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FOREIGN EXCHANGE OPTION AND RETURNS BASED CORRELATION FORECASTS

EVALUATION AND TWO APPLICATIONS

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In 2005 all ECB publications will feature a motif taken from the €50 banknote.

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Abstract: We compare option-implied correlation forecasts from a dataset consisting of over 10 years of daily data on over-the-counter (OTC) currency option prices to a set of return-based correlation measures and assess the relative quality of the correlation forecasts. We find that while the predictive power of implied correlation is not always superior to that of returns based correlations measures, it tends to provide the most consistent results across currencies. Predictions that use both implied and returns-based correlations generate the highest adjusted R^2s, explaining up to 42 per cent of the realised correlations. We then apply the correlation forecasts to two policy-relevant topics, to produce scenario analyses for the euro effective exchange rate index, and to analyse the impact on cross-currency co-movement of interventions on the JPY/USD exchange rate.

Keywords: Correlation forecasts, currency options data, effective exchange rate.

JEL classification: F31, F37, G15.

Non-technical summary

The purpose of this study is to investigate the extent to which it is possible to use returns based measures and foreign exchange options based measures to predict the correlation between bilateral exchange rates. In particular, we study whether the forward-looking information contained in the OTC currency options data can provide good forecasts of the future realised correlation between exchange rates by themselves or in addition to various correlation forecasts derived from returns based measures. Armed with the results from the correlation forecast analysis, we then illustrate two different applications of the methodology for policy related purposes.

There is ample anecdotal evidence that over time, certain currency pairs tend to move in tandem. In other words, when one of the two exchange rates appreciates (depreciates), the other tends to follow a similar pattern. In economic terms, these patterns are interesting from several points of view. First, the reason why two currency pairs show a positive correlation over time could be that their dynamics is driven by the same economic fundamentals. Second, a sudden fall in an otherwise relatively steady degree of correlation could be indicative of attempts by policymakers to try to influence the dynamics of some particular exchange rate. Third, a set of correlations among several exchange rates could provide an idea about which currencies are facing excess demand in the foreign exchange market. And fourth, if we have a reliable forecast of the correlation relationship between, say, the euro and the currencies of two or more euro area major trading partner economies, then the impact of an assumed future movement in one of the bilateral exchange rates on the future movements in the other bilateral exchange rates can be assessed using these correlation forecasts. For a central bank that uses exchange rates mainly as an indicator for future inflationary risks, it is important to have a forecast of as many of the bilateral exchange rates entering into the effective exchange rate basket as possible. Forecasts of correlation provide one way of expanding the information on future developments received from individual bilateral exchange rates.

We find that the implied correlation calculated from currency options prices shows predictive power for the future realised correlation among most major currency pairs. However, for the exchange rate pairs that show correlation predictability, implied

correlation is not the only one that produces good forecasts. Both GARCH and RiskMetrics correlation forecasts show substantial predictive power. In substance, the two types of correlations forecasts seem to nicely complement each other in that the best forecasts are often produced when implied and return-based correlations are used jointly. The highest adjusted R^2 is almost invariably obtained from the encompassing (multivariate) regressions. The total predictability obtained using a combination of forecasts ranges from 18 to 38 per cent for the entire sample and from 20 to 42 per cent for the post-January 1999 sample.

After assessing the relative forecasting properties of the various methodologies, we apply the correlations measures on two policy relevant cases. In the first study, the correlation forecasts are employed to generate scenario analysis for the euro effective exchange rate conditional on assumptions on the future evolution of the JPY/USD exchange rate. In the second case, we study whether the interventions by the Japanese authorities on the JPY/USD exchange rate in the 1990s and 2000s have affected the patterns of co-movement among the JPY/EUR and USD/EUR exchange rates. We find that when included as an additional explanatory variable in the correlation forecast regressions, interventions improve upon the explanatory power of the model. Therefore, it cannot be excluded that interventions on the JPY/USD rate tend to increase the co-movement among the euro cross rates (JPY/EUR and USD/EUR).

1. Introduction

The purpose of this study is to investigate the extent to which it is possible to use returns based measures and foreign exchange options based measures to predict the correlation between bilateral exchange rates. In particular, we study whether the forward-looking information contained in the OTC currency options data can provide good forecasts of the future realised correlation between exchange rates by themselves or in addition to various correlation forecasts derived from returns based measures. Armed with the results from the correlation forecast analysis, we then illustrate two different applications of the methodology for policy related purposes.

There is ample anecdotal evidence that over time, certain currency pairs tend to move in tandem. In other words, when one of the two exchange rates appreciates (depreciates), the other tends to follow a similar pattern. In economic terms, these patterns are interesting from several points of view. First, the reason why two currency pairs show a positive correlation over time could be that their dynamics is driven by the same economic fundamentals. Second, a sudden fall in a historically stable correlation relationship could be indicative of attempts by policymakers to try to influence the dynamics of some particular exchange rate. Third, a set of correlations among several exchange rates could provide an idea about which currencies are facing excess demand in the foreign exchange market. And fourth, if we have a reliable forecast of the correlation relationship between, say, the euro and the currencies of two or more euro area major trading partner economies, then the impact of an assumed future movement in one of the bilateral exchange rates on the future movements in the other bilateral exchange rates can be assessed using these correlation forecasts. For a central bank that uses exchange rates mainly as an indicator for future inflationary risks, it is important to have a forecast of as many of the bilateral exchange rates entering into the effective exchange rate basket as possible. Forecasts of correlation provide one way of expanding the information on future developments received from individual bilateral exchange rates.

There is a substantial literature investigating the informational content of options in relation to asset price returns. Several early contributions use market-based options data with mixed results to investigate conditional second moments, but they almost invariably concentrate on volatility rather than correlation. Beckers (1981) finds that not all available information is reflected in the current option price and questions the efficiency of the option markets. Canina and Figlewski (1993) find that implied volatility is a poor forecast of subsequent realized volatility. Lamoureux and Lastrapes (1993) provide evidence against restrictions of option pricing models which assume that variance risk is not priced. However, Jorion (1995) finds that statistical models of volatility based on returns are dominated by implied volatility forecasts even when the former are given the advantage of *ex post* in sample parameter estimation. He also finds evidence of bias. More recently, Christensen and Prabhala (1998) use longer time series and non-overlapping data and find that implied volatility outperforms past volatility in forecasting future volatility. Fleming (1998) finds that implied volatility dominates historical volatility in terms of *ex ante* forecasting power and suggests that a linear model which corrects for the bias present in implied volatility forecasts can provide a useful market-based estimator of conditional volatility. Blair, Poon, and Taylor (2001), find that nearly all relevant information is provided by the VIX index and there is not much incremental information in high-frequency index returns. Neely (2003) finds that econometric projections supplement implied volatility in out-of-sample forecasting and delta hedging. He also provides some explanations for the bias and inefficiency pointing to autocorrelation and measurement errors in implied volatility. Pong, Shackleton, Taylor and Xu (2004) find that high-frequency historical forecasts are superior to implied volatilities using OTC data for horizons up to one week. Covrig and Low (2003) use OTC data to find that quoted implied volatility subsumes the information content of historically based forecasts at shorter horizons, while the former is as good as the latter at longer horizons. Finally, Christoffersen and Mazzotta (2004) systematically assess the quality of option based volatility, interval and density forecasts for the major currencies 1992-2003. They find that implied volatilities explain a large share of the variation in realized volatility and that wide-range interval and density forecasts are often misspecified whereas narrow-range interval forecasts are well specified.

It is of course striking that all of the above studies investigate options informational content with regard to *volatility* forecasts. Studies investigating exchange rate *correlations* implied by market data are, on the contrary, rather sparse.[1] The contributions perhaps closest related to our work are Siegel (1997), Campa and Chang (1998) and Lopez and Walter (2000), who specifically focus on exchange rate correlations. Campa and Chang find that implied correlation among the DEM/USD, USD/JPY and DEM/JPY currency pairs from January 1989 to May 1995 outperform alternative forecasts at one- and three month horizons. In addition, they find that when included in joint forecast regressions, implied correlation always incrementally improves the performance of other forecasts.

In this study, we extend upon the results by Campa and Chang by looking at several other currencies in a larger sample that also covers the first five years of the single European currency. In particular, we focus our attention on the correlations between the following exchange rate pairs: USD/EUR-JPY/EUR; USD/EUR-GBP/EUR; GBP/EUR-JPY/EUR; USD/GBP-JPY/GBP; USD/JPY-GBP/JPY; USD/EUR-PLN/EUR; and USD/EUR-CZK/EUR.[2] Our sample starts in January 1992 and ends in March 2004, except for the Polish zloty and the Czech koruna currency pairs for which the sample period commences at January 2001. Prior to the launch of the euro in January 1999, we use data on D-mark currency pairs. This is reflected in our estimations in that all regressions are run in two samples, the full sample and the post-January 1999 sample. In the case of the full sample the notation, for simplicity, refers only to the euro.

We find that the implied correlation calculated from currency options prices shows predictive power for the future realised correlation among all currency pairs except the GBP/EUR-JPY/EUR. However, for the exchange rate pairs that show correlation predictability, implied correlation is not the only one that produces good forecasts. Both GARCH and RiskMetrics correlation forecasts show substantial predictive power. In substance, the two types of correlations forecasts seem to nicely complement each other in that the best forecasts are often produced when implied and return-based correlations are used jointly. The highest adjusted R^2 is almost invariably

[1] However, there exists a more generous literature in correlations among stock and bond markets. Good reviews of such studies are provided Kroner and Ng (1998) and Cappiello, Engle and Sheppard (2003).

[2] The choice of the particular correlation pairs is partially dictated by data availability on the currency options, as will be discussed in more detail below.

obtained from the encompassing (multivariate) regressions. The total predictability obtained using a combination of forecasts ranges from 18 to 38 per cent for the entire sample and from 20 to 42 per cent for the post-January 1999 sample.

After assessing the relative forecasting properties of the various methodologies, we apply the correlations measures on two policy relevant cases. In the first study, the correlation forecasts are employed to generate scenario analysis for the euro effective exchange rate conditional on assumptions on the future evolution of the JPY/USD exchange rate. In the second case, we study whether the interventions by the Japanese authorities on the JPY/USD exchange rate in the 1990s and 2000s have affected the patterns of co-movement among the JPY/EUR and USD/EUR exchange rates.

The rest of this study is organised as follows. Section 2 introduces the framework in which the various correlation measures will be analysed. Section 3 specifies the estimated equations and the reports the results. Section 4 presents the two applications and Section 5 concludes.

2. Correlation Forecast Evaluation

2.1. Data issues

The currency options data used in this study consists of 1-month implied volatilities on a large number of exchange rates, obtained from Citigroup. Traditionally, the bulk of trading in options is on OTC basis and not at centralised futures/options exchanges. Christensen, Hansen and Prabhala (2001) argue that in terms of forecasting properties, OTC options data could be of superior quality relative to exchange traded options. This is because OTC prices are quoted daily with fixed “moneyness“ (the distance between the forward rate and the option’s strike price) in contrast with market-traded options, which have fixed strike prices and thus time-varying moneyness as the forward exchange rate changes. Moreover, the trading volume in OTC options is often much larger than in the corresponding market traded contracts. The underlying liquidity on OTC quotes is therefore deeper, which makes the OTC quotes a more reliable source for information extraction. The fact that the currency options market is heavily concentrated on a few global players does that the liquidity problems can be reduced further if data from these institutions is available. Citigroup has a significant market share both in options on major exchange rates as well as on the emerging currencies.

2.2. The Forecasting Object of Interest

The methodologies we adopt for this study are in several ways similar to those used to investigate volatility predictability from OTC currency options in Christoffersen and Mazzotta (2004), with some major differences. The particular object of interest of our study is forecasting the realised future sample correlation of an exchange rate pair over the horizon of the following $h = 21$ trading days.

There exists substantial literature regarding the use of realized volatility as a measure of equity and foreign exchange variability (see e.g. Andersen and Bollerslev (1998) and Andersen et al. (2001a, 2001b, 2003)). The common thread of this literature is the idea that one can sum squared log returns at a frequency higher than that of interest to obtain a measure of the realized quadratic variation over the frequency of interest. For instance, one can compute the monthly variance as the sum of squared daily log returns or the daily variance as the sum of intraday squared log returns. In this theoretical framework, by increasing the sampling frequency it is possible to construct ex post realized volatility measures for the integrated latent volatilities that are asymptotically free of measurement error. In practice, the benefit of increasing the frequency is offset by the microstructure noise which is invariably included in the observed market quotes.

One approach commonly taken is to strike a balance between the horizon of interest and the number of sub-periods in which such horizon is divided for the purpose of computing the squared returns. In the case of daily variance estimates, whereas early work suggests using 5-minute returns more recent contributions indicate that 30-minute returns (i.e. about 16-18 data points per trading day) provide a measure of daily volatility relatively robust to microstructure noise. In our case, since we want a measure of monthly correlation, the sum of own and cross products of demeaned[3] daily log return over the 21 trading days can be considered a sufficiently robust measure of monthly realized co-variation. The measure of correlation we obtain is nothing but the ex-post sample correlation over the next 21 trading days. Following the conventions established in the above mentioned literature, we call this measure "realised correlation", henceforth RC. We define RC for the next h days as follows

$$\rho(R_1, R_2)^{RC}_{t,h} = \frac{\frac{1}{h}\sum_{i=1}^{h}(R_{1,t+i} - \overline{R}_{1,t+1,t+h})(R_{2,t+i} - \overline{R}_{2,t+1,t+h})}{\sigma_{R1}\sigma_{R2}}, \qquad (2.1)$$

where

$$\sigma_{R,j} = \sqrt{\frac{1}{h}\sum_{i=1}^{h} R^2_{j,t+i}} \qquad (2.2)$$

and

$$R_{1,t+i} = \ln(S_{1,t+i} / S_{1,t+i-1}) \qquad (2.3)$$

are the FX spot return of exchange rate S_1 on day t+i.

The plots of all correlation measures are illustrated in Appendix 3 (note that we have labelled the realised correlation as "historical correlation" as the latter is simply a lagged realised correlation as will be explained in more detail below). The charts show that on daily basis, the measures are very volatile. In particular, it seems that the correlations between the USD/EUR and JPY/EUR currency pairs, between the USD/EUR and GBP/EUR currency pairs, between the USD/GBP and JPY/GBP currency pairs, and between the USD/JPY and GBP/JPY currency pairs have fluctuated in the positive territory most of the time. Moreover, the positive correlation seems to be higher in the post-euro subsample.

[3] Although asymptotically the mean should be irrelevant and in practice is very close to zero almost always in the case of correlation it is a good empirical practice to subtract the sample mean from each 21-day sample to constrain the realised correlation to be between minus one and one.

2.3. The Measures of Correlation

To forecast future realised correlation, four alternative correlation measures are applied. First, we calculate the implied correlation from options implied volatility. To do so it is necessary to assume that in addition to the Black and Scholes model also the triangular parity condition between exchange rate cross rates holds.

Being based on options data, implied correlation provides a forward-looking perspective to the analysis of co-movements between currency pairs. Because exchange rate options provide information on the currency options market's uncertainty about the price of one currency in terms of another, with three currencies and options on each of the possible exchange rate pairings we can derive an estimate of the market's expected future, or implied, correlation between any two of the exchange rates. To put it in another way, implied correlation represents the degree of co-movement between two currencies using a third currency as a numeraire.

The implied correlations are derived using the well-known Black-Scholes pricing formula as well as exploiting the arbitrage condition on currencies. The Black-Scholes formula allows one to derive implied volatilities for the underlying asset. The no-arbitrage condition provides, given the proportional changes in returns of two exchange rates, R_1 and R_2, the proportional change in the return of a third exchange rate R_3 simply as $R_3 = R_1 - R_2$. It then follows that

$$Var\left(R_3\right) = Var\left(R_1\right) + Var\left(R_2\right) - 2Cov\left(R_1, R_2\right), \qquad (2.4)$$

whereby it is straightforward to derive the implied correlation (IC) between R_1 and R_2 knowing $Var(R_1)$, $Var(R_2)$, and $Var(R_3)$.[4] The implied correlation is then defined as

$$\rho(R_1, R_2)_{t,h}^{IC} = \frac{\sigma_{1,t}^2 + \sigma_{2,t}^2 - \sigma_{3,t}^2}{2\sigma_{1,t}\sigma_{2,t}}. \qquad (2.5)$$

[4] See Malz (1997), Butler and Cooper (1997) and Brandt and Diebold (2003) for further details.

In (2.5), $\sigma^2_{1,t} = Var_t(R_1)$ and $\sigma^2_{2,t} = Var_t(R_2)$, can be measured by the square of the implied volatility on each of the currency pairs. The implied correlation for a particular date can then be calculated simply by inserting values for the implied volatilities in the equation.

Bollerslev and Zhou (2003)[5] point out that if the volatility risk is priced in the options markets then implied volatility is a biased predictor of realized volatility. In fact, implied volatilities are often empirically found to be upward biased estimates of the objective volatility. In a standard stochastic volatility set up, it can be shown that if the price of volatility risk is zero, the process followed by the volatility is identical under the objective and the risk neutral measures. In such a case there would be no bias. However, the volatility risk premium is generally estimated to be negative which in turn implies that the volatility process under the risk neutral measure will have higher drift. These theoretical arguments do apply to the computation of implied correlation as well. However, because such a potential bias could affect all variances used in the computation of the implied correlation in (2.5), it is not clear *a priori* that the bias for implied correlations is a problem as severe as it is for volatilities. We will show below that bias is indeed present in correlations computed from options.

The other three volatility forecasts are derived from historical FX returns only. The simplest possible forecast is the historical *h*-day volatility, defined as

$$\rho_{t,h}^{HC(1,2)} = \rho_{t-h,h}^{RC(1,2)} . \tag{2.6}$$

The historical correlation is simply the lagged realized correlation. Alternatively, we can consider second moments that apply an exponential weighting scheme putting progressively less weight on distant observations. The simplest measure using such a scheme is the Exponential Smoother or RiskMetrics correlation. Daily variance and covariance then evolve as

$$\begin{aligned} \tilde{\sigma}^2_{(1),t+1} &= (1-\lambda)\sum_{i=1}^{\infty}\lambda^{i-1}R^2_{1,t-i+1} = \lambda\tilde{\sigma}^2_{(1),t} + (1-\lambda)R^2_{1,t} \\ \tilde{\sigma}^2_{(1,2),t+1} &= (1-\lambda)\sum_{i=1}^{\infty}\lambda^{i-1}R_{1,t-i+1}R_{2,t-i+1} = \lambda\tilde{\sigma}^2_{(1,2),t} + (1-\lambda)R_{1,t}R_{2,t} \end{aligned} . \tag{2.7}$$

[5] See also Bandi and Perron (2003), Chernov (2003), and Bates (2002).

Following JP Morgan we simply fix λ=0.94 for all the daily FX returns. The forecast for h-day correlation is therefore

$$\rho_{t+1}^{RM(j,k)} = \frac{\tilde{\sigma}_{(j,k),t+1}^{2}}{\tilde{\sigma}_{j,t+1}\tilde{\sigma}_{k,t+1}}. \tag{2.8}$$

The third estimate for correlation based on past exchange rate returns that is considered here is the GARCH correlation. The GARCH methodology permits the calculation of time-varying second moments for the universe of assets that are considered by the researcher. According to this approach, variances and correlations are conditional on a time-varying information set that allows one to update the estimated second moments at each point in time when new information becomes available. We have adopted a bivariate GARCH model where R_t is defined as the vector of returns

$$R_t = \left(R_{1,t}, R_{2,t}\right). \tag{2.9}$$

We assume that R_t follows a GARCH process

$$R_t = H_t^{1/2}\varepsilon_t\,. \tag{2.10}$$

In (2.10) ε_t is an identical and independently distributed vector sequence with mean zero and unit variance. The conditional covariance H_t evolves according to a diagonal BEKK GARCH process[6]

$$\begin{aligned}
&H_t = \Omega\Omega' + \mathrm{B}H_{t-1}\mathrm{B}' + \mathrm{A}R_{1,t-1}R_{2,t-1}\mathrm{A}' \\
&\text{where} \\
&\mathrm{H} = 2 \times 2, \quad \mathrm{A, B} = 2 \times 2 \text{ diagonal}, \quad \Omega = 2 \times 2 \text{ lower triangular} \\
&\mathrm{H}(1,1) = \text{variance of exchange rate 1} \\
&\mathrm{H}(2,2) = \text{variance of exchange rate 2} \\
&\mathrm{H}(1,2) = \text{covariance of currency 1 and currency 2}
\end{aligned} \tag{2.11}$$

[6] See Engle and Kroner (1995) for further details.

The next day GARCH correlation is thus defined as

$$\rho(R_1, R_2)_{t+1}^{GARCH} = \frac{H(1,2)_{t+1}}{\sqrt{H(1,1)_{t+1}}\sqrt{H(2,2)_{t+1}}}. \qquad (2.12)$$

In contrast to the RiskMetrics model, which implies a random walk volatility process, to forecast the 21 days ahead correlation with GARCH it is necessary to consider the mean reversion of the model and iteratively forecast variances and covariances. The computations to obtain the GARCH correlation forecasts are detailed in Appendix 1. The plots of the GARCH correlations (GC) for the various exchange rate pairs are found in Appendix 3. The plots are substantially smoother than those obtained from historical correlations.

3. Correlation Forecast Evaluation Methodology and Results

To compare the forecasting capability of the different correlation measures, we run simple linear predictability regressions. These are carried out in-sample, by using different windows for the realised correlation (the left-hand side variable) and for the right-hand side variables. In other words, we assess how various estimates of monthly exchange rate correlations have in the past predicted realised correlation one month ahead in time. More specifically, the following univariate regressions are first run for each correlation

$$\rho_{t,h}^{RC} = a + b\rho_{t,h}^{j} + \varepsilon_{t,h}^{j} \qquad (3.1)$$

for $j = IC, HC, GC.$

These univariate regressions[7] serve to assess the fit through the adjusted R^2 and to check how close the estimates of a are to 0 and how close the estimates of b are to 1. In addition, bivariate regressions are performed, including the implied correlation and the two return-based forecasts in turn, as follows

$$\rho_{t,h}^{RC} = a + b\rho_{t,h}^{IC} + c\rho_{t,h}^{j} + \varepsilon_{t,h}^{IC,j} \qquad (3.2)$$

for $j = HC, GC$.

These bivariate regressions shed some light into whether the return-based correlation forecasts add anything to the market-based forecast implied from currency options. Finally, a regression will be run including all three correlation forecasts in the same equation, in order to asses the relative merits of the different correlation forecasts.

The results are reported in Tables 1 and 2 of Appendix 4 where both regression point estimates as well as standard errors corrected for heteroskedasticity and autocorrelation, using GMM, are included. The robust Newey-West weighting matrix with a pre-specified bandwidth equal to 21 days is applied. The regression fit is reported using adjusted R^2. Table 2 of Appendix 4 includes the same regressions than Table 1, but now using the sample period beginning from January 1999.

We find that correlation between foreign exchange pairs is predictable to a substantial extent. The adjusted R^2 of the GMM regressions[8] ranges from 18 to 38 per cent for the entire sample and from 20 to 42 per cent for the post-January 1999 sample. However, for the exchange rate pairs that show correlation predictability, implied correlation is only in a few cases the best univariate forecast. Both GARCH and RiskMetrics correlation forecasts show considerable predictive power, too.

When comparing these results with predictability regressions for volatility forecasts, one difference we find is, therefore, that information from currency options prices does not always seem to be as helpful in predicting correlation as it is in predicting volatility. Returns based measures sometimes perform better than correlation measures based on options data. We note however that the return based measures also sometimes perform very poorly. This is in contrast with the implied correlation, which seems to be more consistent as it shows less variability in the predictive power from one pair of exchange rates to the other. In substance, the two types of correlations forecasts seem to nicely complement each other. The best forecasts obtain when

[7] See e.g. Fleming et al (1995)

[8] For the technicalities regarding the GMM implementation refer to Christoffersen Mazzotta (2004).

return based measures are used jointly with market based measures, as the highest adjusted R^2 is almost invariably obtained from the encompassing (multivariate) regressions.

For the entire sample implied correlation and GARCH correlation generally show good predictive power and typically outperform historical correlations. Implied and GARCH correlations between the most important currency pairs from the euro area perspective, *i.e.* the correlations between the USD/EUR; GBP/EUR and the USD/EUR; JPY/EUR exchange rates, provide reliable forecasts of future correlation. They can thus be useful in assessing near-term future inflationary risks that originate from exchange rate movements. Perhaps surprisingly, in the post-1999 sample the best forecasts are RiskMetrics and implied correlation, both winning the race in 3 out of 7 cases. It is possible that RiskMetrics displays a better ability to model the extremely high persistence of typical forex correlations. However, we conjecture that the fact that RiskMetrics outperform GARCH may be due to the choice of the adjusted R^2 as the metric to determine the best forecast.[9] We leave an in-depth analysis of this and related issues for future research.

3.1. Efficiency and Bias

To study the merit of each correlation forecasts with regard to the relative efficiency and bias we perform a Mincer-Zarnowitz (1969) decomposition of the MSE into bias squared, inefficiency and random variation.[10] The decomposition is as follows: MSE $= [E[y] - E[\hat{y}]]^2 + (1 - \beta)^2 \, Var(\hat{y}) + (1 - R^2) \, Var(y)$, where y is the variable of interest, in our case the realised correlation, and $\hat{y}$ is each correlation forecast in turn. From the regression of y on $\hat{y}$ and a constant, we obtain the slope coefficient β and the regression fit, R^2. The Mincer-Zarnowitz regressions are run for each of the currency pairs and for each of the currency forecasts. Table 3 in Appendix 4 reports the MSEs in absolute value and their decomposition into bias squared, inefficiency, and residual variation, in percentage of the total MSE. It appears that bias is generally higher for the implied correlation than is for all the other correlation forecasts, with the only exception of the RiskMetrics correlation for the USD/EUR-JPY/EUR pair for the entire sample. In the same sample, historical correlation is shown to be the least

[9] For the importance of the loss function see e.g. Christoffersen and Jacobs (2004)

[10] We thank an anonymous referee for pointing us in this direction.

efficient of the correlation forecasts. In the post 1999 sample however, implied correlation bias becomes less of an issue, almost disappearing for the USD/EUR-GBP/EUR and GBP/EUR-JPY/EUR pairs. A notable exception to this pattern is the USD/JPY-GBP/JPY implied correlation bias which almost doubles to 47.29 per cent. In the post 1999 sample the historical correlation is shown to be rather inefficient but substantially unbiased. RiskMetrics correlation appears to be somewhat inefficient for some currency pair and biased for others. GARCH often perform better than the other forecasts under one measure but not the other.

In summary, although in general implied correlation from options is more efficient but biased and return based measures are less biased but also less efficient, the ranking does not hold for all the currency pairs in both sample periods. In other words, the decomposition reinforces the idea that different measures of correlation may have different informational content and therefore they may contribute to provide the best forecasts when used jointly.

4. Two applications of correlation forecasts

Measures of correlation were above shown to provide effective forecasts of future realised correlation. A question that arises from the practical perspective is then whether such measures can contribute to enhance our understanding on exchange rate developments beyond the simple co-movement among various bilateral exchange rates. In this section we propose and illustrate two applications where correlation forecasts can be useful when monitoring and assessing exchange rate developments.

4.1. Scenario analysis for the euro nominal effective exchange rate index

The nominal effective exchange rate (NEER) index of a currency is commonly calculated as a weighed average whereby the various bilateral exchange rates of the most important trading partner currencies are aggregated using the respective trade shares as weights. The resulting index would then better reflect the possible future inflationary risks originating from exchange rate movements in so far as diverging movements of bilateral exchange rates would partially cancel each other out. Many central banks therefore use the NEER among indicators of medium-term risks to price stability. In addition, the price-deflated real effective exchange rates (REERs) provide an insight to the economy's overall price competitiveness in the medium to long term.

In the context of forward-looking monetary policy, various scenarios for the likely future developments of the NEER index could prove useful in assessing the risks to a given baseline model. Due to the known near-impossibility of forecasting bilateral exchange rates it should be clear that assessing the future level of an index that consists of a large number of bilateral rates should, if anything, multiply the difficulty of the task. However, by using measures of correlation it is, in principle, possible to construct consistent scenarios for future movements in a NEER index conditional on an assumption of a future change in one bilateral exchange rate only.

As an example, we take the euro nominal effective exchange rate index with the narrow group of trading partner currencies, calculated by the ECB.[11] Since the weights in the euro NEER are rather concentrated on the currencies of the three largest trading partner countries of the euro area (the United States, the UK and Japan), we analyse how the changes in these currencies, conditional on an assumed movement in another major world exchange rate, the Japanese yen-US dollar rate, are reflected in the NEER index. We consider here the sample period starting from January 1999 only. To this end, we exploit the property of conditional expectation under bivariate normal distribution that can be written as follows.

$$E_t(X_{i,t+1} | Y_{t+1} = \vartheta) = E_t(X_{i,t+1}) + \rho_{Xi,Y,t} \frac{\sigma_{Xi,t}}{\sigma_{Y,t}} (\vartheta - E_t(Y_{t+1})) \qquad (4.1)$$

i= USD/EUR, JPY/EUR, GBP/EUR

In (4.1), the left-hand side captures the level expected to be realised at time t+1 of the bilateral exchange rate of the euro against the dollar, the pound or the yen (X_i), given an assumption ϑ made at time t about the level of the JPY/USD exchange rate (Y) to be realised at t+1. The right-hand side of (4.1) shows how this conditional expectation on X_i differs from the unconditional expectation of that exchange rate that is provided at time t by the t+1 horizon forward exchange rate $E_t(X_{i,t+1})$. In particular, under the

[11] A detailed overview of the methodology used to calculate the euro effective exchange rate indices is provided by Buldorini, Makrydakis and Thimann (2002).

horizon of 1 month, the spread between the assumed future level ϑ of the JPY/USD exchange rate and the 1-month forward JPY/USD rate $E_t(Y_{t+1})$ is multiplied by the forecast correlation between the JPY/USD and the relevant bilateral euro exchange rate, scaled by the ratio of forecast volatilities. After having calculated the conditional expectations for the three main euro bilateral exchange rates, the conditional expectation of the NEER index can be calculated by multiplying the former with the relevant trade weights, and aggregating across currencies.[12]

In Appendix 2, we run regressions à la Fama and find that the conditional expectations on the bilateral USD/EUR, JPY/EUR or GBP/EUR exchange rates as calculated using equation 4.1 produce estimates that outperform the forecasts provided by the forward exchange rates. We can now construct a framework for scenario analysis on the euro NEER index. To this end, the particular question we want to ask is the following. What is the impact on the expectation of the euro NEER one-month ahead, given that today the Japanese yen is expected to appreciate by 10% against the US dollar over one month's horizon? Clearly, since the measures of correlation are time-varying the impact on the euro NEER of an expected yen appreciation against the US dollar vary across different dates. For instance, a scenario where the euro NEER would be expected to move significantly following an expected 10% move in the JPY/USD rate would presuppose that the euro would be expected to move in the same direction against all three major currencies. In that case, the USD/JPY rate would need to be positively correlated against all three major bilateral euro exchange rates.[13] Table A illustrates the scenarios on the bilateral euro exchange rates and on the euro NEER for four selected dates using GARCH correlation forecasts.

[12] Note that since the calculation of the expectation of the euro NEER requires as input the correlation between the GBP/EUR and the JPY/USD exchange rates, which do not enter the same exchange rate "triangle", the correlation forecasts using the implied correlation approach cannot be used for this exercise.

[13] The results have to be qualified in so far as the three main currencies "only" represent some 70% of the weight in euro NEER basket. In the calculations it is assumed that the other bilateral rates do not change, although some of them could be rather sensitive to movements in the JPY/USD rate.

Table A: Scenarios for the euro exchange rates one month ahead (GARCH correlation)

	Assumption: 10% JPY appreciation against US dollar in 1 month's time			
	USD/EUR	GBP/EUR	JPY/EUR	Euro NEER
27 Sep 2000	-7.21%	-2.95%	-22.6%	-6.24%
21 Jan 2002	0.47%	-1.01%	-11.88%	-2.01%
22 Jul 2002	6.89%	0.67%	-0.85%	1.71%
12 Dec 2003	2.48%	1.58%	-1.37%	0.76%

Positive (negative) reading denotes euro appreciation (depreciation).

The forecast co-movements of the various bilateral euro exchange rates conditional on the assumed 10% appreciation of the yen vis-à-vis the US dollar vary substantially across episodes. This is also reflected by the fact that the euro NEER depreciates in some occasions, while it appreciates in others. Therefore, expectations on a stronger yen against the US dollar could contribute to higher or lower expected import prices and inflationary pressures in the euro area, depending on the particular correlation configuration in the FX market at the time when the scenario is conducted.

Looking at the conditional expectations of the bilateral rates, a general observation is that the conditional expectations on the movements in the euro bilateral exchange rates have changed over time. In particular, there is a tendency from expected euro weakness against the US dollar and the pound towards expected euro strength as a response to the assumed 10% appreciation of the yen against the US dollar. Moreover, there is a tendency from a sharp towards more moderate projected future euro depreciation against the yen. What could be the factors contributing to the constellation during the early years of the single currency whereby an appreciation of the yen against the dollar would have contributed to a stronger dollar against the euro, rather than to a general weakness of the US currency? Soon after its launch in January 1999, the euro entered a protracted period of broad-based depreciation that by fall of 2000 was considered to have brought the single currency out of line of the underlying fundamentals. The euro exchange rates subsequently stabilised but remained weak throughout 2001. From 2002 Q2 onwards the US dollar started depreciating against all major currencies amid growing concerns regarding the large US current account deficit. This seems to have changed also the correlations that measure the interplay among the various bilateral exchange rates and, consequently, the conditional expectations regarding future movements in the euro NEER as a response to a hypothetical yen appreciation vis-à-vis the US currency. Finally, throughout 2003 the Japanese authorities markedly increased the intervention activity to retard the pace of yen appreciation against the US dollar. In that context, a sudden switch in policy to "tolerate" a 10% appreciation of the yen could have been seen as reducing the pressure on the euro to appreciate against the US currency. This would explain the conditional expectation indicating a more moderate appreciation of the euro relative to the US dollar than was the case in mid-2002.

4.2. Exchange rate intervention and correlation among cross-rates

In the 1990s and in the early 2000s, the attempts by Japanese authorities to counter the pressures of yen appreciation against the US dollar were often seen as a potential factor affecting G3 exchange rate dynamics.[14]

How is foreign exchange market intervention supposed to affect exchange rates and their cross-rates? According to the standard monetary or portfolio balance approach to interventions, an increased supply of a currency (or bonds denominated in that currency) in the context of an open market operation should imply a depreciation of that currency against all other currencies in the market until the equilibrium is restored. For example, an intervention operation by the Japanese authorities where the yen is sold against the US dollar should imply a depreciation of the yen not only against the US dollar but also against the euro, the pound and so on. Conversely, the purchase of US dollars should exert a general upward pressure on the US currency in the market. Therefore, a yen-selling intervention against the US dollar should, *ceteris paribus,* contribute to a weaker yen and a stronger US dollar also against the euro.

However, as argued by Sarno and Taylor (20001), the daily trading volumes in the foreign exchange markets are so large that even relatively sizeable interventions are unlikely to affect the levels of major currencies through the monetary or the portfolio channels. On the other hand, if the interventions are repeated and follow a systematic

[14] See Castrén (2004) and Ito (2002) for analyses of the Japanese interventions using official Japanese intervention data.

strategy, possibly combined with oral communication, they are likely to affect the market's expectations regarding the "desired" level of the USD/JPY rate. In such a constellation, the adjustment pressures in the FX market are likely to be channelled increasingly through currency pairs that are not actively managed.[15] Following the previous example, with the USD/JPY rate "managed" by systematic intervention any pressure on the US dollar to depreciate – for instance due to the large US current account deficit – would imply that the euro would be expected to appreciate over time both against the dollar and, due to the interventions on the JPY/USD rate, against the yen. If this hypothesis were correct, the implications of interventions should demonstrate themselves in increased correlation between the euro cross rates.

We will augment our earlier correlation forecast regressions by incorporating a variable that measures the daily purchases of Japanese yen carried out by the Bank of Japan in the FX market between April 1992 and March 2004. Our goal is to analyse whether data on the interventions on the JPY/USD exchange rate can improve the forecasts of correlation between the USD/EUR and JPY/EUR exchange rates. In other words, we want to find out whether interventions can work as an additional explanatory factor for realised correlation between the two cross rates of the particular exchange rate that is the focus of the market operation. The particular equation we estimate is

$$\rho_{t,h}^{RC} = a + b\rho_{t,h}^{j} + cINT_t + \varepsilon_{t,h}^{j} \tag{4.2}$$

for $j = HC, RMC, GC, IC$

The regressions serve to assess whether the coefficients of the intervention variable are positive and significant and whether the adjusted R^2 improves relative to standard correlation forecast equations.

The results are summarized in Table B. The regressions show that the variable measuring the BoJ yen-purchasing interventions receives the negative and statistically

[15] BIS (2004) reports evidence from Reuters and EBS trading systems suggests that in 2002-2004, there was a marked reduction in absolute trading volumes in the JPY/USD exchange rate while the absolute volumes on the USD/EUR and the USD/GBP exchange rates sharply increased. The period incorporates some of the most pronounced episodes of interventions by the BoJ that could have reduced the traders' appetite to take large yen positions.

significant coefficient in all regressions. The interpretation of the negative coefficient is that yen-selling interventions (almost all observations in the data set were yen sales) have a positive impact on the forecasts of future realised correlation. In all cases, the adjusted R^2s improve; the increase is particularly marked in the case of implied correlation forecast (15% in the full sample). Hence, an intervention strategy that aims at systematically stabilising a particular exchange rate over time could increase the expected future co-movement among its cross exchange rates as reflected in particular by the currency options prices.

Table B: Japanese interventions on JPY/USD and forecasts of correlation between USD/EUR and JPY/EUR (standard errors in parenthesis)

	Full sample			Post euro sample		
	Correlation *(b)*	Intervention *(c)*	R^2	Correlation *(b)*	Intervention *(c)*	R^2
Implied	0.747* (0.105)		0.205	0.924* (0.113)		0.359
	0.745* (0.067)	-0.28* (0.053)	0.220	0.920* (0.069)	-0.013* (0.044)	0.365
Historical	0.564* (0.053)		0.314	0.583* (0.076)		0.343
	0.561* (0.037)	-0.197* (0.045)	0.326	0.581* (0.050)	-0.013* (0.044)	0.349
RiskMetrics	0.874* (0.112)		0.235	1.168* (0.143)		0.382
	0.871* (0.079)	-0.022* (0.046)	0.242	1.163* (0.093)	-0.011* (0.044)	0.387
GARCH	0.858* (0.066)		0.329	0.834* (0.094)		0.362
	0.854* (0.049)	-0.020* (0.045)	0.341	0.832* (0.094)	-0.014* (0.045)	0.370

**Denotes a significant estimate at 5% level*

5. Concluding remarks

The various estimations of correlation between the major bilateral exchange rates show distinctive fluctuations over time. The correlations generally increased soon after the introduction of the euro, but have more recently returned closer to their longer-term average levels. This development reflects the episode of broad-based euro depreciation 1999-2000, followed in 2002-early 2003 by euro appreciation that was somewhat more prominent against the US dollar than against the pound sterling and the Japanese yen.

Regarding the ability to forecast future correlation, implied correlation can predict up to 36% of future realized correlation. Nevertheless, it is not univocally the best predictor of future correlation as GARCH and RiskMetrics correlations yield occasionally very good predictive power, too. When used together, implied correlation, GARCH correlation and RiskMetrics correlation are particularly useful in predicting future correlation between the major euro currency pairs at the one-month horizon. The predictive power seems to have strengthened after the introduction of the euro.

When applying the estimated correlation measures, we found that using correlation forecasts to analyse scenarios for effective exchange rates is useful as an expected movement in one currency pair seems to indicate a very different impact on the effective exchange rate in various points in time. The time-varying correlation forecasts take into account the market's current perception of the relative adjustment of various exchange rates as a response to a sudden movement in one major exchange rate. Mapping these bilateral movements into the NEER index provides conditional forecasts that could be a useful input in analysing scenarios for future inflationary risks. Furthermore, we show that data on interventions on the JPY/USD exchange rate improve the ability of implied correlation in particular to forecast future realised correlation. This effect, that is not consistent with the monetary or portfolio channels of interventions, suggests that systematic intervention might be capable of affecting the options market's perception about future co-movement among the cross-rates of the currency pair that is on the focus of the market operation.

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Appendix 1

In contrast with the RiskMetrics model, the GARCH model implies a non-constant term structure of variance and covariance. To compute the GRACH forecast it is necessary to take into account the mean reverting nature of the process.

In particular, the persistence of currency 1 is $A(1,1)^2 + B(1,1)^2$; similarly, the persistence of currency 2 $= A(2,2)^2 + B(2,2)^2$; the persistence of the covariance is $=$ $A(1,1)*A(2,2) + B(1,1)*B(2,2)$.

The unconditional second moment for forex rates can be computes as:

$$Ucvar(1) = \Omega(1,1)^2/(1 - persistence(1))$$

$$Ucvar(2) = (\Omega\ (2,2)^2 + \Omega\ (2,1)^2)/(1 - persistence(2))$$

$$Ucovar(1,2) = \Omega\ (1,1)^*\ \Omega\ (2,1)/(1 - persistence(1,2))$$

The term structures are:

$$Term(1) = \sum_{i=1}^{21} Persistence(j)^{(i-1)}$$

$$Term\ (2) = \sum_{i=1}^{21} Persistence(k)^{(i-1)}$$

$$Term\ (1,2) = \sum_{i=1}^{21} Persistence(j,k)^{(i-1)}$$

The variance 21 days ahead forecasts are:

GARCHterm(1) = (21*Ucvar(1) – Ucvar(1)*Term(1) + H(1,1)*Term(1));

GARCHterm(2) = (21*Ucvar(2) – Ucvar(2)*Term(2) + H(2,2)*Term(2));

GARCHterm(1,1) = (21*Ucvar(1,2) – Ucvar(1,2)*Term(1,2) + H(1,2)*Term(1,2).

Finally, the 21 days ahead GARCH correlation forecast is

$$\rho(R_1, R_2)_{t+1,21}^{GARCH} = \frac{\text{GARCHterm}(1,2)}{\sqrt{\text{GARCHterm}(1)}\sqrt{\text{GARCHterm}(2)}} \quad .$$

Appendix 2

In this appendix, we assess the relative forecasting performance of the conditional expectations, as calculated using equation 4.1, against the forward exchange rates. To this end, we run regressions à la Fama for all three currency pairs, where the realised change in the exchange rate over 1-month horizon is regressed on a constant and a forecast error. The forecast future rate is either the forward exchange rate $E_t(X_{i,t+1})$ or the conditional expectation $E_t(X_{i,t+1} | Y_{t+1} = \vartheta)$, where ϑ is chosen to be the realised value of the JPY/EUR exchange rate 1 month ahead in time:[16]

$$
\begin{aligned}
X_{i,t+1} - X_{i,t} &= \alpha + \beta(E_t(X_{i,t+1}) - X_{i,t}) + e \\
X_{i,t+1} - X_{i,t} &= \alpha + \beta(E_t(X_{i,t+1} | Y_{t+1} = \vartheta) - X_{i,t}) + e
\end{aligned}
\qquad \text{(A2.1)}
$$

The null hypothesis is that α is equal to a possibly non-zero constant (including Jensen's inequality term) and β equals positive unity. The sample period covers the period of euro exchange rates only, *i.e.* it stretches from 4 January 1999 to 31 March 2004.

Table C summarises the results of the Fama regressions for the forward exchange rates and for the conditional expectations where three different correlation forecasts are applied (historical, RiskMetrics and GARCH correlation).

Table C: Results from the regressions of equation

	Forward rate	Conditional expectation based on correlation		
		Historical	RiskMetrics	GARCH
USD/EUR	-0.553* (0.058)	0.085* (0.037)	0.121* (0.013)	0.125* (0.041)
GBP/EUR	0.204* (0.095)	0.264* (0.054)	0.193* (0.023)	0.397* (0.070)
JPY/EUR	0.593* (0.003)	0.616* (0.029)	0.035* (0.008)	0.779* (0.037)

**Denotes a significant estimate at 5% level*

[16] *We make this choice purely arbitrarily; alternatively, we also used today's spot exchange rates as a proxy for* υ and found that the results were little changed.

The results show that, in line with several earlier studies, for the USD/EUR the β coefficient is significant and *negative* in the forward rate regression, amounting to the familiar forward bias puzzle. However, the coefficients from the regressions including conditional expectations are all correctly signed (positive) indicating that the estimators do not suffer from the bias. Regarding the GBP/EUR and the JPY/EUR exchange rates, the coefficients are also all correctly signed (positive) and significant. Moreover, apart from the RiskMetrics correlation, the coefficients from the regressions using conditional expectations are higher than from the regressions that use forward exchange rates. The constant terms were small and significant for most of the regressions involving the USD/EUR and JPY/EUR rates, and significant in some cases for the GBP/EUR rate. The R^2s were generally higher for the regressions that use conditional expectations.

All in all, it cannot be excluded that expectations conditional on future developments in the JPY/USD exchange rate, that use the information on correlation forecasts, can improve upon the forecasting power of forward exchange rates in the case of the bilateral euro exchange rates.

Appendix 3 Charts

1: Correlations pre-January 1999[17]

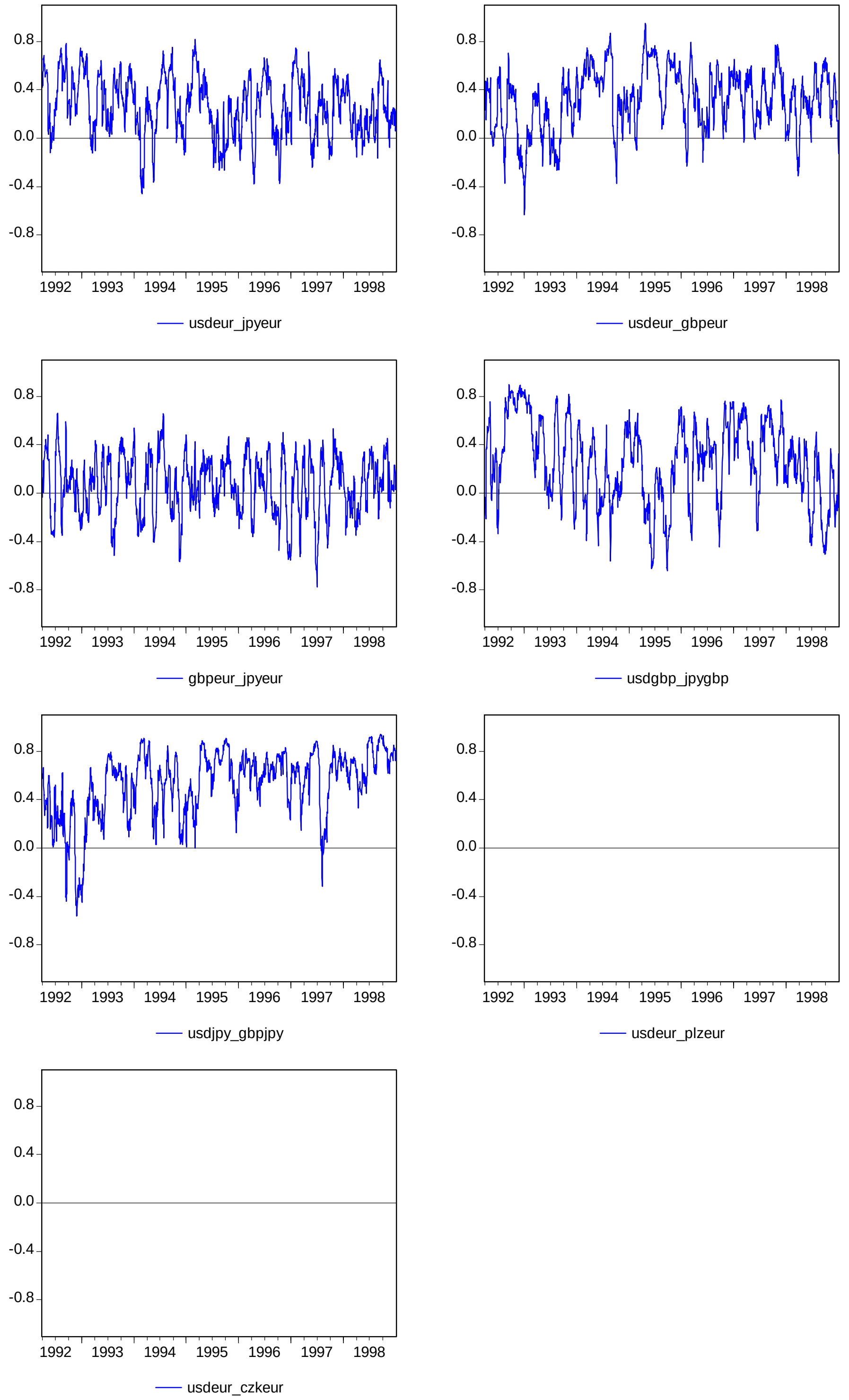

Chart 1. Historical Correlation. Pre 1999.

[17] For all the pre-1999 period or the full sample period the DEM proxies for the EUR.

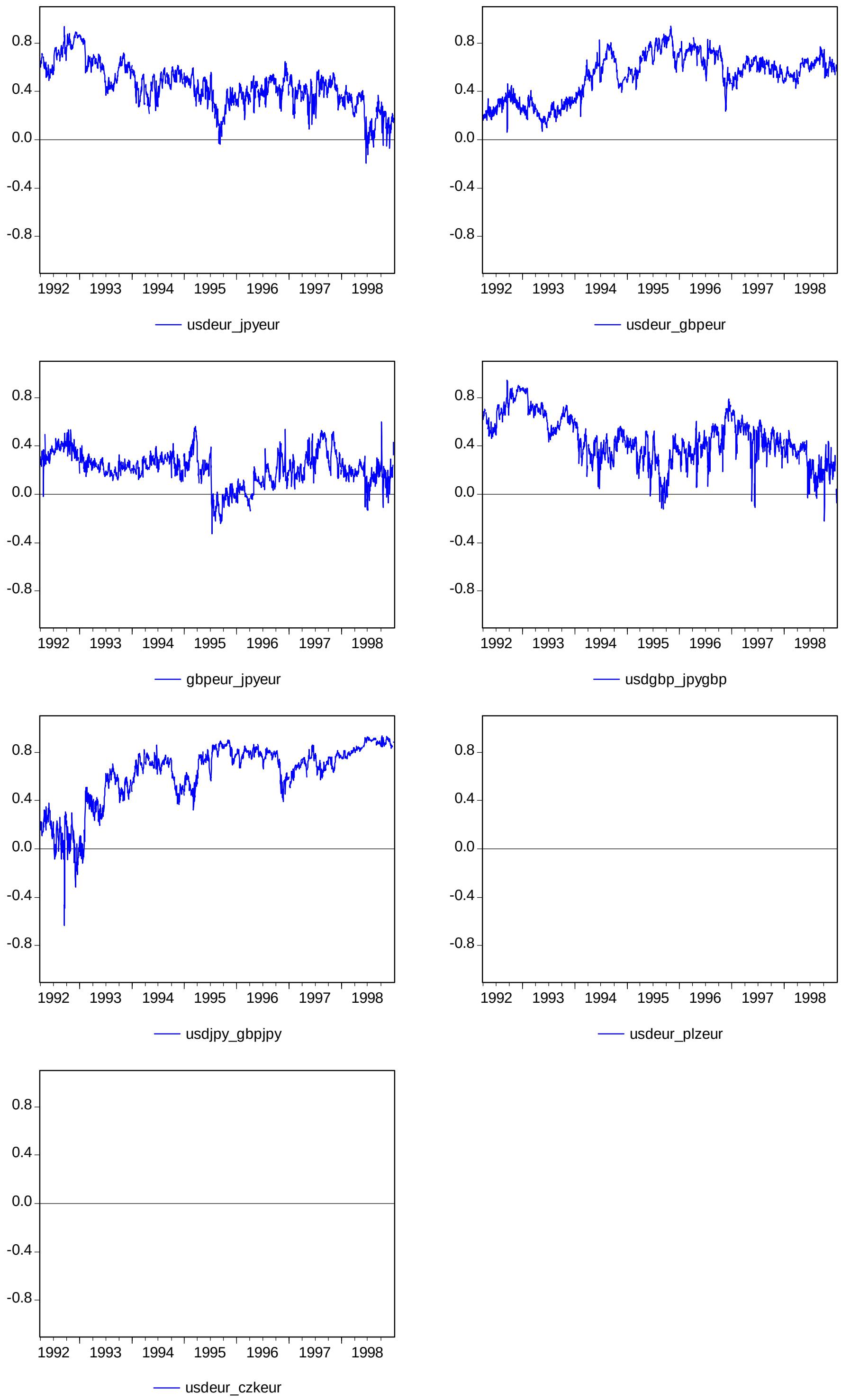

Chart 2. Implied Correlation. Pre 1999.

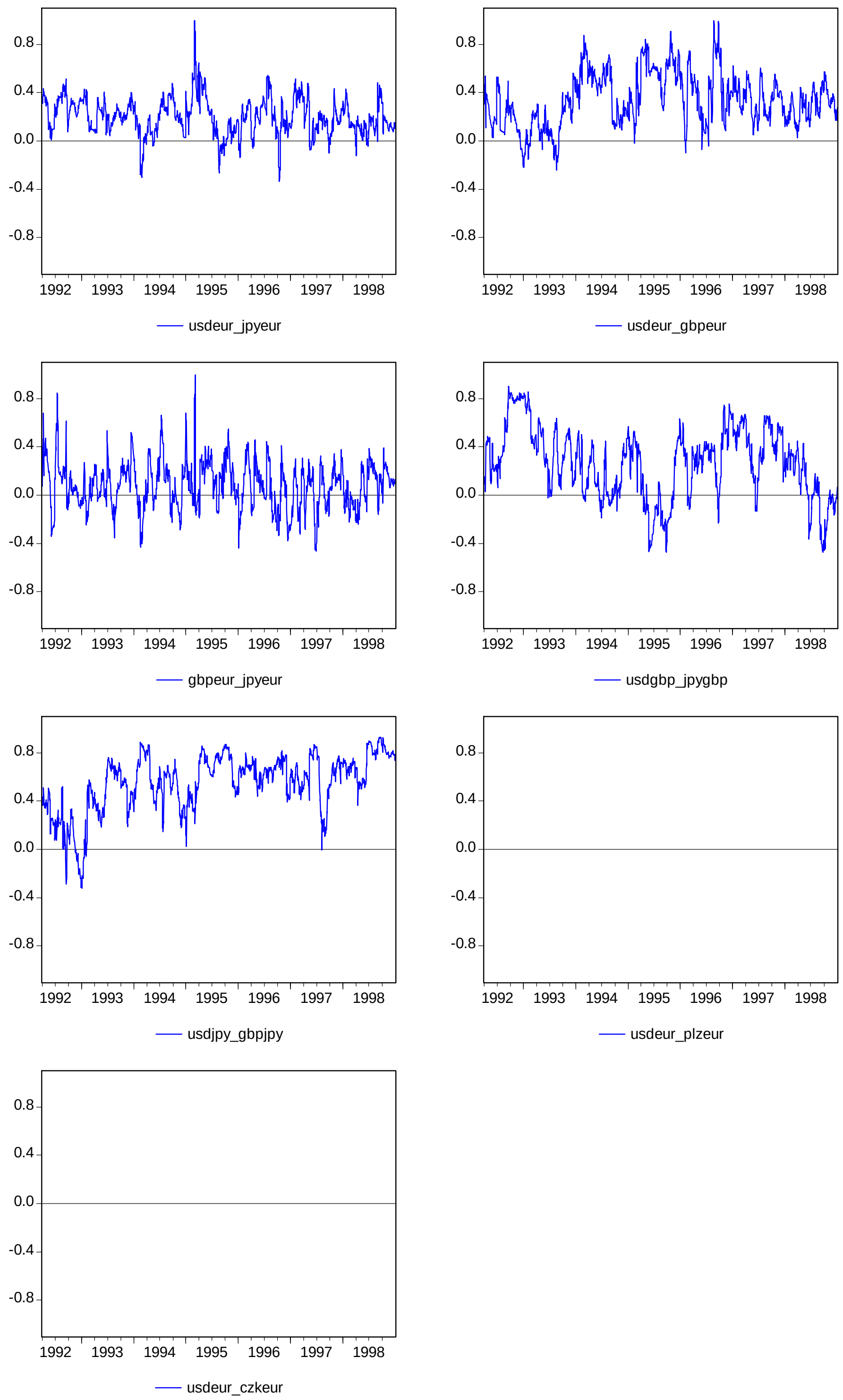

Chart 3. RiskMetrics Correlation. Pre 1999.

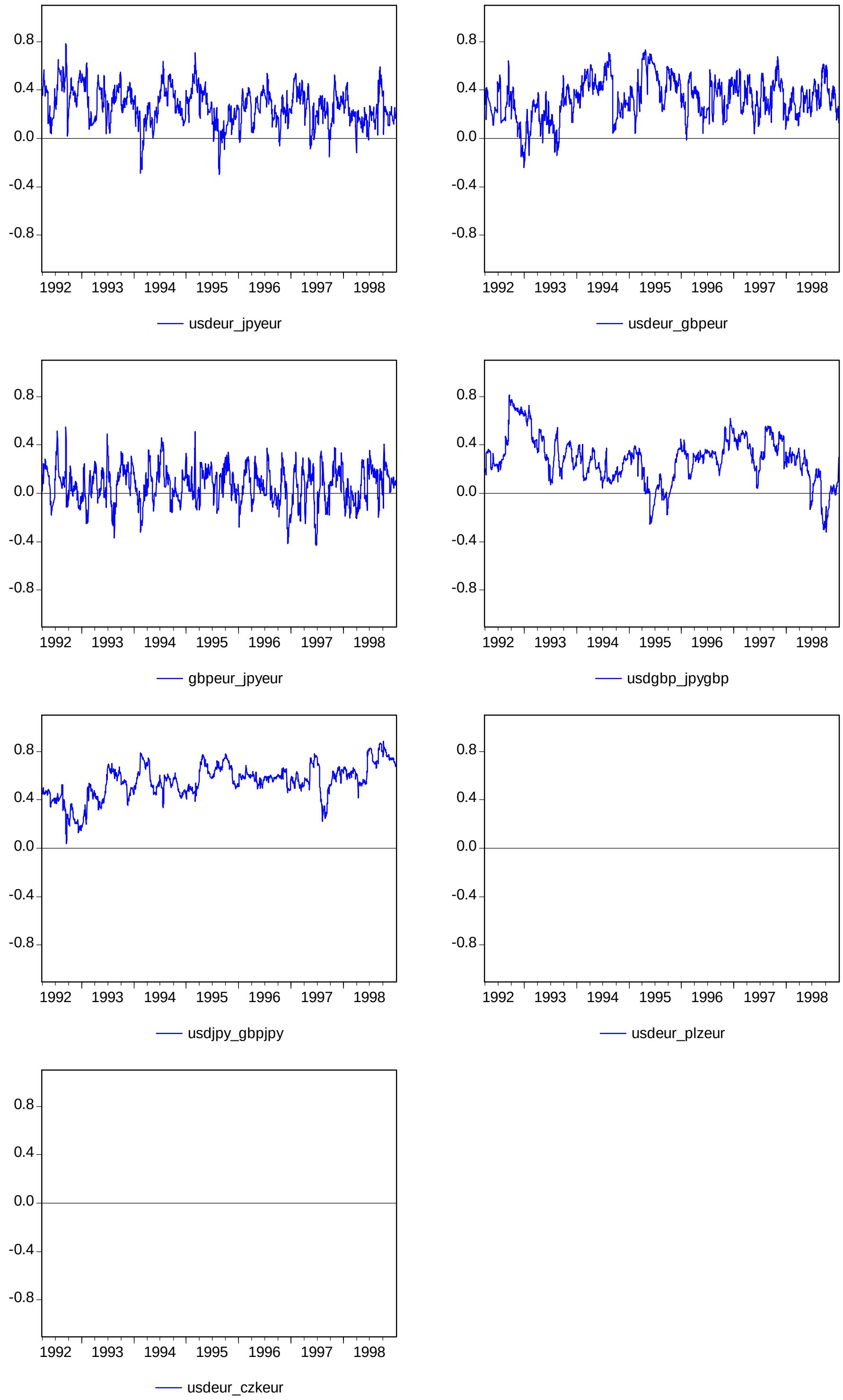

Chart 4. GARCH correlation. Pre 1999.

2: Correlations post-January 1999

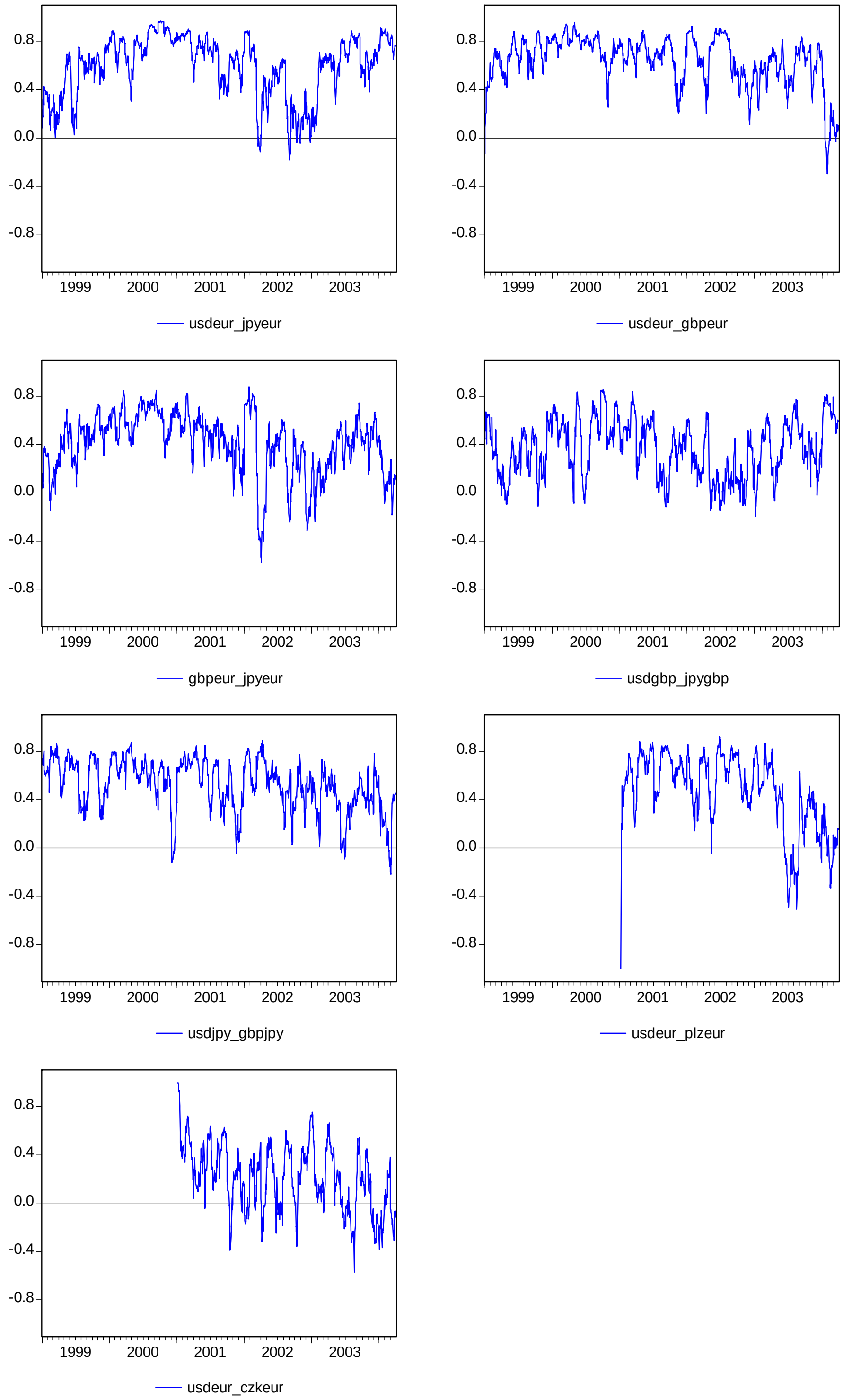

Chart 5. Historical Correlation. Post 1999.

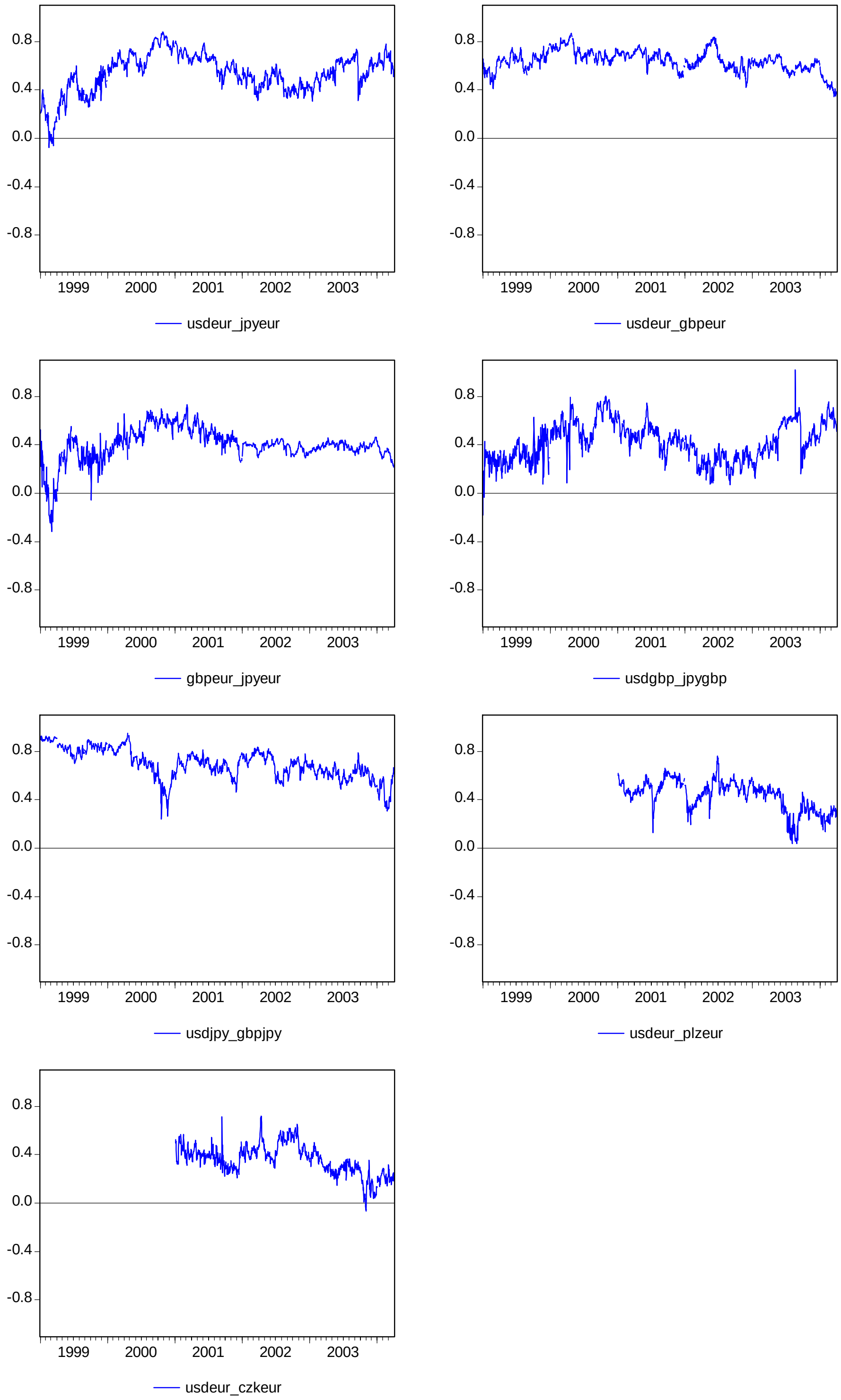

Chart 6: Implied Correlation. Post 1999.

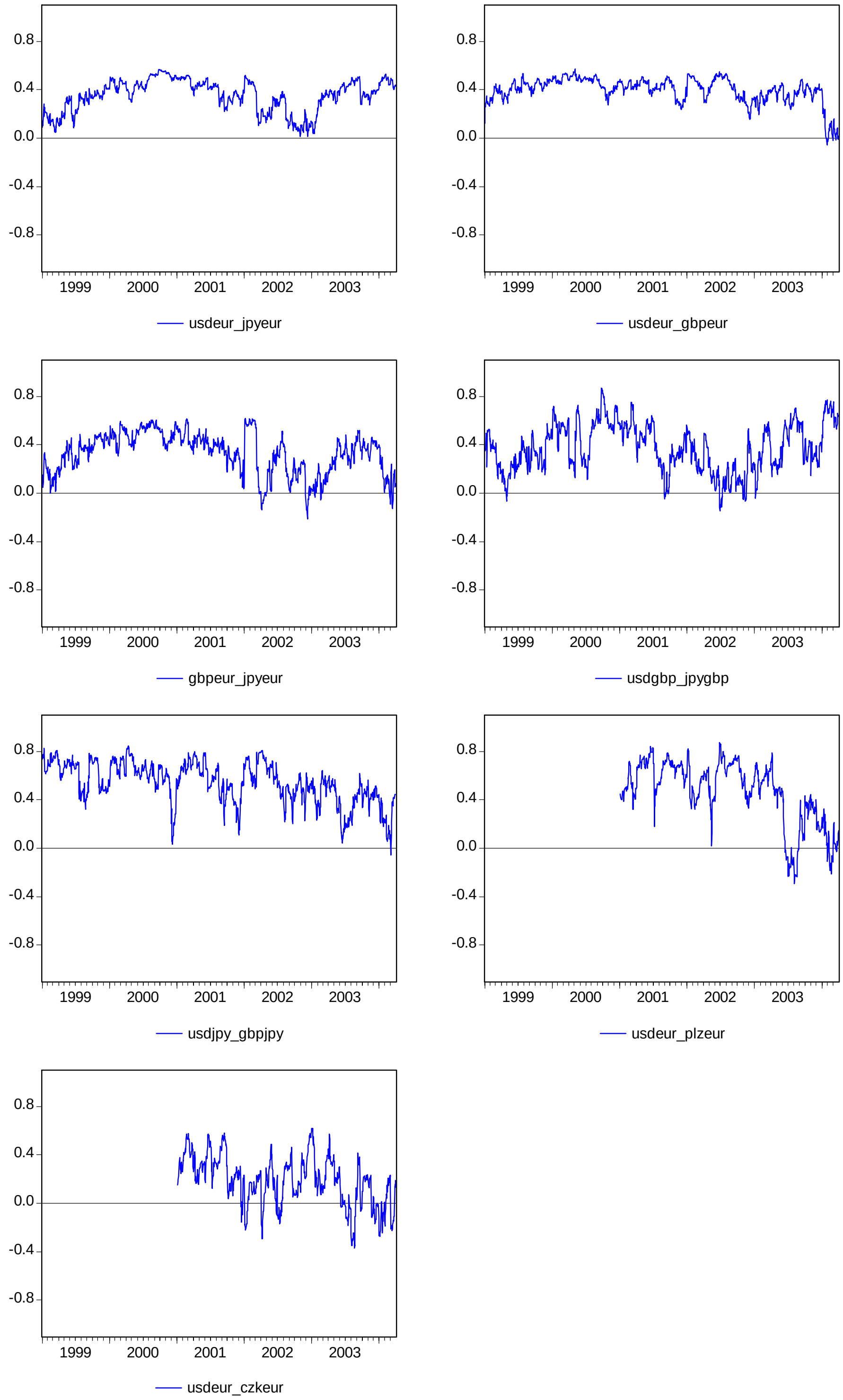

Chart 7. RiskMetrics Correlation. Post 1999.

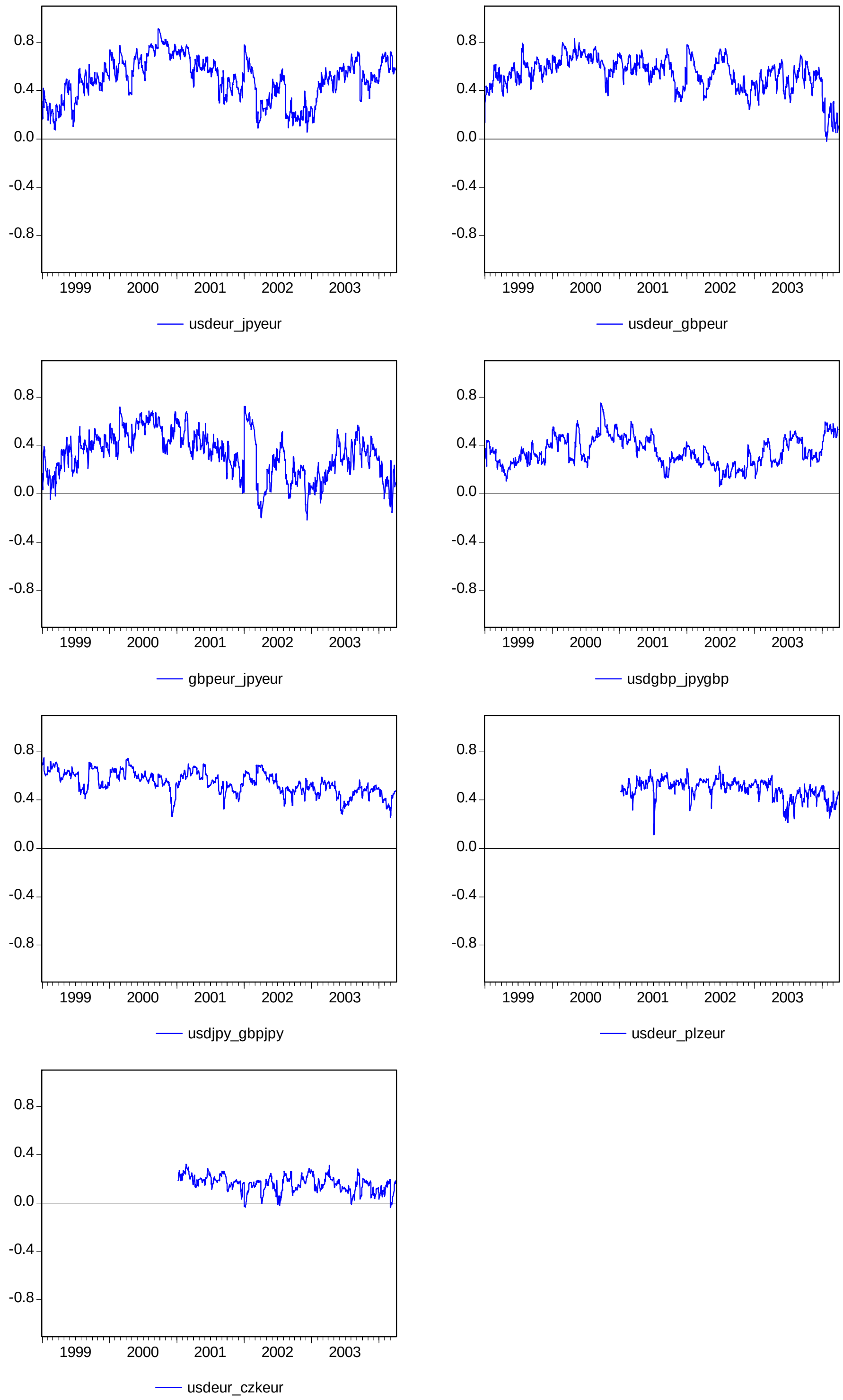

Chart 8. GARCH Correlation. Post 1999.

Appendix 4 Tables (coefficient estimates above; standard errors below)

Table 1. Correlation Predicatability Regressions. All sample: January 1992 - March 2003

USD/EUR-JPY/EUR

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
0.045	0.747				0.205
0.052	0.105				
0.178		0.564			0.314
0.027		0.053			
0.174			0.874		0.229
0.03			0.112		
0.095				0.858	0.329
0.029				0.066	
0.053	0.356	0.446			0.346
0.04	0.095	0.059			
0.028	0.452		0.602		0.282
0.044	0.103		0.115		
0.022	0.263			0.706	0.343
0.038	0.096			0.085	
0.048	0.29	0.367	-0.55	0.59	0.366
0.037	0.094	0.105	0.167	0.165	

USD/EUR-GBP/EUR

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
0.009	0.79				0.207
0.073	0.124				
0.209		0.548			0.295
0.034		0.058			
0.257			0.563		0.125
0.048			0.12		
0.093				0.875	0.324
0.04				0.079	
0.045	0.384	0.426			0.33
0.065	0.12	0.059			
0.009	0.653		0.216		0.219
0.07	0.126		0.113		
-0.022	0.323			0.708	0.347
0.062	0.12			0.086	
-0.008	0.425	0.182	-0.493	0.765	0.383
0.062	0.123	0.11	0.137	0.164	

GBP/EUR-JPY/EUR

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
0.021	0.661				0.136
0.039	0.117				
0.14		0.351			0.123
0.021		0.063			
0.133			0.43		0.101
0.022			0.091		
0.109				0.584	0.171
0.021				0.077	
0.027	0.47	0.232			0.179
0.035	0.112	0.063			
0.017	0.506		0.259		0.166
0.036	0.115		0.089		
0.024	0.385			0.43	0.206
0.033	0.108			0.084	
0.034	0.376	-0.044	-0.476	0.945	0.227
0.032	0.103	0.096	0.127	0.173	

USD/GBP-JPY/GBP

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
-0.02	0.751				0.203
0.049	0.101				
0.211		0.315			0.099
0.031		0.068			
0.166			0.46		0.156
0.033			0.075		
0.078				0.745	0.174
0.042				0.107	
-0.011	0.662	0.097			0.209
0.048	0.113	0.067			
0.001	0.56		0.204		0.221
0.046	0.12		0.084		
-0.033	0.517			0.372	0.227
0.047	0.125			0.132	
-0.059	0.491	-0.19	0.012	0.669	0.234
0.05	0.126	0.102	0.141	0.279	

USD/JPY-GBP/JPY

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
0.075	0.698				0.337
0.068	0.093				
0.316		0.406			0.161
0.048		0.075			
0.221			0.565		0.222
0.057			0.091		
-0.008				0.982	0.228
0.095				0.16	
0.075	0.674	0.031			0.338
0.067	0.111	0.075			
0.07	0.629		0.092		0.34
0.068	0.117		0.097		
0.016	0.604			0.219	0.342
0.083	0.108			0.158	
-0.057	0.603	-0.145	-0.013	0.507	0.346
0.107	0.114	0.135	0.154	0.275	

USD/EUR-PLZ/EUR

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
-0.101	1.316				0.285
0.139	0.261				
0.237		0.496			0.234
0.085		0.12			
0.143			0.69		0.296
0.091			0.143		
-0.409				1.786	0.187
0.278				0.519	
-0.047	0.955	0.215			0.306
0.113	0.252	0.149			
-0.016	0.675		0.405		0.318
0.109	0.328		0.236		
-0.324	1.065			0.672	0.3
0.299	0.283			0.67	
-0.125	0.647	-0.032	0.388	0.293	0.319
0.245	0.342	0.125	0.232	0.595	

USD/EUR-CZK/EUR

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
-0.132	0.906				0.186
0.082	0.217				
0.175		0.109			0.012
0.047		0.116			
0.177			0.114		0.007
0.051			0.154		
0.16				0.238	0.002
0.081				0.423	
-0.134	0.888	0.039			0.186
0.084	0.202	0.086			
-0.134	0.897		0.025		0.185
0.087	0.204		0.121		
-0.144	0.897			0.089	0.184
0.108	0.209			0.305	
-0.131	0.889	0.074	-0.054	-0.008	0.183
0.104	0.205	0.113	0.233	0.568	

Table 2. Correlation Predicatability Regressions. Sample: March 1999 - March 2004

USD/EUR-JPY/EUR

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
0.1	0.924				0.359
0.075	0.113				
0.254		0.583			0.343
0.052		0.076			
0.189			1.168		0.382
0.056			0.143		
0.19				0.834	0.362
0.056				0.094	
0.097	0.577	0.32			0.411
0.062	0.139	0.107			
0.091	0.473		0.717		0.417
0.062	0.17		0.254		
0.088	0.512			0.477	0.406
0.062	0.14			0.151	
0.093	0.485	0.106	0.55	-0.025	0.418
0.062	0.154	0.123	0.557	0.307	

USD/EUR-GBP/EUR

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
-0.085	1.139				0.217
0.188	0.274				
0.34		0.473			0.206
0.071		0.096			
0.253			0.979		0.218
0.088			0.198		
0.272				0.692	0.206
0.1				0.164	
-0.012	0.728	0.294			0.27
0.145	0.205	0.082			
-0.023	0.661		0.608		0.265
0.14	0.191		0.196		
-0.035	0.713			0.412	0.261
0.151	0.211			0.149	
-0.013	0.67	0.185	0.169	0.075	0.27
0.144	0.186	0.173	0.386	0.317	

GBP/EUR-JPY/EUR

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
0.222	0.467				0.067
0.069	0.141				
0.251		0.387			0.146
0.038		0.081			
0.217			0.581		0.152
0.044			0.127		
0.218				0.586	0.193
0.041				0.095	
0.179	0.229	0.334			0.158
0.057	0.13	0.087			
0.167	0.178		0.513		0.16
0.057	0.138		0.147		
0.177	0.131			0.543	0.199
0.053	0.117			0.107	
0.191	0.147	-0.049	-0.309	0.855	0.204
0.05	0.12	0.14	0.471	0.423	

USD/GBP-JPY/GBP

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
0.077	0.682				0.198
0.054	0.119				
0.274		0.252			0.063
0.043		0.094			
0.224			0.382		0.1
0.047			0.108		
0.153				0.612	0.09
0.068				0.18	
0.077	0.67	0.014			0.197
0.054	0.129	0.094			
0.076	0.646		0.041		0.198
0.054	0.139		0.12		
0.08	0.696			-0.026	0.197
0.065	0.144			0.207	
0.149	0.697	0.026	0.335	-0.61	0.204
0.064	0.139	0.109	0.157	0.314	

USD/JPY-GBP/JPY

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
-0.075	0.856				0.242
0.106	0.144				
0.412		0.213			0.043
0.061		0.097			
0.305			0.404		0.103
0.071			0.115		
0.105				0.771	0.112
0.118				0.203	
-0.087	0.943	-0.092			0.247
0.108	0.179	0.107			
-0.077	0.876		-0.022		0.242
0.107	0.201		0.152		
-0.068	0.878			-0.042	0.242
0.118	0.213			0.289	
-0.167	0.79	-0.337	0.192	0.39	0.258
0.138	0.216	0.147	0.187	0.425	

USD/EUR-PLZ/EUR

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
-0.128	1.368				0.314
0.131	0.248				
0.225		0.514			0.267
0.075		0.105			
0.14			0.694		0.329
0.078			0.123		
-0.456				1.872	0.219
0.244				0.458	
-0.06	0.965	0.228			0.338
0.107	0.254	0.134			
-0.024	0.677		0.416		0.351
0.104	0.316		0.203		
-0.358	1.083			0.719	0.332
0.262	0.28			0.611	
-0.135	0.649	-0.03	0.392	0.302	0.351
0.229	0.328	0.125	0.21	0.571	

USD/EUR-CZK/EUR

Intercept	Implied	Historical	RiskMetrics	GARCH	Adj-rbar2
-0.156	0.956				0.206
0.077	0.21				
0.166		0.114			0.012
0.046		0.115			
0.166			0.126		0.008
0.05			0.151		
0.152				0.233	0.002
0.08				0.423	
-0.158	0.94	0.037			0.205
0.079	0.196	0.085			
-0.158	0.947		0.027		0.205
0.081	0.197		0.118		
-0.165	0.948			0.074	0.204
0.103	0.203			0.304	
-0.145	0.937	0.068	-0.02	-0.095	0.203
0.1	0.2	0.114	0.227	0.558	

Table 3. Mincer Zarnowitz Decomposition of MSE in Percentage

	All sample				Post 1999			
	USD/EUR-GBP/EUR				USD/EUR-GBP/EUR			
	MSE	Bias	Inefficiency	Residual	MSE	Bias	Inefficiency	Residual
Implied	0.077	16.379	1.513	82.108	0.030	0.061	0.411	99.528
Historical	0.073	0.008	22.189	77.803	0.042	0.040	24.315	75.645
RiskMetrics	0.085	10.684	7.093	82.223	0.091	65.952	0.005	34.043
GARCH	0.056	2.789	0.936	96.275	0.044	24.434	3.717	71.848
	USD/EUR-JPY/EUR				USD/EUR-JPY/EUR			
	MSE	Bias	Inefficiency	Residual	MSE	Bias	Inefficiency	Residual
Implied	0.078	7.778	2.656	89.566	0.044	8.144	0.347	91.509
Historical	0.077	0.005	21.491	78.504	0.052	0.089	21.072	78.839
RiskMetrics	0.088	22.581	0.476	76.943	0.101	61.291	0.489	38.220
GARCH	0.062	3.089	1.294	95.618	0.052	22.414	1.704	75.882
	USD/EUR-PLZ/EUR				USD/EUR-PLZ/EUR			
	MSE	Bias	Inefficiency	Residual	MSE	Bias	Inefficiency	Residual
Implied	0.071	2.171	2.200	95.629	0.070	1.565	3.166	95.269
Historical	0.095	0.136	23.985	75.878	0.094	0.041	24.650	75.310
RiskMetrics	0.072	0.133	7.875	91.991	0.071	0.049	8.714	91.237
GARCH	0.080	0.358	4.284	95.358	0.081	0.738	5.730	93.532
	USD/EUR-CZK/EUR				USD/EUR-CZK/EUR			
	MSE	Bias	Inefficiency	Residual	MSE	Bias	Inefficiency	Residual
Implied	0.082	33.949	0.165	65.885	0.083	35.371	0.035	64.593
Historical	0.124	0.132	46.572	53.295	0.123	0.324	45.211	54.465
RiskMetrics	0.099	0.029	33.026	66.945	0.099	0.001	31.743	68.256
GARCH	0.070	1.932	3.623	94.445	0.071	1.124	3.553	95.323
	GBP/EUR-JPY/EUR				GBP/EUR-JPY/EUR			
	MSE	Bias	Inefficiency	Residual	MSE	Bias	Inefficiency	Residual
Implied	0.088	7.151	3.708	89.141	0.065	0.084	8.620	91.296
Historical	0.118	0.001	32.415	67.584	0.077	0.013	29.989	69.998
RiskMetrics	0.099	0.537	16.411	83.052	0.065	9.191	7.749	83.060
GARCH	0.085	1.299	9.349	89.352	0.064	10.222	9.597	80.182
	USD/GBP-JPY/GBP				USD/GBP-JPY/GBP			
	MSE	Bias	Inefficiency	Residual	MSE	Bias	Inefficiency	Residual
Implied	0.091	18.372	2.238	79.390	0.051	6.521	4.774	88.705
Historical	0.124	0.007	34.097	65.896	0.085	0.000	37.359	62.641
RiskMetrics	0.096	0.001	20.289	79.711	0.066	0.040	22.535	77.425
GARCH	0.077	0.000	2.405	97.595	0.054	0.612	3.863	95.525
	USD/JPY-GBP/JPY				USD/JPY-GBP/JPY			
	MSE	Bias	Inefficiency	Residual	MSE	Bias	Inefficiency	Residual
Implied	0.061	24.794	6.556	68.650	0.065	47.291	0.475	52.235
Historical	0.074	0.009	29.115	70.875	0.070	0.047	38.557	61.396
RiskMetrics	0.057	0.691	14.431	84.877	0.051	0.885	19.967	79.148
GARCH	0.049	0.677	0.010	99.313	0.041	0.901	1.095	98.004

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