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In 2004 all publications will carry a motif taken from the €100 banknote.

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2 Dipartimento di Economia Politica e Metodi Quantitativi, University degli Studi di Pavia, Via S. Felice, 5, 27100 Pavia, Italy, gascari@eco.unipv.it.
3 Department of Economics, University of Warwick, Coventry CV4 7AL, UK, n.rankin@warwick.ac.uk.

Address
Kaiserstrasse 29
60311 Frankfurt am Main, Germany

Postal address
Postfach 16 03 19
60066 Frankfurt am Main, Germany

Telephone
+49 69 1344 0

Internet
http://www.ecb.int

Fax
+49 69 1344 6000

Telex
411 144 ecb d

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Abstract

In the "perpetual youth" overlapping-generations model of Blanchard and Yaari, if leisure is a "normal" good then some agents will have negative labour supply. We suggest a solution to this problem by using a modified version of Greenwood, Hercowitz and Huffman's utility function. The modification incorporates real money balances, so that the model may be used to analyse monetary as well as fiscal policy. In a Walrasian version of the economy, we show that increased government debt and increased government spending raise the interest rate and lower output, while an open-market operation to increase the money supply lowers the interest rate and raises output.

JEL classification: D91, E63

Keywords: Blanchard-Yaari overlapping generations, endogenous labour supply

Non-Technical Summary

Building on work by Yaari, Blanchard (*Journal of Political Economy* 1985) showed how a simple model of overlapping generations (OLG) in which agents have uncertain lifetimes could be constructed. The advantages of Blanchard's framework over other OLG models are that the average length of life of a typical agent can be parameterised, and that the special case of infinite lives is nested within it. This is achieved while retaining elegance and simplicity at the aggregate level. A key assumption is that an agent's probability of death is a constant, independent of the agent's age. This has caused the model sometimes to be referred to as one of "perpetual youth".

The perpetual youth framework has found many useful applications. In particular it implies that "Ricardian Equivalence" does not hold, and so provides a fruitful structure for analysing the effects of government debt and deficits. So far it has most often been used in the long-run context of growth and capital accumulation, but it is also beginning to be used in the short-run context of business cycles. Here, changes in labour inputs are important, and hence it becomes desirable to model households as having endogenous labour supply. In this paper, we point out a potential problem with making labour supply endogenous in a perpetual youth model. The problem is that some agents' labour supply is then likely to be negative. Negative labour supply makes no sense, and of course should be ruled out by imposing a non-negativity constraint. However, if this is done, then the fact that the constraint is binding for some agents and not for others causes the perpetual youth model to lose its elegance and simplicity at the aggregate level.

The intuitive explanation for why there is a "negative labour supply" problem is as follows. With most specifications of agents' preferences, leisure is a "normal" good, i.e. the wealthier the agent is, the more leisure she demands. In the perpetual youth model, the older the agent is, the more financial wealth she has. Moreover, the age distribution is such that there is no upper bound on age. Therefore at any point in time there are always some arbitrarily wealthy agents in the population. If leisure is a normal good, then there are guaranteed to exist agents who want to consume more leisure than is feasible given their time endowment. Otherwise stated, there are bound to be some agents who want to supply negative amounts of labour.

The main part of the paper is devoted to suggesting a solution to this problem. Our proposal is to use a specification of households' preferences adapted from Greenwood, Hercowitz and Huffman (*American Economic Review*, 1988) (GHH) which makes labour supply independent of wealth. We extend GHH by including real balances in the utility function. This is so that the model can be used to address questions of monetary policy, and not just of fiscal policy. We first explain the new preference structure, derive its implications for aggregate relationships in a "perpetual youth" environment, and embed these relationships in a simple flexible-price, dynamic general equilibrium model. We then explore the model's implications for some standard fiscal and monetary policy experiments.

We find that, with the proposed utility function, additive separability between leisure and real balances cannot be preserved. In consequence, policy changes which increase real balances also increase labour supply. This provides a channel for policy to affect output, even though prices and wages are fully flexible. A permanent increase in the stock of government debt, for example, partially crowds out real balances, because (given that "Ricardian Equivalence" does not hold) it permanently raises the real and nominal interest rate. Thereby it also causes a contraction in output. On the other hand, an open market operation to increase the money supply acts just like a permanent decrease in the stock of government debt, and so raises output. The model hence exhibits a "liquidity effect" of monetary policy. Thirdly, we show that a permanent balanced-budget increase in government spending (which leaves the stock of debt unchanged) raises the interest rate and contracts output.

1. Introduction

Building on Yaari (1965), Blanchard (1985) showed how a simple model of overlapping generations (OLG) in which agents have uncertain lifetimes could be constructed. The advantages of Blanchard's framework over other models of overlapping generations (such as that of Diamond (1965)) are that the average length of life of a typical agent can be parameterised, and that the special case of infinite lives is nested within it. This is achieved while retaining elegance and simplicity at the aggregate level. A key assumption in achieving this elegance is that an agent's probability of death is a constant, independent of the agent's age. This has caused the model sometimes to be referred to as one of "perpetual youth" (e.g. by Blanchard and Fischer, 1989). The perpetual youth framework has found many useful applications. Most obviously, and like other OLG models, it implies that "Ricardian Equivalence" does not hold, and so it provides a fruitful structure for analysing fiscal policy, in particular the effects of government debt and deficits. Many of these applications (as in Blanchard's original 1985 paper itself) have been in the "long-run" context of growth and capital accumulation. However, as microeconomic foundations have become more widely used also in "short-run" models of business cycle fluctuations, a number of researchers have seen the advantage of departing from the infinitely-lived agent (sometimes called the "representative agent") assumption in this context too.

In the short-run context, changes in labour inputs are typically at least as important as changes in capital inputs in explaining output movements, and hence it becomes desirable to model the labour market, and as part of this, the labour supply, in some detail. In the long-run growth context, labour supply is commonly treated as exogenous and fixed by the population size. That is, households are assumed to obtain no utility from leisure, and so they supply their given time endowments to the labour market completely inelastically. By contrast in short-run models it is desirable to assume that households obtain utility from leisure, so that their optimal labour supply decision is then in general a function of current and future real wages, interest rates, and wealth levels. In this paper, we point out a potential problem with making the labour supply decision endogenous in a perpetual youth model. The problem can be explained very simply: it is that some agents' labour supply is likely then to be negative. Negative labour supply makes no sense, and of course should be ruled out by imposing a non-negativity condition. However, if this is done, then the fact that the non-negativity constraint is sometimes binding and sometimes not causes the perpetual youth model to lose its elegance and simplicity at the aggregate level.

The intuitive explanation for why there is a "negative labour supply" problem is as follows. With most specifications of agents' preferences, leisure is a "normal" good, i.e. the wealthier the agent is, the more leisure she demands, and therefore the less labour she supplies. In the perpetual youth model, the older the agent is, the more "nonhuman" wealth she has. Moreover, the age distribution is such that there is no upper bound on age: there are always some arbitrarily old agents alive at any point in time. Therefore at any point in time there are always some arbitrarily wealthy agents in the population, and, if leisure is a normal good, these agents will want to consume more leisure than is feasible given their time endowment. Otherwise stated, such agents will want to supply negative amounts of labour. This problem is particular to the perpetual youth model because of the lack of an upper bound on age. In a Diamond-type model, with *n*-period lives, then while there may be a question of negative labour supply, it is not guaranteed that there are agents for whom the preferred labour supply is negative.

The main part of the present paper is devoted to suggesting a solution to this problem. Our premise is that there are many potential interesting applications for a perpetual youth model with endogenous labour supply, and that it is hence desirable and important to have a version of this model which avoids the negative labour supply problem. Indeed, there is a small but growing number of published papers which already make use of perpetual youth models with endogenous labour supply (for example, Heijdra and Ligthart (2000), Cavallo and Ghironi (2002), Smets and Wouters (2002)). These are all, at least potentially, vulnerable to the negative labour supply problem: it appears to have gone unrecognised in the literature so far.

The objection may at this point be raised that the problem has been overstated. Even if leisure is a normal good, it does not follow from this alone that very wealthy agents must have negative labour supply. For example, depending on preferences, labour supply may tend asymptotically to zero (or to some positive lower bound) as wealth tends to infinity. However, here it needs to be remembered that a number of restrictions are already placed on agents' preferences by the requirement of the perpetual youth model that aggregate equivalents of individual demand and supply functions should exist. In particular, agents' demand functions for goods and leisure need to be linear in wealth. This ensures that aggregate demands for consumption and leisure can be written as functions only of aggregate wealth (and of relative prices), and that they are independent of the wealth distribution across agents of different ages. Such a condition rules out preferences which would make labour supply tend asymptotically to zero, since labour supply would then be nonlinear in wealth. A further objection may be that all that is needed is to take proper account of the non-negativity condition on labour supply. However, this will introduce a "kink" in the individual's labour supply as a function of wealth, and is thus itself a form of nonlinearity. It would then no longer be possible to write aggregate labour supply as a function of aggregate wealth alone: there would exist a threshold level of wealth above which an individual's labour supply would be zero, and in solving for the general equilibrium we would need to keep track of the fraction of agents whose wealth exceeded the threshold level.

Our proposed solution to the problem is to use a specification of households' preferences which makes labour supply independent of wealth. The specification which we shall use is adapted from Greenwood, Hercowitz and Huffman (1988) (henceforth "GHH"). It extends GHH in that it incorporates real balances into the utility function. Our motive for this extension is that, in keeping with the aim of providing a framework which can potentially be applied to short-run, business cycle, issues, we want to be able to address questions of monetary policy, and not just of fiscal policy. Moreover we want to create a setting into which nominal rigidities can ultimately be introduced, which means that money and nominal variables need to be brought into the model. What the rest of the paper does is to explain the new preference structure, derive its implications for aggregate relationships in a "perpetual youth" environment, and then embed this OLG structure into a simple flexible-price, dynamic general equilibrium model. The implications of this type of economy for some standard fiscal and monetary policy experiments are then explored. Our aim is mainly to establish the feasibility of constructing a perpetual youth model with endogenous labour supply which avoids the problem mentioned above, and to show that it provides a basis for a richer analysis of monetary and fiscal policy than is possible in infinitely-lived agent models. Our specific results concerning monetary and fiscal policy we interpret not as final statements about the real world, but as merely laying down some baseline properties against which those of a more fully-featured business cycle model, containing for example nominal rigidities, could be compared.

The main findings are that, first, "monetary GHH preferences" do make possible a tractable OLG model which avoids the negative labour supply problem. When we examine the effects of policy, the central experiment of interest concerns the impact of a step increase in the stock of government debt, injected via a temporary tax cut. Since "Ricardian Equivalence" does not hold, we would expect this to have real effects, as is indeed the case.

We show that it raises the real interest rate, since, as in any typical OLG model, it raises current demand for goods relative to future demand for goods, by redistributing wealth from future to current generations. However, it might have been thought that there would be no impact on output, because, although government debt now adds to households' net wealth, labour supply in our model is independent of wealth, and hence the level of the only input to production would be unchanged. In fact, we find that output falls. The cause is the way money enters the model. It turns out that the only reasonable way to bring real balances into the utility function is in such a manner that real balances and consumption are complements. The rise in the real (and nominal) interest rate depresses equilibrium holdings of real balances, which in our model lowers the marginal utility of consumption and discourages labour supply. Although we do not argue that such an effect is likely to be empirically large, we think it is of a plausible nature and demonstrates an interesting interaction between the fiscal and monetary sides of the economy.

A second policy measure examined is an open market operation (OMO). In dynamic general equilibrium models with infinitely-lived agents the mechanism by which the money supply is typically changed is via a tax handout. This is acknowledged to be much less realistic than the alternative of the OMO, but there is no point in modelling the latter, since an OMO can be decomposed into a tax-financed money supply increase plus a tax-financed reduction in government debt, and the second has no additional effect when Ricardian Equivalence holds. In the present model, raising the money supply by a tax cut and by an OMO are not equivalent. The former turns out to have no real effect - unsurprisingly, since prices are flexible. The latter is just the reverse of the policy change described in the previous paragraph. Hence the present model provides an example of how a money supply increase can lower the interest rate (the "liquidity effect") and also raise output, through the complementarity mechanism mentioned above.

The final policy shock we look at is a balanced-budget increase in government spending on goods and services. This is found to increase the real interest rate and depress output. It hence crowds out consumption by more than 100%. The mechanism causing the increase in the interest rate turns out to be the difference in effect of the fall in consumption on, on the one hand, the demand for money as a medium of exchange, and, on the other hand, the demand for money as a store of value. The distinction between these two roles of money is something which lies at the heart of the model. Our method of solution draws attention to these roles, in attempting to reveal the workings of the economy more intuitively.

The structure of the remainder of the paper is as follows. In Section 2 we present the utility function which we use, and derive individual and aggregate household behaviour from it; in Section 3 we define the general equilibrium of the model and show how it is determined; and in Section 4 we study the effects of a variety of monetary and fiscal policy measures. Section 5 concludes.

2. Households with Uncertain Lifetimes and Wealth-Independent Labour Supply

We use a discrete-time version of Blanchard's (1985) overlapping-generations structure, as in Frenkel and Razin (1987). Agents have an exogenous probability, q ($0 < q \leq 1$), of surviving to the next period. This is the only source of uncertainty in the model. Each period, $1\text{-}q$ new agents are born and $1\text{-}q$ die, so that the population remains constant. The number of agents of age a alive in any period is thus $(1\text{-}q)q^a$. This illustrates the point made in the Introduction, that there are always some agents alive of any arbitrarily chosen age: the cross-section distribution of the population by age is a declining geometric distribution, with an unbounded support. Under these assumptions, the total population size is 1.

The expected utility over her remaining lifetime of an agent who is alive in period n and was born in period $s \leq n$, is hence:

$$E_n U_{s,n} = \Sigma_{t=n}^{\infty} (\beta q)^{t-n} u_{s,t} \qquad 0 < \beta < 1 \tag{1}$$

where $u_{s,t}$ is flow utility. The novelty in our specification lies in the form proposed for $u_{s,t}$. We assume:

$$u_{s,t} = \ln(c_{s,t}^{1-\delta} m_{s,t}^{\delta} - d(l_{s,t})) \qquad 0 < \delta < 1 \tag{2}$$

where $d(l_{s,t})$ is a function giving disutility of labour supply, with d', $d'' > 0$. In a model without money, in order to make labour supply wealth-independent the term inside the log operator must take the form $c_{s,t}$ - $d(l_{s,t})$, as Greenwood, Hercowitz and Huffman (1988) show. Here we wish to include real balances, $m_{s,t} \equiv M_{s,t}/P_t$, in utility to represent the liquidity services which money provides in helping households to buy goods. It is not plausible to introduce $m_{s,t}$ into $c_{s,t}$ - $d(l_{s,t})$ in an additively separable way, because this would make not only labour supply, but also money demand, wealth-independent, and the latter seems especially unreasonable. Since money is used for buying goods, we combine $m_{s,t}$ with $c_{s,t}$. To preserve wealth-independence of labour supply we then need to make the combined function linear-homogeneous in $(c_{s,t},m_{s,t})$, as done in (2).

The agent has the following single-period budget constraint:

$$P_t c_{s,t} + M_{s,t} + B_{s,t} = (1/q)[M_{s,t-1} + (1+i_{t-1})B_{s,t-1}] + W_t l_{s,t} + \Pi_t - T_t \tag{3}$$

Here, P_t is the price level, W_t is the money wage, Π_t is profit receipts from firms, T_t is a lump-sum tax, and $M_{s,t}$, $B_{s,t}$ are the stocks of money and government bonds the agent holds at the end of period t.[1] Note that, as well as receiving the nominal interest rate i_t on bonds, the agent receives an "annuity" at the gross rate $1/q$ on her total financial wealth, so long as she stays alive. If she dies, on the other hand, all her financial wealth passes to the insurance company.

[1] Profit receipts and taxes, which are exogenous to the agent, are assumed to be the same for all agents irrespective of age.

This is the actuarially fair insurance scheme which operates in Blanchard's (1985) OLG structure. We can rewrite the budget constraint in real terms to get:

$$c_{s,t} + \frac{i_t}{1+i_t} m_{s,t} + \frac{q}{1+r_t} v_{s,t} = v_{s,t-1} + w_t l_{s,t} + \pi_t - \tau_t . \tag{4}$$

This introduces the concepts of the real interest rate and real financial wealth, defined as:

$$1+r_t \equiv (1+i_t)\frac{P_t}{P_{t+1}},\ \ v_{s,t-1} \equiv \frac{1}{q}[M_{s,t-1} + (1+i_{t-1})B_{s,t-1}]\frac{1}{P_t}, \tag{5}$$

while (w_t, π_t, τ_t) are the values of their upper-case counterparts deflated by P_t.

Maximising the function obtained by combining (1) and (2), subject to (4) for $t = n,\ldots,\infty$, we may derive the first-order conditions for the problem. These take the form:

$$c_{s,t+1} - (c_{s,t+1}/m_{s,t+1})^{\delta} d(l_{s,t+1}) = \beta(1+r_t)[c_{s,t} - (c_{s,t}/m_{s,t})^{\delta} d(l_{s,t})], \tag{6}$$

$$\frac{c_{s,t}}{m_{s,t}} = \frac{1-\delta}{\delta}\frac{i_t}{1+i_t} \tag{7}$$

$$w_t = (1-\delta)^{-1}(c_{s,t}/m_{s,t})^{\delta} d'(l_{s,t}) \tag{8}$$

It is worth emphasising that (6)-(8) are conditional, in the sense that they obviously only apply if the agent remains alive for the periods concerned. A first point which they reveal is that certain choices will be the same for all agents irrespective of birthdate, s. Since all agents face the same nominal interest rate i_t, (7) shows that all will choose the same ratio of real balances to consumption. (7) may be interpreted as a demand-for-money function, in which the demand is proportional to consumption and inversely related to the nominal interest rate. It arises from the "medium of exchange" role of money, here captured by the presence of real balances in the utility function. For later reference, it is helpful to define z_t as money demand per unit of consumption, so that (7) then implies:

$$z_{s,t} (\equiv m_{s,t}/c_{s,t}) = \frac{\delta}{1-\delta}\frac{1+i_t}{i_t} . \tag{9}$$

$z_{s,t}$ is the same for all generations, even though $m_{s,t}$ and $c_{s,t}$ are not. Having noted this, we may then observe from (8) that labour supply will be the same for all agents. This, of course, is as intended: our utility function is designed to remove the effect of wealth on labour supply, and since financial wealth is the only respect in which agents differ, they should then all choose the same labour supply. The way in which the wealth effect on labour supply would normally manifest itself is by the presence of $c_{s,t}$ in (8) (other than through $z_{s,t}$), but it is clear that in the present case $c_{s,t}$ is absent. On the other hand, the presence of $z_{s,t}$ introduces a link between the labour market and the money market, and is due to the lack of additive separability in the flow utility function mentioned above.

Turning to the choice of consumption, (6) shows that the usual Euler equation for consumption is now modified by subtraction of the term $(c_{s,t}/m_{s,t})^{\delta} d(l_{s,t})$ from $c_{s,t}$ (ditto for $c_{s,t+1}$). From what has just been seen, this term is the same for all generations s, and so independent of $c_{s,t}$. To interpret it, notice that the flow utility function can be rewritten as:

$$u_{s,t} = \ln[c_{s,t} - (c_{s,t}/m_{s,t})^{\delta} d(l_{s,t})] + \delta \ln(m_{s,t}/c_{s,t}) . \tag{10}$$

Hence $(c_{s,t}/m_{s,t})^{\delta} d(l_{s,t})$ acts like a "shift of origin" for consumption. It shows a resemblance of our function to the Stone-Geary utility function, in which the new origin can thought of as a "subsistence" level of consumption. For future reference, we label the surplus of consumption over its subsistence level as "adjusted" consumption, $a_{s,t}$:

$$a_{s,t} \equiv c_{s,t} - (c_{s,t}/m_{s,t})^{\delta} d(l_{s,t}) . \tag{11}$$

As pointed out in the Introduction, an essential requirement of the perpetual youth model is that the agent's consumption be linear in his lifetime wealth. To derive consumption as a function of lifetime wealth, we combine the agent's first-order conditions with his intertemporal budget constraint. The latter is obtained by integrating (4) from n to infinity and imposing a "No Ponzi Game" condition, resulting in:

$$\Sigma_{t=n}^{\infty} q^{t-n} \alpha_{n,t} [c_{s,t} + (i_t/(1+i_t)) m_{s,t}] = v_{s,n-1} + h_{s,n} \equiv \omega_{s,n} \tag{12}$$

where

$$h_{s,n} \equiv \Sigma_{t=n}^{\infty} q^{t-n} \alpha_{n,t} [w_t l_{s,t} + \pi_t - \tau_t],$$

$$\alpha_{n,t} \equiv (1+r_n)^{-1}(1+r_{n+1})^{-1} ... (1+r_{t-1})^{-1} \quad \text{(for } t > n;\ \alpha_{n,n} \equiv 1).$$

Thus $h_{s,n}$ is "human wealth", i.e. the discounted present value of labour income and profits minus taxes; while $\omega_{s,n}$ is total wealth, comprising human and "nonhuman", or financial, wealth. Since we have seen that $l_{s,t}$ is the same for all age cohorts, it is clear that human wealth is also the same for all. Financial wealth, $v_{s,n-1,}$ on the other hand, generally increases with age. Combining (12) with repeated applications of (6) and (7), we arrive at:

$$c_{s,t} - z_t^{-\delta} d(l_t) = (1-\delta)(1-\beta q)[\omega_{s,t} - \Sigma_{i=0}^{\infty} q^i \alpha_{t,t+i} (1-\delta)^{-1} z_{t+i}^{-\delta} d(l_{t+i})] \tag{13}$$

where we have dropped s subscripts on (z_t, l_t), for the reasons explained. (13) says that an agent's adjusted consumption is a constant fraction of his total wealth net of the present value of future subsistence consumption levels[2]. The important point is that consumption is still linear in wealth. This means that aggregate consumption can be written as the same function of aggregate wealth and that the distribution of aggregate wealth across agents is immaterial.

So far we have derived the behaviour of the choice variables of an individual household born in period s. We turn now to look at the behaviour of aggregate household variables. For any variable x_t, the relationship of aggregate to individual values is $x_t = \Sigma_{s=-\infty}^{t} (1-q) q^{t-s} x_{s,t}$, where the lack of an "$s$" subscript indicates an aggregate value.[3] Moreover, since the

[2] In fact, these are the subsistence levels of "full" consumption, where full consumption is consumption plus the foregone interest on money balances, $fc_{s,t} \equiv c_{s,t} + (i_t/(1+i_t)) m_{s,t}$. This explains the presence of $(1-\delta)^{-1}$.

[3] While this is also true for asset holdings $M_{s,t}$, $B_{s,t}$, for $v_{s,t}$ the relationship is $\Sigma_{s=-\infty}^{t} (1-q) q^{t-s} v_{s,t} = (1/q) v_t$. This is because our definition of $v_{s,t}$ in (5) includes the annuity payout in the definition of $v_{s,t}$, and this, being a pure redistribution from those who die to the survivors, does not apply to aggregate holdings.

population size is 1, aggregate and average values coincide. As regards the behaviour of z_t and l_t, we have already shown that individual choices are independent of age, whence $z_t = z_{s,t}$, $l_{s,t} = l_t$, for all s. For c_t, as further noted, (13) can be applied, where $\omega_{s,t}$ is replaced by its aggregate counterpart, ω_t. It is also useful to derive an aggregate counterpart of the individual Euler equation for consumption, (6). We do this by first deriving a relationship of ω_{t+1} to ω_t bringing in aggregate financial wealth, v_t, and then we use the aggregate version of (13) to link total wealth levels to consumption levels. After some manipulation, we obtain:

$$c_{t+1} - z_{t+1}^{-\delta} d(l_{t+1}) = \beta(1+r_t)[c_t - z_t^{-\delta} d(l_t)] - (1-\delta)(1-\beta q)(1/q-1)v_t \,. \tag{14}$$

(14) shows that not only is the growth rate of aggregate adjusted consumption positively related to r_t - as is the case for individual adjusted consumption - but it is also negatively related to aggregate financial wealth. Intuitively, the reason for this is the "generational turnover effect".[4] Between t and t+1, some already-existing agents are replaced by newborn agents. To the extent that those who have just been born have lower consumption than the average consumption of those who die, this compositional effect will reduce aggregate consumption between the two periods. Now, as seen, an agent's consumption is increasing in her financial wealth, and moreover, agents are born with zero financial wealth. Therefore the newborn do have lower consumption than the average of the rest of the population, and it is a randomly chosen sample of the latter who die.

3. General Equilibrium in a Flexible-Price Economy with Money and Government Debt

We now combine the OLG household structure as described in Section 2 with some simple assumptions about firms and the government and study their interaction in a Walrasian

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[4] This effect is highlighted by Heijdra and Ligthart (2000).

environment. The representative firm has an increasing, concave production function, $y_t = f(l_t)$, and so maximises profits where:

$$w_t = f'(l_t). \tag{15}$$

The government has a single-period budget constraint of the form:

$$P_t(g_t - \tau_t) + i_{t-1}B_{t-1} = (B_t - B_{t-1}) + (M_t - M_{t-1}) \tag{16}$$

where g_t denotes government spending on firms' output. We can re-write this in real terms as:

$$g_t - \tau_t = [b_t - (1 + r_{t-1})b_{t-1}] + [m_t - (P_{t-1} / P_t)m_{t-1}] \tag{17}$$

in which $b_t \equiv B_t/P_t$. There are many policy regimes which can be considered, but we wish to focus on the simplest types of monetary and fiscal policy experiment. Hence we will treat g_t and M_t as exogenous and constant over time, except for the possibility of a once-and-for-all change in their values. The level of government debt will be treated as a third independent policy instrument, leaving τ_t to balance the budget as an endogenous residual. There is more than one way to fix the stock of government debt exogenously: it can be set in real or nominal terms, exclusive or inclusive of current interest obligations. Here we choose to set it in real terms, inclusive of interest. Hence the debt policy instrument is taken to be:

$$b_t' \equiv (1 + r_t)b_t. \tag{18}$$

Under this assumption a government bond is like an indexed treasury bill: it is a promise of one unit of goods in one period's time.

To determine the general equilibrium, we begin in the labour market. Equating the "supply wage" as given by (8) to the "demand wage" as given by (15), we have:

$$f'(l_t) = d'(l_t)/(1 - \delta)z_t^{\delta}. \tag{19}$$

This equation implicitly determines l_t (and thus y_t) as a function of z_t. Note that in a non-monetary version of our economy, in which we would therefore have $\delta = 0$, (19) would tie down l_t by itself. The non-monetary version of the economy thus has a "natural rate" property, in the sense that employment is independent of monetary and fiscal policy. However, when money is included, there is a positive relationship between the level of real balances per unit of consumption and employment (as can be seen from (19), recalling $f'' < 0$, $d'' > 0$). The "classical dichotomy" between real and monetary sectors does not hold here. The cause of this is complementarity between consumption and real balances in households' utility, associated with the fact that the cross-partial derivative in (2), u_{cm}, is positive. This indicates that the higher the individual's consumption, the more useful are real balances to the agent, which makes intuitive sense given that the purpose of money is to facilitate transactions. In (19), then, a rise in z (or m, at given c) raises the marginal utility of consumption and so makes it worthwhile for the agent to work an extra hour. In other words, higher real balances stimulate labour supply. This type of effect has also been highlighted in the "shopping time" approach to the demand for money (see, e.g., Walsh, 1998). In that approach, real balances are assumed to economise on the agent's shopping time, rather than provide utility. An increase in real balances releases time for work or leisure, and so generally stimulates labour supply.

Turning next to the goods market, the equilibrium condition is:

$$y_t = c_t + g_t. \tag{20}$$

Having seen that y_t can be found as an implicit function of z_t, it is then clear that c_t can be expressed as an implicit function of (z_t, g_t). Moreover, in the special case where $g_t = 0$, c_t obviously coincides with y_t and is then likewise a function of z_t alone.

It is helpful for what follows to adopt specific functional forms for the production and disutility-of-work functions. Thus, suppose that $f(.)$ and $d(.)$ are constant-elasticity functions:

$$f(l_t) = l_t^{\sigma} \qquad 0 < \sigma \leq 1, \tag{21}$$

$$d(l_t) = \eta l_t^{\varepsilon} \qquad \varepsilon \geq 1. \tag{22}$$

Using these in (19), we obtain explicit solutions for l_t and y_t as functions of z_t:

$$l_t = [\sigma(1-\delta)/\eta\varepsilon]^{1/(\varepsilon-\sigma)} z_t^{\delta/(\varepsilon-\sigma)}, \tag{23}$$

$$y_t = [\sigma(1-\delta)/\eta\varepsilon]^{\sigma/(\varepsilon-\sigma)} z_t^{\delta\sigma/(\varepsilon-\sigma)}. \tag{24}$$

We next consider equilibrium in the asset markets. The aggregate Euler equation, (14), relates aggregate adjusted consumption growth to the real interest rate and the aggregate stock of financial wealth, where the latter is given by:

$$v_t = (1+r_t)b_t + M_t / P_{t+1}. \tag{25}$$

Under a policy of holding b'_t (see (18)) and M_t constant over time at values b' and M, this equals:

$$v_t = b' + m_{t+1}. \tag{26}$$

Using the definition of adjusted consumption (11) (in its aggregate version), we can hence write (14) as:

$$a_{t+1} = \beta(1+r_t)a_t - (1-\delta)(1-\beta q)(1/q-1)(b'+m_{t+1}). \tag{27}$$

We now proceed to develop this into a difference equation which will be the central equation of the model. We will initially assume that there is no government spending. This means, as seen, that not only l_t and y_t, but also c_t, can be written as functions of z_t alone. Moreover the same is true of m_t, since $m_t \equiv z_t c_t$. As regards the endogenous variables in (27), we can hence see that a_t can be expressed as just a function of z_t, and a_{t+1} and m_{t+1} just as functions of z_{t+1}. This leaves $1+r_t$. $1+r_t \equiv (1+i_t)P_t/P_{t+1}$, and i_t is simply related to z_t by (9). P_t/P_{t+1} can furthermore be eliminated as m_{t+1}/m_t, so that $1+r_t$ is seen also to depend only on z_t and z_{t+1}. We hence reduce the model to an implicit first-order difference equation in z_t.

To make the stages of this transformation a little more explicit, first use the substitutions for $1+r_t$ to rearrange (27) as:

$$\frac{a_{t+1}}{m_{t+1}} = \beta \frac{1}{1-\delta/(1-\delta)z_t} \frac{a_t}{m_t} - (1-\delta)(1-\beta q)(1/q-1)\left(1+\frac{b'}{m_{t+1}}\right). \tag{28}$$

We may now show that, with the constant-elasticity functions (21)-(22),

$$\frac{m_t}{a_t} = \frac{\varepsilon}{\varepsilon - (1-\delta)\sigma} z_t \,, \tag{29}$$

$$m_{t+1} = [\sigma(1-\delta)/\eta\varepsilon]^{\sigma/(\varepsilon-\sigma)} z_{t+1}^{[\varepsilon-(1-\delta)\sigma]/(\varepsilon-\sigma)} . \tag{30}$$

Substituting (29) and (30) into (28) yields:

$$\frac{1}{z_{t+1}} + \frac{\varepsilon(1-\delta)(1-\beta q)(1/q-1)}{\varepsilon-(1-\delta)\sigma}\left[1 + b'\left(\frac{\eta\varepsilon}{\sigma(1-\delta)}\right)^{\frac{\sigma}{\varepsilon-\sigma}} z_{t+1}^{-[\varepsilon-(1-\delta)\sigma]/(\varepsilon-\sigma)}\right] = \frac{\beta}{z_t - \delta/(1-\delta)} \tag{31}$$

Thus we arrive at the law of motion for the economy.

Since z_t is a non-predetermined variable, (31) must be solved in a "forward-looking" manner. This means that, for a unique non-explosive solution (or, at least, locally unique, disregarding solutions which may be generated by the nonlinearity of the equation), we need (31) to have a unique, locally unstable, steady state. In this case z_t jumps straight to its steady-state value. We therefore proceed to study the steady-state version of (31). Setting $z_t = z_{t+1} = z$, multiplying through by z/β, and subtracting 1 from both sides, we have:

$$\frac{\delta/(1-\delta)}{z-\delta/(1-\delta)} = (1/\beta - 1) + \frac{\varepsilon(1-\delta)(1/\beta q - 1)(1-q)}{\varepsilon-(1-\delta)\sigma}\left[z + b'\left(\frac{\eta\varepsilon}{\sigma(1-\delta)}\right)^{\frac{\sigma}{\varepsilon-\sigma}} z^{-\frac{\delta\sigma}{\varepsilon-\sigma}}\right]. \tag{32}$$

Note that the left-hand side (LHS) also gives the value of i, and is just the inverse of the money demand function, (9). This is plotted in Figure 1 as the "ME" curve, since it gives a relationship between z and i which derives from the "medium of exchange" role of money. ME is clearly negatively sloped, with horizontal and vertical asymptotes at $i = 0$ and $z = \delta/(1-\delta)$, respectively.

Turning to the right-hand side of (32), first note that in the steady state, inflation is zero, and hence $i = r$. The RHS can be interpreted as giving the value of r consistent with various levels of financial wealth per unit of adjusted consumption, *v*/*a*, where *v*/*a* is in turn related to *z*. To see this, consider the steady-state version of (27), rearranged as an expression for *r*:

$$r = (1/\beta - 1) + (1-\delta)(1/\beta q - 1)(1-q)v/a \,. \qquad (33)$$

(33) shows that in the steady state there is a positive relationship between the real interest rate and the level of financial wealth per unit of adjusted consumption. To understand this intuitively, note that the role of financial wealth as a whole, from households' point of view, is as a "store of value". By accumulating or decumulating $v_{s,t}$, a household is able match the time profile of its "labour" income (i.e. $w_t l_t + \pi_t - \tau_t$) to the desired time profile of its consumption.[5] The greater is *r*, the steeper will be the desired growth path of consumption and the lower the initial level, so households will be accumulating financial wealth more rapidly. Across the population as a whole, therefore, a higher *r* is associated with a higher demand for financial wealth (in relation to the average level of consumption). The RHS of (32) further links *v*/*a* to *z*. This link arises firstly because the numerator of *z*, i.e. *m*, is one of the components of *v* (= *b*′+ *m*), but also because the other endogenous terms, *c* and *a*, can be related to *z* using the labour and goods market linkages. In view of its derivation from the store-of-value role of financial wealth, we label the RHS of (32) as the "SV" curve. In the special case *b*′ = 0, it is clear that the SV curve is just an upward-sloping straight line, with a vertical intercept at $r = 1/\beta - 1$.

The equilibrium of the model in the absence of any government debt is therefore at point A in Figure 1. At this point, the real and nominal interest rates are equal, and the level of real balances per unit of consumption, *z*, is the value consistent both with the demand for real

[5] In fact, decumulation will never be done willingly, in an equilibrium of the perpetual youth model. It occurs only when the agent dies. At this point decumulation happens abruptly, as all the agent's financial wealth passes to the insurance company.

balances as a medium of exchange, and with the demand for real balances as a store of value. We can see from the diagram that in general r exceeds $1/\beta$-1, the pure time preference rate. Notice that when q = 1, i.e. when agents live forever, the SV line is horizontal. The real interest rate is then identical to the pure time preference rate - a standard feature of "representative agent" models. The introduction of overlapping generations, i.e. the reduction of q below 1, pivots upwards the SV line and hence raises the real interest rate, moving the equilibrium from point C to point A.

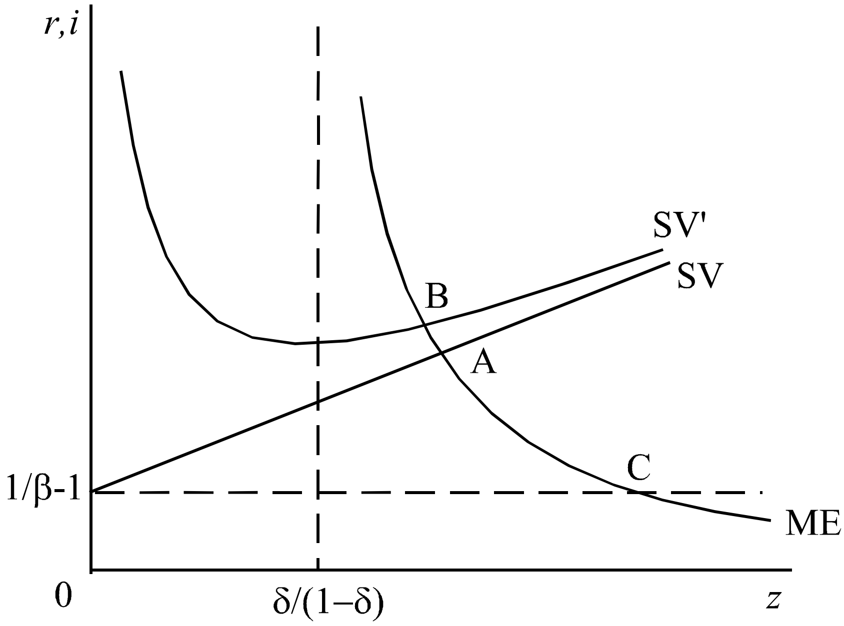

Figure 1

Notice also that, even when q < 1, the equilibrium real interest rate would still be $1/\beta$-1 if it were not for the existence of money. If we let δ (the exponent on real balances in the utility function) tend to zero, then the vertical asymptote of the ME curve shifts left until it coincides with the vertical axis, and the ME curve shrinks in towards its asymptotes, so that the equilibrium in Figure 1 then occurs at the vertical intercept of the SV line. Thus, in the absence of any financial assets, the presence of overlapping generations makes no difference to the interest rate. It is only when money or (as we discuss below) government debt is introduced that the real interest rate is raised above the pure time preference rate. The reason for this is that when r exceeds $1/\beta$-1, the individual wants to choose a rising lifetime profile of consumption and thus, on average, positive holdings of financial wealth. By contrast when r = $1/\beta$-1, the individual just consumes her labour income period by period and does not try to accumulate wealth. The existence of a positive supply of financial wealth therefore requires r > $1/\beta$-1 in order that agents willingly hold this wealth. Conversely, in the absence of a stock of financial wealth, $r = 1/\beta$-1 is needed in order that the demand for wealth be zero.

When $b' = 0$ we can in fact solve for z explicitly. (32) can be arranged as the following quadratic equation in z:

$$\frac{\varepsilon(1-\delta)(1/\beta q-1)(1-q)}{\varepsilon-(1-\delta)\sigma}z^2+\left[\frac{1}{\beta}-1-\frac{\delta\varepsilon(1/\beta q-1)(1-q)}{\varepsilon-(1-\delta)\sigma}\right]z-\frac{1}{\beta}\frac{\delta}{1-\delta}=0\,. \qquad (34)$$

This has two solutions. Since the coefficient on z^2 and the constant term have opposite signs, one solution must be negative and the other positive. Obviously the "relevant" solution is the positive one, and this is the solution depicted in Figure 1. A negative value of z does not seem to make sense. Moreover the negative solution would be associated with a negative value of i, as could be seen if we extended Figure 1 into the negative quadrant, and this also does not make sense. Despite this, we can gain some insight into why the equations tend to generate two steady-state solutions by looking again at the case $\delta = 0$. $\delta = 0$ means that there is no demand for money as a medium of exchange. (34) then yields the following solutions for z:

$$z \;=\; 0 \quad \text{or} \quad -\left(\frac{1}{\beta}-1\right)\frac{\beta q(\varepsilon-\sigma)}{(1-\beta q)(1-q)\varepsilon}\,. \qquad (35)$$

Intuitively, $z = 0$ is the "right" solution in this case, and it is puzzling that there is any other solution at all. Notice, however, that when $\delta = 0$ the economy is in principle similar to Samuelson's (1958) OLG model of a monetary economy, in which money still potentially has a role to play as a pure store of value. Here it should be recalled that Samuelson found that money would only be valued in equilibrium if the resource allocation in the economy before the introduction of money was dynamically inefficient, i.e. if "$r < n$" was satisfied. Otherwise, the level of real balances required to generate a monetary steady state would be negative, corresponding to a desire by agents to be borrowers rather than lenders. Negative real balances are of course impossible, so in that case money would not be valued. The $\delta = 0$ case of our model is like the second case: if a supply of financial wealth were assumed absent from our economy, the equilibrium real interest rate would have to be $1/\beta$-1, as noted above. This is greater than the population growth rate, "n", (namely zero), so that the equilibrium of our economy prior to introducing money is dynamically efficient. When a supply of money is introduced, we then find, as Samuelson did, that the only non-zero real value of this stock consistent with steady state conditions is negative.

So far we have shown that the perpetual youth model with monetary GHH preferences appears to possess a steady state which is economically relevant. However, two further questions must be asked about it. It is easiest to answer these initially if we continue to work with the benchmark case where $b' = 0$. The first question is whether the steady state is locally unstable. It is straightforward to investigate the dynamics of the difference equation (31) using a phase diagram approach. We do this in Appendix A, where we show that the steady state is indeed locally unstable. The second question arises because the utility function (2) restricts the set of consumption values for which an individual's preferences are well defined. As (10) makes clear, they are undefined for consumption values below the "subsistence" level. The question is therefore whether $a_{s,t}$ is positive for all s,t in the steady state. Now, since $1+r > 1/\beta$, an individual's consumption grows as long as she stays alive (see (6)), which means that her total wealth, and within this, her financial wealth, also grow (see (13)). Thus we see that the agents most "at risk" of having negative adjusted consumption are the newborn, who have zero financial wealth. Hence if we can show that $a_{t,t} > 0$ in the steady state, then $a_{s,t} > 0$ is assured for agents of all ages, t-s. In Appendix B we investigate the sign of $a_{t,t}$ in the steady state. We are able to prove that it is indeed positive.

4. Effects of Monetary and Fiscal Policy

A key property which we expect the model to possess is lack of "Ricardian Equivalence", so to test this it is natural to consider, as the first policy experiment, the effect of a once-and-for-all increase in government debt. Suppose, then, that b' is raised from zero to a positive value by a tax cut which lasts for one period only. Otherwise stated, the government runs a temporary budget deficit. If the change occurs in period t, the only difference which this makes to the law of motion (31) is that b' now becomes positive. The economy thus jumps immediately to its new steady state, as given by (32) with positive b'. In Figure 1, this is shown as an upward shift of the SV curve to SV′. It is clear that the introduction of a positive b' adds a new component to the RHS of (32), a component which considered by itself is a decreasing function of z, and which has the horizontal and vertical axes as its asymptotes. It therefore shifts up the SV curve and makes it non-linear.

The effect of the increase in the debt is hence to move the economy to point B in Figure 1. As can be seen, the interest rate (real and nominal) increases, and the level of real balances per unit of consumption decreases. Associated with the fall in z is a reduction in output and employment. Government debt thus has real effects in this economy, i.e. Ricardian Equivalence does not hold. It is easy to verify that this is due to overlapping generations, because if we set q=1 then the SV curve becomes horizontal at $1/\beta$-1, and it is clearly unaffected by b' in this case. The reasons for the real effects when $q < 1$ are familiar ones: although today's tax cut is matched by permanently higher future taxes of equal present value, these being necessary to enable the government to pay the interest on the permanently higher debt, the currently-alive generations who benefit from the tax cut do not bear all the burden of the future increases, since some of the increases fall on agents not yet born. Hence the currently alive feel wealthier and attempt to increase their consumption spending. To the extent that output cannot increase, the real interest rate has to rise to choke off demand. In fact, output falls. If the wealth effect on labour supply had not been eliminated, we might have suspected that this was the explanation for the fall in output, with the increase in perceived lifetime wealth encouraging households to demand more leisure. However, as is by now clear, the mechanism is instead the fall in real money balances, and its effect on labour supply via the complementarity of money and consumption. The reduction in real balances is induced both by the increase in demand for goods, which raises the general price level in the face of an unchanged money supply; and by the increase in the nominal interest rate, which reduces demand for money as a medium of exchange and so motivates a portfolio shift.

In summary, the effects of government debt in this OLG model with an endogenous supply of labour are not dissimilar to the better known effects of debt in OLG models with an endogenous supply of capital (Diamond (1965), Blanchard (1985)). In both types of model, the real interest rate is increased and output is reduced. Moreover, in both types of model, government debt "crowds out" another asset from private portfolios. In Diamond and Blanchard it is physical capital which is crowded out, whereas here real money balances, m, are crowded out. Although, unlike physical capital, real balances are not directly an input to production, they are nevertheless linked to such an input, namely labour, by their effect on labour supply; and hence their reduction is likewise part of the causal chain whereby output is reduced.[6]

[6] A further question is what happens as we let the level of debt tend to infinity. Intuitively, there must exist some maximum sustainable level of government debt: for example, Rankin and Roffia (2003) analyse this in Diamond's (1965) economy. In the present model, the maximum occurs where the adjusted consumption of the newborn is driven to zero.

A second policy experiment on which our model can shed light is an open-market operation (OMO) to change the money supply. As pointed out in the Introduction, the standard way of modelling a money supply change in dynamic general equilibrium models is to assume that its counterpart is a tax cut. In practice, however, the main mechanism open to a central bank is an asset-swap operation, in which bonds are bought or sold in exchange for money. If, for example, the central bank wants to increase the money supply by ΔM_t in period t, then (16) implies that it needs to change the stock of debt in public hands by $\Delta B_t = -\Delta M_t$, i.e. it needs to purchase (in absolute value) this quantity of debt on the open market. Equivalently, it needs to change b_t' by $-((1+i_t)/P_{t+1})\Delta M_t$. The easiest way to study the impact of an OMO is to break it into two separate operations, namely an increase in M_t financed by a cut in τ_t, and a reduction in b_t' financed by an increase in τ_t, such that, overall, τ_t is unchanged. Now, since the first of these operations leaves b' unchanged, it causes no alteration in the equilibrium condition (32). This way of increasing M_t is just the standard "helicopter drop" method, and it has no real effects, simply causing an equi-proportional rise in all nominal variables. The second of the operations reduces b', and so is just the reverse of the government debt increase studied above. It can therefore be considered as a movement from B to A in Figure 1, which lowers the real and nominal interest rates and raises z. We therefore find that an increase in the money supply through an OMO does have real effects: by raising z, which stimulates labour supply, it expands output and employment. The analysis here moreover shows how a "liquidity effect" of monetary policy can be generated. In many dynamic general equilibrium models, an increase in the money supply is associated with no change, or even with a rise, in the nominal interest rate; whereas the usual view is that in reality, as in the IS-LM model of the textbooks, monetary expansion causes the nominal interest rate to fall. Our model explains this fall by the associated reduction in the bond stock and the lack of Ricardian Equivalence. Therefore, both on the grounds of the way it allows us to represent monetary control, and on the grounds of its effect on the economy, the model has some advantages for the modelling of monetary policy.[7]

Let us lastly restore government spending. A tax-financed increase in spending is represented by an increase in g_t at unchanged b'. (We can also consider a debt-financed increase in spending. However, a spending increase cannot be debt-financed forever, since the debt stock would explode; hence such a policy is most naturally broken into a combination of a balanced-budget spending increase and a separate debt increase, where the latter has already been analysed.) In the labour market, (19) holds whether or not $g_t = 0$, so there are still unique positive relationships between l_t and z_t, and y_t and z_t. That is, (23) and (24) remain valid if the production and disutility-of-work functions take their constant-elasticity forms. The relationship (28), between a_{t+1}/m_{t+1} and a_t/m_t, also still applies. However, m_t/a_t can no longer be expressed as a function of z_t alone, since in linking m_t/a_t to z_t we need to use the goods market clearing condition (20), in which g_t now drives a wedge between y_t and c_t. In consequence m_t/a_t becomes a function of g_t as well as of z_t (cf. (29)). Similarly m_{t+1} becomes a function of g_{t+1} as well as of z_{t+1} (cf. 30)). We can nevertheless proceed to derive a first-order difference equation in z_t as the central equation of the model, as we did before (cf. (31)). This equation now contains (g_t, g_{t+1}) as additional exogenous variables. For brevity, we shall not present these steps here; instead we advance to the steady-state equation, the counterpart of (32). It is simplest to study this when $b' = 0$, when it is given by:

$$\frac{\delta/(1-\delta)}{z-\delta/(1-\delta)} = (1/\beta - 1) + \frac{\varepsilon(1-\delta)(1/\beta q - 1)(1-q)}{\varepsilon-(1-\delta)\sigma} z\Gamma(z,g),$$

$$\text{where } \Gamma(z,g) \equiv 1 - \frac{\sigma(1-\delta)}{\varepsilon} + \frac{\sigma(1-\delta)}{\varepsilon}\left[1 - \frac{\varepsilon}{\varepsilon-(1-\delta)\sigma}\left(\frac{\eta\varepsilon}{\sigma(1-\delta)}\right)^{\frac{\sigma}{\varepsilon-\sigma}} z^{-\frac{\delta\sigma}{\varepsilon-\sigma}} g\right]^{-1} \qquad (36)$$

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[7] The broad idea that a money supply change implemented through an open-market operation may be non-neutral is not new - it goes back at least to Metzler (1951). Another recent demonstration of it in an OLG model is provided by Benassy (2003).

As before, we seek to plot the LHS and RHS of (36) as functions of z (see Figure 2). The LHS is unaffected by the presence of g, and so is represented by the same downward-sloping "ME" curve as in Figure 1. As regards the RHS, first note that the new term $\Gamma(z,g)$ equals 1 if $g = 0$. The RHS, depicted as the "SV" curve, is then just an upward-sloping straight line with intercept $1/\beta$-1, as in Figure 1. When $g > 0$, $\Gamma(z,g)$ is clearly greater than one. We can also see that it is decreasing in z, that it tends to 1 as z tends to infinity, and that it tends to infinity as z falls towards some strictly positive lower bound. The curve SV′, which takes its appearance from the product of z and $\Gamma(z,g)$, therefore has the U-shape depicted in Figure 2. Its asymptotes are the SV line associated with $g = 0$, and a vertical line at some strictly positive value of z.

From this it follows that the effect of an increase in government spending, starting from zero, is to move the economy from point A to point B. It therefore raises r and lowers z. The fall in z means that output and employment also fall. These impacts on r and z are clearly associated with the presence of overlapping generations, because if $q = 1$ the SV curve is horizontal and unaffected by a change in g. To understand better why the interest rate has to increase, suppose that it were to remain the same. The demand for z, which we can read off from the ME curve, would then be unchanged, and output would hence be unchanged too. In this case c would have to fall by the amount of the increase in g, i.e. there would be 100% "crowding out" of consumption. At unchanged z, a fall in c means an equiproportional fall in demand for real balances, m (recall $m \equiv zc$). However, to keep the market for financial wealth in equilibrium at the old value of r, the fall in m (remember $m = v$, since $b' = 0$) must be matched by an equiproportional fall in adjusted consumption, a, as (33) reminds us. But a falls by proportionally more than c in the case posited, since it equals c net of subsistence consumption. Therefore the rise in g reduces the demand for money as a store of value by more than it reduces the demand for money as a medium of exchange, and this fact means the interest rate has to rise, to restore equality between them. The interest rate rise is thus driven by the fact that the demand for money as a store of value is proportional to *adjusted* consumption, whereas the demand for money as a medium of exchange is proportional to actual consumption.

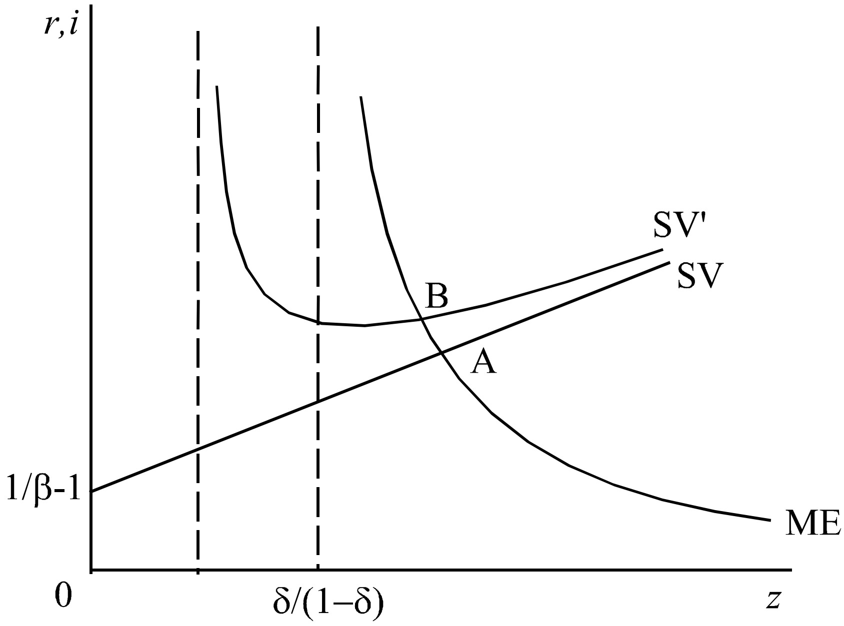


Figure 2

As with the effects of government debt, the effects of government spending in our perpetual youth model with endogenous labour supply bear some resemblance to the effects of government spending in Blanchard's (1985) perpetual youth model with an endogenous capital stock: in both, the real interest rate rises and output falls.

It may be of interest to compare the effects just discussed with the effects of the same policy experiments in a model with a more conventional utility function: for example, utility which is additively separable and logarithmic over consumption, real balances and leisure. For reasons of space, and because of its unsatisfactory microeconomic implications, we shall not present details of the calculations for the "conventional" model. In such a model, a permanent increase in government debt is found still to raise the real (and nominal) interest rate, but no longer to affect output. An open market operation to raise the money supply, on the other hand, lowers the real interest rate but again does not affect output. Lastly, a balanced-budget increase in government spending in the log-utility model has no effect on the real interest rate, but it now raises, rather than lowers, output. “Monetary GHH preferences” thus have some notably different macroeconomic implications compared to a more standard utility function. Changes in government debt now affect output, and not just the real interest rate, because they drive real balances in the opposite direction to debt, and real balances impact on labour supply via non-separable preferences. A balanced-budget fiscal expansion lowers output in our model because it raises the interest rate and thence lowers real balances, as just explained; whereas with a more standard utility function it raises output because it increases the present value of the tax burden on households, and this fall in wealth stimulates labour supply - an effect which is by construction absent in our model.

5. Conclusions

In this paper we have drawn attention to a problem which is likely to be encountered as soon as one tries to extend the “perpetual youth” model of overlapping generations to incorporate endogenous labour supply, namely the problem that some agents will have negative labour supply. We have proposed a solution, which is to use a form of preferences adapted from Greenwood, Hercowitz and Huffman (1988). We chose a utility function which includes real money balances, in order to generate an OLG framework with potential for monetary as well as fiscal analysis. To demonstrate the logical coherence of the framework, we have shown how it permits a general equilibrium to exist in a simple Walrasian economy where the only input to production is labour. To demonstrate its usefulness, we applied it to the study of three types of fiscal and monetary policy shock. Increases in the stock of government debt, or in government spending financed by taxation, were shown to raise the interest rate and lower output. An increase in the money supply through an open-market purchase of government debt was shown to lower the interest rate and raise output.

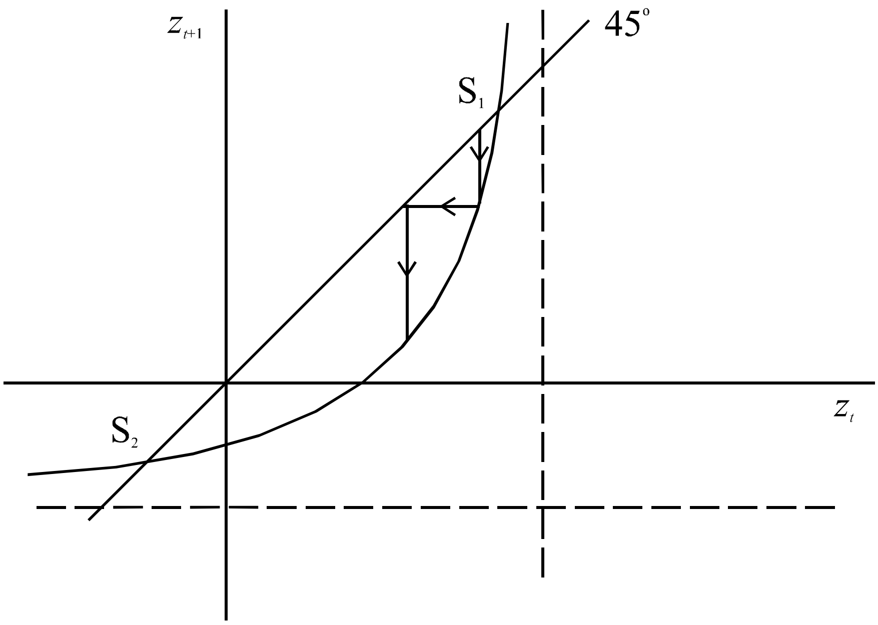
We hope that the framework may also prove useful in more sophisticated models of short-run business cycle behaviour, for example models which incorporate market imperfections such as imperfect competition and nominal rigidities. Since its key feature is that Ricardian Equivalence does not hold, it should offer greater scope for the modelling of fiscal policy issues involving government debt and deficits. Although we have carried out some basic analysis of these issues here, we see this as just a benchmark against which more realistic analyses could be compared. It should also be possible to apply a similar structure in an open-economy context, where OLG models are known to have the advantage of tying down a country's long-run net foreign asset position.

Appendix A

To study the stability of the difference equation (31) in the case $b' = 0$, we rearrange it as:

$$z_{t+1} = -\zeta - \frac{\beta\zeta^2}{z_t - (\beta\zeta + \delta/(1-\delta))} \quad \text{where} \quad \zeta \equiv \frac{\varepsilon - (1-\delta)\sigma}{\varepsilon(1-\delta)(1-\beta q)(1/q-1)}.$$

From this, z_{t+1} as a function of z_t is clearly a rectangular hyperbola, with a horizontal asymptote at $-\zeta$ and a vertical asymptote at $\beta\zeta + \delta/(1-\delta)$. It intersects the horizontal axis at $\delta/(1-\delta)$. It can hence be sketched as:



As discussed in the main text, there are two steady states, i.e. intersections with the 45° line: one at positive z (S_1) and one at negative z (S_2). Using the diagram to trace out the dynamics in the usual way, it is clear that S_1 is locally unstable, and S_2 is locally stable. Time paths which converge on S_2 are however invalid as equilibrium paths, since they violate the condition that z_t be always non-negative.

Appendix B

To determine the sign of the adjusted consumption of the newborn, $a_{t,t}$, we first use (13) to express it as:

$$a_{t,t} = (1-\delta)(1-\beta q)\left[h_{t,t} - \frac{1+r}{1+r-q}(1-\delta)^{-1} z^{-\delta} d(l)\right].$$

Here we have set $v_{t,t-1} = 0$ (see main text) and have assumed that the economy is in a steady state. In a steady state, human wealth is given by:

$$h_{t,t} = \frac{1+r}{1+r-q}(wl+\pi)$$

where τ is zero since $b' = g = 0$ in the present case. Now, $wl + \pi = y = c$, so we then have:

$$a_{t,t} = (1-\beta q)\frac{1+r}{1+r-q}\left[(1-\delta)c - z^{-\delta} d(l)\right]$$

The sign of $a_{t,t}$ therefore depends on the sign of $(1\text{-}\delta)c$ - $z^{-\delta}d(l)$. Appealing to the constant-elasticity production and disutility-of-work functions, and thus using (23) and (24), this can be expressed as:

$$(1-\delta)c - z^{-\delta} d(l) = \left(\frac{\sigma(1-\delta)}{\eta\varepsilon}\right)^{\frac{\sigma}{\varepsilon-\sigma}}(1-\delta)(1-\sigma/\varepsilon) z^{\frac{\delta\sigma}{\varepsilon-\sigma}}.$$

The RHS term is unambiguously positive, which hence implies $a_{t,t} > 0$.

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