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ASYMPTOTIC CONFIDENCE BANDS FOR THE ESTIMATED AUTOCOVARIANCE AND AUTOCORRELATION FUNCTIONS OF VECTOR AUTOREGRESSIVE MODELS

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Address	**Kaiserstrasse 29**
	D-60311 Frankfurt am Main
	GERMANY
Postal address	**Postfach 16 03 19**
	D-60066 Frankfurt am Main
	Germany
Telephone	**+49 69 1344 0**
Internet	**http://www.ecb.int**
Fax	**+49 69 1344 6000**
Telex	**411 144 ecb d**

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Abstract

This paper provides closed-form formulae for computing the asymptotic standard errors of the estimated autocovariance and autocorrelation functions for stable VAR models by means of the δ-method. These standard errors can be used to construct asymptotic confidence bands for the estimated autocovariance and autocorrelation functions in order to assess the underlying estimation uncertainty. A Monte Carlo experiment gives evidence on the small-sample performance of these asymptotic confidence bands compared with that obtained using bootstrap methods. The usefulness of the asymptotic confidence bands for empirical work is illustrated by two applications to euro area data on inflation, output and interest rates.

JEL Classification System: C13, C32, E31, E43

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1 Introduction

Vector autoregressive (VAR) models are one of the most popular classes of models in applied econometrics. They provide a simple tool for characterising the dynamic interaction of the data, which can be displayed either by their autocovariance and autocorrelation functions or by their impulse response functions. Whereas the latter may be sensitive to the validity of a set of assumptions used to identify particular structural shocks in the data (see Bernanke and Mihov (1998) and Christiano, Eichenbaum and Evans (1999) for a review of this issue in the context of measuring the effects of monetary policy), the former are not, because of their purely descriptive nature. Therefore, in order to avoid the need to identify structural shocks, McCallum (1999) has recently advocated the use of autocovariance and autocorrelation functions as the more appropriate device for confronting economic models with the data.

Although the computation of the autocovariance and autocorrelation functions of VAR models is straightforward from a technical point of view, there remains a fundamental shortcoming in applied work. The autocovariance and autocorrelation functions are computed from coefficients of VAR models which are estimated from the data. The former are therefore also estimates and, hence, affected by uncertainty. This estimation uncertainty is not properly taken into account when only reporting the point estimates. Extending common practice, we therefore argue that the underlying uncertainty should be assessed by also reporting their confidence bands. These can be set up either by means of bootstrap methods or by relying on asymptotic theory. Focusing on the latter approach, this paper provides some simple formulae for computing the asymptotic standard errors of the estimated autocovariance and autocorrelation functions of stable VAR models. These can be used to construct asymptotic confidence bands, thus saving the practitioner the computational costs of the bootstrap.

It is well known that asymptotic confidence bands for the autocovariances and the autocorrelations of the data — as estimated by their sample moments — could alternatively be derived under the null hypothesis that the data are generated by a white-noise process. In this case, the sample autocovariances and the sample autocorrelations are asymptotically normal (see Hannan (1970) and Anderson (1971) among others), with the standard errors of the sample autocorrelations being approximately equal to $1/\sqrt{T}$. Tests based on the sample autocorrelations are thus very easy to conduct, but it has been shown by Dufour and Roy (1985) that these tests may reject the null hypothesis less frequently than is consistent with their nominal size. Instead, we establish the asymptotic normality of the estimated autocovariance and autocorrelation functions under the null hypothesis that the data are generated by a VAR process. This approach would appear to be more appropriate if the true data-generating process is more closely approximated by a VAR than by a white-noise process.

The remainder of the paper is organised as follows. In Section 2 we state the asymptotic distribution of the estimated autocovariance and autocorrelation functions of stable VAR models by relying on the δ-method. Section 3 presents some Monte Carlo evidence on the small-sample performance of the confidence bands computed from the asymptotic standard errors compared with that obtained using bootstrap methods. In Section 4 we illustrate the usefulness of the asymptotic confidence bands for empirical work by two applications to euro area data on inflation, output and interest rates. Section 5 concludes the paper, and the closed-form formulae of the partial derivatives of the autocovariance and autocorrelation functions, which are needed to compute the asymptotic standard errors, are provided in an appendix.

2 The Asymptotic Distribution of the Estimated Autocovariance and Autocorrelation Functions of Stable VAR Models

Before stating the asymptotic distribution of the estimated autocovariance and autocorrelation functions, we briefly review some results on the estimation of stable VAR models and their autocovariance and autocorrelation functions, which are referred to later on.

2.1 The Stable VAR Model

Let $\{ y_t : t = 0, \pm 1, \dots \}$ be a sequence of a k-dimensional vector of variables which is generated by an unrestricted vector autoregressive (VAR) process of order p,

$$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \qquad t = 0, \pm 1, \dots \tag{1}$$

where u_t is serially uncorrelated with mean zero and positive definite covariance matrix Σ_u.

The VAR(p) model (1) is assumed to be stable, i.e.

$$\det(I_k - A_1 z - \cdots - A_p z^p) = 0 \quad \Rightarrow \quad | z | > 1,$$

where $| \cdot |$ denotes the absolute value operator.

Let $f(y_{-p+1},\dots,y_0;\beta)\prod_{t=1}^{T} f(y_t \mid y_{t-p},\dots,y_{t-1};\beta)$ be the density of a sample $\{\, y_t : t=-p+1,\dots,T \,\}$ generated by the VAR(p) process (1). Then, for fixed initial values $y_{-p+1},\dots,y_0$, the conditional quasi-maximum-likelihood (QML) estimator for $\beta \in B$ is

$$\hat\beta_T \quad = \quad \arg\max_{\beta\in B} \sum_{t=1}^{T} \ln f(y_t \,|\, y_{t-p},\dots,y_{t-1};\beta),$$

where

$$\beta \quad = \quad \left[\, \mathrm{vec}(A_1,\dots,A_p)', \mathrm{vech}(\Sigma_u)' \,\right]'$$

is the n-dimensional parameter vector of the VAR(p) model with $n = pk^2 + k(k+1)/2)$ and $B \subset \mathbf{R}^n$ denotes the feasible parameter space.[1] The vec($\cdot$)-operator stacks the columns of a matrix in a column vector and the vech($\cdot$)-operator stacks the elements on and below the principal diagonal of a square matrix.

Under general regularity conditions the QML estimator $\hat\beta_T$ converges in probability to the "true" parameter vector β_0 as $T \to \infty$,

$$\operatorname*{plim}_{T\to\infty} \hat\beta_T \quad = \quad \beta_0,$$

and is asymptotically normal,

$$\sqrt{T}\,(\hat\beta_T - \beta_0) \quad \stackrel{d}{\longrightarrow} \quad \mathrm{N}\left[0, \Sigma_{\hat\beta}(\beta_0)\right], \tag{2}$$

where $\Sigma_{\hat\beta}(\beta_0) \;=\; \mathcal{H}(\beta_0)^{-1}\mathcal{I}(\beta_0)\,\mathcal{H}(\beta_0)^{-1}$ is the asymptotic covariance matrix of $\sqrt{T}\,(\hat\beta_T - \beta_0)$. $\mathcal{I}(\beta_0)$ denotes the asymptotic information matrix and $\mathcal{H}(\beta_0)$ is the asymptotic expected Hessian of the appropriately normalised quasi-log-likelihood function evaluated at β_0.[2]

2.2 The Estimated Autocovariance and Autocorrelation Functions

In order to estimate the autocovariance and autocorrelation functions of the stable VAR(p) model (see, e.g., Lütkepohl (1991), Chapter 2.1.4), it is convenient to start from its VAR(1) representation

$$Y_t \quad = \quad A\,Y_{t-1} + U_t, \qquad t = 0, \pm 1, \dots$$

with

$$Y_t \quad = \quad \begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-p+1} \end{bmatrix}, \qquad U_t \;=\; \begin{bmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

[1] Closed-form expressions for $\hat\beta_T$ are available, for instance, from Lütkepohl (1991), Chapter 3.4.

[2] See White (1994) for a thorough treatment of QML theory and covariance estimation.

and

$$A \;=\; \begin{bmatrix} A_1 & A_2 & \cdots & A_{p-1} & A_p \\ I_k & 0 & \cdots & 0 & 0 \\ 0 & I_k & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_k & 0 \end{bmatrix}, \qquad \Sigma_U \;=\; \begin{bmatrix} \Sigma_u & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}.$$

The autocovariance function implied by the VAR(1) model, $\{\,\Gamma_{Y,h} : h = 0, \pm 1, \dots\}$ with $\Gamma_{Y,h} = \Gamma_{Y,h}(\beta) = \mathrm{E}[\,Y_t\,Y_{t-h}'\,]$, can then be obtained as follows. First, the stacked contemporaneous covariance matrix fulfills the equation

$$\mathrm{vec}(\Gamma_{Y,0}) \;=\; \Big(I_{(kp)^2} - A \otimes A\Big)^{-1} \mathrm{vec}(\Sigma_U),$$

where $\otimes$ denotes the Kronecker product. And second, the higher order autocovariance matrices are given recursively by the Yule-Walker equation of the VAR(1) model,

$$\Gamma_{Y,h} \;=\; A\,\Gamma_{Y,h-1}, \qquad h = 1, 2, \dots .$$

Finally, the autocovariance function of the VAR(p) model, $\{\,\Gamma_{y,h} : h = 0, \pm 1, \dots\}$ with $\Gamma_{y,h} = \Gamma_{y,h}(\beta) = \mathrm{E}[\,y_t\,y_{t-h}'\,]$, is easily recovered from the autocovariance function of its VAR(1) representation by applying appropriately defined $(0,1)$ selection matrices, since

$$\Gamma_{Y,h} \;=\; \begin{bmatrix} \Gamma_{y,h} & \Gamma_{y,h+1} & \cdots & \Gamma_{y,h+p-1} \\ \Gamma_{y,h-1} & \Gamma_{y,h} & \cdots & \Gamma_{y,h+p} \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{y,h-p+1} & \Gamma_{y,h-p} & \cdots & \Gamma_{y,h} \end{bmatrix}, \qquad h = 0, \pm 1, \dots$$

with $\Gamma_{y,h} = \Gamma_{y,-h}'$.

Given the autocovariance function $\{\,\Gamma_{y,h} : h = 0, \pm 1, \dots\}$, the autocorrelation function, $\{\,R_{y,h} : h = 0, \pm 1, \dots\}$ with $R_{y,h} = R_{y,h}(\beta)$, is defined by

$$R_{y,h} \;=\; D^{-1}\Gamma_{y,h}\,D^{-1}, \qquad h = 0, \pm 1, \dots$$

where D is a diagonal matrix with its diagonal elements being the square roots of the diagonal elements of $\Gamma_{y,0}$.

Replacing the unknown parameter vector β with its QML estimate $\hat{\beta}_T$, we obtain the estimated autocovariance and autocorrelation functions $\{\,\hat{\Gamma}_{y,h} : h = 0, \pm 1, \dots\}$ and $\{\,\hat{R}_{y,h} : h = 0, \pm 1, \dots\}$ with $\hat{\Gamma}_{y,h} = \Gamma_{y,h}(\hat{\beta}_T)$ and $\hat{R}_{y,h} = R_{y,h}(\hat{\beta}_T)$, respectively.

2.3 The Asymptotic Distribution of the Estimated Autocovariance and Autocorrelation Functions

The asymptotic distribution of the estimated autocovariance and autocorrelation functions can be obtained by applying the δ-method. Specifically, under general regularity conditions (see Serfling (1980), Theorem 3.3.A) the following proposition is true.

Proposition: *Let $\{ y_t : t = -p+1, \ldots, T \}$ be generated by a stable VAR(p) process as represented by (1) and let $\hat{\beta}_T$ be the QML estimator of the VAR parameter vector β, which is assumed to be asymptotically normal according to (2); the estimators of the autocovariance and autocorrelation functions, $\{ \hat{\Gamma}_{y,h} : h = 0, \pm 1, \ldots \}$ and $\{ \hat{R}_{y,h} : h = 0, \pm 1, \ldots \}$, are then asymptotically normal:*

$$i. \qquad \sqrt{T} \left(\mathrm{vec}(\hat{\Gamma}_{y,h} - \Gamma_{y,h}(\beta_0)) \right) \xrightarrow{d} \mathrm{N} \left[0, \Sigma_{\mathrm{vec}(\hat{\Gamma}_{y,h})}(\beta_0) \right], \qquad h = 0, \pm 1, \ldots$$

where

$$\Sigma_{\mathrm{vec}(\hat{\Gamma}_{y,h})}(\beta_0) = \frac{\partial \mathrm{vec}(\Gamma_{y,h})}{\partial \beta'} \Sigma_{\hat{\beta}}(\beta_0) \frac{\partial \mathrm{vec}(\Gamma_{y,h})'}{\partial \beta},$$

and

$$ii. \qquad \sqrt{T} \left(\mathrm{vec}(\hat{R}_{y,h} - R_{y,h}(\beta_0)) \right) \xrightarrow{d} \mathrm{N} \left[0, \Sigma_{\mathrm{vec}(\hat{R}_{y,h})}(\beta_0) \right], \qquad h = 0, \pm 1, \ldots$$

where

$$\Sigma_{\mathrm{vec}(\hat{R}_{y,h})}(\beta_0) = \frac{\partial \mathrm{vec}(R_{y,h})}{\partial \beta'} \Sigma_{\hat{\beta}}(\beta_0) \frac{\partial \mathrm{vec}(R_{y,h})'}{\partial \beta},$$

with the partial derivates of the autocovariance and autocorrelation matrices being evaluated at the true parameter vector β_0.

Note that the elements on the principal diagonal of the autocorrelation matrix of order $h = 0$ are one per construction. Hence, their row vectors of partial derivatives are zero. In this case, the δ-method, which assumes among its regularity conditions that the rows of the matrices of partial derivatives are non-zero when evaluated at the true parameter vector β_0, would not be applicable. The obvious violation of the regularity conditions, however, could easily be dealt with by introducing appropriately defined $(0, 1)$ selection matrices when stating the asymptotic normality result above. To simplify notation this was omitted here. The distribution of the elements on the principal diagonal of the estimated autocorrelation matrix of order $h = 0$ must instead be considered degenerate, with their variances and covariances equal to zero.

The appendix of the paper provides closed-form formulae for computing the partial derivatives of the autocovariance and autocorrelation matrices with respect to the parameter vector β by applying matrix differential calculus. Using these closed-form formulae, the covariance matrices of the estimated autocovariance and autocorrelation matrices can be computed by replacing the unknown parameter vector β_0 with its QML estimate $\hat{\beta}_T$ and by using an appropriate estimate of the covariance matrix of the latter. The estimated asymptotic standard errors of the autocovariance and autocorrelation functions are the square roots of the elements on the principal diagonal of these matrices.

3 Monte Carlo Evidence

It is well known that the asymptotic normal approximation to the distribution of the estimated autocovariance and autocorrelation functions of VAR models may not perform very reliably in small samples. In this section, we therefore aim at presenting some Monte Carlo evidence on the small-sample performance of the confidence bands derived from the asymptotic standard errors provided above. Since bootstrap methods have gained increased popularity in applied research recently, we also present some evidence on the performance of bootstrap confidence bands but confine ourselves to standard bootstrap techniques.[3]

3.1 The Design of the Monte Carlo Experiment

In designing the Monte Carlo experiment we closely follow Kilian (1998) who extensively explores the performance of small-sample confidence bands for the estimated impulse response functions of VAR models. The data-generating process is the stable bivariate VAR(1) model

$$y_t = \begin{bmatrix} a_{11} & 0 \\ 0.5 & 0.5 \end{bmatrix} y_{t-1} + u_t, \qquad u_t \sim \text{IIN}\left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix}\right]$$

with the parameter $a_{11} \in \{0.5, 0.7, 0.9\}$ governing its persistence. The sample sizes considered are $T \in \{50, 100, 200\}$. For each simulated sample, 200 initial observations have been discarded to minimise the effect of the starting values which are set to zero. For each Monte Carlo design point $R = 2000$ replications have been carried out, and for each single replication 200 bootstrap samples have been drawn.

[3]See Li and Maddala (1996) for a survey of recent developments in bootstrap techniques and their application in time series models.

It is beyond the scope of our Monte Carlo experiment to provide a comprehensive assessment of the small-sample confidence bands for the entire estimated autocovariance and autocorrelation functions of the data-generating process. Instead, we restrict our investigation to an assessment of the small-sample confidence bands for its first order autocovariances. Specifically, let $\Gamma^{ij}_{y,1}$ denote the element in the ith row and the jth column of its first order autocovariance matrix $\Gamma_{y,1}$, and let $\hat{\Gamma}^{ij}_{y,1}$ denote the associated estimate computed under either the asymptotic or the bootstrap method. We then assess the small-sample performance of the asymptotic confidence bands compared with that of the bootstrap confidence bands by evaluating the size properties of testing

$$\mathrm{H}_0 : \hat{\Gamma}^{ij}_{y,1} = \Gamma^{ij}_{y,1} \qquad \text{against} \qquad \mathrm{H}_1 : \hat{\Gamma}^{ij}_{y,1} \neq \Gamma^{ij}_{y,1}, \qquad i, j = 1, 2$$

under both methods.

The decision whether the null hypothesis is rejected or not is based on the studentised test statistic $\mathcal{T} = (\hat{\Gamma}^{ij}_{y,1} - \Gamma^{ij}_{y,1})/\hat{\sigma}_{\hat{\Gamma}^{ij}_{y,1}}$ obtained using either the asymptotic or the bootstrap method, with $\hat{\sigma}_{\hat{\Gamma}^{ij}_{y,1}}$ denoting the estimated standard error of $\hat{\Gamma}^{ij}_{y,1}$. Under appropriate regularity conditions, both test statistics are asymptotically distributed as standard normal.

3.2 The Results of the Monte Carlo Experiment

The results of the Monte Carlo experiment are presented by means of the probability (P-) value plots suggested by Davidson and MacKinnon (1998). These plots are based on the empirical distribution function of the P-values associated with the simulated realisations τ_r ($r = 1, \ldots, R$) of the test statistic $\mathcal{T}$ and provide a simple graphical tool for evaluating the size properties of the above hypothesis tests for a continuous range of nominal sizes. The P-value associated with a fixed value τ_r is the probability $P_r = P(\{\tau : \tau \geq \tau_r > 0\} \cup \{\tau : \tau \leq \tau_r < 0\})$ of observing a value of $\mathcal{T}$ being as extreme or more extreme than τ_r, i.e. the probability of rejecting the null hypothesis for a critical value equal to τ_r.

Noting that the test statistic $\mathcal{T}$ is asymptotically distributed as standard normal under the null, the P-value associated with τ_r amounts to $P_r = 2\,(1 - F_{\mathrm{N}}(|\tau_r|; 0, 1))$ with $F_{\mathrm{N}}(\,\cdot\,; 0, 1)$ denoting the cumulative distribution function of the standard normal distribution. For a randomly varying τ_r it then follows from a probability integral transform that the probability $U = P_r$ has a uniform distribution on the unit interval $[\,0, 1\,]$ with the cumulative distribution function $F_{\mathrm{U}}(u; 0, 1) = u$. Therefore, when plotting the empirical distribution function of the simulated P-values point by point against $F_{\mathrm{U}}(u; 0, 1) = u$, the resulting graph should be close to the 45°-line if the test statistic $\mathcal{T}$ were well-behaved in small samples. Points above the 45°-line would indicate that the test's relative frequency of rejection is too high compared with the nominal size u; points below the line would reveal that the frequency of rejection is too low.

Figures 1 to 3 display the P-value plots of the four hypothesis tests under investigation for each of the Monte Carlo design points and for both the asymptotic and the bootstrap method. For convenience, the P-value plots are truncated at $u = 0.4$. The four hypothesis tests behave quite similarly under the two methods. For both methods, however, the tests over-reject the null hypotheses for nominal sizes being standard for hypothesis testing. For $a_{11} = 0$ and a sample size of $T = 50$, for instance, the frequency of rejection of the null hypothesis associated with $\Gamma_{y,1}^{11}$ is 0.101 (0.053, 0.129) under the asymptotic method and 0.081 (0.048, 0.111) under the bootstrap method, compared with a nominal size of 0.05 (0.01, 0.10). By contrast, the tests under-reject for large nominal sizes under both methods.

As expected, the size properties of the tests improve with the sample size under both methods. Also as expected, the size properties under both methods deteriorate for data-generating processes with a higher degree of persistence. This latter finding reflects that a rise in the persistence of the data-generating process increases the bias and the skewness of the small-sample distribution of the estimated autocovariances. The bias and the skewness of the small-sample distribution, in turn, adversely affect the performance of the asymptotic confidence bands the construction of which is based on the assumption of a symmetric distributional shape. Somewhat surprisingly, the non-parametric bootstrap confidence bands are found to be distorted to almost the same extent as the asymptotic bands.[4]

Overall, a comparison of the results under the asymptotic method with those under the bootstrap method shows that for large nominal sizes the distortions under the bootstrap method are even more severe than those under the asymptotic method. For small nominal sizes there is no clear advantage to using the bootstrap method for samples of the size $T = 100$ or $T = 200$, whereas the bootstrap method obviously outperforms the asymptotic method for a sample size of $T = 50$. In view of these results, the use of the asymptotic confidence bands for the estimated autocovariance and autocorrelation functions seems very much justified. Beyond that, it also saves the practitioner the computational costs of the bootstrap.

[4] It is recognised, although beyond the scope of the present paper, that the use of the bootstrap-after-bootstrap technique proposed by Kilian (1998) would improve on the performance of the bootstrap confidence bands. This two-step bootstrap technique accounts for the bias of the small-sample distribution indirectly by bias-correcting the estimated parameters of the VAR model before bootstrapping the confidence bands.

4 Empirical Applications

In this section we illustrate the usefulness of the proposed asymptotic method for applied work by constructing asymptotic confidence bands for the autocovariance and autocorrelation functions estimated from two euro area data sets. The first dataset comprises inflation and output data, and the second data on the yield spread and the short-term real interest rate.

4.1 Inflation and Output

In a widely quoted paper, Fuhrer and Moore (1995) investigated the dynamic characteristics of the inflation and output gap processes for the US economy by means of the estimated autocorrelation function of a VAR model. They pursued two objectives. First, they used the autocorrelation function as a descriptive device to investigate the lead-lag relationship between inflation and the output gap which traditionally underlies structural modelling of the short-run Phillips curve trade-off. Second, in the spirit of McCallum (1999), they used the estimated autocorrelation function as a benchmark against which the capacity of alternative structural models to explain the inflation persistence in the US data was evaluated.

In this application, we focus on the first of the two objectives and explore the inflation and output gap dynamics for the euro area, whereas the second objective is pursued in Coenen and Wieland (1999). Specifically, we estimate the autocovariance and autocorrelation functions of a VAR model fitted to quarterly data on the annualised quarterly change in the log of the euro area GDP deflator, π, and the log of euro area real GDP, q. The time series span the period from the first quarter of 1974 to the fourth quarter of 1998. The graphs of the series are depicted in Figure 4.

In fitting the VAR model we allow for deterministic components in the data. Specifically, we assume that the data $\{\, y_t^* : t = -p+1, \ldots, T \,\}$ are a sample of the 2-dimensional vector of variables $y^* = [\,\pi,\, q\,]'$, being generated by the linear model

$$y_t^* \;=\; \alpha_0 + \alpha_1\, t + y_t, \qquad t = 0, \pm 1, \ldots \tag{3}$$

with $\{\, y_t : t = 0, \pm 1, \ldots \,\}$ following a VAR(p) process as represented by equation (1) above.

This general linear model was advocated by Toda and Yamamoto (1995) for conducting statistical inference in vector autoregressions with possibly integrated processes without pre-testing for unit roots or cointegration.[5]

We proceed in two steps. First, we detrend the data using a projection technique to account for the downward trend in the inflation rate within our sample and to

[5]Substituting (3) into (1), it becomes obvious that $\{\, y_t^* \,\}$ is assumed to follow a VAR(p) process

obtain a simple measure of the output gap.[6] Second, using the detrended data $\{ y_t : t = -p+1, \ldots, T \}$, we estimate the parameters of the VAR(p) model, i.e. the coefficient matrices $A_1, \ldots, A_p$ and the covariance matrix Σ_u employing QML methods.

We chose a lag order of 2, using a standard lag selection procedure based on the HQ and SC criteria. The Ljung-Box Q(12) statistic indicates serially uncorrelated residuals with a probability value of 42.8%. The QML estimates of the parameters of the VAR(2) model are reported in Table 1. The point estimates imply that the smallest root of the characteristic equation $\det(I_2 - A_1 z - A_2 z^2) = 0$ is 1.2835, thereby suggesting that the deviations of inflation from trend and the output gap are stationary and, hence, that the autocovariance and autocorrelation functions are well-defined.[7]

Figures 5 and 6 show the point estimates (solid line) and the estimated 95%-confidence bands (dotted lines) for the autocovariance and autocorrelation functions of the VAR(2) model for inflation as a deviation from trend and the output gap. The diagonal panels pertain to the autocovariances and autocorrelations of the detrended inflation rate and the output gap, the off-diagonal panels to the lagged cross covariances and cross correlations. The autocovariances and autocorrelations are indicative of a rather high degree of persistence in both the inflation and output gap processes. The cross correlations in the upper right-hand panel show that the output gap leads the inflation rate by about four quarters, thereby suggesting the existence of a short-run Phillips curve trade-off. This trade-off proves to be significant, as revealed by the estimated confidence bands. By contrast, the lower left-hand panel displays that the lagged inflation rate is negatively, albeit not significantly correlated with the output gap.

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around a deterministic linear trend,

$$y^*_t - \alpha_0 - \alpha_1 t \quad = \quad A_1 (y^*_{t-1} - \alpha_0 - \alpha_1 (t-1)) + \cdots + A_p (y^*_{t-p} - \alpha_0 - \alpha_1 (t-p)) + u_t$$

which, in turn, can be rewritten as

$$y^*_t \quad = \quad \tilde{\alpha}_0 + \tilde{\alpha}_1 t + A_1 y^*_{t-1} + \cdots + A_p y^*_{t-p} + u_t$$

with $\tilde{\alpha}_0 = A(1) \alpha_0 - A'(1) \alpha_1$ and $\tilde{\alpha}_1 = A(1) \alpha_1$, where $A(z) = I_k - A_1 z - \cdots - A_p z^p$. If each series of $\{ y_t \}$ were integrated, with none of the individual series being cointegrated with any of the others, then $A(1) = 0$ and, hence, $\tilde{\alpha}_1 = 0$. This could also occur if $\{ y_t \}$ were cointegrated since then $A(1)$ would be of reduced rank (see Toda and Yamamoto (1995), p. 228).

[6]Let $X^* = [x^*_1, \ldots, x^*_T]'$ with $x^*_t = [y^{*\prime}_t, y^{*\prime}_{t-1}, \ldots, y^{*\prime}_{t-p}]'$ and $\mathcal{T} = [\tau_1, \ldots, \tau_T]'$ with $\tau_t = [1, t]'$, then $M X^* = X$, where $M = I_T - \mathcal{T} (\mathcal{T}' \mathcal{T})^{-1} \mathcal{T}'$ is a $(T \times T)$-dimensional projection matrix and $X = [x_1, \ldots, x_T]'$ with $x_t = [y'_t, y'_{t-1}, \ldots, y'_{t-p}]'$.

[7]Our findings are also supported by the results of univariate augmented Dickey-Fuller tests for unit roots in the detrended series. The values of the t-statistics for inflation in deviation from trend and the output gap are -3.93 and -2.64, which are significant at the 5% and 10% levels, respectively.

4.2 Spread and Short-Term Real Interest Rate

In this application, we explore the dynamic interaction of the spread between the long-term and the short-term nominal interest rates and the ex post short-term real interest rate for the euro area. We start by fitting a VAR model to quarterly data on the spread between the euro area long-term government bond yield and the euro area three-month money market rate, $s = i^l - i^s$, and the differential of the euro area three-month money market rate and the annualised quarterly change in the log of the euro area GDP deflator, $r = i^s - \pi$. The two time series range from the first quarter of 1980 to the fourth quarter of 1998. Their accompanying graphs are shown in Figure 7.

Again, we use the linear model (3), (1), but restrict the parameter α_1 to zero and, thus, exclude linear trends from the spread and the real interest rate data. The HQ and SC criteria suggest a lag order of 2, with the probability value of the Ljung-Box Q(12) statistic amounting to 31.9%. The QML estimates of the VAR(2) model are reported in Table 2. The minimum root of the characteristic equation is 1.267, so we treat the spread and the real interest rate as stationary, with well-defined autocovariance and autocorrelation functions.[8]

Figures 8 and 9 display the point estimates (solid line) and the estimated 95%-confidence bands (dotted lines) for the autocovariance and autocorrelation functions of the VAR(2) model for the spread and the short-term real interest rate. The diagonal panels reveal a rather high degree of persistence in both the spread and the real interest rate. The off-diagonal panels indicate that the spread and the short-term real rate are negatively and significantly correlated to lags of around five quarters. Hence, the yield curve flattens following an increase in the short-term real rate. Interestingly, significance is detected only by means of the cross autocorrelations, i.e. after correcting the cross autocovariances for their estimated scale.

Overall our findings are consistent with the expectation theory of the term structure. Assuming a monetary contraction, for instance, emanating from a temporary increase in the short-term nominal interest rate, the short-term real interest rate will rise almost to the same amount (given the sluggishness of inflation), whereas the increase in the short-term nominal rate will feed into the long-term nominal interest rate by less. Of course, to investigate the term structure more rigourosly would require a structural approach which, however, is beyond the scope of the present example, which merely aims at providing some stylised facts.

[8] See Coenen and Vega (1999) for empirical evidence that the spread and the real rate for the euro area constitute cointegrating relationships, i.e. that they are stationary.

5 Concluding Remarks

In this paper we have shown how to derive asymptotic confidence bands for the estimated autocovariance and autocorrelation functions of stable VAR models. We argue that plotting the point estimates of the autocovariance and autocorrelation functions together with their asymptotic confidence bands provides a useful tool for assessing the estimation uncertainty involved. The usefulness of these asymptotic confidence bands for applied work has been demonstrated by two illustrative examples. An application to inflation and output data for the euro area indicated that there is a significant short-run Phillips curve trade-off. This finding constitutes a first but important explorative step in investigating the Phillips curve trade-off, which is built upon in Coenen and Wieland (1999). An application to interest rate data for the euro area revealed that, in line with the expectation theory of the term structure, the yield curve flattens significantly following an increase in the short-term real interest rate which, in turn, may be the outcome of a tightening of monetary conditions.

By means of a Monte Carlo experiment we have provided evidence that the asymptotic confidence bands perform quite well in small samples when compared with bootstrap confidence bands obtained using standard techniques. However, it has also been recognised that the use of more efficient bootstrap techniques, such as the bias-corrected bootstrap method proposed by Kilian (1998) for instance, may improve on the relative performance of the latter. Notwithstanding this possible improvement, which is considered an interesting topic for future research, the use of the asymptotic confidence bands seems very much justified by the results of this paper, not at least because it is very easy to implement and saves the practitioner the computational costs of the bootstrap.

Appendix: The Partial Derivatives of the Autocovariance and Autocorrelation Functions

In order to derive the partial derivatives of the autocovariance matrices in part *i.* of the proposition stated in Section 2, note first that by repeatedly applying rules (5) and (7) in Lütkepohl (1996), Chapter 7.2,

$$\begin{aligned}
\mathrm{vec}(\Gamma_{y,0}, \Gamma_{y,1}, \ldots, \Gamma_{y,p-1}) &= \mathrm{vec}(S_1\, \Gamma_{Y,0}) \\
&= (I_{kp} \otimes S_1)\, \mathrm{vec}(\Gamma_{Y,0}), \qquad \text{(A.1)}
\end{aligned}$$

$$\begin{aligned}
\mathrm{vec}(\Gamma_{y,h}) &= \mathrm{vec}(S_1\, \Gamma_{Y,h-p+1}\, S_2) \\
&= (S_2' \otimes S_1)\, \mathrm{vec}(\Gamma_{Y,h-p+1}), \qquad h = p, p+1, \ldots \qquad \text{(A.2)}
\end{aligned}$$

$$\begin{aligned}
\mathrm{vec}(\Gamma_{Y,0}) &= \Big(I_{(kp)^2} - A \otimes A \Big)^{-1} \mathrm{vec}(\Sigma_U) \\
&= \Big(\mathrm{vec}(\Sigma_U)' \otimes I_{(kp)^2} \Big)\, \mathrm{vec}\Big(\Big(I_{(kp)^2} - A \otimes A \Big)^{-1} \Big) \qquad \text{(A.3)}
\end{aligned}$$

and

$$\begin{aligned}
\mathrm{vec}(\Gamma_{Y,h+1}) &= \mathrm{vec}(A\, \Gamma_{Y,h}) \\
&= (I_{kp} \otimes A)\, \mathrm{vec}(\Gamma_{Y,h}) \\
&= (\Gamma_{Y,h}' \otimes I_{kp})\, \mathrm{vec}(A), \qquad h = 0, 1, \ldots \qquad \text{(A.4)}
\end{aligned}$$

where

$$S_1 = \left[\begin{array}{cccc} I_k & 0_{k,k} & \cdots & 0_{k,k} \end{array} \right], \qquad S_2 = \left[\begin{array}{cccc} 0_{k,k} & \cdots & 0_{k,k} & I_k \end{array} \right]'.$$

Then, starting from the identities (A.1) and (A.3), straightforward application of the chain rule and the product rule of matrix differentiation, hereby using the rule for differentiating the inverse of a matrix (see Lütkepohl (1996), Chapter 10.6, rule (1)) and the rule for differentiating the Kronecker product of two matrices (see Lütkepohl (1996), Chapter 10.5.2, rule (1.b)), the partial derivatives of the autocovariance matrices of order $h = 0, 1, \ldots, p-1$ are given by

$$\frac{\partial \mathrm{vec}(\Gamma_{y,0}, \Gamma_{y,1}, \ldots, \Gamma_{y,p-1})}{\partial \beta'} = (I_{kp} \otimes S_1)\, \frac{\partial \mathrm{vec}(\Gamma_{Y,0})}{\partial \beta'},$$

where

$$\begin{aligned}
\frac{\partial \mathrm{vec}(\Gamma_{Y,0})}{\partial \beta'} &= \Big(\mathrm{vec}(\Sigma_U)' \otimes I_{(kp)^2} \Big) \frac{\partial \mathrm{vec}\Big(\Big(I_{(kp)^2} - A \otimes A \Big)^{-1} \Big)}{\partial \mathrm{vec}\Big(I_{(kp)^2} - A \otimes A \Big)'} \\
&\quad \times \frac{\partial \mathrm{vec}\Big(I_{(kp)^2} - A \otimes A \Big)}{\partial \mathrm{vec}(A \otimes A)'}\, \frac{\partial \mathrm{vec}(A \otimes A)}{\partial \mathrm{vec}(A)'}\, \frac{\partial \mathrm{vec}(A)}{\partial \beta'} \\
&\quad + \Big(I_{(kp)^2} - A \otimes A \Big)^{-1} \frac{\partial \mathrm{vec}(\Sigma_U)}{\partial \beta'}
\end{aligned}$$

with

$$\frac{\partial \text{vec}\Big(\big(I_{(kp)^2} - A \otimes A\big)^{-1}\Big)}{\partial \text{vec}\big(I_{(kp)^2} - A \otimes A\big)'} = -\Big(I_{(kp)^2} - A \otimes A\Big)^{\prime -1} \otimes \Big(I_{(kp)^2} - A \otimes A\Big)^{-1},$$

$$\frac{\partial \text{vec}\big(I_{(kp)^2} - A \otimes A\big)}{\partial \text{vec}(A \otimes A)'} = -I_{(kp)^4}$$

and

$$\frac{\partial \text{vec}(A \otimes A)}{\partial \text{vec}(A)'} = (I_{kp} \otimes K_{kp,kp} \otimes I_{kp})\Big[\Big(I_{(kp)^2} \otimes \text{vec}(A)\Big) + \Big(\text{vec}(A) \otimes I_{(kp)^2}\Big)\Big],$$

where $K_{kp,kp}$ denotes the $((kp)^2 \times (kp)^2)$-dimensional commutation matrix defined such that $\text{vec}(A) = K_{kp,kp}\,\text{vec}(A')$ (see Lütkepohl (1996), Chapter 1.5).

Finally, when taking into account the particular structure of the VAR(1) coefficient matrix A,

$$\frac{\partial \text{vec}(A)}{\partial \beta'} = \Big[\ (I_{pk} \otimes \big[\ 1\ 0_{1,p-1}\ \big]' \otimes I_k) \quad 0_{(pk)^2,k(k+1)/2}\ \Big]$$

and, using the identity

$$\text{vec}(\Sigma_U) = \text{vec}\left(\begin{bmatrix} 1 & 0_{1,p-1} \\ 0_{p-1,1} & 0_{p-1,p-1} \end{bmatrix} \otimes \Sigma_u\right)$$

and again the rule for differentiating the Kronecker product of two matrices,

$$\frac{\partial \text{vec}(\Sigma_U)}{\partial \beta'} = \Bigg[\ 0_{(pk)^2,(pk)^2} \quad (I_k \otimes K_{k,k} \otimes I_k) \\ \times \text{vec}\left(\begin{bmatrix} 1 & 0_{1,p-1} \\ 0_{p-1,1} & 0_{p-1,p-1} \end{bmatrix}\right) \otimes \text{vec}(D_k)\Bigg],$$

where D_k denotes the $(k^2 \times k(k+1)/2)$-dimensional duplication matrix defined such that $\text{vec}(\Sigma_u) = D_k\,\text{vech}(\Sigma_u)$ (see Lütkepohl (1996), Chapter 1.5).

Using the identities (A.2) and (A.4) and applying the product rule of matrix differentiation, the partial derivatives of the autocovariance matrices of order $h = p, p-1, \ldots$ are given by

$$\frac{\partial \text{vec}(\Gamma_{y,h})}{\partial \beta'} = (S_2' \otimes S_1)\,\frac{\partial \text{vec}(\Gamma_{Y,h-p+1})}{\partial \beta'},$$

where $\partial \text{vec}(\Gamma_{Y,h-p+1})/\partial \beta'$ is obtained by the simple recursion

$$\frac{\partial \text{vec}(\Gamma_{Y,h-p+1})}{\partial \beta'} = (I_{kp} \otimes A)\,\frac{\partial \text{vec}(\Gamma_{Y,h-p})}{\partial \beta'} + \Big(\Gamma_{Y,h-p}' \otimes I_{kp}\Big)\,\frac{\partial \text{vec}(A)}{\partial \beta'}$$

which is initialised with $\mathrm{vec}(\Gamma_{Y,0})$ and $\partial\mathrm{vec}(\Gamma_{Y,0})/\partial\beta'$.

In order to derive the partial derivatives of the autocorrelation matrices in part *ii.* of the proposition, note that by again applying rules (5) and (7) in Lütkepohl (1996), Chapter 7.2,

$$\begin{aligned}
\mathrm{vec}(R_{y,h}) &= \mathrm{vec}(D^{-1}\Gamma_{y,h}\, D^{-1}) \\
&= \left(D^{-1} \otimes D^{-1}\right) \mathrm{vec}(\Gamma_{y,h}) \\
&= \left(I_k \otimes D^{-1}\Gamma_{y,h}\right) \mathrm{vec}(D^{-1}).
\end{aligned} \tag{A.5}$$

Using the identity (A.5) and applying the chain and product rules of matrix differentiation and the rule for differentiating the inverse of a matrix, the partial derivatives of the autocorrelation matrices of order $h = 0, 1, \ldots$ are given by

$$\frac{\partial\mathrm{vec}(R_{y,h})}{\partial\beta'} = \frac{\partial\mathrm{vec}(R_{y,h})}{\partial\mathrm{vec}(\Gamma_{y,h})'}\,\frac{\partial\mathrm{vec}(\Gamma_{y,h})}{\partial\beta'},$$

where $\partial\mathrm{vec}(\Gamma_{y,h})/\partial\beta'$ is derived above and where

$$\begin{aligned}
\frac{\partial\mathrm{vec}(R_{y,h})}{\partial\mathrm{vec}(\Gamma_{y,h})'} &= \left(D^{-1} \otimes D^{-1}\right) I_{k^2} \\
&\qquad + 2\left(I_k \otimes D^{-1}\Gamma_{y,h}\right) \frac{\partial\mathrm{vec}(D^{-1})}{\partial\mathrm{vec}(D)'}\,\frac{\partial\mathrm{vec}(D)}{\partial\mathrm{vec}(\Gamma_{y,h})'}
\end{aligned}$$

with

$$\frac{\partial\mathrm{vec}(D^{-1})}{\partial\mathrm{vec}(D)'} = -\,D^{-1} \otimes D^{-1}$$

and

$$\frac{\partial\mathrm{vec}(D)}{\partial\mathrm{vec}(\Gamma_{y,h})'} = \begin{cases} 0.5\,\mathrm{diag}\left(\mathrm{vec}(D^{-1}) \otimes \iota_{k^2}'\right), & h = 0 \\ 0_{k^2,k^2}, & h = 1, 2, \ldots \end{cases}$$

with ι_{k^2} denoting a k^2-dimensional column vector of ones and exploiting the structure of the diagonal matrix D and the definition of the elements on its diagonal.

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Table 1: QML Estimates of the Parameters of the VAR(2) Model for $y^* = [\,\pi,\, q\,]'$

A_1		A_2		$\Sigma_u \times 10^4$	
0.4879 (0.0963)	0.3890 (0.1709)	0.0989 (0.0899)	-0.2190 (0.1688)	0.9871 (0.1518)	
0.0481 (0.0571)	1.1236 (0.0928)	-0.2159 (0.0366)	-0.1605 (0.1094)	-0.0686 (0.0528)	0.2736 (0.0584)

Note: Estimates of the asymptotic standard errors are given in parentheses, with the asymptotic information matrix being estimated by the Newey-West (1987) estimator with the lag truncation parameter set equal to 3.

Table 2: QML Estimates of the Parameters of the VAR(2) Model for $y^* = [\, s,\, r\,]'$

A_1		A_2		$\Sigma_u \times 10^4$	
1.1601	-0.0617	-0.3359	0.0170	0.1681	
(0.1241)	(0.0457)	(0.1282)	(0.0340)	(0.0292)	
-0.2605	0.5295	-0.0652	0.0774	-0.1160	0.8971
(0.2547)	(0.1105)	(0.1880)	(0.0953)	(0.0560)	(0.1498)

Note: Estimates of the asymptotic standard errors are given in parentheses, with the asymptotic information matrix being estimated by the Newey-West (1987) estimator with the lag truncation parameter set equal to 3.

Figure 1: P-Value Plots for Testing $\mathrm{H}_0 : \hat{\Gamma}^{ij}_{y,1} = \Gamma^{ij}_{y,1}$ with $a_{11} = 0.5$

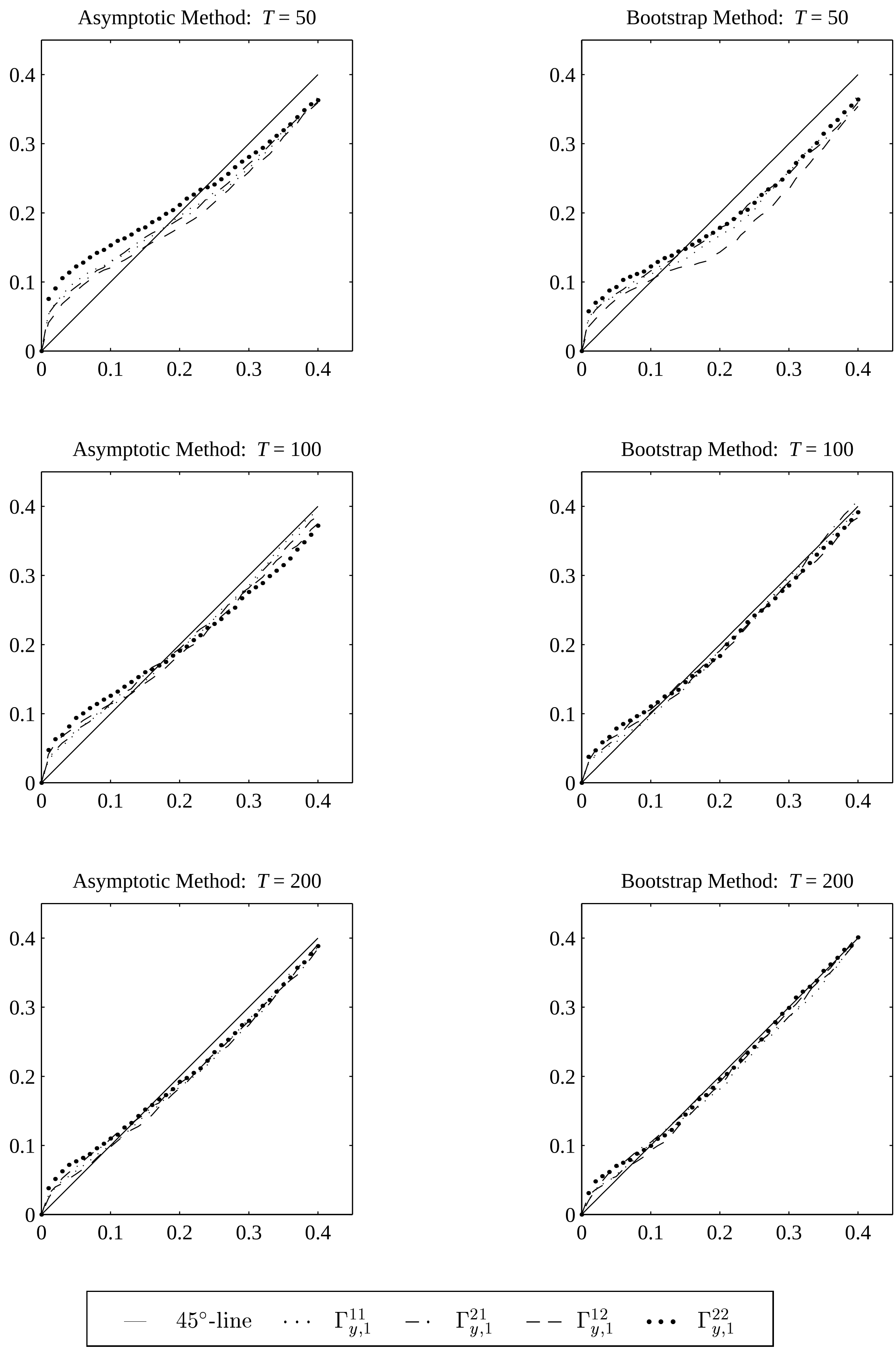


Figure 2: P-Value Plots for Testing $\mathrm{H}_0 : \hat{\Gamma}^{ij}_{y,1} = \Gamma^{ij}_{y,1}$ with $a_{11} = 0.7$

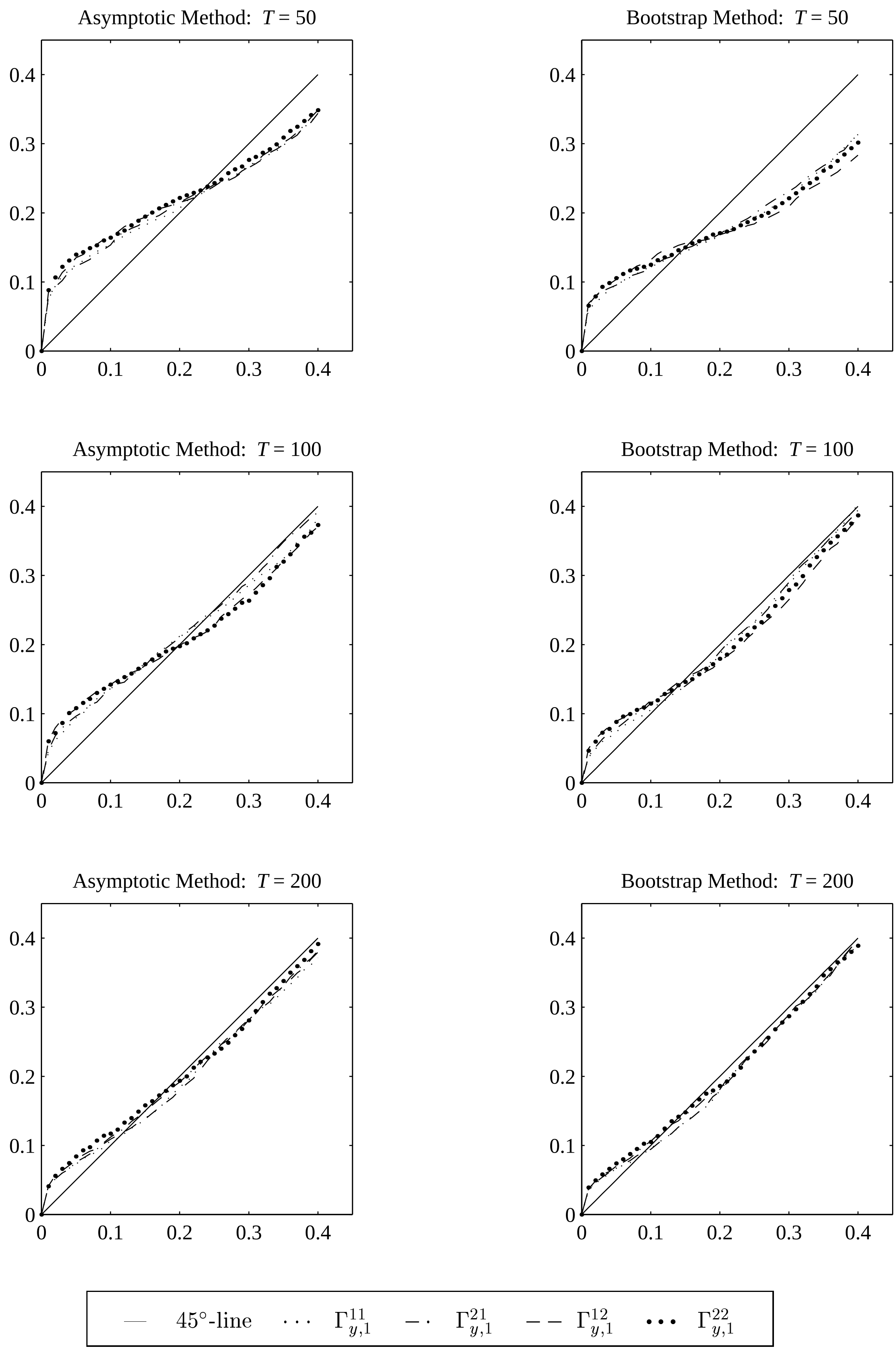


Figure 3: P-Value Plots for Testing $\mathrm{H}_0 : \hat{\Gamma}^{ij}_{y,1} = \Gamma^{ij}_{y,1}$ with $a_{11} = 0.9$

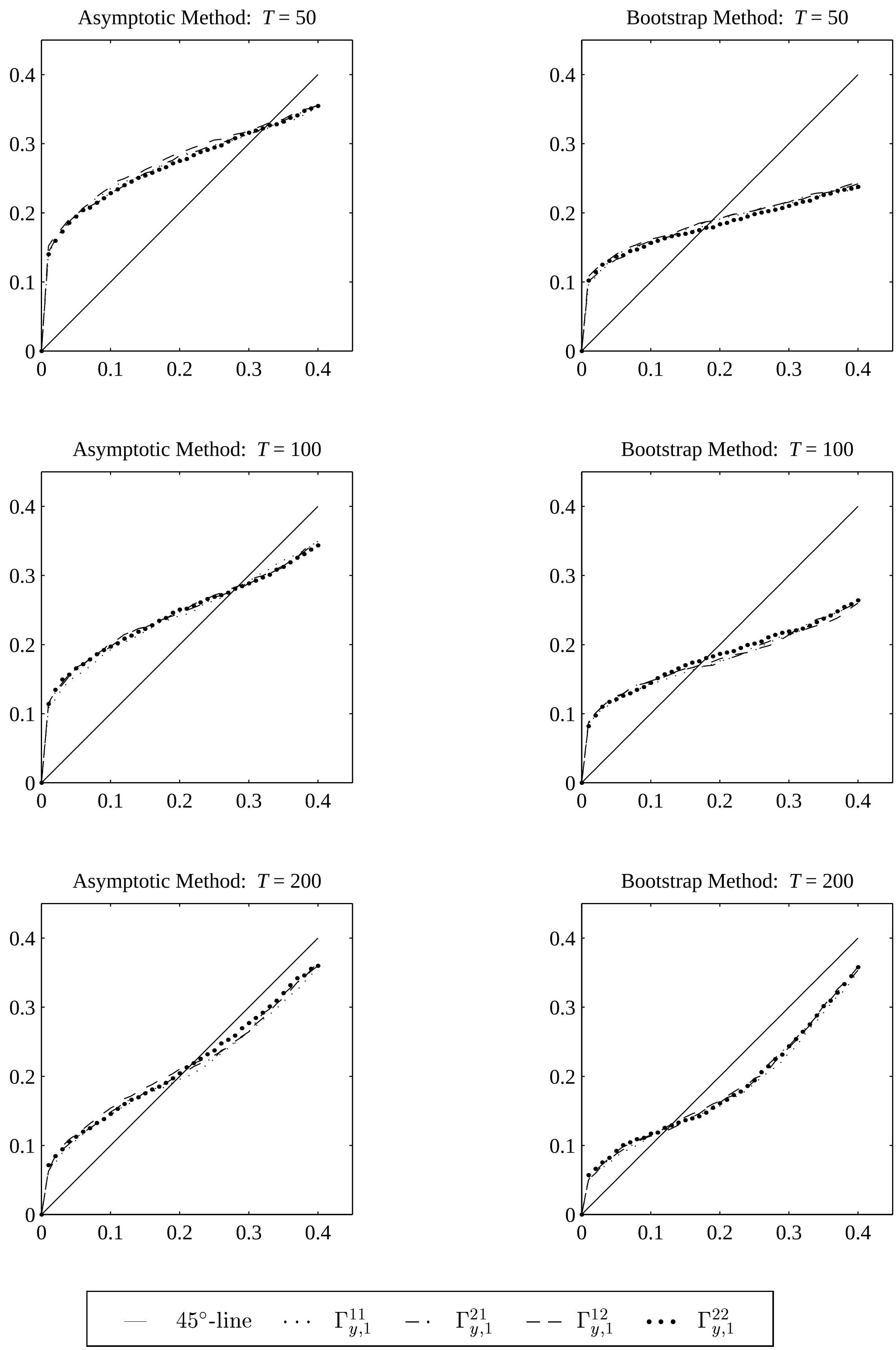
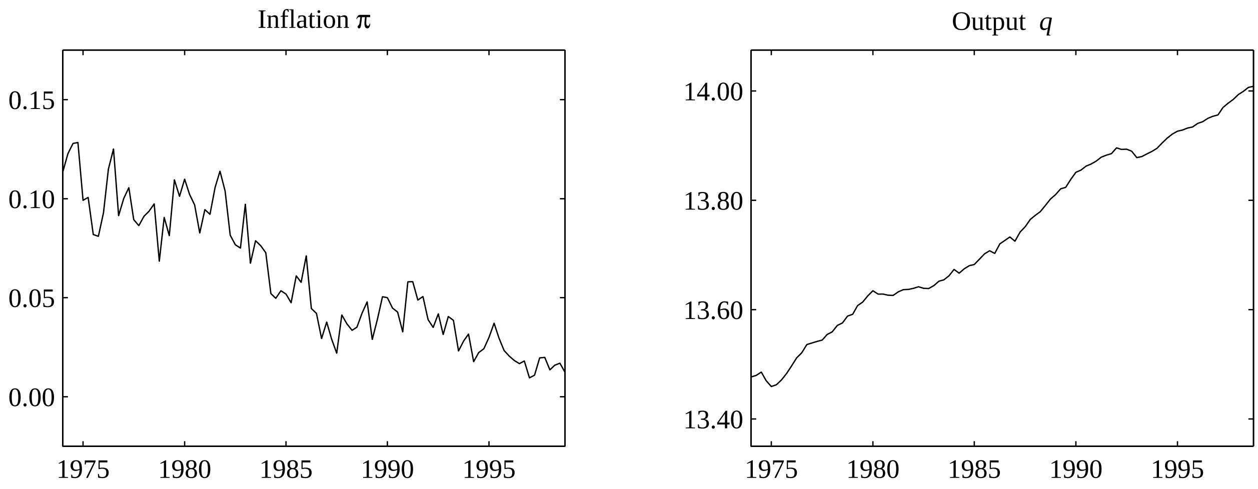
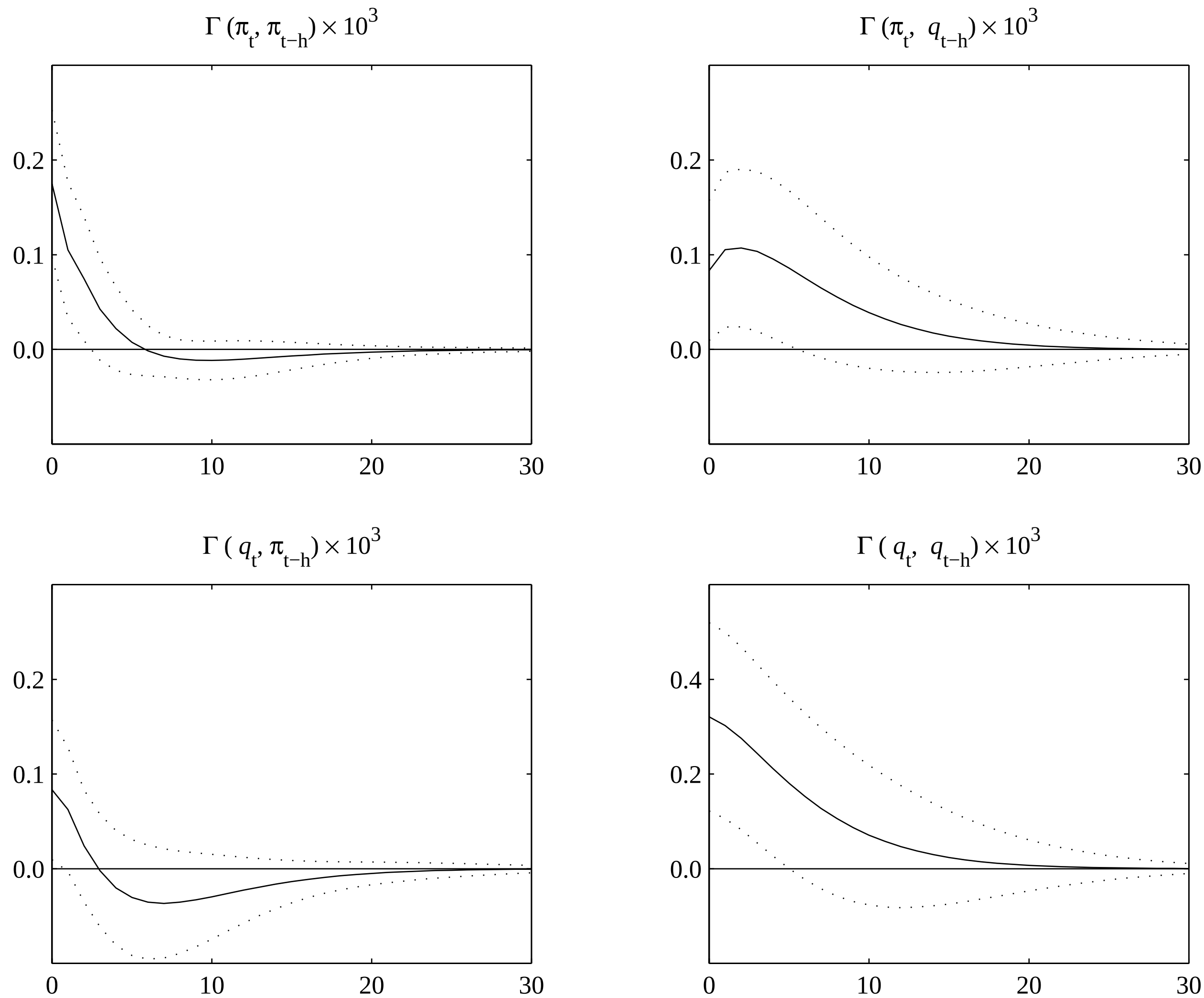


Figure 4: Graphs of the Time Series for $y^{*} = [\,\pi,\, q\,]'$



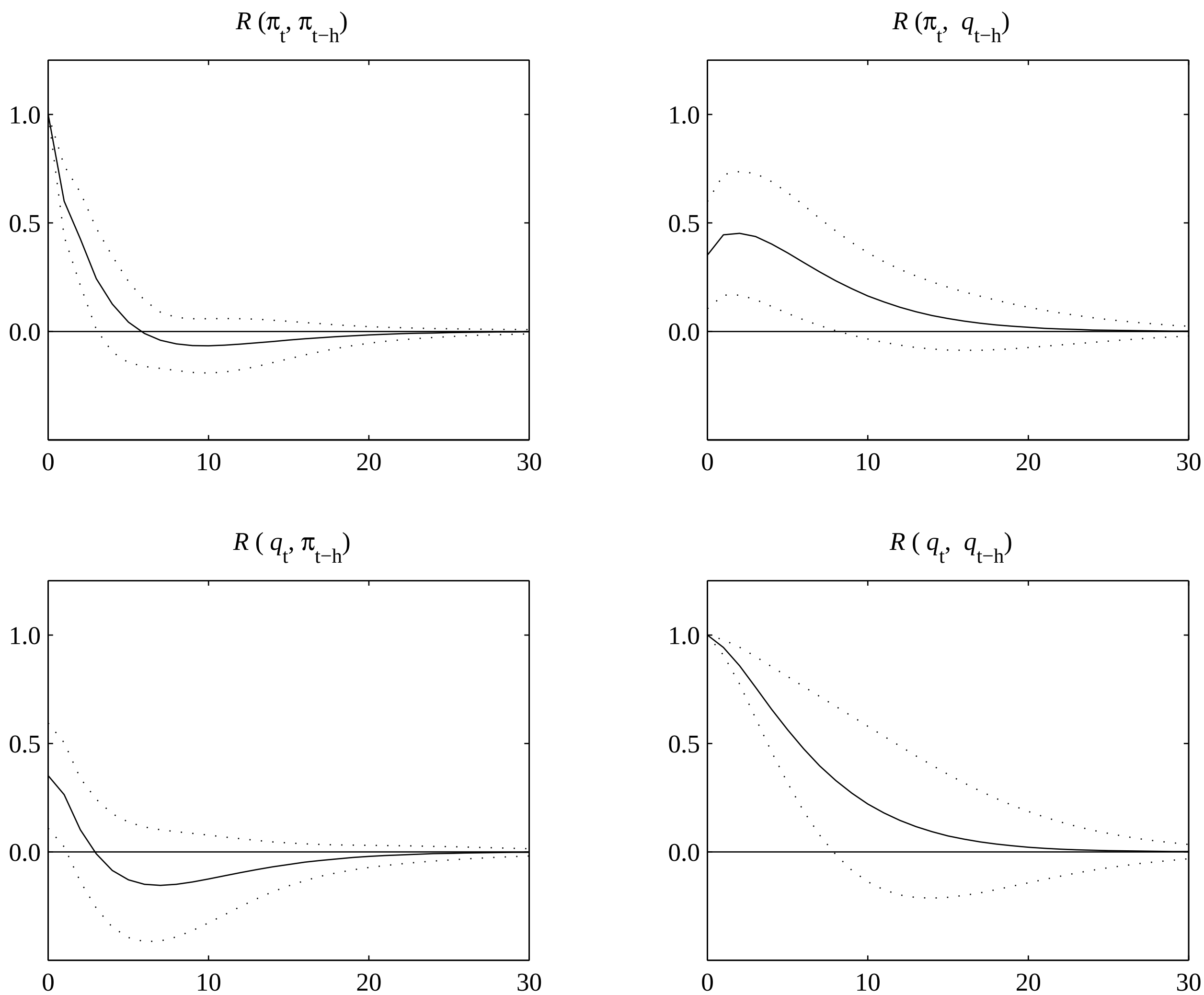
Source: ECB area-wide model database (see Fagan et al. (1999)). Aggregation of data for the countries of the euro area using fixed 1995 GDP weights at PPP rates.

Figure 5: Estimated Autocovariances of the VAR(2) Model for $y^* = [\,\pi,\, q\,]'$



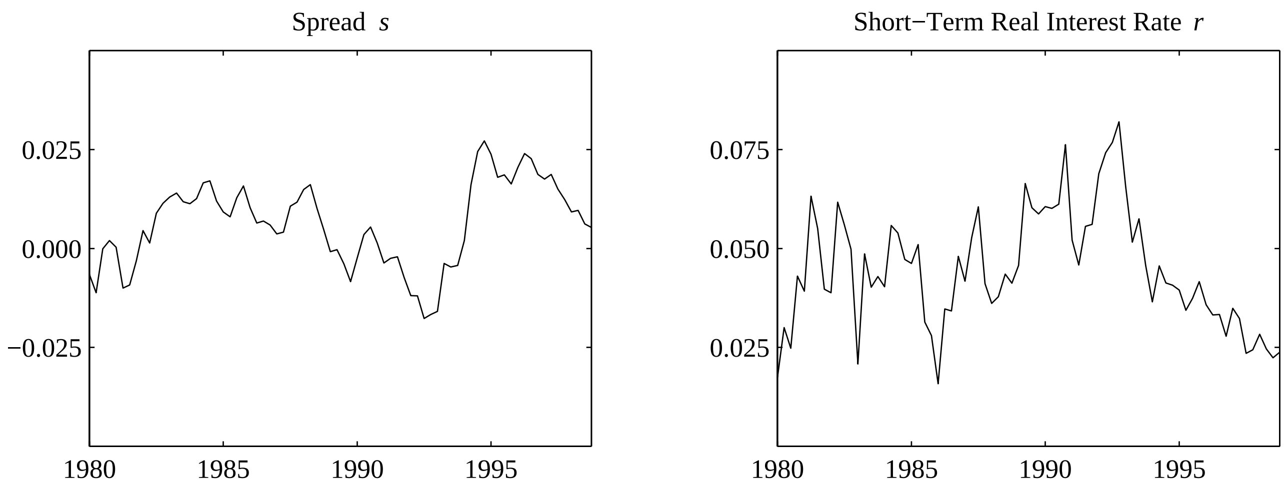
Note: Solid line: estimated autocovariances. Dotted lines: estimated autocovariances plus/minus twice their estimated asymptotic standard errors.

Figure 6: Estimated Autocorrelations of the VAR(2) Model for $y^* = [\,\pi,\, q\,]'$



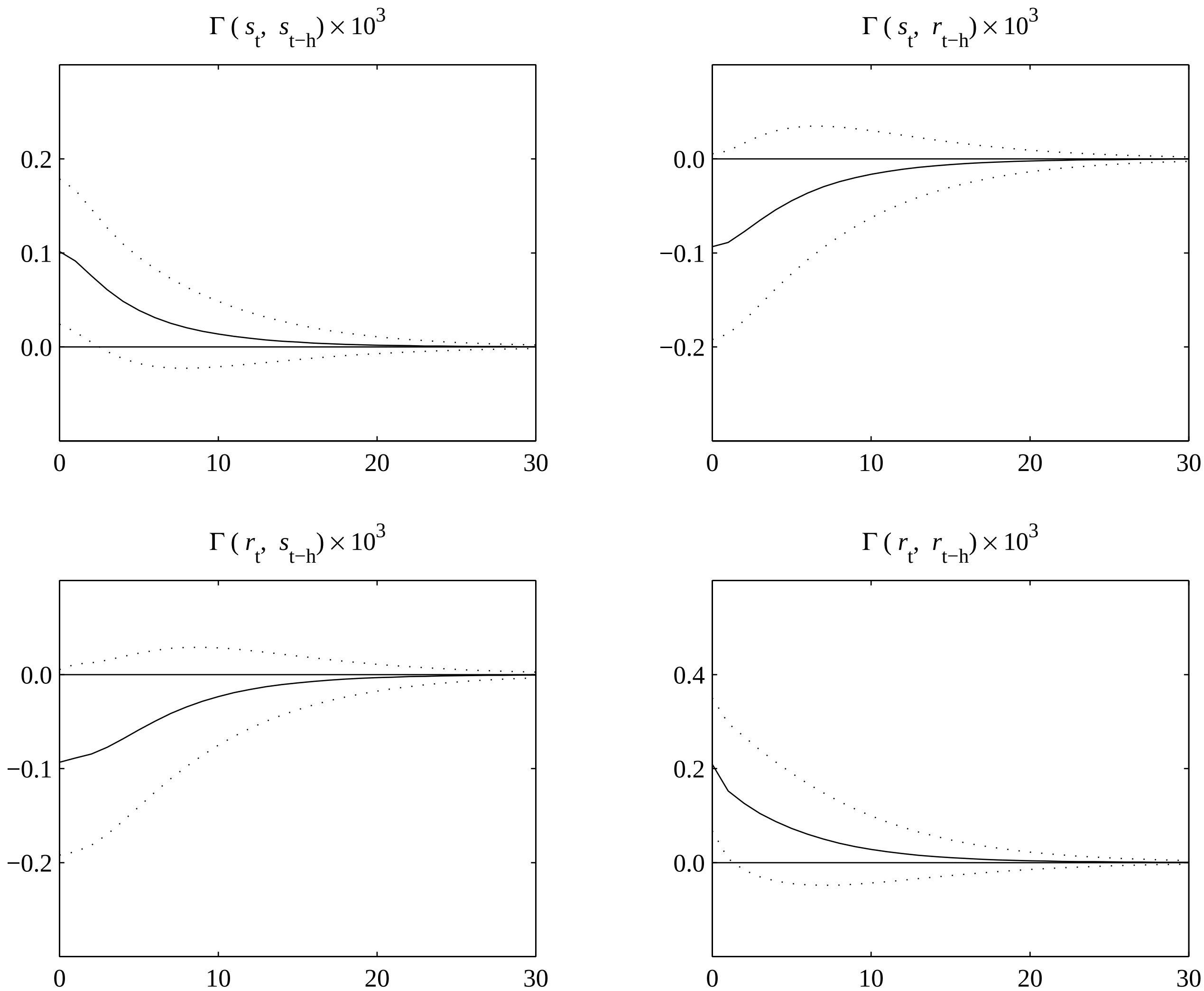
Note: Solid line: estimated autocorrelations. Dotted lines: estimated autocorrelations plus/minus twice their estimated asymptotic standard errors.

Figure 7: Graphs of the Time Series for $y^* = [\, s,\, r\,]'$



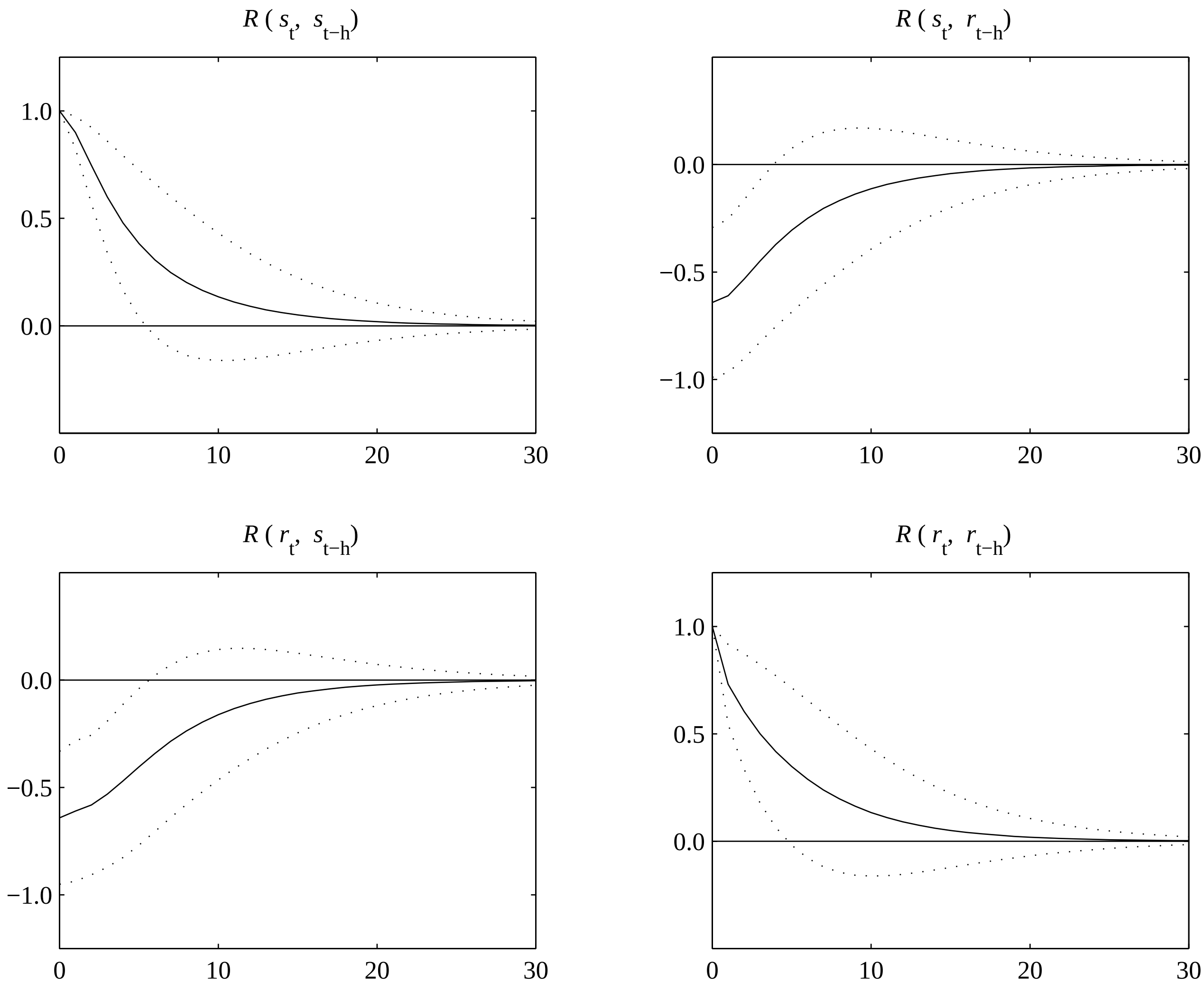
Source: ECB area-wide model database (see Fagan et al. (1999)). Aggregation of data for the countries of the euro area using fixed 1995 GDP weights at PPP rates.

Figure 8: Estimated Autocovariances of the VAR(2) Model for $y^* = [\,s,\, r\,]'$



Note: Solid line: estimated autocovariances. Dotted lines: estimated autocovariances plus/minus twice their estimated asymptotic standard errors.

Figure 9: Estimated Autocorrelations of the VAR(2) Model for $y^* = [\, s,\, r\,]'$



Note: Solid line: estimated autocorrelations. Dotted lines: estimated autocorrelations plus/minus twice their estimated asymptotic standard errors.

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