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COUNTERFEITING AND INFLATION

by Cyril Monnet

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In 2005 all ECB publications will feature a motif taken from the €50 banknote.

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2 DG-Research, European Central Bank, Postfach 16 03 19, D-60066 Frankfurt am Main, Germany; e-mail: cyril.monnet@ecb.int

Address
Kaiserstrasse 29
60311 Frankfurt am Main, Germany

Postal address
Postfach 16 03 19
60066 Frankfurt am Main, Germany

Telephone
+49 69 1344 0

Internet
http://www.ecb.int

Fax
+49 69 1344 6000

Telex
411 144 ecb d

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The public can be confident of the quality of the euro banknotes and their security features. However, the Eurosystem, i.e. the European Central Bank (ECB) and the 12 national central banks of the euro area, continues to advise the public to be alert to the possibility of receiving a counterfeit. The vast majority of counterfeit euro banknotes can be easily distinguished from genuine ones by using the simple FEEL-LOOK-TILT test described in the Eurosystems information material. (European Central Bank, Biannual information on the counterfeiting of the euro, 2005.)

Abstract

In this paper I show that a lax anti-counterfeiting policy is inconsistent with price stability. I use a deterministic matching model with no commitment and no enforcement. An intrinsically worthless but perfectly durable object called a 'note' can be produced by banks at a given cost, but also by nonbanks at a (possibly) higher cost. Counterfeiting occurs when nonbanks produce notes in equilibrium. When it is cheap for nonbanks to produce notes, or the technology used to detect counterfeits is poor, counterfeits are circulating in equilibrium and trade is only implemented with a growing stock of notes (thus creating inflation). Finally, I show that the highest welfare level is achieved when counterfeiting is costly, or when the detection of counterfeits is of high quality.

JEL Classification: D8, E5

Keywords: Counterfeiting, Inflation, Money, Limited Commitment.

Executive Summary

Central banks being responsible for the integrity of the money stock can take three main measures to preserve it. First, information and training on the security features of banknotes can be increased. Second, law enforcement activities can be enhanced in collaboration with enforcement authorities. Third, a central bank can upgrade banknote features. The European Central Bank takes all three measures and continuously invests in new technology to discourage counterfeiters. As a result, the total value of circulating counterfeit euro notes is minimal in the euro area. A telling statistic is that a European citizen can expect to receive 1 counterfeit euro every time he/she receives approximately 14,000 euro.

In light of this figure, it is surprising that the European Central Bank keeps investing in new, costly, technology to prevent fraudulent activities when losses caused by counterfeiting activities are so minimal. This paper relates the responsibility for the integrity of the money stock to price stability, the primary objective of the European Central Bank and most central banks. More precisely, I show that investing in new technology is vital, as a lax anti-counterfeiting policy is inconsistent with the goal of maintaining price stability.

I use a model where money is needed for trade to take place. There are two types of agents, banks and nonbanks. Banks can print banknotes at a cost, while nonbanks can produce counterfeits (at a higher cost). Banknotes all have the same denomination. The cost to produce a counterfeit is interpreted as the technological cost of production, and is also used as a proxy for the degree of law enforcement. Therefore, as the cost of a counterfeit approaches the cost of a banknote, the degree of law enforcement diminishes.

While goods prices are set, by say a market mechanism, nonbanks have an incentive to sell their goods at a lower price if the cost of producing counterfeits is low. The reason is that lower sales revenues can be compensated by the production of counterfeits. Since counterfeiting is an unproductive activity, I find that welfare is maximized when there is none in equilibrium.

Therefore, when the cost of producing counterfeits is low, nonbanks should be given the incentive to sell at the market price. Inflation realigns incentives as the number of notes in exchange for goods increases (each note has equal denomination): while the cost of counterfeit is fixed, each note has less value in an inflationary environment. Furthermore, the higher the inflation is, the lower is the value of a single note. Hence, there is a threshold of inflation above which selling below the market price (i.e. receiving a lower number of notes) cannot be compensated by the production of counterfeits any more. Therefore, a low cost of counterfeit, that is a lax anti-counterfeiting policy, is inconsistent with price stability.

I also analyze the results of enhancing law enforcement whereby counterfeits are detected and confiscated by some authority. This reduces incentives to accept counterfeits. However, the results continue to hold.

1 Introduction

Generally, central banks are responsible for the integrity of the money stock. There are three main measures to preserve it. First, information and training on the security features of banknotes can be increased. Second, law enforcement activities can be enhanced in collaboration with enforcement authorities. Third, a central bank can upgrade banknote features. The European Central Bank takes all three measures and continuously invests in new technology to discourage counterfeiters. As a result, the total value of circulating counterfeit euro notes is minimal in the euro area. A telling statistic is that a European citizen can expect to receive 1 counterfeit euro every time he/she receives approximately 14,000 euro.[1] In light of this figure, why should the European Central Bank, or indeed any central bank, keep investing in new technology to prevent fraudulent activities when losses caused by counterfeiting activities are so minimal ? In this paper, I relate the integrity of the money stock to price stability, the primary objective of most central banks. More precisely, I show that investment aiming at reducing counterfeiting is of utmost importance, as a lax anti-counterfeiting policy is inconsistent with the goal of maintaining price stability.

I use a deterministic matching model with no commitment and no enforcement, populated with banks and nonbanks. Banks produce at a cost intrinsically worthless objects (banknotes) that can be replicated (at a possibly higher cost) by nonbanks. As in Kultti (1996), I define counterfeiting as the private provision of money. Hence, counterfeiting occurs whenever nonbanks produce banknotes in equilibrium.

There are two types of nonbanks. As in Cavalcanti (2004) and Monnet (2001), nonbanks' status alternates each period between 'consumer' and 'producer'. Following Monnet (2001), a consumer bilaterally meets a producer each period, but the identity of the producer is uncertain. In each match, there is an absence of double coincidence of wants, as consumers do not produce goods of value to producers. Consumers also bilaterally meet banks each period. In addition to an absence of a double coincidence of wants, the environment is characterized by limited enforcement, implying that money is essential.[2]

[1] In 2004, 594,000 counterfeit euro banknotes were confiscated, with an approximate value of 35 million euro. This can be compared with the 9 billion legitimate euro notes circulating in 2004, with an approximate value of 501 billion euro.

[2] See Kocherlakota (1996).

To model limited enforcement, I consider mechanisms where agents can walk away from any exchange. This lack of commitment is also reinforced by the ability of consumers to offer fewer notes to producers. An allocation is defined as the quantity of goods being exchanged in a match. Within this class of no-commitment mechanisms, I consider all symmetric stationary allocations that are implementable, i.e. for which banks and nonbanks have no incentive to walk away or accept offers.

One important feature of this framework is that a monetary equilibrium always exists, unless counterfeiting is a costless activity. Additionally, when it is relatively cheap for non-banks to produce banknotes, i.e when few anti-counterfeiting measures are adopted, I show that allocations can only be implemented with counterfeiting and a high degree of inflation. Moreover, I show that if banks' money is costly enough to replicate, then allocations can be implemented with a low degree of, or even no, inflation. In this sense, a lax anti-counterfeiting policy is inconsistent with price stability. Furthermore, I show that welfare is maximized when there is little or no counterfeiting.

The intuition behind these results is simple. When counterfeiting is cheap, nonbanks are more prone to accept offers consisting of fewer banknotes, as they can counterfeit them. Incentives are realigned when nonbanks find it too expensive to produce notes. This is the case when they produce so many notes in equilibrium that producing an additional note eliminates all benefits from the exchange of goods. Therefore, to realign incentives, there must be counterfeiting in equilibrium and the stock of money must grow.

In other words, inflation reduces the value of counterfeit activities, through the following channel. While the cost of produce a note is unaffected by inflation, it reduces the value of each note. As a consequence, the higher the inflation, the less attractive it is to produce counterfeits. With inflation, while counterfeits can exist they are limited, and monetary exchange exists. However, as counterfeiting is costly (and unproductive) and trades can be intermediated by banknotes, welfare is maximized when counterfeiting activities are minimized.

I also analyze the results of enhancing law enforcement activities whereby counterfeits are detected and confiscated by some authority. This reduces the incentives to accept counterfeits. However, in this environment as well, I find that the main result continues to hold.

There are very few models of counterfeiting. Three notable exceptions are Green and

Weber (1996), Kultti (1996) and the recent work by Nosal and Wallace (2004). In Green and Weber (1996) and Kultti (1996), counterfeiting can potentially be welfare-improving as it can alleviate a shortage of money. Nosal and Wallace (2004) propose a more general setup, where counterfeiting is harmful. However they do not find equilibrium with counterfeiting. The main contribution of the current paper is that, contrary to the previous literature, it analyzes an environment in which money holdings are not restricted to be either 0 or 1. Among other things, this allows the possibility for inflation, whereby a growing number of notes are necessary to purchase the same quantity of goods. I find that equilibrium with counterfeiting can exist if the counterfeiting cost is low. Still an equilibrium with no counterfeiting always Pareto-dominates an equilibrium with counterfeits. Therefore, when agents are not liquidity constraint, counterfeiting is certainly welfare diminishing.

The paper is structured as follows. Section 2 outlines the environment. In Section 3 I define implementable allocations, which I characterize in Section 4. There, I also analyze the level of welfare that is achievable when counterfeiting is possible. I present some extensions in Section 5, while Section 6 makes some final remarks.

2 The environment

I consider the same economy as in Monnet (2001), with a continuum of infinitely lived agents. Time is infinite and discrete with $t = 1, 2, ...$ There are three types of agents with equal measure. One type will be referred as banks, and the two other types (labelled 1 and 2) as nonbanks.

The main difference between banks and nonbanks is that nonbanks are endowed with production technology, while banks are not. Type 1 (type 2) agents can only operate the technology in even (odd) periods. They can produce perishable goods that are only consumed by type 2 (type 1) agents. Hence, in odd periods type 1 agents will be called consumers while agents of type 2 will be called producers, inversely in even periods. Nonbanks can produce either 0 or $q > 0$ units of good. Each unit of the good costs 1 to produce. Consumption of q units gives utility $u(q)$. For later reference, q^* will denote the socially optimal production level ($u'(q^*) = 1$).

Banks and nonbanks can also produce non-distinguishable *notes* in positive integer amounts. A note is a durable and intrinsically worthless good. While the production of notes is unrestricted, the cost of production varies across banks and nonbanks. Banks can produce notes at a unit cost of $\varepsilon > 0$.[3] Nonbanks can replicate banknotes at unit cost $\gamma \geq \varepsilon$. I do not impose any upper bound on holdings of notes.

Consumers are also endowed with ω units of a perfectly divisible consumer-specific good. Consuming one unit of this good increases consumers or banks' utility by one. Producers derives no utility from it. This good is perishable and, as a consequence, cannot be used as commodity money. This good plays no role except that it allows transfers from consumers to banks in a simple way. I further assume $\omega \geq \varepsilon$, so that with a large enough transfer of this good, banks are compensated for the production of one note.

Within each period consumers can choose to be randomly matched with a bank, prior to being randomly matched with a producer. Consumers always meet a producer in each period. However, prior to the match the identity of the producer remains uncertain for any consumer. No information leaks from any bilateral meetings and only those agents taking part in the meeting know the outcome of their match. Agents discount utility by a factor $\beta \in (0, 1)$.

Agents cannot pre-commit to specific actions and there is no enforcement of trades. Therefore, as Kocherlakota (1998) shows, in such environments characterized by an absence of a double coincidence of wants, any exchanges will necessitate the use of a means of exchange or money. Only banknotes and counterfeits can serve as money, as the other goods are perishable.

In what follows, I focus exclusively on equilibria where agents are treated symmetrically. Furthermore, I analyze only stationary allocations where all consumers receive the same amount of goods each period. Notes are the only means of payment, but they are costly to produce. Therefore, I will only consider the cheapest monetary mechanism to get active trading. This implies that notes will only be transferred by consumers (and banks). Also, consumers will always transfer all their money for goods.[4]

Social welfare is the discounted sum of all agents' payoff, including the cost of producing notes. The transfer of consumer-specific goods to banks is a mere redistribution of utility from

[3] ε is assumed small enough so that a monetary equilibrium exists.

[4] Otherwise, since the future is discounted, the payment scheme would be dominated by another where fewer notes are produced earlier on.

nonbanks to banks and does not affect social welfare. Hence, when ρ_t, $\tilde{\rho}_t$ stand respectively for the production of banknotes and counterfeits in period t, and q is exchanged between two nonbanks, welfare is given by

$$W = \sum_{t=1}^{\infty} \beta^{t-1}[\omega + u(q) - q - \varepsilon\rho_t - \gamma\tilde{\rho}_t]$$

Under the assumption that nonbanks produce notes that are perfectly distinguishable from banknotes, Monnet (2001) shows it is socially optimal that banks produce the means of payment for the economy. Here, however when $\gamma > \varepsilon$, banks have a natural comparative advantage in producing notes. Therefore, I will consider the equilibrium where banks (and potentially nonbanks as well) produce notes. In Section 3, I define implementable allocations, that I partially characterize in Section 4. In Section 4, I will also show that welfare is highest when there is no counterfeits circulating.

3 Implementable allocations

At this point of the analysis, a game (or a mechanism) is generally specified. However, since I want results to be independent of a specific game form, I consider a general class of games, without specifying all the details of a game, such as action sets, stages, etc. The result of these games will include notes being produced and transferred in each match, and some quantity of consumer-specific good being relinquished by consumers to banks. Also, in all these games, a quantity q of private goods will be consumed in each match. I refer to this quantity q as an allocation.

I restrict games in this class to satisfy two non-commitment criteria. First, in any match, any agent is given the possibility to walk away, in which case both agents in the match receive the autarkic allocation (i.e. no exchange takes place). Second, I allow consumers and producers to agree on transferring fewer notes than dictated by the mechanism. However, they are not allowed to agree on another quantity to be exchanged.

Agents can contemplate to play any game within this class of no-commitment mechanisms. When they play a specific games, agents expect payoffs for each action they take. I now define what these payoffs are, given the allocation q, the production and transfers of notes, and transfers of consumer-specific goods.

I will let $\nu(m^b, t)$ be the expected lifetime utility of a bank in period t when holding m^b units of money. If an allocation requires the exchange of n_t notes for α_t units of consumer-specific goods, $\nu(m^b, t)$ is:

$$\nu(m^b, t) = \alpha_t - \varepsilon \max\{0, n_t - m^b\} + \beta\nu(\max\{0, m^b - n_t\}, t + 1) \tag{1}$$

Whenever $n_t > m^b$, banks have to produce $\rho_t = n_t - m^b$ banknotes. However following their money transfers, banks have no more notes in hand. Inversely, when $n_t < m^b$, banks have enough to only transfer n_t and they leave the match with $m^b - n_t$ notes. Only off-equilibrium will m^b differ from 0.

$v(m, t)$ denotes the expected lifetime utility of a (nonbank) producer in period t when holding m units of (legitimate and counterfeit) money. When a producer receives $\tilde{n}_t$ notes in exchange for q units of private goods, $v(m, t)$ is

$$v(m, t) = -q + \beta w(m + \tilde{n}_t, t + 1) \tag{2}$$

The lifetime expected utility of (nonbank) consumers in period t is slightly different in its structure, as they meet banks and then nonbanks in every period. In addition to being able to produce counterfeits, they can also give up a share α_t of their specific good to banks. Therefore, the expected utility of a consumer is given by a pair of value functions:

$$w(m, t) = \omega - \alpha_t + \tilde{w}(m + n_t, t) \tag{3}$$

$$\tilde{w}(m, t) = u(q) - \gamma max\{0, \tilde{n}_t - m\} + \beta v(\max\{0, m - \tilde{n}_t\}, t + 1) \tag{4}$$

Equation (3) denotes the expected lifetime payoff of a consumer when relinquishing α_t units of the consumer-specific good to banks in exchange for n_t notes. Equation (4) is the expected lifetime payoff of a consumer with m notes, when q units of goods necessitate the transfer of $\tilde{n}_t$ notes. I will denote $\tilde{\rho}_t = \tilde{n}_t - m$ the production of counterfeits in period t. Since autarky is always a possibility for banks and nonbanks, expected utility has to satisfy the following participation constraints for consumers, producers and banks respectively:

$$w(m,t) \geq \omega + \tilde{w}(m,t) \text{ and } w(m,t) \geq \omega/(1-\beta^2), \tag{5}$$

$$\tilde{w}(m,t) \geq \beta v(m,t+1) \tag{6}$$

$$v(m,t) \geq \beta w(m,t+1) \text{ and } v(m,t) \geq \beta\omega/(1-\beta^2), \tag{7}$$

$$\nu(m^b,t) \geq \beta\nu(m^b,t+1) \geq 0. \tag{8}$$

The left-hand side constraints imposes participation in any date t match, while the right-hand side constraints guarantee that at any time, permanent autarky is not desirable. For instance, the first inequality in expression (5) imposes that, in a meeting with a bank, a consumer is better off subscribing to the allocation than keeping ω and not receiving any banknotes, entering the match with a producer with still m notes. The other expressions read in a similar fashion. Furthermore, since consumers are allowed to offer fewer notes, it must be that producers are better off rejecting such offers than accepting fewer notes and producing q. This should of course be the case for all offers $n'_t < n_t$. Hence, the following incentive constraint has to hold for all t.

$$\beta w(m,t+1) \geq -q + \beta w(m+n'_t,t+1) \quad \forall n'_t < n_t \tag{9}$$

Definition 1. *An allocation q is implementable if there exist $\{\alpha_t, \rho_t, \tilde{\rho}_t, n_t, \tilde{n}_t\}_{t\in\mathbb{N}}$, and functions $\nu(.)$, $v(.,.)$, $w(.,.)$ and $\tilde{w}(.,.)$ such that (1)-(9) hold.*

I now (partially) characterize the set of implementable allocations.

4 Characterization of implementable allocations and welfare

4.1 Characterization

I will suppose for the moment that banks produce banknotes in exchange for some consumer-specific good. More precisely, for each banknote produced, an equivalent share of the consumer-specific good is transferred to banks. Consumers will have no incentive to fail to deliver

$\alpha_t \leq n_t\gamma$ in exchange for n_t banknotes, as long as they need to deliver at least n_t notes to producers. In case $\alpha_t > n_t\gamma$, consumers would rather opt for autarky and produce n_t counterfeits. But α_t should also compensate banks from producing notes, so that $\alpha_t \geq \varepsilon n_t$. Hence, we obtain the first condition for implementability.

Proposition 1. *For all $t \geq 1$, if $n_t > 0$ then $\alpha_t/n_t \in [\varepsilon, \gamma]$ and $\alpha_t = 0$ otherwise.*

We also need a similar condition when consumers meet producers. Producers must find it profitable to deliver q rather than producing in the next period the notes that should have been delivered. Precisely, we obtain the following proposition:

Proposition 2. *If q is implementable then $\tilde{n}_t \geq q/(\beta\gamma)$.*

Proof. Suppose this is not the case, then the (discounted) cost of producing $\tilde{n}_t$ counterfeits the next period is $\beta\tilde{n}_t\gamma$, and this is lower than producing q today. Producers in period t are therefore better off choosing autarky and producing $\tilde{n}_t$ counterfeits in the next period than producing q today. □

The result above says that enough banknotes have to be exchanged between two nonbanks (and the smaller γ, the more notes) for exchange to take place. This has an interesting implication. As banknotes are being exchanged, in the class of mechanisms considered, consumers now have the opportunity to offer fewer banknotes for the same quantity of good. This must be incentive *in*compatible. As shown below, this implies that counterfeiting can emerge if γ is sufficiently low.

Proposition 3. *If q is implementable then $\tilde{\rho}_{t+1} + 1 \geq [u(q) - q/\beta]/\gamma$ for all $t \geq 1$.*

Proof. Suppose that in period t, a consumer proposes to acquire q units of goods for only $n'_t < \tilde{n}_t$ notes. Should the producer accept? If it does then it will have to counterfeit the missing notes next period and incur a cost $\gamma(\tilde{n}_{t+1} - n'_t)$, to be able to consume. Its payoff is then $-q - \beta\gamma(\tilde{n}_{t+1} - n'_t) + \beta(u(q) + \omega) + \beta^2 v(0, t+2)$. If it rejects, then it will remain in autarky today and tomorrow (it will not find profitable to counterfeit the whole package of missing notes tomorrow, as otherwise it would never produce), and its payoff is $\beta\omega + \beta^2 v(0, t+2)$. Therefore, it rejects only if $\gamma(\tilde{n}_{t+1} - n'_t) \geq u(q) - q/\beta$. Now $n'_t \leq \tilde{n}_t - 1$, and $\tilde{n}_{t+1} - \tilde{n}_t = \tilde{\rho}_{t+1}$. Hence it will reject *any* offer n'_t if $\gamma(\tilde{\rho}_{t+1} + 1) \geq u(q) - q/\beta$. □

If γ is relatively low, i.e. $u(q) - q/\beta > \gamma$, $\tilde{\rho}_t > 1$ for all $t > 1$. In words, if counterfeiting is cheap relative to the allocation q and there is no production of counterfeits, producers will find optimal to accept low price offers, i.e. offers consisting of fewer notes than expected. To deter producers from accepting such offers, the cost of missing notes has to increase. This is the case if a mechanism already requires producers to produce a (large) number of counterfeits. Hence counterfeits will circulate in equilibrium, and the stock of (legitimate and counterfeit) notes will inflate. Therefore, a lax anti-counterfeiting policy (a low γ) is inconsistent with price stability.

To complete this section, I now show that there is at least one implementable allocation.

Proposition 4. *Let* $(q, \rho, \tilde{\rho}, n, \tilde{n})$ *be such that* $\alpha_t \in [\rho_t \varepsilon, \rho_t \gamma]$, $\rho_1 > 0$, $[u(q) - q/\beta] - \gamma \leq \gamma \tilde{\rho}_{t+1} \leq [u(q) - q/\beta]$ *and* $\tilde{n}_t = \tilde{\rho}_t + n_t + \tilde{n}_{t-1}$ *for all* $t \geq 1$ *with* $\tilde{n}_0 = 0$ *then* q *is implementable.*

Proof. Consider the following game. If consumers meet banks, they post a quantity α_t of consumer-specific goods. Banks then post a quantity n_t of banknotes. Consumers and bank then either accept of reject. If one rejects, then the outcome of the game is autarky. If both accept, the posted quantities are exchanged. When they meet producers, consumers post a quantity of notes. If producers accept (say 'yes' to) the offer, q is produced and exchanged. Otherwise, the outcome is autarky. Note that this game falls in the class of no-commitment games. The equilibrium strategies are to always post the quantities prescribed by the mechanism, and to always reject offer that do not respect these quantities. In particular, the condition $\alpha_t \in [\rho_t \varepsilon, \rho_t \gamma]$ ensures that in the case $\rho_t > 0$, banks participate and consumers do not choose autarky when they meet banks. $\rho_1 > 0$ states that there is a means of payment in the economy. The two subsequent inequalities imply that nonbanks will not default if they participate. Finally, agents will be willing to participate if the benefits from producing today and incurring the cost $-q$ is compensated by the (discounted) surplus from consumption, $\beta(-\gamma\rho_t + u(q))$. This is the case as $\gamma\rho_t \leq [u(q) - q/\beta]$. □

4.2 Welfare

Given the (partial) characterization of implementable allocations, welfare results are easily obtained. Let $W^*(q)$ be the *maximum* level of welfare attainable when allocation q is implemented.

Corollary 1. *If q, such that $u(q) - q/\beta > \gamma$, is implemented then given $\{\rho_t\}_{t\in\mathbb{N}}$,*

$$W^*(q) = (\omega + \beta\gamma)/(1-\beta) + u(q) - \sum_{t=1}^{\infty} \beta^{t-1}\rho_t\varepsilon.$$

Proof. Note that $\tilde{\rho}_0 = 0$ is feasible. Moreover, as shown above, $\gamma\tilde{\rho}_{t+1} \geq u(q) - q/\beta - \gamma$ for all $t \geq 1$. Replacing this expression with equality in the welfare function gives us the desired result. □

Therefore the easier it is to counterfeit banknotes, i.e. the lower γ, the lower welfare will be. The intuition is simple. As counterfeits are cheaply produced, the incentives to accept fewer notes than envisaged by the mechanism are larger. Hence, counterfeiting is a necessary feature of the mechanism used to implement q. In addition, the cheaper counterfeits are, the more there are. Since counterfeiting is a counter-productive activity, welfare decreases with it. The upper bound on welfare is not a continuous function of γ. For instance, if γ is zero, no monetary equilibrium exists and the highest attainable welfare level is the one of autarky $\omega(1-\beta)^{-1}$.

As γ increases, the attainable welfare approaches $(\omega + u(q) - q)/(1-\beta)$, the welfare level attainable in the frictionless environment. This is the next result.

Corollary 2. *If q, such that $u(q) - q/\beta \leq \gamma$, is implemented, then given $\{q_t\}_{t\in\mathbb{N}}$,*

$$W^*(q) = (\omega + u(q) - q)/(1-\beta) - \sum_{t=1}^{\infty} \beta^{t-1}\rho_t\varepsilon.$$

Proof. When $\gamma \geq u(q) - q/\beta$ then either $\gamma = u(q) - q/\beta$ in which case the result is obtained by replacing this value in the previous welfare expression, or $\gamma > u(q) - q/\beta$ in which case $\tilde{\rho}_t$ can be zero for all t as implied by proposition 3, and the above welfare level equals the given upper bound. □

In both corollaries, the upper bound for the welfare value is attainable. Note that in the present framework, the second-best welfare level is attained when γ is sufficiently high and banks print one note in the first period and then nothing thereafter. In this case, there is no inflation and the first best quantity q^* is implementable. We obtain our third welfare result.

Corollary 3. *Welfare is maximized when $\gamma \geq u(q^*) - q^*/\beta$.*

5 Extensions

In this section, I consider two extensions. The first extension introduces a standing facility, whereby banks are producing banknotes on consumers demand. The second extension introduces a technology to detect and confiscate counterfeits by some authority.

5.1 Standing facility

In the framework analyzed in Section 3 and 4, banks were not ready to provide additional notes to consumers in need, i.e. they were not providing a standing facility. More precisely, if an agent does not have enough notes to consume, it can only counterfeit notes, rather than asking banks for the missing notes, in exchange for a higher share of the consumer specific good ω.

If a standing facility were to be opened, banks could provide x notes in exchange for more units of the consumer-specific good. As long as bank do not require more than γx of the good for these notes, consumers would prefer to use the standing facility rather than produce counterfeit notes.

However, a slightly modified version of proposition 3 would hold, and the stock of money supply would still grow. If banks were to require less than γx for these notes, the stock of money supply would grow more, as these notes are less costly to finance.

Hence, while providing a standing facility helps reduce the number of counterfeits in the economy, it will not reduce inflationary pressure.

5.2 Detecting counterfeits

A legitimate criticism of the present framework is that it assumes that people have no motivation "to avoid receiving counterfeit money" (see Nosal and Wallace, 2004). In particular, no active measures are taken to eliminate counterfeit money, such as monitoring randomly nonbanks' money holdings. In this section, I consider this possibility. More precisely, I assume that banks have the technology to check the quality of consumers' money holdings. I assume that banks check money holdings with a probability $\pi \in [0, 1]$. If consumers hold counterfeits, then banks confiscate and destroy them. Consumers can then decide whether or not they wish to produce counterfeits in meetings with producers.

This resembles very closely the assumption in Green and Weber (1996), where 'If a trader's counterfeit is confiscated by a government agent, the trader can either replace the counterfeit or not. (...) What determines whether or not a trader chooses to produce replacement counterfeit after confiscation is the essence of what [Green and Weber, 1996] study'. This also answers the potential critique of Nosal and Wallace (2004), as producers would rather not receive counterfeits since they may be confiscated when they in turn become consumers.

Now, we have to take into account different histories for consumers. In particular, a mechanism designer might wish to distinguish those "unlucky" consumers who were monitored from the lucky ones. Still, for simplicity, I will restrict my analysis to the case where the price of private goods is the same for all. This implies that unlucky consumers whose counterfeits, if any, have been confiscated may have to produce more counterfeits than lucky consumers. I let $\tilde{\rho}_t^u$ be the production of counterfeits for an unlucky consumers. If a portfolio contains ϕ notes, then either $\tilde{\rho}_t^u = \tilde{\rho}_t + \phi$, allowing consumers to acquire the producers' good or $\tilde{\rho}_t^u = 0$ if the cost of producing $\tilde{\rho}_t + \phi$ notes is greater than the benefit of consuming q.

Working with this new framework only modifies the value function (3) of consumers, which now incorporates the possibility that counterfeits are confiscated. When ϕ is the number of counterfeit notes in an agent's portfolio,

$$w(m,t) = \omega - \alpha_t + \pi\tilde{w}(m - \phi + n_t, t) + (1-\pi)\tilde{w}(m + n_t, t) \tag{10}$$

Then the definition of implementable allocations is modified accordingly.

Definition 2. *An allocation q is implementable if there exist $\{\alpha_t, \rho_t, \tilde{\rho}_t, \tilde{\rho}_t^u, n_t, \tilde{n}_t\}_{t\in\mathbb{N}}$, and functions $\nu(.)$, $v(.,.)$, $\tilde{w}(.,.)$ and $w(.,.)$ such that (1),(2) and (4)-(10) hold.*

Since counterfeits are not recognized as such by nonbanks, propositions 1 and 2 continue to hold here. However, in equilibrium with $\tilde{\rho}_t > 0$ producers know money transfers from consumers contain counterfeits. Therefore, proposition 3 is likely to be modified. The question then becomes: Given they can be confiscated, will producers accept fake notes, and if so, how many?

Lemma 1. *Given an allocation q, producers accept a portfolio of m notes including ϕ counterfeits if $\beta m\gamma > q + \pi\beta\gamma\phi$ and either*

1. $u(q) - \tilde{\rho}_t > q/\beta + \pi\gamma\phi$*; or*

2. $(1-\pi)[u(q) - \tilde{\rho}_t\gamma] > q/\beta$ *and* $\gamma\phi > u(q) - \tilde{\rho}_t$.

Proof. First, m is accepted in exchange for q if consumers find producing m counterfeits next period too expensive, i.e. if $\beta m\gamma > q + \pi\beta\gamma\phi$.

Now we have to distinguish the case where $\tilde{\rho}_t^u > 0$ from the one where $\tilde{\rho}_t^u = 0$. In the first case, a consumer will accept m if the expected gain $\beta[u(q) - \tilde{\rho}_t] - \beta\pi\gamma\phi$ of accepting the portfolio outweighs the cost q of so doing. In the second case, consumers only acquire the producers' good if they are not monitored. Hence, the expected benefit is $(1-\pi)\beta[u(q) - \tilde{\rho}_t\gamma]$, and it should be greater than the cost of accepting m, which is q. Finally, in this case, it should also be the case that consumers do not find it profitable to produce ϕ counterfeits, since $\tilde{\rho}_t^u = 0$. This is guaranteed by $\gamma\phi > u(q) - \tilde{\rho}_t$. □

From the lemma above, we see that the effect of the ability of banks to confiscate counterfeits is to limit the size of $\tilde{\rho}_t$ and, as a consequence, the extent to which the stock of notes is growing. However, as shown in the next proposition, the anti-counterfeiting policy has to be very strict, i.e. π must be relatively high. In other words, in this environment, inflation is only limited by once more relatively serious screening.

Proposition 5. *Let* π *such that* $q/\beta > (1-\pi)u(q)$*. If* q *is implementable, then there exists* T *such that* $\tilde{\rho}_t = 0$ *for all* $t > T$*.*

Proof. Note first that any mechanism implementing q predicts whether or not counterfeits are produced. Hence, in equilibrium, agents know the composition of their holdings of notes, although they cannot distinguish counterfeits from banknotes.

Let ϕ^* be such that $u(q) = q/\beta + \pi\gamma\phi^*$. Then from the lemma above, for all $\phi < \lfloor\phi^*\rfloor$, a portfolio containing ϕ fake notes may be accepted (if $\tilde{\rho}_t$ is low enough). The argument of Proposition 3 therefore applies. If at date t a consumer proposes to exchange q units of the good, but only $n_t' < \tilde{n}_t$ units of notes, the producer rejects only if $q/\beta + \gamma\tilde{\rho}_{t+1} + [(1-\pi) + \pi\phi_t]\gamma > u(q)$, where $\phi_t = \sum_{s=0}^{t} \tilde{\rho}_s$. Hence, either it is necessary that $\tilde{\rho}_{t+1} > 0$, or else the result holds.

If $\tilde{\rho}_{t+1} > 0$, then $\phi_{t+1} > \phi_t$, and approaches $\lfloor\phi^*\rfloor$. When $\phi_t = \lfloor\phi^*\rfloor$, the producer rejects the offer only if $q/\beta > (1-\pi)[u(q) - (\tilde{\rho}_{t+1} + 1)\gamma]$. However since $\phi_t = \lfloor\phi^*\rfloor$, it is not possible that $\tilde{\rho}_{t+1} > 0$ as otherwise producers would not accept the portfolio any more (by the lemma above). It must therefore be that at $\lfloor \phi^* \rfloor$, $q/\beta > (1-\pi)[u(q)-\gamma]$, which is true by assumption. As a consequence, producers will reject any transfer lower than the envisaged one. We can then conclude that there is T such that $\tilde{\rho}_t = 0$ for all $t > T$. □

Finally, in this paper, anti-counterfeiting technologies (the level of γ and π) are given to the authority. An interesting additional focus for future work would be to devise the optimal quality of anti-counterfeiting technologies, and analyze whether in this case counterfeits still circulate.

6 Final remarks

In the present environment, I considered a class of mechanisms that implement trade when private agents can counterfeit banknotes. In this environment, counterfeits may circulate in equilibrium, even if there is a chance that they will be confiscated. As counterfeits become more costly to produce, there are less in equilibrium. Somewhat surprisingly, higher counterfeiting costs are associated with higher welfare, as this reduces incentives to counterfeit, i.e. to invest in a counterproductive activity.

The most striking and perhaps novel aspect of the current framework is that counterfeiting implies inflationary pressures. This result appears a robust finding for the following intuitive reason: if counterfeiting activities are not costly, the money stock will rise, as more counterfeits will be in circulation. If a central bank wishes to take action against this activity without increasing the cost of counterfeiting itself, it will have to inflate the stock of legitimate money. As inflation increases the relative cost of counterfeiting, it reduces counterfeiting incentives. Therefore a lax anti-counterfeiting policy is inconsistent with price stability, although it should of course be noted (Monnet, 2001) that there can be inflation without counterfeiting.

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