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Abstract

This paper aims to address the issue of public pension reforms under demographic ageing that is likely to occur in Europe over the next 50 years. Three possible scenarios are analysed in a Blanchard OLG framework.

These include: i) a decrease both in public pensions and the lump sum labour income tax, ii) a decrease both in public pensions and the distortionary corporate tax, iii) an increase in the retirement age. The analysis focuses on the effects of these fiscal policies on key economic variables such as consumption, private and public debt, output and wages. Quantitative experiments assess the impact of different fiscal policies in terms of public debt sustainability but most importantly suggest policies that smooth the transition of the economy to the new equilibrium. The main results suggest that the adverse effects of pension reforms on consumption are moderated when they are accompanied by appropriate taxation policies. In particular, when the tax response is rapid most of the adverse movement in consumption is avoided while public and national debt reach lower equilibrium levels.

JEL codes: E6; H3; J1; H55

Keywords: Ageing, Pension Reforms, Taxation, Overlapping Generations

Non-technical summary

This paper addresses the issue of pension reforms under demographic change that are likely to occur in Europe over the next 50 years. This demographic change is characterised not only by lower population growth but also by the alteration of the population structure, with a decline in the proportion of young people to retired workers. This has substantial ripple effects on public pension systems, which in Europe are mostly financed in a pay-as-you-go manner, i.e. current pensions are financed by current revenues through payroll taxes on employees. An ageing population puts a burden on current employees as a higher ratio of pensioners to employees necessitates higher tax contributions to maintain the sustainability of the public pension system. In the last forty years, substantial increases in income taxes have been imposed to finance public pension systems causing a number of labour market distortions such as a lower participation rate. Against this backdrop, European governments are considering a series of reforms that will ensure sustainability of public pension systems and may boost the already rigid European labour market.

In this paper, we employ a theoretical framework to analyse different public pension reform schemes and the response of the economy in the short and medium run. We assume that the government can use different taxation schemes to minimize adverse effects (of the pension reform) during the transition of the economy to a new equilibrium with a reformed pension system. We examine different taxation responses that can cushion the initial adverse effect on consumption. To model these effects, we use the workhorse framework of Blanchard (1985) with overlapping generations and finite horizons. To capture life cycle behaviour and introduce pension systems, we adopt a modified version of the framework in Nielsen (1994) and Heijdra and Bettendorf (2005). We depart from the set-up in these studies by allowing for an unbalanced pension system and budget deficits because one of our main aims is to study the dynamics of public debt during pension reforms. In addition, we illustrate the impact of declining population and ageing by allowing for different birth and death rates (Buiter, 1988) and age dependent labour productivity. Another departure from the existing literature is a realistic supply side where firms issue equities and face adjustment costs in investment. This enables us to consider the impact of pension reforms on capital accumulation and investment decisions.

In addition we also examine different fiscal policies that can be used to "soften" the negative impact that reforms may cause. The model employed in this paper is not of the magnitude of large scale simulation models, but our simple framework can be used readily for quick quantitative assessment of fiscal policy experiments. Most importantly it enables us to observe the transition path of the main macroeconomic variables from the current state to the new long run equilibrium featuring

the desired pension system.

Against this backdrop, three policy experiments are analysed: (a) a suspension of the public pension system accompanied by a fall of the lump sum income tax; (b) a suspension of the pension systems accompanied by a fall in the corporate tax rate and (c) the effect of an increase of the pension age on private and public finances.

The findings suggest that when drastic pension cuts are accompanied by a reduction of the corporate distortionary tax on firms' gross output (case (b)), the adverse effects on per capita consumption are minimised in the short run while in the long run output and consumption are higher whilst private and public indebtedness reach lower levels.

The same impact on the demand side can be observed when a lump sum income tax is used instead (case (a)). However, the supply side remains unaffected by these fiscal measures and their impact on the long run consumption is considerably smaller.

The outcome of both these policies is affected by the reaction function that the government uses to adjust to public debt developments. In other words if the government reacts rapidly and adjusts taxes then the negative impact on consumption can be minimised.

The last policy experiment models an increase in the retirement age and leaves the amount of pension benefits unchanged. This policy appears to be unsuccessful in economies with high levels of public debt especially if the aim is lower public indebtedness. Even if this policy measure is able to sustain the current fiscal position, the young generations of declining population eventually have to pay high taxes to sustain the pension system.

1 Introduction

Europe is experiencing a major demographic transition. This demographic change not only includes lower population growth but also alters the population structure with a decline in the proportion of young people to retired workers. These changes are due to both declining birth rates and higher life expectancies. Ageing and a slowdown in population growth has a number of implications for different aspects of the economy and the society– potentially even leading not only to "lower technical efficiency and economic welfare but also to less intellectual and artistic achievements as well" (Report of the Royal Commission on Population, 1949).

Ageing critically affects pension systems which in turn play a crucial role in the welfare of retired and elderly. In most European countries public pensions are a large part of the retirement income. In many cases, especially in Southern Europe, public pensions provide the main source of income to the retired.

In Europe most public pension systems operate under pay as you go (PAYG) plans. In PAYG systems current pensions are usually financed by revenues through taxes on employees. An ageing population puts a burden on current employees as a higher ratio of pensioners to employed means higher payments for sustainable public pension funds. The last forty years have seen a substantial increase in labour income taxes imposed to finance public pension funds causing a number of labour market distortions such as a lower participation rate.

In order to face the challenge of the new demographic map, European governments are considering a series of reforms that will ensure sustainability of public pension systems and may boost the relatively rigid European labour market. However pension reforms as part of fiscal policy have important implications for the rest of the macroeconomy: In the pioneering work of Feldstein (1974) it was shown that the introduction of unfunded social security can lower savings and private wealth. People of working age do not need to save as much as before because they will receive public benefits after a certain age (replacement effect). On the other hand, agents may try to retire earlier and increase their savings more while working (induced effect). Thus the level of private savings depends on which effect prevails. In any case, savings and investment decisions which determine the net asset position of a country are affected. Most theoretical studies agree that a reduction of pension benefits can lead to higher savings levels and an improvement in the net asset position of a country while empirically this effect is not so clear. Nevertheless, the announcement of lower public pension benefits in the future will have adverse effects on the short and medium run consumption levels as agents increase savings to provide for their retirement.

In the present paper we examine different public pension reform schemes and the response of the economy in the short and medium run. We postulate that the government can use different taxation schemes to minimize adverse effects of the pension reform during the transition of the economy to a new equilibrium with a reformed pension system. We examine different taxation responses that can cushion the initial adverse effect on consumption. The workhorse modelling framework of Blanchard (1985) with overlapping generations and finite horizons is employed. In order to capture life cycle behaviour and introduce public pension systems, we adopt a modified version of the framework in Nielsen (1994) and Bettendorf and Heijdra (2005). We depart from the set up in these studies by allowing for an unbalanced pension system and budget deficits as one of our main aims is to study the dynamics of public debt during pension reforms. In addition, we illustrate the impact of a declining population and ageing by allowing for different birth and death rates (Buiter, 1988) and age dependent labour productivity. Another departure from the existing literature is a realistic supply side where firms issue equities and face adjustment costs in investment. This enables us to consider the impact of public pension reforms on capital accumulation and investment decisions. We also employ a simulation algorithm that does not rely on log linearisation near the initial steady state. Although used frequently, this technique can result in serious approximation errors in models where the long run equilibrium after a policy intervention may be very distant in the future[1].

The scope of this paper is not only to examine the effects of pension reforms but also to evaluate different fiscal policies that can be used to "soften" the negative impact that reforms may cause. The model employed in this paper is not of the size of large scale simulation models, but our simple framework can be used readily for a quick quantitative assessment of fiscal policy experiments. Most importantly it enables us to observe the transition path of the main macroeconomic variables from the current state to the new long run equilibrium featuring the desired pension system.

The remainder of the paper is organised as follows: Section 2 sets up the Yaari-Blanchard model which features ageing population and an unfunded public pension system. The laws of motion for main macroeconomic variables and the long run equilibrium of the economy are presented. Section 3 sets out a quantitative analysis of three policy experiments: (a) a suspension of the public pension system accompanied by a fall of the lump sum income tax , (b) a suspension of the pension system combined by a fall in the proportional corporate tax rate and (c) the effect of an increase of the pension age on private and public finances. Section 4 provides concluding remarks.

[1] see Temple 2003.

2 The Model

We consider a small open economy in a one good world with perfect competition and perfect international capital mobility. The single good which is produced domestically is used for both consumption and investment purposes and it is a perfect substitute for foreign goods. The interest rate enters exogenously in the model as we assume a small open economy where the accumulation of assets and capital stock does not affect the interest rate. Therefore $r_{domestic} = r_{world}$. The economy consists of three sectors: households, firms and the government. Households optimize their utility intertemporally and firms maximize their profits according to their respective constraints. The behaviour of households and firms is determined at the microeconomic level. The government is not an optimizing entity and faces an intertemporal budget constraint and a tax reaction function while it maintains an unfunded pension system.

2.1 Households

We begin our analysis by deriving the conditions related to an individual household's optimal behaviour and then proceed to look at the aggregate behaviour patterns. Following Yaari (1965) and Blanchard (1985) a person born at time $s \leq t$ faces a constant, age invariant probability of death γ. The length of life is the only source of uncertainty for the agent while human wealth $h(z)$ and financial wealth $f(z)$, $z \geq t$, are held with certainty. At each time t, the agent derives utility from the consumption of both private and public goods, c and $\bar{G}$ respectively:

$$U_{s,t} \equiv \int_t^{\infty} e^{-(\rho+\gamma)(z-t)} \left(\log c(s,z) + \psi \log G\right) dz \tag{1}$$

where ρ is the pure rate of time preference and ψ $(0 < \psi < 1)$ measures the impact of government consumption on the utility of the household.

The flow budget identity of an individual of age $s+t$ at time t is given by:

$$c_{s,t}(z) = yi_{s,t}(z) + (r+\gamma)f(z) - \dot{f}(z) \tag{2}$$

where yi stands for non-interest income, f for the household's non human or financial wealth and r is the exogenous rate of interest. Agents are not altruistic and do not leave any positive or negative bequests to their descendants. This implies the existence of competitive life insurance companies which offer a premium $(r+\gamma)$ to individuals while alive in exchange for their financial wealth upon death[2]. The financial wealth f is held in the form of equities v, private bonds b^P, and public

[2] For a more detailed analysis, see Blanchard O. and S. Fischer (1989), chapter 3.

bonds b^G. Agents can buy a share of firms' future profits in the stock market in the form of equities. A firm's market value at time t, v, is the present discounted value of the future dividends d paid by the firm to shareholders: $V_t = \int_t^{\infty} e^{-rt} d(z)dz$.

The dividends that a firm has to pay to its shareholders equal the difference between the firm's after tax output and the sum of its investment and wage expenditure: $D_t = (1-\tau)Q_t - y_t L_t - I_t(1+\frac{gI_t}{2K_t})$. Self-fulfilling speculative asset price bubbles are ruled out: $\lim_{z\to\infty} v(z)e^{-\int_t^z (r+\gamma)d\mu} = 0$. Agents can issue debt in the form of private bonds b^P and can buy government debt b^G. The intertemporal No-Ponzi-Game condition is imposed: $\lim_{z\to\infty} b^P(z)e^{-\int_t^z (r+\gamma)d\mu} = 0$. The net financial holdings of a household of generation s at time t is: $f(s,t) = v(s,t) + b^G(s,t) - b^P(s,t)$. All forms of financial assets are perfect substitutes and carry the same rate of return.

Households supply one unit of labour to firms inelastically and receive a wage payment of y which is age dependent. However, employees' productivity is declining with age and a way to model it is to introduce an efficiency index which falls exponentially with age: $E(z-s) = w_o e^{-\xi(z-s)}$. Thus the net wage income of an individual at time t, $y(s,t)$ is :

$$y(s,t) = w_o e^{-\xi(t-s)} MPL(k(t)), \ \ for \ t \geq s,$$

where ξ determines the speed at which efficiency falls by age and w_0 is a positive parameter. The higher is ξ, the faster income wage is falling. If $\xi = 0$ then wage is not age dependent but it remains constant throughout life. This is an important feature of the perpetual youth literature as it enables us to model ageing and the need for life cycle savings and pensions.

Young households receive a wage income (y) from firms and have to pay a lump sum tax ϕ until the age of $s+\pi$ to partly finance government expenditure, debt interest payments and public pensions. After this age and till their death old households receive a lump sum pension benefit σ. The parameter π has been characterised by Bettendorf and Heijdra (2005) as loosely referring to the retirement age since households never really retire: they keep on working during their whole lifetime but they receive decreasing labour income since wage depends on productivity and age (see below).

In this structure the non interest income for a person of generation s at time t has the following profile:

$$\begin{aligned} yi(s,t) &= y(t) - \phi(t) \text{ if } t \leq s+\pi, \\ &= y(t) + \bar{\sigma} \text{ if } t > s+\pi. \end{aligned}$$

Households maximise their utility (1) with respect to budget constraint (2) to derive the optimal consumption plans:

$$\frac{\dot{c}(s,z)}{c(s,z)} = (r-\rho). \tag{3}$$

We can use the Euler equation (3) to determine consumption as a function of total (human and financial) wealth:

$$c(s,t) = (\rho+\gamma)[h(s,t)+f(s,t)], \tag{4}$$

where $\rho+\gamma$ is the marginal propensity to consume out of total wealth. By integrating (2) and using the solvency condition we derive the intertemporal budget constraint:

$$\int_t^\infty e^{-\int_t^z (r+\gamma)d\mu} c(s,z)dz = \int_t^\infty e^{-\int_t^z (r+\gamma)d\mu}[yi(s,z)]dz + f(s,t) = w(s,t).$$

The human wealth of an individual is defined as the net present value of all future non interest income:

$$h(s,t) = \int_t^\infty e^{-\int_t^z (r+\gamma)d\mu}[yi(s,z)]dz.$$

Since the non interest income differs for retirees and young workers, we have to define two cases: The human wealth of a retiree of generation s at period t can be defined as:

$$[h(s,t)]_{t-s>\pi} \equiv \int_t^\infty y(s,z)e^{-\int_t^z (r+\gamma)d\mu}dz + \frac{\sigma}{r+\gamma}.$$

The expression above states that the human wealth of a retiree is his wage income which is very low at this stage plus the public benefit discounted.

The human wealth of a young worker of generation s at time t can be expressed as:

$$[h(s,t)]_{0<t-s\leq\pi} \equiv \int_t^\infty y(s,z)e^{-\int_t^z (r+\gamma)d\mu}dz - \int_t^{s+\pi} \phi e^{-\int_t^z (r+\gamma)d\mu}dz + \int_{s+\pi}^\infty \sigma e^{-\int_t^z (r+\gamma)d\mu}dz$$

$$= \int_t^\infty y(s,z)e^{-(r+\gamma)(z-t)}dz - \frac{\phi}{(r+\gamma)}(1-e^{-(r+\gamma)(s+\pi-t)}) + \frac{\sigma}{(r+\gamma)}e^{-(r+\gamma)(s+\pi-t)}$$

The equation above states that the human wealth of a young worker is the sum of his declining wage income (net of taxes) that he is going to pay until the age of $s + \pi$ and the discounted pension benefit that he is going to receive when he reaches the retirement age. This definition of human wealth is crucial in this set up because it allows us to introduce (a) heterogeneity among agents and (b) a social security system.

2.2 Demographics

At this point we introduce population characteristics to allow for net population dynamics. Migration is not considered. We distinguish the probability of death γ from the birth rate η which are both exogenous. The growth of population L at any point of time t is defined as:

$$\frac{L(\dot{t})}{L(t)} = \eta - \gamma = h_L.$$

The path of the population evolution is: $L(t) = e^{(\eta-\gamma)t}$, given the initial condition $L(0) = 1$.

Following Buiter (1988), the size of a cohort born at time s $(L(s,s))$ is a proportion η of the existing population $L(s)$ at that time: $L(s,s) = \eta L(s)$. At every instance there is also a cohort that dies with a probability γ. At any time t $(t > s)$ the size of a cohort born at time s would be: $L(s,t) = e^{\gamma(s-t)}L(s,s) = e^{\gamma(s-t)}\eta L(s) = \eta e^{-\gamma t}e^{\eta s}$.

In this system the fraction of the young workers who pay a lump sum tax for the social security is $1 - e^{-\eta\pi}$ while the old workers who receive benefits are $e^{-\eta\pi}$. The old age dependency ratio is given by: $dep(\pi) = \frac{e^{-\eta\pi}}{1-e^{-\eta\pi}}$.

2.3 Aggregation

Having derived the household's optimal consumption path we next aggregate across generations. The relation between aggregate and individual variables is given by:

$$X(t) = \int_{-\infty}^{t} L(s,t)x(s,t)ds,$$

where $L(s,t) = \eta e^{-\gamma t}e^{\eta s}$. All aggregate variables are denoted in capital letters. The aggregate consumption in period t is the sum of consumption of individuals from all cohorts:

$$C(t) = \int_{-\infty}^{t} L(s,t)c(s,t)ds = (\rho + \gamma)[H(t) + F(t)]. \tag{5}$$

The expression above states that the aggregate consumption at any period t is a positive function of total wealth (which is the sum of aggregate human wealth $H(t)$ and aggregate financial wealth $F(t)$) where the parameters of the subjective discount factor and the probability of death are the propensity to consume out of wealth. Invariance of the individual spending propensity with respect to age is reflected in the equality between the individual and the aggregate spending propensities.

The aggregate human wealth consists of the human wealth of the old and young households:

$$H(t) = \int_{-\infty}^{t-\pi} L(s,t)h^{old}(s,t)ds + \int_{t-\pi}^{t} L(s,t)h^{young}(s,t)ds.$$

The aggregate human wealth of old households is the net present value of their wage income and of their pension benefits:

$$H^{old}(t) = \int_{-\infty}^{t-\pi} L(s,t)\left[h^{old}(s,t)\right]ds = L(t)\left[\frac{\eta\Omega(t)}{\xi+\gamma}e^{-\pi(\xi+\eta)} + \frac{\sigma}{r+\gamma}e^{-\eta\pi}\right],$$

where $\Omega(t) = \int_{t}^{\infty} e^{(\xi+r+\gamma)(t-z)}w_0 y(s,z)dz$.

Aggregating across the young generations we obtain their human wealth:

$$\begin{aligned} H^{young}(t) &= \int_{t-\pi}^{t} L(s,t)\left[h^{young}(s,t)\right]ds = \\ &= L(t)\left\{\begin{array}{c} \frac{\eta}{\eta+\xi}\Omega_o(t)(1-e^{-\pi(\xi+\eta)}) - \phi(t)\frac{\eta}{(r+\gamma-\eta)(r+\gamma)}(e^{-\pi(\gamma+r)} - e^{-\eta\pi}) \\ -\phi(t)\frac{1}{r+\gamma}(1-e^{-\eta\pi}) - \sigma\frac{\eta}{(r+\gamma-\eta)(r+\gamma)}(e^{-\pi(\gamma+r)} - e^{-\gamma\pi}) \end{array}\right\} \end{aligned}$$

By adding and simplifying the two expressions above we have aggregate human wealth:

$$\begin{aligned} H(t) &= L(t)\left\{\frac{\eta}{\xi+\eta}\Omega_o(t) + \frac{\sigma}{(r+\gamma)}e^{-\eta\pi} - \frac{\phi(t)}{(r+\gamma)}(1-e^{-\eta\pi})\right\} \\ &\quad + L(t)\frac{\eta(e^{-\eta\pi} + e^{-\pi(r+\gamma)})}{(r+\gamma-\eta)(r+\gamma)}(\phi(t)+\sigma). \end{aligned}$$

Note that the aggregate human wealth consists of the aggregate wage income net of tax payments by the young generations plus pensions received by the old generations. The last term reflects the fiscal effect on human wealth caused by changes in population[3].

The dynamic path of the aggregate human wealth is:

$$\begin{aligned}\dot{H}(t) &= (\xi + r + \eta)H(t) - YI(t) + \\ &\quad L(t)\left\{-\frac{\xi}{r+\gamma}\left(\sigma e^{-\eta\pi} - \phi(t)(1-e^{-\eta\pi})\right)\right\} + \\ &\quad L(t)\left\{\frac{\eta(\xi + r + \gamma)}{(r+\gamma-\eta)\,(r+\gamma)}(e^{-\pi(r+\gamma)} - e^{-\eta\pi})(\phi(t)+\sigma)\right\},\end{aligned} \tag{6}$$

where YI stands for the aggregate non interest income and is equal to:

$$YI(t) = L(t)\left\{\frac{\eta w_0}{\eta+\xi}y(t) + \sigma e^{-\eta\pi} - \phi(t)(1-e^{-\eta\pi})\right\}.$$

We define the aggregate financial wealth as: $F(t) = \int_{-\infty}^{t} f(s,t)L(t,s)ds$. The dynamics of the aggregate financial wealth are given by:

$$\dot{F}(t) = rF(t) + YI(t) - C(t). \tag{7}$$

Since the optimal decision for consumption of the aggregate household sector is $C(t) = (\rho+\gamma)[H(t)+F(t)]$, intuitively the evolution of consumption jointly depends on the time paths of human and financial wealth: $\dot{C}(t) = (\rho+\gamma)[\dot{H}(t) + \dot{F}(t)]$. By substituting and simplifying we obtain the dynamics for aggregate consumption:

$$\begin{aligned}\dot{C}(t) &= (\eta + \xi + r - \rho - \gamma)C(t) - (\eta+\xi)(\rho+\gamma)F(t) - \\ &\quad L(t)(\rho+\gamma)\frac{\xi}{r+\gamma}\left[\sigma e^{-\eta\pi} - \phi(t)(1-e^{-\eta\pi})\right] - \\ &\quad L(t)(\rho+\gamma)\frac{\eta(\xi+r+\gamma)(e^{-\pi(r+\gamma)} - e^{-\eta\pi})}{(r+\gamma-\eta)\,(r+\gamma)}(\phi(t)+\sigma).\end{aligned} \tag{8}$$

This is the usual Ramsey equation modified for overlapping generations where the change in consumption depends on financial wealth. Note that the demographic structure and the pension system influence optimal consumption decisions. If the pension system was balanced, i.e. benefits were equal to contributions, there would be no direct effects on aggregate consumption apart from the fiscal effect through the generational turnover (the last term). However, in the above expression we see that the pension system can influence negatively consumption growth.

[3]For example, if population is decreasing or $\eta < r + \gamma$ then $\frac{\eta(e^{-\pi(r+\gamma)} - e^{-\eta\pi})}{(r+\gamma-\eta)(r+\gamma)} > 0$. Since less people are born, the fiscal effect is positive on the aggregate human wealth as less tax is paid while more pensions are given.

2.4 Government

The government provides a lump sum pension benefit to all senior households in the economy and spends on the public good. It finances its total expenditure by taxing the gross output of firms and imposing a lump sum tax on the young households. The government is not a maximizing entity. It only fulfils its intertemporal budget constraint:

$$\dot{B}^G(t) = \bar{G} + L(t)\left[e^{-\eta\pi}\bar{\sigma} - (1 - e^{-\eta\pi})\phi(t)\right] - \tau Q(t) + rB^G(t), \tag{9}$$

where

$$\phi(t) = \frac{\beta}{1 - e^{-\eta\pi}} b(t)^G - \frac{\tau}{1 - e^{-\eta\pi}} o(t). \tag{10}$$

and τ is the proportional tax rate on firms' gross output Q.

The lump sum tax paid by young households adjusts to a certain extent to changes in public debt and output in order to avoid unsustainable situations. The parameter β reflects how quickly the lump sum tax adjusts to changes in public debt and can be an effective fiscal tool because it can smooth or exacerbate transitions of the economy following pension reforms as we will see later on. The No-Ponzi condition applies: $\lim_{t\to\infty} e^{-\int_0^t r d\mu} B^G(t) = 0$

2.5 The Supply Side: Firms

The production sector consists of a large number of identical firms (owned entirely by households) which produce a single good under conditions of perfect competition. The firms maximize their profits and face distortionary taxes on output. Distortionary taxation is another departure from Blanchard's framework where less realistic lump sum taxes are assumed. In our analysis the economy does not experience perpetual growth but production is subject to the law of diminishing returns. Firms invest and buy capital from perfectly integrated capital markets. Labour supply is fixed inelastically at unity and is assumed to be perfectly immobile across countries. Firms employ labour and capital to produce output by using a linearly homogeneous Cobb-Douglas production function which is subject to constant returns of scale:

$$Q(t) = AK(t)^a L(t)^{1-a},$$

where $0 < a < 1$. The firms own their capital stock and households have a claim on the firms' profits in the form of dividends. Capital accumulation is defined by:

$$\dot{K}(t) = I(t) - \delta K(t), \tag{11}$$

where $I(t)$ is gross investment δ is the rate of depreciation. Following the Tobin's q theory of investment, private investment is subject to increasing and convex costs of adjustment. A firm's investment pattern is given by the adjustment cost function: $I(.) = I + \frac{gI^2}{2K}$ where g is the rate at which adjustment costs rise with investment ($I'(.) > 0$ and $I''(.) > 0$). Including installation costs to our model allows us to provide well defined investment and saving decisions where investment is not a passive function of savings[4]. The firm maximizes the present discounted value of its after tax profits by choosing the optimal level of labour and investment:

$$V_o = \int_{t=0}^{\infty} e^{-rt} \left[(1-\tau)Q(t) - y(t)L(t) - I(t)(1 + \frac{gI(t)}{2K(t)}) \right],$$

subject to the evolution of the capital stock, the initial and the transversality conditions for capital:

$$\dot{K}(t) = I(t) - \delta K(t); \quad K_0 = K(0); \quad \lim_{t->\infty} e^{-rt}\mu_t q_t K_t = 0, \tag{12}$$

where $\mu_t q_t$ is the co-state variable or the shadow price of capital in period t. From the three first order conditions we derive the optimal wage and investment level:

$$y(t) = \frac{\eta w_0}{\eta + \xi}(1-\tau)(1-a)AK(t)^a N(t)^{-a}, \tag{13}$$

$$I(t) = \left(\frac{q(t) - 1}{g} \right) K(t). \tag{14}$$

The above relation is the usual profit maximization condition according to which the firm employs labour up to the point where the after tax marginal product of labour is equal to the wage rate. The second condition indicates the investment decision of the firm which depends positively on the shadow price of capital.

Lastly, the first order condition associated with the capital stock states the dynamics for the shadow price of capital:

$$\dot{q}(t) = (\delta + r)\, q(t) - a(1-\tau) \left(\frac{\eta w_0}{\eta + \xi} \right)^{1-a} AK(t)^{a-1} + \frac{g}{2} \left(\frac{I(t)}{K(t)} \right)^2 \tag{15}$$

[4] Note that our set up of capital accumulation is a departure from Bettendorf and Heijdra (2005) who introduce traded and non traded goods and assume that only non traded goods are used for investment. Increasing marginal costs of production in the non traded goods sector results in finite rate of investment. In contrast this paper follows a more mainstream approach by using a simple Cobb-Douglas production function and adjustment costs in investment. In Nielsen (1994) agents hold physical capital and their savings decision directly affect the capital stock in the economy which is not subject to adjustment costs.

According to Hayashi (1982) the shadow price of the installed capital q is also defined as the ratio of the stock market value of capital to the capital replacement cost as long as the production function is subject to constant returns of scale which is the case in this model:

$$q(t) = \frac{V(t)}{K(t)}.$$

In order to find the intertemporal shadow price of capital we can use the dynamics of q. By integrating (15) and using the transversality condition (12) q is equal to:

$$q_t = \int_{t=0}^{\infty} \left[a(1-\tau) \left(\frac{\eta w_0}{\eta + \xi} \right)^{1-a} AK(t)^{a-1} + \frac{g}{2} \left(\frac{I(t)}{K(t)} \right)^2 \right] e^{-rt} dt. \tag{16}$$

Equation (16) states that the intertemporal shadow price of capital is equal to the present discounted value of the marginal product of capital and the marginal reduction in adjustment costs from an increase in the capital stock. Given the interest rate, the investment decision does not depend on the utility function, the consumption decision of the households, or the level of private and public debt. However, since $V = qK$, the shadow price of capital affects not only the investment decision of firms but also the consumption pattern of households through their equity holdings.

2.6 The Current Account and Market Equilibrium

From the dynamics of financial wealth (7), public debt (9), and equities we derive the net foreign asset position of the private sector. In order to capture the full effects of fiscal policies we assume that all government debt is bought by domestic households. Hence the country's net foreign asset position is identical to private sector indebtedness to the rest of the world $B^P(t)$:

$$\dot{B}^P(t) = C(t) + I(.) + \bar{G} + rB^P(t) - Q(t). \tag{17}$$

The following expression provides the clearing condition for the goods market: gross domestic product is equal to domestic absorption and the current account surplus:

$$Q(t) = C(t) + I(.) + \bar{G} - \dot{B}^P(t) + rB^P(t).$$

2.7 Model Summary

In Table 1 we present the dynamics of the main macroeconomic variables in a stationary form relative to population size $L(t)$. All variables are functions of time and have been divided by the population size at time t.

Table 1: Key Equations of the Model

	Laws of motion:	
Capital:	$\dot{k} = (\frac{q-1}{g} - \delta - \eta + \gamma)k$	(S.1)
Tobin's q:	$\dot{q} = (r+\delta)\,q - a(1-\tau)(\frac{\eta w_0}{\eta+\xi})^{1-a} A k^{a-1} + \frac{(q-1)^2}{2g}$	(S.2)
Consumption:	$\dot{c} = (\xi + r - \rho)c - (\eta+\xi)(\rho+\gamma)(b^g + qk - b^p)$	(S.3)
	$-\frac{\xi(\rho+\gamma)}{r+\gamma}\left[\sigma e^{-\eta\pi} - \phi(1-e^{-\eta\pi})\right] + \frac{\eta(\rho+\gamma)(\xi+r+\gamma)(e^{-\pi(r+\gamma)} - e^{-\eta\pi})}{(r+\gamma-\eta)(r+\gamma)}(\phi+\sigma)$	
Private debt:	$\dot{b}^p = c + (r-\eta+\gamma)b^p - \left(\frac{\eta w_0}{\eta+\xi}\right)^{1-a} A k^a + \frac{(q^2-1)}{2g}k + ge$	(S.4)
Public debt:	$\dot{b}^g = ge + \sigma e^{-\eta\pi} - \phi(1-e^{-\eta\pi}) - \tau\left(\frac{\eta w_0}{\eta+\xi}\right)^{1-a} A k^a + (r-\eta+\gamma)b^g$	(S.5)
	Static equations:	
Output:	$o = (\frac{\eta w_0}{\eta+\xi})^{1-a} A k^a$	(S.6)
Dividends:	$d \equiv (1-\tau)\left(\frac{\eta w_0}{\eta+\xi}\right)^{1-a} A k^a - y - \frac{(q^2-1)}{2g}k$	(S.7)
Wage	$y = \left(\frac{\eta w_0}{\eta+\xi}\right)^{1-a}(1-\tau)(1-a)Ak^a$	(S.8)
Tax on young househ.	$\phi = \frac{\beta}{1-e^{-\eta\pi}} b^G - \frac{\tau}{1-e^{-\eta\pi}} o$	(S.9)

Steady State

In the long run the economy reaches its equilibrium and the endogenous variables take the following equilibrium values:

$$\bar{q} = 1 + (\delta + \eta - \gamma)g, \tag{18}$$

$$\bar{k} = (\frac{\eta w_0}{\eta+\xi})\left[\frac{(r+\delta)\,\bar{q} + \frac{(\bar{q}-1)^2}{2g}}{Aa(1-\tau)}\right]^{\frac{1}{a-1}}, \tag{19}$$

$$\begin{aligned}\bar{c} \;=\; & \frac{(\eta+\xi)(\rho+\gamma)}{(\xi+r-\rho)}(\bar{b}^G + \bar{q}\bar{k} - \bar{b}^P) \\ & + \frac{\xi(\rho+\gamma)}{(r+\gamma)(\xi+r-\rho)}\left[\sigma e^{-\eta\pi} - \phi(1-e^{-\eta\pi})\right] \\ & - \frac{\eta(\rho+\gamma)(\xi+r+\gamma)(e^{-\pi(r+\gamma)} - e^{-\eta\pi})}{(\xi+r-\rho)(r+\gamma-\eta)(r+\gamma)}(\phi+\sigma),\end{aligned} \tag{20}$$

$$\begin{aligned}\bar{b}^P \;=\; & \frac{1}{(r-\eta+\gamma)}\left(\frac{\eta w_0}{\eta+\xi}\right)^{1-a} A\bar{k}^a - \frac{(\bar{q}^2-1)}{2g(r-\eta+\gamma)}\bar{k} \\ & - \frac{ge}{(r-\eta+\gamma)} - \frac{1}{(r-\eta+\gamma)}\bar{c},\end{aligned} \tag{21}$$

$$\bar{b}^G = \phi\frac{(1-e^{-\eta\pi})}{(r-\eta+\gamma)} + \frac{\tau}{(r-\eta+\gamma)}\left(\frac{\eta w_0}{\eta+\xi}\right)^{1-a} A\bar{k}^a - \frac{ge}{(r-\eta+\gamma)} - \sigma\frac{e^{-\eta\pi}}{(r-\eta+\gamma)}. \tag{22}$$

The first two equations characterise the equilibrium in the supply side of the economy. The shadow price of capital (eq. (18)) and the capital stock per capita (eq. (19)) take the usual values in models where capital is costly to install. Note that the capital stock per capita depends positively on the efficiency term $(\frac{\eta w_0}{\eta+\xi})$. Equilibrium consumption (eq. (20)) does not only depend on capital and bonds but also on pensions and taxes and on the demographic structure. Private debt per capita (eq. (21)) is the difference between output and total expenses for capital, consumption and public expenditure. Public borrowing per capita (eq. (22)) is the gap between tax revenues and expenditure for public consumption and pensions which are all demography dependent.

3 Quantitative Analysis of Public Pension Reform Options in the Face of a Declining Population

In this section we assess the impact of pension reforms under different fiscal scenarios. First we describe the calibration of the model, then the simulation method, and finally we provide results for three policy experiments.

3.1 Simulation method

The system of differential equations is non-linear and cannot be solved analytically. Numerical methods are required to solve for the transition path.

A number of algorithms have been proposed to solve this boundary value problem. Well known examples include shooting and projection methods (Judd 1992), time elimination (Mulligan and Sala-i-Martin (1991)) and the method proposed by Mercenier and Michel (1994). Trimborn *et al* (2006) provide a critical survey of these methods and introduce a relaxation procedure to solve for the transition path. They argue that this relaxation method outperforms other proposed solution methods in high dimension systems such as those encountered in this paper. Here, we adopt their proposed solution.

Trimborn *et al*'s method works as follows: Consider the following system of N_1 differential equations:

$$\dot{Y} = F \cdot Y,$$

and N_2 static equations

$$Z(.) - K \cdot Y = 0,$$

that are assumed to describe the dynamics of the economic problem at hand. There are n_1 initial conditions and n_2 terminal conditions such that $N_1 = n_1 + n_2$.

The problem is transformed into a finite (discrete) time interval by choosing a set of M mesh (grid) points in time. At each of these mesh points, the system of N_1 differential equations is approximated using the midpoint rule. This gives a system of $M \times N$ equations that have to be solved simultaneously. Trimborn *et al* (2006) propose a Gauss-Newton method that can be used to find a root for the system in an efficient way.

3.2 Calibration

The values assigned to the exogenous parameters are given in Table 2 and are taken from the existing growth, real business cycle and macroeconomic literature. It should be noted that the selected parameter values are representative of a range of economies with emphasis on European economies rather than being specific to one individually.

Following Heijdra and Ligthart (2006), the subjective discount factor is set at 4% with the real interest rate assumed to be 3%. The productivity factor in the production function is set equal to 1 and the share of capital is assumed to be 0.36, which is standard in the literature. Under the benchmark scenario, population is assumed to be decreasing. The death rate of 1% is broadly in line with population data (see United Nations, 2006). We consider birth rates of 0.8% in the experiments with declining population and 1% when population remains constant. The rate of depreciation is 10% as in Collard and Dellas (2004). The adjustment cost parameter is chosen to ensure a realistic profile for capital adjustment. The main implications of the model are unaffected if a higher or lower value is chosen.

We consider a range of values for the pension age, the coefficient of diminishing labour productivity and the reaction coefficient β. This last parameter is particularly important in determining the implications of shocks to the pension system. In the benchmark case π is set to give a total dependency ratio of 0.5 which is in line with the EU statistics (see United Nations, 2006). Similarly the value of β gives a realistic steady state public debt which is around 65% of GDP.

The choice of lump-sum pension (σ) and the government expenditure (G) implies an initial level of public debt to GDP ratio which is similar to the one prevalent in EU countries. Note that the shape of the transition paths of the main variables is unaffected by the calibrated values of G and σ[5].

Table 2: Calibrated Parameter Values

$\rho = 0.04$	subjective discount factor
$r = 0.03$	real rate of interest
$\eta = 0.008$	birth rate
$\gamma = 0.010$	death rate
$a = 0.36$	share of capital
$\pi = 125$	parameter that determines the dependency ratio
$A = 1$	productivity factor
$g = 3$	adjustment costs of investment
$\xi = 0.014$	coefficient of diminishing productivity of labour
$\sigma = 0.40$	lump sum pension
$G = 0.10$	government expenditure
$\delta = 0.10$	depreciation rate of capital
$\tau = 0.08$	corporate tax rate on firms' gross output
$\beta = 0.3$	coefficient of tax response on public debt

3.3 Pension cuts with lump sum taxes response

The first experiment considers a suspension of the public pension benefit σ to old households. The reform takes place after fifty periods and all agents are aware of this fiscal development. The perfect foresight assumption implies that the adjustment process starts immediately. Figure 1 shows the dynamics of main variables when the public pension system is suspended in fifty periods. The shock in this case is modeled as $\sigma \to 0$ at period fifty. The graphs are drawn for the benchmark values of the parameters. Sensitivity analysis for different values of the productivity and tax reaction coefficients (ξ and β), retirement age (π), public benefit (σ), and the birth rate (η) parameters can be seen in the appendix.

Consumption initially falls and hits its lowest point at period fifty: agents increase their savings, especially households who expect to be alive at the period of the reform. Note that as we examine a complete abolition of the pension system in this experiment, we observe large falls in public debt and also an instantaneous fall in the lump sum tax paid by young households after period 50. Due to anticipated tax cuts, private consumption does not fall strongly in period 50 (only 0.8%)) and increases afterwards to its new long run equilibrium level. The fall in

[5] Only the initial and final steady state values change but the shape of the transition path remains the same.

consumption around the time of the public pension cut comes mainly from the generation which is currently working and expects to retire around period 50 This generation increases its savings in order to subsidise its income for the period between the retirement and the fall of the income tax. The shorter is the period of the tax response the lower is the increase in savings and the fall in consumption. The current retired generation does not change its savings pattern as it does not expect to be alive in period 50. Intuitively the higher is the retired population in period 0 the lower will be the fall in the aggregate per capita consumption at period 50.

To ensure fiscal sustainability the lump-sum income tax ϕ adjusts automatically to variations in public debt (see eq.(10)). Nevertheless the government can choose the speed of adjustment and hence it indirectly affects the level of public debt and the tax.

Note that the cut of the public pension and the fall of the lump sum tax do not affect the supply side of the economy[6]. This is because the marginal product of capital (MPK) which affects investment and output does not depend on these parameters. Hence the MPK in steady state depends on population characteristics, adjustment costs and the rates of interest and depreciation:

$$MPK = aA\left(\frac{\eta w_0}{\eta+\xi}\right)\frac{Aa(1-\tau)}{(r+\delta)\,\bar{q}+\frac{(\bar{q}-1)^2}{2g}}.$$

All adjustments take place in the demand side: The levels of consumption, private and public bonds change for the same level of output.

Table 3 and all tables that follow show a percentage change of the main macroeconomic variables for different periods over time compared to the initial steady state at period $t = 0$. They show how much those variables change in 10 periods after the announcement of the pension cut, when this policy is implemented in 50 periods and when the economy has converged to the new steady state.

σ	σ=0		
time	t=10	t=52	t=inf
C	-0.01	-0.23	0.37
Bp	-13.31	-197.90	-157.73
Bg	0.00	0.00	-59.54
K	0.00	0.00	0.00
O	0.00	0.00	0.00
Y	0.00	0.00	0.00
φ	0.00	0.00	-99.92

Table 3: Percentage change from initial steady state. Variables are expressed in per capita terms.

[6] Bettendorf and Heijdra (2005) obtain a change in capital stock in response to this shock because their production sector consists of producers of traded and non traded goods.

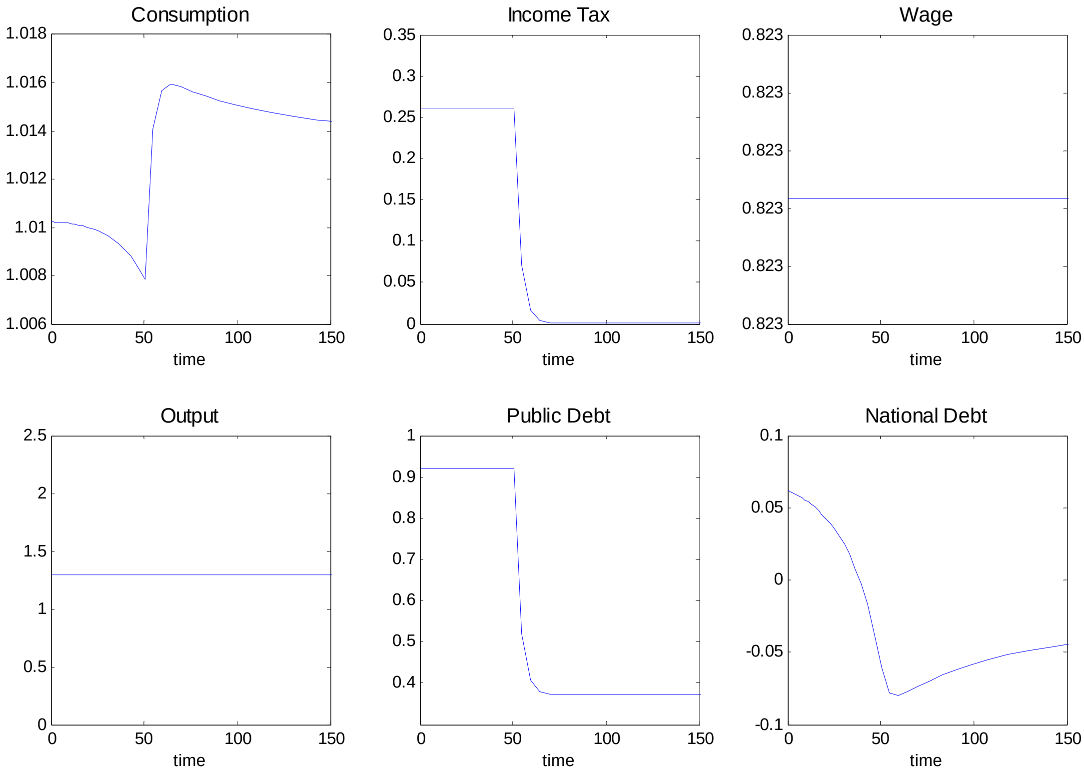

Figure 1: Dynamics following suspension of public pension and decrease of the lump sum tax.

It is important to note that the path of consumption, tax and private debt depends on the tax reaction function that the government has set initially. If government has chosen a quick adjustment process (high β) then lower income taxation commences immediately after the shock and the cost in terms of a fall in consumption is lower. On the other hand, when after shock tax adjustment is delayed, households need to increase savings to smooth consumption for the years after the pension cut. A quick post-shock compensation will not substantially alter the optimal decisions of the current young generation. The speed of the tax response involves redistribution of wealth across generations. An inflexible tax system, for instance, will leave current young generations worse off while future generations will enjoy higher levels of consumption and lower levels of public and national debt. Hence the tax reaction function is an effective policy instrument for wealth reallocation across generations.

This argument can be illustrated in the following experiment where four identical economies start at the same level of public debt but follow different fiscal

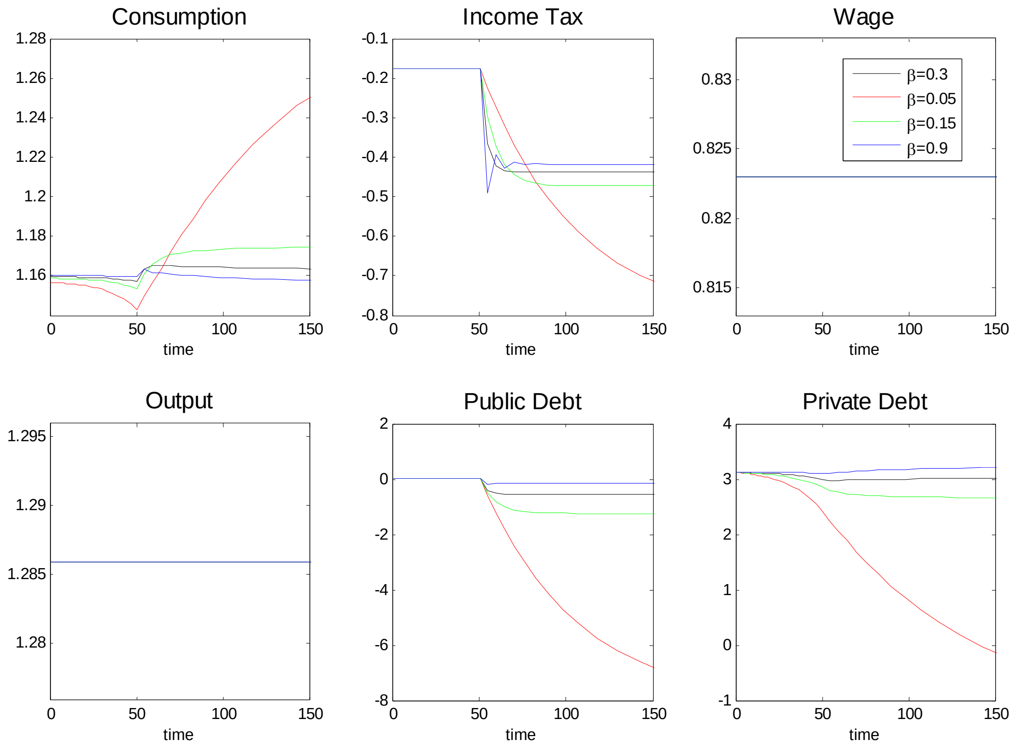

Figure 2: Dynamics following cuts in public pension and in lump sum tax, for different values of the tax reaction coefficient β.

responses. The calibration for Figure 2 is different from the benchmark case: for simplicity public expenditure has been chosen to give zero public debt initially. When β is very low, consumption fluctuates substantially and falls a lot before the reform even though it is higher in the long run. However for certain values of β such as 0.3 consumption and private debt stay almost at the same levels[7]. This indicates that economies that do not need to change their debt levels but wish to proceed to pension reforms can do so without a large negative impact on consumption.

[7] When the tax response is very slow ($\beta = 0.05$) the economy still converges to a new equilibrium but it cannot be seen in this figure as convergence occurs in 350 periods.

β	β=0.3 (default)			β=0.05			β=0.15			β=0.9		
time	t=10	t=50	t=inf	t=10	t=50	t=inf	t=10	t=50	t=inf	t=10	t=50	t=inf
C	-0.01	-0.20	0.32	-0.04	-1.19	11.96	-0.01	-0.45	1.42	0.00	-0.02	-0.29
Bp	-0.27	-3.94	-3.14	-1.55	-22.98	-134.97	-0.59	-8.73	-15.22	-0.02	-0.37	3.43
Bg	0.00	0.00	-96954	0.00	0.00	-96952	0.00	0.00	-96953	0.00	0.00	-96949
K	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
O	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
φ	0.00	0.00	147.54	0.00	0.00	366.94	0.00	0.00	167.58	0.00	0.00	136.64

Table 4: Percentage change from initial equilibrium for various tax responses (β).

3.4 Pension cuts combined with proportional tax response

In this experiment public pensions are again suspended at period fifty. However, in this case the government does not adjust the lump sum tax paid by young households but the proportional tax on gross output paid by firms. In the previous experiment both the pension benefit σ and the income tax ϕ were lump sum and their alteration caused no distortions to the economy. In contrast the use of the proportional tax on output changes the equilibrium level of output, capital stock and real wage. This can be a more realistic case as there is a widespread discussion about linking pensions to firms' profits.

Figure 3 shows the effects of the composite shock. We have used the same parameter values as before with the only exception that the lump sum income tax is exogenous ($\phi = 0.10$) while the corporate tax rate τ is endogenous and adjusts to changes to per capita public debt and output:

$$\tau(t) = \frac{\beta B(t)^g}{(\frac{\eta w_0}{\eta+\xi})^{1-a} A k(t)^a}. \tag{23}$$

In this exercise consumption falls slightly in anticipation of the reform to increase substantially to a higher long run steady state (39% higher than the initial one). The reason for this is that agents internalise future wealth effects coming from higher returns of their equity holdings and do not alter considerably their savings. Zero pensions will decrease public debt which in turn induces a decrease in the tax rate on firms' gross output. The marginal product of capital rises and triggers higher investment, output and wages.

In this model economy the cut of public pensions has occurred at a very small cost: the short run is characterised by a very mediocre fall of consumption (0.2%) (see Table 5) while the long run demonstrates only positive effects: higher per capita consumption, output, and wage levels, while public and private indebtedness have been reduced considerably.

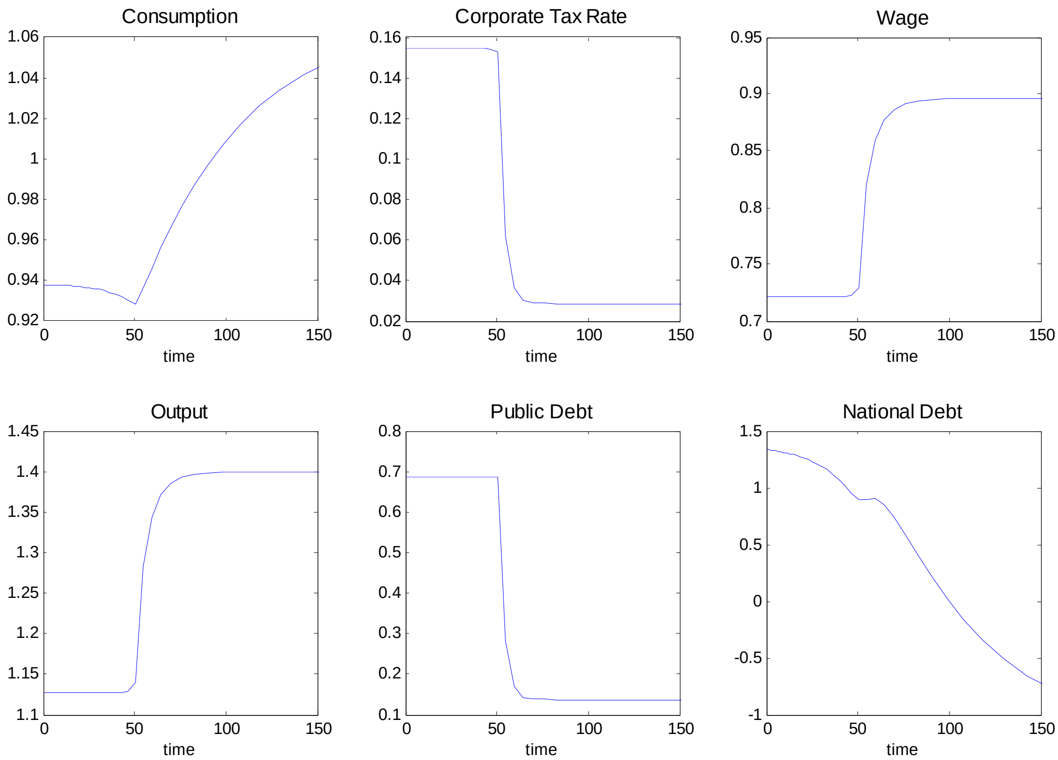

Figure 3: Dynamics following pensions and corporate tax rate cuts.

beta	β=0.15		
time	t=10	t=52	t=inf
C	-0.02	-0.21	39.67
Bp	-2.48	-2.62	-111.12
Bg	0.00	0.00	-80.00
K	0.00	0.00	67.97
O	0.00	0.00	67.97
Y	0.00	0.00	67.97
φ	0.00	0.00	-83.41

Table 5: Percentage change from initial steady state.

3.5 Increase in retirement age

In this experiment we do not implement pension cuts but decrease the age dependency parameter π so that the retirement age increases at period 50. In this set up a higher retirement age does not solve the problem of pension systems unsustainability. Noticeably, public debt falls very little while the lump sum tax required

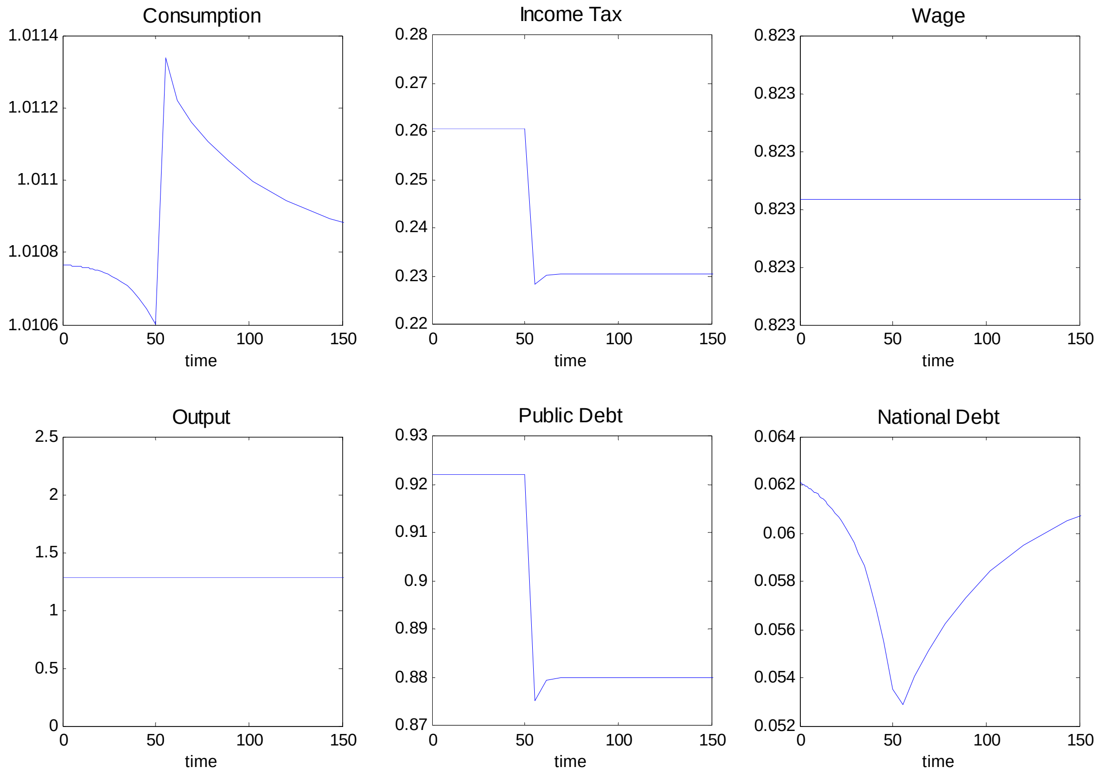

Figure 4: Dynamics following a decrease of the age dependency parameter π.

to support this economy does not decrease much. As expected, per capita consumption does not fluctuate much as individuals will receive pension benefits in the future. This slight fall of consumption that we observe in Figure 4 is due to falling labour income combined with the delay of pension benefits. Agents should increase their savings to smooth their consumption for the time period between very low wage income and retirement.

In a model without diminishing productivity and wages, we would expect to see a pure wealth effect and a rise in per capita consumption as agents work until late in their lives and can afford to decrease their savings holdings. In other words such a policy does not decrease national indebtedness (see Figure 4) as it does not motivate agents to save more. In this model this policy does not promote private savings which may be needed in an ageing economy. In addition, it does not alleviate young households from the income tax they have to pay to support such a system. The public sector also remains heavily indebted.

π	π=135		
time	t=10	t=52	t=inf
C	0.00	0.00	0.00
Bp	-0.29	-1.78	0.52
Bg	0.00	0.00	-4.58
K	0.00	0.00	0.00
O	0.00	0.00	0.00
Y	0.00	0.00	0.00
φ	0.00	0.00	-11.64

Table 6: Percentage change from initial steady state.

Next we repeat the same experiment (implementation of a policy which results in the decrease of the dependency ratio) accompanied by a decrease of the corporate tax rate. This policy mix produces higher output in the long run from the previous one due to the elimination of a distortion. Consumption does not change much until period 50 because the loss from wage income is compensated by the wealth effects coming from higher output and equity holdings. We can see in Table 7 that the long run picture of the main macroeconomic variables does not change substantially: A decrease of the dependency ratio by 8% causes a fall of public debt in the long run by 18.5 % which in turn lowers the tax rate for fiscal sustainability. The lower proportional tax triggers output which reaches a higher level and finally boosts consumption by 1.5%. However the policy of suspending the public pensions and lowering the corporate tax rate in Section 3.4 produces higher levels of consumption, output, wage and lower levels of public and national debt.

π	π=135		
time	t=10	t=52	t=inf
C	0.00	0.10	1.45
Bp	-0.37	-2.50	-18.42
Bg	0.00	-7.21	-7.69
K	0.00	1.32	2.43
O	0.00	1.88	2.43
Y	0.00	1.88	2.43
φ	0.00	-7.65	-8.48

Table 7: Percentage change from initial steady state.

4 Conclusions

This paper studies the effects of alternative macroeconomic policies to reform public pension systems in a Yaari-Blanchard overlapping generations framework that incorporates demographic structure, life cycle behaviour and a public pension system.

We carry out three experiments: in the first two experiments public pensions are suspended and the government does not support the transition period, implying

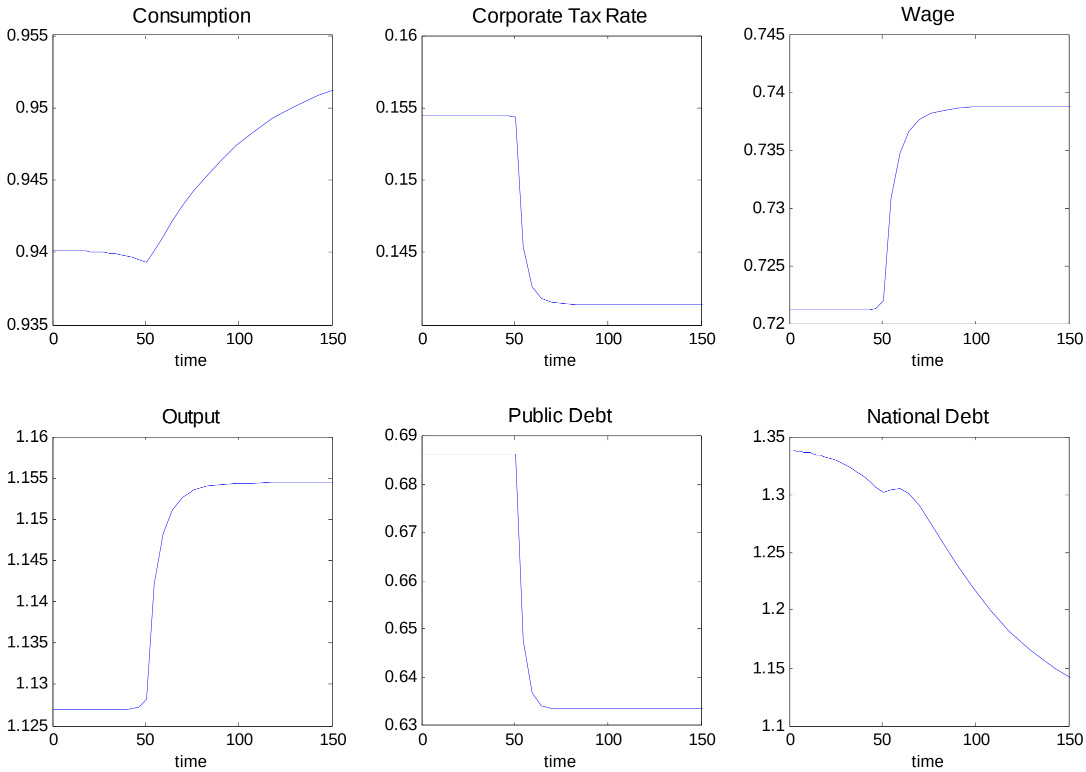

Figure 5: Dynamics following a decrease of the age dependency parameter π accompanied by a decrease of the corporate tax rate.

that the generation alive during the transition will bear the burden by paying for the existing pensioners and saving for its own retirement. In the third case we study the macroeconomy under the hypothesis of a rise in the pension age.

The findings suggest that when drastic pension cuts are accompanied by a reduction of the corporate distortionary tax on firms' gross output, the adverse effects on per capita consumption are minimised in the short run while only positive effects on the economy are displayed in the long run: Output and consumption are higher whilst private and public indebtedness reach lower levels.

The same impact on the demand side can be observed when a lump sum income tax is used instead. However, the supply side remains unaffected by these fiscal measures and their impact on the long run consumption is considerably smaller.

The outcome of both these policies is affected by the reaction function that the government uses to adjust to public debt developments. In other words if the government reacts rapidly and adjusts taxes then the negative impact of lower pensions on consumption can be minimised.

The last policy experiment concerns increasing the retirement age and leaves the amount of pension benefits unchanged. This policy appears to be unsuccessful in economies with high levels of public debt especially if the aim is lower public indebtedness. Even if this policy measure is able to sustain the current fiscal position, the young generations in a declining population world eventually have to pay high taxes to sustain it.

5 Appendix: Sensitivity Analysis of Key Parameters

In this appendix we examine the sensitivity of our results for different values of the diminishing wage factor ξ, the pension benefit σ, the pension age π, the birth ratio η, and the tax reaction on public debt parameter β. In all these experiments public pensions terminate at period fifty ($\sigma_{50} = 0$)and the income lump sum tax ϕ falls according to the movement of the public debt.

5.1 Sensitivity analysis for the coefficient of diminishing labour productivity (ξ)

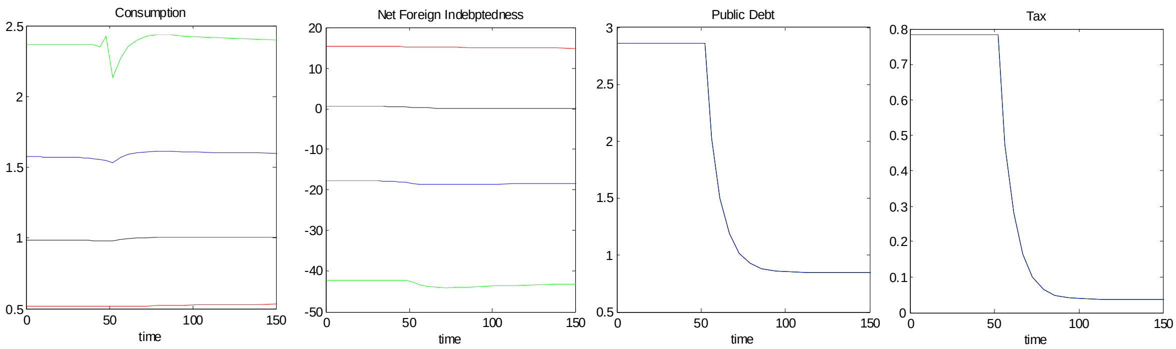

Figure A1: Sensitivity analysis for the coefficient of diminishing labour productivity (ξ)

ξ	ξ=0.014			ξ=0.001			ξ=0.8			ξ=0.05		
time	t=10	t=52	t=inf	t=10	t=52	t=inf	t=10	t=52	t=inf	t=10	t=52	t=inf
C	-0.02	-0.81	2.14	0.01	-0.13	3.93	0.00	-9.82	0.81	-0.04	-2.56	1.27
Bp	-2.63	-51.07	-74.09	-2.70	-0.95	-3.90	0.00	0.68	1.42	-0.78	3.99	3.42
Bg	0.00	0.00	-70.40	0.00	0.00	-70.40	0.00	0.00	-70.40	0.00	0.00	-70.40
K	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
W	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
fi	0.00	0.00	-95.18	0.00	0.00	-95.18	0.00	0.00	-95.18	0.00	0.00	-95.18

Table A1: Percentage change from initial equilibrium values

5.2 Sensitivity analysis for the tax reaction coefficient (π)

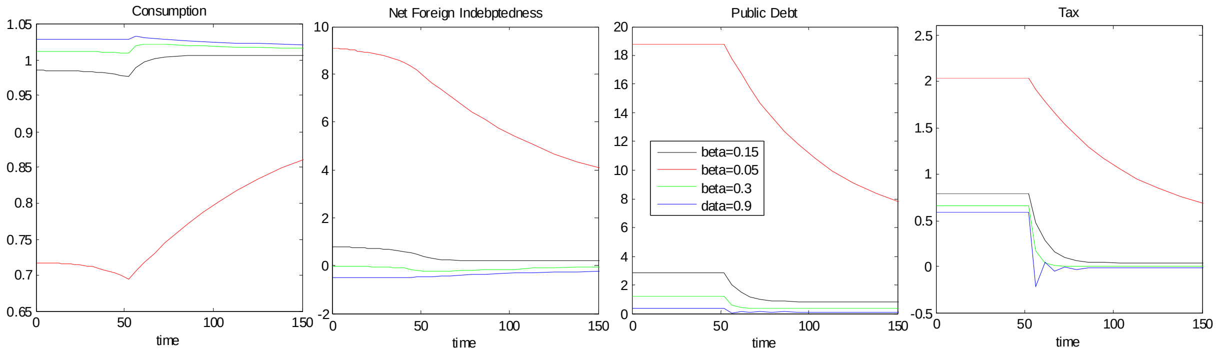

Figure A2: Sensitivity analysis for the tax coefficient β

beta	β=0.15			β=0.05			β=0.3			β=0.9		
time	t=10	t=52	t=inf	t=10	t=52	t=inf	t=10	t=52	t=inf	t=10	t=52	t=inf
C	-0.02	-0.81	2.14	-0.09	-3.08	29.37	-0.01	-0.33	0.12	0.00	0.02	-0.95
Bp	-2.63	-51.07	-74.09	-9.95	-12.59	-70.73	-1.06	852.08	78.89	0.07	-2.33	-63.97
Bg	0.00	0.00	-70.40	0.00	0.00	-70.40	0.00	0.00	-70.40	0.00	0.00	-70.39
K	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
W	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
fi	0.00	0.00	-95.18	0.00	0.00	-79.92	0.00	0.00	-99.95	0.00	0.00	-103.40

Table A2: Percentage change from equilibrium values

5.3 Sensitivity analysis for the lump sum pension (σ)

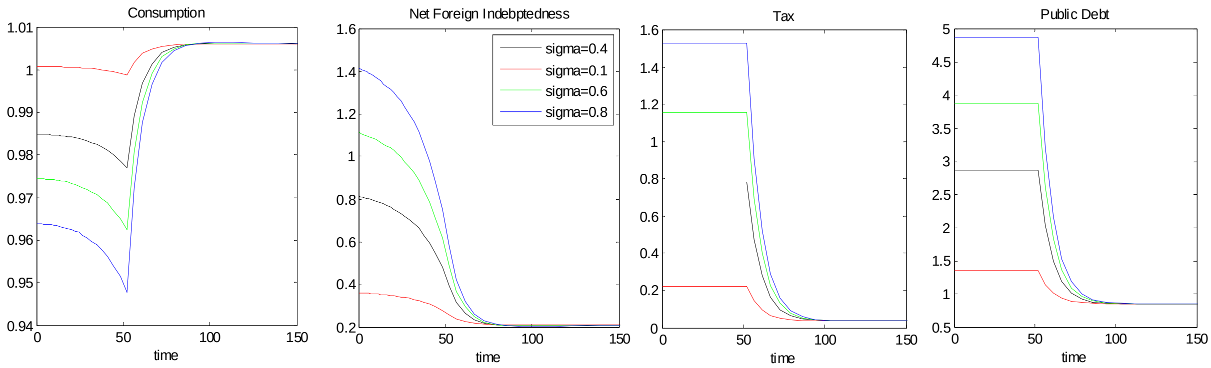

Figure A3: Sensitivity analysis for the pension benefit σ

σ	σ=0.4			σ=0.1			σ=0.6			σ=0.8		
time	t=10	t=52	t=inf	t=10	t=52	t=inf	t=10	t=52	t=inf	t=10	t=52	t=inf
C	-0.02	-0.81	2.14	-0.01	-0.20	0.53	-0.04	-1.23	3.25	-0.05	-1.66	4.38
Bp	-2.63	-51.07	-74.09	-0.65	-28.73	-41.68	-3.99	-55.90	-81.09	-5.38	-58.68	-85.11
Bg	0.00	0.00	-70.40	0.00	0.00	-37.29	0.00	0.00	-78.10	0.00	0.00	-82.63
K	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
W	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
fi	0.00	0.00	-95.18	0.00	0.00	-83.16	0.00	0.00	-96.74	0.00	0.00	-97.53

Table A3: Percentage change from equilibrium values

5.4 Sensitivity analysis for the age dependency parameter (π)

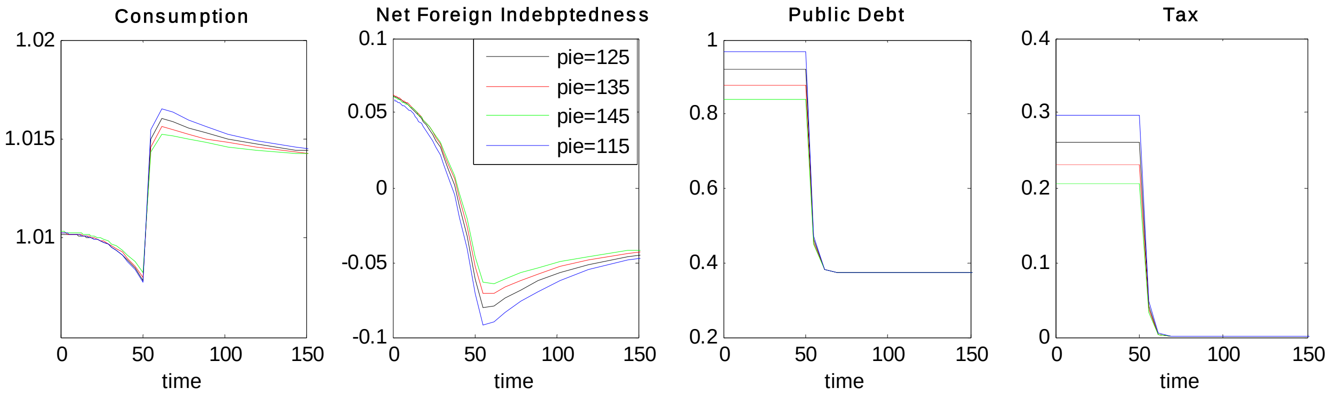

Figure A4: Sensitivity Analysis for the parameter π that determines age dependency.

π	π=125			π=135			π=145			π=115		
time	t=10	t=52	t=inf	t=10	t=52	t=inf	t=10	t=52	t=inf	t=10	t=52	t=inf
C	0.00	-0.02	0.37	0.00	-0.02	0.37	0.00	-0.02	0.36	0.00	-0.02	0.37
Bp	-4.12	-25.49	-157.72	-3.83	-23.73	-157.41	-3.65	-22.62	-158.55	-4.57	-28.29	-160.08
Bg	0.00	0.00	-59.54	0.00	0.00	-57.60	0.00	0.00	-55.63	0.00	0.00	-61.45
K	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
O	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
φ	0.00	0.00	-99.93	0.00	0.00	-99.92	0.00	0.00	-99.91	0.00	0.00	-99.93

Table A4: Percentage change from equilibrium values.

5.5 Sensitivity analysis for the birth rate (η)

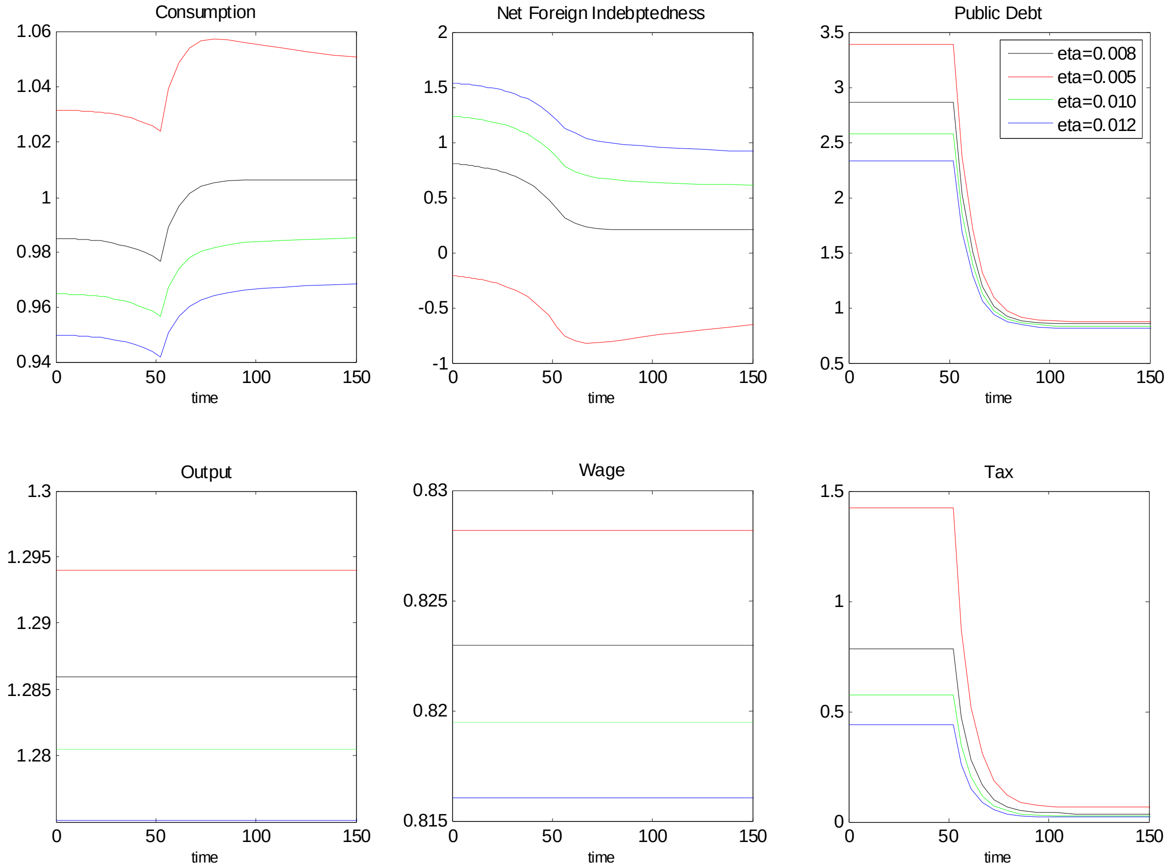

Figure A5: Sensitivity Analysis for the birth rate η

η,γ	η=0.008,γ=0.010			η=0.005, γ=0.010			η=0.010, γ=0.010			η=0.012,γ=0.010		
time	t=10	t=52	t=inf	t=10	t=52	t=inf	t=10	t=52	t=inf	t=10	t=52	t=inf
C	-0.02	-0.81	2.14	-0.02	-0.75	1.44	-0.02	-0.83	2.19	-0.02	-0.83	2.06
Bp	-2.63	-51.07	-74.09	-2.74	226.42	179.67	-2.50	-30.71	-52.17	-2.34	-22.43	-41.73
Bg	0.00	0.00	-70.40	0.00	0.00	-74.29	0.00	0.00	-67.62	0.00	0.00	-64.71
K	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Y	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
W	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
fi	0.00	0.00	-95.18	0.00	0.00	-95.47	0.00	0.00	-95.03	0.00	0.00	-94.92

Table A5: Percentage change from equilibrium values

References

[1] Bettendorf, L. and B. Heijdra, 2005, "Population ageing and pension reform in a small open economy with non-traded goods", Forthcoming: Journal of Economic Dynamics and Control.

[2] Blanchard, O., 1985, "Debts, Deficits and Finite Horizons", Journal of Political Economy Vol.93, pp.223-247.

[3] Blanchard, O. and S. Fischer, 1989, *Lectures in Macroeconomics*, MIT Press.

[4] Buiter, W.H., 1988, "Death, Birth, Productivity Growth and Debt Neutrality", Economic Journal, Vol. 98, pp. 279-293.

[5] Collard, F. and H. Dellas, 2004. "The New Keynesian Model with Imperfect Information and Learning", IDEI Working Papers 273, Institut d'Économie Industrielle (IDEI), Toulouse.

[6] Feldstein, M. S., 1974, "Social Security, Induced Retirement and Aggregate Capital Accumulation", Journal of Political Economy, Vol. 82, pp. 905-926.

[7] Hayashi, F., 1982,"Tobin's Marginal q and Average q: A Neoclassical Interpretation", Econometrica, Econometric Society, Vol. 50(1), pp. 213-24.

[8] Heijdra, B. and J. Ligthart, 2006, "The Macroeconomic Dynamics of Demographic Shocks", Macroeconomic Dynamics, Vol. 10, pp.349-370.

[9] Judd, K. L. 1992, "Projection Methods for Solving Aggregate Growth Models", Journal of Economic Theory, 58, pp.410–452.

[10] Mercenier, J. and P. Michel, 1994, "Discrete-Time Finite Horizon Approximation of Infinite Horizon Optimization Problems with Steady-State Invariance", Econometrica, Vol. 62(3), pp. 635–656.

[11] Mulligan, C. and X. Sala-i-Martin, 1991 "A Note on the Time Elimination Method for Solving Recursive Growth Models", Technical Working Paper No. 116.

[12] Nielsen, S. B., 1994, "Social Security and Foreign Indebtedness in a Small Open Economy", Open Economies Review, Vol. 5, pp. 47-63.

[13] Royal Commission on Population, 1949, Final Report. HMSO, London.

[14] Temple, J., 2003,"The Long-Run implications of Growth Theories", Journal of Economic Surveys, Blackwell Publishing, Vol. 17(3), pp. 497-510, 07.

[15] Trimborn, T., Koch, K.-J., and T. M. Steger, 2006, “Multi-Dimensional Transitional Dynamics: A Simple Numerical Procedure”, CESifo Working Paper Series CESifo Working Paper No., CESifo GmbH.

[16] United Nations, 2006, *World Population Prospects: The 2006 Revision Population Database*, http://esa.un.org/unpp/

[17] Yaari, M., 1965, "Uncertain Lifetime, Life Insurance and the Theory of the Consumer", Review of Economic Studies, Vol. 32, pp. 137-150.

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