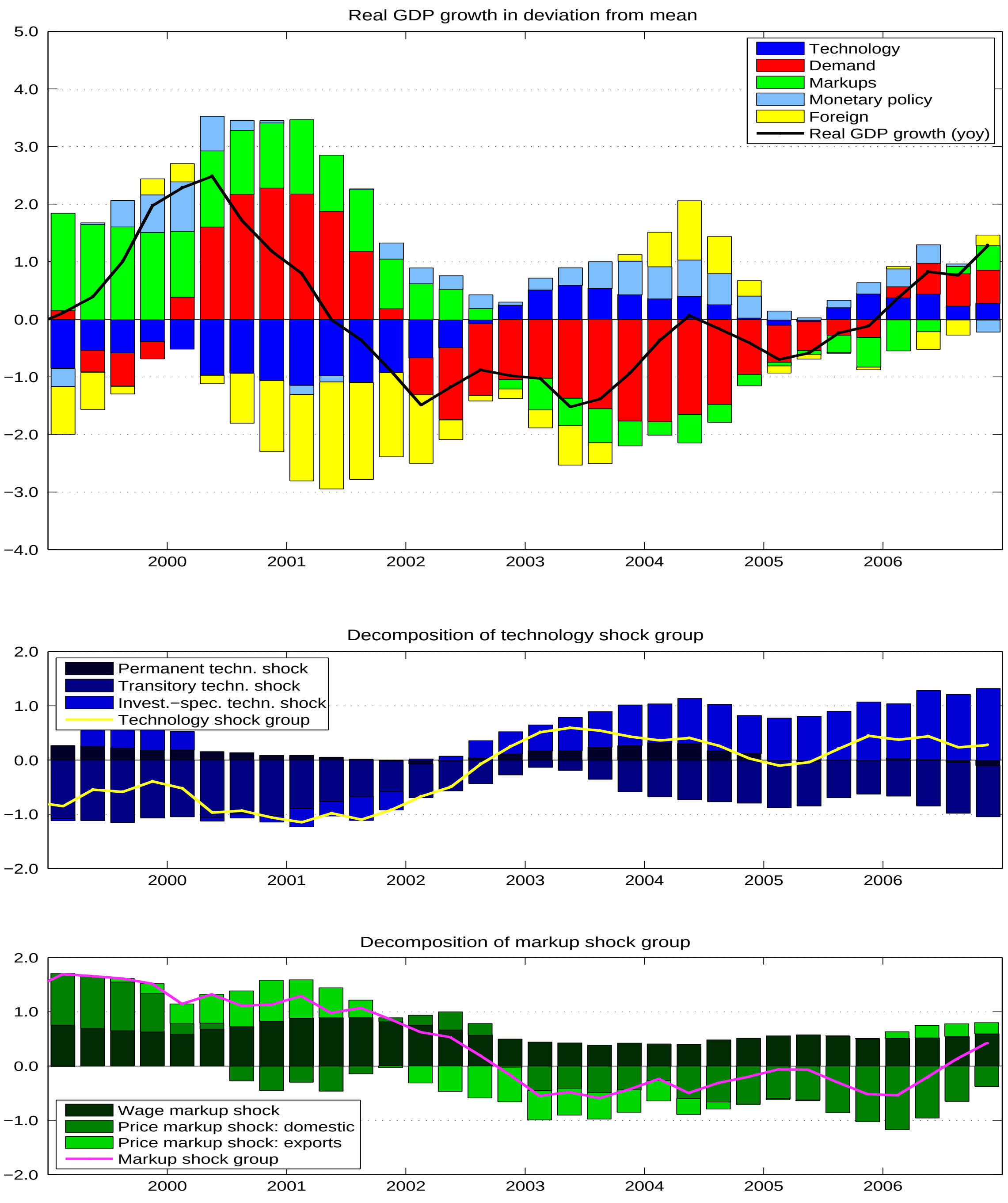
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the NAWM-based sample autocorrelations between real GDP and selected observed variables along with the sample autocorrelations based on the data (red solid line with plus sign markers for the different leads and lags). The results are based on 500 draws from the posterior distribution of the model's parameters using 500 sample paths per parameter draw.

Figure 12: Root-Mean-Squared Errors of Unconditional Forecasts



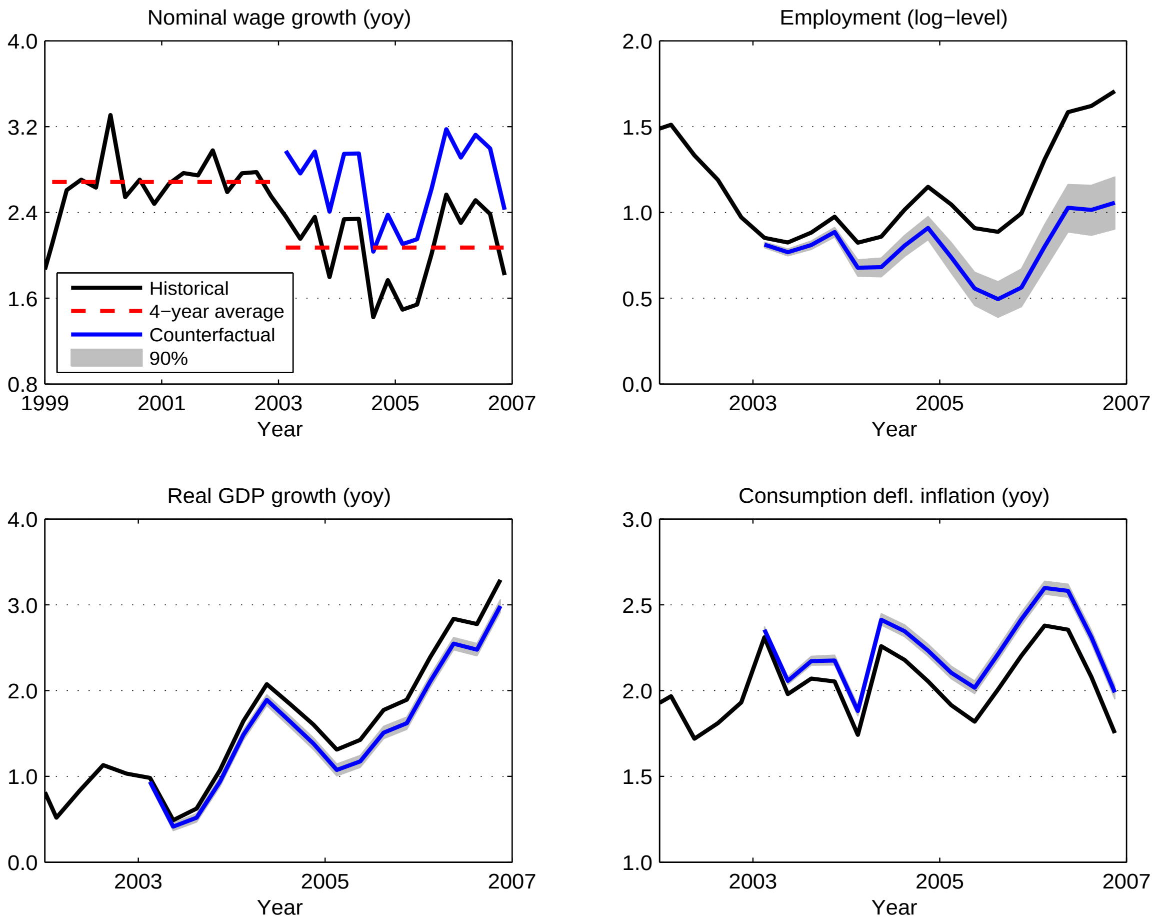
Note: For the NAWM, a BVAR model with steady-state prior and two naïve benchmarks (the random walk and the sample mean), this figure shows the root-mean-squared errors (RMSEs) of unconditional 1 to 8 quarter-ahead forecasts for year-on-year real GDP growth, year-on-year GDP deflator inflation and the annual nominal interest rate. The forecasts have been computed out-of-sample over the period 1998Q4 to 2005Q4, and the point forecasts for computing the RMSEs are given by the means of the predictive distributions. The predictive distributions are based on 500 draws from the posterior distribution of the model's parameters using 500 prediction paths per parameter draw. The models have been re-estimated in the fourth quarter of each calendar year.

Figure 13: Decomposition of Real GDP Growth, 1999-2006



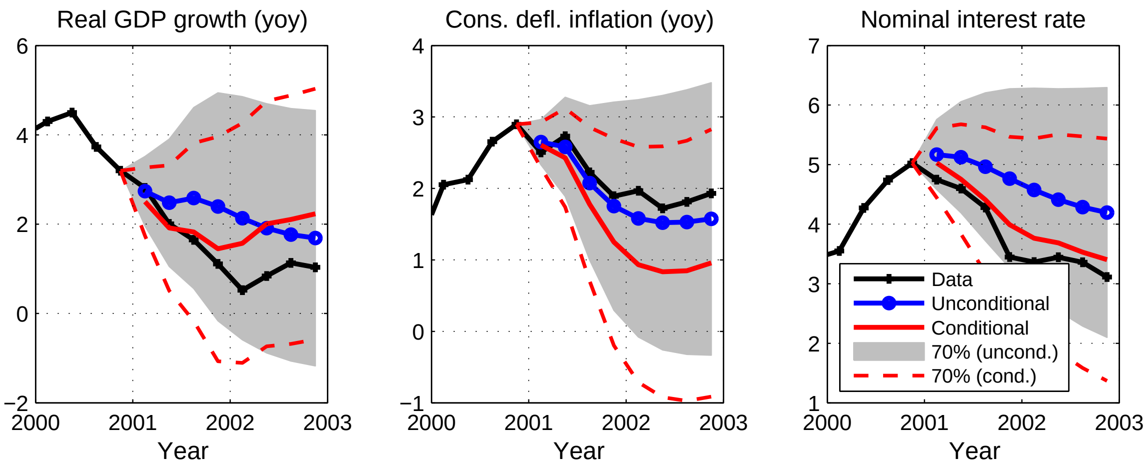
Note: The upper panel of the figure depicts the decomposition of year-on-year real GDP growth over the period 1999-2006 into the contributions of five different shock categories. Real GDP growth is reported in deviation of the steady-state mean growth rate of 2 percent per annum. Residual contributions, which capture the influence of the initial state of the economy and measurement errors, are not shown. The lower two panels show the decomposition of the contributions from two shock groups, namely technology and markup shocks. The decompositions have been computed using the posterior mode estimates of the NAWM's structural parameters.

Figure 14: The Impact of Higher Wage Growth, 2003-2006



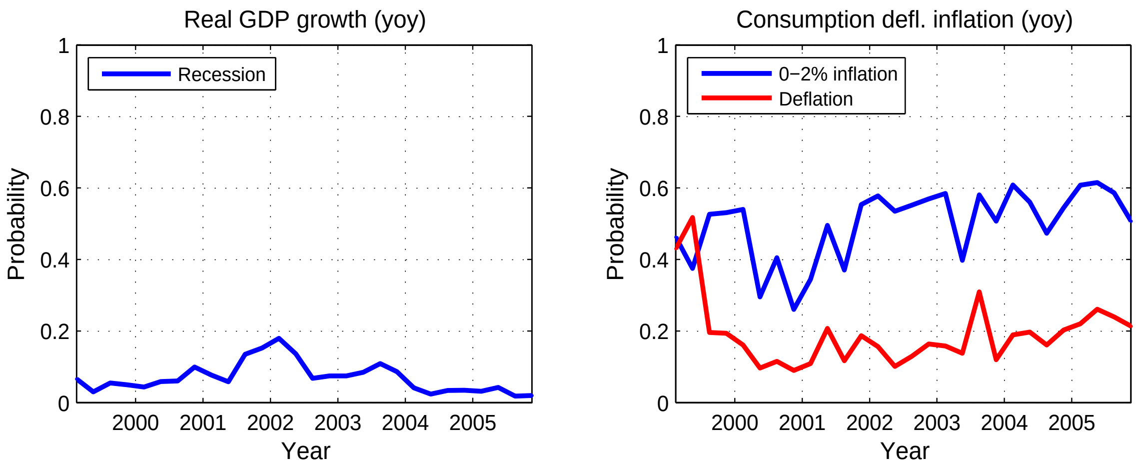
Note: This figure shows the mean effects (blue solid line) and the 90 percent equal-tail uncertainty bands (grey-shaded areas) of a counterfactual increase in year-on-year nominal wage growth over the period 2003-2006. The counterfactual is implemented by adjusting the NAWM's wage markup shock so that the average year-on-year growth rate of nominal wages over the period 2003-2006 equals the average rate of wage growth realised over the 1999-2002 period. All variables are expressed in terms of annual growth rates, except for employment which is reported in log-levels in deviation from a linear trend. The results are based on 5,000 draws from the posterior distribution of the model's structural parameters.

Figure 15: Mean Predictions and Prediction Intervals, 2001Q1-2002Q4



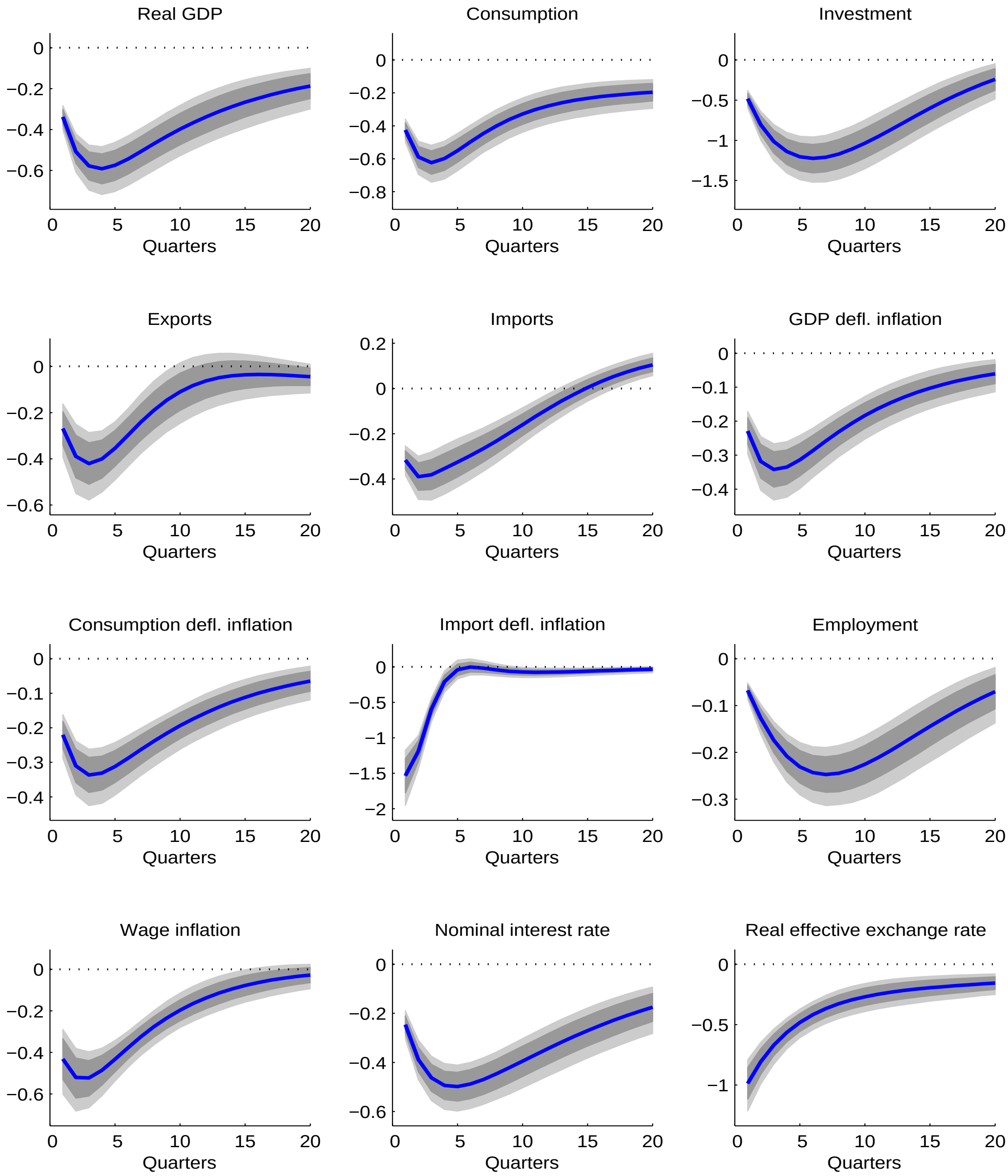
Note: This figure compares the unconditional mean predictions and the unconditional equal-tail 70 percent prediction intervals for year-on-year real GDP growth, year-on-year GDP deflator inflation and the annual nominal interest rate, with those conditioned on developments in the NAWM's foreign variables, including foreign demand. The predictions are made in the period 2000Q4 and extend from 2001Q1 to 2002Q4. The NAWM has been estimated using data until 2000Q4, and the predictive distributions are based on 500 draws from the posterior distribution of the model's structural parameters using 500 prediction paths per parameter draw.

Figure 16: Prediction Event Probabilities, 1999-2005



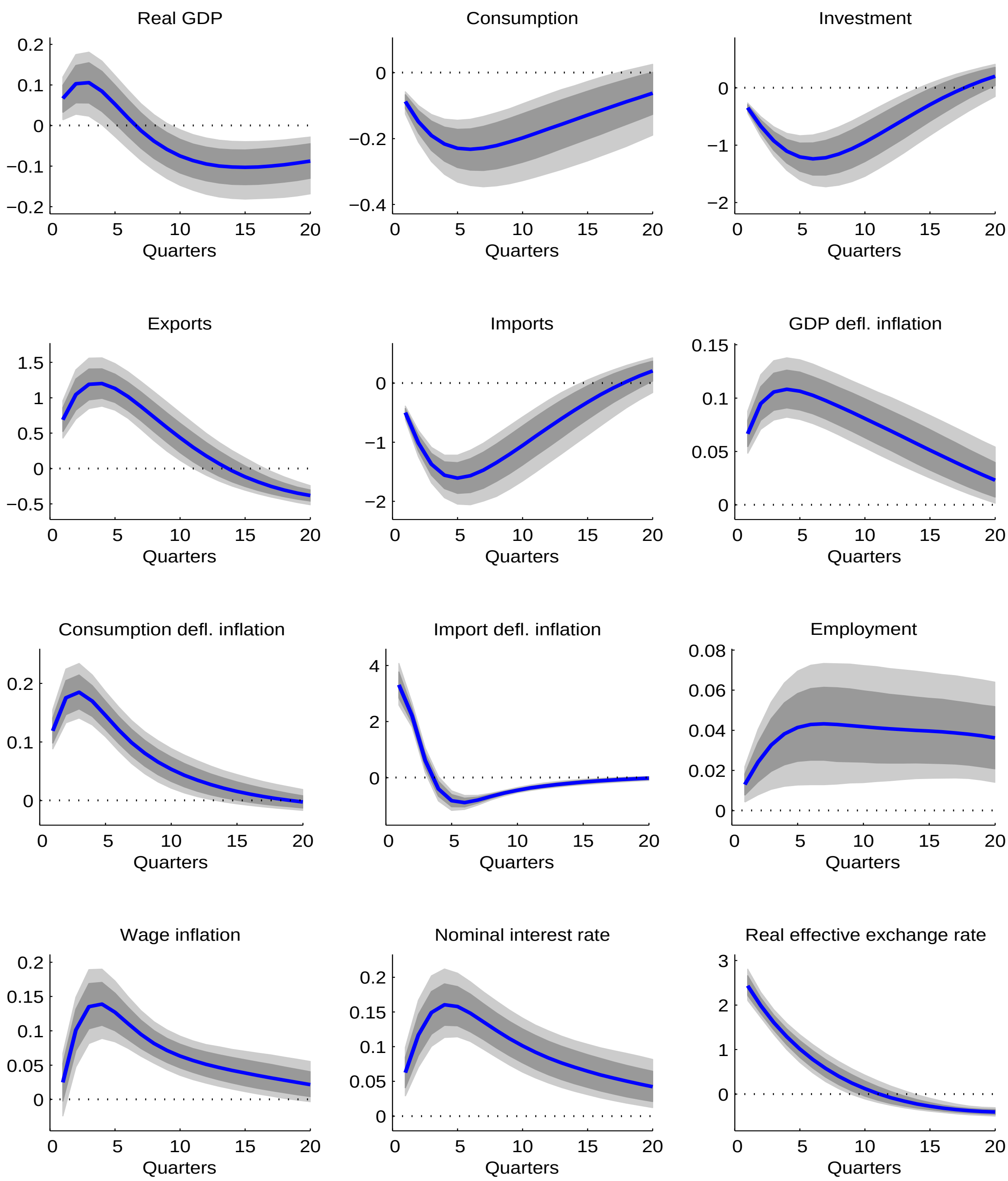
Note: For the predictive densities obtained over the period 1999-2005, this figure shows the probabilities of certain prediction events for year-on-year real GDP growth and year-on-year consumption deflator inflation. The predictions extend up to 8 quarters into the future, and the NAWM has been re-estimated in the 4th quarter of each calendar year. The predictive distributions are based on 500 draws from the posterior distribution of the model's structural parameters using 500 prediction paths per parameter draw.

Figure A.1: Impulse Responses to a Domestic Risk Premium Shock



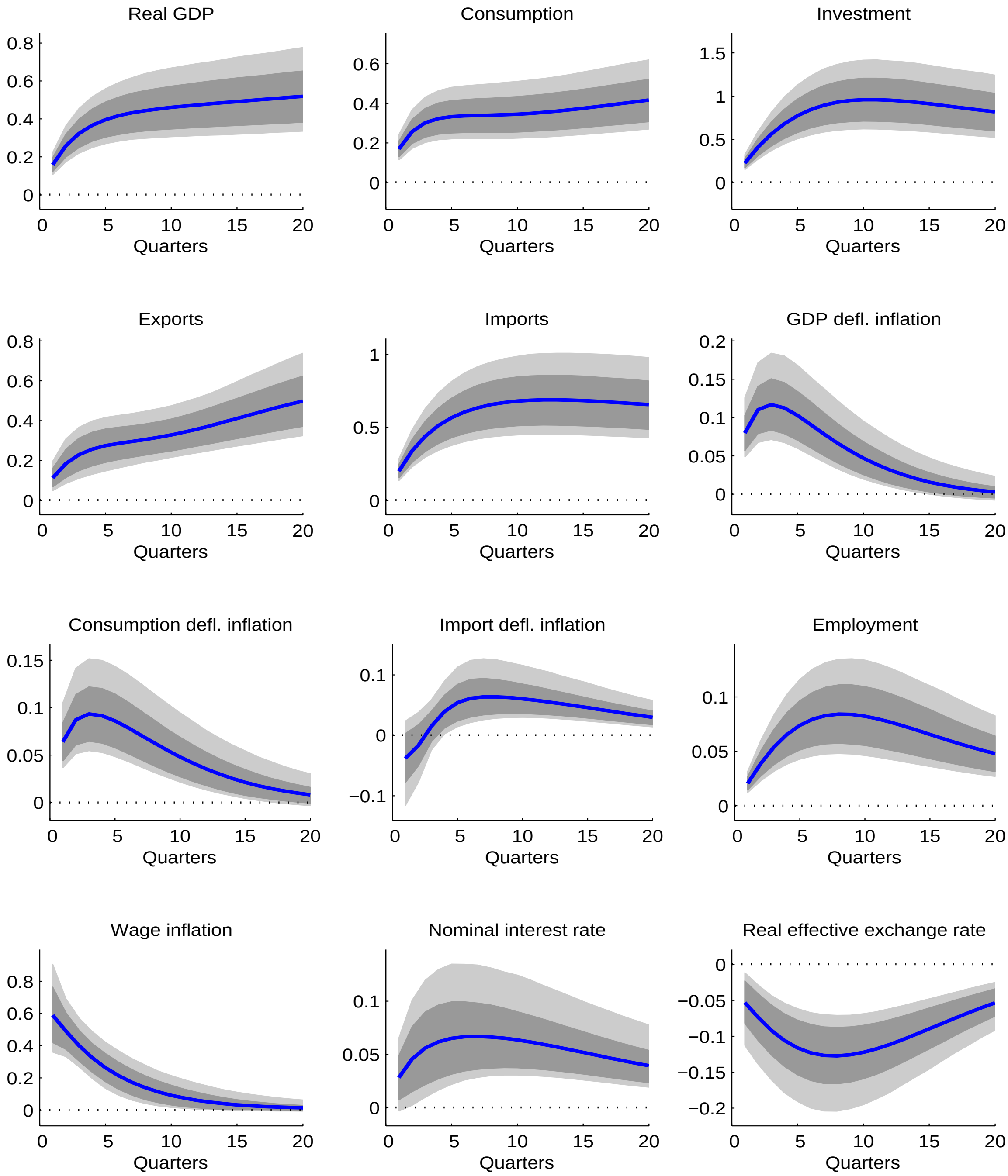
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to a domestic risk premium shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure A.2: Impulse Responses to an External Risk Premium Shock



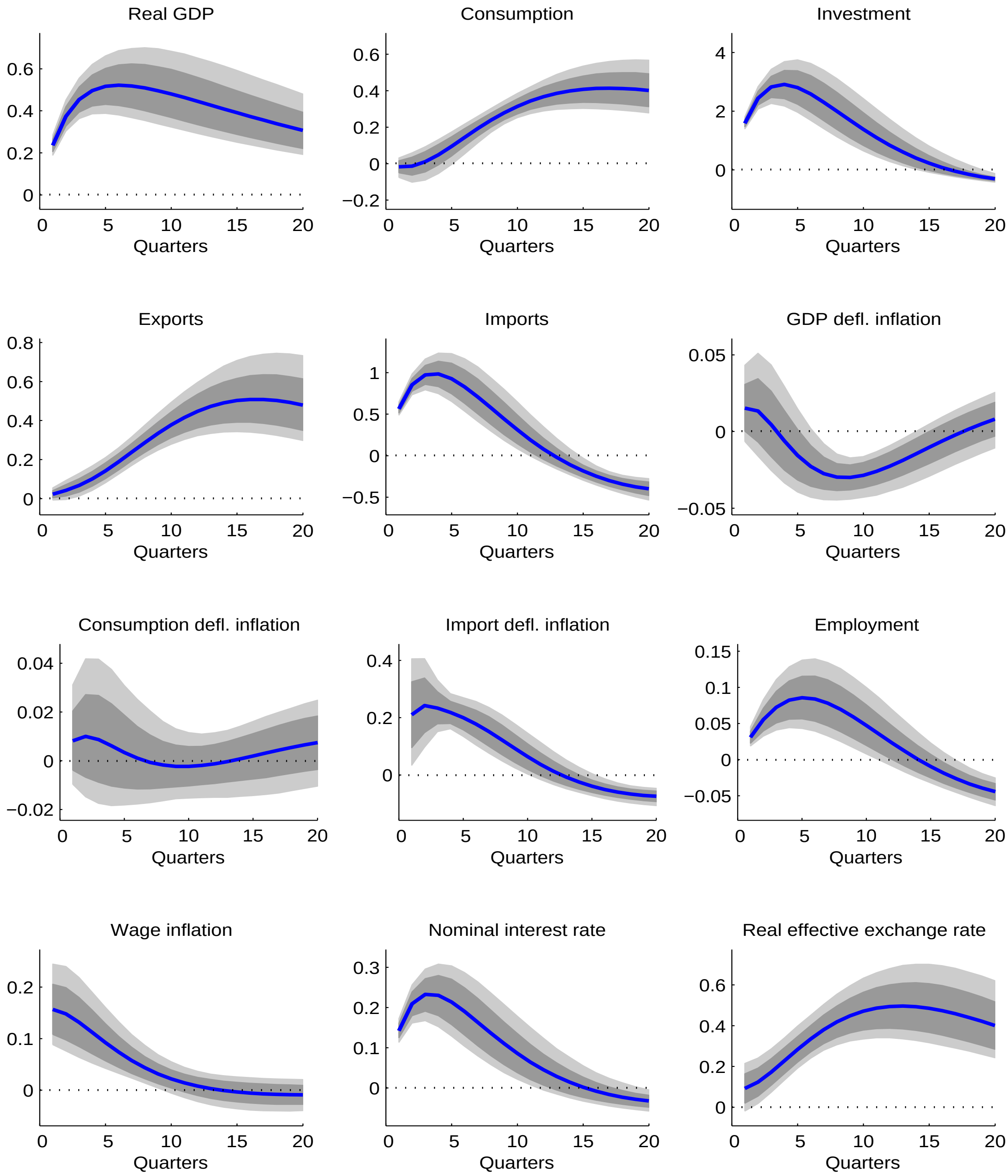
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to an external risk premium shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure A.3: Impulse Responses to a Permanent Technology Shock



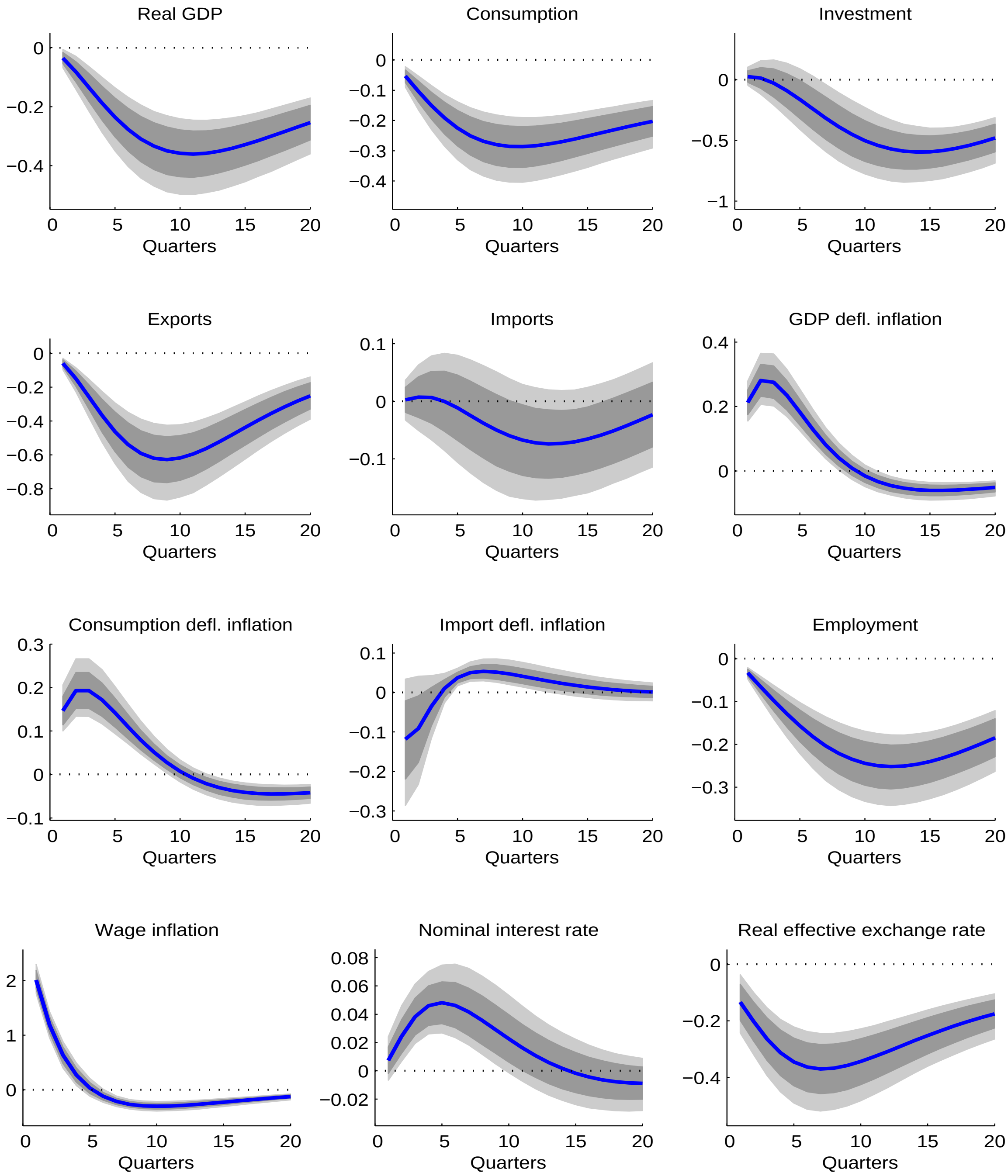
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to a permanent technology shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure A.4: Impulse Responses to an Investment-Specific Technology Shock



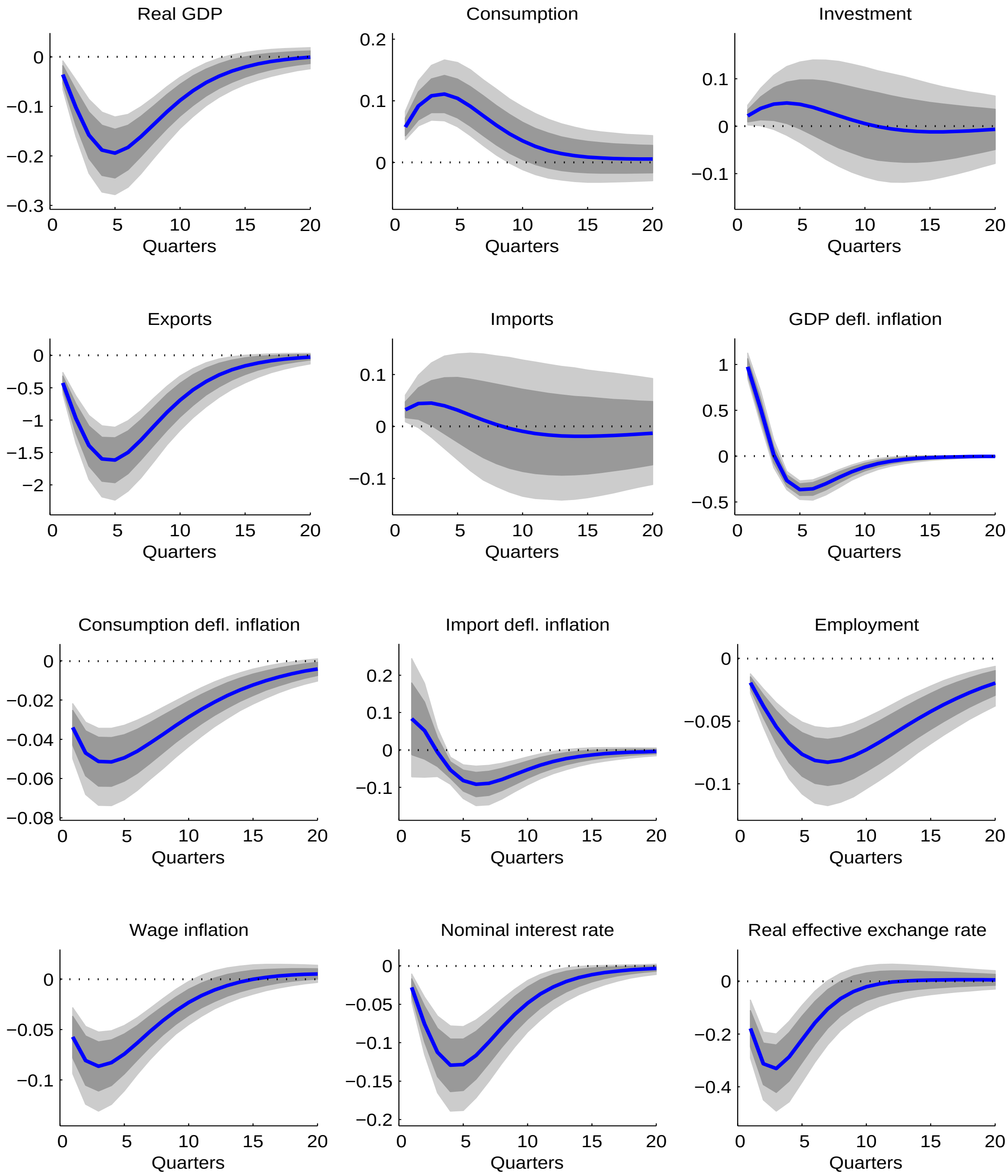
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to an investment-specific technology shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure A.5: Impulse Responses to a Wage Markup Shock



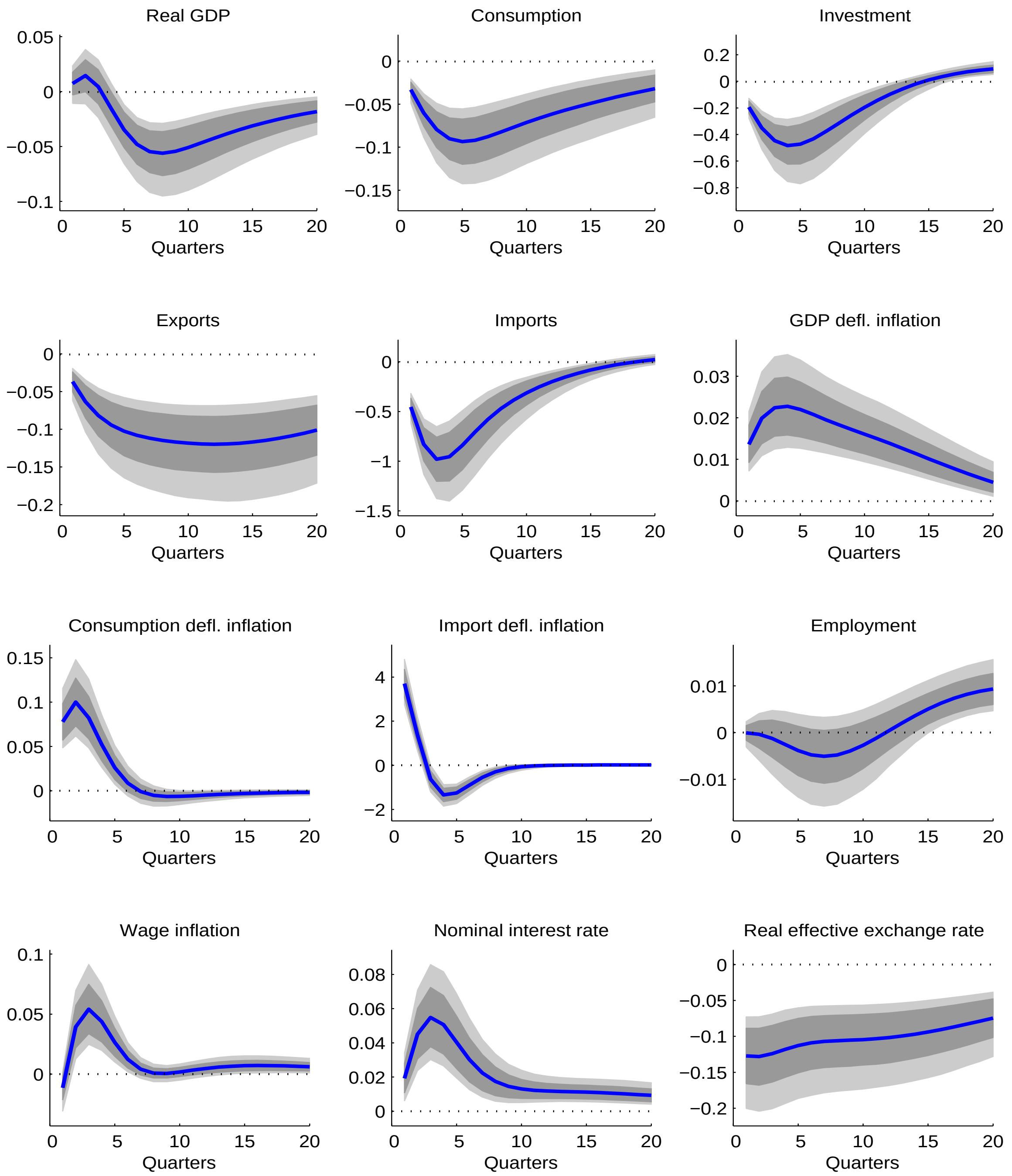
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to a wage markup shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure A.6: Impulse Responses to an Export Price Markup Shock



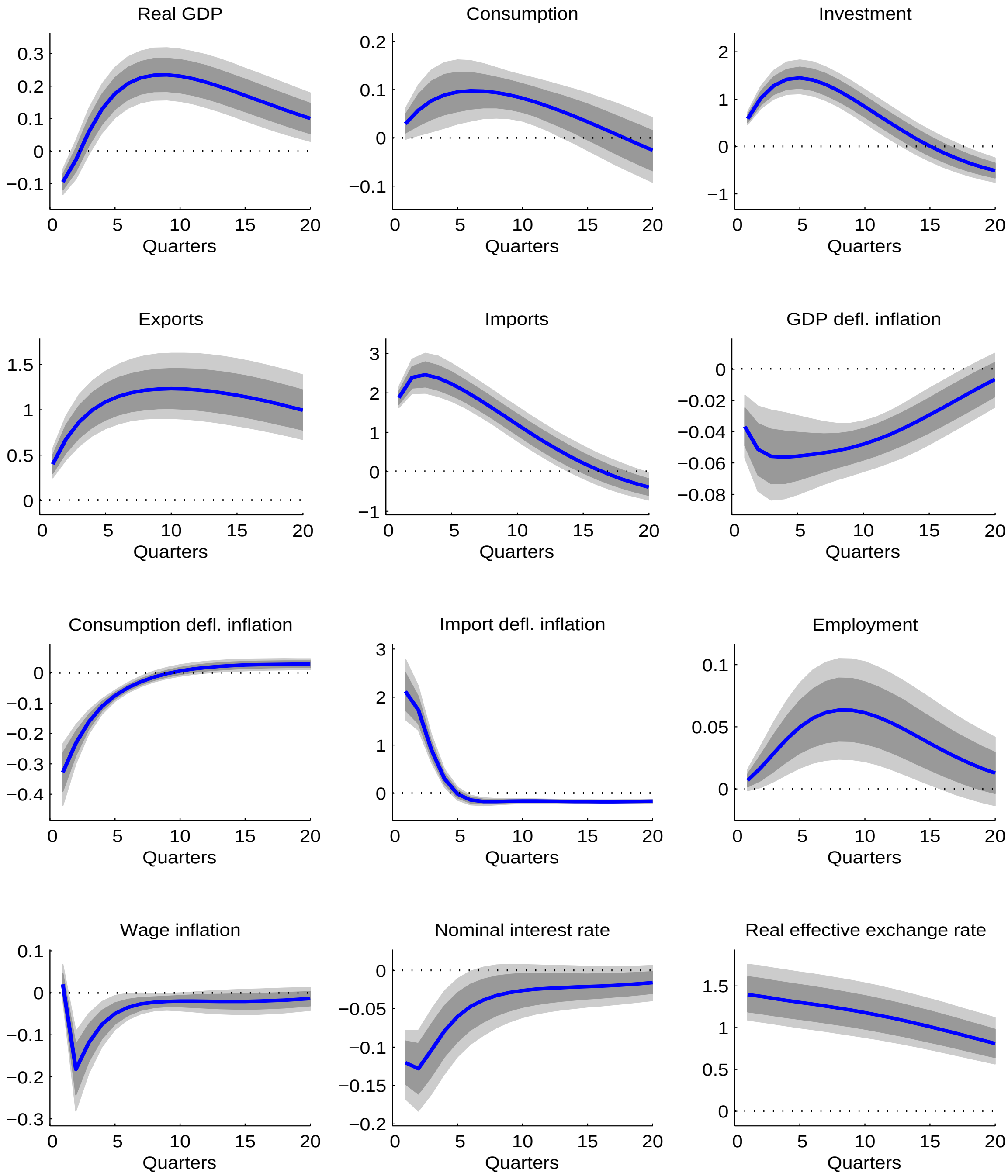
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to an export price markup shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure A.7: Impulse Responses to an Import Price Markup Shock



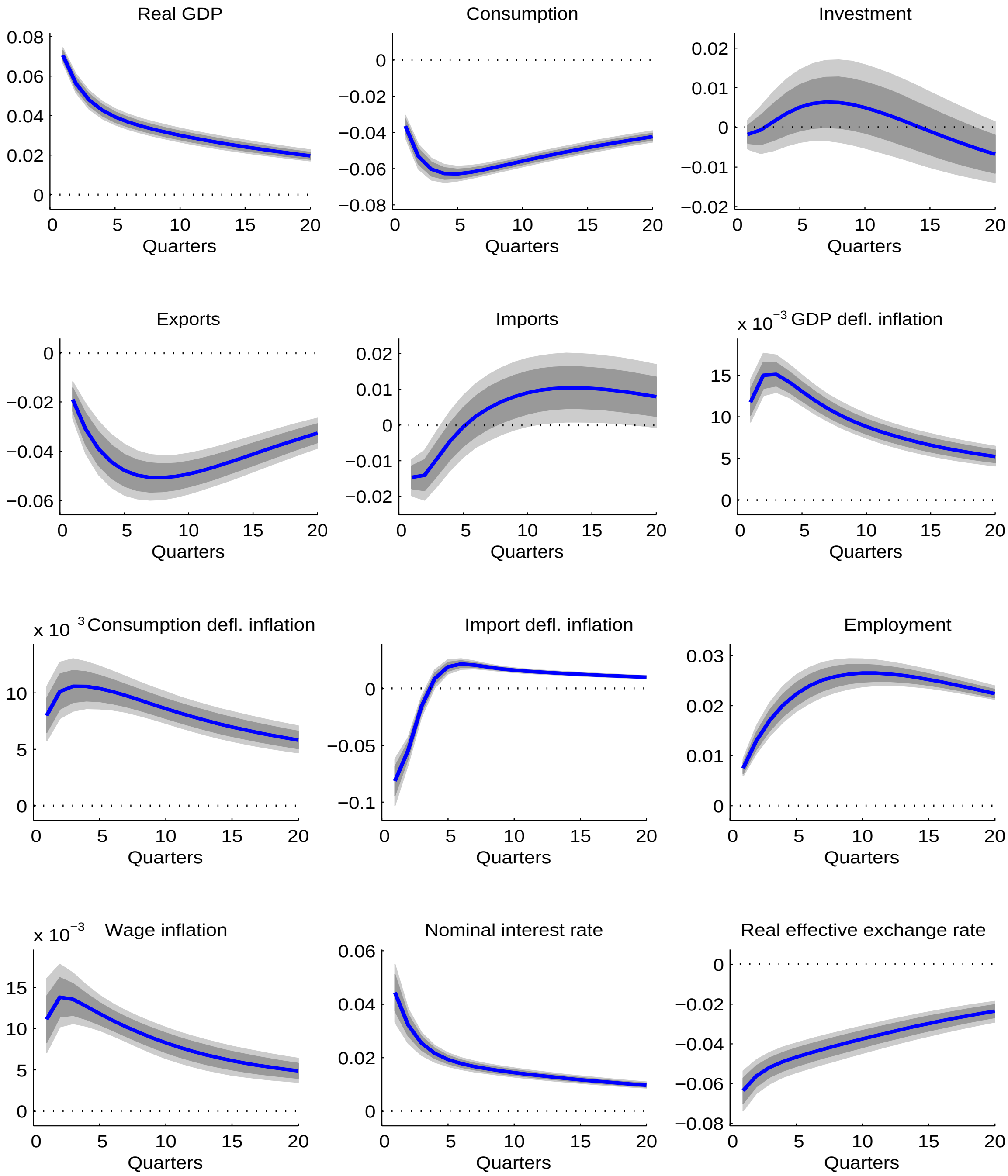
Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to an import price markup shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure A.8: Impulse Responses to an Import Demand Shock



Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to an import demand shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

Figure A.9: Impulse Responses to a Government Consumption Shock



Note: This figure shows the mean (blue solid line) and the 70 and 90 percent equal-tail uncertainty bands (grey-shaded areas) of the impulse responses of selected observed variables to a government consumption shock equal to one standard deviation. All impulse responses are reported as percentage deviations from the NAWM's non-stochastic steady state, except for the impulse responses of the inflation and interest rates which are reported as annualised percentage-point deviations. The results are based on 5,000 draws from the posterior distribution of the model's parameters.

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