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DOES MONEY MATTER IN THE IS CURVE?

THE CASE OF THE UK

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Abstract

Narrow and broad money measures (including Divisia aggregates) have been found to have explanatory power for UK output in backward-looking specifications of the IS curve. In this paper, we explore whether or not real balances enter into a forward-looking IS curve for the UK, building on the theoretical framework of Ireland (2004). To do this, we test for additive separability between consumption and money over a sizeable part of the post-ERM period using non-parametric methods. If consumption and money are not additively separable, then real money balances enter into the forward-looking IS curve (the converse does not hold, however). A main finding is that the UK data seem to be broadly consistent with additive separability for the the more recent period from 1999 to 2007.

Keywords: Additive Separability, IS Curve, Non-Parametric Tests, Measurement Error, Divisia Monetary Aggregates.

JEL Codes: C14, C43, C63, E21, E41.

Non-technical summary

Monetary aggregates would play an important role in macroeconomic models if current and expected future real money balances enter into the forward-looking IS curve (i.e. the relationship describing the inter-temporal allocation of consumption and output). In that case, changes in real balances would directly affect the dynamics of inflation and real output. The forward-looking IS curve is derived from the standard Euler equation for consumption, which implies that (ignoring uncertainty) the marginal rate of substitution between current and future consumption, adjusted for the subjective rate of time discount, equals the gross real interest rate.

In standard money-in-the-utility function models, real balances will generally affect the marginal utility of consumption and, therefore, will enter into the IS curve. This is often referred to as "non-separability" in the literature. This direct role for money is only absent if the marginal utility of consumption does not depend on money. However, testing for non-separability is no straightforward matter.

Ireland (2004) develops a method to test whether or not real balances enter into the IS curve within a stochastic dynamic general equilibrium model. Using this approach, Ireland (2004) and Andres, Lopez-Salido, and Valles (2006) find little evidence for a direct role for real balances in the IS curve using US and euro area data respectively, but Kremer, Lombardo and Werner (2003) find the contrary evidence for Germany. This method has, however, never been applied to the UK data.

The microeconomic concept of additive separability is useful for testing whether or not real balances enter into the IS curve. A utility function is said to be additively separable between consumption and monetary assets, following Varian (1983), if there exists a monotonic transformation of it, which renders it into the sum of two utility functions one containing only consumption and one containing only money. If the instantaneous utility function is not additively separable, then real balances will enter into the forward-looking IS curve. Thus, additive separability between consumption and monetary assets is a necessary, but not sufficient, condition for excluding real money balances from the IS curve.

Non-parametric revealed preference methods have often been used to test whether or not a set of observed data is consistent with the assumption that a group of monetary assets are weakly separable from consumption goods and services and other variables (including, possibly, other monetary assets). In this paper, we use non-parametric methods to test whether or not a set of data is consistent with additive separability between consumption and monetary assets. An innovative aspect of our study is that we use a new method, based on Varian (1985) and Elger and Jones (2008), to determine whether or not violations of additive separability can be attributed to measurement errors in the observed data.

The non-parametric methods used in this study have several advantages relative to parametric tests based on estimating dynamic stochastic general equilibrium models:

- they do not involve the use of linear approximations around steady state;

- they also allow to include different types of monetary assets including both interest-bearing and non interest-bearing ones;
- saddle-path stability and equilibrium determinacy are not an issue.

The main limitation of our approach is that we only test a necessary condition for excluding real balances from the forward-looking IS curve.

There is considerable empirical evidence suggesting some explanatory power for money in explaining overall economic activity for the UK; therefore, an application to the UK monetary data appears particularly promising. In this paper, we test for additive separability using household-sector data on consumption and monetary assets for the UK. In our tests, we use Bank of England data, which are used to construct its household-sector Divisia index. We also run tests using the Bank of England's household-sector Divisia index to measure money.

We test for additive separability over a sizeable part of the post-ERM period from 1994Q1 to 2007Q1 as well as over various sub-periods. A main finding is that the UK data seem to be quite consistent with additive separability for the more recent period from 1999 to 2007 and particularly from 2001 onwards. For the full sample period, the data are much more consistent with additive separability if we measure money using either break-adjusted data or using a Divisia index than if we measure money using non break-adjusted data.

1 Introduction

In 2002, Mervyn King noted that "[m]ost people think economics is the study of money. But there is a paradox in the role of money in economic policy. It is this: that as price stability has become recognised as the central objective of central banks, the attention actually paid by central banks to money has declined." (King, 2002, p. 162) After analyzing the role of money in the economy, he concludes that the disappearance of money from economic models is more apparent than real, but cautions that there are "real dangers in relegating money to this behind-the-scenes role." (King, 2002, p. 173)[1]

Money could play a more interesting role in macroeconomic models if current and expected future real money balances entered into the forward-looking IS curve. In that case, changes in real balances would directly affect the dynamics of inflation and real output; see Ireland (2004). The forward-looking IS curve is derived from the standard Euler equation for consumption, which implies that (ignoring uncertainty) the marginal rate of substitution between current and future consumption, adjusted for the subjective rate of time discount, equals the gross real interest rate.

In standard money-in-the-utility function models, real balances will generally affect the marginal utility of consumption and, hence, will enter into the IS curve. This is often referred to as "non-separability" in the literature. Koenig (1990), for example, found that empirical results strongly suggested that an increase in real money balances raises the marginal utility of consumption.[2] This direct role for money is only absent if the marginal utility of consumption does not depend on money; see, in addition to Koenig (1990), McCallum and Nelson (1999, p. 303) and Ireland (2004). Ireland (2004) develops a method to test whether or not real balances enter into the IS curve within a stochastic dynamic general equilibrium model. Using this approach, Ireland (2004) and Andres, Lopez-Salido, and Valles (2006) find little evidence for a direct role for real balances in the IS curve using US and euro area data respectively, but Kremer, Lombardo and Werner (2003) find the contrary for Germany.[3] This method has, to our knowledge, not been applied to the UK.

The microeconomic concept of additive separability is useful for testing whether

[1] For additional discussion of these issues, see Berry, Harrison, Thomas, and de Weymarn (2007). They argue that "[a]t the very least they [monetary aggregates] provide a cross-check for other economic indicators that are subject to uncertainty. And there may also be channels through which monetary quantities contain incremental information for inflation." For a skeptical view on the importance of money in monetary policy see, among others, Woodford (2006).

[2] In theory, the effect of changes in real money balances on the marginal utility of consumption could be in either direction. Koenig (1990) also lists other variables that might affect the marginal utility of consumption, which he points out is typically measured as expenditures on non-durable goods and services. These variables include, in addition to real balances, stocks of durable goods, government purchases, and leisure. On the latter, see Mankiw, Rotemberg, and Summers (1985).

[3] Gabriel *et al.* (2008) apply a siminar methodology to US data in a more elaborated model, which also accounts for an interest rate channel of monetary policy. Their results are similar to Ireland's.

or not real balances enter into the IS curve. The instantaneous utility function is said to be additively separable between consumption and monetary assets, following Varian (1983), if there exists a monotonic transformation of it, which renders it into the sum of two utility functions one containing only consumption and one containing only money. If the instantaneous utility function is not additively separable, then real balances will enter into the forward-looking IS curve. Thus, additive separability between consumption and monetary assets is a necessary condition for excluding real money balances from the IS curve.

Non-parametric methods are often used to test whether groups of monetary assets are weakly separable from consumption goods and services and other variables (including, possibly, other monetary assets).[4] We use non-parametric methods to test for additive separability between consumption and monetary assets. An innovative aspect of our study is that we use a new method, based on Varian (1985) and Elger and Jones (2008), to determine whether or not violations of the necessary and sufficient non-parametric conditions for additive separability can be attributed to measurement errors in the observed data. The non-parametric methods used in this study have several advantages relative to parametric tests based on estimating dynamic stochastic general equilibrium models: First, they do not involve the use of linear approximations around steady state. Second, they also allow us to include different types of monetary assets including both interest-bearing and non interest-bearing ones. And, finally, saddle-path stability and equilibrium determinacy is not an issue. The main limitation of our approach is that we only test only a necessary condition for excluding real balances from the forward-looking IS curve, but not a sufficient one.

There is considerable empirical evidence suggesting some explanatory power for money in explaining overall economic activity for the UK. Nelson (2002), for example, finds that the growth rate of the real monetary base is statistically significant in a backward-looking specification of the IS curve. Elger *et. al.* (2008) find that the same is true for household-sector Divisia monetary aggregates. These results corroborate similar findings for the US (Nelson, 2002, and Hafer, Haslag and Jones, 2006) and for the euro area (Stracca, 2004, and Binner *et. al.*, 2009). In addition, Goodhart and Hofmann (2005) find that broad money growth (M4) is significant in a backward-looking specification of the UK IS curve (unique among G7 countries). This evidence suggests that it would be interesting to test whether or not money and consumption are additively separable in the UK.[5]

In this paper, we test for additive separability using household-sector data on consumption and monetary assets for the UK. In our tests, we use Bank of England data, which are used to construct its household-sector Divisia index (Hancock,

[4]For some recent examples, see Jones, Dutkowsky and Elger (2005), Drake and Fleissig (2006), Elger *et. al.* (2008), and Binner *et. al.* (2009).

[5]As noted by Nelson (2002), Koenig (1990) found that money balances entered consumption regressions significantly, a result he interpreted as supportive of nonseparable utility.

2005). Household-sector data are well suited for testing for additive separability between consumption and monetary assets. In our tests, we use disaggregate data on the different types of household-sector monetary assets and explicitly account for the interest paid on them. In that regard, we also use the Bank of England's tax adjustment in order to correctly measure net of tax interest rates. We also run tests using the Bank of England's household-sector Divisia index to measure money. For consumption, we run tests using data on both total consumption and disaggragate data for non-durables and services.

We test for additive separability over a sizeable part of the post-ERM period from 1994Q1 to 2007Q1 as well as over various sub-periods. A main finding is that the UK data seem to be broadly consistent with additive separability for the the more recent period from 1999 to 2007.

The paper is structured as follows: In Section 2, we provide the theoretical background behind testing for additive separability. In Section 3, we discuss non-parametric tests for additive separability. In Section 4, we discuss the UK data. In Section 5, we reports the results of our tests. Section 6 concludes.

2 Methodology

2.1 Theoretical Model

We begin with a theoretical model to motivate the additive separability test, which builds upon Ireland (2004). The model generalizes upon Ireland (2004) to allow for multiple monetary assets, some of which may be interest-bearing, in the spirit of Barnett (1980). The model is discussed in somewhat more detail in Jones and Stracca (2006).

Consider a representative household who maximizes the expected value of a strongly time separable lifetime utility function defined as follows:

$$E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} \left(u(c_{t+\tau}, \mathbf{m}_{t+\tau}) - \eta h_{t+\tau} \right) \right] \tag{1}$$

where E_t denotes conditional expectations, β is a discount factor, c is real consumption, $\mathbf{m} = (m_1, ..., m_N)$ is a vector of N monetary assets expressed in real terms, and h is labour supply ($\eta > 0$). Under standard budget constraints, the following optimality conditions can be obtained:

$$\eta = u_c(c_t, \mathbf{m}_t) w_t \tag{2}$$

$$\frac{u_{m_i}(c_t, \mathbf{m}_t)}{u_c(c_t, \mathbf{m}_t)} = \frac{r_{B,t} - r_{i,t}}{1 + r_{B,t}} \text{ for all } i = 1, ..., N \tag{3}$$

$$u_c(c_t, \mathbf{m}_t) = \beta(1 + r_{B,t})E_t\left[\frac{u_c(c_{t+1}, \mathbf{m}_{t+1})}{1 + \pi_{t+1}}\right] \quad (4)$$

where w_t is the real wage, $r_{B,t}$ is the nominal interest rate on a non-monetary benchmark asset, $r_{i,t}$ is the nominal rate of remuneration on the i^{th} monetary asset, π_t is the one-period inflation rate (*i.e.* $\pi_t = (p_t^c - p_{t-1}^c)/p_{t-1}^c$, where p_t^c is the price of consumption), u_{m_i} denotes the partial derivative of u with respect to the i^{th} monetary asset, and u_c denotes the partial derivative of u with respect to consumption.

We define $p_{i,t}^m = p_t^c(r_{B,t} - r_{i,t})/(1 + r_{B,t})$, which is the *nominal user cost* of the i^{th} monetary asset (Barnett, 1978). Using this notation, the conditions in (3) are equivalent to the following:

$$\frac{u_{m_i}(c_t, \mathbf{m}_t)}{u_c(c_t, \mathbf{m}_t)} = p_{i,t}^m/p_t^c \text{ for all } i = 1, ..., N \quad (5)$$

2.2 Additive Separability

Equation (4) is the standard inter-temporal Euler equation for consumption, which can be interpreted as an IS curve that contains current and future real money balances. Real balances are excluded from this IS curve if the instantaneous utility function is the sum of two utility functions, U and V, such that $u(c, \mathbf{m}) = U(c) + V(\mathbf{m})$. Under this assumption, the IS curve takes the following form:

$$U'(c_t) = \beta E_t\left[\frac{1 + r_{B,t}}{1 + \pi_{t+1}}U'(c_{t+1})\right]$$

where $(1 + r_{B,t})/(1 + \pi_{t+1})$ is the gross real interest rate.

The basic idea behind testing for additive separability is that (5) are not only optimality conditions from a forward-looking household's expected lifetime utility maximization problem, but are also first-order necessary conditions from a *static* utility maximization problem of the following form:

$$\underset{c,\mathbf{m}}{MAX}\,\{u(c, \mathbf{m}) : p_t^c c + \mathbf{p}_t^m \mathbf{m} = Y_t\} \quad (6)$$

where $\mathbf{p}_t^m = (p_{1,t}^m, ..., p_{N,t}^m)$ is the vector of nominal user costs and Y_t is the optimal expenditure on current-period consumption and monetary assets as determined from the household's lifetime utility maximization problem. In this problem, the instantaneous utility function, u, is said to be *additively separable* if there is some monotonic transformation of it, f, such that

$$f(u(c, \mathbf{m})) = U(c) + V(\mathbf{m}) \quad (7)$$

This definition follows from Varian (1983).

If the instantaneous utility function, u, is not additively separable in this sense, then real money balances will enter into the IS curve, since no monotonic transformation of the utility function can render it into the sum of two utility functions U

and V separating consumption and real balances.[6] Thus, additive separability is a necessary, but not sufficient, condition to exclude real money balances from the IS curve.

3 Non-Parametric Tests

Additive separability is defined similarly when there is a vector of consumption goods and services $\mathbf{c}$. In that case, the utility function $u(\mathbf{c}, \mathbf{m})$ is additively separable between the block of consumption goods and services, $\mathbf{c}$, and the block of monetary assets, $\mathbf{m}$, if there is some monotonic transformation of it, f, such that $f(u(\mathbf{c}, \mathbf{m})) = U(\mathbf{c}) + V(\mathbf{m})$.[7]

For later reference, we note that the utility function is *weakly separable* in monetary assets if there exists a macro function, $\bar{u}$, and a sub-utility function, V, such that $u(\mathbf{c}, \mathbf{m}) = \bar{u}(\mathbf{c}, V(\mathbf{m}))$. Additive separability implies that the utility function is *blockwise weakly separable*: *i.e.* that it is simultaneously weakly separable in both consumption goods and services and in monetary assets.

3.1 Necessary and Sufficient Conditions

Theorem 6 of Varian (1983) provides necessary and sufficient non-parametric conditions for a dataset to be rationalized by an additively separable utility function. Let $(\mathbf{p}_t^c, \mathbf{c}_t)$ and $(\mathbf{p}_t^m, \mathbf{m}_t)$ represent observed data on the prices and quantities for the two blocks, where $t = 1, ..., T$ indexes the observations. The notation $\mathbf{c}_t = (c_{1,t}, ..., c_{K,t})$ denotes the observed quantities for a set of K consumption goods and services with corresponding prices $\mathbf{p}_t^c = (p_{1,t}^c, ..., p_{K,t}^c)$. Similarly, $\mathbf{m}_t$ denotes the real quantities of a set of N monetary assets with corresponding nominal user costs $\mathbf{p}_t^m$.

Theorem 6 states that there exist two concave, monotonic, continuous utility functions whose sum rationalizes the data (*i.e.* the observed data can be rationalized by an additively separable utility function) if and only if there exist numbers U_t, V_t, $\lambda_t > 0$ such that

$$U_t - U_s - \lambda_s \mathbf{p}_s^c(\mathbf{c}_t - \mathbf{c}_s) \leq 0 \text{ for all } t, s = 1, ..., T \tag{8}$$

[6] To avoid confusion, we note that in some of the relevant literature the term additively separable is used in a different sense. For example, Mankiw, Rotemberg, and Summers (1985) consider the following utility function for consumption, C, and leisue, L:

$$U(C, L) = \frac{1}{1-\gamma}\left(\frac{C^{1-\alpha}-1}{1-\alpha} + d\frac{L^{1-\beta}-1}{1-\beta}\right)^{1-\gamma}.$$

They refer (on page 232) to additive separability between consumption and leisure as the condition that $\gamma = 0$.

[7] Of course, additive separability could be defined in other contexts, so the two blocks need not represent consumption and money. We refer to the two blocks in this way, just to be consistent with the rest of the paper.

$$V_t - V_s - \lambda_s \mathbf{p}_s^m (\mathbf{m}_t - \mathbf{m}_s) \leq 0 \text{ for all } t, s = 1, ..., T \tag{9}$$

Two necessary conditions for the data to be rationalized by an additively separable utility function are that the quantity and price data for the consumption goods and services and the quantity and user cost data for the monetary assets must both satisfy the Generalized Axiom of Revealed Preference (GARP). If the data for either block violates GARP, then a solution will not exist for the constraints in (8) or (9) corresponding to that block.[8]

3.2 Testing the Observed Data

Diewert and Parkan (1985) propose a method to determine whether a set of observed data satisfy the necessary and sufficient conditions for additive separability using linear programming techniques. A simplified linear programming method is used by Fleissig and Whitney (2007, pp. 216-217). Their method is carried out as follows: Minimize F subject to the following constraints:

$$U_t - U_s - \lambda_s \mathbf{p}_s^c (\mathbf{c}_t - \mathbf{c}_s) \leq F \text{ for all } t, s = 1, ..., T \tag{10}$$

$$V_t - V_s - \lambda_s \mathbf{p}_s^m (\mathbf{m}_t - \mathbf{m}_s) \leq F \text{ for all } t, s = 1, ..., T \tag{11}$$

$$\lambda_t > 0 \text{ for all } t = 1, ..., T \tag{12}$$

$$F \geq 0 \tag{13}$$

Intuitively, (10) relaxes all of the constraints in (8) and, similarly, (11) relaxes all of the constraints in (9), since $F \geq 0$. The procedure finds values for U_t, V_t, λ_t, for $t = 1, ..., T$, and F satisfying the inequalities. Let $\hat{F}$ denote the value of F obtained by the procedure. A feasible solution with $\hat{F} = 0$ implies that the data satisfy the necessary and sufficient conditions for additive separability.

3.3 Accounting for Measurement Error

Non-parametric revealed preference methods can be used to test for optimizing behavior as well as separability. For example, a dataset can be rationalized by a well-behaved and non-degenerate utility function if and only if it satisfies GARP. As discussed by Varian (1985, p. 445), such tests are very stringent, however, since they

[8]Let $\mathbf{p}_t^x$ and $\mathbf{x}_t$ be price and quantity data for some group of goods. GARP is defined through the following revealed preference relations (see Varian, 1982, p. 947):

$\mathbf{x}_t$ is *directly revealed preferred* to $\mathbf{x}$, $\mathbf{x}_t R^0 \mathbf{x}$, if $\mathbf{p}_t^x \mathbf{x}_t \geq \mathbf{p}_t^x \mathbf{x}$.

$\mathbf{x}_t$ is *strictly directly revealed preferred* to $\mathbf{x}$, $\mathbf{x}_t P^0 \mathbf{x}$, if $\mathbf{p}_t^x \mathbf{x}_t > \mathbf{p}_t^x \mathbf{x}$.

$\mathbf{x}_t$ is *revealed preferred* to $\mathbf{x}$, $\mathbf{x}_t R \mathbf{x}$, if $\mathbf{p}_t^x \mathbf{x}_t \geq \mathbf{p}_t^x \mathbf{x}_{t_1}$, $\mathbf{p}_{t_1}^x \mathbf{x}_{t_1} \geq \mathbf{p}_{t_1}^x \mathbf{x}_{t_2}$, ..., $\mathbf{p}_{t_k}^x \mathbf{x}_{t_k} \geq \mathbf{p}_{t_k}^x \mathbf{x}$ for some sequence of observations $(\mathbf{x}_{t_1}, ..., \mathbf{x}_{t_k})$.

The data satisfy GARP if $\mathbf{x}_t R \mathbf{x}_s$ implies not $\mathbf{x}_s P^0 \mathbf{x}_t$. Varian (1982) provides a proof of Afriat's theorem, which states that the data satisfy GARP if and only if there exists numbers U_t, $\lambda_t > 0$ satisfying $U_t - U_s - \lambda_s \mathbf{p}_s^x (\mathbf{x}_t - \mathbf{x}_s) \leq 0$ for all t, s.

do not account for the possibility of measurement errors in the data. Various methods have been proposed to determine whether violations of GARP can be attributed to measurement errors in the observed data; See, for examples, Varian (1985) and Fleissig and Whitney (2005).

The same problem applies to non-parametric separability tests. For example, Diewert and Parkan (1985) and Swofford and Whitney (1994) propose non-parametric tests for weak separability, but those tests also do not account for the possibility of measurement errors in the data. Elger and Jones (2008) propose a non-parametric method to determine if violations of weak separability can be attributed to measurement errors in the observed quantity data, building upon Varian (1985). In this paper, we apply their method to additive separability. We explain the method in the remainder of this section.[9]

3.3.1 Computing the Minimal Perturbation

The method is based on computing minimally perturbed quantity data that satisfy the necessary and sufficient conditions for additive separability. This is done by minimizing the following objective function:

$$G = \sum_{t=1}^{T}\sum_{k=1}^{K}\left(\frac{\hat{c}_{k,t} - c_{k,t}}{c_{k,t}}\right)^2 + \sum_{t=1}^{T}\sum_{n=1}^{N}\left(\frac{\hat{m}_{n,t} - m_{n,t}}{m_{n,t}}\right)^2 \tag{14}$$

in perturbed quantities for the two blocks, $\hat{\mathbf{m}}_t = (\hat{m}_{1,t}, ..., \hat{m}_{N,t})$ and $\hat{\mathbf{c}}_t = (\hat{c}_{1,t}, ..., \hat{c}_{K,t})$, and in the numbers, U_t, V_t, λ_t, subject to the following constraints:

$$U_t - U_s - \lambda_s \mathbf{p}_s^c(\hat{\mathbf{c}}_t - \hat{\mathbf{c}}_s) \leq 0 \text{ for all } t, s = 1, ..., T \tag{15}$$

$$V_t - V_s - \lambda_s \mathbf{p}_s^m(\hat{\mathbf{m}}_t - \hat{\mathbf{m}}_s) \leq 0 \text{ for all } t, s = 1, ..., T \tag{16}$$

$$\hat{m}_{n,t} > 0 \text{ for all } n = 1, ..., N, \text{ and } t = 1, ..., T \tag{17}$$

$$\hat{c}_{k,t} > 0 \text{ for all } k = 1, ..., K, \text{ and } t = 1, ..., T \tag{18}$$

$$\lambda_t > 0 \text{ for all } t = 1, ..., T \tag{19}$$

The constraints in (15) and (16) correspond to (8) and (9) respectively, but are applied to the perturbed quantities rather than the observed ones. (17) and (18) just make sure that the minimally perturbed quantities are strictly positive.

[9] Fleissig and Whitney (2007) propose a different method to determine if violations of additive separability are due to measurement error, building upon Fleissig and Whitney (2005). Their method is to perturb the observed quantity data with random measurement errors and compute $\hat{F}$ for the perturbed data. This is done 1,000 times. If more than α% of the values of $\hat{F} = 0$, then you fail to reject the null hypothesis of additive separability.

3.3.2 Statistical Test

Following Varian (1985) and Elger and Jones (2008), we relate the minimal perturbation to a test of the following null hypothesis: (H_0) *the true data, i.e. the data measured without error, can be rationalized by the sum of two concave, monotonic, continuous utility functions.* That is to say, the null hypothesis is that the true data satisfy the necessary and sufficient conditions for additive separability.

We use the notation $\mathbf{c}_t^*$ and $\mathbf{m}_t^*$ to denote the true consumption and monetary asset quantity data. We assume that the true data are related to the observed quantity data, $\mathbf{c}_t$ and $\mathbf{m}_t$, by the following proportional measurement error specification:

$$c_{k,t}^* = (1 + \varepsilon_{k,t}^c)c_{k,t} \text{ for } k = 1, ..., K \text{ and } t = 1, ..., T \tag{20}$$

$$m_{n,t}^* = (1 + \varepsilon_{n,t}^m)m_{n,t} \text{ for } n = 1, ..., N \text{ and } t = 1, ..., T \tag{21}$$

where $\varepsilon_{k,t}^c$ is a random term representing measurement errors in the quantities of the k^{th} consumption good and, similarly, $\varepsilon_{n,t}^m$ represents measurement errors in the quantities of the n^{th} monetary asset. Prices are assumed to be measured without error.

Let $\hat{G}$ denote the minimized value of the objective function G. Under the null hypothesis, both the true data and the minimally perturbed data satisfy the conditions for additive separability, which implies that

$$\frac{\hat{G}}{\sigma^2} \leq \frac{1}{\sigma^2} \sum_{t=1}^{T} \sum_{k=1}^{K} \left(\frac{c_{k,t}^* - c_{k,t}}{c_{k,t}} \right)^2 + \frac{1}{\sigma^2} \sum_{t=1}^{T} \sum_{n=1}^{N} \left(\frac{m_{n,t}^* - m_{n,t}}{m_{n,t}} \right)^2 \tag{22}$$

If we assume that the measurement errors in the data are *i.i.d.* $N(0, \sigma^2)$ random variables, then the right-hand side of (22) equals

$$\sum_{t=1}^{T} \sum_{k=1}^{K} \left(\varepsilon_{k,t}^c / \sigma \right)^2 + \sum_{t=1}^{T} \sum_{n=1}^{N} \left(\varepsilon_{n,t}^m / \sigma \right)^2 \tag{23}$$

and has a chi-square distribution with $T\,(N+K)$ degrees of freedom. Let C_α be the critical value for a chi-square distribution with $T(N+K)$ degrees of freedom at the α significance level.

If we reject the null whenever $\hat{G}/\sigma^2 > C_\alpha$, as suggested by Varian (1985, p. 448), then the resulting test will have at least the desired level of significance: *i.e.* the probability of rejecting H_0, given that it is true, will be less than or equal to α. Furthermore, if the observed data actually satisfy the necessary and sufficient conditions for additive separability, then the minimally perturbed quantities coincide with the observed ones and, consequently, $\hat{G} = 0$. In that case, the test will not reject the null for any α.

3.3.3 Bound Statistic

Obviously, the main issue with our proposed test is that one must postulate a value for the unknown standard deviation of measurement error, σ, to run it. The test is algebraically equivalent to rejecting the null of additive separability whenever

$$\bar{\sigma}^2 \equiv \hat{G}/C_\alpha > \sigma^2 \tag{24}$$

We refer to $\bar{\sigma}$ as the *bound statistic*, following Elger and Jones (2008, p. 46). If the econometrician believes that the standard deviation of measurement errors in the data, σ, is greater than or equal to the bound statistic, then the null hypothesis of additive separability should not be rejected, See, Varian (1985, pp. 450-451) for additional discussion and interpretations. Varian (1985), Jones, Dutkowsky, and Elger (2005) and Elger and Jones (2008) all report empirical results based on minimal perturbation methods in terms of analogous bound statistics.[10]

Less rigorously, the bound statistic measures "how close" the observed quantity data are to satisfying the hypothesis of additive separability given the observed price data.[11] Consequently, if the bound statistic is extremely small, then the observed dataset is very close to a perturbed dataset that satisfies the necessary and sufficient conditions for additive separability. On the other hand, if the bound statistic is large, then only datasets which are very different from the observed one satisfy the conditions for additive separability. Thus, the bound statistic can be used to judge how consistent the data are with additive separability.

4 Data Description

We test for additive separability between consumption and money using UK data for the post-ERM period from 1992Q4 to 2007Q1. In this section, we provide details concerning the data.

4.1 Consumption and Population Data

We test using data on both total UK domestic household final consumption and using disaggregate data for the components of non-durable goods and services. Specifically, we use data on the 8 components of non-durable goods and the 10 components of services, which are listed in Tables NDG.CS and SER.CS of Office for National Statistics (2007), following Elger *et. al.* (2008). As noted by Koenig (1990, p. 399), when considering nonseparability of the household utility function it is typical to measure

[10] For additional empirical results based on minimal perturbation methods, see Jones and de Peretti (2005).

[11] The minimized value of the objective, $\hat{G}$, also provides such a measure, but it will not be directly comparable in different cases if the number of perturbed quantities, $T(N+K)$, differs.

consumption in terms of expenditures on non-durable goods or on both non-durable goods and services, but not including durable goods.[12]

We use the seasonally adjusted chained volume measures, for reference year 2003, as quantities. Prices are the implicit price deflators calculated by dividing seasonally adjusted expenditure at current prices by the appropriate chained volume measure. The data are described in Office for National Statistics (2007).[13] We convert quantities to per-capita terms using an estimate of the population of the UK. Quarterly values for the UK population data are interpolated from mid-year population estimates for each year from 1992 to 2006; see Office for National Statistics (2007a, 2007b).[14]

4.2 Background on UK Monetary Data

The monetary data used in this paper are described in Table A.6.1 of Bank of England (2007). We use data on the Bank of England's household-sector Divisia index and the underlying data that is used to construct it. The Bank of England recently made a number of changes to its Divisia indices, which are described by Hancock (2005). There are some important differences in how the monetary data are constructed up through 1998Q3 and from 1999Q2 onwards and there are breaks in between.

First, the household-sector Divisia index is based on quoted rates of return until 1999, but is based on effective rates of return thereafter (see Hancock, 2005, pp. 41-42). Quoted rates measure the average rate of interest offered on new customer deposits. There are a number of problems associated with the use of quoted rates: Deposits from existing customers may yield a different rate from those offered to new customers and the rates paid on similar accounts may differ. In contrast, effective rates are calculated as the value of interest paid divided by the outstanding level of balances, which is more appropriate for constructing user costs. Hancock (2005,

[12]In fact, as he notes on p. 400, durable goods are better thought of as an alternative candidate for non-separability with consumption. We also note that studies in the literature on non-parametric weak separability testing typically either exclude durable goods (see, Jones, Dutkowsky, and Elger, 2005, and Elger *et. al.*, 2008 for examples) or else use sophisticated methods to handle them appropriately (see Patterson, 1991, Drake, 1997, and Drake and Fleissig, 2006 for examples). As noted by Patterson (1991, p. 1107), "[t]he appropriate price for a durable good is its user cost and not the implicit deflator for new purchases, but the appropriate quantity is the net stock and not the expenditure flow over a given period."

[13]For total UK domestic household final consumption, the data for expenditure at current prices (seasonally adjusted) correspond to Table 0GS.CS and the data for chained volume measures (seasonally adjusted) correspond to Table 0GS.KS. The data for the components of non-durable goods correspond to Tables NDG.CS and NDG.KS and the data for the components of services correspond to Tables SER.CS and SER.KS.

[14]The interpolation is done in FORTRAN using the double precision IMSL routine DCSINT. The mid-year estimates are assumed to correspond to the mid-point of the corresponding year. The mid-year estimate for years from 1993 to 2005 are in Table 1.2 of Office for National Statistics (2007a) and the mid-year estimate for 2006 is from Office for National Statistics (2007b). We note that the value for 1992 is available from the ONS website, but is not in Table 1.2.

p. 42) notes that the decision to use effective rates where possible "leads to a small break in the Divisia indices between 1998Q4 and 1999Q2."

A second difference concerns the benchmark rate of return. Conceptually, the Bank of England uses an envelope approach to construct the benchmark rate. In this approach, "...it is assumed that the benchmark asset is the M4 component that pays the highest interest rate." (Hancock, 2005, p. 40) Beginning in 1991, the benchmark rate for the household-sector is the rate of return on Tax Exempt Special Savings Accounts (TESSA) until 1999Q2, when it becomes the rate of return on Individual Savings Accounts (ISA). The rates of return on TESSAs and ISAs are graphed in **Figure 1**. In the figure, the ISA rate is dashed and the TESSA rate is solid. A consequence of this envelope approach is that TESSAs serve as the benchmark asset until 1999Q2, but have a positive expenditure share weight in the household-sector Divisia index from then on until 2004Q1.[15]

Finally, the Bank of England began separating out household building society instant access accounts from accounts requiring a period of notice beginning in 1998Q4.

In addition to these factors, we must also consider the issue of "break-adjustment". Monetary statistics for the UK must be adjusted for breaks, which largely occur when a building society demutualises and changes classification to become a bank (see Hancock, 2005, p. 43). When this happens, non break-adjusted levels data show large flows out of building societies and into banks.

Figure 2 graphs interest-bearing bank time deposits for 1992Q4 to 2007Q1. **Figure 3** graphs interest bearing bank sight deposits and **Figure 4** graphs building society deposits for comparison. In the figures, the solid series are the non break-adjusted levels and the dashed series are break-adjusted. The break-adjustment procedure adjusts the back data to be consistent with the new classification. Thus, when a building society demutualises to become a bank, the past data for that building society are reallocated to the new classification.[16] The algorithm for constructing break-adjusted levels data is described in the Appendix to this paper.

Since 2005, the Bank of England's Divisia indices, D_t, are constructed as follows:

$$\frac{\Delta D_t}{D_{t-1}} = \sum_{i=1}^{N} \frac{1}{2}(W_{i,t} + W_{i,t-1})\frac{\Delta M_{i,t}}{M_{i,t-1}} \tag{25}$$

where

$$W_{i,t} = \frac{M_{i,t}(r_{B,t} - r_{i,t})}{\sum_{j=1}^{N} M_{j,t}(r_{B,t} - r_{j,t})}$$

is the expenditure share of the i^{th} asset. The Bank of England applies these formulas using non break-adjusted levels data for M_i, but using break-adjusted flows data for

[15] From April 1999, it is has not been possible to open new TESSA accounts. Consequently, from April 2004, there are no deposits recorded in TESSAs. See Note (e) to Table A6.1 of Bank of England (2007).

[16] For a list of building society conversions from the Bank of England's website, see http://www.bankofengland.co.uk/statistics/test.htm.

ΔM_i; see Hancock (2005, p. 45). Before 2005, the Bank of England's Divisia indices had been based on break-adjusted levels data.

The argument against using break-adjusted levels data is that "[w]hen a building society becomes a bank, *past* deposits at that building society were still remunerated at *past* building society interest rates, and must be measured as such." (Hancock, 2005, p. 43) On the other hand, we must also consider the fact that the user costs of the monetary assets are based on quoted interest rates before 1999. Thus, if we use non break-adjusted levels data, then (before 1999) we are implicitly assuming that when a building society demutualises it switches (at the time of the change) from paying the average rate on new building society deposits to paying the average rate on new bank deposits.

4.3 Monetary Data: Components

We test for additive separability using disaggregate data on the components of the household-sector Divisia index and using the Divisia index itself. When using the components, the household-sector monetary assets are listed in **Table 1**.

In order to facilitate testing with data for the components, we need to consider a consistent set of monetary assets over the entire sample period. Thus, we aggregate together instant access and notice and term building society deposits when separate data are available. We also exclude TESSAs from our set of monetary assets for the same reason. Finally, we aggregate *NC* and *NIB* together, since both are non interest-bearing. These decisions closely follow Elger *et. al.* (2008).

The monetary asset quantities are converted to real per-capita terms. When total UK domestic household final consumption is used, we use the corresponding implicit deflator to convert the monetary assets to real terms. When the disaggregated components of non-durable goods and services are used, we use a price index computed from those components to convert the monetary assets to real terms.

Nominal user costs for each series are defined as follows: $p^m_{i,t} = p^c_t(r_{B,t} - r_{i,t})/(1 + r_{B,t})$, where $r_{B,t}$ (the benchmark rate) is the rate of return on ISAs beginning in 1999Q2 and the rate of return on TESSAs for earlier periods, $r_{i,t}$ are the own rates of return of the monetary assets (zero for *NC* and *NIB*), and p^c_t is the implicit price deflator used to convert the monetary assets to real terms. The own rates of the monetary assets are net of tax and are based on effective rates where available as described in Hancock (2005, p.40).

We run tests using both non break-adjusted and break-adjusted levels in order to investigate the effects of breaks in the series on our test results. As can be seen from Figures 2-4, break-adjustment has a relatively modest effect on the data after 1999, but it has much larger effects on the data beforehand.

As discussed previously, the switch from quoted to effective rates leads to break in the Divisia index between 1998Q4 and 1999Q2. To expand on this point, **Figure 5** graphs the expenditure share of the household-sector monetary assets. Specifically, it is the ratio of monetary expenditures, the sum of quantities multiplied by user costs for all monetary assets, to monetary expenditures plus expenditures on non-durable goods and services. The share based on non break-adjusted data is solid and the share based on break-adjusted data is dashed. In the figure, the vertical lines indicate 1998Q3 and 1999Q2. The effect of the break in the series can be clearly seen in the graph. To account for this break, we omit the two quarters 1998Q4 and 1999Q1 from the dataset when running our tests.[17]

4.4 Monetary Data: Divisia Index

We also test using the household-sector Divisia quantity index (in real per-capita terms) to measure money. In order to run the test, we also need a corresponding measure of the opportunity cost of money. The usual procedure when using Divisia indices is to compute a dual price index by dividing total expenditure on the monetary assets (*i.e.* the sum of quantities multiplied by user costs) by the Divisia quantity index (see, for example, Anderson, Jones, and Nesmith, 1997). The Bank of England does not publish a dual price index, so we computed one. We compute total expenditure on the monetary assets using non break-adjusted data to be consistent with how the expenditure share weights are computed by the Bank of England. We also include TESSAs from 1992Q2 to 2004Q1 when computing expenditure, since TESSAs have a non-zero expenditure share weight in the Bank of England's household-sector Divisia index over that period.

5 Results

5.1 Previous Literature on Weak Separability on UK Data

Many studies have investigated whether or not groups of monetary assets are weakly separable from consumption goods and services and other variables using UK data.[18] In particular, Drake and Fleissig (2006) and Elger *et. al.* (2008) focus on the post-ERM period beginning in 1992Q4, which both characterize as a period with a stable monetary policy regime. Weak separability of the monetary assets is primarily interesting due to its implications for money demand and monetary aggregation (see Barnett, 1982, and Belongia, 1996).

Both Drake and Fleissig (2006) and Elger *et. al.* (2008) use break-adjusted monetary data.[19] Drake and Fleissig (2006) find that the monetary assets in Table 1 are weakly separable from non-durable goods, services, and durable goods using data from 1992Q4 to 2003Q3. Elger *et. al.* (2008) test whether the monetary assets in Table 1 are weakly separable from the components of non-durable goods and services

[17] This decision also closely follows Elger *et. al.* (2008).

[18] See, for some examples, Patterson (1991), Belongia and Chrystal (1991), and Drake (1997).

[19] See footnote 7 of Drake and Fleissig (2006, p. 685) and Section 3.2 of Elger *et. al.* (2008).

using data from 1992Q4 to 2005Q1. They find that the monetary assets are weakly separable if the test is run using only the data from 1994Q1 onwards, but not if the test is run on the full dataset. We repeated the weak separability tests from Elger *et. al.* (2008), using break-adjusted monetary data and data for the components of non-durable goods and services, for the slightly longer sample period from 1992Q4 to 2007Q1 and obtained the same result. Weak separability of the monetary assets is a necessary condition for additive separability between consumption goods and services and monetary assets, since additive separability implies blockwise weak separability. Thus, we focus on the period from 1994Q1 to 2007Q1 in our subsequent empirical analysis.

5.2 Results for Break-Adjusted Monetary Asset Data

In this section, we test for additive separability between consumption and monetary assets using break-adjusted data for the assets in Table 1. We will treat the results in this section as a benchmark to which later results will be compared.

5.2.1 Total UK Domestic Household Final Consumption

We begin by presenting test results, where consumption is measured as total UK domestic household final consumption, similar to Jones and Stracca (2006). The results for these data are presented in **Panel A** of **Table 2**.

We start with the test results for the full sample period: 1994Q1-2007Q1. We remind the reader that 1998Q4 and 1999Q1 have been removed from the dataset as discussed in Section 4.3. The first step of the analysis is to test whether or not the observed data satisfy the necessary and sufficient conditions for additive separability. This is done by solving the linear programming problem in Section 3.2 to obtain $\hat{F}$.[20] The observed data satisfy the conditions for additive separability if and only if $\hat{F} = 0$. The values of $\hat{F}$ are reported in the second column of the table. The value of $\hat{F}$ for the full sample period is 0.02224 indicating that the observed data do not satisfy the conditions for additive separability.

The next step in the analysis is to apply the minimal perturbation method, described in Section 3.3, which is designed to account for measurement errors in the quantity data. We calculate minimally perturbed quantity data for the consumption and monetary variables, which satisfy the necessary and sufficient conditions for additive separability (see Section 3.3.1 for details). The minimized value objective function, $\hat{G}$, obtained from this calculation is reported in the third column of the table (multiplied by 10,000).[21] We use this minimized value to compute the corresponding

[20] We solve the linear programming problem in FORTRAN using the IMSL 6.0 subroutine DENSE_LP.

[21] We compute the minimally perturbed data in FORTRAN using the subroutine NLPQLP, which was written by Klaus Schittkowski.

bound statistic, $\bar{\sigma} \equiv \sqrt{\hat{G}/C_\alpha}$, where C_α is the critical value for a chi-square test with $T(N+K)$ degrees of freedom (see Sections 3.3.2 and 3.3.3 for further discussion). The bound statistic for the 10% significance level, multiplied by 100, is reported in the fourth column of the table.

We remind the reader that N is the number of monetary assets and K is the number of consumption goods and services. For Panel A of Table 2, $N = 4$ (since CC and NIB are aggregated together) and $K = 1$ (since we are using a single variable to measure consumption). $T = 51$ is the total number of observations for the full sample period. As explained by Elger and Jones (2008, p. 46), the bound statistic is a transparent way to report the results from their test, since it can be easily compared to one's own subjective prior regarding the standard deviation of measurement errors in the data.

For the full sample period, the bound statistic (multiplied by 100) is 0.15885. To interpret this bound statistic, recall that measurement errors have the following form: $\varepsilon^c_{k,t} = (c^*_{k,t} - c_{k,t})/c_{k,t}$ and $\varepsilon^m_{n,t} = (m^*_{n,t} - m_{n,t})/m_{n,t}$ (see equations 20 and 21). Thus, when multiplied by 100, measurement errors represent the percentage differences between the true data and the observed data for each good or asset. The bound statistic is interpreted by assuming that measurement errors are *i.i.d.* and normally distributed with mean zero and variance σ^2. The null hypothesis of additive separability should not be rejected if the true standard deviation of measurement errors, σ, is believed to be greater than or equal to the bound statistic, $\bar{\sigma}$. If the standard deviation of measurement errors was equal to the bound statistic for the full sample, then this would imply that measurement errors of between $\pm 0.32\%$ of the observed data would lie within two standard deviations of the mean.

As discussed in Section 4.2, there are several differences in how the monetary data are constructed up through 1998Q3 and from 1999Q2 onwards and there are breaks in between. These include the switch from TESSAs to ISAs as the benchmark asset and the switch from quoted to effective rates. Thus, we also ran the additive separability tests on two sub-periods 1994Q1-1998Q3 and 1999Q2-2007Q1.

The observed data do not satisfy the conditions for additive separability over either of these sub-periods, since $\hat{F}$ is non-zero in both cases. We again computed minimally perturbed data satisfying the additive separability conditions and report the corresponding bound statistics in the table. The bound statistic (multiplied by 100) for 1999Q2-2007Q1 is 0.03287, which is considerably smaller than the bound statistic for the full sample. In contrast, the bound statistic (multiplied by 100) for 1994Q1-1998Q3 is 0.23905, which is larger than the one for the full sample.

Finally, we also tested for additive separability over the sub-period from 2001Q1 to 2007Q1. For this sub-period, we found that the observed data actually satisfy the necessary and sufficient conditions for additive separability, since $\hat{F} = 0$. Consequently, the bound statistic for this sub-period is exactly zero.

5.2.2 Components of Non-Durables and Services

Next, we present test results, where consumption is measured as the components of non-durable goods and services. These data are essentially updated versions of the data used by Elger *et. al.* (2008).[22] The results for these data are presented in **Panel B** of **Table 2**. For purposes of comparison, we report the test results for the same sample periods as in Panel A.

We again found that the observed data do not satisfy the necessary and sufficient conditions for additive separability for 1994Q1-2007Q1, since $\hat{F}$ is non-zero. The corresponding bound statistic (multiplied by 100) is 0.07988, which is lower than the one from Panel A.[23] The observed data also do not satisfy the conditions for additive separability over any of the three sub-periods considered.

As in Panel A, the bound statistic for 1994Q1-1998Q3 is higher than the bound statistic for the full sample period. The bound statistics for the recent sub-periods, 1999Q2-2007Q1 and 2001Q1-2007Q1, are approximately equal to each other and are lower than the bound statistic for the full sample period (the values, multiplied by 100, are 0.03529 and 0.03711 respectively). Moreover, the bound statistics for these two sub-periods are very close to the bound statistic for 1999Q2-2007Q1 from Panel A.

5.2.3 Discussion of Results

Taken together, the results in Table 2 show that the data seem to be quite consistent with additive separability if we consider the recent sub-periods beginning in either 1999Q2 or in 2001Q1. When total consumption is used, the observed data satisfy the necessary and sufficient conditions for additive separability for the period beginning in 2001Q1. If the sample period is extended back to 1999Q2, then the observed data violate the conditions for additive separability, but the bound statistic is quite low.

When the components of non-durables and services are used to measure consumption, the observed data violate the conditions for additive separability for both 1999Q2-2007Q1 and 2001Q1-2007Q1, but the bound statistics are very close to the bound statistic obtained using total consumption for 1999Q2-2007Q1. Specifically, the bound statistics (multiplied by 100) for 1999Q2-2007Q1 are both less than 0.036. If the standard deviation of measurement errors was equal to this value, then measurement errors of between $\pm 0.072\%$ of the observed quantity data would lie within two standard deviations of the mean. To reject the null hypothesis of additive separability for 1999Q2-2007Q1, one would have to believe that the true standard deviation of measurement errors in the data is less than these bound statistics and, consequently, that measurement errors in the data are quite small.

[22] One minor difference is that Elger *et. al.* (2008) weight together the TESSA and ISA rates to produce a benchmark rate, when both rates are available. Our benchmark rate is identical to the one used by the Bank of England.

[23] Note that for Panel B, $K = 18$ when computing the degrees of freedom for the chi-square test.

Judging from bound statistics, the data are less consistent with additive separability for the full sample period from 1994Q1 to 2007Q1. Moreover, the results suggest that the violations of additive separability in the full sample period are largely attributable to the inclusion of observations from 1994Q1 to 1998Q3. Nevertheless, the bound statistics for the full sample period are still not very high, particularly when the components of non-durables and services are used to measure consumption.

5.3 Additional Results

5.3.1 Results for Non Break-Adjusted Monetary Asset Data

In this section, we test for additive separability between consumption and monetary assets using non break-adjusted data for the assets in Table 1. We begin by presenting test results, where consumption is measured as total UK domestic household final consumption. These results are reported in **Panel A** of **Table 3**.

For 1994Q1-2007Q1, the quantity and user cost data for the monetary assets violate GARP. By itself, this implies that the observed data violate the conditions for additive separability (see Section 3.1). The corresponding value of $\hat{F}$ is 2.90928. We investigated further and found that the monetary data satisfy GARP for 1997Q3-2007Q1, but that 14 GARP violations occurred if we extended it back to 1997Q1 or earlier. Similarly, the monetary data satisfy GARP for 1994Q1-1998Q3, but violations occurred if we extended the sample further. All 14 violations are detected when testing for GARP from 1997Q1 to 2000Q4. Thus, we can conclude that interactions between observations in early 1997 and observations in 1999 and 2000 are causing the violations.

The bound statistic (multiplied by 100) for 1994Q1-2007Q1 using non break-adjusted data is 1.54214. The corresponding value from Panel A of Table 2, using break-adjusted data, was just 0.15885. Clearly, the non break-adjusted monetary data are much less consistent with additive separability than are the break-adjusted data over the full sample period.

We also ran the additive separability tests over the same sub-periods as in Table 2. As can be seen in Figures 2-4, break adjustment has very little effect on the data after 2000. Not surprisingly, therefore, the observed dataagain satisfy the necessary and sufficient conditions for additive separability for 2001Q1-2007Q1, as in Panel A of Table 2. The observed data do not satisfy the conditions for additive separability for 1999Q2-2007Q1 and the bound statistic is approximately the same as the corresponding value in Table 2 (the values, multiplied by 100, are 0.03372 and 0.03287 respectively). The observed data also violate the conditions for additive separability for 1994Q1-1998Q3, but in this case the bound statistic is lower than the corresponding value in Table 2 (the values, multiplied by 100, are 0.09569 and 0.23905 respectively).

In **Panel B** of **Table 3**, we present test results using non-break-adjusted data, where consumption is measured as the components of non-durable goods and services.

For 1994Q1-2007Q1, the observed data again do not satisfy the conditions for additive separability: the monetary data violate GARP and the corresponding value of $\hat{F}$ is 3.18430. In this case, our attempt to compute minimally perturbed quantity data was unsuccessful and, consequently, we are unable to report a bound statistic for the full sample period.[24]

The observed data do not satisfy the conditions for additive separability for any of the three sub-periods and the values of the bound statistics are very similar to the corresponding values in Panel B of Table 2, which are based on break-adjusted data.

Taken together, these results reinforce the conclusion from the previous section that the data seem to be quite consistent with additive separability if we consider the recent sub-periods beginning in either 1999Q2 or in 2001Q1.

5.3.2 Results for the Divisia Index

In this section, we complete our empirical analysis by testing for additive separability using the household-sector Divisia index to measure money. We start with the test results reported in **Panel A** of **Table 4**, where consumption is measured as total UK domestic household final consumption.

The observed data do not satisfy the conditions for additive separability for the full sample or for any of the three sub-periods we have been considering. The bound statistic for the full sample period (multiplied by 100) is 0.22648. This value is slightly higher than the value obtained using break-adjusted data (0.15885), but is much lower than the value obtained using non break-adjusted data (1.54214). Thus, for the full sample period, the results using the Divisia index to measure money are much more like the results using break-adjusted data than the ones using non break-adjusted data. Intuitively, the Divisia index takes care of breaks in the underlying assets by aggregating over them.

The bound statistic for 1999Q2-2007Q1, multiplied by 100, is 0.10948, which is higher than the corresponding values in Panel A of both Tables 2 and 3. In addition, the observed data also violate the conditions for additive separability for 2001Q1-2007Q1, unlike in Panel A of the previous two tables.

In **Panel B** of **Table 4**, we present test results, where consumption is measured as the components of non-durable goods and services. Again, the observed data do

[24] The values of $\hat{F}$ for 1994Q1-2007Q1 in Panels A and B of Table 3 (2.90928 and 3.18430 respectively) are substantially higher than for all other cases in Tables 2 and 3. If values of U_t, V_t, and $\lambda_t > 0$ exist such that all constraints in (8) and (9) are satisfied, then the observed data are consistent with additive separability and $\hat{F} = 0$. Otherwise, $\hat{F} > 0$ measures the largest violation of the constraints in (8) and (9) for the corresponding values of U_t, V_t, and $\lambda_t > 0$ obtained from the solution to the linear programming problem in Section 3.2. In that sense, $\hat{F}$ is a measure of how consistent the observed data are with additive separability. In addition, the only instances in either Tables 2 or 3 where the monetary asset data violate GARP are for 1994Q1-2007Q1 in Panels A and B of Table 3. Taken together, these results suggest that the observed data are relatively inconsistent with additive separability for 1994Q1-2007Q1 in Panel B of Table 3.

not satisfy the conditions for additive separability for the full sample or for any of the three sub-periods. The bound statistic for the full sample period, multiplied by 100, is 0.07812, which is substantially lower than the one in Panel A (similar to what we found in Table 2). In contrast, the bound statistics for the two recent sub-periods, beginning in either 1999Q2 or in 2001Q1, are very similar to the ones in Panel A and are higher than the corresponding bound statistics in Panel B of Tables 2 and 3. The bound statistic for 1994Q1-1998Q3, multiplied by 100, is 0.01120, which is very low.

6 Conclusions

In this paper, we use non-parametric methods to test for additive separability between consumption and money over the period from 1994Q1 to 2007Q1 using data for the UK, building on Jones and Stracca (2006). If consumption and money are not additively separable, then real money balances enter into the forward-looking IS curve (the converse does not hold, however).

An innovative aspect of this study is that we use a new method, based on Varian (1985) and Elger and Jones (2008), to determine whether or not violations of additive separability can be attributed to measurement errors in the observed data. The method is based on constructing minimally perturbed quantity data that satisfy the necessary and sufficient non-parametric conditions for additive separability from Varian (1983). A bound statistic can be computed from the minimal perturbation, which indicates "how close" the observed data are to satisfying the additive separability conditions. A relatively small bound statistic indicates that there exists perturbed data satisfying the additive separability conditions, which are relatively close to the observed data. Thus, the bound statistic can be used to gauge how consistent the data are with additive separability over different sample periods and using different measures of consumption and money.

A main finding is that the UK data are broadly consistent with additive separability for the the more recent period from 1999 to 2007. For this period, the bound statistics are quite low when either break-adjusted or non break-adjusted components data are used to measure money. The bound statistics are higher when the Divisia index is used to measure money, but they are still not very high.

For the full sample period from 1994 to 2007, the data seem to be much less consistent with additive separability when using non break-adjusted components data than when using either break-adjusted data or the Divisia index to measure money. In our view, the similarity of the results obtained using either break-adjusted data or the Divisia index for 1994 to 2007 suggests that the results obtained using non break-adjusted data for this period are not very compelling. In any case, for thinking about the relevance of monetary aggregates in a policy making strategy today, it would be the results for the most recent period that matter most.

We conclude by noting that the non-parametric approach used here, and initially developed in Jones and Stracca (2006), could potentially be applied to other interesting economic problems, such as non-separabilities between consumption and leisure. This appears to be a promising avenue for future research.

7 Appendix: Break Adjustment

The formula for constructing break adjusted levels data was provided to us by the Bank of England. Let, $\Delta M_{i,t}$ denote the break-adjusted flows for the i^{th} monetary asset and let $M_{i,t}$ denote the corresponding non break-adjusted level. These data are available from the Bank of England's website. Break adjusted levels, $M_{i,t}^{BA}$, can be constructed using the following procedure:

1) Start from the most recent period T. Create an index I_t and a multiplier X_t both with a value of 1 in the most recent period, *i.e.* $X_T = I_T = 1$. Also, let $M_{i,T}^{BA} = M_{i,T}$.

2) For all other periods, calculate $I_{t-1} = \dfrac{M_{i,t}}{M_{i,t-1} + \Delta M_{i,t}}$ and $X_{t-1} = X_t I_{t-1}$.

3) The break adjusted level is then calculated as $M_{i,t-1}^{BA} = M_{i,t-1} X_{t-1}$.

This procedure implies that the growth rate of the break-adjusted level equals the ratio of the break adjusted flow to the non break-adjusted level from the previous period: *i.e.* $\dfrac{M_{i,t}^{BA} - M_{i,t-1}^{BA}}{M_{i,t-1}^{BA}} = \dfrac{\Delta M_{i,t}}{M_{i,t-1}}$, which provides the motivation behind (25).

8 Acknowledgements

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Table 1
Monetary Assets

Notes and coin (NC)
Non interest-bearing bank deposits (NIB)
Interest-bearing bank sight deposits (SD)
Interest-bearing bank time deposits (TD)
Building society deposits (BSD)

Notes:
NC and NIB are aggregated together in our empirical analysis

Table 2
Additive Separability Tests
Break Adjusted Data on Monetary Assets

Sample Period	*Observed Data*	*Minimally Perturbed Data*	
	$\hat{F}$	$\hat{G}$ * 10,000	$\bar{\sigma}$ * 100[1]
A. Total UK Domestic Household Final Consumption			
1994Q1-1998Q3	0.01422	6.45962	0.23905
1994Q1-2007Q1[2]	0.02224	7.17480	0.15885
1999Q2-2007Q1	0.00348	0.19804	0.03287
2001Q1-2007Q1	0.00000*	0.00000	0.00000
B. Components of Non-Durable Goods and Services			
1994Q1-1998Q3	0.04526	3.92078	0.09278
1994Q1-2007Q1[2]	0.09064	7.54867	0.07988
1999Q2-2007Q1	0.04774	0.93733	0.03529
2001Q1-2007Q1	0.04774	0.81651	0.03711

Notes:
1. $\bar{\sigma} = \sqrt{\hat{G}/C_{0.1}}$, where $C_{0.1}$ is the 10% critical value for a χ^2 with $T(N+K)$ degrees of freedom.
2. 1998Q4 and 1999Q1 are omitted from full sample period.
* Observed data satisfy the conditions for additive separability.

Table 3
Additive Separability Tests
Non Break Adjusted Data on Monetary Assets

Sample Period	*Observed Data*	*Minimally Perturbed Data*	
	$\hat{F}$	$\hat{G}$ * 10,000	$\bar{\sigma}$ * 100[1]
A. Total UK Domestic Household Final Consumption			
1994Q1-1998Q3	0.00747	1.03512	0.09569
1994Q1-2007Q1[2]	2.90928†	676.20390	1.54214
1999Q2-2007Q1	0.00379	0.20844	0.03372
2001Q1-2007Q1	0.00000*	0.00000	0.00000
B. Components of Non-Durable Goods and Services			
1994Q1-1998Q3	0.02228	4.03872	0.09417
1994Q1-2007Q1[2]	3.18430†		N/A††
1999Q2-2007Q1	0.04134	0.98240	0.03613
2001Q1-2007Q1	0.04134	0.72664	0.03501

Notes:

1. $\bar{\sigma} = \sqrt{\hat{G}/C_{0.1}}$, where $C_{0.1}$ is the 10% critical value for a χ^2 with $T(N+K)$ degrees of freedom.
2. 1998Q4 and 1999Q1 are omitted from full sample period.

* Observed data satisfy the conditions for additive separability.
† Quantity and user cost data for monetary assets violate GARP.
†† We were unable to compute the minimal perturbation.

Table 4
Additive Separability Tests
Household-Sector Divisia Index

Sample Period	*Observed Data*	*Minimally Perturbed Data*	
	$\hat{F}$	$\hat{G}$ * 10,000	$\bar{\sigma}$ * 100[1]
A. Total UK Domestic Household Final Consumption			
1994Q1-1998Q3	0.00805	0.85056	0.13107
1994Q1-2007Q1[2]	0.01225	6.18988	0.22648
1999Q2-2007Q1	0.01099	0.94515	0.10948
2001Q1-2007Q1	0.00185	0.29174	0.06796
B. Components of Non-Durable Goods and Services			
1994Q1-1998Q3	0.00203	0.04965	0.01120
1994Q1-2007Q1[2]	0.03011	6.25998	0.07812
1999Q2-2007Q1	0.03011	6.15010	0.09704
2001Q1-2007Q1	0.03011	2.58935	0.07091

Notes:

1. $\bar{\sigma} = \sqrt{\hat{G}/C_{0.1}}$, where $C_{0.1}$ is the 10% critical value for a χ^2 with $T(N+K)$ degrees of freedom.
2. 1998Q4 and 1999Q1 are omitted from full sample period.

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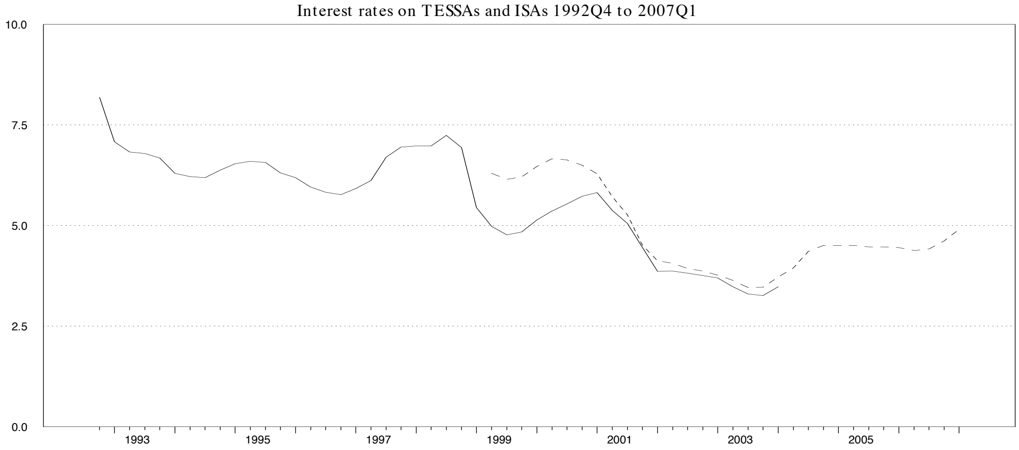
Figure 1: Benchmark Rates of Return

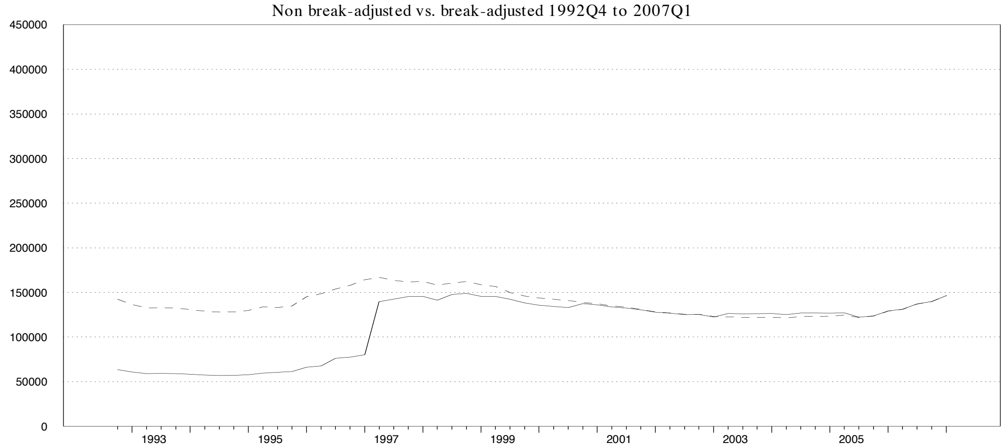
Interest rates on TESSAs and ISAs 1992Q4 to 2007Q1



Annual Rates on TESSAs (Solid) and ISAs (Dashed).

Figure 2: Interest-Bearing Bank Time Deposits

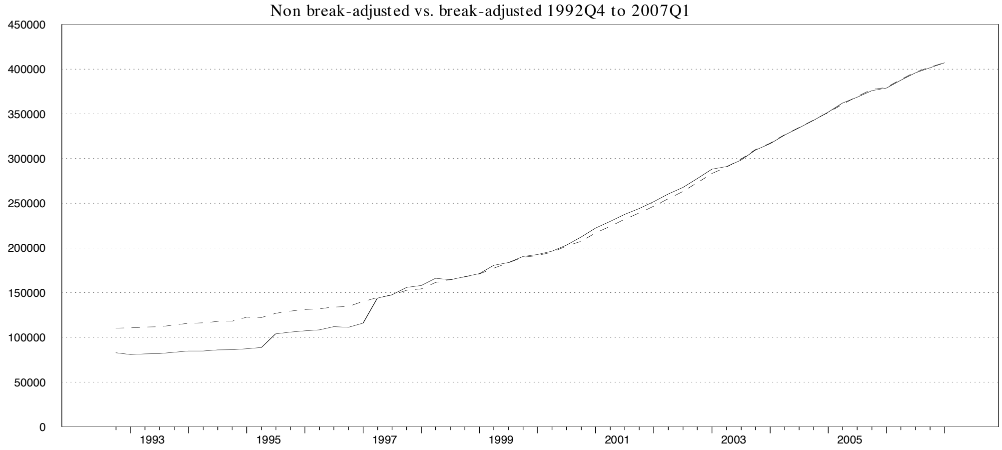
Non break-adjusted vs. break-adjusted 1992Q4 to 2007Q1



Interest-Bearing Bank Time Deposits: Non Break-adjusted (Solid) and Break-adjusted (Dashed) in millions of £, seasonally adjusted.

Figure 3: Interest-Bearing Bank Sight Deposits

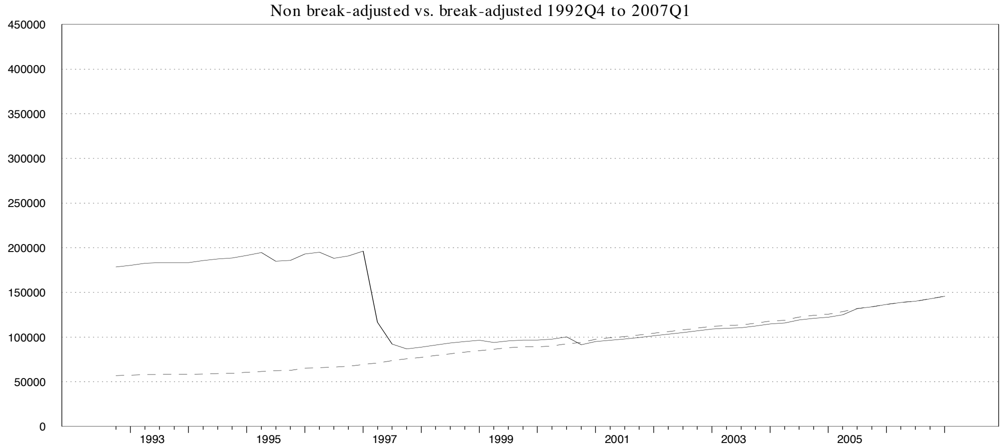
Non break-adjusted vs. break-adjusted 1992Q4 to 2007Q1



Interest-Bearing Bank Sight Deposits: Non Break-adjusted (Solid) and Break-adjusted (Dashed) in millions of £, seasonally adjusted.

Figure 4: Building Society Deposits

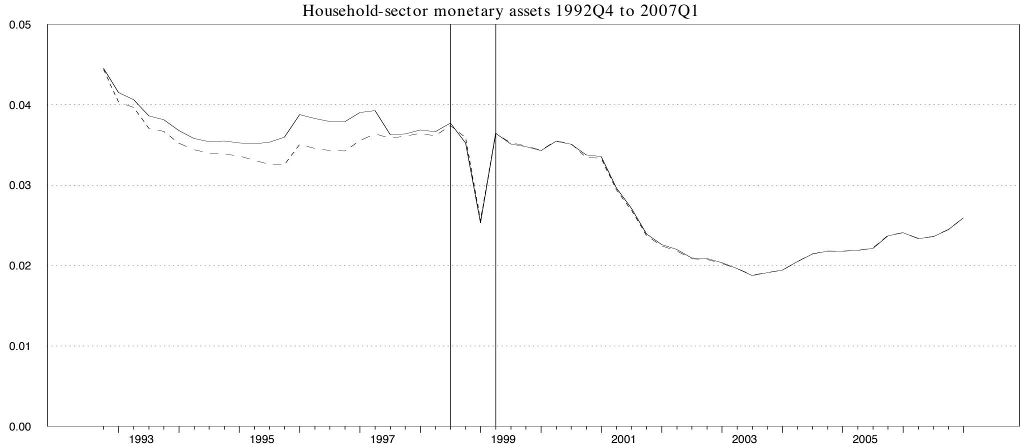
Non break-adjusted vs. break-adjusted 1992Q4 to 2007Q1



Building Society Deposits: Non Break-adjusted (Solid) and Break-adjusted (Dashed) in millions of £, seasonally adjusted.

Figure 5: Expenditure Share

Household-sector monetary assets 1992Q4 to 2007Q1



Expenditure share is calculated using non break-adjusted data (solid) or break-adjusted data (dashed). The share is expenditure on monetary assets divided by expenditure on monetary assets plus expenditure on non-durable goods and services. The vertical lines indicate 1998Q3 and 1999Q2.

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