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ECB WORKSHOP
ON THE ANALYSIS OF
THE MONEY MARKET

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PREDICTIONS OF SHORT-TERM RATES AND THE EXPECTATIONS HYPOTHESIS OF THE TERM STRUCTURE OF INTEREST RATES

by Massimo Guidolin
and Daniel L. Thornton

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In 2008 all ECB publications feature a motif taken from the €10 banknote.

1 The views expressed here are the author's and do not necessarily reflect the views of the Board of Governors of the Federal Reserve System, the Federal Reserve Bank of St. Louis or the European Central Bank. I would like to thank Michael Dueker, Victor Gaspar, Jeremy Piger, Lucio Sarno, Martin Sola, Jacky So, and Tao Wu for helpful comments on an earlier draft of this paper and John Zhu and John McAdams for valuable research assistance.

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ECB WORKSHOP ON THE ANALYSIS OF THE MONEY MARKET

On 14 and 15 November 2007, Alain Durré, Huw Pill and Diego Rodriguez-Palenzuela of the ECB's Monetary Policy Stance Division organised a central bank workshop titled "The Analysis of the Money Market: Role, Challenges and Implications from the Monetary Policy Perspective". This workshop provided an opportunity for participating central bank experts to exchange views and foster debate, also in interaction with international organizations and academic institutions. The first day of the workshop addressed issues related to the macro-perspective of the money market, drawing on the experiences of a large number of countries. The second day adopted a micro-perspective on the money market, looking in particular at trading behaviour in the overnight money market and its implications for the evolution of spreads.

A first version of this paper was presented at this workshop. The papers presented at the time of the workshop did not consider the potential implications of the financial turmoil for the results of the paper, given that the tensions in money markets emerged in August 2007. The published version of these papers represents an update of the original paper, which incorporates the discussion which took place at the workshop and in most cases a discussion on the developments in the money markets since August 2007.

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CONTENTS

Abstract

Despite its important role in monetary policy and finance, the expectations hypothesis (EH) of the term structure of interest rates has received virtually no empirical support. The empirical failure of the EH was attributed to a variety of econometric biases associated with the single-equation models used to test it; however, none account for it. This paper analyzes the EH by focusing on its fundamental tenet - the predictability of the short-term rate. This is done by comparing *h*-month ahead forecasts for the 1- and 3-month Treasury yields implied by the EH with the forecasts from random-walk, Diebold and Lei (2006), and Duffee (2002) models. The evidence suggests that the failure of the EH is likely a consequence of market participants' inability to predict the short-term rate.

Keywords: expectations theory, random walk, time-varying risk premium.

JEL Classification: E40, E52

Non-technical summary

The expectations hypothesis (EH) of the term structure of interest rates—the proposition that the long-term rate is determined by the market's expectation of the short-term rate over the holding period of the long-term asset plus a constant risk premium—has been tested and rejected using a wide variety of interest rates, over a variety of time periods and monetary policy regimes. It is often suggested that the failure of the EH is not because the EH is invalid, but rather because of problems associated with commonly used tests of the EH. Because of these unresolved issues, this paper investigates the validity of the EH by analyzing the fundamental tenet of the EH, namely, the predictability of the short-term interest rate. If the future short-term rate is essentially unpredictable, there would be no particular reason for economic agents to use their prediction of the short-term rate to determine the long-term rate. The approach taken here is unique in that we not only use several widely used models to predict the future short-term rate but we make predictions of the short-term rate that the market must have been making if the EH were true. That is, we use the actual observed values of the current short-term and long-term rates to determine the market's expectation for the future short-term rate if the EH held. Obtaining these "EH-consistent" forecasts of the future short-term rate requires an identification restriction. However, we show that this restriction can be weakened considerably and is innocuous.

We analyze data on zero-coupon U.S. Treasury bond yields on Treasury securities with maturities of 1 month to 10-years over the sample period January 1970 through December 2003. We then generate true out-of-sample forecasts for horizons from 1 month ahead to to 15 months ahead. Our analysis of the forecast errors from the interest rate forecasting models and our EH-consistent forecasts shows that none of the models considered nor our EH-consistent forecasts can produce significantly smaller forecast errors than a simple model that says the best forecast of the short-term rate *h*-months is the current level of the short-term rate, i.e., a forecast that the short-term rate will be unchanged. We note that our results do not invalidate the idea that long-term rates incorporate all of the information that is useful for forecasting the short-term rate. Rather our results—which are consistent with the efficient markets hypothesis—suggest that the dominant factor in the change in the short-term rate between now and *h*-months from now is news which is not forecastable.

The most commonly given reason for the failure of the EH is that the risk premium is not constant as the EH requires, but is time-varying. Our analysis shows that the EH-consistent forecasts of the future short-term rate are very similar whether one assumes that the risk premium is constant or permitted to vary considerably over time. These findings strongly suggest that the time-variation in the risk premium has little to do with the failure of the EH. The EH fails because short-term interest rates are not predictable to any significant degree.

"The forecasting of short term interest rates by long term interest is, in general, so bad that the student may well begin to wonder whether, in fact, there really is any attempt to forecast."—Macaulay (1938, p. 33)

1. Introduction

The expectations hypothesis (EH) of the term structure of interest rates—the proposition that the long-term rate is determined by the market's expectation of the short-term rate over the holding period of the long-term asset plus a constant risk premium—has been tested and rejected using a wide variety of interest rates, over a variety of time periods and monetary policy regimes (e.g., Campbell and Shiller, 1991, Sarno, Thornton, and Valente, 2007). A number of hypotheses about the empirical failure of the EH have been proposed. One possibility is econometric problems associated with the single-equation models that have been most often used to test the EH. It is well known that estimates can be biased away from those under the null hypothesis due to (i) a time-varying risk premium, (ii) irrational expectations (e.g., the overreaction hypothesis), and (iii) statistical biases (e.g., peso problems, measurement error, etc.). These explanations are unable to account for the empirical shortcoming of the EH, however (e.g., Simon, 1990; Campbell and Shiller, 1991; Hardouvelis, 1994, Dotsey and Otrok, 1995; Balduzzi, et al., 1997; Roberds and Whiteman, 1999; Bekaert, et al., 1997b, 2001). Moreover, Bekaert, Hodrick, and Marshall (1997a) show that because of a positive and very persistent small-sample bias in the estimate from single-equation models, common estimates from these models are even less favorable to the EH than asymptotic distribution theory would suggest. Estimates from these models are further complicated by Thornton's (2006) demonstration that these models can yield results that are favorable to the EH when the null hypothesis is false.

Given the strong rejections of the EH, we investigate that the failure of the EH is fundamental rather than econometric. Specifically, we analyze the fundamental tenet of the EH; namely, the predictability of the short-term rate. This investigation is motivated in part by recent evidence indicating that it is difficult to improve on random walk forecasts of interest rates (e.g., Duffee, 2002; Dai and Singleton, 2000; and Chen and Scott, 1993). We investigate the possibility that the lack of support for the EH stems from the market's inability to adequately predict the future short-term rate by calculating the theoretical t-period expectation of the short-term rate for periods $t+h$, $h=1,2,\ldots H$, under the assumption that the EH holds. This approach has the desirable feature that no assumption is made about how expectations are formed.[1] Because these forecasts depend only on observed values of the long-term and short-term rates; however, they reflect the market's expectation for the short-term rate at each horizon. We compare these forecasts with forecasts from the random walk model, Diebold and Li's (2006) three factor term structure model, and Duffee's (2002) "essentially affine" term structure model. The Diebold and Li and Duffee models are used because of their relative success in forecasting short-term rates.

To anticipate the results, the estimated theoretical forecasts of the short-term rate do not differ significantly from the forecasts obtained from a random walk model of the short-term rate or from forecasts of the short-term rate from Diebold and Li's three factor model of the term structure. These results are shown to be robust to whether the risk premiums are taken to be constant over the sample period or permitted to vary considerably over time. Consistent with these results, it is shown that the current rate spread has power for predicting the *expected change* in the short-term rate, as measured by Diebold and Li's three factor term structure model. Unfortunately, these expectations contain relatively little incremental information about the future short-term rate relative to the current short-term rate. Consequently, the behavior of the long-term rate or the rate spread has little power for predicting the actual change in the short-term rate.

The outline of the paper is as follows. Section 2 presents the EH and the predictability of the spread between the long-term and short-term rate. Section 3 presents the methodology for estimating the t-period expectation for 1- and 3-month rates h periods ahead. The theoretical expectations are compared with the forecasts with those from the random walk model. Section 4 presents the forecasts from econometric models, specifically, Diebold and Li's (2006) three factor term structure model and affine and essentially affine modeld. Forecasts from all of the models are compared and analyzed in Section 5. Section 6 concludes.

2. The EH and the Predictability of the Short-term Rate

The EH asserts that

(1) $$r_t^n = (1/k)\sum_{i=0}^{k-1} E_t r_{t+mi}^m + \pi^{n,m},$$

where r^n denotes the long-term (n-period) rate and r^m denotes the short-term (m-period) rate, $k = n/m$ is an integer, and $\pi^{n,m}$ denotes a term-specific but constant risk premium.

[1] Conventional tests of the EH assume that expectations are rational in the sense that $E_t i_{t+j} = i_{t+j} + v_{t+j}, \quad v_{t+j} \sim iid(0, \sigma_j^2)$, for all j.

The most widely used test of the EH is obtained by subtracting r_t^m from both sides of (1) and rearranging terms to yield

(2) $$(1/k)\sum_{i=0}^{k-1} E_t r_{t+i}^m - r_t^m = -\pi^{n,m} + (r_t^n - r_t^m) .$$

The test of the EH is obtained by assuming that market participants' expectations are rational in the sense that

(3) $$E_t r_{t+i}^m = r_{t+i}^m + \nu_t^i ,$$

where ν_t^i is distributed i.i.d. (0, σ_i^2). Substituting (3) into (2) and parameterizing the resulting expression yields

(4) $$(1/k)\sum_{i=0}^{k-1} r_{t+i}^m - r_t^m = \alpha + \beta(r_t^n - r_t^m) + \varpi_t ,$$

where $\omega_t = -(1/k)\sum_{i=0}^{k-1} \nu_{t+i}$. The EH is tested by estimating (4) and testing the hypothesis $\beta = 1$, its value if the EH holds.

Estimates of β are frequently positive and statistically significant from zero; however, the null hypothesis $\beta = 1$ is nearly always rejected at a very low significance level. Moreover, estimates of the adjusted R-square are typically very small (frequently less than 10 percent), suggesting that the spread between the longer-term and the short-term rates is a relatively poor predictor of future changes in the short-term rate. The low predictive power is consistent with predictive experiments by Carriero et al. (2003). They suggest that the common practice of using the actual short-term rate as a proxy for the tth-period expectation of the short-term rate (i.e., the assumption given by Equation 3) is inappropriate. The EH per se places no restrictions on how the market participants' expectations of the short-term rate are formed. Consequently it is useful to investigate the EH using a procedure that does not require an assumption about how expectations formed, but, nevertheless, reflects market participants' expectations for the future short-term rate.

3. Estimating the Theoretical Expected Future Short-Term Rate

This section shows that it is possible to "theoretical expectations" of the future short-term rate under the assumption that the EH holds, or more generally, that long-term rates are based on the market participants' forecast of the future short-term rate. That is, we identify the theoretical expected short-term rate under the EH. To see how the expected rate can be estimated it is convenient to consider the case where $n = 2$ and $m = 1$, so that (1) is rewritten as

$$(5) \qquad 2r_t^2 - r_t^1 = E_t r_{t+1}^1 + 2\pi^{2,1}.$$

Since both r_t^2 and r_t^1 are observed $E_t r_{t+1}^1$ can be estimated up to a constant risk premium under the assumption that the EH holds.

In general,

$$(6) \qquad jr_t^j - (j-1)r_t^{j-1} = E_t r_{t+j-1}^1 + j\pi^{j,1} - (j-1)\pi^{(j-1),1},$$

for all $j \geq 2$, where $\pi^{1,1} = 0$.

3.1 *Identifying the Constant Risk Premiums*

Because (5) identifies $\tilde{E}_t r_{t+1}^1$ only up to a constant risk premium, an additional identifying assumption is required. Note that the mean forecast error for $E_t r_{t+1}^1$ is given by

$$(7) \qquad (1/T)\sum_{t=1}^{T}[r_{t+1}^1 - (2r_t^2 - r_t^1)] = (1/T)\sum_{t=1}^{T}[r_{t+1}^1 - E_t r_{t+1}^1] - 2\pi^{2,1}$$

If it is assumed that expectations are unbiased on average over a long period of time, i.e.,

$$(8) \qquad (1/T)\sum_{t=1}^{T}[r_{t+1}^{1} - E_t r_{t+1}^{1}] = 0,$$

the constant risk premium can be estimated as

$$(9) \qquad \hat{\pi}^{2,1} = -(1/2T)\sum_{t=1}^{T}[r_{t+1}^{1} - (2r_t^2 - r_t^1)].$$

Estimates of the other risk premiums can be obtained sequentially in the same way.

Generally,

$$(10) \qquad \hat{\pi}^{j,1} = -(1/jT)\sum_{t=1}^{T}[r_{t+j-1}^{1} - (jr_t^j - (j-1)r_t^{j-1})] + ((j-1)/j)\hat{\pi}^{j-1,1}.$$

Given estimates of the risk premiums, Er_{t+j}^1 can be estimated as

$$(11) \qquad \tilde{E}_t r_{t+j-1}^1 = jr_t^j - (j-1)r_t^{j-1} - j\hat{\pi}^{j,1} + (j-1)\hat{\pi}^{j-1,1}.$$

3.2 *Comparison of Theoretical and Random Walk Forecasts*

We calculate the theoretical expected 1-month rate for 1-, and 2-,month horizons and the theoretical expected 3-month rate for horizons from 3- to 15-months The data are end-of-period monthly observations on continuously compounded yields on riskless pure discount bonds for the U.S. The raw data are from Bloomberg. The riskless pure discount bond yields were obtained using FORTRAN codes provided by Robert Bliss and Dan Waggoner based on Biss (1997) and Waggoner (1997)the period 1952.01 through 1991.02. The yields were calculated for bonds with maturities of 1, 2, 3, 6, 9, 12, 15, 18, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months for the period January 1970 through December 2003. .

Estimates of the risk premiums are $\hat{\pi}^{2,1} = 0.149$, $\hat{\pi}^{3,1} = 0.282$, $\hat{\pi}^{6,3} = 0.238$, $\hat{\pi}^{9,3} = 0.353$, $\hat{\pi}^{12,3} = 0.469$, $\hat{\pi}^{15,3} = 0.601$, and $\hat{\pi}^{18,3} = 0.712$. These estimates are reasonable and, as expected, they increase at a decreasing rate as the term to maturity lengthens.

Table 1 presents summary statistics for the monthly theoretical forecast errors, $r_{t+i}^{1} - \tilde{E}_t r_{t+i}^{1}$, for all horizons along with the corresponding forecast errors from the random walk model. Not surprisingly, the theoretical forecasts have zero means. The medians are small and positive at shorter horizons and small and negative at longer horizons. The forecast errors are sometime large in absolute value. Not surprisingly, the largest forecast errors occur in the late 1970s and early 1980s. This is shown in Figure 1, which presents the theoretical (solid lines) and random walk (dashed lines) forecast errors for the 1-month rate at the 1- and 2-month horizons and for the 3-month rate and the 3-month and 15-month horizons. The absolute size of the forecast errors and their standard deviation tend to increase monotonically with the length of the forecast horizon.

The summary statistics for the forecast errors from the random walk model are similar to theoretical forecast errors. The mean forecast errors are slightly negative, indicating a tendency of the random walk model to under predict the short-term rate, and the under-prediction increases monotonically as the forecast horizon lengthens. The similarity in summary statistics suggests a high degree of correspondence between the theoretical and random walk forecasts over the period, which is reflected in Figure 1.

3.3 *Time-Varying Risk Premiums*

Perhaps the most frequent explanation for the failure of the EH is that the risk premium is time varying; not constant was the EH requires. With the exception of Dai and Singleton (2002) and Tzavalis and Wickens (1997), whose approaches are flexible enough to account for nearly all of the time variation in the observed risk premiums, time-varying-risk-premium explanations for the lack of empirical success of the EH have been relatively unsuccessful (e.g., Hardouvelis, 1994; Dotsey and Otrok, 1995; Rudebusch, 1995; Bekaert, et al., 1997; and Roberds and Whiteman, 1999).[2]

The analysis in the preceding section is based on the assumption that the risk premium is constant over the sample period. Since this assumption is at odds with historical massive rejections of the EH, it is important to investigate how the forecast errors are affected by it. To this end, the theoretical forecast errors were estimated by assuming that the forecast errors average to zero over a rolling window of P observations. It is obvious from (10) that the estimated risk premiums are likely to vary considerably when estimated over short samples. A number of window sizes were considered. While the degree of time variation in the estimated risk premiums varied considerably with the choice of P, the estimated forecast errors were relatively insensitive to the window size. Consequently, the results are presented for a very short window of ten months. The estimated time-varying risk premiums for the 1-month rate at the 1- and 2-month horizons are presented in Figure 2 along with the corresponding estimate of the constant risk premiums over the entire sample period. These estimates allow for considerable time variation in the risk premiums over this period, with the risk premiums declining below their full-sample average during the period of the great moderation. The estimated time-

[2] Dai and Singleton (2002) show this using a dynamic term structure model that allows for considerable flexibility in the specification of the market price of risk. Tzavalis and Wickens, on the other hand, assume that the term premiums associated with different maturities are determined by a common factor. Tzavalis and Wickens show that when their estimate of the risk premium is included in (4), the estimate of β is insignificantly different from unity. Dai and Singleton show an analogous result for an alternative single-equation test of the EH.

varying risk premiums for the 3-month rate show a similar degree of time variation. The risk premiums are stationary, however.[3]

Figures 3 and 4 compare the forecast errors under the constant and time-varying risk premium assumptions for the 1- and 15-month horizons, respectively. Despite the high degree of variability of the estimated time-varying risk premiums, there is little difference in the forecast errors at the 1-month horizon. The differences become larger as the forecast horizon increases; however, even at the 15-month horizon, the differences appear to be relatively modest. The fact that the forecast errors change little when the risk premiums are allowed to vary over time suggests that the effect of variation in the risk premium on the forecast errors is modest relative to the new information reflected in the observed value of the short-term rate. That is, the forecast errors are dominated by news, which the market is unable to forecast.

4. Econometric Forecasts

To further investigate the possibility that the empirical shortcomings of the EH may be due to market participants' inability to improve on the random walk forecasts we estimate several econometric interest rate forecasting models. Diebold and Li (2006) generate out-of-sample forecasts of the short-term rate using a three factor model of the Nelson and Siegel (1987) exponential components framework for estimating the yield curve. Diebold and Li (2006) an improvement over random walk forecasts at longer forecast horizons; however, Carriero, et al. (2003) report some improvement over the

[3] This is confirmed by augmented Dickey-Fuller tests, where the null hypothesis of a unit root is easily rejected at the 1-percent significance level for every horizon.

random walk model at short horizons using the Diebold-Li approach. Carriero, et al. (2003) provide no formal statistical analysis of the improvement, however.[4]

4.1 The Diebold and Li Model

Diebold and Li (2006) use the following modified version of the Nelson and Siegel (1987, 1988) forward rate curve to approximate the yield curve

$$(12) \qquad r_t^j = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t j}}{\lambda_t j}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t j}}{\lambda_t j} - e^{-\lambda_t j}\right).$$

The parameter λ_t governs the exponential decay rate. Small values produce slow decay and a better fit at longer maturities, while large values tend to provide a better fit at short maturities. λ_t also governs where the loading on β_{3t} achieves it maximum. Because the loading on β_{1t} is 1 and, hence, does not decay, Diebold and Li interpret it to be the long-term factor corresponding to the *level* of the term structure. Because the factor loading on β_{2t} decays monotonically from 1 to zero, it is viewed as the short-term factor, corresponding to the *slope* of the yield curve. In contrast, the factor loading on β_{3t} rises from zero and then decays back to zero. Hence, Diebold and Li suggest that this factor corresponds to the *curvature* of the yield curve.[5]

Rather than estimating (12) by nonlinear least squares, Diebold and Li fix the value of λ_t and estimate (12) for each period using ordinary least squares. They argue that this not only greatly simplifies the estimation, but likely results in more trustworthy estimates of the level, slope and curvature factors. Diebold and Li set $\lambda = 0.0609$, precisely the value where the loading on the curvature factor reaches it maximum on the assumption that the curvature of the yield curve reaches its maximum at 30 months.

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[4] Carriero, et al. (2003) find essentially no improvement in the forecasts when the model is augmented with economic variables, specifically, the CPI-inflation and unemployment rates.

Diebold and Li make out-of-sample forecasts of rates at all maturities along the yield curve by estimating (12) over the first N observations and then forecasting the yields going forward according to,

$$(13) \qquad \hat{r}^{j}_{t+h} = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left(\frac{1-e^{-\lambda j}}{\lambda j}\right) + \hat{\beta}_{3,t+h}\left(\frac{1-e^{-\lambda j}}{\lambda j} - e^{-\lambda j}\right),$$

where forecasts of the three factors are obtained by estimating

$$(14) \qquad \beta_{i,t} = c + \gamma\beta_{i,t-1}, \quad i = 1,2,3$$

over the first N observations. The process is updated recursively to generate T-N out-of-sample forecasts of each rate over horizons, $h = 1,2,\ldots,H$.

To estimate the three factor model, all of the available rates along the term structure are used, i.e., rates with maturities 1, 2, 3, 6, 9, 12, 15, 18, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months. The procedure is initialized using the period January, 1972 - December, 1981, and updated recursively. While not shown here, these factors correspond very closely to estimates of the level, slope, and curvature factors obtained from the first three principal components obtained from these rates.

The theoretical and Diebold-Li forecast errors for 1-month and 15-month horizons are presented in Figures 5 and 6. The theoretical forecast errors correspond closely to the Diebold-Li forecast errors for the 1-month horizon. The differences get larger as the forecast horizon increases. However, the differences appear to be relatively modest even at the 15-month horizon.

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[5] See Diebold and Li (2003) for a more detailed analysis of and interpretation of these three factors.

4.2 Affine and Essential Affine Term Structure Models

Duffee (2002) shows that a class of "essentially affine" models can improve on random walk forecasts by this criterion, where the improvement generally increases with the length of the forecast horizon. Duffee (2002) does not test whether the differences in forecasts are statistically significant.

INSERT THE MODEL AND RESULTS FOR DUFFEE HERE

5. Test of Differences in Out-of-Sample Forecasts

We test for the statistical significance of differences between the theoretical and random walk and econometric model forecasts using the Diebold and Mariano (1995) test

(15) $$DM = \frac{\bar{d}}{\sqrt{V(\bar{d})}},$$

where $\bar{d}$ is an average over T observations of a differential loss function, d_t, and $V(\bar{d})$ is the variance of $\bar{d}$. The *DM* statistic has an asymptotic standard normal distribution under the null hypothesis that $\bar{d} = 0$. Following standard practice, the variance of $\bar{d}$ is estimated using a heteroskedastic-autocorrelation consistent estimator

(16) $$\hat{V}(\bar{d}) = T^{-1}[\hat{\varphi}_0 + 2T^{-1}\sum_{j=1}^{K}(T-k)\hat{\varphi}_j],$$

where $\varphi_j = (T-j)^{-1}\sum_{t=j+1}^{T}(d_t - \bar{d})(d_{t-j} - \bar{d})$. Based on the findings of Harvey, et al. (1997) the modified Diebold-Mariano test,

(17) $$MDM = \left[\frac{T+1-2K+T^{-1}K(k-1)}{T}\right]DM$$

is used. The *MDM* statistic corrects for size distortions associated with the *DM* statistic.[6]

[6] Harvey, et al, (1997, 1998) also recommend using the critical values from the Student's t distribution rather than those from the normal distribution. The sample sizes used here are large enough, however, that the distinction is trivial.

Two differential loss functions are considered—the absolute forecast error and the squared forecast error. Table 2 presents the *MDM* statistics for both differential loss functions for all forecast horizons.[7] The theoretical forecast errors were larger than the random walk forecast errors for most forecast horizons; however, in no case was the difference statistically significant for either loss function. Likewise, the random walk forecast errors were small than the Diebold-Li forecast errors. Hence, for these rates and this sample period, the Diebold-Li model was unable to beat the random walk model in out-of-sample forecasting, even at relatively long forecast horizons. The Diebold-Li model also unable to improve on the theoretical forecasts.

Particularly surprising is the fact that, with two exceptions, the theoretical forecasts allowing for significant time variation in the risk premiums were generally not statistically significantly different from those based on a constant risk premium. This suggests strongly that it is the new information, which is essentially unpredictable, that dominates the forecast errors.

6. Conclusions and Implications

Recent empirical work suggests the low predictive power of the long-term/short-term rate spread may stem from the inability of market participants to predict the future short-term rate significantly beyond its current level. This paper investigated this possibility by estimating the theoretical expected 1- and 3-month rates at various forecast horizons using a relatively mild identifying restriction. The theoretical forecast errors are then compared with forecast errors from a random walk model and the three factor term structure model of Diebold and Li (2006).

[7] In all cases the truncation lag, K, is 20.

The evidence suggests that the theoretical forecasts implied by the EH do not differ appreciably from the random walk or term structure forecasts. Moreover, it is shown that, just as the EH implies, long-term rates reflect significant information about the markets' expectation for the short-term rate. That is, to the extent that the market is able to forecast the future short-term rate, long-term rates reflect that information. The difficultly arises from the fact that the observed short-term rates are dominated by new information that appears to be essentially unpredictable. For this reason, the spread between the long-term and short-term rate is a relatively poor predictor of the future short-term rate.

Hence, while the EH is fundamentally correct—longer-term rates incorporate the markets' expectation for the future short-term rate—its usefulness for financial market analysts and policymakers is doubtful. Of course, policymakers targeting short-term interest rates might increase the predictability of the rate spread by making short-term rates more predictable. Indeed, some recent evidence (e.g., Lange, et al., 2003; Poole, et al., 2002; and Watson, 2002) indicates that the predictability of the federal funds rate has increased since the Fed began announcing its funds rate target in 1994.

It is important to note that the fact that the markets' expectations for the short-term rate differ significantly from the observed future short-term rate is not a violation of rational expectations. Indeed, it may be that the markets' expectations incorporate all of the relevant information for forecasting the future short-term rate except the news that affects the rate that cannot be forecast. This possibility is supported by the fact that the theoretical forecast errors are relatively unaffected by allowing for greater time variation in the risk premiums and by the fact that the spread between the long-term and short-term rate is a relatively good predictor of the model-based expected change in the short-term rate.

The findings presented here also support Carriero, et al.'s (2003) conclusion that researchers should be wary about using the ex-post short-term rate to proxy for the markets' ex-ante expectation is common practice, e.g., using (4) to test the EH.

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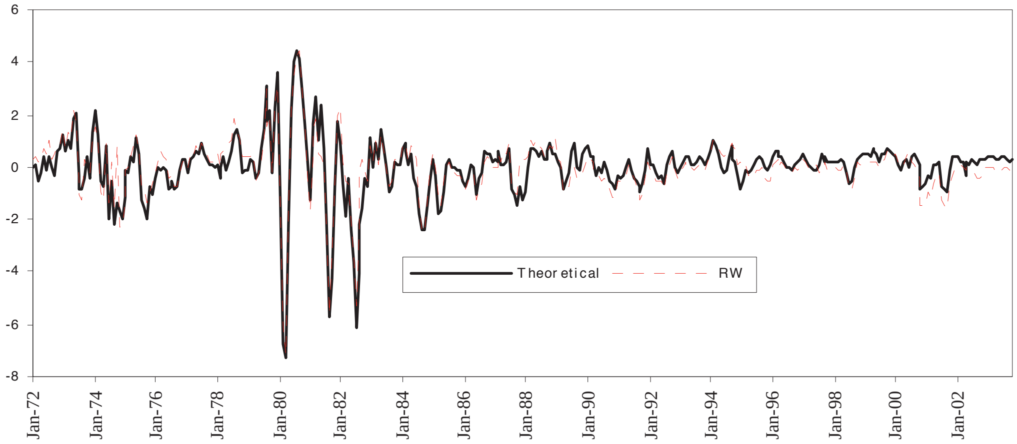
Table 1: Summary Statistics for Theoretical and Random Walk Forecast Errors							
Statistic	Theoretical Forecast Errors						
	(t+1)	**(t+2)**	**(t+3)**	**(t+6)**	**(t+9)**	**(t+12)**	**(t+15)**
Mean	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Median	0.1112	0.1939	0.1244	0.0869	-0.0118	-0.0731	-0.0677
Max.	2.3012	4.3679	4.4434	6.9774	7.4722	7.5094	7.8973
Min.	-4.7378	-7.8311	-7.2986	-5.3906	-5.6868	-6.8026	-6.8647
S.D.	0.6879	1.0489	1.1744	1.5236	1.7830	2.1396	2.4260
	Random Walk Forecast Errors						
Mean	-0.0058	-0.0114	-0.0209	-0.0441	-0.0711	-0.1025	-0.1418
Median	-0.0080	0.0210	0.0180	-0.0365	-0.1360	-0.2580	-0.3410
Max.	2.6260	3.8000	4.4410	6.7310	6.2910	7.6870	8.1350
Min.	-4.7280	-7.7690	-7.2190	-5.8710	-5.5830	-7.6710	-7.7910
S.D.	0.6858	1.0139	1.1070	1.5079	1.7102	2.0466	2.3121

Table 2: MDM Test							
Horizon	**(t+1)**	**(t+2)**	**(t+3)**	**(t+6)**	**(t+9)**	**(t+12)**	**(t+15)**
Theoretical: Constant Versus Time-Varying							
Absolute	-0.155	-0.617	-1.966	-1.795	-1.412	-1.817	-2.246
Squared	-0.137	-0.855	-1.304	-1.309	-1.102	-1.222	-1.468
Theoretical Versus RW							
Absolute	-0.251	-0.305	0.603	-0.213	0.746	0.618	0.455
Squared	0.069	0.444	0.984	0.226	0.728	0.764	0.581
Theoretical Versus DL							
Absolute	-1.105	-0.699	-1.059	-1.223	-1.150	-0.849	-0.969
Squared	-0.470	-0.122	-0.455	-0.682	-0.981	-0.860	-0.920
RW Versus DL							
Absolute	-1.111	-0.849	-1.819	-1.522	-1.198	-0.890	-0.894
Squared	-1.366	-1.531	-1.804	-1.373	-1.180	-1.050	-0.998

Forecast Errors, Theoretical Versus RW
3-month Horizon



Forecast Errors: Theoretical Versus RW
15-month Horizon

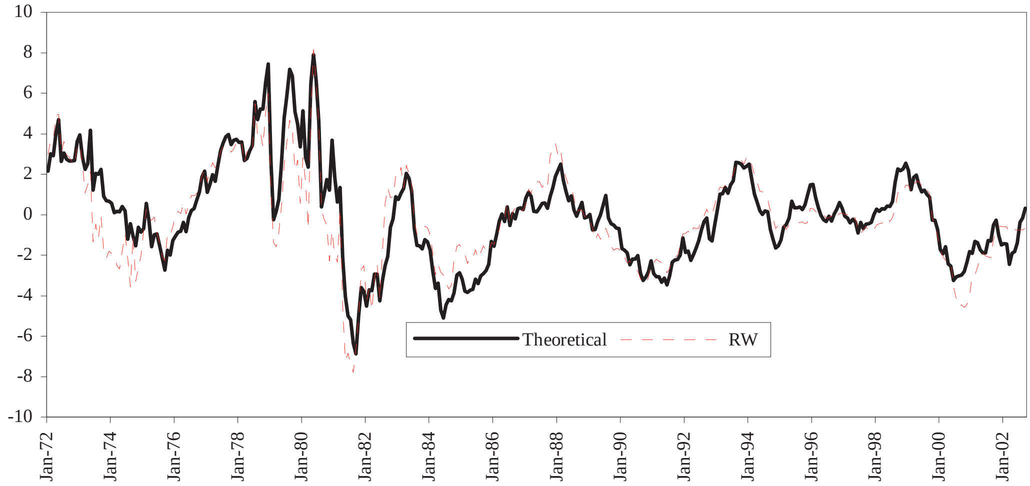


Figure 2: Constant and Time-Varying Risk Premiums for the 1-Month Rate

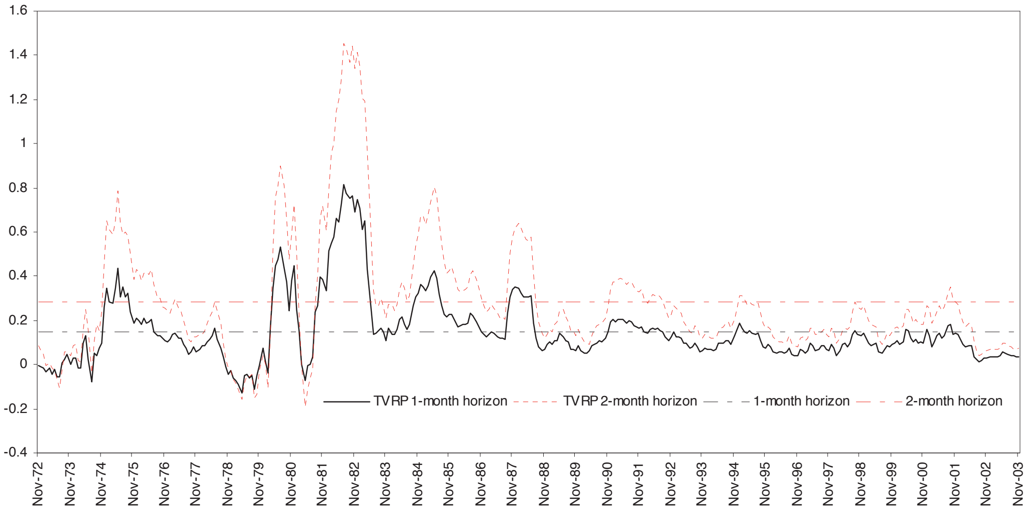


Figure 3: Theoretical Forecast Errors, Constant Versus Time-Varying Risk Premiums
1-month horizon

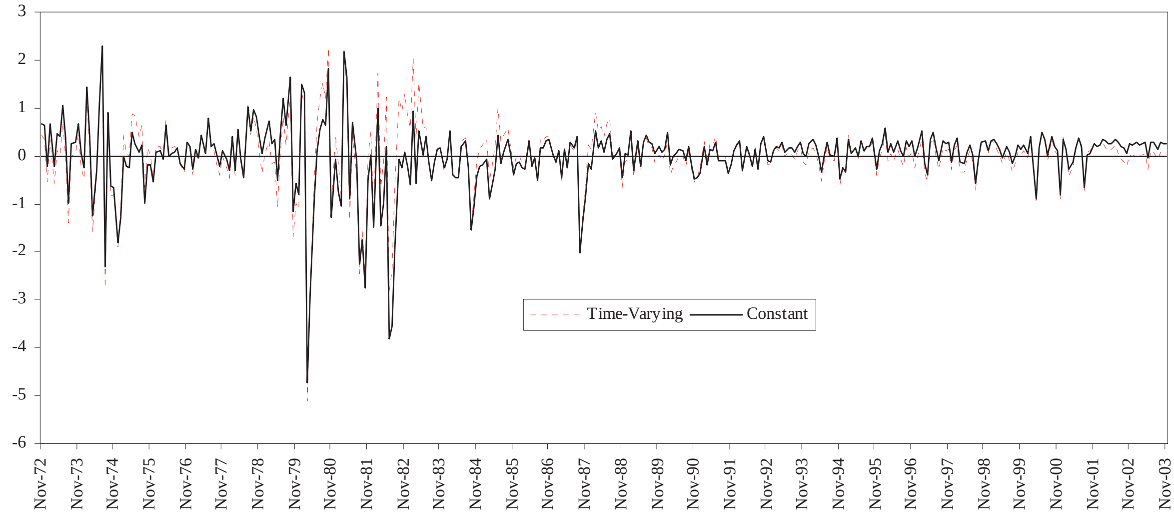


Figure 4: Theoretical Forecast Errors, Constant Versus Time-Varying Risk Premiums
15-month horizon

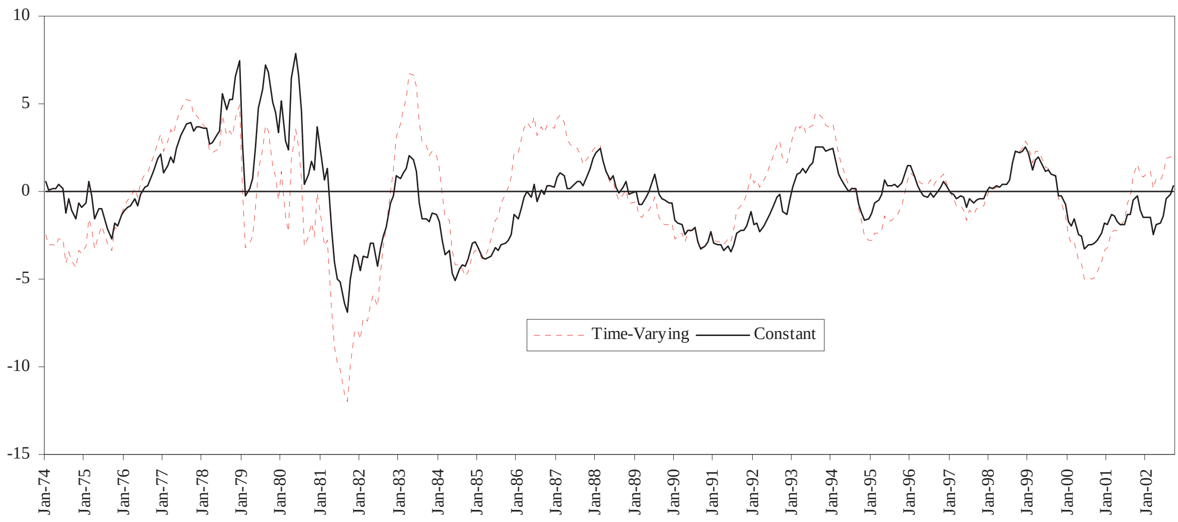


Figure 5: Forecast Errors, Theoretical Versus DL
1-month Horizon

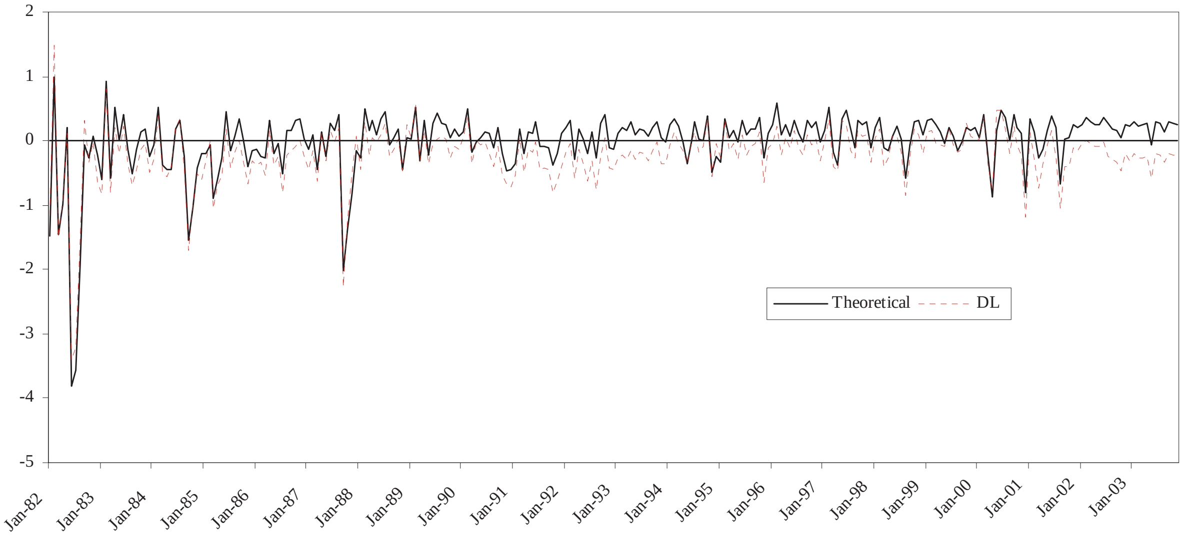
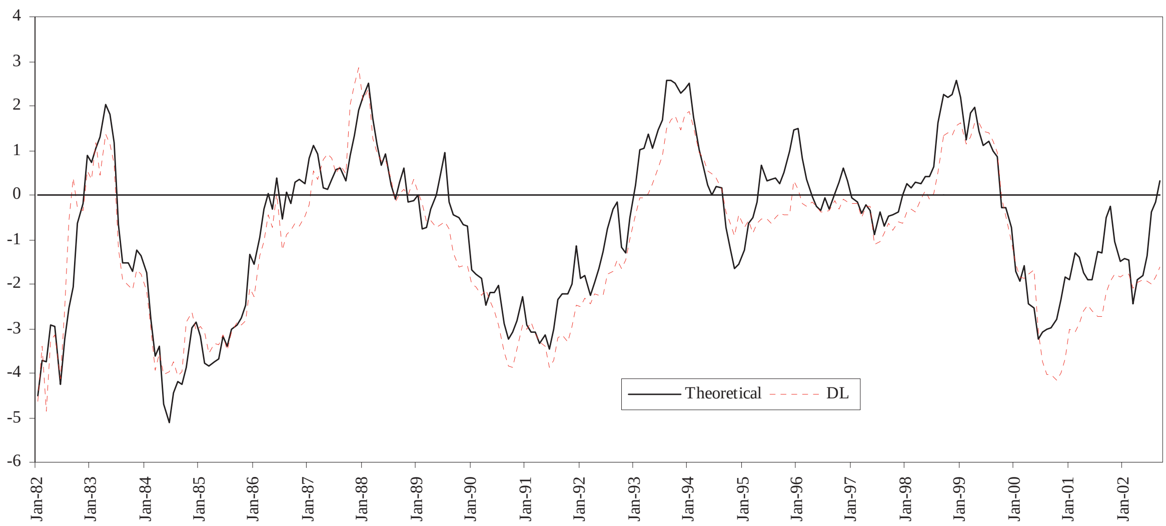


Figure 6: Forecast Errors, Theoretical Versus DL
15-month Horizon



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