Agency Costs, Risk Management, and Capital Structure

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Abstract

The joint determination of capital structure and investment risk is examined. Optimal capital structure reflects both the tax advantages of debt less default costs (Modigliani-Miller), and the agency costs resulting from asset substitution (Jensen-Meckling). Agency costs restrict leverage and debt maturity and increase yield spreads, but their importance is relatively small for the range of environments considered.

Risk management is also examined. Hedging permits greater leverage. Even when a firm cannot precommit to hedging, it will still do so. Surprisingly, hedging benefits often are greater when agency costs are low.

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The choice of investment financing, and its link with optimal risk exposure, is central to the economic performance of corporations. Financial economics has a rich literature analyzing the capital structure decision in qualitative terms. But it has provided relatively little specific guidance. In contrast with the precision offered by the Black and Scholes (1973) option pricing model and its extensions, the theory addressing capital structure remains distressingly imprecise. This has limited its application to corporate decision-making.

Two insights have profoundly shaped the development of capital structure theory. The arbitrage argument of Modigliani and Miller (*M-M*) (1958, 1963) shows that, with fixed investment decisions, nonfirm claimants must be present for capital structure to affect firm value. The optimal amount of debt balances the tax deductions provided by interest payments against the external costs of potential default.

Jensen and Meckling (*J-M*) (1976) challenge the *M-M* assumption that investment decisions are independent of capital structure. Equityholders of a levered firm, for example, can potentially extract value from debtholders by increasing investment risk after debt is in place: the "asset substitution" problem. Such predatory behavior creates agency costs which the choice of capital structure must recognize and control.

A large volume of theoretical and empirical work has built upon these insights.[1] But to practitioners and academics alike, past research falls short in two critical dimensions.

First, the two approaches have not been fully integrated. While higher risk may transfer value from bondholders, it may also limit the ability of the firm to reduce taxes through leverage. A general theory must explain how both *J-M* and *M-M* concerns interact to determine the joint choice of optimal capital structure and risk.

Second, the theories fail to offer *quantitative* advice as to the amount (and maturity) of debt a firm should issue in different environments. A principal obstacle to developing quantitative models has been the valuation of corporate debt with credit risk. The pricing of risky debt is a precondition for determining the optimal amount and maturity of debt. But risky debt is a complex instrument. Its value will depend on the amount issued, maturity, call provisions, the determinants of default, default costs, taxes, dividend payouts, and the structure of riskfree rates. It will also depend upon the risk strategy chosen by the firm—which in turn will depend on the amount and maturity of debt in the firm's capital structure.

Despite promising work two decades ago by Merton (1974) and Black and Cox (1976), subsequent progress was slow in finding analytical valuations for debt with realistic features. Brennan and Schwartz (1978) formulated the problem of risky debt valuation and capital structure in a more realistic environment, but required complex numerical techniques to find solutions for a few specific cases.

Recently some important progress has been made. Kim, Ramaswamy, and Sundaresan (1993) and Longstaff and Schwartz (1995) provide bond pricing with credit risk, although they do not focus on the choice of capital structure.[2] Leland (1994) and Leland and Toft (1996) consider optimal static capital structure. But the assumption of a static capital structure is limiting: firms can and do restructure their financial obligations through time.

Building on work by Kane, Marcus, and McDonald (1984), by Fischer, Heinkel, and Zechner (1989), and by Wiggins (1990), Goldstein, Ju, and Leland (1997) develop closed form solutions for debt value when debt can be dynamically restructured. These studies retain the *M-M* assumption that the firm's cash flows are invariant to debt choice. In doing so, the key *J-M* insight—that the firm's choice of risk may depend on capital structure—is ignored.

Another line of research, again using numerical valuation techniques, examines the potential feedback between investment/production decisions and capital structure. Brennan and Schwartz (1984) present a very general formulation of the problem, but one in which few general results can be derived. In a much more specific setting, Mello and Parsons (1992) extend the Brennan and Schwartz (1985) model of a mine to contrast the production decisions of a mine with and without debt in place. Mauer and Triantis (1994) analyze the interactions of production and financing decisions when debt covenants constrain choices to maximize total firm value. These covenants by assumption remove the potential incentive conflicts between stockholders and bondholders.[3]

This paper seeks to encompass elements of both the *M-M* and *J-M* approaches to optimal capital structure in a unified framework.[4] The model reflects the interaction of financing decisions and investment risk strategies. When investment policies are chosen to maximize equity value after *(*i.e., *ex post)* debt is in place, stockholder-bondholder conflicts will lead to agency costs as in *J-M*. The initial capital structure choice, made *ex ante*, will balance agency costs with the tax benefits of debt less default costs. Thus the optimal capital structure will reflect both *M-M* and *J-M* concerns.

The paper focuses on two interrelated sets of questions:

(i) *How does **ex post** flexibility in choosing risk affect optimal capital structure? In particular, how do leverage, debt maturity, and yield spreads depend upon risk flexibility?*

(ii) *How does the presence of debt distort a firm's **ex post** choice of risk? At the optimal capital structure and risk choices, how large are agency costs?*

The extant literature on firm risk-taking centers on increasing risk by asset substitution. This focus results from the analogy between equity and a call option on the firm.[5] One-period models examining asset substitution include Barnea, Haugen and Senbet (1980), Gavish and Kalay (1983), and Green and Talmor (1986). Barnea, Haugen and Senbet suggest that shorter maturity debt will be used when agency costs are high, a contention which has received only mixed empirical support.[6] In the analysis which follows, the role of debt maturity as well as leverage in controlling asset substitution is examined. The relative importance of agency considerations and tax benefits are also studied.

The framework equally permits the study of potential *decreases* in risk: risk management. Increasingly firms are using derivatives and other financial products to control risk. But our current understanding of why firms hedge is incomplete.[7] It is also unclear whether hedging is *ex post* incentive compatible with equity value maximization in the presence of risky debt. This paper provides a methodology to examine these and related questions.

In Section I below, the model of asset value dynamics and capital structure is described. Section II examines *ex post* selection of risk and introduces a measure of agency costs. Closed form values of debt and equity are derived. Section III considers the extent of asset substitution and agency costs in a set of examples, and shows how risk flexibility affects capital structure. Section IV extends the previous results to examine optimal risk management. Section V concludes.

I. The Model

A. The Evolution of Asset Value

Consider a firm whose unlevered asset value V follows the process

$$\frac{dV(t)}{V(t)} = (\mu - \delta)dt + \sigma dw(t), \tag{1}$$

where μ is the total expected rate of return, δ is the total payout rate to all security holders, σ is the risk (standard deviation) of the asset return, and $dw(t)$ is the increment of a standard Wiener process. Expected return, payout, and volatility may be functions of V, although restrictions are placed on these functions later. Initial asset value $V(0) = V_0$.

The value V represents the value of the net cash flows generated by the firm's activities (and excludes cash flows related to debt financing). It is assumed that these cash flows are spanned by the cash flows of marketed securities.

A riskfree asset exists that pays a constant continuously compounded rate of interest r. While Kim, Ramaswamy, and Sundaresan (1993) and other studies have assumed that r is stochastic, this increase in complexity has a relatively minor quantitative impact on their results.

B. Initial Debt Structure

The firm chooses its initial capital structure at time $t = 0$. The choice of capital structure includes the amount of debt principal to be issued, coupon rate, debt maturity, and call policy. This structure remains fixed without time limit until either (i) the firm goes into default (if asset value falls to the default level) or (ii) the firm calls its debt and restructures with newly-issued debt (if asset value rises to the call level).

Let P denote initial debt principal, C the continuous coupon paid by debt, M the average maturity of debt (discussed below), and V_U $(> V_0)$ the asset level at which debt will be called.

Default occurs if asset value falls to a level V_B prior to the calling of debt.[8] Different environments will lead to alternative default-triggering asset values. A "positive net worth" covenant in the bond indenture triggers default when net worth falls to zero, or $V_B = P$. If net cashflow is proportional to asset value, at a level λV, a cashflow-triggered default implies $V_B = C/\lambda$. Finally, default may be initiated endogenously when shareholders are no longer willing to raise additional equity capital to meet net debt service requirements. This determines V_B by the smooth-pasting condition utilized in Black and Cox (1976), Leland (1994), and Leland and Toft (1996). It is the default condition assumed here.

If default occurs, bondholders receive all asset value less default costs, reflecting the "absolute priority" of debt claims. Default costs are assumed to be a proportion α of remaining asset value V_B. Alternative specifications are possible. Different priority rules or default cost functions would change the boundary condition of debt value at $V = V_B$.

Although the finite-maturity debt framework of Leland and Toft (1996) could be used here, the approach introduced by Leland (1994b) and subsequently used by Ericsson (1997) and Mauer and Ott (1996) provides a much simpler analysis that admits finite average debt maturity. In this approach, debt has no stated maturity but is continuously retired at par at a constant fractional rate m. Debt retirement in this fashion is similar to a sinking fund that continuously buys back debt at par.

Debt is initially issued at time $t = 0$ with principal P and coupon payment rate C. At any time $t > 0$, a fraction e^{-mt} of this debt will remain outstanding, with principal $e^{-mt}P$ and coupon rate $e^{-mt}C$. Neglecting calls or bankruptcy, Leland (1994b) shows that the average maturity of debt $M = 1/m$.[9] Thus higher debt retirement rates lead to shorter average maturity.

Between restructuring points (and prior to bankruptcy), retired debt is continuously replaced by the issuance of new debt with identical principal value, coupon rate, and seniority. The firm's total debt structure (C, P, m) remains constant through time until restructuring or default, even though the amounts of previously-issued debt are declining exponentially through time through retirement.[10] New debt is issued at market value, which may diverge from par value.[11] Net refunding cost occurs at the rate $m(P - D(V))$, where $D(V)$ is the market value of total debt, given current asset value V. Higher retirement rates incur additional funding flows and raise the default value V_B. Debt retirement and replacement incurs a fractional cost k_2 of the principal retired.

C. Capital Restructuring

When $V(t)$ reaches V_U without prior default, debt will be retired at par value and a new debt will be issued as in Goldstein, Ju, and Leland (1997). The time at which debt is called is termed a "capital restructuring point." At the first restructuring point, P, C, V_B, and V_U will be scaled up by same proportion ρ that asset value has increased, where $\rho = V_U/V_0$. Debt and equity values will similarly increase by ρ. Subsequent restructurings will again scale up these variables by the same ratio. Initial debt and equity values will reflect the fact that capital restructurings potentially can occur an unlimited number of times. Initial debt issuance, and subsequent debt issuance at each restructuring point, incurs a fractional cost k_1 of the principal issued.

Downside restructurings prior to default are not explicitly considered. In principle such restructurings could be included (given a specification of how asset value would be split between bondholders and stockholders at the restructure point).[12] Note that if a downside restructuring were to take place at some value $V_L > V_B$, subsequent debt and the new bankruptcy-triggering value would be scaled downward by the factor $\gamma = V_L/V_0$. Repeated restructurings would always take place before default, and default would never occur. As default is not uncommon, this approach is not pursued. But observe that the model encompasses firms being restructured on a smaller scale *after* default; the costs of such restructuring (less future tax benefits) are subsumed in the parameter α.

II. Ex Post Selection of Risk and Agency Costs

With the few exceptions noted above, past studies of capital structure have assumed that risk σ and payout rate δ are exogenously fixed and remain constant through time. This paper extends previous work to allow the firm to *choose* its risk strategy.[13] The extension allows the analysis of two important and closely-related topics: asset substitution and risk management. It further permits an examination of the interaction between capital structure and risk choice.

To capture the essential element of agency, it is assumed that risk choices are made *ex post*, (that is, *after debt is in place),* and that the risk strategy followed by the firm cannot be pre-contracted in the debt covenants or otherwise precommitted. The analysis presumes rational expectations, in that both equityholders and the debtholders will correctly anticipate the effect of debt structure on the chosen risk strategy, and the effect of this strategy on security pricing.

The environment with *ex post* risk choice can be contrasted with the hypothetical situation where the risk strategy as well as the debt structure can be contracted *ex ante* (or otherwise credibly precommitted). In this situation the firm simultaneously chooses its risk strategy and its debt structure to maximize initial firm value. The difference in maximal values between the *ex ante* and *ex post* cases serves as a measure of agency costs, because it reflects the loss in value that follows from the risk strategy maximizing equity value rather than firm value. Ericsson (1997) uses a similar measure.

To keep the analysis as simple as possible, it is assumed that firms can choose continuously (and without cost) between a low and a high risk level: σ_L and σ_H, respectively.[14] Similar to Ross (1997), the risk strategy considered here determines a time-independent "switch point" value V_S, such that when $V < V_S$, the firm chooses the high risk level σ_H, and when $V \geq V_S$, the firm chooses the low risk level.[15]

In the subsections below, closed form solutions for security values are developed given the switch point V_S, the capital structure $X = (C, P, m, V_U)$, the default level V_B, and the exogenous parameters. Subsequent subsections determine the default level V_B and the optimal switch point V_S when the risk strategy is determined *ex ante* or *ex post*.

A. Debt Value D

Given constant risk σ over an interval of values $[V_1, V_2]$, Goldstein, Ju, and Leland (1997) (following Merton (1974)), show that $D^0(V,t)$, the value of debt issued at time $t = 0$, will satisfy the partial differential equation

$$\frac{1}{2}\sigma^2 V^2 D^0_{VV} + (r-\delta)VD^0_V - rD^0 + D^0_t + e^{-mt}(C+mP) = 0, \qquad V_1 \leq V \leq V_2 \tag{2}$$

where subscripts indicate partial derivatives. This reflects the fact that the original debtholders receive a total payment rate (coupon plus return of principal) of $e^{-mt}(C+mP)$.

Define $D(V) = e^{mt}D^0(V,t)$. Observe that $D(V)$ is the value of total outstanding debt at any future time t prior to restructuring. Because $D(V)$ receives a constant payment rate $(C+mP)$, it is independent of t. Substituting $e^{-mt}D(V)$ for $D^0(V,t)$ in equation 2, it follows that $D(V)$ satisfies the ordinary differential equation

$$\frac{1}{2}\sigma^2 V^2 D_{VV} + (r-\delta)VD_V - (r+m)D + (C+mP) = 0 \tag{3}$$

with general solution

$$D(V) = \frac{C + mP}{r + m} + a_1 V^{y_1} + a_2 V^{y_2}, \tag{4}$$

where

$$y_1 = \frac{-(r - \delta - \frac{\sigma^2}{2}) + \sqrt{(r - \delta - \frac{\sigma^2}{2})^2 + 2\sigma^2(r + m)}}{\sigma^2} \tag{5}$$

$$y_2 = \frac{-(r - \delta - \frac{\sigma^2}{2}) - \sqrt{(r - \delta - \frac{\sigma^2}{2})^2 + 2\sigma^2(r + m)}}{\sigma^2}, \tag{6}$$

and $a = (a_1, a_2)$ is determined by the boundary conditions at $V = V_1$ and $V = V_2$.

The risk strategy characterized by V_S specifies $\sigma = \sigma_L$ when $V_S \leq V \leq V_U$, and $\sigma = \sigma_H$ when $V_B \leq V < V_S$. From equation 4, the solutions to this equation in the high and low risk regions are given by

$$\begin{aligned} D(V) = DL(V) &= \frac{C + mP}{r + m} + a_{1L} V^{y_{1L}} + a_{2L} V^{y_{2L}}, \quad V_S \leq V \leq V_U, \\ = DH(V) &= \frac{C + mP}{r + m} + a_{1H} V^{y_{1H}} + a_{2H} V^{y_{2H}}, \ V_B \leq V < V_S \end{aligned} \tag{7}$$

with (y_{1H}, y_{2H}) given by equations 5 and 6 with $\sigma = \sigma_H$, and (y_{1L}, y_{2L}) given by equations 5 and 6 with $\sigma = \sigma_L$.

The coefficients $a = (a_{1H}, a_{2H}, a_{1L}, a_{2L})$ are determined by four boundary conditions. At restructuring,

$$DL(V_U) = P, \tag{8}$$

reflecting the fact that debt is called at par. At default,

$$DH(V_B) = (1 - \alpha) V_B, \tag{9}$$

recognizing that debt receives asset value less the fractional default costs α.[16]

Value matching and smoothness conditions at $V = V_S$ are

$$DH(V_S) = DL(V_S)$$
$$DH_V(V_S) = DL_V(V_S), \tag{10}$$

where subscripts of the functions indicate partial derivatives. In Appendix A, these four conditions are used to derive closed form expressions for the coefficients a, as functions of the capital structure X, the initial and bankruptcy values V_0 and V_B, the risk-switching value V_S, and the exogenous parameters including σ_L and σ_H.

B. Firm Value, Equity Value, and Endogenous Bankruptcy

Total firm value $v(V)$ is the value of assets, plus the value of tax benefits from debt $TB(V)$, less the value of potential default costs $BC(V)$ and costs of debt issuance $TC(V)$:

$$v(V) = V + TB(V) - BC(V) - TC(V). \tag{11}$$

These value functions include the benefits and costs in all future periods, and reflect possible future restructurings as well as possible default. They are time-independent because their cash flows and boundary conditions are not functions of time. Again following Merton (1974), any time-independent value function $F(V)$ with volatility σ will satisfy the ordinary differential equation

$$\frac{1}{2}\sigma^2 V^2 F_{VV} + (r - \delta) V F_V - rF + CF(V) = 0, \tag{12}$$

where $CF(V)$ is the time-independent rate of cash flow paid to the security. If the cash flow rate is a constant CF, equation 12 has solution

$$F(V) = \frac{CF}{r} + c_1 V^{x_1} + c_2 V^{x_2}, \tag{13}$$

where

$$x_1 = \frac{-(r - \delta - \frac{\sigma^2}{2}) + \sqrt{(r - \delta - \frac{\sigma^2}{2})^2 + 2\sigma^2 r}}{\sigma^2},$$

$$x_2 = \frac{-(r - \delta - \frac{\sigma^2}{2}) - \sqrt{(r - \delta - \frac{\sigma^2}{2})^2 + 2\sigma^2 r}}{\sigma^2}. \tag{14}$$

and $c_1$and c_2 are constants determined by boundary conditions.

If the cash flow $CF(V) = \kappa V$, equation 12 has solution

$$F(V) = \frac{\kappa V}{\delta} + c_1 V^{x_1} + c_2 V^{x_2}. \tag{15}$$

B.1. The Value of Tax Benefits TB

When the firm is solvent and profitable, debt coupon payments will shield income from taxes, producing a net cash flow benefit of τC. When earnings before interest and taxes ($EBIT$) are less than the coupon, tax benefits are limited to $\tau(EBIT)$.

Two simplifications permit closed form results: that $EBIT = \lambda V$ (earnings before interest and taxes are proportional to asset value), and that losses cannot be carried forward. Under these assumptions, the cash flows associated with tax benefits are

$$\begin{aligned} CF &= \tau C, \qquad & V_T \leq V \leq V_U \\ CF &= \tau \lambda V, \qquad & V_B \leq V \leq V_T \end{aligned}$$

where $V_T = C/\lambda$ is the asset value below which the interest payments exceed $EBIT$, and full tax benefits will not be received.

There are several possible regimes for the value of tax benefits, depending on the ordering of the values V_T, V_S, and V_0. Here it is assumed that $V_B < V_T < V_S < V_0 < V_U$.[17]

Using equations 13 and 15,

$$\begin{aligned} TB(V) = TBL(V) &= \tau C/r + b_{1L}V^{x_{1L}} + b_{2L}V^{x_{2L}}, \quad V_S \leq V \leq V_{U,} \\ = TBH(V) &= \tau C/r + b_{1H}V^{x_{1H}} + b_{2H}V^{x_{2H}}, \quad V_T \leq V < V_S, \\ = TBT(V) &= \tau \lambda V/\delta + b_{1T}V^{x_{1H}} + b_{2T}V^{x_{2H}}, \; V_B \leq V < V_T, \end{aligned} \tag{16}$$

where (x_{1H}, x_{2H}) and (x_{1L}, x_{2L}) are given by equation 14 with $\sigma = \sigma_H$ and $\sigma = \sigma_L$, respectively.

Boundary conditions are $TBL(V_U) = \rho TBL(V_0)$, reflecting the scaling property of the valuation functions at V_U, and $TBT(V_B) = 0$, reflecting the loss of tax benefits at bankruptcy. In addition, there are value-matching and smoothness requirements at V_S and V_T. These six conditions determine the coefficient vector $b = (b_{1L},\, b_{2L},\, b_{1H},\, b_{2H},\, b_{1T},\, b_{2T})$. A closed form expression for b is provided in Appendix A.

B.2. The Value of Default Costs BC

There is no continuous cash flow associated with default costs, and $CF = 0$ in equation 13. It follows that

$$\begin{aligned} BC(V) = BCL(V) &= c_{1L}V^{x_{1L}} + c_{2L}V^{x_{2L}}, \qquad V_S \leq V \leq V_U, \\ = BCH(V) &= c_{1H}V^{x_{1H}} + c_{2H}V^{x_{2H}}, \qquad V_B \leq V \leq V_S. \end{aligned} \tag{17}$$

Boundary conditions are $BCL(V_U) = \rho BCL(V_0)$, $BCH(V_B) = \alpha V_B$, and the value matching and smoothness conditions at V_S. Appendix A provides a closed form solution for the coefficients $c = (c_{1L},\, c_{2L},\, c_{1H},\, c_{2H})$.

B.3. The Value of Debt Issuance Costs

Debt issuance is costly. Initial debt issuance and subsequent restructurings incur a fractional cost k_1 of the principal value issued. The continuous retirement and reissuance of debt, which (prior to restructurings) occur at the rate mP, incurs a fractional cost k_2. It is presumed that k_1 and k_2 represent the after-tax costs of debt issuance.

Following Goldstein, Ju, and Leland (1997), consider the function $T\hat{C}(V)$, the value of transactions costs exclusive of the initial issuance cost at time $t = 0$. Noting that the flow of transactions costs associated with continuous debt retirement and replacement is $CF = k_2 mP$, and using equation 13 yields the function

$$\begin{aligned} T\hat{C}(V) = T\hat{C}L(V) &= \frac{k_2 mP}{r} + d_{1L}V^{x_{1L}} + d_{2L}V^{x_{2L}}, \quad V_S \leq V \leq V_U, \\ = T\hat{C}H(V) &= \frac{k_2 mP}{r} + d_{1H}V^{x_{1H}} + d_{2H}V^{x_{2H}}, V_B \leq V \leq V_S. \end{aligned} \tag{18}$$

with boundary conditions $T\hat{C}L(V_U) = \rho(T\hat{C}L(V_0) + k_1 P)$, $T\hat{C}H(V_B) = 0$, and the value matching and smoothness conditions at V_S. The coefficients $d = (d_{1L}$, d_{2L}, d_{1H}, $d_{2H})$ are derived in Appendix A.

Debt issuance costs $TC(V)$ are the sum of $T\hat{C}(V)$ and initial issuance costs $k_1 P$:

$$\begin{aligned} &TC(V) = \\ &TCL(V) = k_1 P + \frac{k_2 mP}{r} + d_{1L}V^{x_{1L}} + d_{2L}V^{x_{2L}}, \; V_S \leq V \leq V_U, \\ &TCH(V) = k_1 P + \frac{k_2 mP}{r} + d_{1H}V^{x_{1H}} + d_{2H}V^{x_{2H}}, V_B \leq V \leq V_S. \end{aligned} \tag{19}$$

B.4. Firm Value v

Firm value from equation 11 can now be expressed as

$$\begin{aligned} &v(V) = \\ &vL(V) = V + TBL(V) - BCL(V) - TCL(V), \quad V_S \leq V \leq V_U, \\ &vH(V) = V + TBH(V) - BCH(V) - TCH(V), \quad V_T \leq V \leq V_S, \\ &vT(V) = V + TBT(V) - BCH(V) - TCH(V), \quad V_B \leq V \leq V_T, \end{aligned} \tag{20}$$

where $TBL(V)$, $TBH(V)$, and $TBT(V)$ are given in equation 16, $BCL(V)$ and $BCH(V)$ are given in equation 17, and the $TCL(V)$ and $TCH(V)$ are given in equation 19.

B.5. Equity Value and Endogenous Bankruptcy

Equity value $E(V)$ is the difference between firm value $v(V)$ from equation 20 and debt value $D(V)$ from equation 7:

$$
\begin{aligned}
&E(V) = \\
&EL(V) = vL(V) - DL(V), && V_S \leq V \leq V_U, \\
&EH(V) = vH(V) - DH(V), && V_T \leq V \leq V_S, \\
&ET(V) = vT(V) - DH(V), && V_B \leq V \leq V_T.
\end{aligned} \tag{21}
$$

All security values are now expressed in closed form as functions of the debt choice parameters $X = (C, P, m, V_U)$, the default value V_B, the risk-switching point V_S, and the exogenous parameters $(\alpha, \delta, \lambda, r, \sigma_L, \sigma_H, \tau, V_0)$. It can be verified that debt and equity values are homogeneous of degree one in $(V, C, P, V_B, V_S, V_U, V_0)$.

The default V_B is chosen endogenously *ex post* to maximize the value of equity at $V = V_B$, given the limited liability of equity and the debt structure $X = (C, P, m, V_U)$ in place. This requires the smooth pasting condition

$$h(X, V_B, V_S) \equiv \frac{\partial ET(V, V_S)}{\partial V}|_{V=V_B} = 0, \tag{22}$$

where the remaining arguments of the functions ET and h have been suppressed.[18] While $h(X, V_B, V_S)$ can be expressed in closed form, a closed form solution for V_B satisfying condition 22 is not available. However, root finding algorithms can readily find V_B, given V_S and X.

C. The Choice of The Optimal Risk Switching Value

The optimal switching point between low and high volatility, V_S, will depend on whether it can be contracted *ex ante* or will be determined *ex post*, after debt is already in place. The difference in maximal firm value between these two cases will be taken as a measure of agency costs.

When the risk switching point can be committed *ex ante*, the firm will choose its capital structure $X = (C, P, m, V_U)$, default value V_B, and risk-switching point V_S to maximize the initial value of the firm:

$$\max_{X, V_B, V_S} \quad v(V, X, V_B, V_S)|_{V=V_0} \tag{23}$$

subject to

$$h(X, V_B, V_S) = 0, \tag{24}$$

$$P = D(V_0), \tag{25}$$

where equation 24 is the required smooth pasting condition at $V = V_B$ and equation 25 is the requirement that debt sell at par.

When the risk switching point V_S cannot be precommitted, it will be chosen *ex post* to maximize equity value E given the debt structure X which is in place. Consider the derivative

$$\begin{aligned} z(V_S, V_B, X) &= \frac{dEL}{dV_S}|_{V=V_S} \\ &= \frac{\partial EL}{\partial V_S}|_{V=V_S} + \frac{\partial EL}{\partial V_B}|_{V=V_S} \frac{\partial V_B}{\partial V_S} \end{aligned} \tag{26}$$

where

$$\frac{\partial V_B}{\partial V_S} = \frac{-\partial h / \partial V_S}{\partial h / \partial V_B}.$$

The function $z(V_S, V_B, X)$ measures the change in equity value that would result from a small change of the switch point at $V = V_S$, recognizing that V_B will change with V_S but capital structure X will not.[19] If z is nonzero, it will be possible to increase equity value by changing V_S. Therefore a necessary condition for V_S to be *ex post* optimal is that

$$z(V_S, V_B, X) = 0. \tag{27}$$

The optimal *ex ante* capital structure X and the optimal *ex post* risk switching point V_S will solve problem 23 subject to constraints 24, 25, and 27. Note that time homogeneity assures that V_S will not change through time until restructuring, at which point the scaling property implies V_S will be increased by the factor ρ.

The caveat that condition 27 is a necessary but not a sufficient condition is appropriate. Numerical examination of examples suggests that there are at most two locally optimal solutions to this problem, one with $V_S \leq V_0$, and one with $V_S = V_U$. In the latter case the firm always uses the high risk strategy σ_H.[20] When two locally optimal solutions exist, the solution with the larger initial firm value is chosen. The capital structure of that solution will induce its associated risk switching point.

Agency costs are measured by the difference in firm value between the *ex ante* optimal case, the maximum of 23 subject to constraints 24 and 25, and the *ex post* optimal case, the maximum of 23 subject to constraints 24, 25, and 27.

D. The Expected Maturity of Debt

Expected debt maturity EM depends upon two factors: the retirement rate m, and the possible calling of debt if V reaches V_U or default if V falls to V_B. Because there are two volatility levels, analytic measures of expected maturity are difficult to obtain.

Appendix 2 computes approximate bounds for expected debt maturity using two assumptions: default can be ignored, and risk is a constant σ. For most examples considered below, the likelihood of restructuring far exceeds the likelihood of default, so ignoring the latter may not be a significant problem. While risk is not constant, *average* risk is bounded above by σ_H and below by σ_L. Expected debt maturity $EM(\sigma)$ is monotonic in risk σ for the range of parameters considered. Therefore the computed bounds on expected maturity are given by $EM_{\max} = \text{Max}[EM(\sigma_L), EM(\sigma_H)]$ and $EM_{\min} = \text{Min}[EM(\sigma_H), EM(\sigma_L)]$.

III. The Significance of Agency Costs

This section applies the methodology of the previous section to examine properties of the optimal capital structure and the optimal risk strategy, and to estimate agency costs. Several examples are studied. In all cases, initial asset value is normalized to $V_0 = 100$. Base case parameters are:[21]

Default Costs:	α	=	.25
Payout Rate:	δ	=	.05
Cashflow Rate:	λ	=	.10
Tax Rate:	τ	=	.20
Riskfree Interest Rate:	r	=	.06
Restructuring Cost:	k_1	=	.01
Continuous Issuance Cost:	k_2	=	.005
Low Risk Level:	σ_L	=	0.20
High Risk Level:	σ_H	=	0.30

The low asset risk level is typical of an average firm; with leverage, equity risk will be somewhat greater than 30 percent per year.[22] The high asset risk level (which is varied below) reflects potential opportunities for "asset substitution". The rate of debt retirement m is a choice variable. For realism it is assumed that $m \geq 0.10$: at least 10 percent of debt principal must be retired per year, implying $M \leq 10$ years. The effects of relaxing this constraint are examined later.

Table 1 shows the optimal capital structure and risk switch points for the

	v	**V**$_S$	**V**$_U$	**EM**$_{\max}$ **(yrs)**	**EM**$_{\min}$ **(yrs)**	**V**$_B$	**LR (%)**	**YS (bp)**	**AC (%)**
Base Case: Ex Ante	108.6	44.7	201	5.65	5.53	33.6	49.4	69	-
Base Case: Ex Post	107.2	79.1	187	5.26	5.14	29.9	45.8	108	1.37
$\sigma_L = \sigma_H =$ **0.20**	107.4	-	196	5.52	5.52	32.4	42.7	48	-

Table 1: Choice of Risk Strategy and Capital Structure

base case, for both *ex ante* and *ex post* determination of the risk switching point V_S. For comparison, the case where the firm has no risk flexibility ($\sigma_L = \sigma_H = 0.20$) is also included. LR is the optimal leverage ratio, and AC measures agency costs as the percentage difference in firm value between optimal ex ante and optimal ex post risk determination. In all cases the minimum constraint $m \geq 0.10$ is binding. Thus debt with the lowest annual rate of principal retirement (here 10 percent) is always preferred.

The following observations can be made:

(i) When the firm's risk policy can be committed *ex ante* to maximize firm value, it nonetheless will increase risk when asset value is low (and therefore leverage is high). For asset values between $V_B = 33.6$ and $V_S = 44.7$, the high risk strategy is chosen. Increasing risk exploits the firm's option to continue the realization of potential tax benefits and avoid default. Leverage actually rises relative to the firm with no risk flexibility. This reiterates the fact that optimal risk strategies do not merely pit stockholders vs. bondholders, but stockholders vs. the government (and bankruptcy lawyers) as well.

(ii) When the firm's risk policy is determined *ex post* to maximize equity value, the firm will switch to the high-risk level at a much greater asset value: V_S increases to 79.1. Higher V_S imply that the firm operates with higher average risk, and reflects the "asset substitution" problem.

(iii) Agency costs are modest: 1.37 percent, less than one fifth of the tax benefits associated with debt.[23] Note that agency costs *when measured against the firm which has no risk flexibility* are even lower: 0.20 percent instead of 1.37 percent. Thus covenants which restrict the firm from (ever) adopting the high risk strategy will have very little value in the environment considered.

(iv) Capital structure shifts in the presence of agency costs. Leverage and the restructure level V_U both decrease relative to the ex ante case. Expected maturity falls, confirming the predictions of Myers (1977) and Barnea, Haugen, and Senbet (1980). The debt structure adjustments are not large in the base case, however.

(v) The yield spread on debt rises by a very significant amount, from 69 to 108 basis points, reflecting the greater average firm risk. Thus agency costs, even when small, may have a significant effect on the yields of corporate debt. Earlier models of risky debt pricing (e.g. Jones, Mason, and Rosenfeld (1984)) predicted yield spreads that were too small; the results here suggest that even relatively modest agency costs may provide an explanation.

A. Comparative Statics

Figure 1 charts *ex post* firm value v, the risk-switching point V_S, the optimal leverage ratio LR and yield spread YS, the restructure point V_U, the default asset value V_B, and agency costs AC as functions of the high-risk level σ_H. All other parameters, including the low risk level σ_L, remain as in the base case. Larger σ_H can be associated with a greater potential for asset substitution.

Not surprisingly, the risk switching point V_S and agency costs increase with σ_H. Less expected is that the leverage ratio and the maximal firm value rise slightly, despite the increase in agency costs. This can be understood in light of the fact that, with *ex ante* risk determination, both firm value and leverage increase significantly with σ_H. Therefore, *relative* to their levels in the *ex ante* case, firm value and leverage in the *ex post* case are falling as σ_H increases.

Yield spreads increase rapidly, reflecting the rise in average risk.

Figure 2 charts the effect of different default costs α. For $\alpha > 0.0625$, the risk switching point V_S is less than V_0 and decreases with α. Higher default costs imply lower average risk. Leverage falls with α, but agency costs are relatively flat. Several papers have sought to find a positive relationship between leverage and agency costs; this result suggests that such a relationship may be hard to identify if default costs are a principal source of leverage variations. The restructure point V_U, and expected debt maturity, are relatively stable. Thus expected maturity will not necessarily be inversely related to leverage.

When default costs are low ($\alpha < 0.0625$ in the base case), risk switching occurs immediately if asset value drops ($V_S = V_0 = 100$). As α falls further, V_S would rise above V_0 if V_U does not fall significantly. But there is no stable V_S level between V_0 and V_U, implying that V_S will jump to V_U if V_U remains high. $V_S = V_U$ is a stable local optimum. But there is a second local optimum, when V_U is reduced, and $V_S = V_0$ remains optimal. The smaller is α, the lower V_U must be to keep $V_S = 100$. In comparing the two local optima, the second gives a higher firm value for the parameters of the base case, and hence it will be chosen.[24] The effect can be seen in Figure 2: as α approaches zero, V_U and expected debt maturity declines significantly to provide incentives for bounding V_S at $V_0 = 100$.

Figure 3 considers changes in the payout rate δ. Lower payouts produce higher firm value v, because a higher leverage ratio can be supported when more assets remain in the firm. Despite higher leverage, yield spreads are smaller, for two reasons: more assets remain in the firm to reduce the likelihood of default, and average firm risk is lower since the risk switching value V_S is lower. Agency costs are relatively flat across a wide range of payout ratios.

Figure 4 considers the effects of alternative debt retirement rates m. Here leverage ratios are positively correlated with agency costs. As m falls towards zero, V_S and average risk rise, and the restructuring value V_U falls dramatically. Nonetheless, expected debt maturity will rise, reflecting the lower debt retirement rate m.

Note that maximal firm value increases as m falls. Despite higher agency costs, the resulting capability to maintain higher leverage ratios induces firms to minimize their principal retirement rate.

IV. Risk Management

The analysis above can be applied to risk management in a straightforward manner. A firm has an exogenously-given normal asset risk, now denoted by σ_H. However, at any time it is assumed that the firm can choose to reduce its risk costlessly to a given level σ_L, perhaps by using derivatives to hedge exposures.[25] A lower σ_L indicates a more effective available hedging strategy.

The firm can cease hedging at any time. As before, the strategies considered specify a risk-switching asset value V_S. When $V \geq V_S$, the firm chooses to hedge, with resultant risk σ_L. When $V < V_S$, the firm abandons its hedge and operates with normal risk σ_H.

Two environments are again considered. In the first, the firm can pre-contract its hedging strategy (summarized by V_S). It will choose both its capital structure and hedging strategy *ex ante* to maximize market value. In the second, it cannot pre-commit to any hedging strategy. It will choose its capital structure *ex ante* to maximize market value, subject to the constraint that the choice of hedging strategy maximizes the value of equity *ex post*, given the debt in place.

These environments are contrasted with two other scenarios: when the firm can do no hedging whatsoever; and when the firm can pre-commit to hedge under all circumstances. The benefit of hedging is measured by the percent increase in firm value from using optimal hedging strategies compared with the no-hedging case. Even though the always-hedging case is suboptimal, the difference in firm value between always hedging and never hedging is often (and incorrectly) proposed as "the" measure of the benefits of hedging.

A. An Example

Exogenous parameters are as in Section III, but with volatility of the unhedged firm $\sigma_H = 0.20$. **Table 2** lists firm value v, the risk switching point (or "hedge abandonment point") V_S, optimal leverage LR, and other variables for the *ex ante* and *ex post* hedging cases. Comparable numbers are listed when no hedging is possible ($\sigma_L = \sigma_H = 20$ percent). The benefits of hedging (ignoring possible costs of hedging) are measured by HB, the percent increase in firm value in comparison with no hedging. Agency cost, AC, measures the percent difference between *ex ante* and *ex post* optimal firm values.

Two possible levels of hedging effectiveness are considered. **Panel A** examines the base case when risk can be reduced to $\sigma_L = 15\%$. The *ex ante* optimal strategy, the *ex post* optimal strategy, and the "always hedge" strategy are compared. **Panel B** has similar comparisons when risk can be reduced to $\sigma_L = 10\%$.

Hedging provides modest benefits, even when the hedging strategy cannot be precommitted.[26] Benefits in the *ex post* base case are 1.44 percent of firm value, excluding possible costs of hedging. More effective hedging (lower σ_L) produces gains of 3.73 percent, as seen in Panel B. These gains result principally from the fact that lower average volatility allows higher leverage, with consequently greater tax benefits. This may be contrasted with earlier studies such as Smith and Stulz (1985) which have emphasized lower expected costs of default given *fixed* leverage. But some benefits come from lower expected default rates, as evidenced by lower yield spreads in Panels A and B despite the greater leverage.

The extent to which the firm hedges is directly related to the magnitude of V_S, the asset value at which the firm ceases to hedge. Higher V_S imply less hedging on average. Compared with the optimal *ex ante* hedging strategy, V_S is higher and hedging is abandoned "too quickly" in the *ex post* case, the result of equity value maximization rather than firm value maximization. In the base case, the inability to pre-commit to the optimal hedging strategy loses about a third of potential hedging benefits. Nonetheless, the *ex post* optimal strategy performs almost as well as an *ex ante* commitment by the firm to *always* hedge.

Finally, the case where risk management might be used for *speculative* as well as hedging purposes is considered. **Panel C** sets $\sigma_L = 15\%$, but assumes that the same instruments which can reduce risk can be used to *increase* risk to $\sigma_H = 30\%$. Note that firm value in the *ex ante* case increases with σ_H. A firm that can increase risk to a higher level can "play the option" to continue in business. But the possibility of incurring higher risk creates greater agency costs in the *ex post* case, and the net benefits to hedging are substantially reduced. Nonetheless they remain positive.

In comparison with the no-hedging case, leverage increases but expected debt maturity falls. In comparison with the *ex ante* optimal strategies, *ex post* optimal strategies have both lower leverage and shorter expected debt maturity. This again confirms the contention of Myers (1977) and Barnea, Haugen, and Senbet (1980) that shorter maturity is used to control agency costs.

B. Comparative Statics

Table 3 examines optimal *ex post* risk strategies and optimal capital structure for a range of parameter values, when $\sigma_L = 15\%$. The Table assumes all exogenous parameters remain at their base case levels except for the parameter heading each row. HB as before measures the benefits of hedging as the per-

	v	$\mathbf{V}_S$	$\mathbf{V}_U$	$\mathbf{EM}_{\max}$ **(yrs)**	$\mathbf{EM}_{\min}$ **(yrs)**	$\mathbf{V}_B$	**LR (%)**	**YS (bp)**	**HB (%)**
No Hedging	107.4	-	195	5.49	5.49	32.4	42.7	48	-
Panel A: Base Case: Hedging to σ_L= 15%									
Ex Ante Optimal	109.7	48.6	175	4.93	4.87	40.6	51.7	33	2.08
Ex Post Optimal	108.9	69.2	171	4.79	4.73	38.1	50.0	41	1.44
Always Hedge	109.0	-	173	4.86	4.86	40.2	48.5	27	1.46
Panel B: Hedging to σ_L= 10%									
Ex Ante Optimal	112.4	61.1	154	4.13	4.03	52.3	62.4	19	4.66
Ex Post Optimal	111.3	80.1	146	3.73	3.63	46.6	60.6	36	3.60
Always Hedge	111.4	-	152	4.03	4.03	52.7	57.4	13	3.77
Panel C: Hedging to σ_L= 15%; Speculation to σ_H= 30%									
Ex Ante Optimal	113.4	65.2	182	5.22	4.98	48.5	69.7	82	5.59
Ex Post Optimal	108.5	84.9	162	4.48	4.26	35.4	53.8	105	1.02
Always Hedge	109.0	-	173	4.86	4.86	40.2	48.5	27	1.46

Table 2: Optimal Hedging Strategies and Capital Structure

	v	$\mathbf{V}_S$	$\mathbf{V}_U$	$\mathbf{EM}_{\max}$ **(yrs)**	$\mathbf{EM}_{\min}$ **(yrs)**	$\mathbf{V}_B$	**LR** (%)	**YS** **(bp)**	**HB** (%)	**AC** (%)
Ex Post Hedging; $\sigma_L = 15\%$										
Base Case	108.9	69.2	171	4.79	4.73	38.1	50.0	41	1.44	0.65
$\alpha = 0.10$	111.1	85.9	172	4.82	4.76	46.3	61.5	76	0.95	0.83
$\alpha = 0.50$	106.8	51.2	171	4.79	4.73	30.6	38.1	21	1.89	0.32
$\delta = 0.04$	112.0	67.2	172	4.49	4.46	41.1	52.8	36	2.19	0.66
$\delta = 0.06$	106.9	71.6	172	5.20	5.09	35.1	46.9	46	0.96	0.64
$m = 0.05$	110.1	82.7	158	5.34	5.19	38.0	53.8	72	0.85	1.22
$m = 0.25$	106.8	53.1	175	3.04	2.92	36.1	41.4	10	3.22	0.15
$\lambda = 0.05$	108.8	63.4	170	4.76	4.70	34.6	46.3	29	1.83	0.61

Table 3: Comparative Statics: Ex Post Hedging

centage increase in value v relative to an otherwise-identical firm that cannot hedge (i.e. $\sigma_L = \sigma_H = 20\%$). AC measures agency costs by comparing the maximal firm value when V_S is chosen *ex post* with that of an otherwise-identical firm which can choose V_S *ex ante.*

As might be expected, the extent of hedging and hedging benefits increase with default costs α. In contrast with the no-hedging case with $\alpha = 0.50$, hedging permits the firm to raise optimal leverage substantially, from 28 percent to 38 percent. But even so, leverage and yield spread are relatively small when α is large. It would be erroneous to presume that firms will hedge less when they have lower leverage and less risky debt. Indeed the opposite is true when default costs α are the source of variation. It is therefore not surprising that empirical tests of the relationship between leverage and hedging by Block and Gallagher (1986), Dolde (1993), and Nance, Smith, and Smithson (1993) find no significant relationship. In contrast with optimal leverage, optimal debt maturity is relatively insensitive to changes in α.

Lower payout rates δ lead to greater leverage and benefits from hedging, but shorter expected maturity. Lower retirement rates m also lead to greater leverage and expected debt maturity (despite the fall in V_U), but hedging and hedging benefits fall dramatically. Hedging benefits are sizable when short term debt is mandated ($m = 0.25$). This reflects the large increase in leverage which the reduced risk from hedging allows. The results show that short term debt is more incentive-compatible with hedging than long term debt.

Lowering net cash flow λ from 10 percent to 5 percent of asset value has two effects. Smaller $EBIT$ reduces the potential for interest payments to shelter taxable income, and maximal value decreases slightly. But with smaller $EBIT$, taxes become a more convex function of asset value. Greater convexity means that expected taxes will be reduced more by hedging. Thus the benefits to hedging are larger, as anticipated by Mayers and Smith (1982) and Smith and Stulz (1985).

A somewhat surprising result is that agency costs and the benefits to hedging are *inversely* related in many cases. High bankruptcy costs, short average debt maturity, and low cash flows are all associated with large hedging benefits but low agency costs. These results challenge the presumption that greater agency costs necessarily imply greater benefits tohedging.

V. Conclusion

Equityholders control the firm's choice of capital structure and investment risk. In maximizing the value of their claims, equityholders will choose strategies which reduce the value of other claimants, including the government (tax collector), external claimants in default, and debtholders. Modigliani and Miller *(*1963*)* emphasize the importance of taxes and default costs in determining leverage. Jensen and Meckling *(*1976*)* emphasize the importance of bondholders' claims in determining risk. But all claimants must be jointly recognized in the determination of capital structure and investment risk.

The model developed above examines optimal firm decisions. It provides quantitative guidance on the amount and maturity of debt, on financial restructuring, and on the firm's optimal risk strategy. Both asset substitution and risk management are studied. Agency costs and the potential benefits of hedging are calculated for a range of environments. For realistic parameters, the agency costs of debt related to asset substitution are far less than the tax advantages of debt. Relative to an otherwise-similar firm which can pre-contract risk levels before debt is issued, the firm will choose a strategy with higher average risk. Leverage will be lower and debt maturity will be shorter. Yield spreads rise as the potential for asset substitution increases. But relative to an otherwise-similar firm which has no potential for asset substitution, optimal leverage may actually rise. This contradicts the presumption that optimal leverage will fall when asset substitution is possible.

Conventional wisdom is challenged by a number of other results. Asset substitution will occur even when there are no agency costs (the *ex ante* case), albeit to a lesser degree than when agency costs are present (the *ex post* case). Agency costs may not be positively associated with optimally-chosen levels of leverage. Greater hedging benefits are not necessarily related to environments with greater agency costs. And equityholders may voluntarily agree to hedge after debt is in place, even though it benefits debtholders: the tax advantage of greater leverage allowed by risk reduction more than offsets the value transfer to bondholders.

The model is restrictive in a number of dimensions. Managers are assumed to behave in shareholders' interests. Dividend (payout) policies and investment scale are treated as exogenous. And information asymmetries are ignored. Relaxing these assumptions remains a major challenge for future research.

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Appendix A

A.1. Debt Coefficients

Boundary conditions include the value-matching and smoothness conditions 10 at $V = V_S$:

$$a_{1L}V_S^{y_{1L}} + a_{2L}V_S^{y_{2L}} - a_{1H}V_S^{y_{1H}} - a_{2H}V_S^{y_{2H}} = 0$$

$$y_{1L}a_{1L}V_S^{y_{1L}-1} + y_{2L}a_{2L}V_S^{y_{2L}-1} - y_{1H}a_{1H}V_S^{y_{1H}-1} - y_{2H}a_{2H}V_S^{y_{2H}-1} = 0$$

The boundary condition 8 at V_U with $\sigma = \sigma_L$ is

$$a_{1L}V_U^{y_{1L}} + a_{2L}V_U^{y_{2L}} = P,$$

and boundary condition 9 at default with $\sigma = \sigma_H$:

$$a_{1H}V_B^{y_{1H}} + a_{2H}V_B^{y_{2H}} = (1-\alpha)V_B.$$

Solving for a gives

$$\begin{bmatrix} a_{1L} \\ a_{2L} \\ a_{1H} \\ a_{2H} \end{bmatrix} = \begin{bmatrix} V_S^{y_{1L}} & V_S^{y_{2L}} & -V_S^{y_{1H}} & -V_S^{y_{2H}} \\ y_{1L}V_S^{y_{1L}-1} & y_{2L}V_S^{y_{2L}-1} & -y_{1H}V_S^{y_{1H}-1} & -y_{2H}V_S^{y_{2H}-1} \\ V_U^{y_{1L}} & V_U^{y_{2L}} & 0 & 0 \\ 0 & 0 & V_B^{y_{1H}} & V_B^{y_{2H}} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ P - \frac{C+mP}{r+m} \\ (1-\alpha)V_B - \frac{C+mP}{r+m} \end{bmatrix} \tag{A1}$$

A.2. Tax Benefit Coefficients

Boundary conditions include the scaling condition

$$TBL(V_U) = (V_U/V_0)TBL(V_0);$$

the default condition

$$TBT(V_B) = 0;$$

and the smoothness and value-matching conditions at V_S and at V_T:

$$TBL_V(V_S) = TBH_V(V_S)$$

$$TBL(V_S) = TBH(V_S)$$

$$TBH_V(V_T) = TBT_V(V_T)$$

$$TBH(V_T) = TBTV_T).$$

Substituting the appropriate equations for TBL, TBH,and TBT from 16 into the boundary conditions and recalling $\rho = V_U/V_0$ leads to the following solution for the coefficients b:

$$\begin{bmatrix} b_{1L} \\ b_{2L} \\ b_{1H} \\ b_{2H} \\ b_{1T} \\ b_{2T} \end{bmatrix} = \Omega^{-1}\eta \qquad \text{(A2)}$$

where

$$\Omega =$$

$$\begin{bmatrix} V_U^{x_{1L}} - \rho V_0^{x_{1L}} & V_U^{x_{2L}} - \rho V_0^{x_{2L}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & V_B^{x_{1H}} & V_B^{x_{2H}} \\ x_{1L} V_S^{x_{1L}-1} & x_{2L} V_S^{x_{2L}-1} & -x_{1H} V_S^{x_{1H}-1} & -x_{2H} V_S^{x_{2H}-1} & 0 & 0 \\ V_S^{x_{1L}} & V_S^{x_{2L}} & -V_S^{x_{1H}} & -V_S^{x_{2H}} & 0 & 0 \\ 0 & 0 & x_{1H} V_T^{x_{1H}-1} & x_{2H} V_T^{x_{2H}-1} & -x_{1H} V_T^{x_{1H}-1} & -x_{2H} V_T^{x_{2H}-1} \\ 0 & 0 & V_T^{x_{1H}} & V_T^{x_{2H}} & -V_T^{x_{1H}} & -V_T^{x_{2H}} \end{bmatrix}$$

$$\eta =$$

$$\begin{bmatrix} (\rho - 1)\frac{\tau C}{r} \\ -\tau \lambda V_B / \delta \\ 0 \\ 0 \\ \tau \lambda / \delta \\ \tau C/\delta - \tau C / r \end{bmatrix}$$

A.3. Default Cost Coefficients

Under the assumption that the risk-switching value $V_S < V_0$, boundary conditions include the scaling property

$$BCL(V_U) = \rho BCL(V_0)$$

and default condition

$$BCH(V_B) = \alpha V_B.$$

Substituting for BCL and BCH from equation 17 into the equations above, together with the smoothness and value-matching conditions at V_S, gives

$$\begin{bmatrix} c_{1L} \\ c_{2L} \\ c_{1H} \\ c_{2H} \end{bmatrix} = \begin{bmatrix} V_U^{x_{1L}} - \rho V_0^{x_{1L}} & V_U^{x_{2L}} - \rho V_0^{x_{2L}} & 0 & 0 \\ 0 & 0 & V_b^{x_{1H}} & V_b^{x_{2H}} \\ x_{1L} V_S^{x_{1L}-1} & x_{2L} V_S^{x_{2L}-1} & -x_{1H} V_S^{x_{1H}-1} & -x_{2H} V_S^{x_{2H}-1} \\ V_S^{x_{1L}} & V_S^{x_{2L}} & -V_S^{x_{1H}} & -V_S^{x_{2H}} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \alpha V_B \\ 0 \\ 0 \end{bmatrix} \tag{A3}$$

A.4. Debt Reissuance Cost Coefficients

The scaling property at the restructure point implies

$$T\hat{C}L(V_U) = \rho(T\hat{C}L(V_0) + k_1 P)$$

and the default boundary condition is

$$T\hat{C}H(V_0) = 0.$$

Substituting for the functions $T\hat{C}L$ and $T\hat{C}H$ from equation 18 into the equations above, together with the smoothness and value-matching conditions at V_S, gives

$$\begin{bmatrix} d_{1L} \\ d_{2L} \\ d_{1H} \\ d_{2H} \end{bmatrix} = \begin{bmatrix} V_U^{x_{1L}} - \rho V_0^{x_{1L}} & V_U^{x_{2L}} - \rho V_0^{x_{2L}} & 0 & 0 \\ 0 & 0 & V_b^{x_{1H}} & V_b^{x_{2H}} \\ x_{1L} V_S^{x_{1L}-1} & x_{2L} V_S^{x_{2L}-1} & -x_{1H} V_S^{x_{1H}-1} & -x_{2H} V_S^{x_{2H}-1} \\ V_S^{x_{1L}} & V_S^{x_{2L}} & -V_S^{x_{1H}} & -V_S^{x_{2H}} \end{bmatrix}^{-1} \begin{bmatrix} (\rho - 1)k_2 mP/r + \rho k_1 P \\ -k_2 mP/r \\ 0 \\ 0 \end{bmatrix} \tag{A4}$$

Appendix B

Recall that debt issued in amount $P(0)$ at time $t = 0$ is redeemed at the rate $mP(t)$, where $P(t) = e^{-mt}P(0)$. Thus the average maturity of debt $M(T)$, if debt is called at par at time T, is given by

$$\begin{aligned} M(T) &= \int_0^T \frac{tmP(t)}{P(0)} dt + \frac{TP(T)}{P(0)} \\ &= \int_0^T tme^{-mt} dt + Te^{-mT}, \\ &= \frac{1 - e^{-mT}}{m} \end{aligned}$$

The call time T is random, with first passage time to V_U density (ignoring default) given by

$$f(T) = \frac{b}{\sigma(2\pi T^3)^{1/2}} \exp\left(-\frac{1}{2}\left(\frac{b - (\mu - \delta - .5\sigma^2)T}{\sigma T^{1/2}}\right)^2\right),$$

where $b = \text{Log}(V_U/V_0)$. Expected maturity of the debt, therefore, is given by

$$\begin{aligned} EM &= \int_0^\infty M(T)f(T)dT \\ &= \frac{1}{m}(1 - \left(\frac{V_U}{V_0}\right)^h), \end{aligned}$$

where

$$h = \frac{(\mu - \delta - 0.5\sigma^2) - ((\mu - \delta - 0.5\sigma^2)^2 + 2m\sigma^2)^{1/2}}{\sigma^2}.$$

Footnotes

* Haas School of Business, University of California, Berkeley. The author thanks Samir Dutt, Nengjiu Ju, Michael Ross, and Klaus Toft both for computer assistance and for economic insights. My intellectual debts to professional colleagues are too numerous to list, but are clear from the references cited. Any errors remain the sole responsibility of the author.

1. See survey articles by Harris and Raviv (1991) and Brennan (1995). A third important approach to corporate finance has emphasized the role of asymmetric information between insiders and outside investors. This paper does not address informational asymmetries.

2. Other related work includes Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997), who focus on strategic debt service. Zhou (1996) and Duffie and Lando (1997) have extended the stochastic process of asset value, V, to include jumps and imperfect observation, respectively, in models examining credit spreads. An alternative approach to valuing credit risks, different in nature from that pursued here, has been pioneered by Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), Madan and Unal (1994), Duffie and Singleton (1995), Das and Tufano (1996), and Nielsen and Ronn (1996).

3. Three recent papers have analyzed capital structure and investment/operating decisions jointly. Ericsson (1997) offers an elegant analysis of asset substitution in a related setting; his model is compared with this work in Section III. Mauer and Ott (1996) consider the effect of growth options on capital structure. Decamps and Faure-Grimaud (1997) examine a firm which can choose when to shut down operations.

4. The focus of this paper is on agency costs generated by stockholder-bondholder conflicts. Conflicts between managers and stockholders are not considered in this paper, but in principle could be included if a managerial objective

function were specified.

5. Long (1974) questions the exactness of the options analogy for equity. See also Chesney and Gibson-Asner (1996).

6. Barclay and Smith (1995) find a link between debt maturity and measures of agency cost related to growth opportunities; Stohs and Mauer (1996) find the linkage ambiguous. Empirical analysis has been made more difficult because few theoretical models which determine both the optimal amount and maturity of debt are available to formulate hypotheses. Stohs and Mauer (1996) suggest that leverage should be an explanatory variable when regressing debt maturity on measures of agency costs. But the theoretical model developed here suggests that leverage, maturity, and agency costs are *jointly* determined by exogenous variables, leading to potential misspecification if leverage is considered exogenous.

7. Reasons offered include the convexity of tax schedules and reduction in expected costs of financial distress (Mayers and Smith (1982), Smith and Stulz (1985)), reducing stockholder-bondholder conflicts (Mayers and Smith (1987)), costly external financing (Froot, Scharfstein, and Stein (1993)), managerial risk aversion (Smith and Stulz (1985) and Tufano (1996)), and the ability to realize greater tax advantages from greater leverage (Ross (1996)). Mian (1996) finds empirical support is ambiguous for all hypotheses except that hedging activities exhibit economies of scale–big firms are more likely to hedge.

8. What happens to the firm in default is not modelled explicitly. It could range from an informal workout to liquidation in bankruptcy, depending upon the least-cost feasible alternative.

9. The average maturity of debt when principal is retired at the rate $mP(t)$ is given by

$$M = \int_{t=0}^{\infty} t\frac{mP(t)}{P}dt = \int_{t=0}^{\infty} t\frac{me^{-mt}P}{P}dt = \frac{1}{m}.$$

10. It is not unreasonable that total debt remains constant prior to the next restructuring or bankruptcy. Currently-outstanding debt is regularly protected from increases in debt of similar or greater seniority; here, debt must be called before the amount of debt is increased at restructuring points. And reduction of debt prior to bankruptcy may not be in the interest of shareholders even if firm value would be increased: see Leland (1994), Section VIII.

11. To avoid path-dependent tax savings from debt, the tax consequences resulting from bonds selling below or above par are assumed negligible.

12. See Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) for a discussion of strategic debt service.

13. While δ is assumed here to be exogenous, straightforward extensions of this approach would enable an examination of payout (or dividend) policies as well. In related models with static debt structure, Fan and Sundaresan (1997) consider payout policies, and Ross (1997) examines joint risk/payout policies using numerical techniques. The extension to the choice of payout policies is not pursued here, however.

14. In a closely related environment, numerical optimization by Ross (1997) indicates that if there exists an *interval* of risk levels $[\sigma_L, \sigma_H]$, the firm will choose one extreme or the other: a "bang-bang" control is optimal. Ericsson (1997) also studies a related case: when the firm can make an irreversible one-time decision at a value $V = K$ to raise risk from σ_L to σ_H.

15. A single risk-switching point is assumed. In a related context, Leland (1994b) shows that debt value becomes relatively less sensitive to changes in risk than equity value, as V increases. This implies that if it does not benefit equityholders to exploit debtholders by increasing risk at $V = V_S$, the optimal policy will not increase risk when $V > V_S$. Ross (1997) does not find reversals in his numerical optimizations.

16. This condition could be changed to reflect alternative formulations of

priorities and costs in default.

17. In some examples below, alternative orderings characterize the optimum. It is left to the interested reader to extend the analysis to such alternative orderings.

18. If multiple solutions exist to equation 22, the largest solution for V_B is chosen. This is the only solution consistent with the limited liability of equity, i.e. that $E(V) \geq 0$ for $V \geq V_B$.

19. Equation 26 is invariant to whether EL or EH is the function used; this follows from smoothness at V_S.

20. As noted above, the equations for security values derived above presume $V_B < V_T < V_S < V_0$. Obviously this condition is not satisfied if $V_S = V_U$, and appropriately modified equations for security values must be used.

21. These parameters roughly reflect a typical Standard and Poor's 500 firm. The default cost α is at the upper bound of recent estimates by Andrade and Kaplan (1997), although their sample of firms may have lower default costs than average, since these firms initially had high leverage, and high leverage is more likely to be optimal for firms with low costs of default. Payout rates and cashflow rates as a proportion of asset value are consistent with average levels, and the tax rate τ reflects personal tax advantages to equity returns which reduce the net advantage of debt to below the corporate tax rate of 35 percent: see Miller (1977).

22. For computing expected maturity bounds, the expected asset total rate of return μ is needed. A annual risk premium of 7 percent above the riskfree rate is assumed, a level consistent with historical returns on the market portfolio. Higher risk premia will typically yield lower expected maturities.

23. Ericsson (1997) finds higher agency costs (approximating 5%) in his model, which assumes a one-time permanent shift to a higher risk level. While exact comparisons are rendered difficult, the higher costs appear to follow from

his assumptions of a static capital structure, and no lower bound on the parameter m.

24. Examples can be constructed (e.g., when $m = 0$) where the local optimum $V_S = V_U$ gives a higher value than the local optimum when $V_S = V_0$ (which of course requires a different V_U). In this case, agency considerations induce the firm always to operate at σ_H.

25. Such hedging will incur no value costs if derivatives are fairly priced and transactions costs are minimal.

26. Smith and Stulz (1985) question whether *ex post* hedging is ever in the stockholders' best interests. The answer is clearly "yes", although less hedging will occur than with an *ex ante* commitment to hedging.

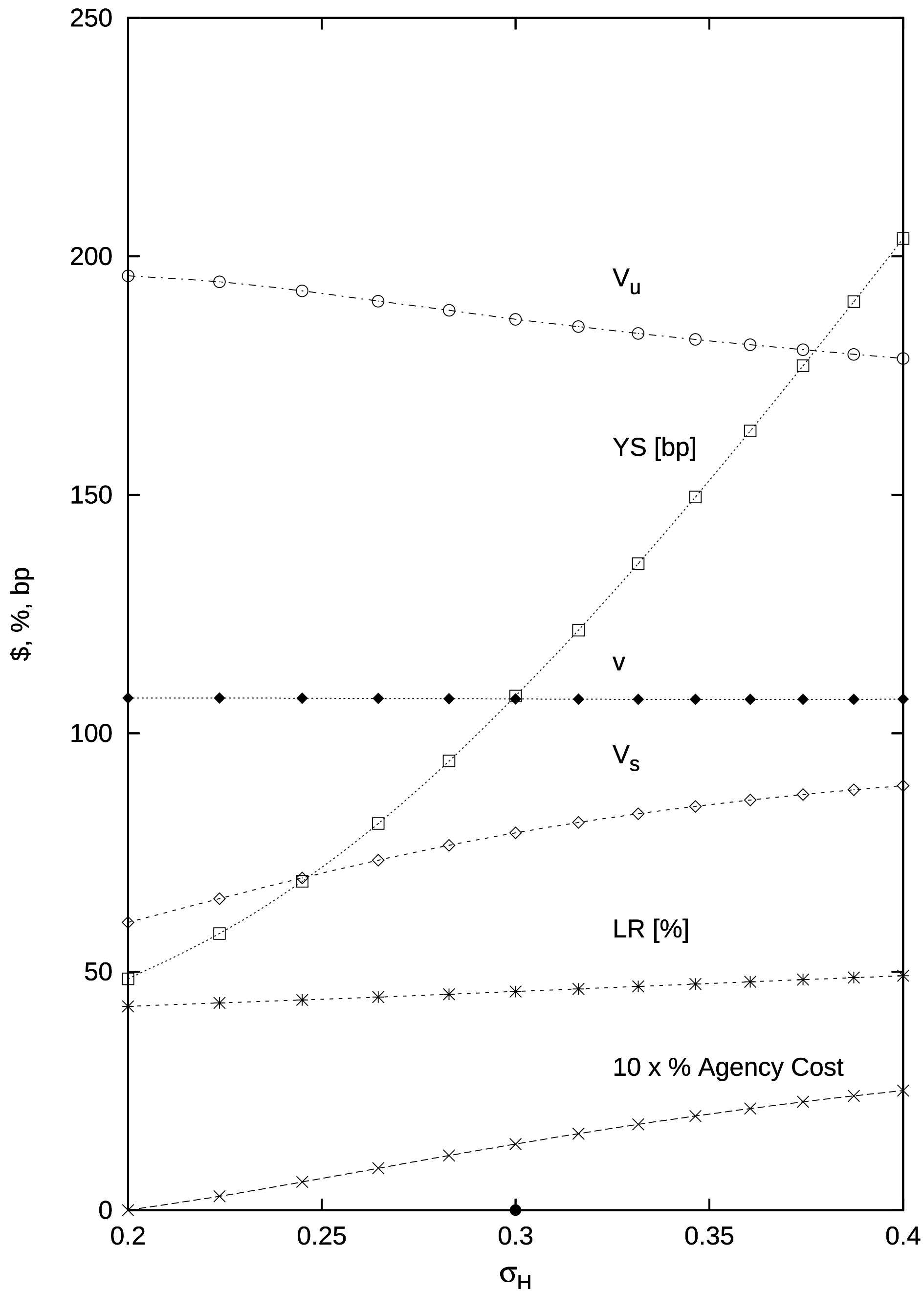

Figure 1: Variation of optimal corporate financial structure with σ_H for baseline parameter values of $m = 0.1$, $\delta = 0.05$, $\tau = 0.2$, $\gamma = 1.0, r = 0.06, \sigma_L = 0.2$, $\alpha = 0.25$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$. The solid dot on the horizontal axis denotes the baseline value of σ_H.

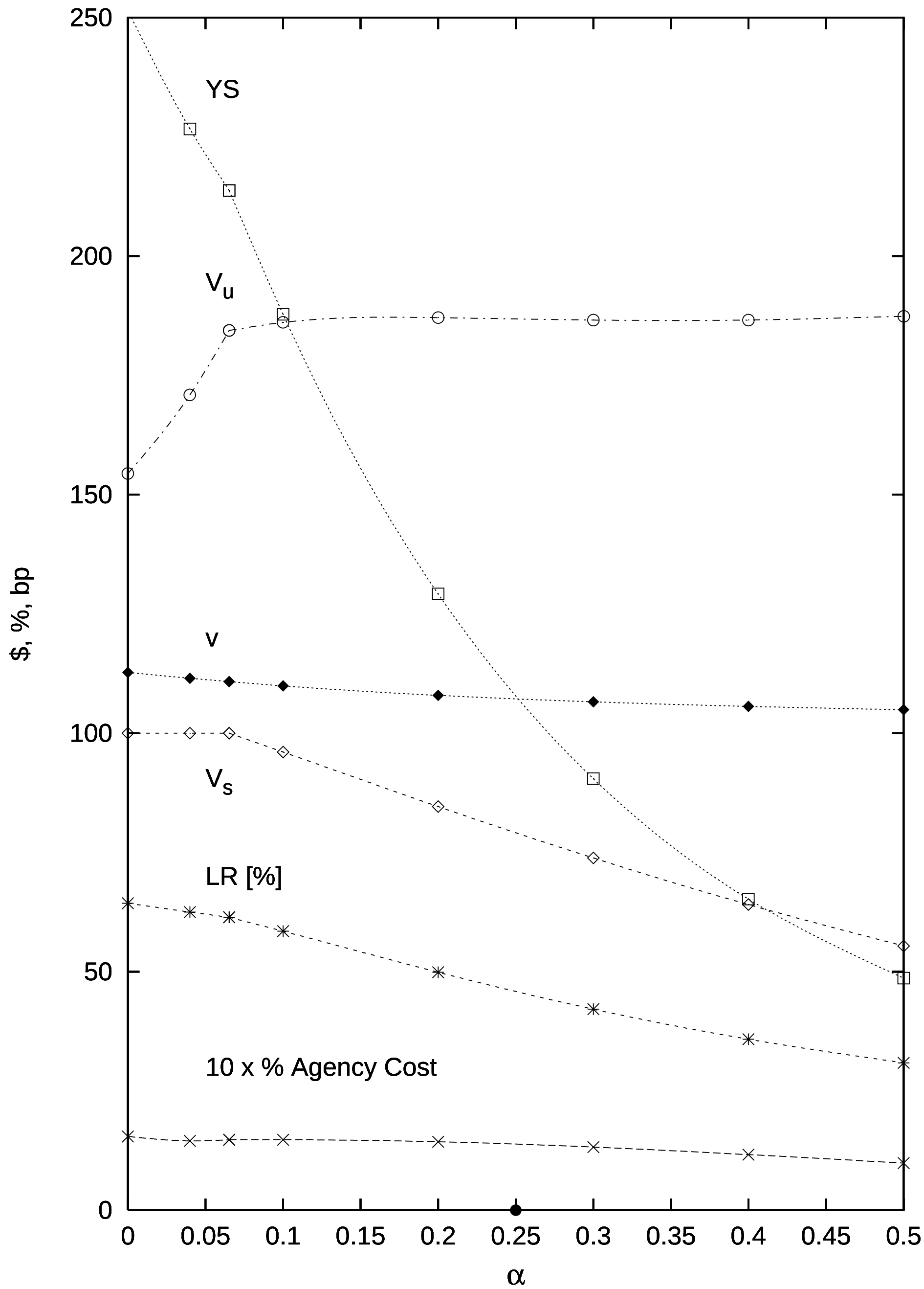

Figure 2: Variation of optimal corporate financial structure with α for baseline parameter values of $m = 0.1$, $\delta = 0.05$, $\tau = 0.2$, $\gamma = 1.0$, $r = 0.06$, $\sigma_L = 0.2$, $\sigma_H = 0.3$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$. The solid dot on the horizontal axis denotes the baseline value of α.

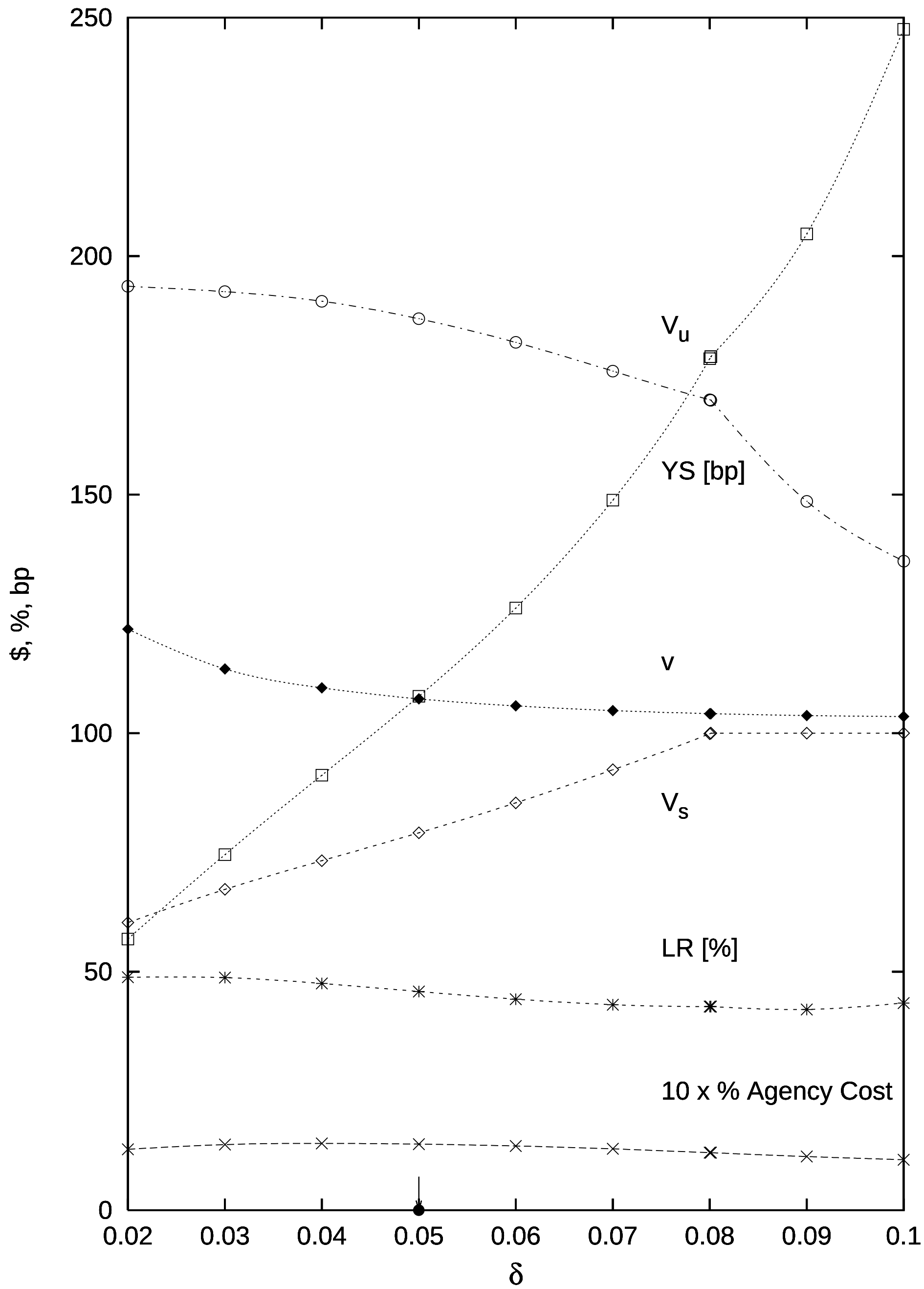

Figure 3: Variation of optimal corporate financial structure with δ for baseline parameter values of $m = 0.1$, $\tau = 0.2$, $\gamma = 1.0$, $r = 0.06$, $\sigma_L = 0.2, \sigma_H = 0.3$, $\alpha = 0.25$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$. The solid dot on the horizontal axis denotes the baseline value of δ.

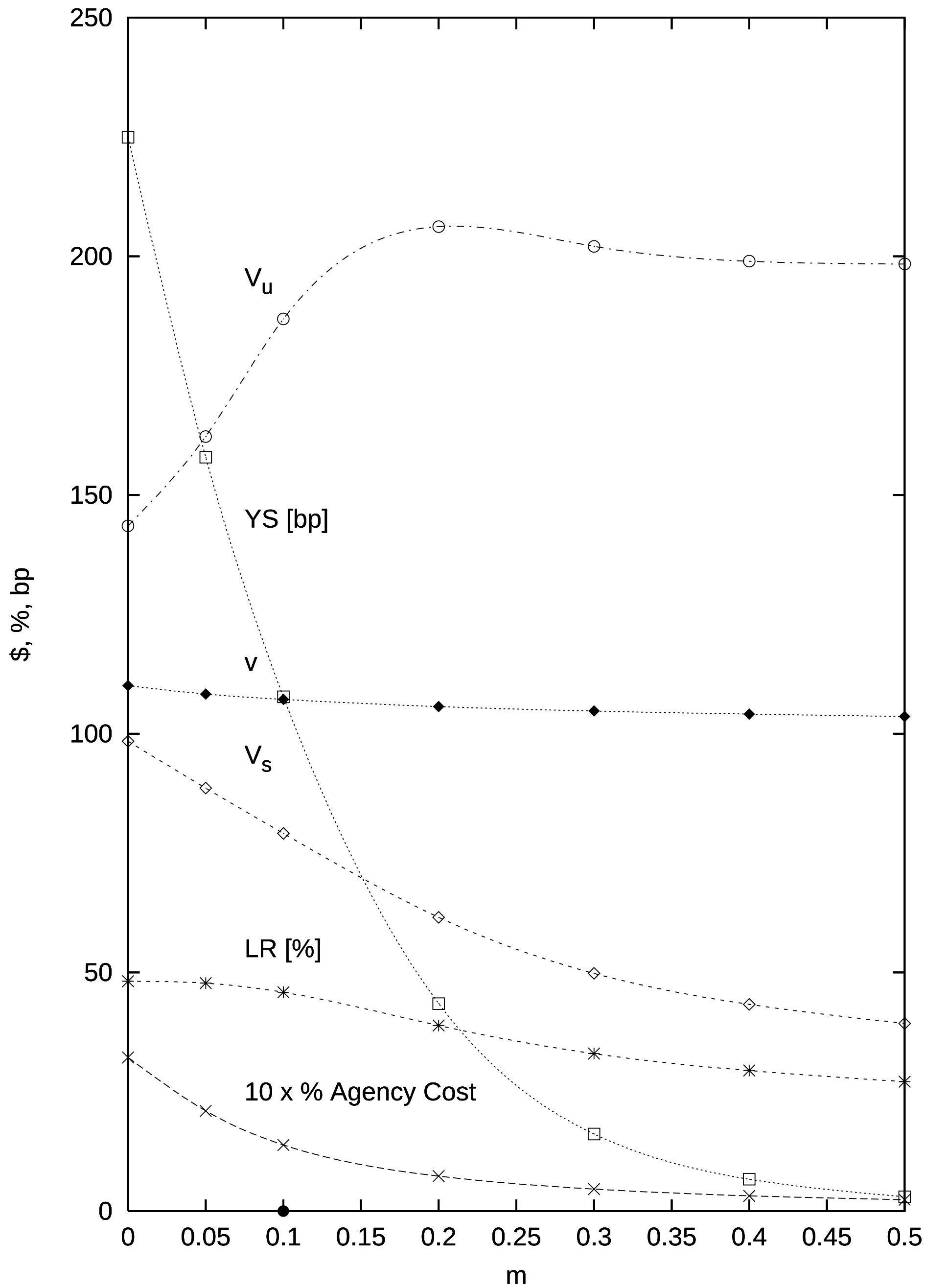

Figure 4: Variation of optimal corporate financial structure with m for baseline parameter values of $\delta = 0.05$, $\tau = 0.2$, $\gamma = 1.0$, $r = 0.06$, $\sigma_L = 0.2$, $\sigma_H = 0.3$, $\alpha = 0.25$, $k_1 = 0.005$, $k_2 = 0.01$, and $V_0 = 100$. The solid dot on the horizontal axis denotes the baseline value of m.