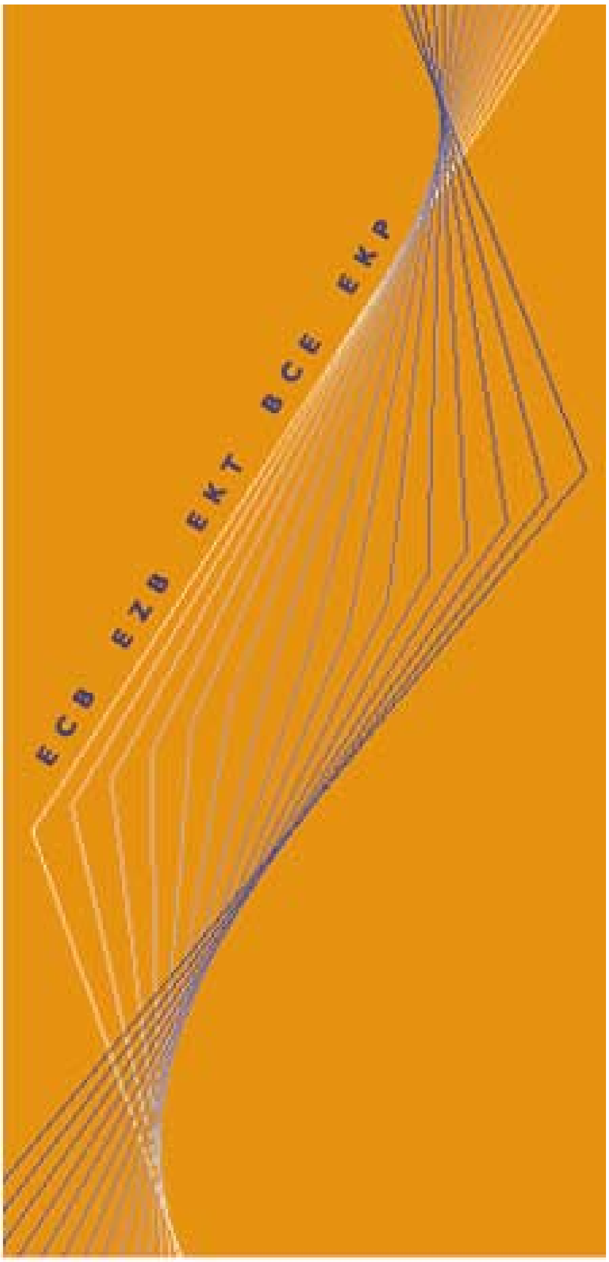
EUROPEAN CENTRAL BANK

WORKING PAPER SERIES



WORKING PAPER NO. 46

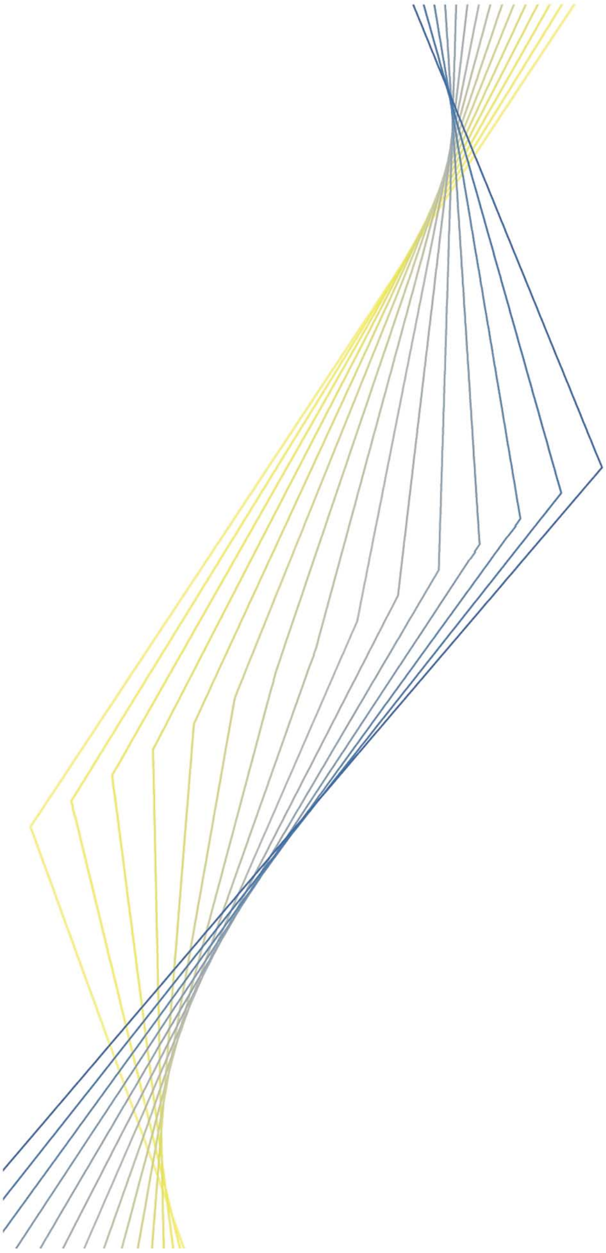
A TWO-FACTOR MODEL OF THE GERMAN TERM STRUCTURE OF INTEREST RATES

BY NUNO CASSOLA
AND JORGE BARROS LUÍS

March 2001

EUROPEAN CENTRAL BANK

WORKING PAPER SERIES



WORKING PAPER NO. 46

A TWO-FACTOR MODEL OF THE GERMAN TERM STRUCTURE OF INTEREST RATES*

BY NUNO CASSOLA**
AND JORGE BARROS LUÍS***

March 2001

* Previous versions of this paper were presented at the ESCB Workshop on Yield Curve Modelling, Frankfurt am Main, 30 March 1999, the European Economics and Financial Centre Conference, Maribor, 30 June 1999, the University of York, 4 November 1999, Universidade Católica Portuguesa, Lisbon, 14 February 2000, Banco de Portugal, Lisbon, 17 February 2000, the CEMAPRE Conference, Lisbon, 6 June 2000, and the Money, Macro and Finance Research Group 2000 Conference, London, 6 September 2000. The authors are grateful to Mike Wickens, Peter N. Smith, Menelaos Karanasos, Jerome Henry and João Pedro Nunes for helpful discussions and suggestions, to Ken Singleton and Karim Abadir for helpful insights on the estimation procedure and to Fátima Silva for research assistance. The second author also acknowledges the support of the Lisbon Stock Exchange. The usual disclaimer applies.
** European Central Bank.
*** University of York and Banif Investimento.

Address	**Kaiserstrasse 29**
	D-60311 Frankfurt am Main
	Germany
Postal address	**Postfach 16 03 19**
	D-60066 Frankfurt am Main
	Germany
Telephone	**+49 69 1344 0**
Internet	**http://www.ecb.int**
Fax	**+49 69 1344 6000**
Telex	**411 144 ecb d**

ISSN 1561-0810

Contents

Abstract:

In this paper we show that a two-factor constant volatility model provides an adequate description of the dynamics and shape of the German term structure of interest rates from 1972 up to 1998. The model also provides reasonable estimates of the volatility and term premium curves. Following the conjecture that the two factors driving the German term structure of interest rates represent the *ex-ante* real interest rate and the expected inflation rate, the identification of one factor with expected inflation is discussed. Our estimates are obtained using a Kalman filter and a maximum likelihood procedure including in the measurement equation both the yields and their volatilities.

JEL Classification codes: E43, G12.

Key words: expectations hypothesis; term premiums; pricing kernels; affine model

1. Introduction

The identification of the factors that determine the time-series and cross section behaviour of the term structure of interest rates is a recurrent topic in the finance literature. Its is a controversial subject that has several empirical and practical implications, namely for assessing the impact of economic policy measures or for hedging purposes (see, for instance, Fleming and Remolona (1998) and Bliss (1997)).

One-factor models were the first step in modelling the term structure of interest rates. These models are grounded on the estimation of bond yields as functions of the short-term interest rate. Vasicek (1977) and Cox *et al.* (1985) (CIR hereafter) are the seminal papers within this literature. However, one factor models do not overcome the discrepancy between the theoretical mean yield curve implied by the time-series properties of bond yields and the observed curves that are substantially more concave than implied by the theory (see, for instance, Backus *et al.* (1998)).

One answer to this question has been provided by multifactor affine models, that consider bond yields as functions of several macroeconomic and financial variables, observable or latent. Affine models are easier to estimate than binomial models,[1] given that the parameters are linear in both the maturity of the assets and the number of factors.[2]

Most papers have focused on the U.S. term structure. The pronounced hump-shape of the US yield curve and the empirical work pioneered by Litterman and Scheinkman (1991) have led to the conclusion that three factors are required to explain the movements of the whole term structure of interest rates. These factors

---

[1] In these models, the state variables can go up or down in any unit of time and a probability is attached to each change (see, e.g., Backus *et al.* (1998, pp. 15-16)).

are usually identified as the level, the slope and the curvature of the term structure. Most studies have concluded that the level is the most important factor in explaining interest rate variation over time.

Moreover, given the apparent stochastic properties of the volatility of interest rates, Gaussian or constant volatility models are often rejected. Therefore, several papers have used 3-factor models with stochastic volatility in order to fit the term structure of interest rates (see, for instance, Balduzzi *et al.* (1996) and Gong and Remolona (1997 a)).

However, stochastic volatility models pose admissibility problems, as the factors determining the volatility of interest rates enter "square rooted" and thus must be positive.[3] Additionally, the parameters of a three-factor model with stochastic volatility are very difficult to estimate. In fact, frequently small deviations of the parameters from the estimated values generate widely different and implausible term structures. Furthermore, some term structures may have properties identifiable with less complex models. For instance, according to Buhler *et al.* (1999), principal component analysis reveals that two factors explain more than 95 percent of the variation in the German term structure of interest rates consistently from 1970 up to 1999.

To motivate our work we start by performing forward rate regressions following Backus *et al.* (1997), in order to assess the adequacy of Gaussian models to estimate the German term structure of interest rates. These models overcome the empirical problems posed by stochastic volatility models and are capable of reproducing a wide variety of shapes of the yield curve, though they face some

---

[2] See Campbell *et al.* (1997, chapter 11) or Backus *et al.* (1998) for graduate textbook presentations of affine models.

[3] For a continuous-time presentation and the discussion of admissibility and classification conditions of the models see Dai and Singleton (1998).

shortcomings regarding the limiting properties of the instantaneous forward rate.[4]

The model is derived in discrete-time: It matches the frequency of the data, allows the identification of the factors with observable macro-economic variables and avoids the problem of estimating continuous-time model with discrete-time data (see, for example, Aït-Sahalia (1996)).

As the data seems supportive of the constant volatility assumption, we estimate a two-latent factor Gaussian model. The specification of the model implies that the short-term interest rate is the sum of a constant with the two latent factors. This is consistent with the idea that nominal interest rates can be (approximately) decomposed into two components: the expected rate of inflation and the expected real interest rate (Fisher hypothesis) or, in the C-CAPM model, the expected growth rate of consumption.

We use the Kalman Filter to uncover the latent factors and a maximum likelihood procedure to estimate the time-constant parameters, following the pioneering work by Chen and Scott (1993a and 1993b).

The two-factor model fits quite well the yield and the volatility curve, providing reasonable estimates for the one-period forward and term premium curves. It also provides a good fit of the time series of bond yields.

As Backus *et al.* (1997) mention, the major outstanding issue is the economic interpretation of the interest rate behaviour approximated with affine models in terms of its monetary and real economic factors.[5]

[4] See, e.g., Campbell *et al.* (1997), pp. 433, on the limitations of a one-factor homoskedastic model.
[5] This is also the major theoretical drawback in arbitrage pricing theory (APT) developed by Ross (1976).

In line with Zin (1997), our conjecture is that one of the two factors driving the German term structure of interest rates is related to expected inflation. Thus after discussing the pros and cons of alternative ways of identifying one factor with the inflation rate process, we present some econometric evidence on the leading indicator properties of the second factor for inflation developments in Germany.

The remainder of paper is structured as follows. In the next section some background on asset pricing is presented. In the third section the theoretical framework of Duffie and Kan (1996) (DK hereafter) affine models is explained. In the fourth section a test of the expectations theory is developed that will be used to empirically motivate the Gaussian model. In the fifth section the Gaussian model to be estimated is fully specified. In the sixth section we discuss alternative ways of identifying the factors in the model. The econometric methodology is presented in the seventh section. Section eighth includes the presentation of the data and the results of the estimation. The main conclusions are stated at the end.

2. Background issues on asset pricing

The main result from modern asset pricing theory states that in an arbitrage-free environment there exists a positive stochastic discount factor (henceforth sdf, denoted by M_t) that gives the price at date t of any traded financial asset providing nominal cash-flows (P_t) as its discounted future pay-off: [6]

[6] Within the consumption-based CAPM framework, the pricing kernel corresponds to the intertemporal marginal rate of substitution in consumption, deflated by the inflation rate (see, Campbell *et al.* (1997) for a detailed presentation). The analysis is usually conducted on a nominal basis, following (1), as financial assets providing real cash-flows are scarce. The best known examples are the inflation-indexed Government bonds that exist only in a few countries. The UK and the US inflation-indexed Government bonds are the most prominent and their information content has been studied in several papers (see, for instance, Deacon and Derry (1994), Gong and Remolona (1997b) and Remolona *et al.* (1998)).

$$P_t = E_t\left[P_{t+1} M_{t+1}\right] \text{ or } 1 = E_t\left[\frac{P_{t+1}}{P_t} M_{t+1}\right] \tag{1}$$

M_t is also known as the pricing kernel, given that it is the determining variable of P_t. In fact, solving forward the basic pricing equation (1), a model of asset pricing requires the specification of a stochastic process for the pricing kernel:

$$P_t = E_t\left[M_{t+1} \ldots M_{t+n}\right] \tag{2}$$

Denoting the one-period nominal gross returns $\left(P_{t+1}/P_t\right)$ by $\left(1+i_{t+1}\right)$, and given that $E_t\left[\left(1+i_{t+1}\right)M_{t+1}\right] = E_t\left[\left(1+i_{t+1}\right)\right]E_t\left[M_{t+1}\right] + Cov_t\left[i_{t+1}, M_{t+1}\right]$ we get from (1):

$$E_t\left[1+i_{t+1}\right] = \frac{1}{E_t\left[M_{t+1}\right]}\left(1 - Cov_t\left[i_{t+1}, M_{t+1}\right]\right) \tag{3}$$

Therefore a risk-free asset, with gross return equal to $\left(1+i_{t+1}^f\right)$, that has a future pay-off known with complete certainty verifies:

$$1 + i_{t+1}^f = \frac{1}{E_t\left[M_{t+1}\right]} \tag{4}$$

Thus, the excess return of any asset over a risk-free asset, measured as the difference between (3) and (4) is:

$$\Lambda_t = E_t\left[i_{t+1}\right] - i_{t+1}^f = -\left(1 + i_{t+1}^f\right)Cov_t\left[i_{t+1}, M_{t+1}\right] \tag{5}$$

Equation (5) illustrates a basic result in finance theory: the excess return of any asset over the risk-free asset depends on the covariance of its rate of return with the stochastic discount factor. Thus, an asset whose pay-off has a negative correlation with the stochastic discount factor pays a risk premium.

Within the C-CAPM framework the sdf is equal to the marginal utility of consumption. When consumption growth is high the marginal utility of consumption is low. Therefore if returns are negatively correlated with the sdf, high returns are associated with high consumption states of nature. A risk premium must be paid for investors to hold such an asset because it fails to provide wealth when it is more valuable for the investor.

3. Duffie-Kan affine models of the term structure

Affine models are built upon a log-linear relationship between asset prices and the sdf, on one side, and the factors or state variables, on the other side. These models were originally developed by Duffie and Kan (1996), for the term structure of interest rates. As referred in Balduzzi *et al.* (1996), "Duffie and Kan (1996) show that a wide range of choices of stochastic processes for interest rate factors yield bond pricing solutions of a form now widely called exponential-affine models".

Let us start by writing equation (1) in logs:

$$p_t = \log\left(E_t\left[P_{t+1}M_{t+1}\right]\right), \tag{6}$$

where lowercase letters denote the logs of the corresponding uppercase letters. With the assumption of joint log-normality of bond prices and the nominal pricing kernel and using the statistical result that if $\log X \sim N(\mu,\sigma^2)$ then $\log E(X) = \mu + \sigma^2/2$, we obtain from equation (6)

$$p_t = E_t\left[m_{t+1} + p_{t+1}\right] + \frac{1}{2} Var_t\left[m_{t+1} + p_{t+1}\right] \tag{7}$$

Duffie and Kan (1996) define a general class of multifactor affine models of the term structure, where the log of the pricing kernel is a linear function of several factors $z_t^T = \left(z_{1,t} \ldots z_{k,t}\right)$. DK models offer the advantage of nesting the most important term structure models, from Vasicek (1977) and CIR one-factor models to three-factor models like the one presented in Gong and Remolona (1997a). An additional feature of these models is that they allow the estimation of the term structure simultaneously on a cross-section and a time-series basis. Furthermore they provide a way of computing and estimating simple closed-form expressions for the spot, forward, volatility and term premium curves.

Expressed in discrete time, the discount factors in DK models are specified as:[7]

$$-m_{t+1} = \xi + \gamma^T z_t + \lambda^T V(z_t)^{1/2} \varepsilon_{t+1}, \tag{8}$$

where $V(z_t)$ is the variance-covariance matrix of the random shocks to the sdf and is defined as a diagonal matrix with elements $v_i(z_t) = \alpha_i + \beta_i^T z_t$. Under certain conditions, the volatility functions $v_i(z_t)$ are positive;[8] β_i has nonnegative elements and ε_t are the independent shocks normally distributed as $\varepsilon_t \sim N(0, I)$. Following (5), the parameters in λ^T are the market prices of risks, as they govern the covariance between the stochastic discount factor and the latent factors of the yield curve. Thus, the higher these parameters are, the higher is the covariance between the discount factor and the asset return and the lower is its expected rate of return or the less risky the asset is (when the covariance is negative).

The k-dimensional vector of factors z_t is defined as follows:

$$z_{t+1} = (I - \Phi)\theta + \Phi z_t + V(z_t)^{1/2} \varepsilon_{t+1}, \tag{9}$$

where Φ has positive diagonal elements which ensure that the factors are stationary and θ is the long-run mean of the factors. Asset prices are also log-linear functions of the factors. Adding a second subscript in order to identify the term to maturity (denoted by n), bond prices are given as follows:

[7] See, e.g., Backus *et al.* (1998).

$$-p_{n,t} = A_n + B_n{}^T z_t \,, \tag{10}$$

where A_n is a parameter and B_n a vector of parameters to be estimated. The parameters in B_n are commonly known as the factor loadings, given that their values measure the impact of a one-standard deviation shock to the factors on the log of asset prices.

In term structure models, the identification of the parameters is easier, considering the restrictions imposed by the maturing bond price. In fact, when the term structure is modelled using zero-coupon bonds paying one monetary unit, the log of the price of a maturing bond must be zero. Consequently, from (10), the common normalisation $A_0 = B_0 = 0$ results. The following recursive restrictions between the parameters are obtained computing the moments in equation (7), using equations (8) and (10), equating the independent terms and the terms in z_t in equation (9) respectively to A_n and B_n in (10) and assuming $p_{0,t}$=0:

$$A_n = A_{n-1} + \xi + B_{n-1}{}^T (I - \Phi)\theta - \frac{1}{2}\sum_{i=1}^{k}\left(\lambda_i + B_{i,n-1}\right)^2 \alpha_i \,, \tag{11}$$

$$B_n{}^T = \left(\gamma^T + B_{n-1}{}^T \Phi\right) - \frac{1}{2}\sum_{i=1}^{k}\left(\lambda_i + B_{i,n-1}\right)^2 \beta_i^T \,, \tag{12}$$

Our empirical analysis is based on interest rates of nominal zero-coupon bonds. Continuously compounded yields to maturity of discount bonds or spot rates ($y_{n,t}$) can be easily computed from bond prices as:

$$y_{n,t} = -\frac{p_{n,t}}{n} \tag{13}$$

Consequently, from (10) and (13), the yield curve is defined as:

[8] See Backus *et al.* (1998).

$$y_{n,t} = \frac{1}{n}\left(A_n + B_n{}^T z_t\right) \tag{14}$$

Using equations (11), (12) and (14), the short-term or one-period interest rate is:

$$y_{1,t} = \xi - \frac{1}{2}\sum_{i=1}^{k}\lambda_i{}^2\alpha_i + \left[\gamma^T - \frac{1}{2}\sum_{i=1}^{k}\lambda_i{}^2\beta_i^T\right]z_t \tag{15}$$

Correspondingly, the expected value of the short rate is:

$$\begin{aligned} E_t\left(y_{1,t+n}\right) &= E_t\left(\xi - \frac{1}{2}\sum_{i=1}^{k}\lambda_i{}^2\alpha_i + \left[\gamma^T - \frac{1}{2}\sum_{i=1}^{k}\lambda_i{}^2\beta_i^T\right]z_{t+n}\right) \\ &= \xi - \frac{1}{2}\sum_{i=1}^{k}\lambda_i{}^2\alpha_i + \left[\gamma^T - \frac{1}{2}\sum_{i=1}^{k}\lambda_i{}^2\beta_i^T\right]E_t\left(z_{t+n}\right) \qquad , \\ &= \xi - \frac{1}{2}\sum_{i=1}^{k}\lambda_i{}^2\alpha_i + \left(\gamma^T - \frac{1}{2}\sum_{i=1}^{k}\lambda_i{}^2\beta_i^T\right)\left[\left(I - \Phi^n\right)\theta + \Phi^n z_t\right] \end{aligned} \tag{16}$$

The volatility curve of the yields is derived from the variance-covariance matrix in the specification of the factors. From equations (9) and (14), the volatility curve is given by:

$$Var_t\left(y_{n,t+1}\right) = \frac{1}{n^2}B_n{}^T V\left(z_t\right)B_n . \tag{17}$$

The instantaneous or one-period forward rate is the log of the inverse of the gross return:

$$f_{n,t} = p_{n,t} - p_{n+1,t} . \tag{18}$$

According to the definition in (18), the price equation in (10) and the recursive restrictions in (11) and (12), the one-period forward curve is:

$$
\begin{aligned}
f_{n,t} &= \left(A_{n+1} + B_{n+1}{}^{T} z_t\right) - \left(A_n + B_n{}^{T} z_t\right) \\
&= \xi + B_n{}^{T}(I-\Phi)\theta - \frac{1}{2}\sum_{i=1}^{k}\left(\lambda_i + B_{i,n}\right)^2 \alpha_i + \left[\gamma^T + B_n{}^{T}(\Phi - I) - \frac{1}{2}\sum_{i=1}^{k}\left(\lambda_i + B_{i,n}\right)^2 \beta_i^T\right] z_t
\end{aligned} \tag{19}
$$

The term premium is usually computed as the one-period log excess return of the n-period bond over the short-rate. Using equations (10), (11), (12) and (15), it is equal to:

$$
\begin{aligned}
\Lambda_{n,t} &= E_t p_{n,t+1} - p_{n+1,t} - y_{1,t} \\
&= -\sum_{i=1}^{k}\left[\lambda_i B_{i,n}\alpha_i + \frac{B_{i,n}{}^2 \alpha_i}{2}\right] - \sum_{i=1}^{k}\left(\lambda_i B_{i,n} + B_{i,n}{}^2\right)\beta_i^T z_t
\end{aligned} \tag{20}
$$

From (16), (19) and (20), one can conclude that the forward rate is equal to the expected future short-term interest rate plus the term premium and a constant term that is related to the mean of the factors. The term premium can alternatively be calculated from the basic pricing equation. In fact, from (1), the following result can be stated:

$$
E_t p_{n,t+1} - p_{n+1,t} = -E_t m_{t+1} - Var_t\left(i_{n,t+1}\right)/2 - Var_t\left(m_{t+1}\right)/2 - COV_t\left(i_{n,t+1}, m_{t+1}\right) \tag{21}
$$

According to (7) and considering the assumption $p_{0t} = 0$, the short-term interest rate is:

$$
p_{1,t} = E_t\left[m_{t+1}\right] + \frac{1}{2} Var_t\left[m_{t+1}\right] \tag{22}
$$

Therefore, computing the short-term interest rate from (13) and using (21), the term premium will be, in general, given by:

$$
\Lambda_{n,t} = -COV_t\left(i_{n,t+1}, m_{t+1}\right) - Var_t\left(i_{n,t+1}\right)/2 \tag{23}
$$

Equation (23) tells us that the term premium is determined by the covariance of the asset's rate of return with the stochastic discount factor and a Jensen's inequality term resulting from the fact that the risk-premium is computed as the log-excess return. Thus, in line with equation (5), the lower the covariance, the higher the term premium is.

As from (10) $i_{n,t+1} = p_{n-1,t+1} - p_{n,t} = -A_n - B_n^T z_{t+1} + A_{n+1} + B_{n+1}^T z_t$, the covariance in (23) is $-B_n^T COV_t(z_{t+1}, m_{t+1})$ and the conditional variances of the factors correspond to $B_n^T Var_t(z_{t+1}) B_n$. Consequently, equation (23) is equivalent to:

$$\Lambda_{n,t} = B_n^T COV(z_{t+1}, m_{t+1}) - B_n^T Var_t(z_{t+1}) B_n \; / \, 2 \tag{24}$$

According to (8) and (9), the term premium may be written from (24) as:

$$\Lambda_{n,t} = -\lambda^T V(z_t) B_n - \frac{B_n^T V(z_t) B_n}{2} \tag{25}$$

Given that $V(z_t)$ was previously defined as a diagonal matrix with elements $v_i(z_t) = \alpha_i + \beta_i^{\,T} z_t$, equation (25) corresponds to (20). The first component in (25) is a pure risk premium, where λ is the price associated with the quantity of risk $V(z_t)B_n$. The parameters in λ determine the signal of the term premium. The second component is a Jensen inequality term.

4. Gaussian affine models

The model we estimate belongs to the class of Gaussian or constant volatility models. It is a generalisation of the Vasicek (1977) one-factor model and a particular case of the DK model, implying that some form of the expectations theory holds. As it will be seen, this model seems to be adequate to fit the German term structure, as the expectations theory is valid to a close approximation in this case.

Following equation (8), the sdf in a two-factor Gaussian model is written as:[9]

$$-m_{t+1} = \delta + \sum_{i=1}^{k}\left(\frac{\lambda_i^2}{2}\sigma_i^2 + z_{it} + \lambda_i\sigma_i\varepsilon_{i,t+1}\right). \tag{26}$$

with k = 2. The factors are assumed to follow a first-order autoregressive order, with zero mean:[10]

$$z_{i,t+1} = \varphi_i z_{it} + \sigma_i\varepsilon_{i,t+1}\text{, with } i = 1,2. \tag{27}$$

Within the DK framework, these models are characterised by:

$$\begin{aligned}
&\theta_i = 0 \\
&\Phi = diag(\varphi_1, \varphi_2) \\
&\alpha_i = \sigma_i^2 \\
&\beta_i = 0 \\
&\xi = \delta + \sum_{i=1}^{k}\frac{\lambda_i^2}{2}\sigma_i^2 \\
&\gamma_i = 1
\end{aligned}. \tag{28}$$

The recursive restrictions are:

$$A_n = A_{n-1} + \delta + \frac{1}{2}\sum_{i=1}^{k}\left[\lambda_i^2\sigma_i^2 - \left(\lambda_i\sigma_i + B_{i,n-1}\sigma_i\right)^2\right], \tag{29}$$

$$B_{i,n} = \left(1 + B_{i,n-1}\varphi_i\right). \tag{30}$$

Notice that, according to the auto-regressive pattern evidenced in equation (30), each $B_{i,n}$ corresponds to the sum of the n-terms of a geometric progression:

---

[9] As it will be seen later, this specification was chosen in order to write the short-term interest rate as the sum of a constant (δ) with the factors.

[10] This corresponds to considering the differences between the "true" factors and their means.

$$B_{i,n} = 1 + \varphi_i + \varphi_i^{\;2} + \ldots + \varphi_i^{\;n} = \sum_{i=1}^{n} \varphi_i^{\;n-1} = \frac{1-\varphi_i^{\;n}}{1-\varphi_i}. \tag{31}$$

As we see from equation (26), in a homoskedastic model, the element in the right-hand side of (8) related to the risk is zero. Thus, there are no interactions between the risk and the factors influencing the term structure, i.e., the term premium is constant.

Given (15) and (28), the short term interest rate is:

$$y_{1,t} = \delta + \sum_{i=1}^{k} z_{it} \;. \tag{32}$$

As referred in Campbell *et al.* (1997),[11] the $B_{i,n}$ coefficients in a Gaussian model measure the sensitivity of the log of bond prices to changes in short-term interest rate. This is different from duration, as it does not correspond to the impact on bond prices of changes in the respective yields, but instead in the short rate.

These models have the appealing feature that the short-term rate is the sum of the factors.[12] Our conjecture is that the yield curve may be determined by two factors, one of them being related to inflation and the other to a real factor, possibly the *ex-ante* real interest rate.

Following (19) and (28), the one-period forward rate is given by:

$$f_{n,t} = \delta + \frac{1}{2}\sum_{i=1}^{k}\left[\lambda_i^2\sigma_i^2 - \left(\lambda_i\sigma_i + \frac{1-\varphi_i^n}{1-\varphi_i}\sigma_i\right)^2\right] + \sum_{i=1}^{k}\left[\varphi_i^n z_{it}\right] \tag{33}$$

This specification of the forward-rate curve accommodates very different shapes. However, the limiting forward rate cannot be simultaneously finite and

[11] Though in a one-factor model setting.
[12] Added by a constant, as the factors are defined as having zero mean.

time-varying. In fact, if φ_i<1, the limiting value will not depend on the factors, corresponding to the following expression:[13]

$$\lim_{n\to\infty} f_{n,t} = \delta + \sum_{i=1}^{k}\left[-\frac{\lambda_i \sigma_i^{\,2}}{(1-\varphi_i)} - \frac{\sigma_i^{\,2}}{2(1-\varphi_i)^2}\right] \tag{34}$$

From (17) and (28), the volatility curve is:

$$Var_t\left(y_{n,t+1}\right) = \frac{1}{n^2}\sum_{i=1}^{k}\left(B_{i,n}^2 \sigma_i^2\right), \tag{35}$$

Notice that as the factors have constant volatility, given by $Var_t\left(z_{i,t+1}\right) = \sigma_i^2$, the volatility of the yields does not depend on the level of the factors.

From (20) and (28), the term premium in these models will be:

$$\begin{aligned}\Lambda_{n,t} &= E_t p_{n,t+1} - p_{n+1,t} - y_{1,t} = \\ &= \frac{1}{2}\sum_{i=1}^{k}\left[\lambda_i^2\sigma_i^2 - \left(\lambda_i\sigma_i + \frac{1-\varphi_i^n}{1-\varphi_i}\sigma_i\right)^2\right] =, \\ &= \sum_{i=1}^{k}\left[-\lambda_i\sigma_i^{\,2}B_{i,n} - \frac{B_{i,n}^{\ \,2}\sigma_i^{\,2}}{2}\right]\end{aligned} \tag{36}$$

According to (33) and (36), the one-period forward rate in these Gaussian models corresponds to the sum of the term premium with a constant and with the factors weighted by the autoregressive parameters of the factors. Once again, the limiting case is worth noting. When φ_i<1, the limiting value of the risk

[13] If $\varphi_i = 1$ interest rates are non-stationary. In that case, the limiting value of the instantaneous forward is time-varying but assumes infinite values. Effectively, according to (31), $\frac{1-\varphi_i^n}{1-\varphi_i} = n$ in this case. Thus, the expression for the instantaneous forward will be given by $f_{n,t} = \delta + \sum_{i=1}^{k}\left[-n\lambda_i\sigma_i^2 - \frac{1}{2}n^2\sigma_i^2\right] + \sum_{i=1}^{k} z_{it}$. Accordingly, even if λ_i <0, the forward rate curve may start by increasing, but at the longer end it will decrease infinitely. Obviously, if λ_i >0, the forward rate curve will decrease monotonously.

premium differs from the forward only by δ. Thus, within constant volatility models, the expected short-term rate for a very distant settlement date is δ, i.e., the average short-term interest rate.

From (16) and (28), we can calculate the expected value of the short-term rate for any future date $t+n$:

$$E_t\left(y_{1,t+n}\right) = \delta + \sum_{i=1}^{k}\left[\varphi_i^n z_{it}\right] \tag{37}$$

Comparing equations (33), (36) and (37) we conclude that the expectations theory of the term structure holds with constant term premiums:[14]

$$f_{n,t} = E_t\left(y_{1,t+n}\right) + \Lambda_n. \tag{38}$$

Thus, assessing the adequacy of Gaussian models correspond to testing the validity of the expectations theory of the term structure with constant term premiums. As in the steady state it is expected that short-term interest rates remain constant, according to (38) the one-period forward curve should be flat under the absence of risk premium ($\Lambda_n = 0$).

5. A forward rate test of the expectations theory

Following Backus *et al.* (1997), the expectations theory holds if in the following forward regression the slope $c_n = 1$:

$$f_{n-1,t+1} - y_{1,t} = constant + c_n\left(f_{n,t} - y_{1,t}\right) + residual \tag{39}$$

This regression may be obtained from equation (38), as according to the latter forward rates are martingales if the term structure of risk premium is flat.[15] In fact, by the Law of iterated expectations,

[14] Also known as the non-pure version of the expectations theory.

$$f_{n,t} = E_t\left(E_{t+1}\left(y_{1,t+n}\right) + \Lambda_n\right) = E_t\left(f_{n-1,t+1}\right) + \left(\Lambda_n - \Lambda_{n-1}\right). \tag{40}$$

Subtracting the short-term interest rate to both sides of (40) and adding a residual term, we get (39).[16] A rejection of this hypothesis can be taken as evidence that term premiums vary over time. It can be confirmed that the theoretical values of c_n implied by the class of Gaussian models under analysis must be equal to one. In fact, by definition, $c_n = \frac{Cov\left(f_{n-1,t+1} - f_{0,t}, f_{n,t} - f_{0,t}\right)}{Var\left(f_{n,t} - f_{0,t}\right)}$, given that $y_{1,t} = f_{0,t}$, where:

$$Cov\left(f_{n-1,t+1} - f_{0,t}, f_{n,t} - f_{0,t}\right) = \left(B_1 + B_n - B_{n+1}\right)^T \Gamma_0\left(B_1 - \Phi^T\left(B_n - B_{n-1}\right)\right), \tag{41}$$

$$Var\left(f_{n,t} - f_{0,t}\right) = \left(B_1 + B_n - B_{n+1}\right)^T \Gamma_0\left(B_1 + B_n - B_{n+1}\right), \tag{42}$$

Γ_0 is the unconditional variance of z and is given by the solution to

$$\Gamma_0 = \Phi\Gamma_0\Phi^T + V, \tag{43}$$

where V is a diagonal matrix with elements equal to the variances. The solution is given by

$$vec\left(\Gamma_0\right) = \left(I - \Phi \otimes \Phi^T\right)^{-1}.vec(V). \tag{44}$$

Therefore, the slope of the forward regression will be:

$$c_n = \frac{\left(B_1 + B_n - B_{n+1}\right)^T \Gamma_0\left(B_1 - \Phi^T\left(B_n - B_{n-1}\right)\right)}{\left(B_1 + B_n - B_{n+1}\right)^T \Gamma_0\left(B_1 + B_n - B_{n+1}\right)}. \tag{45}$$

---

[15] This is a generalisation of the result in Campbell *et al.* (1997), chapter 11, derived assuming null risk premium.

[16] The term related to the slope of the term structure of term premium is included in the constant, as it is assumed to be constant.

For any affine model we have $\lim_{n \to \infty} c_n = \frac{B_1{}^T \Gamma_0 B_1}{B_1{}^T \Gamma_0 B_1} = 1$. In our class of Gaussian models, from (28), $c_n = 1, \forall_n$.

6. Identification

The identification of the factors with macroeconomic variables can, in principle, be achieved by estimating a joint model for the term structure and the macro-economic variable, with a common factor related to the latter.

Assuming that the short-term interest rate can be decomposed into the short-term real interest rate (expressed in deviation from the mean and denoted by $r_{1,t}$) and the expected one-period ahead inflation rate, we have:

$$y_{1,t+1} = r_{1,t+1} + E_t(\pi_{t+1}). \tag{46}$$

In Remolona *et al.* (1998), nominal and indexed-bonds are used in order to estimate the expected inflation rate. In our case, as there were no indexed bonds in Germany, the inflation process has to be estimated jointly with the processes for bond yields. For example, if inflation (in deviation from the mean, denoted by π_t) follows an AR(1) process:

$$\pi_{t+1} = \rho \pi_t + u_{t+1} \tag{47}$$

where u_{t+1} is white noise, the component of the short-term interest rate related to the second factor may be considered as the one-period inflation expectation:

$$z_{2,t} = E_t(\pi_{t+1}) = \rho \pi_t \tag{48}$$

From (48) and (32) the value of the second factor in t+1 is given by:

$$z_{2,t+1} = E_{t+1}(\pi_{t+2}) = \rho(\pi_{t+1}) = \rho(\rho\pi_t + u_{t+1}) = \rho z_{2,t} + \rho u_{t+1} \tag{49}$$

Comparing equations (32) and (49), we can exactly identify the parameters of the second factor with the parameters of the inflation process:

$$\begin{aligned} \rho &= \varphi_2 \\ \rho u_{t+1} &= \sigma_2 \varepsilon_{2,t+1} \end{aligned}, \tag{50}$$

and the following relationship between inflation and the second factor can be obtained:

$$\pi_t = \frac{1}{\varphi_2} z_{2,t} . \tag{51}$$

The main problem with the procedure sketched above is that (47) is not necessarily the optimal model for forecasting inflation. In fact, it is too simple concerning its lag structure and also does not allow for the inclusion of other macroeconomic information that market participants may use to form their expectations of inflation.[17] For example, information about developments in monetary aggregates, commodity prices, exchange rates, wages and unit labour costs, etc, may be used by market participants to forecast inflation and, thus, may be reflected in the bond pricing process. However, a more complex model would certainly not allow a simple identification of the factor.

Given the difficulty and shortcomings of such exercise we suggest, as an alternative, testing for the leading indicator properties of z_{2t} for inflation.

We set up a VAR model with p lags (see Hamilton (1994), chapter 11):

$$x_t = A_1 x_{t-1} + \ldots + A_p x_{t-p} + \mu + u_t \tag{52}$$

where x_t is (2 x 1) and each of the A_i is a (2 x 2) matrix of parameters with generic element denoted $\left[a_{kj}^i\right]$ and $u_t \sim IN(0, \Sigma)$.

[17] A more general ARMA model would perhaps be necessary to model the dynamics of inflation. Nevertheless, given that φ_2 is close to 1, equation (51) implies a loss of leading indicator properties of the second factor regarding inflation.

The vector x_t is defined as $x_t = \begin{bmatrix} \pi_t \\ z_{2t} \end{bmatrix}$.

To test whether z_{2t} has leading indicator properties for inflation we test the hypothesis $H_0 : a_{12}^1 = ... = a_{12}^p = 0$. This is a test for Granger causality, i.e., a test of whether past values of the factor along with past values of inflation better "explain" inflation than past values of inflation alone. This of course does not imply that bond yields cause inflation. Instead it means that z_{2t} is possibly reflecting bond market's expectations as to where inflation might be headed.

In assessing the leading indicator properties of z_{2t}, the Granger causality test can be supplemented with an impulse-response analysis. The vector $MA(\infty)$ representation of the VAR is given by:

$$x_t = \mu + u_t + \Psi_1 u_{t-1} + \Psi_2 u_{t-2} + \dots . \tag{53}$$

Thus the matrix Ψ_s has the interpretation:

$$\frac{\partial x_{t+s}}{\partial u_t'} = \Psi_s ;$$

that is, the row *i*, column *j* element of Ψ_s identifies the consequences of a one-unit increase in the *j*th variable's innovation at date *t* (u_{jt}) for the value of the *i*-th variable at time *t*+*s* ($x_{i,t+s}$), holding all other innovations at all dates constant.

A plot of row *i*, column *j* element of Ψ_s

$$\frac{\partial x_{i,t+s}}{\partial u_{jt}},$$

as a function of *s* is the impulse-response function. It describes the response of ($x_{i,t+s}$) to a one-time impulse in $x_{j,t}$, with all other variables dated *t* or earlier held constant.

Suppose that the date t value of the first variable in the autoregression z_{2t} is higher than expected, so that $u_{1,t}$ is positive. Then

$$\frac{\partial \hat{E}(x_{i,t+s} \mid z_{2t}, x'_{t-1}, x'_{t-2}, \ldots, x'_{t-p})}{\partial z_{2t}} = \frac{\partial x_{i,t+s}}{\partial u_{1t}},$$

when Σ is a diagonal matrix.[18]

Thus if z_{2t} is a leading indicator of inflation, a revision in market expectations of inflation $\partial \hat{E}(x_{i,t+s} \mid z_{2t}, x'_{t-1}, \ldots, x'_{t-p})$ should be captured by the marginal impact of a shock to the innovation process in the equation for z_{2t}.

7. Econometric methodology

Given that the factors determining the dynamics of the yield curve are non-observable, a Kalman filtering and maximum likelihood procedure was the method chosen for the estimation of the model. In order to estimate the parameters, the model must be written in the linear state-space form. According to equation (14) and exploiting the information on the homoskedasticity of yields, the measurement or observation equation for the two-factor Gaussian model may be written as:[19]

$$\begin{bmatrix} y_{1,t} \\ \ldots \\ y_{l,t} \\ Var(y_1) \\ \ldots \\ Var(y_l) \end{bmatrix} = \begin{bmatrix} a_1 \\ \ldots \\ a_l \\ a_{l+1} \\ \ldots \\ a_{2l} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{22} \\ \ldots & \ldots \\ b_{1l} & b_{2l} \\ 0 & 0 \\ \ldots & \ldots \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} + \begin{bmatrix} \upsilon_{1,t} \\ \ldots \\ \upsilon_{l,t} \\ \upsilon_{l+1,t} \\ \ldots \\ \upsilon_{2l,t} \end{bmatrix}, \tag{54}$$

---

[18] We use a Cholesky decomposition of the variance-covariance of the innovations to identify the shocks. We order inflation first in the VAR to reflect the idea that whereas inflation does not respond, contemporaneously, to shocks to expectations, these may be affected by contemporaneous information on inflation.

[19] In this way we adjust simultaneously the yield curve and the volatility curve, avoiding implausible estimates for the latter.

where $y_{1,t},\ldots y_{l,t}$ are the zero-coupon yields at time t with maturities j = 1, … , l periods and $v_{1,t},\ldots,v_{l,t},v_{l+1,t},v_{2l,t}$, are normally distributed i.i.d. errors, with zero mean and standard-deviation equal to e_j^2, of the measurement equation for each interest rate considered, $a_j = A_j/j$, $a_{l+j} = \frac{1}{n^2}\left(B_{1,j}^2\sigma_1^2 + B_{2,j}^2\sigma_2^2\right)$, $b_{1,j} = B_{1,j}/j$ and $b_{2,j} = B_{2,j}/j$.

Following equations (27), the transition or state equation for the same model is:

$$\begin{bmatrix} z_{1,t+1} \\ z_{2,t+1} \end{bmatrix} = \begin{bmatrix} \varphi_1 & 0 \\ 0 & \varphi_2 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \end{bmatrix}, \tag{55}$$

where $\varepsilon_{1,t+1}$ and $\varepsilon_{2,t+1}$ are orthogonal shocks with zero mean and variances equal to 1. In the appendix, the technical details on Kalman filtering and the maximum likelihood procedure are presented. Additional to the recursive restrictions, the parameters are estimated subject to the usual signal restrictions.[20] For further details see, for instance, Hamilton (1994, chapter 13) or Harvey (1990).

8. Data and estimation results

The data used consist of two databases. The first comprises monthly averages of nine daily spot rates for maturities of 1 and 3 months and 1, 2, 3, 4, 5, 7 and 10 years, between January 1986 and December 1998. These spot rates were estimated from euro-mark short-term interest rates, obtained from Datastream, and par yields of German government bonds, obtained from J.P. Morgan, using the Nelson and Siegel (1987) and Svensson (1994) smoothing techniques. One-

[20] The Kalman filtering and the maximum likelihood estimation were carried out using a Matlab code. We are grateful to Mike Wickens for his help on the code.

period forward rates were calculated for this sample, which allowed the forward regressions to be performed.[21]

The second data set covers a longer period, between September 1972 and December 1998.[22] However this sample includes only spot rates for annual maturities between 1 and 10 years, excluding the 9-year maturity.

The inflation rate was computed as the difference to the mean of the yearly inflation rate, obtained from Datastream.

Average yield curve properties

The properties of the German yield curve for both data sets are summarised in Table 1. A number of features are worth noting. Firstly, between 1986 and 1998, the term structure is negatively slopped at the short end, in contrast with the more familiar concave appearance observed for the USA market (see, Backus *et al.* (1997)). Secondly, yields are very persistent, with monthly autocorrelations above 0.98 for all maturities.[23] Thirdly, yields are highly correlated along the curve, but correlation is not equal one, suggesting that non-parallel shifts of the yield curve are important. Therefore, one-factor models seem to be insufficient to explain the German term structure of interest rates. As expected, the volatility curve of yields is downward sloping.

Tests of the expectations theory

Figures 1 (Simple test) and 2 (Forward regressions) show the results of the tests of the expectations theory of the term structure mentioned in the paper, respectively equations (38) and (39). The pure expectations theory is easily rejected: as shown in Figure 1 average one-period short-term forward rates vary with maturity, which contradicts equation (38) in the steady state. By contrast,

[21] We are grateful to Fátima Silva for research assistance on this issue.
[22] We are grateful to Manfred Kremer from the Research Department of the Bundesbank for providing the data.
[23] Interest rates are close to being non-stationary.

forward regressions shown in Figure 2 generate slope coefficients close to one for all maturities with relatively small standard errors,[24] suggesting that the assumption of constant term premiums is a reasonable approximation.

Kalman filtering results

The results of the estimation are shown in Figures 3a and 3b (average nominal yield curves), 4a and 4b (Volatility curves), 5a and 5b (Term premium curves), 6a and 6b (One-period forward curves), 7a and 7b (Expected short-term interest rate curves), 8a and 8b (Time-series yields), 9a and 9b (Factor loadings) and 10a, 10b, 10c, 10d, (Time-series factors). The parameters and respective standard errors are reported in Table 2.

The estimates reproduce very closely both the average yield and the volatility curves and generates plausible term premium curves. The estimates also reproduce very closely the time-series of the yields across the whole maturity spectrum.[25] It is interesting to note that the fit is poorer at the end of the sample, in 1998, when long bond yields fell (more than predicted by the model) as a consequence of the Russian and Asian crisis, whilst short-rates remained relatively stable (and above values predicted by the model).

Focusing on the estimates of the parameters, as the estimated φ's are close to one and exhibit low volatility the factors are very persistent.[26] Note that standard-deviations are very low and thus confidence intervals are extremely

[24] Newey-West standard errors.

[25] The quality of the fit for the yields contrasts with the results in Gong and Remolona (1997c). For the US at least two different two-factor heteroskedastic models are needed to model the whole US term structures of yields and volatilities: one to fit the medium/short end of the curves and another to fit the medium/long term. The lowest time-series correlation coefficients is 0.87, for the 10-year maturity, and the cross-section correlation coefficients are, in most days of the sample, above 0.9.

[26] Nevertheless, given that standard-deviations are low, the unit root hypothesis is rejected and, consequently, the factors are stationary. We acknowledge the comments of Jerome Henry on this issue.

narrow.[27] This confirms the high sensitivity of the shapes of the average and volatility term structures to parameter estimates.[28] It is the second factor that contributes positively to the term premium and exhibits higher persistence.

Figures 9a and 9b show the factor loadings. The first factor is relatively more important for the dynamics of the short end of the curve, and the second is relatively more important for the long end of the curve.

Figures 10a, 10b, 10c, and 10d show the time-series of the unobservable factors and proxies for the economic variables with which they are supposed to be correlated – the real interest rate and the inflation rate. A first feature worth mentioning is the correlation between the factors and the corresponding economic variables.

Using *ex-post* real interest rates as proxies for the *ex-ante* real rates, the correlation coefficients between one-month and three-month real rates and the first factor is around 0.75. In the larger sample the correlation coefficient is 0.6. Correlation between the second factor and inflation is close to 0.7 and 0.5, respectively in the shorter and in the larger sample.

Leading indicator properties of factor 2 for inflation

We start with simple descriptive statistics. Table 3 shows cross-correlation between π_t and z_{2t} in the larger sample. If z_{2t} is a leading indicator for π_t then the highest (positive) correlation should occur between lead values of π_t and z_{2t}. The shaded figures in the table are the correlation coefficients between z_{2t} and lags of π_t. The first figure in each row is k and the next corresponds to the

[27] Standard-deviations were computed from the variance-covariance matrix of the estimators (see Hamilton (1994), pp. 389, for details), using a numerical procedure.
[28] Very low standard-deviations for the parameters have been usually obtained in former Kalman filter estimates of term structure models, as Babbs and Nowman (1998), Gong and Remolona (1997a), Remolona *et al.* (1998) and Geyer and Pichler (1996).

correlation between π_{t-k} and z_{2t} (negative k means lead). The next to the right is the correlation between π_{t-k-1} and z_{2t}, etc.

The correlation analysis is indicative of leading indicator properties of z_{2t}. Firstly, note that the highest correlation (in bold) is at the fourth lead of inflation and, secondly, that correlation increases with leads of inflation up to lead 4 and steadily decline for lags of inflation.

Table 4 presents the Granger causality test and Figure 11 the impulse-response functions. The results strongly support the conjecture that z_{2t} has leading indicator properties for inflation. At the 5% level of confidence one can reject that z_{2t} does not Granger cause inflation. This is confirmed by the impulse-response analysis.

A positive shock to the innovation process of z_{2t} is followed by a statistically significant increase in inflation. However a positive shock to the innovation process of inflation does not seem to be followed by a statistically significant increase in z_{2t}. These results suggest that an innovation to the inflation process is not “news” for the process of expectation formation. This is very much in accordance with the (forward-looking) interpretation of shocks to z_{2t} as reflecting “news” about the future course of inflation.

Overall our results suggest that inflation expectations influence long-term rates. Similar results concerning the information content of the German term structure regarding future changes in inflation rate were obtained in previous papers, namely Schich (1996), Gerlach (1995) and Mishkin (1991), using different samples and testing procedures.[29]

[29] Mishkin (1991) and Jorion and Mishkin (1991) results on Germany are contradictory as, according to Mishkin (1991), the short-end of the term structure does not contain information on future inflation for all OECD countries studied, except for France, United Kingdom and

9. Conclusions

The identification of the factors that determine the time-series and cross-section behaviour of the term structure of interest rates is one of the most challenging research topics in finance. As German yield data seems to support the expectations theory with constant term premiums, we used a constant volatility model to fit the term structure of interest rates in Germany.

We have shown that a two-factor models describes quite well the dynamics and the shape of the German yield curve between 1972 and 1998. Reasonable estimates are obtained also for the term premium and the volatility curves. Two factors seem to drive the German term structure of interest rates: one factor related to the *ex-ante* real interest rate and a second factor linked to inflation expectations.

Germany. Conversely, Jorion and Mishkin (1991) conclude that the predictive power of the shorter rates about future inflation is low in the U.S., Germany and Switzerland.
Fama (1990) and Mishkin (1990a and 1990b) present identical conclusions concerning the information content of U.S. term structure regarding future inflation and state that the U.S. dollar short rates have information content regarding future real interest rates and the longer rates contain information on inflation expectations. Mishkin (1990b) also concludes that for several countries the information on inflation expectations is weaker than for the United States. Mehra (1997) presents evidence of cointegration between the nominal yield on 10-year Treasury Bond and the actual U.S. inflation rate. Koedijk and Kool (1995), Mishkin (1991) and Jorion and Mishkin (1991) supply some evidence on the information content of the term structure concerning the inflation rate in several countries.

Appendix - The Kalman Filter

The Kalman Filter is an algorithm that computes the optimal estimate for the state variables at a moment t using the information available up to t-1. When the parameters of the model are also unknown, as it is the case of our problem, they are usually estimated by a maximum likelihood procedure.

The starting point for the derivation of the Kalman filter is to write the model in state-space form, as in equations (54) and (55):

Observation or measurement equation:

(A1) $$\underset{(2l\times 1)}{y_t} = \underset{(2l\times r)}{A} \cdot \underset{(r\times 1)}{X_t} + \underset{(2l\times k)}{H} \cdot \underset{(k\times 1)}{S_t} + \underset{(l\times 1)}{w_t}$$

State or transition equation:

(A2) $$\underset{(k\times 1)}{S_t} = \underset{(k\times 1)}{C} + \underset{(k\times k)}{F} \cdot \underset{(k\times 1)}{S_{t-1}} + \underset{(k\times k)}{G} \underset{(k\times 1)}{v_t}$$

where $2l$ is the number of variables to estimate (being l the number of terms considered in the estimation), r is the number of observable exogenous variables and k is the number of non-observable exogenous variables (the factors). Besides the parameters that form the elements of A, H, C and F, it is also required to estimate the elements of the variance-covariance matrix of the residuals of equations (A1) and (A2):

(A3) $$\underset{(l\times l)}{R} = E(w_t w_t')$$

(A4) $$\underset{(k\times k)}{Q} = E(v_{t+1} v_{t+1}')$$

In our two-factor model A is a column vector with elements $a_{j,t}$ for the first l rows ($l = 9$), and $\frac{1}{n^2}\left(B_{1,j}^2\sigma_1^2 + B_{2,j}^2\sigma_2^2\right)$ for the next l rows; X_t is a $2l$-dimension column vector of one's ($r = 1$), C is a column vector of zeros and F is a $k \times k$ diagonal matrix, with typical element $F_{ii} = \varphi_i$ ($k = 2$). The values of the elements of these matrices may be time-varying or constant, being in this case known or unknown. In our model, they are constant and unknown.

The estimation departs from assuming that the starting value of the state vector S is obtained from a normal distribution with mean $\widehat{S}_o$ and variance P_0. $\widehat{S}_o$ can be seen as a guess concerning the value of S using all information available up to and including t = 0. As the residuals are orthogonal to the state variables, $\widehat{S}_o$ cannot be obtained using the data and the model. P_0 is the uncertainty about the prior on the values of the state variables.

Using $\widehat{S}_o$ and P_0 and following (A2), the optimal estimator for S_1 will be given by:

(A5) $\quad \hat{S}_{1|0} = C + F\hat{S}_0$

Consequently, the variance-covariance matrix of the estimation error of the state vector will correspond to:

$$\text{(A6)} \qquad \begin{aligned} P_{1|0} &= E\left[\left(S_1 - \hat{S}_{1|0}\right)\left(S_1 - \hat{S}_{1|0}\right)'\right] \\ &= E\left[\left(C + FS_0 + Gv_1 - C - FS_0\right)\left(C + FS_0 + Gv_1 - C - FS_0\right)'\right] \\ &= E\left[\left(Fv_0 + Gv_1\right)\left(v_0'F' + v_1'G'\right)\right] \\ &= E\left(Fv_0v_0'F'\right) + E\left(Gv_1v_1'G'\right) \\ &= FP_{1|0}F' + GQ_1G' \end{aligned}$$

Given that $vec(ABC) = \left(C' \otimes A\right) \cdot vec(B)$, $P_{1|0}$ may be obtained from:

$$
\text{(A7)} \quad \begin{aligned} vec\left(P_{1|0}\right) &= vec\left(FP_{1|0}F'\right) + vec\left(GQ_1G'\right) \\ &= (F \otimes F) \cdot vec\left(P_{1|0}\right) + (G \otimes G) \cdot vec\left(Q_1\right) \\ &= \left[\underset{\left(r^2 \times r^2\right)}{I} - (F \otimes F) \right]^{-1} \left[(G \otimes G) \cdot vec\left(Q_1\right) \right] \end{aligned}
$$

Generalising equations (A5) and (A6), we have the following prediction equations:

(A8) $\hat{S}_{t|t-1} = C + F\hat{S}_{t-1}$

(A9) $P_{t|t-1} = FP_{t|t-1}F' + GQ_tG'$

As w_t is independent from X_t and from all prior information on y and x (denoted by ζ_{t-1}), we can obtain the forecast of y_t conditional on X_t and ζ_{t-1} directly from (A1):

(A10) $E\left(y_t | X_t, \zeta_{t-1}\right) = AX_t + H\hat{S}_{t|t-1}$

Therefore, from (A1) and (A10), we have the following expression for the forecast error:

(A11) $y_t - E\left(y_t | X_t, \zeta_{t-1}\right) = \left(AX_t + HS_t + w_t\right) - \left(AX_t + H\hat{S}_{t|t-1}\right) = H\left(S_t - \hat{S}_{t|t-1}\right) + w_t$

Given that w_t is also independent from S_t and $\hat{S}_{t|t-1}$ and considering (A11), the conditional variance-covariance matrix of the estimation error of the observation vector will be:

(A12)

$$
\begin{aligned} E\left\{\left[y_t - E\left(y_t | X_t, \zeta_{t-1}\right)\right]\left[y_t - E\left(y_t | X_t, \zeta_{t-1}\right)\right]'\right\} &= E\left\{\left[H\left(S_t - \hat{S}_{t|t-1}\right) + w_t\right]\left[H\left(S_t - \hat{S}_{t|t-1}\right) + w_t\right]'\right\} = \\ &= HE\left[\left(S_t - \hat{S}_{t|t-1}\right)\left(S_t - \hat{S}_{t|t-1}\right)'\right]H' + E\left(w_t w_t'\right) \\ &= HP_{t|t-1}H' + R \end{aligned}
$$

In order to characterise the distribution of the observation and state vectors, it is also required to compute the conditional covariance between both forecasting errors. From (A11) we get:

$$\text{(A13)}\quad \begin{aligned} E\left\{\left[y_t - E(y_t | X_t, \zeta_{t-1})\right]\left[S_t - E(S_t | X_t, \zeta_{t-1})\right]'\right\} &= E\left\{\left[H\left(S_t - \hat{S}_{t|t-1}\right) + w_t\right]\left[S_t - \hat{S}_{t|t-1}\right]'\right\} \\ &= HE\left[\left(S_t - \hat{S}_{t|t-1}\right)\left(S_t - \hat{S}_{t|t-1}\right)'\right] \\ &= HP_{t|t-1} \end{aligned}$$

Therefore, using (A10), (A12) and (A13), the conditional distribution of the vector (y_t, S_t) is:

$$\text{(A14)}\quad \begin{bmatrix} y_t | X_t, \zeta_{t-1} \\ S_t | X_t, \zeta_{t-1} \end{bmatrix} \sim N\left(\begin{bmatrix} AX_t + H_{t|t-1} \\ \hat{S}_{t|t-1} \end{bmatrix}, \begin{bmatrix} HP_{t|t-1}H' + R & HP_{t|t-1} \\ P_{t|t-1}H' & P_{t|t-1} \end{bmatrix}\right)$$

According to a well-known result (see, Mood *et al.* (1974, pp. 167-168)), if X_1 and X_2 have a joint normal conditional distribution characterised by

$$\text{(A15)}\quad \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}\right)$$

then the distribution of $X_2 | X_1$ is $N(m, \Sigma)$, with

$$\text{(A16)}\quad m = \mu_2 + \Omega_{21}\Omega_{11}^{-1}(X_1 - \mu_1)$$

$$\text{(A17)}\quad \Sigma = \Omega_{22} - \Omega_{21}\Omega_{11}^{-1}\Omega_{12}$$

Equations (A16) and (A17) correspond to the optimal forecast of X_2 given X_1 and to the mean square error of this forecast respectively. Consequently, following (A14), the distribution of S_t given y_t, X_t and ζ_{t-1} is $N\left(\hat{S}_{t|t}, P_{t|t}\right)$, where $\hat{S}_{t|t}$ and $P_{t|t}$ are respectively the optimal forecast of S_t given $P_{t|t}$ and the mean square error of this forecast, corresponding (using (A16) and (A17)) to the following updating equations of the Kalman Filter:

$$\text{(A18)}\quad \hat{S}_{t|t} = \hat{S}_{t|t-1} + P_{t|t-1}H'\left(HP_{t|t-1}H'+R\right)^{-1}\left[y_t - \left(AX_t + H_{t|t-1}\right)\right]$$

$$\text{(A19)}\quad P_{t|t} = P_{t|t-1} - P_{t|t-1}H'\left(HP_{t|t-1}H'+R\right)^{-1}HP_{t|t-1}$$

After estimating $\hat{S}_{t|t}$ and $P_{t|t}$, we can proceed with the estimation of $\hat{S}_{t+1|t}$ and $P_{t+1|t}$. Considering (A8) and (A9), we obtain:

$$\text{(A20)}\quad \begin{aligned} \hat{S}_{t+1|t} &= C + F\widehat{S}_{t|t} = C + F\left\{\hat{S}_{t|t-1} + P_{t|t-1}H'\left(HP_{t|t-1}H'+R\right)^{-1}\left[y_t - \left(AX_t + H_{t|t-1}\right)\right]\right\} \\ &= C + F\hat{S}_{t|t-1} + FP_{t|t-1}H'\left(HP_{t|t-1}H'+R\right)^{-1}\left[y_t - \left(AX_t + H_{t|t-1}\right)\right] \end{aligned}$$

$$\text{(A21)}\quad \begin{aligned} P_{t+1|t} &= FP_{t|t}F'+GQ_tG' \\ &= F\left[P_{t|t-1} - P_{t|t-1}H'\left(HP_{t|t-1}H'+R\right)^{-1}HP_{t|t-1}\right]F'+GQ_tG' \\ &= FP_{t|t-1}F'-FP_{t|t-1}H'\left(HP_{t|t-1}H'+R\right)^{-1}HP_{t|t-1}F'+GQ_tG' \end{aligned}$$

The matrix $FP_{t|t-1}H'\left(HP_{t|t-1}H'+R\right)^{-1}$ is usually known as the gain matrix, since it determines the update in $\widehat{S}_{t+1|t}$ due to the estimation error of y_t. The equation (A21) is known as a Ricatti equation. Concluding, the Kalman Filter may be applied after specifying starting values for $\widehat{S}_{1|0}$ and $P_{1|0}$ using equations (A10), (A12), (A18), (A19) and iterating on equations (A20) and (A21).

The parameters are estimated using a maximum likelihood procedure. The log-likelihood function corresponds to:

$$\text{(A22)}\quad \log L\left(Y_T\right) = \sum_{t=1}^{T} \log f\left(y_t | I_{t-1}\right)$$

being

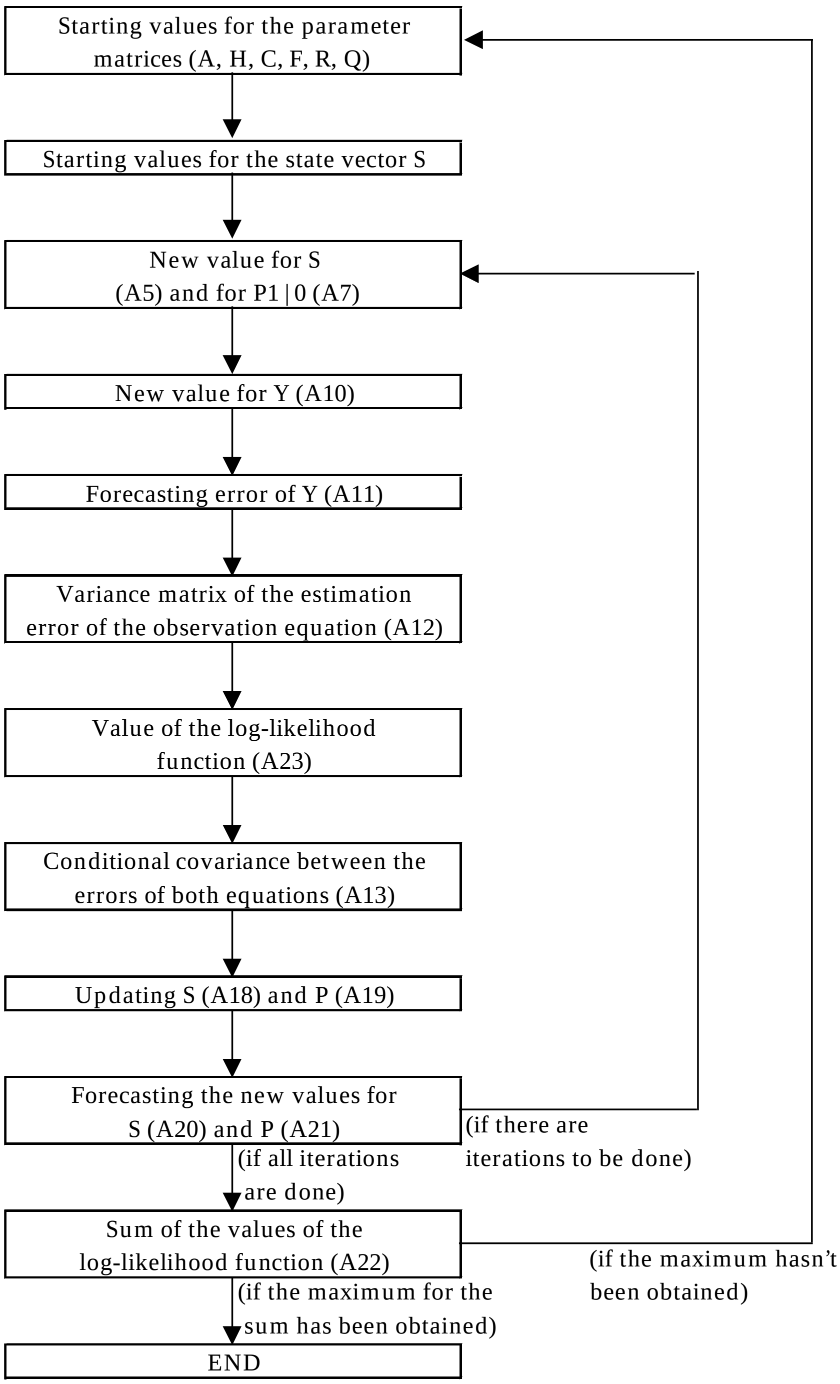
(A23)

$$f(y_t|I_{t-1}) = (2\pi)^{-1/2}\left|HP_{t|t-1}H' + R\right|^{-1/2} \cdot \exp\left[-\frac{1}{2}\left(y_t - A - H\hat{S}_{t|t-1}\right)'\left(HP_{t|t-1}H' + R\right)^{-1}\left(y_t - A - H\hat{S}_{t|t-1}\right)\right]$$

for $t = 1, \ldots, T$.

The estimation procedure may be resumed as follows:

Kalman filter algorithm with unknown parameters



References

Aït-Sahalia, Yacine (1996), "Testing Continuous-Time Models of the Spot Interest Rate", *Review of Financial Studies*, No. 9, pp. 427-70.

Babbs, Simon H. and K. Ben Nowman (1998), "An Application of Generalized Vasicek Term Structure Models to the UK Gilt-edged Market: a Kalman Filtering analysis", *Applied Financial Economics*, No. 8, pp. 637-644.

Backus, David, Silverio Foresi and Chris Telmer (1998), "Discrete-Time Models of Bond Pricing", NBER Working paper No. 6736.

Backus, David, Silverio Foresi, Abon Mozumdar and Liuren Wu (1997), "Predictable Changes in Yields and Forward Rates", *mimeo*.

Balduzzi, Pierluigi, Sanjiv Ranjan Das, Silverio Foresi and R. Sundaram (1996), "A Simple Approach to Three Factor Affine Term Structure Models", *Journal of Fixed Income*, No.6, pp. 43-53, December.

Bliss, Robert (1997), "Movements in the Term Structure of Interest Rates", Federal Reserve Bank of Atlanta, *Economic Review*, Fourth Quarter.

Buhler, Wolfgang, Marliese Uhrig-Homburg, Ulrich Walter and Thomas Weber (1999), "An Empirical Comparison of Forward-Rate and Spot-Rate Models for Valuing Interest-Rate Options", *The Journal of Finance*, Vol. LIV, No.1, February.

Campbell, John Y., Andrew W. Lo and A. Craig MacKinlay (1997), *The Econometrics of Financial Markets*, Princeton University Press.

Campbell, John Y. (1995), "Some Lessons from the Yield Curve", *Journal of Economic Perspectives*, Vol. 9, No. 3, pp. 129-152.

Chen, R. and L. Scott (1993a), "Maximum Likelihood Estimations for a Multi-Factor Equilibrium Model of the Term Structure of Interest Rates", *Journal of Fixed Income*, No. 3, pp. 14-31.

Chen, R. and L. Scott (1993b), "Multi-Factor Cox-Ingersoll-Ross Models of the Term Structure: Estimates and Test from a Kalman Filter", Working Paper, University of Georgia.

Cox, John, Jonathan Ingersoll and Stephen Ross (1985), "A Theory of the Term Structure of Interest Rates", *Econometrica*, No. 53, pp. 385-407.

Dai, Qiang and Kenneth J. Singleton (1998), "Specification Analysis of Affine Term Structure Models", *mimeo*.

Deacon, Mark and Andrew Derry (1994), "Deriving Estimates of Inflation Expectations from the Prices of UK Government Bonds", Bank of England *Working Paper* 23.

De Jong, Frank (1997), "Time-Series and Cross-section Information in Affine Term Structure Models", Center for Economic Research Working Paper.

Duffie, Darrell (1992), *Dynamic Asset Pricing Theory*, Princeton University Press.

Duffie, Darrell and Rui Kan (1996), "A Yield Factor Model of Interest Rates", *Mathematical Finance*, No. 6, pp. 379-406.

Fama, Eugene F. (1990), "Term Structure Forecasts of Interest Rates, Inflation and Real Returns", *Journal of Monetary Economics*, No. 25, pp. 59-76.

Fleming, Michael J. and Eli M. Remolona (1998), "The Term Structure of Announcement Effects", *mimeo*.

Fung, Ben Siu Cheong, Scott Mitnick and Eli Remolona (1999), "Uncovering Inflation Expectations and Risk Premiums from Internationally Integrated Financial Markets", W.P. 99-6, Bank of Canada.

Gerlach, Stefan (1995), "The Information Content of the Term Structure: Evidence for Germany", BIS Working Paper No. 29, September.

Geyer, Alois L. J. and Stefan Pichler (1996), "A State-Space Approach to Estimate and Test Multifactor Cox-Ingersoll-Ross Models of the Term Structure", *mimeo*.

Gong, Frank F. and Eli M. Remolona (1997a), "A Three-factor Econometric Model of the US Term Structure" FRBNY *Staff Reports*, No. 19, Jan.1997.

Gong, Frank F. and Eli M. Remolona (1997b), "Inflation Risk in the U.S. Yield Curve: The Usefulness of Indexed Bonds", Federal Reserve Bank of New York, June, *mimeo*.

Gong, Frank F. and Eli M. Remolona (1997c), "Two factors along the yield curve" *The Manchester School Supplement*, pp. 1-31.

Hamilton, James D. (1994), *Time Series Analysis*, Princeton (NJ), Princeton University Press.

Harvey, Andrew C. (1990), *Forecasting, Structural Time-Series Models and the Kalman Filter*, Cambridge University Press.

Jorion, P. and Frederic Mishkin (1991), "A multicountry comparison of term-structure forecasts at long horizons", *Journal of Financial Economics*, No. 29, pp. 59-80.

Koedijk, K. G. and C. J. M. Kool (1995), "Future inflation and the information in international term structures", *Empirical Economics*, No. 20, pp. 217-242.

Litterman, Robert and José Scheinkman (1991), "Common Factors Affecting Bond Returns", *Journal of Fixed Income* 1, June, pp. 49-53.

Mehra, Yash P. (1997), "The Bond Rate and Actual Future Inflation", Federal Reserve Bank of Richmond, Working Paper Series, W.P. 97-3, March.

Mishkin, Frederic (1990a), "What does the term structure tell us about future inflation", *Journal of Monetary Economics*, No. 25, pp. 77-95.

Mishkin, Frederic (1990b), "The information in the longer maturity term structure about future inflation", *Quarterly Journal of Economics*, No. 55, pp. 815-828.

Mishkin, Frederic (1991), "A multi-country study of the information in the shorter maturity term structure about future inflation", *Journal of International Money and Finance*, No. 10, pp. 2-22.

Mood, A. *et al.* (1974) (3rd ed.) *Introduction to the Theory of Statistics*, International Student Edition, McGraw-Hill.

Nelson, Charles R. and Andrew F. Siegel (1987) - "Parsimonious Modelling of Yield Curves", *Journal of Business*, 1987, Vol. 60, No. 4.

Remolona, Eli, Michael R. Wickens and Frank F. Gong (1998), "What was the market's view of U.K. Monetary Policy? Estimating Inflation Risk and Expected Inflation with Indexed Bonds", FRBNY *Staff Reports*, No. 57, December 1998.

Ross, Stephen A. (1976), "The arbitrage theory of capital asset pricing", *Journal of Economic Theory*, No. 13, pp. 341-360.

Schich, Sebastian T. (1996), "Alternative Specifications of the German Term Structure and its Information Content Regarding Inflation", Deutsche Bundesbank, Discussion Paper 8/96.

Svensson, Lars E.O. (1994) - "Estimating and Interpreting Forward Interest Rates: Sweden 1992-4", CEPR *Discussion Paper Series* No. 1051.

Vasicek, Oldrich (1977), "An equilibrium characterisation of the term structure", *Journal of Financial Economics*, No. 5, pp. 177-188.

Wong, F. (1964), "The construction of a class of stationary Markov processes", in Bellman, R. (Ed.), *Stochastic Processes in Mathematical Physics and Engineering*, Proceedings of Symposia in Applied Mathematics, Vol. 16, American Mathematical Society, Providence, R.I., pp. 264-276.

Zin, Stanley (1997), "Discussion of Evans and Marshall", *Carnegie-Rochester Conference on Public Policy*, November.

Table 1A

Properties of German Government Bond Yields: 1986–1998

Maturity	1	3	12	24	36	48	60	84	120
Mean	5.732	5.715	5.681	5.823	6.020	6.203	6.359	6.587	6.793
St.Dev.	2.267	2.224	2.087	1.882	1.686	1.524	1.398	1.228	1.095
Skew.	0.450	0.449	0.490	0.522	0.519	0.487	0.432	0.282	0.049
Kurt.	-1.258	-1.267	-1.139	-1.051	-0.975	-0.871	-0.737	-0.426	0.001
Autocorrel.	0.991	0.992	0.991	0.990	0.989	0.989	0.988	0.987	0.983

Normality Tests

Maturity	1	3	12	24	36	48	60	84	120
$\chi2(2)$	15.56	15.67	14.68	14.27	13.19	11.11	8.39	3.25	0.06
P(X>x)	0.000	0.000	0.001	0.001	0.001	0.004	0.015	0.197	0.969

Correlation matrix

y1	1.000	0.998	0.969	0.936	0.913	0.891	0.867	0.816	0.743
y3		1.000	0.982	0.955	0.934	0.912	0.889	0.839	0.765
y12			1.000	0.993	0.980	0.964	0.944	0.898	0.829
y24				1.000	0.996	0.986	0.972	0.935	0.875
y36					1.000	0.997	0.988	0.961	0.911
y48						1.000	0.997	0.980	0.940
y60							1.000	0.992	0.962
y84								1.000	0.989
y120									1.000

Table 1 B

Properties of German Government Bond Yields: 1972–1998

Maturity	12	24	36	48	60	72	84	96	120
Mean	6.348	6.609	6.856	7.051	7.202	7.321	7.414	7.487	7.594
St.Dev.	2.290	2.053	1.891	1.770	1.674	1.598	1.535	1.483	1.401
Skew.	0.545	0.349	0.224	0.151	0.108	0.084	0.075	0.077	0.112
Kurt.	-0.513	-0.671	-0.677	-0.636	-0.575	-0.495	-0.397	-0.281	-0.006
Autocorrel.	0.981	0.985	0.985	0.985	0.985	0.985	0.985	0.984	0.981

Normality Tests

Maturity	12	24	36	48	60	72	84	96	120
$\chi2(2)$	19.28	12.44	8.76	6.59	5.01	3.64	2.39	1.37	0.67
P(X>x)	0.000	0.002	0.013	0.037	0.082	0.162	0.302	0.505	0.715

Correlation matrix

y12	1.000	0.986	0.964	0.942	0.919	0.896	0.874	0.853	0.812
y24		1.000	0.993	0.979	0.962	0.943	0.924	0.906	0.870
y36			1.000	0.996	0.986	0.973	0.958	0.943	0.912
y48				1.000	0.997	0.990	0.980	0.968	0.943
y60					1.000	0.998	0.992	0.984	0.964
y72						1.000	0.998	0.994	0.979
y84							1.000	0.999	0.989
y96								1.000	0.995
y120									1.000

Table 2 A

Parameter estimates

	δ	$\sigma 1$	$\varphi 1$	$\lambda 1 \sigma 1$	$\sigma 2$	$\varphi 2$	$\lambda 2 \sigma 2$
1986-1998	0.00489	0.00078	0.95076	0.22192	0.00173	0.98610	-0.09978
1972-1998	0.00508	0.00127	0.96283	0.10340	0.00165	0.99145	-0.10560

Table 2 B

Standard deviation estimates

	δ	$\sigma 1$	$\varphi 1$	$\lambda 1 \sigma 1$	$\sigma 2$	$\varphi 2$	$\lambda 2 \sigma 2$
1986-1998	0.00009	0.00001	0.00142	0.00648	0.00003	0.00100	0.00084
1972-1998	0.00001	0.00001	0.00000	0.00000	0.00249	0.00084	0.00013

Table 3

Cross correlations of series π_t and z_{2t}

Monthly Data From 1972:09 To 1998:12

Correlation between π_{t-k} and z_{2t}(i.e. negative k means lead)

-25:	0.3974759	0.4274928	0.4570631	0.4861307	0.5129488	0.5394065
-19:	0.5639012	0.5882278	0.6106181	0.6310282	0.6501370	0.6670820
-13:	0.6829368	0.6979626	0.7119833	0.7235875	0.7324913	0.7385423
-7:	0.7432024	0.7453345	0.7457082	**0.7458129**	0.7443789	0.7411556
-1:	0.7368274	0.7311008	0.7144727	0.6954511	0.6761649	0.6561413
5:	0.6360691	0.6159540	0.5949472	0.5726003	0.5478967	0.5229193
11:	0.4960254	0.4706518	0.4427914	0.4156929	0.3870690	0.3590592
17:	0.3311699	0.3011714	0.2720773	0.2425888	0.2126804	0.1829918
23:	0.1527293	0.1226024	0.0927202	0.0636053	0.0362558	0.0085952
29:	-0.0167756	-0.0421135	-0.0673895	-0.0911283	-0.1139021	-0.1356261

Table 4

Granger causality tests

Table 4: Granger causality tests

Inflation does not Granger cause Z_2		
Variable	F-Statistic	Significance
π_t	5.8	0.03 *
Z_2 does not Granger cause inflation		
Variable	F-Statistic	Significance
Z_2	6.12	0.03 *

* Means rejection at 5% level

Figure 1

Simple test of the pure expectations theory of interest rates
Forward rate average curve 1986–1998

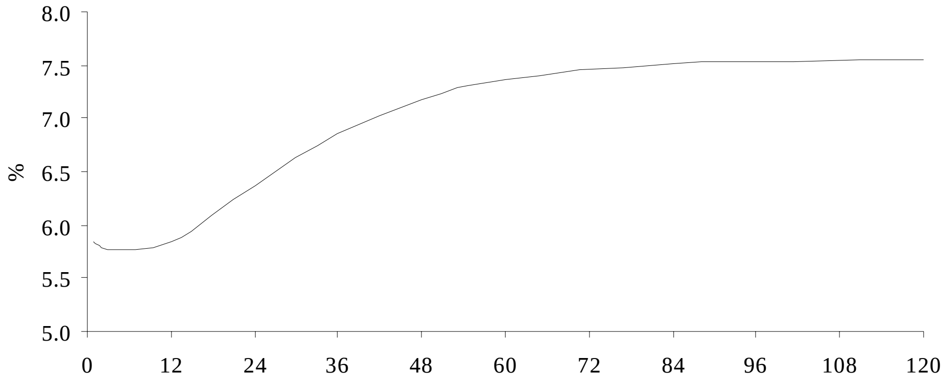
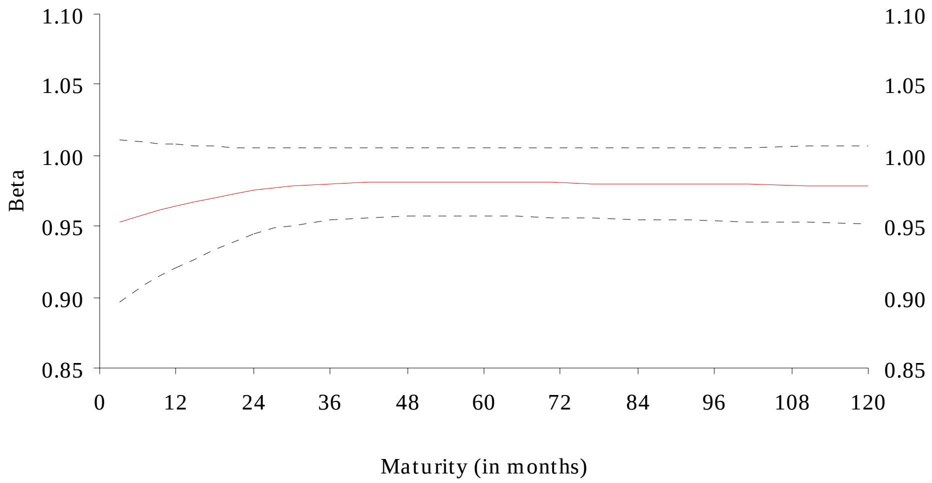


Figure 2

Test of the expectations theory of interest rates
Forward regression coefficient 1986–1998



m	3	12	24	36	48	60	84	120
β	0.953	0.965	0.975	0.980	0.981	0.981	0.980	0.979
σ(β)	0.029	0.022	0.015	0.013	0.012	0.012	0.013	0.014
α	-0.003	0.005	0.020	0.029	0.034	0.038	0.043	0.046
σ(α)	0.007	0.019	0.025	0.030	0.033	0.036	0.040	0.043

Figure 3A

Average Nominal Yield Curve 1986–1998

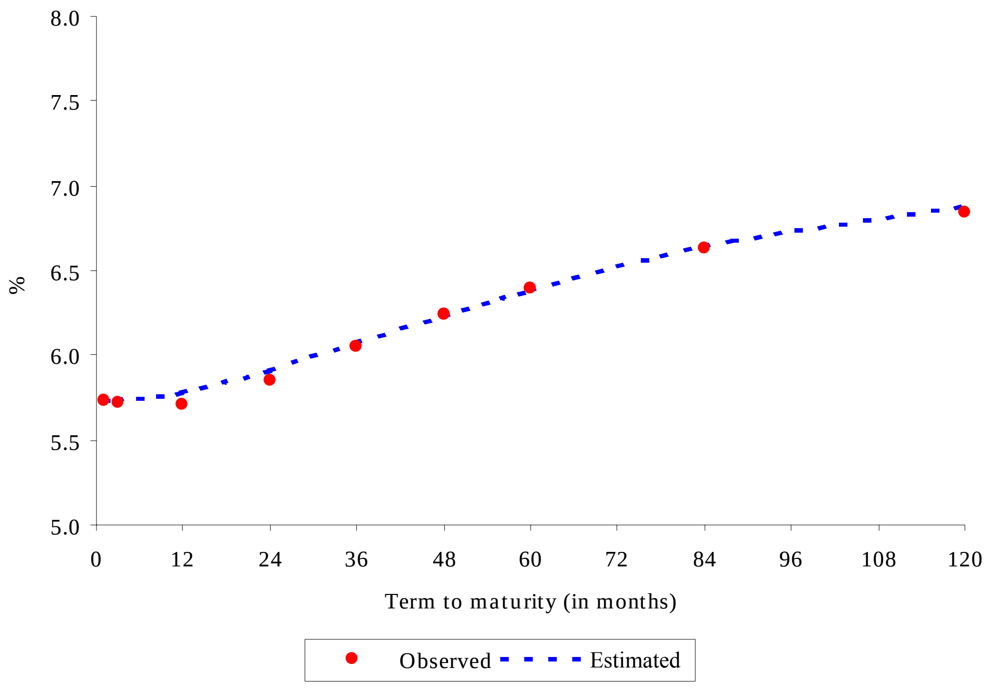


Figure 3B

Average Yield Curve 1972–1998

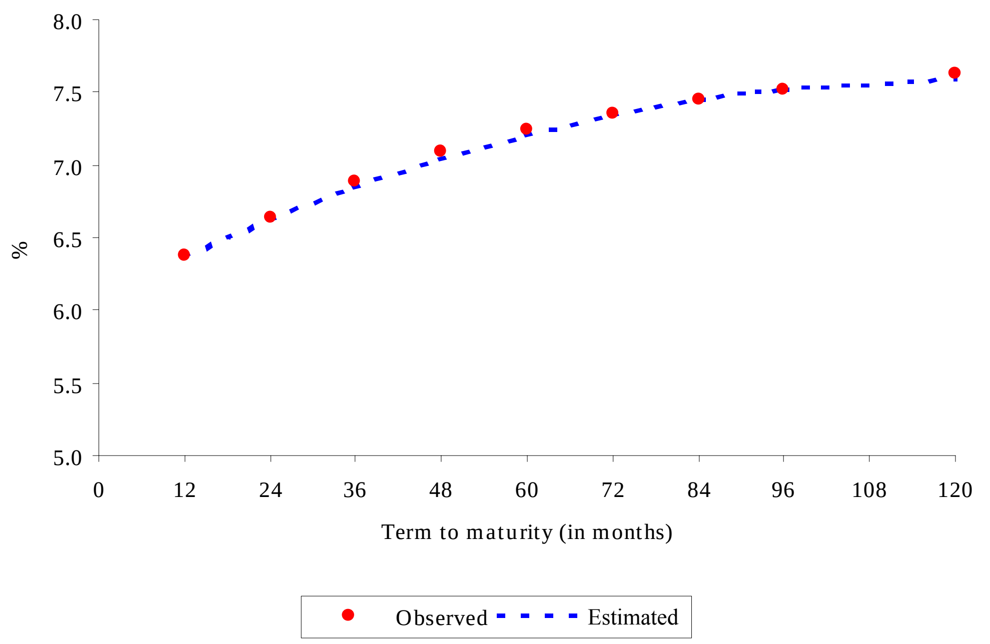


Figure 4A

Volatility Curve 1986–1998

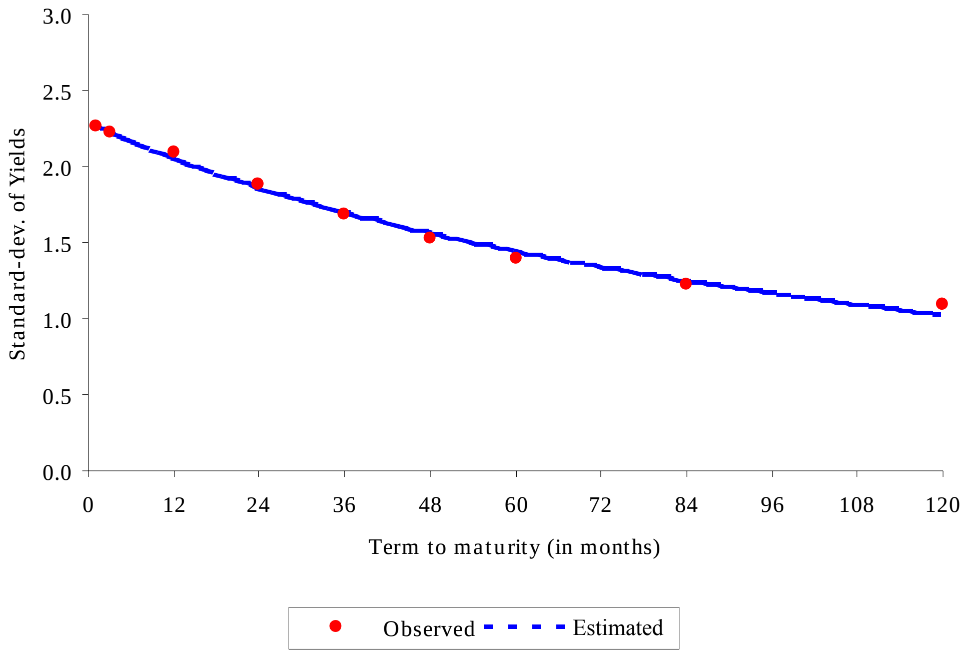


Figure 4B

Volatility Curve 1972–1998

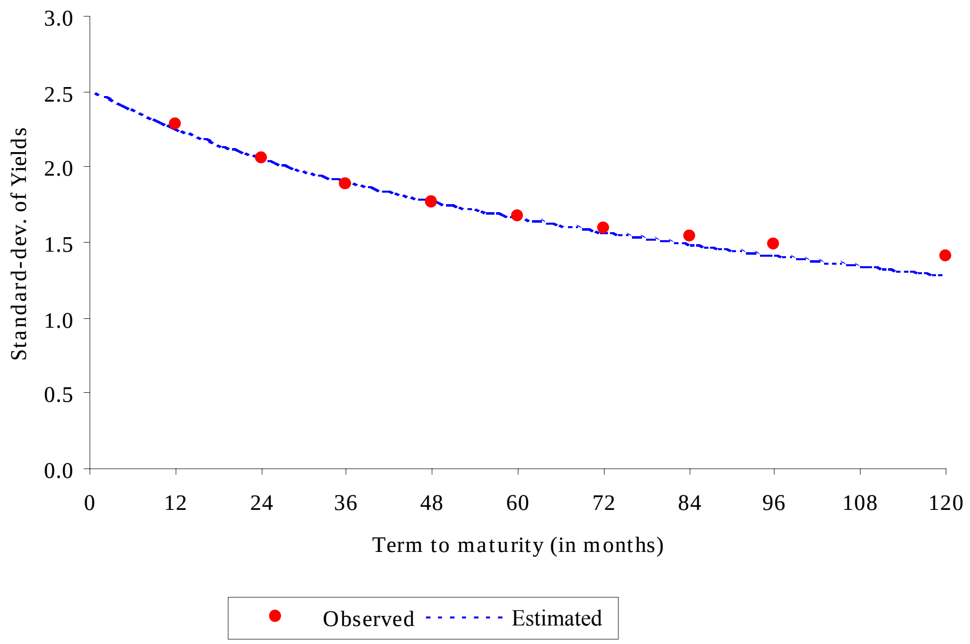


Figure 5A

The Premium Curve 1986–1998

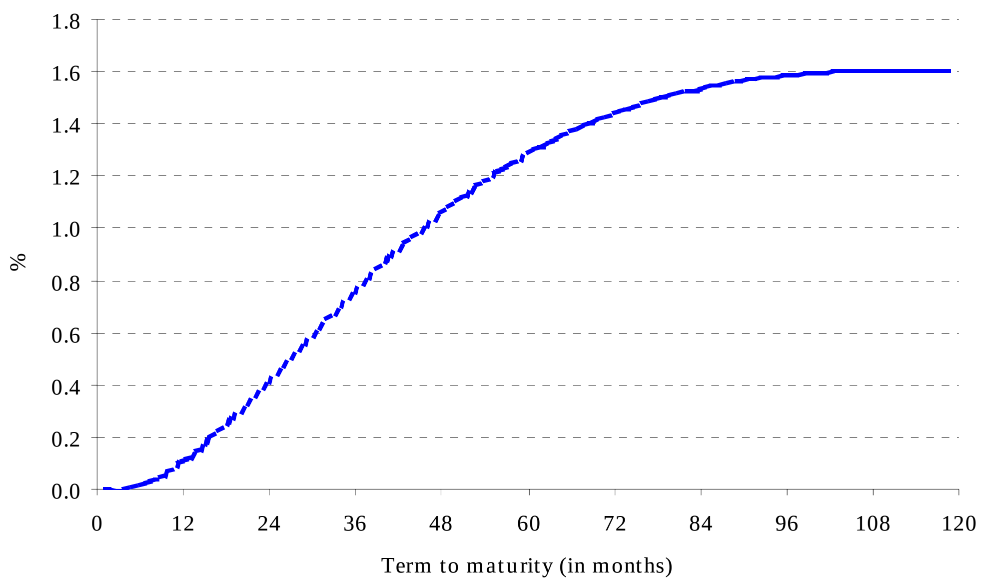


Figure 5B

Term Premium Curve 1972–1998

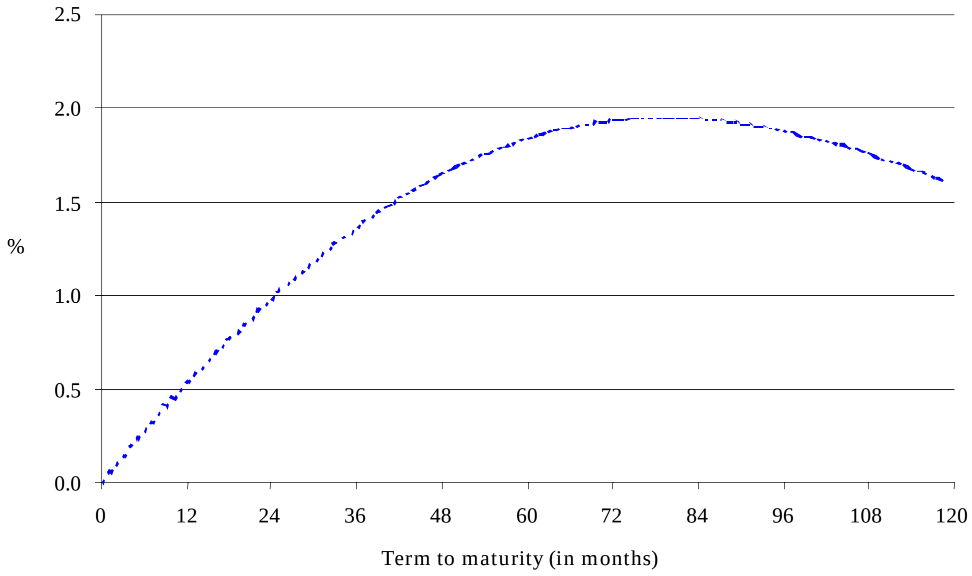


Figure 6A

Average Forward Curve 1986–1998

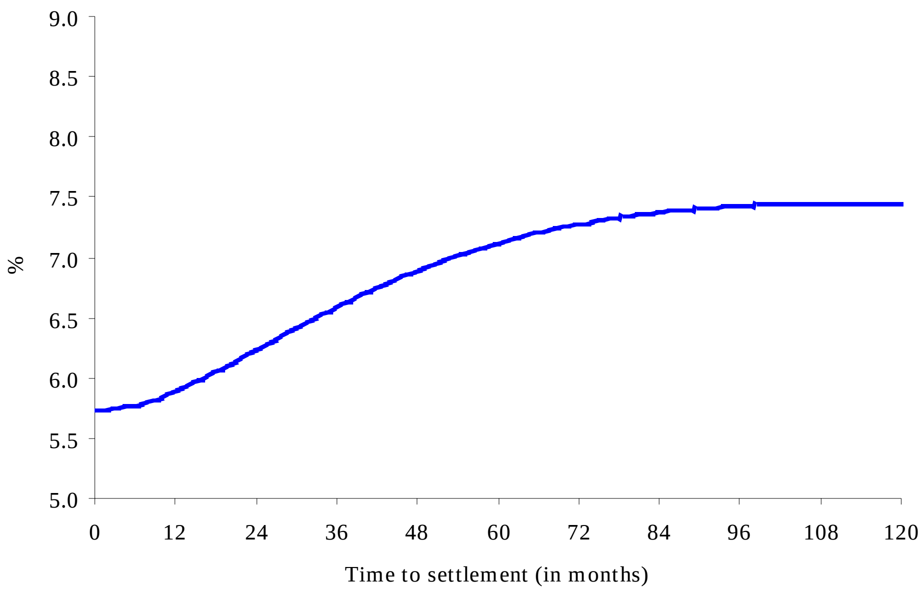


Figure 6B

Average Forward Curve 1972–1998

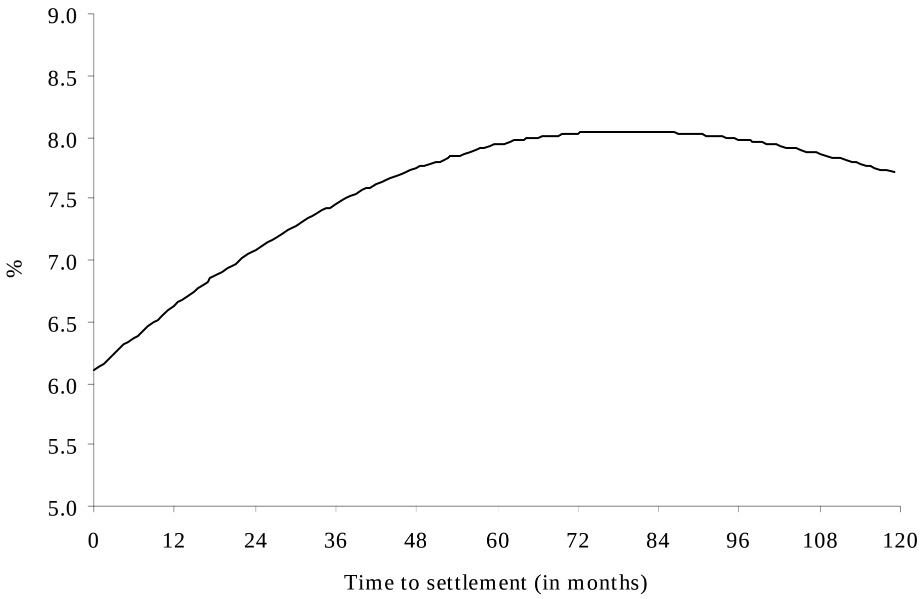


Figure 7A
Average Expected Short-term Interest Rate 1986–1998

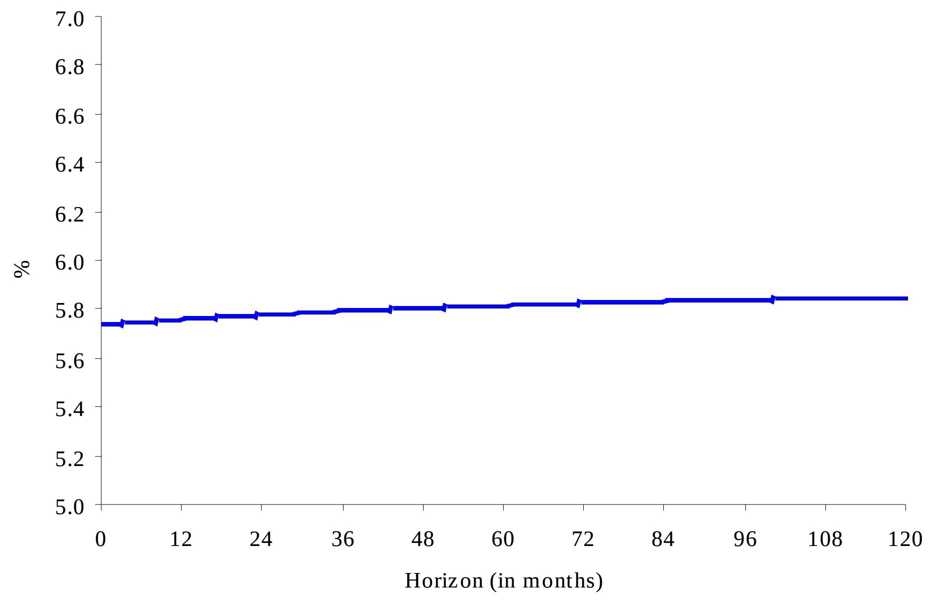


Figure 7B
Average Expected Short-term Interest Rate 1972–1998

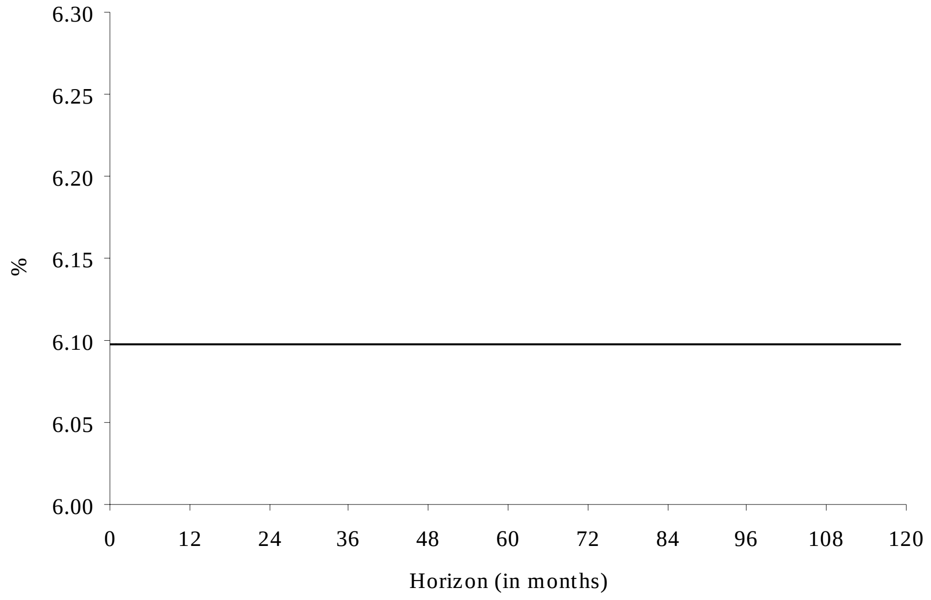


Figure 8A

Time-series yield estimation results 1986–1998

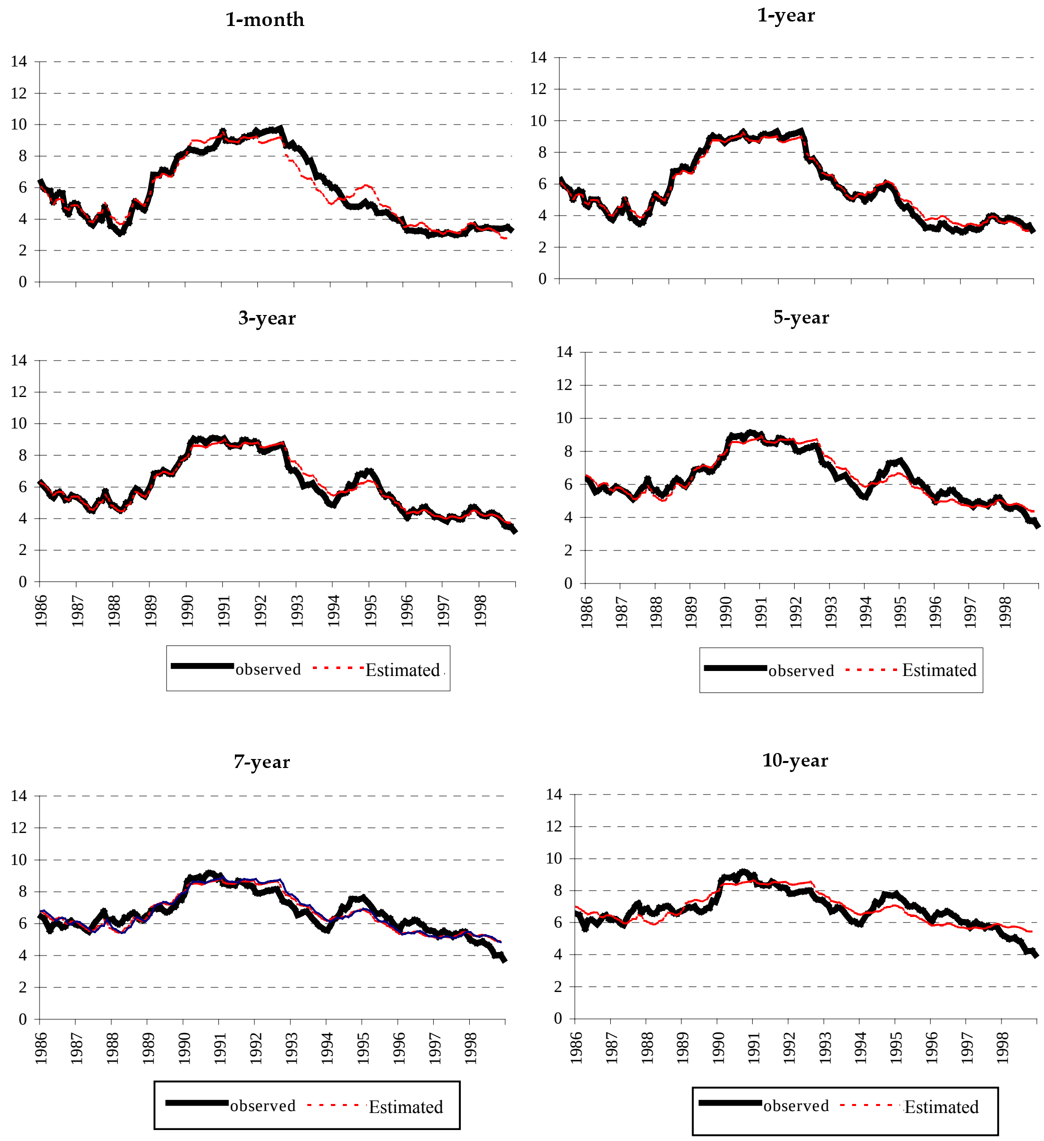


Figure 8B

Time-series yield estimation results 1972–1998

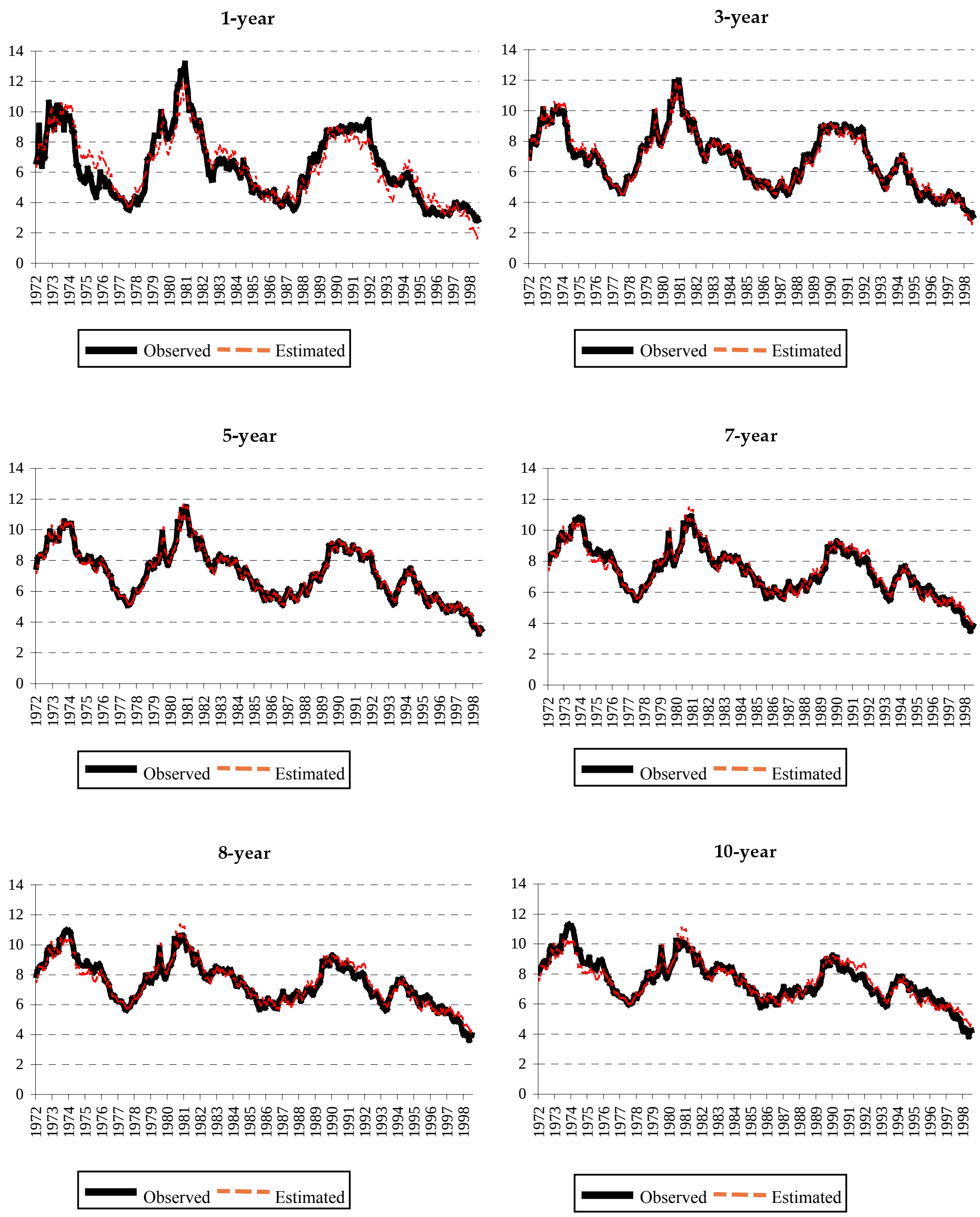


Figure 9A
Factor loadings in the two-factor models 1986–1998

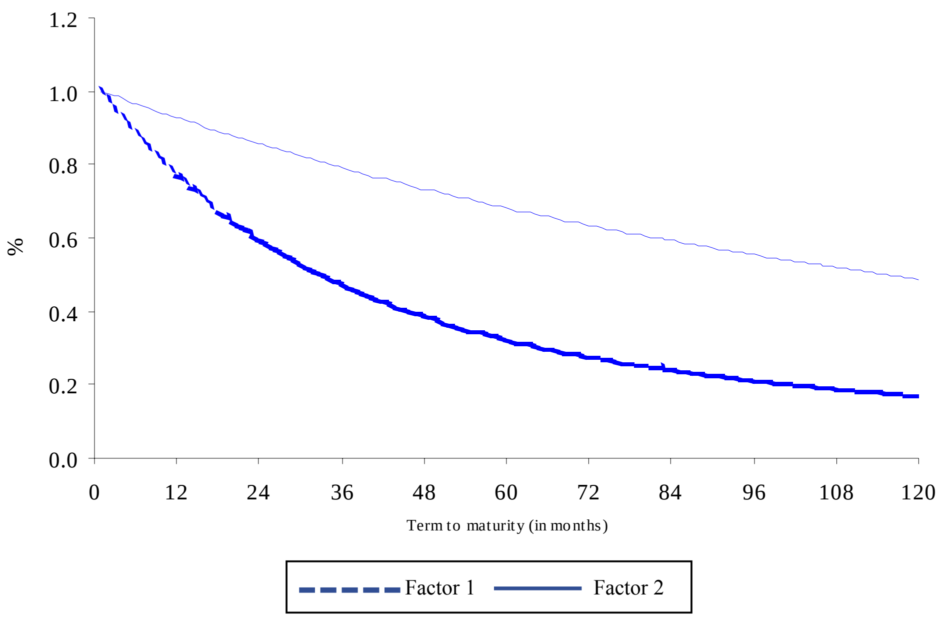


Figure 9B
Factor loadings in the two-factor models 1972–1998

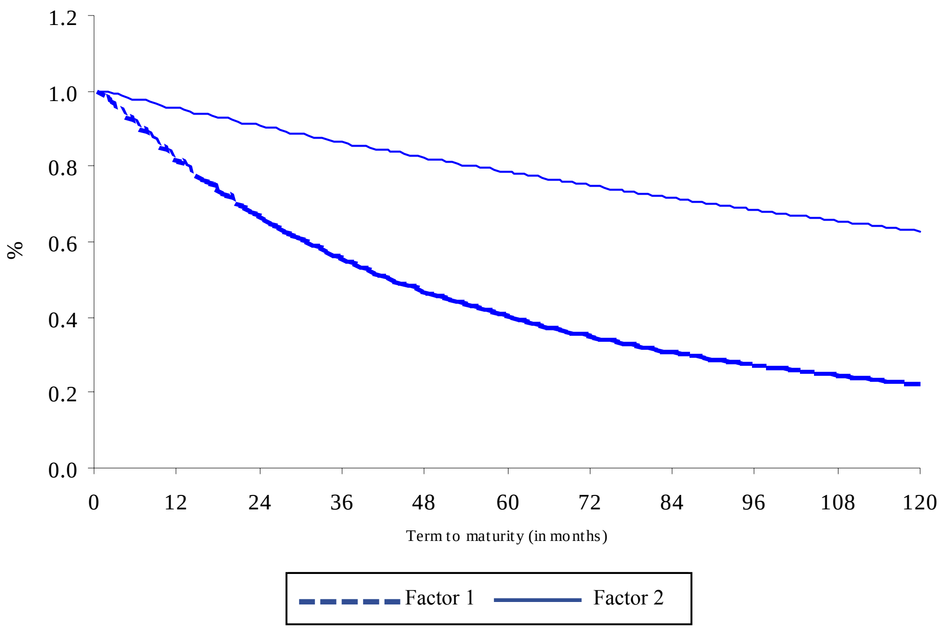


Figure 10A

Time-series evolution of the 1st. factor 1986–1998

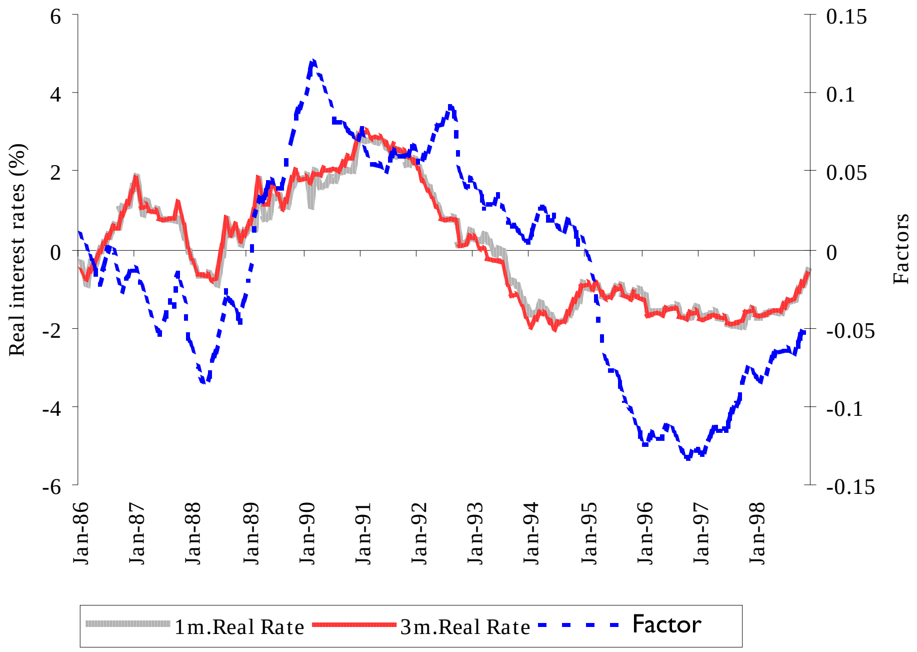


Figure 10B

Time-series evolution of the 1 st. factor 1972–1998

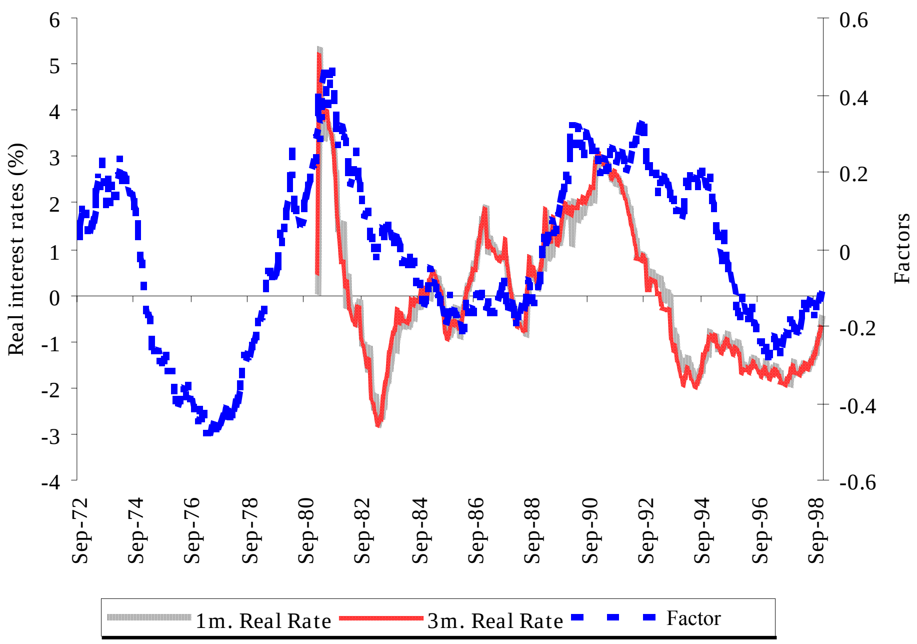


Figure 10C

Time-series evolution of the 2nd. factor 1986–1998

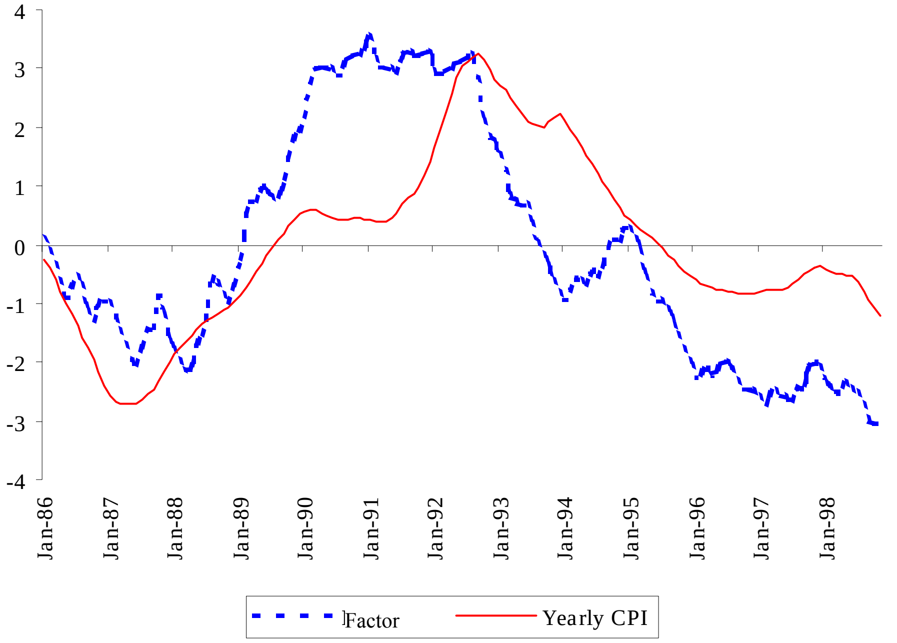


Figure 10D

Time-series evolution of the 2nd. factor 1972–1998

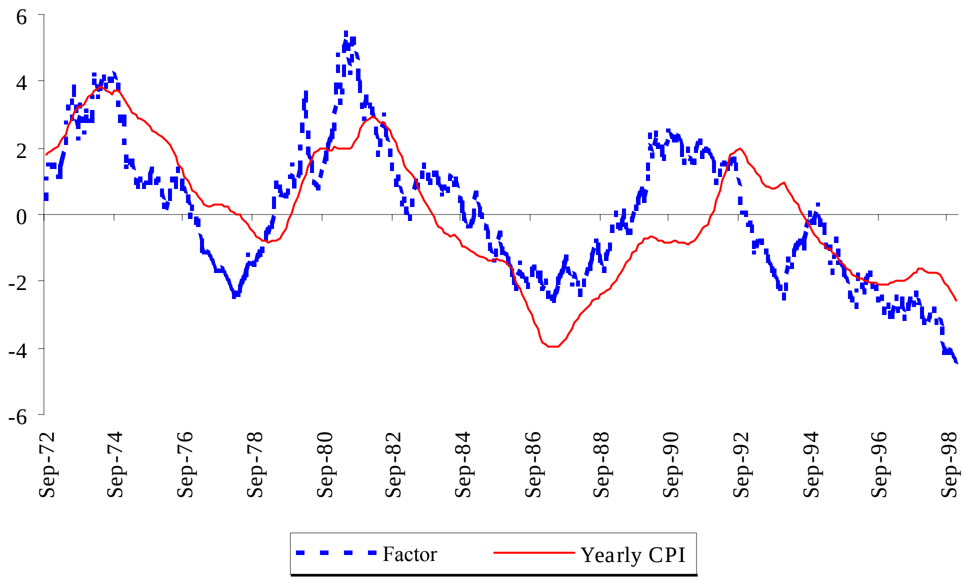
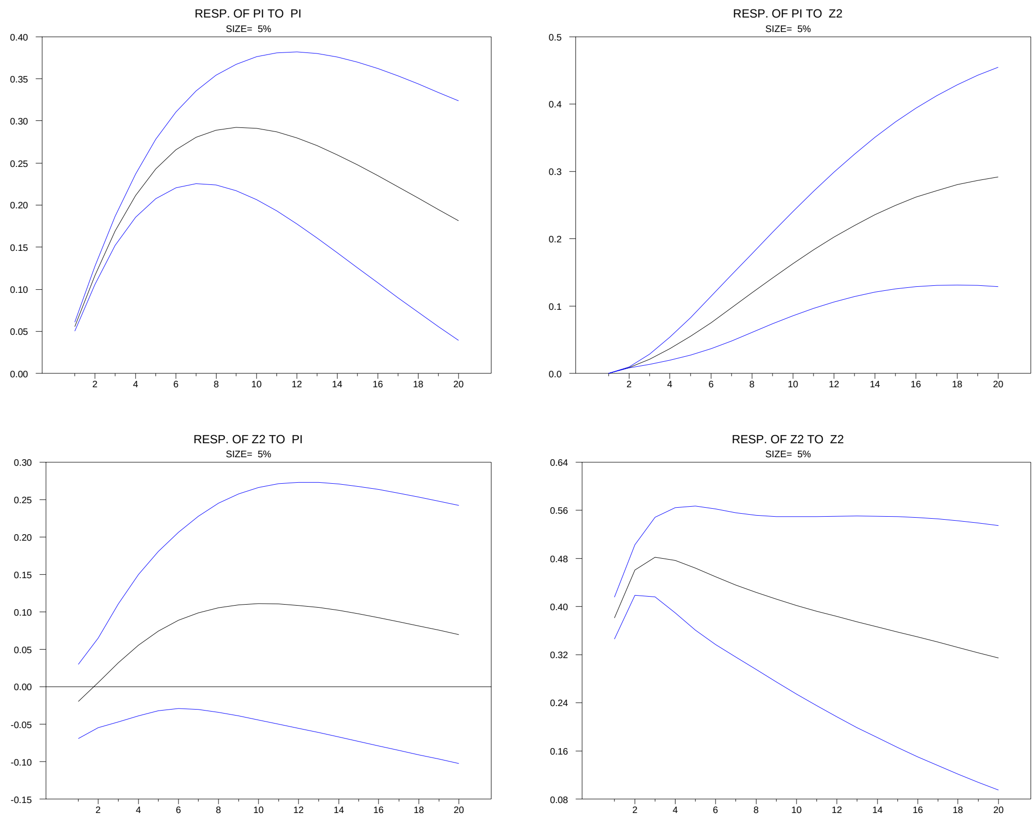


Figure 11

Impulse-response functions and two standard error bankds



European Central Bank Working Paper Series

1 "A global hazard index for the world foreign exchange markets" by V. Brousseau and F. Scacciavillani, May 1999.

2 "What does the single monetary policy do? A SVAR benchmark for the European Central Bank" by C. Monticelli and O. Tristani, May 1999.

3 "Fiscal policy effectiveness and neutrality results in a non-Ricardian world" by C. Detken, May 1999.

4 "From the ERM to the euro: new evidence on economic and policy convergence among EU countries" by I. Angeloni and L. Dedola, May 1999.

5 "Core inflation: a review of some conceptual issues" by M. Wynne, May 1999.

6 "The demand for M3 in the euro area" by G. Coenen and J.-L. Vega, September 1999.

7 "A cross-country comparison of market structures in European banking" by O. de Bandt and E. P. Davis, September 1999.

8 "Inflation zone targeting" by A. Orphanides and V. Wieland, October 1999.

9 "Asymptotic confidence bands for the estimated autocovariance and autocorrelation functions of vector autoregressive models" by G. Coenen, January 2000.

10 "On the effectiveness of sterilized foreign exchange intervention" by R. Fatum, February 2000.

11 "Is the yield curve a useful information variable for the Eurosystem?" by J. M. Berk and P. van Bergeijk, February 2000.

12 "Indicator variables for optimal policy" by L. E. O. Svensson and M. Woodford, February 2000.

13 "Monetary policy with uncertain parameters" by U. Söderström, February 2000.

14 "Assessing nominal income rules for monetary policy with model and data uncertainty" by G. D. Rudebusch, February 2000.

15 "The quest for prosperity without inflation" by A. Orphanides, March 2000.

16 "Estimating the implied distribution of the future short term interest rate using the Longstaff-Schwartz model" by P. Hördahl, March 2000.

17 "Alternative measures of the NAIRU in the euro area: estimates and assessment" by S. F. and R. Mestre, March 2000.

18 "House prices and the macroeconomy in Europe: Results from a structural VAR analysis" by M. Iacoviello, April 2000.

19 "The euro and international capital markets" by C. Detken and P. Hartmann, April 2000.

20 "Convergence of fiscal policies in the euro area" by O. De Bandt and F. P. Mongelli, May 2000.

21 "Firm size and monetary policy transmission: evidence from German business survey data" by M. Ehrmann, May 2000.

22 "Regulating access to international large value payment systems" by C. Holthausen and T. Rønde, June 2000.

23 "Escaping Nash inflation" by In-Koo Cho and T. J. Sargent, June 2000.

24 "What horizon for price stability" by F. Smets, July 2000.

25 "Caution and conservatism in the making of monetary policy" by P. Schellekens, July 2000.

26 "Which kind of transparency? On the need for clarity in monetary policy-making" by B. Winkler, August 2000.

27 "This is what the US leading indicators lead" by M. Camacho and G. Perez-Quiros, August 2000.

28 "Learning, uncertainty and central bank activism in an economy with strategic interactions" by M. Ellison and N. Valla, August 2000.

29 "The sources of unemployment fluctuations: an empirical application to the Italian case" by S. Fabiani, A. Locarno, G. Oneto and P. Sestito, September 2000.

30 "A small estimated euro area model with rational expectations and nominal rigidities" by G. Coenen and V. Wieland, September 2000.

31 "The disappearing tax base: Is foreign direct investment eroding corporate income taxes?" by R. Gropp and K. Kostial, September 2000.

32 "Can indeterminacy explain the short-run non-neutrality of money?" by F. De Fiore, September 2000.

33 "The information content of M3 for future inflation" by C. Trecroci and J. L. Vega, October 2000.

34 "Capital market development, corporate governance and the credibility of exchange rate pegs" by O. Castrén and T. Takalo, October 2000.

35 "Systemic risk: A survey" by O. De Bandt and P. Hartmann, November 2000.

36 "Measuring core inflation in the euro area" by C. Morana, November 2000.

37 "Business fixed investment: Evidence of a financial accelerator in Europe" by P. Vermeulen, November 2000.

38 "The optimal inflation tax when taxes are costly to collect" by F. De Fiore, November 2000.

39 "A money demand system for euro area M3" by C. Brand and N. Cassola, November 2000.

40 "Financial structure and the interest rate channel of ECB monetary policy" by B. Mojon, November 2000.

41 "Why adopt transparency? The publication of central bank forecasts" by P. M. Geraats, January 2001.

42 "An area-wide model (AWM) for the euro area" by G. Fagan, J. Henry and R. Mestre, January 2001.

43 "Sources of economic renewal: from the traditional firm to the knowledge firm" by D. R. Palenzuela, February 2001.

44 "The supply and demand for eurosystem deposits – The first 18 months" by U. Bindseil and F. Seitz, February 2001.

45 "Testing the Rank of the Hankel matrix: a statistical approach" by G. Camba-Mendez and G. Kapetanios, February 2001.

46 "A two-factor model of the German term structure of interest rates" by N. Cassola and J. B. Luís, February 2001.