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INFORMATION, HABITS, AND CONSUMPTION BEHAVIOR

EVIDENCE FROM MICRO DATA

by Mika Kuismanen and Luigi Pistaferri





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CONTENTS

Abstract						
Non-technical summary						
1	Introduction					
2	The	model	8			
	2.1 Approximating the Euler equation					
	2.2 Superior information					
	2.3	The consumption function	13			
3	Empirical approach					
	3.1	Identification of ψ	15			
4	The data					
5	Results					
6	5 Conclusions					
Tables						
Appendices						
Appendix A: Second order approximation						
		of the Euler equation	23			
Appendix B: Proof of the constancy of κ						
References						
European Central Bank Working Paper Series						

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Abstract

Most of the empirical literature on consumption behaviour over the last decades has focused on estimating Euler equations. However, there is now consensus that data-related problems make this approach unfruitful, especially for answering policy relevant issues. Alternatively, many papers have proposed using the consumption function to forecast behaviour. This paper follows in this tradition, by deriving an analytical consumption function in the presence of intertemporal non-separabilities, "superior information", and income shocks of different nature, both transitory and permanent. The results provide evidence for durability, and show that people are relatively better at forecasting short-term rather than long-term shocks.

JEL classification: D11,D12,D82,E21

Keywords: Consumption, Superior Information, Durability, Habit Persistence, Panel Data

Non-technical summary

Up to recent years many researchers have studied consumer choice under the assumption that preferences are time separable. Also, most of the empirical literature on consumption behaviour over the last decades has focused on estimating the first order condition of the intertemporal maximization problem under uncertainty. However, recent literature has shown that given the lack of cross-sectional variability in interest rates, lack of information on individual discount rates, and reliable measures of uncertainty, the results using this approach are mixed at best. The problem of delivering credible estimates of structural parameters, such as the elasticity of intertemporal substitution or the coefficient of prudence, as well as its inadequacy to answer policy-relevant questions, has made the search for alternative approaches a priority in macroeconomics.

Our approach is to study the consumption function. We show that it is possible to approximate the first order condition of the intertemporal maximization problem under uncertainty and the constant relative risk aversion preferences without resorting to log-linearization. In addition we show how such approximation can be implemented even in the presence of a general form of intertemporal non-separability, covering both habit persistence and durability. We relax the assumption that the individual knows no more than the ecometrician regarding future income prospects. In particular, we assume that people may be able to forecast the actual realizations of income shocks (transitory and permanent) with a certain degree of confidence, and that their expectations become increasingly less precise as the forecast horizon widens.

Most of the consumption function literature uses aggregate data while our empirical analysis is conducted on individual data. This means that we avoid a number of difficult aggregation issues. The main difficulty of implementing our approach is to find a data set that satisfies two principal requirements: it features a longitudinal component, and it contains information on household consumption, income, and assets. The PSID (Panel Study of Income Dynamics) satisfies the first requirement fully and the second partly. In particular, data on assets are available only at five-year intervals (starting in 1984), and the only consumption data available is for food. As it is not clear how well food consumption behavior generalizes to non-durable consumption behavior, we use imputed non-durable consumption data. The idea is to impute consumption to all PSID households combining PSID data with consumption data from repeated CEX (Consumer Expenditure Survey) cross-sections. The final sample used in our analysis is composed of 1,125 households.

We estimate a dynamic consumption function using microeconomic data. We show that the dynamics hinge upon a number of structural parameters: the degree of intertemporal separability in preferences (which may indicate habit persistence or durability), the slope of the intertemporal consumption path (which captures the importance of the precautionary motive for savings), the impact of advance information about future income prospects (which signals a discrepancy of information between the individual and the econometrician), and the forecastability of permanent shocks relative to transitory shocks. Our findings point to the existence of durability rather than habit persistence. This means that the conventional definition of non-durable consumption most likely includes goods that provide services for longer than one year, thus obscuring (or offsetting) any habit persistence that may exist in the data. We find evidence for a positively sloped consumption path, meaning that consumers delay spending in response to the risks they face. As for the superior information issue we find that consumers have substantial information regarding near future income changes, especially short-lived ones.

1 Introduction

Most of the empirical literature on consumption behaviour over the last decades has focused on estimating the Euler equation, i.e., the first order condition of the intertemporal maximization problem under uncertainty (Hall (1978)). However, given the lack of cross-sectional variability in interest rates, lack of information on individual discount rates, and reliable measures of uncertainty, the results using this approach are mixed at best (Attanasio and Low (2004), Browning and Lusardi (1995), Attanasio (2000) and Carroll (2001)). The inability of the Euler equation approach to deliver credible estimates of structural parameters, such as the elasticity of intertemporal substitution or the coefficient of prudence, as well as its inadequacy to answer policy-relevant questions, has made the search for alternative approaches a priority in macroeconomics.

A popular alternative to the Euler equation is to study the consumption function. This approach is complicated by the fact that realistic assumptions about preferences and uncertainty prevent finding a closed form solution, and hence the model can only be simulated (Zeldes (1989)). To overcome such difficulty, many approximations have been proposed (Campbell and Mankiw (1987), Fuhrer (2001)). Our paper follows in this tradition, although it departs from it on a number of important ways. First, we show that, as long as the higher moments of the conditional distribution of consumption are constant over time, we can approximate the Euler equation under CRRA preferences without resorting to log-linearization. Secondly, we are able to derive such approximation even in the presence of a general form of intertemporal non-separability, covering both habit persistence and durability (Constantinides and Ferson (1991)). Thirdly, we relax the assumption that the individual knows no more than the ecometrician regarding future income prospects. In particular, we assume that people may be able to forecast the actual realizations of income shocks (transitory and permanent) with a certain degree of confidence, and that their expectations become increasingly less precise as the forecast horizon widens. Fourthly, most of the consumption function literature uses aggregate data, while our empirical analysis is conducted on individual data drawn from the US PSID, thus avoiding a number of aggregation issues.

Using our assumptions, we derive an expressions that relates current consumption to once and twice lagged consumption, lagged assets, and lagged income. We deal with the issue of endogeneity using a conventional instrumental variable procedure, whereby lags of income and consumption act as excluded variables. The reduced form coefficients of this relationship are complicated, non-linear functions of all the parameters of the model. A minimum distance procedure maps these reduced form coefficients into the structural parameters, while the covariance between unexplained consumption and unexplained income changes permits identification of one remaining crucial parameter, the forecastability of permanent shocks relative to transitory shocks. The rest of the paper is organized as follows. In Section 2 we present the model, discuss the approximation of the Euler equation and derive an analytical expression for the consumption function under the assumption of "superior information". In Section 3 we discuss the empirical approach and the identification issues. Section 4 deals with the data, while Section 5 presents the results. Section 6 concludes.

2 The Model

Households are assumed to maximize the expected discounted utility of current and future consumption. We assume that the utility function is of the constant-relative-risk-aversion (CRRA) form, and so the problem is to:

$$\max E_t \left[\sum_{i=0}^{\infty} \left(\frac{1}{1+\rho} \right)^i \frac{c_{t+i}^{* 1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}} \right] \tag{1}$$

where E_t indicates that the expectation is conditional to the information set available to the individual at time t, ρ is the rate of subjective time preference, and φ is the elasticity of intertemporal substitution. Preferences are defined over the composite term $c_{t+i}^* = c_{t+i} - hc_{t+i-1}$. This allows for non-separability in utility over time. The parameter h measures habit persistence (if h > 0) or durability (h < 0). Habit persistence implies that higher is the previous consumption (bigger is the habit), higher also must be the current consumption to deliver the same effect. Also note that habits wear off over time. On the other hand, not all nondurables or services are perishable so that consumption and expenditure can be equated. Some goods and services are durables in their nature thus adding to utility over time, see Costantinides and Ferson (1991).

The maximization problem is subject to the per-period budget constraint:

$$a_{t+1} = (1+r)(a_t + y_t - c_t) \tag{2}$$

where a_t is financial wealth at the beginning of period t. The real interest rate is assumed to be constant over time,¹ y_t is income, and c_t consumption. This translates into a lifetime budget constraint of the form:

$$\sum_{i=0}^{\infty} \frac{E_t \left(c_{t+i} \right)}{(1+r)^i} = a_t + \sum_{i=0}^{\infty} \frac{E_t \left(y_{t+i} \right)}{(1+r)^i} \tag{3}$$

where $\sum_{i=0}^{\infty} \frac{E_t(y_{t+i})}{(1+r)^i} = H_t$ represents the expected discounted value of human wealth.

The Euler equation of this problem for a generic period i is:

¹Time-varying interest rate can be easily added to the analysis.

$$E_{t+i}\left[\left(\frac{c_{t+i+1}^*}{c_{t+i}^*}\right)^{-\frac{1}{\varphi}}\right] = \frac{1+\rho}{1+r}$$

and by the law of iterated expectations:

$$E_t \left[\left(\frac{c_{t+i+1}^*}{c_{t+i}^*} \right)^{-\frac{1}{\varphi}} \right] = \frac{1+\rho}{1+r}$$

$$\tag{4}$$

Following Hall's (1978) seminal paper, the majority of the empirical literature on consumption has used Euler equations derived from the intertemporal optimization problem to estimate structural parameters (primarily, the elasticity of intertemporal substitution, and, in some instances, the rate of subjective time preference). One reason for this is that the non-linearities induced by stochastic labour income and CRRA preferences prevent obtaining a closed form solution for consumption. The Euler equation, a first order condition implied by the optimization problem, does not present such problems and yet allows estimation of some of the structural behavioural parameters of the problem. A clear drawback of the Euler equation approach is its inability to address questions such as how consumption reacts to unexpected changes in income and tax reform, for example. Thus, the Euler equation allows estimation of intertemporal substitution effects, but is silent regarding income effects. This limits researchers' ability to answer policy-relevant questions.

The approach we take in this paper is to take the Euler equation as a first step in the derivation of an approximate closed form solution for consumption. We consider a second order Taylor approximation of the Euler equation and substitute this into the budget constraint to derive a dynamic relationship linking current consumption to lagged consumption, income, assets, and permanent and transitory shocks to income. The dynamics depend on all the parameters of the problem, not just intertemporal substitution. While the idea of approximating the Euler equation is not new, the solution we propose is novel in that we are able to derive a workable approximation when preferences are characterized by intertemporal non-separabilities. The only requirement for this solution to work is that higher moments of the distribution of consumption are constant over time.

2.1Approximating the Euler equation

As mentioned above, our problem is to approximate the Euler equation (4). We consider a second order Taylor expansion of $\left(\frac{c_{t+i+1}^*}{c_{t+i}^*}\right)^{-\frac{1}{\varphi}}$ around $\left(E_t\left(c_{t+i+1}^*\right), E_t\left(c_{t+i}^*\right)\right)$, and take expectations to yield²

 $^{^{2}}$ See the Appendix for a complete derivation.

$$E_{t}\left[\left(\frac{c_{t+i+1}^{*}}{c_{t+i}^{*}}\right)^{-\frac{1}{\varphi}}\right] = \left(\frac{E_{t}\left(c_{t+i+1}^{*}\right)}{E_{t}\left(c_{t+i}^{*}\right)}\right)^{-\frac{1}{\varphi}} \times \underbrace{\left[1 + \frac{1+\varphi}{2\varphi^{2}}CV_{t}\left(c_{t+i+1}^{*}\right)^{2} + \frac{1-\varphi}{2\varphi^{2}}CV_{t}\left(c_{t+i}^{*}\right)^{2} - \frac{1}{2\varphi^{2}}\varrho CV_{t}\left(c_{t+i+1}^{*}\right)CV_{t}\left(c_{t+i}^{*}\right)\right]}_{\omega_{t}}$$

$$= \omega_{t}\left(\frac{E_{t}\left(c_{t+i+1}^{*}\right)}{E_{t}\left(c_{t+i}^{*}\right)}\right)^{-\frac{1}{\varphi}}$$
(5)

where $CV_t\left(c_{t+i+1}^*\right) = \frac{\sqrt{Var_t(c_{t+i+1}^*)}}{E_t(c_{t+i+1}^*)}$ is the coefficient of variation of c_{t+i+1}^* (conditional on infomation available at t), and ϱ is the (conditional) coefficient of correlation between c_{t+i+1}^* and c_{t+i}^* .

In what follows, we assume $\omega_t = \omega$ for all t. This assumption is satisfied by many distributions, including the log-normal (a proof for the log-normal case can be found in the Appendix; there, we prove that the only requirement is that the higher order moments of the consumption distribution are stationary).³ From now on, we define $\kappa = \omega^{-1}$. Precautionary saving is consistent with $\kappa < 1$ (i.e., $\omega > 1$). This coefficient captures a precautionary motive for saving; if preferences are characterized by prudence, $\kappa < 1$. Indeed, κ measures the slope of the intertemporal consumption path: prudent consumers delay spending in response to uncertainty.

Using the recursion implicit in (5), the Euler equation (4) can be rewritten as

$$E_t\left(c_{t+i}^*\right) = c_t^* \left[\frac{1+r}{\kappa\left(1+\rho\right)}\right]^{i\varphi} \tag{6}$$

Substituting $c_{t+i}^* = c_{t+i} - hc_{t+i-1}$ and equation (6) in the LHS of the lifetime expected budget constraint (3), we obtain:

$$\sum_{i=0}^{\infty} \frac{E_t \left(c_{t+i} \right)}{\left(1+r \right)^i} = \frac{\left(1+r \right) \left(\kappa \left(1+\rho \right) \right)^{\varphi} c_t - h \left(1+r \right)^{\varphi} c_{t-1}}{\left(1+r-h \right) \left[\left(\kappa \left(1+\rho \right) \right)^{\varphi} - \left(1+r \right)^{\varphi-1} \right]}$$

Solving for c_t in (3), we finally obtain:

$$c_{t} = \frac{1+r-h}{1+r} \left(1 - \frac{(1+r)^{\varphi-1}}{(\kappa (1+\rho))^{\varphi}} \right) (a_{t} + H_{t}) + \frac{(1+r)^{\varphi-1}}{(\kappa (1+\rho))^{\varphi}} hc_{t-1}$$
(7)

Clearly, assuming $r = \rho$, no habits or durability (h = 0), and quadratic preferences $(\kappa = 1)$, one obtains the traditional consumption function solution (see Deaton, (1992), p. 82):

$$c_t = \frac{r}{1+r}(a_t + H_t)$$



 $^{^{3}}$ Krueger and Perri (2003) provide partial evidence in support of this assumption, by showing that the variance of consumption has been fairly constant over the last 20 years.

2.2 Superior Information

One of the puzzling findings in the Euler equation literature is that consumption appears to be excessively sensitive to (predictable) income changes (see Deaton (1992) and Browning and Lusardi (1995)). It has been suggested that excess sensitivity (as well as its mirror image of excess smoothness) can be explained by the fact that the econometrician's information set differs from the one held by the agent, i.e., that households forecast future realizations of income on the basis of a larger information set than the econometrician's. In other words, agents hold *superior information* (see Shiller (1972); Quah (1990); Flavin (1993); and Pistaferri (2001)).

In what follows we will assume that people observe the history of the income process (as the econometrician does), but may also have private information concerning future income realizations. For example, people may have advance notices regarding promotions, firm closure, etc. We also assume that income evolves according to the process:

$$y_t = P_t + v_t \tag{8}$$

$$P_t = P_{t-1} + u_t \tag{9}$$

where P_t is a permanent component, following a random walk process with i.i.d. innovation u_t , and v_t is a transitory i.i.d. component. If the individual's information set coincides with the econometrician's, then

$$E_t^H \Delta y_{t+i} = E_t^H (u_{t+i} + v_{t+i} - v_{t+i-1})$$

= $-v_t \mathbf{1} \{i = 1\}$

where $\mathbf{1}\{.\}$ is the indicator function and we indicate with E_t^H an expectation that conditions only on the history of the income process. However, in the presence of superior information, the expectation is

$$E_t^S \Delta y_{t+i} = E_t^S u_{t+i} + E_t^S v_{t+i} - E_t^S v_{t+i-1}$$

where we indicate with E_t^S the individual's expectation when the information set includes private information about future income realizations, transitory and permanent, as well as the history of the income process (i.e., 8 and 9). Call $\gamma_{t,t+i}$ the probability that at time t the individual has superior information regarding income realizations at time t + i (with probability $1 - \gamma_{t,t+i}$ the individual's forecasts are no better than the econometrician's, i.e., they are only based on the history of the income process). Clearly, $\gamma_{t,t} \equiv 1$. Thus we can write, using the law of iterated expectations

$$E_t \Delta y_{t+i} = \gamma_{t,t+i} E_t^S \Delta y_{t+i} + \left(1 - \gamma_{t,t+i}\right) E_t^H \Delta y_{t+i}$$

The special (and conventional) case in which the individual's and econometrician's information sets coincide is when $\gamma_{t,t+i} = 0$, and therefore $E_t(.) = E_t^H(.)$. Suppose to facilitate intuition that there is no transitory component, so that income follows a pure random walk, and that the individual is working at a firm that is contemplating laying off part of its labor force. The individual's expectation concerning next year's income growth in the absence of advance information concerning the lay-offs ($\gamma_{t,t+1} = 0$) is $E_t \Delta y_{t+1} = 0$ because of the random walk assumption inherited by the history of the process. However, if there is some advance information, $\gamma_{t,t+1} > 0$ could measure the probability of being laid-off, and $E_t^S \Delta y_{t+1}$ could measure income growth when moving from employment to unemployment (for example, in the absence of unemployment benefits and lack of alternative job opportunities, $E_t^S \Delta y_{t+1} = -y_t$).

To obtain a workable case, we will make the (admittedly strong) assumption that the superior information expectations are centered around the actual realization of the income shocks, i.e., $E_t^S u_{t+i} = u_{t+i}$ and $E_t^S v_{t+i} = v_{t+i}$. This gives

$$E_t \Delta y_{t+i} = \gamma_{t,t+i} \left(u_{t+i} + v_{t+i} \right) - \gamma_{t,t+i-1} v_{t+i-1} \tag{10}$$

for $i \ge 1$. Thus, for example when i = 1:

$$E_t \Delta y_{t+1} = \gamma_{t,t+1} \left(u_{t+1} + v_{t+1} \right) - v_t$$

and when i = 2,

$$E_t \Delta y_{t+2} = \gamma_{t,t+2} \left(u_{t+2} + v_{t+2} \right) - \gamma_{t,t+1} v_{t+1}$$

and so forth.

Since $y_{t+i} = y_t + \sum_{j=1}^{i} \Delta y_{t+j}$, we can use (10) to write

$$E_t y_{t+i} = y_t + \sum_{j=1}^{i} \left(\gamma_{t,t+j} \left(u_{t+j} + v_{t+j} \right) - \gamma_{t,t+j-1} v_{t+j-1} \right)$$
(11)

We assume that the probability of holding superior information decays geometrically with time, or $\gamma_{t,s} = \gamma^{s-t4}$. This is an approximation to having the probability of superior infomation distributed exponential. This assumption allows us to rewrite (11) as:



⁴This assumption, as well as the general flavor of the private information modelling, is also in Willman (2003).

$$E_t y_{t+i} = y_t + \sum_{j=1}^{i} \left(\gamma^j \left(u_{t+j} + v_{t+j} \right) - \gamma^{j-1} v_{t+j-1} \right)$$
(12)

Substitution of (12) into $\sum_{i=0}^{\infty} \frac{E_t(y_{t+i})}{(1+r)^i} = H_t$ gives:

$$H_t = y_t \frac{1+r}{r} - \frac{1}{r} v_t + \frac{1+r}{r} \sum_{i=1}^{\infty} \left(\frac{\gamma}{1+r}\right)^i u_{t+i} + \sum_{i=1}^{\infty} \left(\frac{\gamma}{1+r}\right)^i v_{t+i}$$
(13)

2.3 The consumption function

Substitution of (13) into (7) gives:

$$c_t - \phi c_{t-1} = \theta a_t + y_t \frac{\theta (1+r)}{r} - \frac{\theta}{r} v_t + H_u + H_v$$
(14)

where $\phi = \frac{(1+r)^{\varphi-1}}{(\kappa(1+\rho))^{\varphi}}h$, $\theta = \frac{1+r-h}{1+r}\left(1 - \frac{(1+r)^{\varphi-1}}{(\kappa(1+\rho))^{\varphi}}\right)$, $H_u = \frac{\theta(1+r)}{r}\sum_{i=1}^{\infty}\left(\frac{\gamma}{1+r}\right)^i u_{t+i}$ and $H_v = \theta\sum_{i=1}^{\infty}\left(\frac{\gamma}{1+r}\right)^i v_{t+i}$.

To derive an empirically tractable expression we need to eliminate H_u and H_v . This can be done by considering the lag-equivalent of (14), using the income process (8)-(9), and then taking first differences to obtain the consumption function:

$$c_{t} = \frac{\left[(1+r)\left(1-\theta\gamma\right)+\gamma\phi\right]}{\gamma}c_{t-1} - \frac{\phi\left(1+r\right)}{\gamma}c_{t-2} - \frac{\theta\left(1-\gamma\right)\left(1+r\right)}{\gamma}a_{t-1} - \frac{\theta\left(1+r\right)^{2}\left(1-\gamma\right)}{r\gamma}y_{t-1} + \frac{\theta\left(1-\gamma\right)\left(1+r\right)}{r\gamma}v_{t-1}$$
(15)

Making the traditional assumptions $r = \rho$, no habits (h = 0), no superior information $(\gamma = 0)$, and quadratic preferences $(\kappa = 1)$ (which give $\phi = 0$, $\theta = \frac{r}{1+r}$), one obtains (after some manipulations):

$$c_t = c_{t-1} + u_t + \frac{r}{1+r}v_t$$

which gives the familiar prediction that consumption is a martingale, and that it reacts on a one-to-one basis to permanent shocks to income, and to a much lower extent (i.e., the annuity value) to transitory shocks (Deaton (1992) p.147).

It is possible that people are better able to forecast future permanent income shocks rather than future transitory shocks. For example, a lottery win (a typical example of transitory, albeit possibly large, shock) may be more difficult to predict than promotions or lay-offs. Alternatively, the opposite might be true. For example, an individual working in a firm that is expanding production may predict with a certain degree of confidence that the firm will ask her to work overtime, but may be much less able to predict, say, a disabling

accident. In other words, this is an empirical issue. We can modify (in an *ad-hoc* way) equation (10) to rewrite it (for $i \ge 1$) as:

$$E_t \Delta y_{t+i} = \gamma_{t,t+i}^u u_{t+i} + \gamma_{t,t+i}^v v_{t+i} - \gamma_{t,t+i-1}^v v_{t+i-1}$$
(16)

where $\gamma_{t,t+i}^{u}$ ($\gamma_{t,t+i}^{v}$) measures the probability of holding superior information regarding the permanent (transitory) shock. Furthermore, we make an assumption of proportionality, i.e., that $\gamma_{t,t+j}^{u} = \psi \gamma_{t,t+j}^{v} = \psi \gamma_{t,t+j}^{v}$. The parameter ψ measures the knowledge of the individual's permanent shock relative to the transitory shock. Thus, if the predictive power is the same, $\psi = 1$, while if the individual knows more about the permanent shock than about the transitory shock, $\psi > 1$ (vice versa if she knows more about the transitory than the permanent shock).

Our consumption function (15) rewrites as:

$$c_{t} = \frac{(1+r)(1-\theta\gamma)+\gamma\phi}{\gamma}c_{t-1} - \frac{\phi(1+r)}{\gamma}c_{t-2} - \frac{\theta(1-\gamma)(1+r)}{\gamma}a_{t-1} - \frac{\theta(1+r)^{2}(1-\gamma)}{r\gamma}y_{t-1} + \frac{\theta(1-\gamma)(1+r)}{r\gamma}v_{t-1} + \frac{\gamma\theta(1+r)(1-\psi)}{r\gamma}u_{t}$$
(17)

If $\gamma^u = \gamma^v$, $\psi = 1$ and we go back to the case of equation (15).

3 Empirical approach

In the absence of measurement error (and regardless of whether $\psi = 1$ or $\psi \neq 1$), the following moment condition could be used to estimate (some of) the parameters of interest:

$$E_{t-2}\left[c_t - \frac{\left[(1+r)\left(1-\theta\gamma\right)+\gamma\phi\right]}{\gamma}c_{t-1} + \frac{\phi\left(1+r\right)}{\gamma}c_{t-2} + \frac{\theta\left(1-\gamma\right)\left(1+r\right)}{\gamma}a_{t-1} + \frac{\theta\left(1+r\right)^2\left(1-\gamma\right)}{r\gamma}y_{t-1}\right] = 0$$

With classical measurement error in consumption ($\tilde{c} = c + \varepsilon$), the error term of (17) is:

$$\zeta_{t} = \frac{\theta \left(1-\gamma\right) \left(1+r\right)}{r\gamma} v_{t-1} + \frac{\gamma \theta \left(1+r\right) \left(1-\psi\right)}{r\gamma} u_{t} + \varepsilon_{t} - \frac{\left[\left(1+r\right) \left(1-\theta\gamma\right)+\gamma\phi\right]}{\gamma} \varepsilon_{t-1} + \frac{\phi \left(1+r\right)}{\gamma} \varepsilon_{t-2} \quad (18)$$

and so the correct moment condition to be used is:

$$E_{t-3}\left[\tilde{c}_{t} - \frac{\left[(1+r)\left(1-\theta\gamma\right)+\gamma\phi\right]}{\gamma}\tilde{c}_{t-1} + \frac{\phi\left(1+r\right)}{\gamma}\tilde{c}_{t-2} + \frac{\theta\left(1-\gamma\right)\left(1+r\right)}{\gamma}a_{t-1} + \frac{\theta\left(1+r\right)^{2}\left(1-\gamma\right)}{r\gamma}y_{t-1}\right] = 0$$

⁵This is necessary if we want to be able to eliminate H_u and H_v from (14).



where \tilde{c} is what we observe, c is the true value, and ε the measurement error. This is the specification we bring to the data, requiring the use of instruments dated t-3 and earlier. Note that ψ cannot be estimated from here. However, we will show how one can use the residuals of these regressions, coupled with income growth residuals, to identify ψ .

An IV regression of c_t onto c_{t-1} , c_{t-2} , a_{t-1} , and y_{t-1} (using instruments dated t-3 and earlier), followed by a minimum distance mapping between reduced form and structural parameters identifies most of the structural parameters. That is, suppose we run the regression:

$$c_t = \alpha_0 + \alpha_1 c_{t-1} + \alpha_2 c_{t-2} + \alpha_3 a_{t-1} + \alpha_4 y_{t-1} + \zeta_t \tag{19}$$

Then we identify the "reduced form" parameters α_j (j = 0, 1, 2, 3, 4). There is a mapping between the reduced form parameters and the structural parameters h, γ and κ given by (20):

$$\begin{pmatrix} \alpha_{1} \\ \alpha_{2} \\ \alpha_{3} \\ \alpha_{4} \end{pmatrix} = \begin{pmatrix} \frac{(1+r)}{\gamma} - (1+r-h) \frac{(\kappa(1+\rho))^{\varphi} - (1+r)^{\varphi-1}}{(\kappa(1+\rho))^{\varphi}} + \frac{(1+r)^{\varphi-1}}{(\kappa(1+\rho))^{\varphi}} h \\ -\frac{(1+r)^{\varphi} - h}{(\kappa(1+\rho))^{\varphi}} - \frac{(1+r-h)(1-\gamma)}{\gamma} \frac{(\kappa(1+\rho))^{\varphi} - (1+r)^{\varphi-1}}{(\kappa(1+\rho))^{\varphi}} \\ -\frac{(1+r-h)(1+r)(1-\gamma)}{r\gamma} \frac{(\kappa(1+\rho))^{\varphi} - (1+r)^{\varphi-1}}{(\kappa(1+\rho))^{\varphi}} \end{pmatrix}$$

$$\alpha = f(\boldsymbol{\lambda})$$
(20)

The minimum distance method finds λ by minimizing:

$$\min\left(\hat{\boldsymbol{\alpha}} - f\left(\boldsymbol{\lambda}\right)\right)' \mathbf{W}\left(\hat{\boldsymbol{\alpha}} - f\left(\boldsymbol{\lambda}\right)\right)$$

where $\mathbf{W} = var(\hat{\boldsymbol{\alpha}})^{-1}$. Since dim $(\hat{\boldsymbol{\alpha}}) > \dim(\boldsymbol{\lambda})$, we have identifying restrictions that can be used to test the goodness of fit of the model. While this approach is less conventional and less efficient than standard nonlinear GMM, it is asymptotically equivalent to it, and less subject to convergence problems. See Malinvaud (1980) and Chamberlain (1984) for technical details.

3.1 Identification of ψ

The minimum distance procedure identifies the parameters h, γ and κ . However, we still need to identify ψ . We do this by exploiting the restrictions imposed by the model on the joint behavior of consumption and income errors. In order to understand how this is done, consider the IV residual of equation (19). Given (18), this is a consistent estimate of

$$g_t^c = \left(\begin{array}{c} \frac{\theta(1-\gamma)(1+r)}{r\gamma}v_{t-1} + \frac{\gamma\theta(1+r)(1-\psi)}{r\gamma}u_t + \\ \varepsilon_t - \frac{(1+r)(1-\theta\gamma)+\gamma\phi}{\gamma}\varepsilon_{t-1} + \frac{\phi(1+r)}{\gamma}\varepsilon_{t-2} \end{array}\right)$$

As for the income change residual, it is given by:

$$g_t^y = \Delta y_t = u_t + v_t - v_{t-1}$$

As in Meghir and Pistaferri (2004), notice that we can identify the variances of the transitory and permanent shocks in panel data:

$$-E\left(g_{t}^{y}g_{t-1}^{y}\right) = \sigma_{v}^{2}$$
$$E\left(g_{t}^{y}\left(g_{t-1}^{y}+g_{t}^{y}+g_{t+1}^{y}\right)\right) = \sigma_{u}^{2}$$

Now note that the covariance between consumption residual and income change residual is given by:

$$E\left(g_{t}^{c}g_{t}^{y}\right) = \frac{\gamma\theta\left(1+r\right)\left(1-\psi\right)}{r\gamma}\sigma_{u}^{2} - \frac{\theta\left(1-\gamma\right)\left(1+r\right)}{r\gamma}\sigma_{v}^{2}$$

This suggests that, if we knew the true values of the structural parameters h, γ and κ , we could run the following regression:

$$\frac{r}{\theta(1+r)}g_t^c g_t^y - \frac{(1-\gamma)}{\gamma}g_t^y g_{t-1}^y = (1-\psi)g_t^y \left(g_{t-1}^y + g_t^y + g_{t+1}^y\right) + \xi_t \tag{21}$$

where ξ_t is sampling variability satisfying $E(\xi_t) = 0$. This regression would identify $(1 - \psi)$. We replace the unknown strucural parameters and unobserved errors in consumption and income change by their consistent estimates in (21); the estimate of $(1 - \psi)$ is also consistent under usual assumptions. Since these are residuals in levels, this regression is estimated excluding influential values (i.e., eliminating the lower and upper percentiles of the income distribution).⁶

4 The data

The main difficulty of implementing our tests is to find a data set that satisfies two principal requirements: it features a longitudinal component, and it contains information on household consumption, income, and assets. The PSID (Panel Study of Income Dynamics) satisfies the first requirement fully and the second partly. In particular, data on assets are available only at five-year intervals (starting in 1984), and the only

$$\frac{E\left(\frac{r}{\theta(1+r)}g_t^c g_t^y - \frac{(1-\gamma)}{\gamma}g_t^y g_{t-1}^y\right)}{E\left(g_t^y \left(g_{t-1}^y + g_t^y + g_{t+1}^y\right)\right)} = (1-\psi)$$

and replacing unobserved means with empirical analogs. When we do this, we obtain similar results.

⁶One could also estimate $(1 - \psi)$ non-parametrically noting that

consumption data available is for food. It is not clear how well food consumption behavior generalizes to nondurable consumption behavior. For this reason, we use imputed non-durable consumption data following the method proposed in Blundell, Pistaferri and Preston (2004). The idea is to impute consumption to all PSID households combining PSID data with consumption data from repeated CEX cross-sections.⁷ The approach consists of writing the demand for food (a consumption item available in both surveys) as a function of prices, total non-durable expenditure, and a host of demographic and socio-economic characteristics of the household. Food expenditure and total expenditure are modeled as jointly endogenous. Under monotonicity (normality) of food demands, these functions can be inverted to obtain a measure of non-durable consumption in the PSID. Blundell, Pistaferri and Preston (2004) review the conditions that make this procedure reliable and show that it is able to reproduce remarkably well the trends in the consumption distribution. We refer the interested reader to that paper for more technical details.⁸

Since the PSID has been widely used for microeconometric research, we shall only sketch the description of its structure in this section.⁹ The PSID started in 1968 collecting information on a sample of roughly 5,000 households. Of these, about 3,000 were representative of the US population as a whole (the core sample), and about 2,000 were low-income families (the Census Bureau's Survey of Economic Opportunities, or SEO sample). Thereafter, both the original families and their split-offs (children of the original family forming a family of their own) have been followed.

The PSID includes a variety of socio-economic characteristics of the household, including age, education, labor supply, and income of household members. Questions referring to income and wages are retrospective; thus, those asked in 1993, say, refer to the 1992 calendar year. In contrast, many researchers have argued that the timing of the survey questions on food expenditure is much less clear (see Hall and Mishkin (1982) and Altonji and Siow (1987), for two alternative views). Typically, the PSID asks how much is spent on food in an average week. Since interviews are usually conducted around March, it has been argued that people report their food expenditure for an average week around that period, rather than for the previous calendar year as is the case for family income. We assume that food expenditure reported in survey year t refers to the previous calendar year.

All monetary variables are deflated using the CPI (1982-84). Education level is computed using the PSID

Working Paper Series No. 572 January 2006

⁷Previous studies (Skinner (1987)) have imputed non-durable consumption data in the PSID using CEX regressions of non durable consumption on consumption items (food, housing, utilities) and demographics available in both the PSID and the CEX.

⁸The definition of total non-durable consumption we use is similar to Attanasio and Weber (1995). It includes food (at home and away from home), alcoholic beverages and tobacco, services, heating fuel, transports (including gasoline), personal care, clothing and footwear, and rents. It excludes expenditure on various durables, housing (furniture, appliances, etc.), health, and education. Unlike Attanasio and Weber, we also includes services from housing and vehicles (data kindly provided by David Johnson at BLS).

⁹See Hill (1992) for more details about the PSID.

variable "grades of school finished". Individuals who changed their education level during the sample period are allocated to the highest grade achieved. We construct a PSID panel data set using data on households continuously present from 1980 to 1985.¹⁰ To avoid dealing with a number of complicated issues having to do with family formation, family dissolution, human capital accumulation and retirement choices, we select a sample of demographically stable households. Thus, ours is a sample of continuously married couples headed by a male (with or without children). We eliminate households facing some dramatic family composition change over the sample period. In particular, we keep only those with no change, and those experiencing changes in members other than the head or the wife (children leaving parental home, say). We next eliminate households headed by a female. We also eliminate households with missing report on education,¹¹ and those with topcoded income. We drop some income outliers.¹² We then drop those born before 1920 or after 1959. As noted above, the initial 1967 PSID contains two groups of households. The first is representative of the US population (61 percent of the original sample); the second is a supplementary low income subsample (also known as SEO subsample, representing 39 percent of the original 1967 sample). To account for the changing demographic structure of the US population, starting in 1990 a representative national sample of 2,000 Latino households has been added to the PSID database. We exclude both Latino and SEO households and their split-offs. Finally, we drop those aged less than 25 or more than 65. This is to avoid problems related to changes in family composition and education, in the first case, and retirement, in the second. The final sample used in the exercise below is composed of 1,125 households.

Our measure of income includes earnings of all household members and income from assets; it is net of federal taxes (available in the survey); it excludes transfers. Our measure of assets is the sum of assets from own farm or business, checking/savings accounts, other real estate assets, stocks and IRAs, home equity, and other savings, net of household liabilities. We also experimented with a definition of assets that excludes home equity (see below). Descriptive statistics on the main variables of interest are reported in Table 1.

5 Results

Table 2 reports the estimates of the structural parameters of the model using the strategy outlined in Section 3. Column (1) reports the results for the entire sample, while columns (2) and (3) reports results separately for the high educated and the low educated, respectively. In all specifications, the overidentifying restriction

¹⁰To estimate equation (19), we need data on consumption for three consecutive years (t, t-1, and t-2), lagged income (t-1), and lagged assets (t-1). Moreover, we need instruments (consumption and income data) dated t-3 to t-5. The only year sequence in which these conditions are all satisfied is 1980-85 (recall that data on assets are available only at five-year intervals). Unfortunately, the year sequence 1985-90 cannot be used because questions on food consumption were not asked in 1987 and 1988. Data after 1992 are still in "early release" format and thus not used.

 $^{^{11}{\}rm When}$ possible, we impute values for education using adjacent records on this variable.

 $^{^{12}}$ An income outlier is defined as a household with an income growth above 500 percent, below -80 percent, or with a level of income below \$100 a year or below the amount spent on food.

is not rejected by the data. All the reduced form regressions also include a variety of demographic controls (age, family size, number of children, grades of schooling and race).

Starting from κ , we find that its estimate is quite similar across groups and displays very low standard errors. Recall that κ is an indicator of the slope of the intertemporal consumption path. A finding $\kappa < 1$ supports the precautionary motive for saving. This is indeed what we find.

Next, we turn to h, the parameter that measures the extent of habit persistence/durability in preferences for consumption. Given that h < 0, we find evidence that durability dominates habit persistence. The effect is stronger among the low educated. The standard errors are high, however, and so inference must be taken with caution.

We now turn to γ . According to the interpretation given above, it should measure the probability that individuals draw their expectation about future realizations of the income shocks from some "superior information" distribution rather than from historical realizations. The estimate in the whole sample is close to 1 (statistically, the hypothesis that $\gamma = 1$ cannot be rejected). We find somewhat higher estimates among the high educated, and lower estimates among the low educated. A few things must be remarked. First, we strongly rejects the hypothesis of *no* superior information ($\gamma = 0$). Second, we find that γ is very high. This could, however, also be interpreted as an indication that people's superior information are not necessarily centered around the actual realization of the shock. That is, people may tend to exaggerate (in one sense or another) their expectation of the shock, behaving over-optimistically in the case of a positive shock and over-pessimistically in the case of a negative shock. With a longer panel this effect would probably be averaged out.

The final parameter we estimate is ψ . Recall that ψ measures the difference in (relative) prediction power for the permanent and the transitory shock. Our results show that people have better predictive power about the transitory shock than the permanent shock. The effect is magnified when we estimate the parameters separately by education.

In Table 3 we perform some sensitivity analysis to check the robustness of our results. One common feature is the remarkable similarity among experiments. In column (1) we replicate our baseline estimates of Table 2. In columns (2)-(4) we adopt different definitions of wealth, consumption, and income (excluding home equity; excluding services from housing and vehicles; and including transfers, respectively). Interestingly, excluding durable services from our definition of consumption gives a lower estimate of h, the extent of "durability" in preferences for consumption. Including transfers in our definition of consumption reduces the probability of predicting permanent shocks relative to transitory shocks. In columns (5) and (6) we check the sensitivity of our results with respect to assumptions made about the parameters r, ρ , and φ . In column (5) we assume $r = \rho = 0.02$. In column (6) we assume $\varphi = 0.5$, which corresponds to a coefficient of relative risk aversion of 2, a standard benchmark outside the log-utility case ($\varphi = 1$). This last case predictably increases the estimate of κ , as consumers are now assumed to be more prudent, hence engaging more in precautionary savings behavior.

6 Conclusions

In this paper we estimate a dynamic consumption function using microeconomic data drawn from the PSID. We show that the dynamics hinge upon a number of structural parameters: the degree of intertemporal separability in preferences (which may indicate habit persistence or durability), the slope of the intertemporal consumption path (which captures the importance of the precautionary motive for savings), the impact of advance information about future income prospects (which signals a discrepancy of information between the individual and the econometrician), and the forecastability of permanent shocks relative to transitory shocks.

Our findings point to the existence of durability rather than habit persistence. This agrees with findings by Hayashi (1985) and Mankiw (1982), and in general with the lack of evidence for habit persistence found in Dynan (2001) and others. This means that the conventional definition of non-durable consumption most likely includes goods that provide services for longer than one year, thus obscuring (or offsetting) any habit persistence that may exist in the data. See Hayashi (1985).

As predicted by models with prudent consumers facing uncertain income, we find evidence for a positively sloped consumption path, meaning that consumers delay spending in response to the risks they face. As for the superior information issue, our results, taken at face value, would suggest that consumers have substantial information regarding near future income changes, especially short-lived ones. These results are consistent with two different explanations. The first is that most of the uncertainty is concentrated in the far distant future, and so it still generates precautionary savings. Alternatively, it may represent a violation of the assumption that consumers' expectations are centered around the actual realizations of the shocks. More likely, consumers are exaggerating (in one sense or another) their expectations, and therefore what should be interpreted as exaggeration is interpreted as excess information. This is an identification problem that cannot be solved in the context of our model. Nevertheless, the forecastability of transitory shocks relative to permanent shocks does not suffer from this identification problem (as long as "exaggeration" in predicting transitory shocks and permanent shocks is similar), and the results reveal that consumers are more able to predict the arrival of temporary shocks that persistent ones.

Future work should be directed towards trying to solve these identification problems, perhaps through the use of subjective expectations data (Dominitz and Manski (1997)), or through alternative specifications of the information set of the individuals.

Table 1: Descriptive statistics for 1985

Variable	Mean
Food expenditure	5,206
(Imputed) non-durable consumption	$25,\!365$
Income	$42,\!608$
Assets	$4,\!171$
Family size	3.54
Number of children	1.26
Age	44.37
College	0.52
White	0.93

Table 2

Estimates of structural parameters.

Baseline specification.

	(1)	(2)	(3)
κ	$\begin{array}{c} 0.9795 \\ (0.0194) \end{array}$	0.9735 (0.0008)	0.9738 (0.0042)
h	-0.9248 (0.4505)	-0.5021 (0.3866)	-0.8855 (0.6383)
γ	1.0202 (0.0494)	1.1167	0.9323 (0.0442)
ψ	0.9498 (0.0240)	0.7875 (0.1091)	0.8712 (0.1078)
Goodness of fit	$\begin{array}{c} 0.0022 \\ [1; \ 0.9623] \end{array}$	$0.0435 \\ [1; 0.8347]$	$\begin{array}{c} 0.0416 \\ [1; \ 0.8384] \end{array}$

Note: Column (1) refers to the whole sample specification; Columns (2) and (3) refer to the high- and loweducated sample, respectively. The estimates of κ , γ , a and λ are obtained from the minimum distance procedure, while that of ψ comes from imposing restrictions on the joint behavior of consumption and income change residuals (see the text for more details). In all cases, we assume r = 1%, $\rho = 3\%$, and $\varphi = 1$.

Table 3

Estimates of structural parameters.

Sensitivity analysis

	(1)	(2)	(3)	(4)	(5)	(6)
κ	$\underset{(0.0194)}{0.9795}$	$\underset{(0.0212)}{0.9797}$	$\underset{(0.0008)}{0.9727}$	$\underset{(0.0113)}{0.9783}$	$\underset{(0.0194)}{0.9924}$	$\underset{(0.0387)}{0.9783}$
h	-0.9248 $_{(0.4505)}$	-0.9239 (0.4593)	-0.7582 (0.4140)	-0.8556 $_{(0.4337)}$	-0.9322 (0.4519)	-0.9248 $_{(0.4505)}$
γ	$\underset{(0.0494)}{1.0202}$	$\underset{(0.0513)}{1.0198}$	$\underset{(0.0376)}{1.0687}$	$\underset{(0.0432)}{1.0247}$	$\underset{(0.0492)}{1.0275}$	$\underset{(0.0494)}{1.0202}$
ψ	$\underset{(0.0240)}{0.9498}$	$\underset{(0.0234)}{0.9510}$	$\underset{(0.0714)}{0.8271}$	$\underset{(0.0297)}{0.8917}$	$\underset{(0.0345)}{0.9277}$	$\underset{(0.0240)}{0.9498}$
Goodness of fit	0.0022 [1; 0.9623]	0.0099 [1; 0.9206]	$\begin{array}{c} 0.1021 \\ [1; \ 0.7493] \end{array}$	0.0931 [1; 0.7602]	0.0004 $[1; 0.9850]$	0.0022 [1; 0.9623]

Note: Column (1) is our baseline specification. In column (2) we use a measure of wealth that excludes housing. In column (3) we use a measure of consumption that excludes services from housing and vehicles. In column (4) we use a measure of income that includes transfers. refer to the high- and low-educated sample, respectively. In column (5) we set r = 2%, $\rho = 2\%$. In column (6) we set $\varphi = 0.5$. The estimates of κ , γ , a and λ are obtained from the minimum distance procedure, while that of ψ comes from imposing restrictions on the joint behavior of consumption and income change residuals (see the text for more details).

A Appendix: Second order approximation of the Euler equation

As explained in section 2 we consider a second order Taylor expansion of $\left(\frac{c_{t+i+1}^*}{c_{t+i}^*}\right)^{-\frac{1}{\varphi}}$ around $\left(E_t\left(c_{t+i+1}^*\right), E_t\left(c_{t+i}^*\right)\right)$. This gives:

$$\begin{pmatrix} c_{t+i+1}^{*} \\ c_{t+i}^{*} \end{pmatrix}^{-\frac{1}{\varphi}} = \left(\frac{E\left(c_{t+i+1}^{*}\right)}{E\left(c_{t+i}^{*}\right)} \right)^{-\frac{1}{\varphi}} - \frac{1}{\varphi} \left(\frac{E\left(c_{t+i+1}^{*}\right)}{E\left(c_{t+i}^{*}\right)} \right)^{-\frac{1}{\varphi}} \frac{\left(c_{t+i+1}^{*} - E\left(c_{t+i+1}^{*}\right)\right)}{E\left(c_{t+i}^{*}\right)} + \frac{1}{\varphi} \left(\frac{E\left(c_{t+i+1}^{*}\right)}{E\left(c_{t+i}^{*}\right)} \right)^{-\frac{1}{\varphi}} \frac{\left(c_{t+i}^{*} - E\left(c_{t+i}^{*}\right)\right)}{E\left(c_{t+i}^{*}\right)} + \frac{1 + \varphi}{2\varphi^{2}} \frac{1}{E\left(c_{t+i+1}^{*}\right)^{2}} \left(\frac{E\left(c_{t+i+1}^{*}\right)}{E\left(c_{t+i}^{*}\right)} \right)^{-\frac{1}{\varphi}} \left(c_{t+i+1}^{*} - E\left(c_{t+i+1}^{*}\right)\right)^{2} + \frac{1 - \varphi}{2\varphi^{2}} \frac{1}{E\left(c_{t+i+1}^{*}\right)^{2}} \left(\frac{E\left(c_{t+i+1}^{*}\right)}{E\left(c_{t+i}^{*}\right)} \right)^{-\frac{1}{\varphi}} \left(c_{t+i}^{*} - E\left(c_{t+i+1}^{*}\right)\right)^{2} - \frac{1}{2\varphi^{2}E\left(c_{t+i+1}^{*}\right)E\left(c_{t+i}^{*}\right)} \left(\frac{E\left(c_{t+i+1}^{*}\right)}{E\left(c_{t+i}^{*}\right)} \right)^{-\frac{1}{\varphi}} \left(c_{t+i+1}^{*} - E\left(c_{t+i+1}^{*}\right)\right) \left(c_{t+i}^{*} - E\left(c_{t+i}^{*}\right)\right) \right)^{2}$$

Taking expectations give equation (5) in the text.

B Appendix: Proof of the constancy of κ

We rewrite our expression for ω_t (omitting the time condition for simplicity):

$$\omega_{t} = \left[1 + \frac{1+\varphi}{2\varphi^{2}}CV\left(c_{t+i+1}^{*}\right)^{2} + \frac{1-\varphi}{2\varphi^{2}}CV\left(c_{t+i}^{*}\right)^{2} - \frac{1}{2\varphi^{2}}\varrho_{c_{t+i+1}^{*},c_{t+i}^{*}}CV\left(c_{t+i+1}^{*}\right)CV\left(c_{t+i}^{*}\right)\right]$$

Our problem is to show that the term ω is time independent. Note that in what follows we define c_{t+i+1}^* as x and c_{t+i}^* as y.

Let us start by assuming joint log-normality for both x and y (i.e., joint normality for x and y):

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim \log N \begin{pmatrix} e^{\mu_x + \frac{1}{2}\sigma_x^2} \\ e^{\mu_y + \frac{1}{2}\sigma_y^2} \end{pmatrix} \begin{pmatrix} e^{2\mu_x + 2\sigma_x^2} - e^{2\mu_x + \sigma_x^2} & \cos\left(x, y\right) \\ e^{2\mu_y + 2\sigma_y^2} - e^{2\mu_y + \sigma_y^2} \end{pmatrix}$$

where $\mu_a = E(\log a)$ and $\sigma_a^2 = var(\log a)$.

It follows that the squared coefficient of variation of x can be expressed as:

$$CV(x)^{2} = \frac{var(x)}{E(x)^{2}} = \frac{e^{2\mu_{x}+2\sigma_{x}^{2}} - e^{2\mu_{x}+\sigma_{x}^{2}}}{e^{2\mu_{x}+\sigma_{x}^{2}}} = e^{\sigma_{x}^{2}} - 1$$

which is *not* a function of the mean μ_x . This is symmetrically also true for $CV(y)^2$. Thus, assuming σ_x^2 and σ_y^2 are constant over time is enough to prove time-invariance of $CV(x)^2$ and $CV(y)^2$.

The next step is to show that the term $\rho_{c_{t+i+1}^*, c_{t+i}^*} CV(c_{t+i+1}^*) CV(c_{t+i}^*)$ also does not vary over time. Note that using our notation:

$$\varrho_{xy}CV(x)CV(y) = \frac{cov(x,y)}{E(x)E(y)} = \frac{E(xy)}{e^{(\mu_x + \mu_y) + \frac{1}{2}(\sigma_x^2 + \sigma_y^2)}} - 1$$

Utilising the results above we can show that

$$E(xy) = E(e^{\ln x}e^{\ln y}) = E(e^{\ln x + \ln y})$$
$$= e^{E(\ln x + \ln y) + \frac{1}{2}var(\ln x + \ln y)}$$
$$= e^{(\mu_x + \mu_y) + \frac{1}{2}(\sigma_x^2 + \sigma_y^2 + 2\tilde{\varrho}_{xy}\sigma_x\sigma_y)}$$

where $\tilde{\varrho}_{xy} = cov (\ln x, \ln y)$, and thus

$$\varrho_{xy}CV(x)CV(y) = \frac{e^{\left(\mu_x+\mu_y\right)+\frac{1}{2}\left(\sigma_x^2+\sigma_y^2+2\tilde{\varrho}_{xy}\sigma_x\sigma_y\right)}}{e^{\left(\mu_x+\mu_y\right)+\frac{1}{2}\left(\sigma_x^2+\sigma_y^2\right)}} - 1$$
$$= e^{\frac{1}{2}2\tilde{\varrho}_{xy}\sigma_x\sigma_y} - 1$$

again, not a function of the means. If the covariance between $\ln x$ and $\ln y$ is also constant over time, then κ is constant over time.



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