WORKING PAPERS

Johannes Holler

On the Role of Pension Systems in Economic Development and Demographic Transition

June 2008

Working Paper No: 0812

DEPARTMENT OF ECONOMICS

UNIVERSITY OF VIENNA

All our working papers are available at: http://mailbox.univie.ac.at/papers.econ

On the Role of Pension Systems in Economic Development and Demographic Transition

J. Holler*

June 2008

Abstract

In this paper we examine whether different pension systems affect the set of initial human capital conditions capturing an economy in a low steady state equilibrium income. To analyze this problem, we employ a three period overlapping generations model where fertility and investments into the children's education are chosen endogenously. We show that education investments are higher and start at lower income levels for a pay-as-you-go pension system economy compared to an informal, fertility related one. The income threshold needed to escape the "poverty trap" is therefore lower if a pay-as-you-go pension system is employed. Moreover, unless the economy is caught in the low income steady state, a pay-as-you-go pension system supports higher equilibrium income. We further highlight that pension systems influence the timing of demographic transition through their different valuation of fertility, contributing to the explanation for observed differences between developed and developing countries.

JEL classification: H55, J13, O23

Key words: Pension systems, poverty trap, demographic transition

---

*Department of Economics, University of Vienna, Hohenstaufengasse 9, A-1010 Vienna, Austria, Phone: +43 (0)1 427737427, Fax: +43 (0)1 42779374, johannes.holler@univie.ac.at

1 Introduction

Many countries around the world suffer from persistent underdevelopment. In the year 2001 about 21% of the world population lived below the poverty line of 1$/day (World Bank (2005)). While political and ecological reasons can be responsible for this tragedy we are focusing on economic explanations. A "poverty trap", the economic term for the situation of an economy captured in a low equilibrium per capita income can be caused by a variety of factors like corruption, search externalities (Diamond (1982)), learning-by-doing externalities (Brezis, Krugman, Tsiddon (1993)) or human capital externalities (Azariadis, Drazen (1990)). Our work is focusing on the situation where a demographic trap (Becker, Murphy, Tamura (1990)) is causing a situation where an economy is caught in a vicious circle of low human capital and high population growth which supports low equilibrium income and low education. Recent studies show the importance of the amount of pension payments (Boldrin, De Nardi, Jones (2005)) as well as the type of pension system (Groezen, Leers and Meijdam (2003), Sinn (2004), Holler (2007)) on agents' fertility decisions. We pick up the idea that pension systems play an important role in fertility dynamics aiming to analyze their effect on long-run per capita income and the income threshold needed to escape a poverty trap.

By including a subsistence level of retirement consumption as in Galor and Weil (2000), we reproduce the historically observed inverted U-shaped fertility dynamics corresponding to increasing income levels.[1] This allows us to additionally study the influence of different pension systems on the timing of demographic transition.

Starting point of our analysis is a model discussed by Ken Tabata (2003) which emphasizes the importance of public education investments on human capital accumulation and the possibility of being caught in a poverty trap. In contrast to this paper we focus on the role of different pension systems on demographic transition and the determination of the human capital threshold level needed to approach a high steady state equilibrium income.

To analyze the impacts of a change from one pension system to the other, this paper is comparing equilibrium per capita income and fertility rates corre-

[1] Works by Dyson and Murphy (1985), Kremer (1993), Lucas (1999) and Lee (2003) give a detailed explanation of fertility reactions along increasing income levels.

sponding to an informally financed pension system and a pay-as-you-go pension system. This enables us to answer the question whether the introduction of a pay-as-you-go pension system can help developing countries to escape a poverty trap. Additionally we check the viability of a fertility-related pension system introduction to diminish a decrease in fertility rates and analyze the corresponding cost.

In the past major changes in pension schemes were mainly due to sociological or demographic changes. Bismarck's social security system introduced in 1889 was for example only a reaction to the brake of familial ties due to the offsetting of industrial revolution. The generous pension schemes after the second World War for almost all western welfare states were also a reaction to missing contributions from a whole generation. This highlights that changes in the pension scheme were not seen as a tool to change economic variables but were only adjusted to the changing environment. Inspired by Wigger (1999) who showed that public pension system contributions are crowding out private gifts from children to their parents we assume that a public pension systems introduction triggers a break for interfamily transfers from adults to the old. This revises the causality between economic variables and pension design and shows that it can be used as a tool in development economics. A variety of African countries that are about to reform their pension systems could use this insight not only to react on demographic changes but also to influence economic development. As the first African country Nigeria performed in 2004 a structural pension system reform by introducing a multi-pillar scheme with mandatory pension contributions. Countries such as Kenya, Senegal and Uganda will soon follow, showing that the political and social structure of the countries seems to be mature enough to impose structured public pension systems.

Our work compares optimal education and fertility decisions for a traditional, informally organized pension system economy with a public pay-as-you-go pension system economy. We choose to examine a pay-as-you-go pensions system as - especially in developing countries - capital markets are not very well established making a fully funded pension system difficult to introduce. Note that the results derived from our comparison would also be true if we exchanged the unfunded with a funded pension system. Only the magnitude of the effects would change. The break of intrafamily ties that takes place for any type of public pension system is the reason for the lower marginal benefit of fertility that drives our results.

2 The Model

We assume a standard neoclassical constant returns to scale production sector for a small open economy. The interest rate is exogenously given and constant. Capital is perfectly mobile implying that the capital labor ratio k and the wage rate w are fixed and constant. The Diamond type OLG economy is populated by finitely living homogenous agents. Individuals live for three periods: childhood, adulthood and retirement. During childhood individuals consume $\theta w h_t$, where θ is a fixed fraction of adult working time needed to rear one child and h_t is the amount of human capital an adult at period t is holding. Human capital is determining the effectiveness of labor. Total working income wh_t is therefore increasing in human capital. During adulthood households decide on quantity n_t and quality of children represented by education investments e_t. Education and fertility decisions are implicitly determining the amount of savings s_t. Child quality investments are like quantity investments expressed as working time cost. Following Galor and Weil (2000) we use a Cobb-Douglas utility function which allows us to abstract from adult consumption without changing the qualitative results. The population dynamics for the productive adult population are described by $N_{t+1} = N_t n_t$. Retired people only consume and have no influence on household optimization. They are assumed to use up their whole savings plus pension benefits. Bequests are therefore excluded from the model.

We assume that individuals preferences are hierarchic in the sense that individuals draw utility solely from retirement consumption as long as a certain subsistence level $\underline{c} > 0$ is not secured. Utility from having children is only derived if the adult income level supports retirement consumption above $\underline{c}$. Along the lines of De La Croix and Doepke (2003) we assume that adults are drawing utility from the existence and the future human capital of their children which is determining future adult income and well being.

Individuals utility is represented by the following logarithmic additive separable function:

$$u^t = \beta \log(c_{t+1}) + (1-\beta)\log(n_t h_{t+1}) \quad (1)$$

The discount factor β which is assumed to be smaller than 1 is determining time preference as well as adult altruism toward children.

Throughout the paper we refer to the second part of the utility function which reflects the consumption good value of a child as the altruistic value of child investments. While the word altruism implies that actions are made despite own utility considerations which is not the case in our model we stick to the word to pay tribute to earlier work we are building upon. Next to the consumption good motive of fertility we additionally model the old age security motive of fertility by incorporating ascending altruism (Wigger (1999)). Following Morand (1999) ascending altruism of individual's preferences is captured through gifts from adult children to their parents during retirement. The ascending altruistic part of preferences is therefore captured in the composition of pension payments π_{t+1} provided for the third period of life. Inspired by the "intergenerational flow theory" (Caldwell (1982)) we assume that ascending altruism is only present for countries without a mandatory public pension system.[2] Two different pension system scenarios are examined.

Informal pension system: This scenario is describing the situation of developing countries where children are socially responsible for the wellbeing of their retired parents. Pension benefits are therefore dependent on own fertility decisions. Nevertheless the pension contributions are socially mandatory (World Bank (1991)) we use the terminology of ascending altruism to describe the private intrafamilial transfer from adult children to their old parents. The transfers τ are assumed to be lump sum. We further assume that the pension system is always budget balanced implying:

$$\pi^I_{t+1} = n^I_t \tau$$

Pay-as-you-go pension system: This case examines economies with a functioning public mandatory pay-as-you-go pension system. In the absence of bequests old age support is the only motivation for private interfamilial gifts.[3] If the state takes over the role of supporting the old generation private gifts are therefore fully crowded out. A mandatory public pension system further implies that pension payments do not depend on individual fertility but on average fertility of the whole economy $\overline{n}_t$. Due to the nature of a pay-as-you-go pension system being managed by a public authority bureaucracy and corruption cost arise. In order to capture this especially for developing countries important

[2] A more detailed description of this argument is provided by Holler (2007).

[3] Positive bequests would lead to children contributing to their parents pensions through private gifts in expectation of bequests from their parents at the end of their lives. For a detailed description of the different bequest motives see Zhang and Zhang (2001) or (WB 1991).

fact, we introduce the parameter $B \in (0,1)$ capturing the efficiency of the public pension system in our pension formula. High bureaucracy or corruption is represented by a low level of B and vice versa.

$$\pi_{t+1}^{P} = B\overline{n}_t^P \tau$$

We start by considering an efficient system with $B = 1$. In subsection 6 we will relax this assumption and discuss its implications. Adults endowed with a human capital level h_t divide their after tax income $h_t w - \tau$ between child cost (rearing cost $\theta n_t h_t w$ + child education cost $e_t n_t h_t w$) and savings since they do not draw utility from consumption when adult. The adulthood budget is therefore constrained by:

$$wh_t n_t(\theta + e_t) + \tau + s_t \leqslant wh_t \tag{2}$$

Retirement consumption is financed through the value of savings at period $t+1$ plus pension benefits. Agents consume their whole retirement income since we assume that bequests are zero. Following Galor and Weil (2000) minimum retirement consumption is limited by a subsistence level $\underline{c}$ that secures survival when old.

$$c_{t+1} = Rs_t + \pi_{t+1} \tag{3}$$

$$c_{t+1} \geqslant \underline{c} \tag{4}$$

Economic growth is solely determined by the evolution of human capital over time. Following Tabata (2006) human capital accumulation is determined by education investments, adult human capital level h_t and productivity of the education sector determined by the parameters a, b, η and σ.

$$h_{t+1} = \eta(a + be_t h_t)^{\sigma}; \eta, a, b > 0; 0 < \sigma < 1 \tag{5}$$

Adult human capital is entering the accumulation formula to capture the positive influence of parental human capital on the child's future skills. The positive a parameter is securing that future human capital is positive in the case of zero education investments. Since σ is smaller than one each additional unit of education investment pays less in terms of additional future human capital.

Equation (1) subject to (2), (3), (4) and (5) describe the household optimization problem. For sufficiently high income supporting consumption above the subsistence level the optimization leads to first order conditions 1 to 4. Superscript P and I specify pay-as-you-go and informal pension system variables.

$$e_t^I : \beta \frac{wh_t R n_t^I}{c_{t+1}^I} = (1-\beta)\frac{bh_t\sigma}{a + be_t^I h_t} \tag{FOC 1}$$

$$n_t^I : \beta \frac{wh_t R(\theta + e_t^I)}{c_{t+1}^I} = (1-\beta)\frac{1}{n_t^I} + \beta\frac{\tau}{c_{t+1}^I} \tag{FOC 2}$$

$$e_t^P : \beta \frac{wh_t R n_t^P}{c_{t+1}^P} = (1-\beta)\frac{bh_t\sigma}{a + be_t^P h_t} \tag{FOC 3}$$

$$n_t^P : \beta \frac{wh_t R(\theta + e_t^P)}{c_{t+1}^P} = (1-\beta)\frac{1}{n_t^P} \tag{FOC 4}$$

Adults can either invest in child quantity (n_t) or child quality (e_t). At the optimum, marginal benefit of the investments have to equal marginal cost. FOC 1 and 3, describing optimal education decisions for both pension systems, state that the marginal value of education measured in terms of additional future human capital has to equal marginal cost of education measured in terms of retirement consumption. In other words at the point where marginal altruistic utility of additional future child income equals marginal cost of reduced retirement consumption education investments are optimal. Optimal Fertility decisions covered in FOC 2 and FOC 4 demand that marginal cost of a child are equal to marginal benefits. FOC 2 further shows that marginal child utility is split into an altruistic and a retirement consumption part for the informal pension system. This is due to the existence of positive intrafamilial gifts. For an economy with a pay-as-you-go pension system marginal child utility is solely determined by altruism (see FOC 4). The first order conditions further highlight that for both pension system cases, a quality quantity trade-off à la Becker and Barro (1988), is in place. High investments in child education are implying low fertility and vice versa.

After describing the situation of relatively high income levels, supporting retirement consumption above or equal to the subsistence level, we focus toward the low income cases. If income levels cannot support subsistence, retirement consumption condition (4) becomes binding and optimal decisions are described by FOC 5.

$$\underline{c} = Rwh_t - (wh_t R(\theta + e_t) - \tau)n_t - \tau R \tag{FOC 5}$$

2.1 Education

In the described model, education investments are solely driven by altruism because they do not create any benefit in the form of retirement consumption. Therefore we assume parents to choose positive education investments only if parental income is supporting a retirement consumption level above subsistence. In other words investments in the quality of children only take place if old age survival is secured. The parameter assumptions connected to this assumption are summarized in Lemma 1.

Lemma 1 *If* $a > \frac{(R\beta\tau + \underline{c})b\theta\sigma}{Rw\beta}$ *the human capital level supporting the subsistence level of consumption* $\underline{h}$ *is lower than the human capital level that supports positive education investments* $\overline{h}$ *for both pension systems.*

To make things easier we skip the proof of Lemma 1 to subsection 2.3. Due to our parameter assumptions we can observe two optimal education results depending on whether income is below or above the positive education investment threshold $\overline{h}$.

$$e_t^P = \begin{cases} 0 & \text{if } h_t \leqslant \overline{h}^P \\ \frac{b\theta\sigma h_t - a}{b(1-\sigma)h_t} & \text{if } h_t \geqslant \overline{h}^P = \frac{a}{b\theta\sigma} \end{cases}$$

$$e_t^I = \begin{cases} 0 & \text{if } h_t \leqslant \overline{h}^I \\ \frac{bRw\theta\sigma h_t - aRw - b\sigma\tau}{b(1-\sigma)Rwh_t} & \text{if } h_t \geqslant \overline{h}^I = \frac{aRw + b\sigma\tau}{bRw\theta\sigma} \end{cases}$$

From optimal education decisions we follow that the threshold level needed to make education decisions positive is different for both pension systems.

Proposition 1 *The positive education threshold* $\overline{h}$ *is higher for the informal pension system case* ($\overline{h}^I > \overline{h}^P$) *implying that it takes higher income levels to make education investments positive. From* $\overline{h}^P$ *onwards pay-as-you-go education investments are higher than informal ones.*

Proof. $\overline{h}^I = \overline{h}^P + \underbrace{\frac{\tau}{Rw\theta}}_{>0}; e_t^P = e^I + \underbrace{\frac{\sigma\tau}{(1-\sigma)Rwh_t}}_{>0}.$ ■

2.2 Fertility

Optimal fertility decisions are again dependent on the level of adult income. Due to our parameter assumptions, we have to differentiate between the following three cases. The first case of human capital below the subsistence threshold $\underline{h}$ describes the situation where the subsistence retirement consumption assumption is binding and education investments are zero. $\underline{h} \leqslant h_t \leqslant \overline{h}$ corresponds to the second case where investments in child quality are still zero but the income level is already high enough to lead to retirement consumption above the subsistence level. The third case $h_t \geqslant \overline{h}$ is reflecting the situation of relatively high human capital supporting positive education investments.

Based on the fact that average fertility is equal to individual fertility because agents are homogenous, optimal pay-as-you-go fertility decisions are described by:

$$n_t^P = \begin{cases} \frac{\underline{c}+R\tau-Rwh_t}{\tau-Rw\theta h_t} & \text{if } h_t \leqslant \underline{h}^P \\ \frac{R(\beta-1)(\tau-wh_t)}{(\beta-1)\tau+Rw\theta h_t} & \text{if } \underline{h}^P \leqslant h_t \leqslant \overline{h}^P \\ \frac{bR(\beta-1)(\sigma-1)(\tau-wh_t)}{aRw+b(\beta-1)(\sigma-1)\tau-bRw\theta h_t} & \text{if } h_t \geqslant \overline{h}^P \end{cases}$$

Optimal informal fertility decisions are represented by:

$$n_t^I = \begin{cases} \frac{\underline{c}+R\tau-Rwh_t}{\tau-Rw\theta h_t} & \text{if } h_t \leqslant \underline{h}^I \\ \frac{R(\beta-1)(\tau-wh_t)}{Rw\theta h_t-\tau} & \text{if } \underline{h}^I \leqslant h_t \leqslant \overline{h}^I \\ \frac{bR(1-\beta)(\sigma-1)(\tau-wh_t)}{bRw\theta h_t-aRw-b\tau} & \text{if } h_t \geqslant \overline{h}^I \end{cases}$$

From equation (5) and Proposition 1, we know that for income levels $h_t < \overline{h}^P$ human capital is constant at ηa^σ. This enables us to directly compare optimal fertility decisions for this income range. We follow that informal and pay-as-you-go fertility are equal for income levels below the subsistence threshold ($h_t < \underline{h}$). Optimal fertility results further imply that the income level needed to surpass minimum retirement consumption is different for both pension systems.

$$\underline{h}^I = \frac{R\beta\tau + \underline{c}}{Rw\beta}$$

$$\underline{h}^P = \frac{\underline{c}Rw\theta + R^2w\beta\theta\tau \pm \sqrt{4\underline{c}R^2w^2(\beta-1)\beta\theta\tau + (\underline{c}Rw\theta + R^2w\beta\theta\tau)^2}}{2R^2w^2\beta\theta}$$

As long as income is below the threshold $\underline{h}$, retirement consumption is constant at $\underline{c}$. In this case parents would like to give up retirement consumption in order to have more children. This is not possible because retirement consumption is at a level needed for survival and condition (4) becomes binding. Implicitly the existence of this case demands that children are a costly investment. In other words opportunity cost of children have to be higher than benefits ($Rw\theta h_0 > \tau$). If this would not be the case condition (4) could never become binding because having more children would not decrease but increase retirement consumption. The assumption that retirement consumption has to be high enough to secure survival together with the fact that fertility has to be positive limits initial human capital to:

$$h_0 > \max\left\{\frac{\tau}{Rw\theta}, \frac{\underline{c}+R\tau}{Rw}\right\}$$

We allow fertility for the lowest possible income level to be smaller than one. In these cases adults can only secure old age survival by choosing fertility rates less than 1. Our model therefore also captures income cases corresponding to shrinking adult population due to a lack of resources. $n_0 < 1$ is only possible if $0 < \underline{c}\theta + (R\theta - 1)\tau$.[4] This changes the minimum human capital condition to:

$$h_0 > \frac{\underline{c}+R\tau}{Rw}$$

As income increases and surpasses $\underline{h}$ agents enjoy retirement consumption above the subsistence level. Education investments are still zero ($\underline{h}^P < h_t < \overline{h}^P$). In this income range individuals use resources above the subsistence level not only to have children but also to increase retirement consumption through higher savings. Households therefore weight marginal utility of children against marginal utility of consumption through higher savings (FOC 2 and FOC 4 with $e_t = 0$). Both pension systems still face the same level of human capital. Comparing optimal decisions highlights that pay-as-you-go fertility is smaller than informal fertility.

If the income level is high enough ($h_t > \overline{h}$) human capital starts to grow due to positive education investments. The income level needed to impose positive education investments and the amount of education investments is different for the two pension systems. From $\overline{h}^P$ onwards pay-as-you-go human capital is higher than informal human capital. This is the reason why a simple comparison of informal and pay-as-you-go fertility decisions can not be performed for the high income case. We skip this exercise to section 3 which focuses on a detailed examination of fertility dynamics.

[4] If $0 \geqslant \underline{c}\theta + (R\theta - 1)\tau : h_t > \frac{\tau}{Rw\theta} \geqslant \frac{\underline{c}+R\tau}{Rw}$. Reformulation gives us $h_t - \frac{\tau}{Rw\theta} \leqslant h_t - \frac{\underline{c}+R\tau}{Rw}$ and $Rw\theta h_t - \tau \leqslant \theta(Rwh_t - \underline{c} - R\tau) < Rwh_t - \underline{c} - R\tau$. This shows that $n_t = \frac{\underline{c}+R\tau-Rwh_t}{\tau - Rw\theta h_t}$ can only be smaller than 1 if $0 < \underline{c}\theta + (R\theta - 1)\tau$.

2.3 Consumption

Budget constraints (2) and (3) together with optimal education and optimal fertility decision determines retirement consumption.

$$c_{t+1}^I = \begin{cases} \underline{c} & \text{if } h_t \leqslant \underline{h}^I \\ R\beta(wh_t - \tau) & \text{if } h_t \geqslant \underline{h}^I \end{cases}$$

$$c_{t+1}^P = \begin{cases} \underline{c} & \text{if } h_t \leqslant \underline{h}^P \\ \frac{R^2 w\beta\theta h_t(wh_t-\tau)}{(\beta-1)\tau+Rw\theta h_t} & \text{if } \overline{h}^P \geqslant h_t \geqslant \underline{h}^P \\ \frac{R^2 w\beta(\tau-wh_t)(-a+b\theta h_t)}{aRw+b(\beta-1)(\sigma-1)\tau-bRw\theta h_t} & \text{if } h_t \geqslant \overline{h}^P \end{cases}$$

Proposition 2 *A pay-as-you-go pension system economy demands lower income levels to support consumption above a subsistence level than an informal pension system economy* ($\underline{h}^P < \underline{h}^I$).

Proof. Assume $\underline{h}^I \leqslant \underline{h}^P$ and $\underline{h} = \underline{h}^P$. Informal retirement consumption is therefore equal or bigger than subsistence retirement consumption ($c_{t+1}^I = R\beta(w\underline{h} - \tau) \geqslant \underline{c}$) and pay-as-you-go retirement consumption is equal to retirement consumption ($c_{t+1}^P = \frac{R^2 w\beta\theta\underline{h}(w\underline{h}-\tau)}{(\beta-1)\tau+Rw\theta\underline{h}} = \underline{c}$). It follows that $R\beta(w\underline{h} - \tau) \geqslant \frac{R^2 w\beta\theta\underline{h}(w\underline{h}-\tau)}{(\beta-1)\tau+Rw\theta\underline{h}}$. Reformulation gives us $1 \geqslant \frac{Rw\theta\underline{h}}{(\beta-1)\tau+Rw\theta\underline{h}}$. Because the right hand side of this expression is bigger than 1, $\underline{h}^I \leqslant \underline{h}^P$ can not be true, proofing that $\underline{h}^I > \underline{h}^P$. ■

At low income levels retirement consumption is constant at the subsistence level. As a certain income threshold is surpassed individuals start to increase consumption. Lower income levels are needed to increase pay-as-you-go retirement consumption above $\underline{c}$ than in the informal case. The result is driven by the fact that marginal benefit of having a child is lower for the pay-as-you-go pension system since pension benefits are independent on own fertility decisions. Therefore for each income level the demand for children is lower than in the informal pension system making it easier for retirement consumption to increase a subsistence level. All income levels above $\underline{h}^P$ support higher retirement consumption for a pay-as-you-go pension system economy. This is the case because savings are higher due to lower fertility investments.

Through the help of the already derived insights we are now in the position to proof Lemma 1 which is securing that education investments only take place if income surpasses the subsistence level of consumption.

Proof of Lemma 1. $\underline{h} \leqslant \overline{h}$ demands that $\underline{h}^I \leqslant \overline{h}^I$ and $\underline{h}^P \leqslant \overline{h}^P$. Because $\underline{h}^I > \underline{h}^P$ and $\overline{h}^I > \overline{h}^P$, $\underline{h} \leqslant \overline{h}$ is true if $\underline{h}^I < \overline{h}^P$. Now plug in $\underline{h}^I = \frac{R\beta\tau+\underline{c}}{Rw\beta}$ and $\overline{h}^P = \frac{a}{b\theta\sigma}$ to see that this is the case if $a > \frac{(R\beta\tau+\underline{c})b\theta\sigma}{Rw\beta}$. ■

2.4 Savings

Education investment and fertility decisions are fully describing the behavior of savings. Nevertheless in order to completely describe the model we produce the following results for optimal savings:

$$s_t^P = \begin{cases} \frac{\tau^2+(-w\tau+w\theta\underline{c})h_t}{Rw\theta h_t-\tau} & \text{if } h_t \leqslant \underline{h}^P \\ \frac{(wh_t-\tau)((\beta-1)\tau+Rw\beta\theta h_t)}{(\beta-1)\tau+Rw\theta h_t} & \text{if } \underline{h}^P \leqslant h_t \leqslant \overline{h}^P \\ \frac{(wh_t-\tau)(aRw\beta+b(\beta-1)(\sigma-1)\tau-bRw\beta\theta h_t)}{aRw+b(\beta-1)(\sigma-1)\tau-bRw\theta h_t} & \text{if } h_t \geqslant \overline{h}^P \end{cases}$$

$$s_t^I = \begin{cases} \frac{\tau^2+(-w\tau+w\theta\underline{c})h_t}{Rw\theta h_t-\tau} & \text{if } h_t \leqslant \underline{h}^I \\ \frac{(wh_t-\tau)(\tau-Rw\beta\theta h_t)}{\tau-Rw\theta h_t} & \text{if } \underline{h}^I \leqslant h_t \leqslant \overline{h}^I \\ \frac{(wh_t-\tau)(aRw\beta+b(1+(\beta-1)\sigma)\tau-bRw\beta\theta h_t)}{aRw+b\tau-bRw\theta h_t} & \text{if } h_t \geqslant \overline{h}^I \end{cases}$$

Corresponding to fertility decisions informal and pay-as-you-go savings are equal for income levels below the pay-as-you-go subsistence threshold $\underline{h}^P$ and lower for the range $\underline{h}^P < h_t < \overline{h}^P$ because informal fertility is higher. Due to the fact that h_t is different for human capital levels larger than $\overline{h}^P$ a simple direct comparison of the optimal decisions for this high income range can not be performed.

2.5 Human Capital Accumulation and the Poverty Trap

Now we are focuse on the differences in human capital accumulation due to differences in education investments for the two pension systems. Equation (5) and optimal education decisions determine human capital accumulation:

$$h_{t+1}^P = \begin{cases} \eta a^\sigma \equiv \Gamma_1(h_t) & \text{if } h_t \leqslant \overline{h}^P \\ \eta\left(a + \frac{b\theta\sigma h_t - a}{(1-\sigma)}\right)^\sigma \equiv \Gamma_2^P(h_t) & \text{if } h_t \geqslant \overline{h}^P \end{cases}$$

$$h_{t+1}^I = \begin{cases} \eta a^\sigma \equiv \Gamma_1(h_t) & \text{if } h_t \leqslant \overline{h}^I \\ \eta\left(a + \frac{-aRw-b\sigma\tau+bRw\theta\sigma h_t}{(1-\sigma)Rw}\right)^\sigma \equiv \Gamma_2^I(h_t) & \text{if } h_t \geqslant \overline{h}^I \end{cases}$$

$\Gamma_1(h_t)$ is a line and $\Gamma_2(h_t)$ is a concave function ($\frac{\partial \Gamma_2(h_t)}{\partial h_t} > 0, \frac{\partial^2 \Gamma_2(h_t)}{\partial h_t^2} < 0$).

Notice that in our framework pay-as-you-go pension contributions are not reducing investments in human capital. Depending on the parameter values the described model has different steady state equilibria. We focus on the case of a poverty trap[5] since we wish to observe whether the different pension systems have an influence on the income level needed to escape the low long-run per capita income equilibrium.

Proposition 3 *If min* $\left\{ \left(\frac{1-\sigma}{b\eta\theta\sigma^2}\right)^{\frac{\sigma}{(\sigma-1)\sigma}} + b\left(\eta\theta\left(\frac{1-\sigma}{b\eta\theta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}} - \frac{\tau}{Rw}\right),\right.$ $\left. b\sigma\left(\eta\theta\left(\frac{1-\sigma}{b\eta\theta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}} - \frac{\tau}{Rw}\right)\right\} > a > (b\theta\sigma\eta)^{\frac{1}{1-\sigma}}$ *the model generates two stable and one instable steady state equilibria for both pension systems. Initial income lower than the poverty trap threshold* $\overline{h}^P_{trap}$ *leads to steady state equilibria that equal each other for both pension systems. If initial human capital levels are higher or equal to* $\overline{h}^P_{trap}$ *a pay-as-you-go public pension system supports a higher steady state equilibrium than the informal pension system.*

Proof. The assumption $a > (b\theta\sigma\eta)^{\frac{1}{1-\sigma}}$ implying that $\eta a^{\sigma} < \overline{h}^P$ secures that a stable low income steady state (E_1) exists for both pension system cases because $\Gamma_1(h)$ intersects the 45° line. Now rearrange $\Gamma_2^I(h) = h$ to separate a linear and a power function. This gives us: $\underbrace{bRw\theta h}_{L(h)} = \underbrace{\left(\frac{h}{\eta}\right)^{\frac{1}{\sigma}} Rw\frac{1-\sigma}{\sigma} + aRw + b\tau}_{R(h)}$. The value $h^{'}$ that equals the slopes of the two functions ($L^{'}(h^{'}) = R^{'}(h^{'})$) is given by $\eta\left(\frac{1-\sigma}{b\theta\eta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}}$. Now compare the functional value of the two curves at $h^{'}$. If the functional value of the power function is lower than the functional value of the line $R(h)$ intersects $L(h)$ twice because $R(0) > 0$ and $\frac{\partial R(h)}{\partial h} > 0$. These two intersects are also solutions to $\Gamma_2^I(h) = h$ implying that $\Gamma_2^I(h)$ has two intersects with the 45° line. $R(h^{'}) = bRw\theta\eta\left(\frac{1-\sigma}{b\theta\eta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}}$; $L(h^{'}) = Rw(1-\sigma)\left[\left(\frac{1-\sigma}{b\theta\eta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{1}{\sigma}} +$ $aRw + b\tau$. $R(h^{'}) < L(h^{'})$ if $\frac{(\sigma-1)}{\sigma}\left(\frac{1-\sigma}{b\eta\theta\sigma^2}\right)^{\frac{\sigma}{(\sigma-1)\sigma}} + b\left(\eta\theta\left(\frac{1-\sigma}{b\eta\theta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}} - \frac{\tau}{Rw}\right) > a$. The lower steady state equilibrium (E_2^I) is unstable, the higher one is stable (E_3^I). Pay-as-you-go education investments are always higher than informal ones. Therefore the described parameter restrictions also produce an unstable steady state equilibrium (E_2^P) and a stable steady state equilibrium (E_3^P) for the

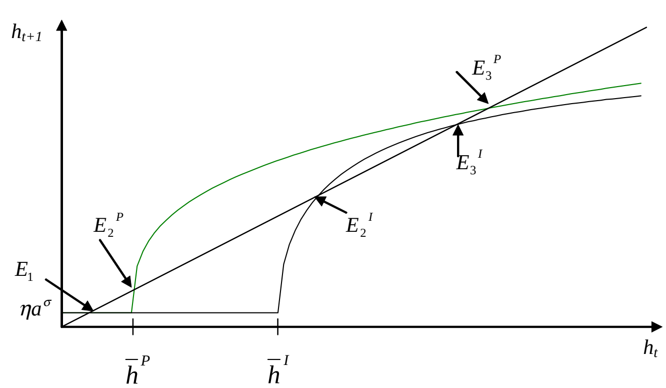
[5] Situation where at least two stable (one low and one high) and one unstable steady states exist. The variable value supporting the unstable equilibrium forms a threshold in reaching the high stable steady state equilibrium.

pay-as-you-go pension system. The last step of the proof secures the existence of E_2 and E_3. In order to exist, the corresponding human capital levels have to be larger than the threshold $\overline{h}$. This is the case if $h^{'} > \overline{h}^I$ and $\eta a^{\sigma} < \overline{h}^P$. $h^{'} > \overline{h}^I$ if $a < b\sigma\left(\eta\theta\left(\frac{1-\sigma}{b\eta\theta\sigma^2}\right)^{\frac{\sigma}{\sigma-1}} - \frac{\tau}{Rw}\right)$. ■

Proposition 4 *A pay-as-you-go pension system economy is featured by a smaller set of initial income conditions supporting a poverty trap equilibrium because the human capital threshold needed to approach the high steady state equilibrium is lower* ($\overline{h}^P_{trap} < \overline{h}^I_{trap}$).

As long as income is below the education threshold $\overline{h}^P$ both pension systems have an identical steady state equilibrium at $h^* = \eta a^{\sigma}$. From $\overline{h}^P$ onwards pay-as-you-go human capital is bigger than informal human capital because education investments are always higher in the pay-as-you-go case (see Proposition 1). The unstable steady state equilibrium determining the poverty trap threshold level ($\overline{h}_{trap}$) is therefore lower for the pay-as-you-go case ($E_2^P < E_2^I$). Additionally the stable pay-as you go positive education steady state is higher $E_3^P > E_3^I$. It follows that lower initial human capital is needed ($\overline{h}^P_{trap} < \overline{h}^I_{trap}$) to approach an even higher stable steady state in the case of a pay-as-you-go pension system. Figure 1 fully describes the behavior of both human capital accumulation equations and the corresponding equilibria.

Figure 1: Human Capital Accumulation



As long as education investments are zero future human capital of both pension systems is constant and equal ($E_1^P = E_1^I$). As income determining human capital reaches the positive education threshold ($\overline{h}$) which is lower for the pay-as-you-go case, future human capital starts to rise. If initial human capital is below (above) the level $\overline{h}_{trap}$ which is corresponding to E_2 the economy approaches the stable low (high) steady state equilibrium $E_1 (E_3)$. Throughout the literature initial human capital lower than $\overline{h}_{trap}$ is known as a "poverty trap" scenario.

Besides the described 3 steady state "poverty trap" scenario different parameter values support a variety of equilibria[6]. While we do not examine each case in detail, one can state that all cases with different stable equilibria support a lower poverty trap threshold for the pay-as-you-go pension system. All income levels above this threshold lead to higher long-run per capita income for the pay-as-you-go pension system. This is the case because education investments start at lower income levels and are always higher in the pay-as-you-go case because marginal utility of procreation is lower.

Proposition 5 *For all possible parameter values supporting different stable equilibria the set of initial conditions leading to a stable high equilibrium is larger for an economy with pay-as-you-go pension system compared to an economy with an informally financed pension system. All parameter values corresponding to a stable steady state equilibrium which is different to $h^* = \eta a^\sigma$ lead to higher long-run per capita income for the pay-as-you-go case. Only if ηa^σ is the unique stable steady state equilibrium both pension systems imply the same long-run per capita income.*

After the comparison of equilibria and connected thresholds we focus on the role of pension systems for demographic transition.

3 Demographic Transition

While the term demographic transition describes the behavior of fertility and mortality over time we only focus on fertility dynamics. Following other economic studies we substitute time for income because historically, income is increasing over time. One can observe that income and fertility are positively

[6]Examples of further existing cases can be examined in Apendix A.

related for low income regions while the opposite is true for high income regions[7]. In order to enable our model to cover the empirical fact that fertility rates are negatively dependent on income increases for high income levels, we assume that parameters satisfy

$$R\theta + \beta + \sigma < 1 + \beta\sigma. \tag{6}$$

This additional parameter assumption enables our model to nicely cover the pattern of historic fertility dynamics for the three income cases $h_t \leqslant \underline{h}, \underline{h} \leqslant h_t \leqslant \overline{h}$ and $h_t \geqslant \overline{h}$.

3.1 Malthusian State: $h_t \leqslant \underline{h}$

This low income scenario describes the situation where the subsistence level of retirement consumption assumption is binding. Because individuals primarily have to secure their survival and use income above the subsistence level only to procreate, education investments are zero. The term Malthusian state well describes this situation because additional income is directly translated to higher fertility while consumption per capita stays constant. To prove that this is the case, take the first derivative of fertility with respect to human capital.

$$\frac{\partial n_t^P}{\partial h_t} = \frac{\partial n_t^I}{\partial h_t} = \frac{Rw\left(\left(R\theta - 1\right)\tau + \theta\underline{c}\right)}{\left(\tau - Rw\theta h_t\right)^2} > 0$$

The second order derivative highlights that the fertility increases take place at a decreasing rate.

$$\frac{\partial^2 n_t^P}{\partial h_t^2} = \frac{\partial^2 n_t^I}{\partial h_t^2} = \frac{2R^2 w^2 \theta\left(\left(R\theta - 1\right)\tau + \theta\underline{c}\right)}{\left(\tau - Rw\theta h_t\right)^3} < 0$$

Proof. As already explained, initial fertility is assumed to be lower than 1 implying that $\underline{c}\theta + (R\theta - 1)\tau > 0$. Therefore $\frac{\partial n_t}{\partial h_t} > 0$ and $\frac{\partial^2 n_t}{\partial h_t^2} < 0$. ■

For low income levels our model reproduces the Malthusian view of an economy that cannot prosper because income increases are only used for additional procreation. As already outlined fertility is equal for both pension systems if retirement consumption is fixed at $\underline{c}$. In this economic stage only the threshold level of income needed to support consumption above the subsistence level depends on the pension system. A lower level of income is needed to induce

[7] For a detailed description of the Demographic Transition see for example Lee 2003.

additional savings for the pay-as-you-go pension system due to the fact that marginal benefit of fertility is lower. This translates to lower fertility and higher savings thus enabling consumption to surpass the subsistence level at a lower income level.

3.2 Post-Malthusian State: $\overline{h} \geq h_t \geq \underline{h}$

At the level $\underline{h}$ income becomes sufficiently high to support an optimal amout of children under the constraint of a minimum retirement budget at the subsistence level. This is implying that marginal utility of fertility and savings are equal. Agents still do not contribute working time to educate their children. Income increases drive down marginal utility of children because marginal cost of children, a fixed part of adult income, is also increasing. Therefore different to the Malthusian state during this economic stage additional income leads to lower fertility rates because the alternative investment opportunity of additional retirement consumption through higher savings becomes more attractive. Calculate the first order derivative of fertility with respect to human capital and use equation (6) to prove that this is the case:

$$\frac{\partial n_t^P}{\partial h_t} = \frac{Rw(1-\beta)\left(\beta+R\theta-1\right)\tau}{\left((\beta-1)\tau+Rw\theta h_t\right)^2} < 0; \frac{\partial n_t^I}{\partial h_t} = \frac{Rw(1-\beta)(R\theta-1)\tau}{\left(\tau-Rw\theta h_t\right)^2} < 0$$

The second derivative further show that for both pension systems the decrease in fertility connected to increasing income takes place at a decreasing rate.

$$\frac{\partial^2 n_t^P}{\partial h_t^2} = \frac{2R^2w^2\theta(\beta-1)\left(\beta+R\theta-1\right)\tau}{\left((\beta-1)\tau+Rw\theta h_t\right)^3} > 0; \frac{\partial^2 n_t^I}{\partial h_t^2} = \frac{2R^2w^2\theta(1-\beta)\left(R\theta-1\right)\tau}{\left(\tau-Rw\theta h_t\right)^3} > 0$$

As already mentioned, children pay less in the pay-as-you-go pension system leading to higher opportunity cost of not investing in savings. Therfore an economy with a pay-as-you-go pension system enters the Post-Malthusian state at lower income levels than economies with informal pension systems. Lower marginal benefit of fertility for the pay-as-you-go pension system further leads to lower demand for children and higher savings.

The combination of the Malthusian and Post-Malthusian state without education investments already outlines the main features of demographic transition: Income increases lead to increasing (decreasing) fertility for low (high) income regions.

3.3 Post-Malthusian State with positive education investments: $h_t \geqslant \overline{h}$

Now education investments become positive. During this stage of economic development fertility is not only competing against additional retirement consumption but also against investments in the quality of children. From the first order derivative of fertility with respect to human capital together with equation (6) we follow that the correlation between income and fertility is still negative.

$$\frac{\partial n_t^P}{\partial h_t} = \frac{bRw(\beta-1)(\sigma-1)(-aRw+b(\beta+R\theta+\sigma-\beta\sigma-1)\tau)}{(aRw+b(\beta-1)(\sigma-1)\tau-bRw\theta h_t)^2} < 0$$

$$\frac{\partial n_t^I}{\partial h_t} = \frac{bRw(\beta-1)(\sigma-1)(-aRw+b(R\theta-1)\tau)}{(aRw+b\tau-bRw\theta h_t)^2} < 0$$

The decrease in fertility due to increasing income is again decreasing for the informal pension scheme.

$$\frac{\partial^2 n_t^I}{\partial h_t^2} = \frac{2b^2R^2w^2\theta(\beta-1)(\sigma-1)(-aRw+b(R\theta-1)\tau)}{(aRw+b\tau-bRw\theta h_t)^3} > 0$$

Proof. Plug in $\overline{h}^I$ which is the smallest viable human capital level connected to positive education investments to obtain:

$\frac{\partial^2 n_t^I}{\partial h_t^2} = \frac{\overbrace{2b^2R^2w^2\theta(\beta-1)(\sigma-1)(-aRw+b(R\theta-1)\tau)}^{<0}}{\underbrace{(aRw-\frac{aRw}{\sigma})^3}_{<0 \text{ because } \sigma<1}}$. The numerator and the denominator are smaller than zero implying that $\frac{\partial^2 n_t^I}{\partial h_t^2} > 0$. This is also true for all human capital levels larger than $\overline{h}^I$. ■

The same is true for the pay-as-you-go pension scheme:

$$\frac{\partial^2 n_t^P}{\partial h_t^2} = \frac{2b^2R^2w^2\theta(\beta-1)(\sigma-1)(-aRw+b(\beta+R\theta+\sigma-\beta\sigma-1)\tau)}{(aRw+b(\beta-1)(\sigma-1)\tau-bRw\theta h_t)^3} > 0$$

Proof. Plug in $\overline{h}^P$ to get $\frac{\partial^2 n_t^P}{\partial h_t^2} = \frac{\overbrace{2b^2R^2w^2\theta(\beta-1)(\sigma-1)(-aRw+b(R\theta-1)\tau)}^{<0}}{(aRw+b(\beta-1)(\sigma-1)\tau-Rw\frac{a}{\sigma})^3}$. The numerator of the expression is negative implying that the second derivative is positive if $a > \frac{b\sigma\tau(1-\beta)}{Rw}$. From the minimum initial human capital constraint we can follow that $\overline{h}^P > \frac{\tau}{Rw\theta}$. Rewrite this condition to obtain $a > \frac{b\sigma\tau}{Rw}$. Therefore $a > \frac{b\sigma\tau(1-\beta)}{Rw}$ has to be true and the second derivative is positive. ■

Now we are in the position to compare pay-as-you-go and informal fertility for all income levels. From optimal fertility decisions and $\frac{\partial n_t^P}{\partial h_t} < 0$ we follow that income levels above the pay-as-you-go education threshold $\overline{h}^P$ support lower fertility for the pay-as-you-go pension system.

Proof. We distinguish between two cases: $\overline{h}^I > h_t \geqslant \overline{h}^P$ and $h_t \geqslant \overline{h}^I$. Simple algebraic reformulation of optimal fertility decisions shows that for the first case pay-as-you-go fertility evaluated at the point $\overline{h}^P$ is lower than informal fertility. Because fertility is decreasing with increasing human capital and human capital is always higher for the pay-as-you-go pension system, all income levels in the range support lower pay-as-you-go fertility. Comparison of optimal fertility results for the income range $h_t > \overline{h}^I$ again shows that fertility is lower in the pay-as-you-go pension system if human capital levels equal each other. Because fertility is again negatively dependent on human capital which is higher for the pay-as-you-go pension system, we can state that pay-as-you-go fertility is lower than informal fertility for all human capital levels above $\overline{h}^P$. ■

Proposition 6 *Pay-as-you-go fertility is equal to informal fertility for all income levels below the pay-as-you-go subsistence threshold $\underline{h}^P$. At the income level $\underline{h}$ both pension systems reach their fertility maximum. Pay-as-you-go fertility is lower than informal fertility for income levels above $\underline{h}^P$.*

As already discussed, the positive education threshold levels depend on the pension system. Positive pay-as-you go education investments are supported by lower human capital levels than informal education investments. The lower threshold is again based on the lower marginal benefit of a child in the pay-as-you-go pension system making it easier for education investments to compete. If the threshold is surpassed, higher income leads to higher education investments at a decreasing rate.

$$\frac{\partial e_t^P}{\partial h_t} = \frac{a}{b(1-\sigma)h_t^2} > 0; \frac{\partial^2 e_t^P}{\partial h_t^2} = -\frac{2a}{b(1-\sigma)h_t^3} < 0$$

$$\frac{\partial e_t^I}{\partial h_t} = \frac{aRw + b\sigma\tau}{bRwh_t^2(1-\sigma)} > 0; \frac{\partial^2 e_t^I}{\partial h_t^2} = -\frac{2(aRw + b\sigma\tau)}{bRwh_t^3(1-\sigma)} < 0$$

For all income levels above the pay-as-you-go education threshold, agents allocate more time to child quality in the pay-as-you-go pension system case. As income is increasing the difference in education investments of the two pension system cases is decreasing because $\frac{\partial e_t^P}{\partial h_t} < \frac{\partial e_t^I}{\partial h_t}$.

The three analyzed income cases also cover information on the observed differences in the timing of demographic transition between developing and developed countries.

Proposition 7 *The introduction of a mandatory pay-as-you-go public pension system to a country with an informal, fertility related pension system shifts down the inverted U-shaped demand for children connected to income increases. Therefore lower levels of income support an escape of the first stage of demographic transition where income increases lead to increasing fertility.*

Lucas (2002) shows that while the demographic transition in the USA and Western Europe already started at the end of the 19^{th} century it took until the 1950s to start in the African countries. While of course several factors connected to the industrial revolution like mortality declines play a role in explaining the different timing of demographic transition, the introduction of a public pension system that first took place in 1889 in Germany appears to play a significant role.

Our model also suggests that developing countries aiming to reduce population growth should introduce a pay-as-you-go pension system as one pillar of their pension scheme. The fertility reduction is additionally accompanied by a lower income threshold needed to escape a poverty trap equilibrium (see proposition 4).

Proposition 8 *Post-Malthusian income levels support a trade-off between fertility and per capita income. A shift from a pay-as-you-go public pension system to a fertility related informal pension system increases fertility rates but decreases long-run per capita income.*

Countries experiencing strong fertility declines due to income increases can weaken the effect, by introducing a fertility related clause in their pension scheme. While such a policy will increase the demand for children, long-run per capita income will decrease, highlighting the existing trade-off between fertility and per capita income.

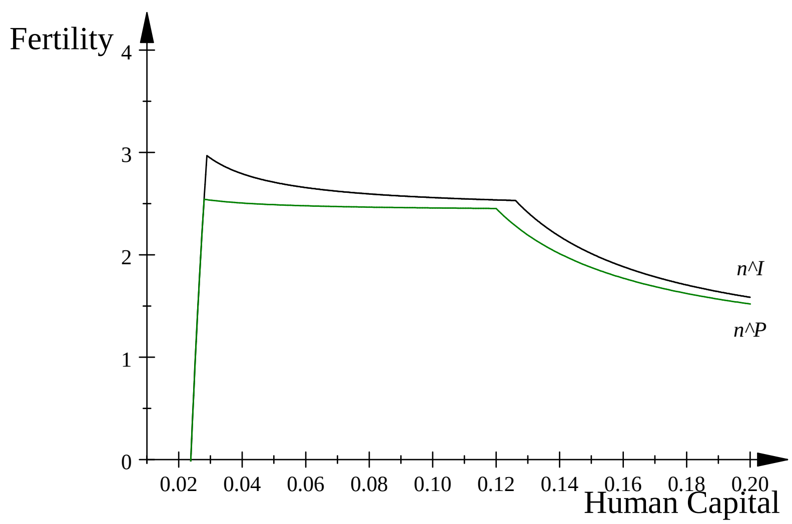
The following subsection presents a numerical example to additionally clarify and summarize fertility dynamics.

3.4 Numerical example

The last subsection already produced all necessary insights to compare the fertility rates of the two pension systems for all development stages. Additionally, we are already able to outline the behavior of fertility over time. The last missing insight to fully describe fertility dynamics is the difference in the strength of fertility reductions observed connected to income increases for the two Post-Malthusian stages. Because an analytic comparison of the partial derivatives does not produce a clear result, a numerical example is performed.

Parameter values are set to satisfy the parameter conditions of a multiple equilibria case (see Proposition 3).[8]

Figure 2: Fertility dynamics



For very low human capital levels, income is too low to secure old age survival. Individuals do not procreate at all and the population becomes extinct in one generation. We excluded these cases ($h_t < 0.024$) from our analysis by assuming that initial human capital is high enough to produce positive fertility. Due to hierarchic preferences only income above the level needed to secure a subsistence level of retirement consumption is invested in procreation. In the

[8] Parameter values are set to:

a	b	w	R	$\underline{c}$	β	θ	η	σ	τ
0.027	5	100	1.04^{20}	1.6	0.99^{20}	0.075	1	0.6	0.1

low income region ($0.024 \leq h_t^I \leq 0.029; 0.024 \leq h_t^P \leq 0.028$) marginal utility of fertility is larger than marginal utility of retirement consumption. Individuals would like to give up retirement consumption in order to have more children which is not possible since retirement budget is already at a minimum. In other words, life-time income is too low to equal marginal utility of both investment opportunities. As human capital is increasing, the cost of having children is also increasing. This, together with higher fertility drives down the marginal benefit of procreation. At the human capital level $\underline{h}$ ($\underline{h}^P = 0.028; \underline{h}^P = 0.29$) individual income is high enough to equal marginal utility of fertility and savings. Further income increases reduce marginal utility of fertility by higher marginal cost. The income level equalizing marginal utility of fertility and retirement consumption is lower for the pay-as-you-go pension system since the marginal benefit of children is lower. Additional income further drives down marginal utility of retirement consumption and fertility. At the level $\overline{h}$ ($\overline{h}^P = 0.12; \overline{h}^I = 0.126$) the marginal benefit of education equals the marginal benefit of the two other investment opportunities and education investments become positive. The income level needed to make education a competing investment is lower in the pay-as you-go pension system since marginal benefit of children is lower. Positive child quality investments increase the increasing effect of higher income on the marginal cost of fertility resulting in an even stronger decline of fertility.

4 Bureaucracy and Corruption

Transferring income from the working generation to retirees via a public system clearly causes cost. While part of these are transparent such as operating cost, others like bureaucracy or corruption are difficult to measure. These costs are country specific and can be considered especially significant for least developed countries, where legal security is low and corruption is soaring. While informal decisions are equal to the former case, positive bureaucracy cost ($B < 1$) change optimal pay-as-you-go fertility decisions and retirement consumption to:

$$n_t^P = \begin{cases} \frac{\underline{c}+R\tau-Rwh_t}{B\tau-Rw\theta h_t} & \text{if } h_t \leqslant \underline{h}^P \\ \frac{R(\beta-1)(\tau-wh_t)}{B(\beta-1)\tau+Rw\theta h_t} & \text{if } \underline{h}^P \leqslant h_t \leqslant \overline{h}^P \\ \frac{bR(\beta-1)(\sigma-1)(\tau-wh_t)}{aRw+b(\beta-1)(\sigma-1)\tau B-bRw\theta h_t} & \text{if } h_t \geqslant \overline{h}^P \end{cases}$$

$$c_{t+1}^P = \begin{cases} \underline{c} & \text{if } h_t \leqslant \underline{h}^P \\ \frac{R^2 w\beta\theta h_t(wh_t-\tau)}{B(\beta-1)\tau+Rw\theta h_t} & \text{if } \overline{h}^P \geqslant h_t \geqslant \underline{h}^P \\ \frac{R^2 w\beta(\tau-wh_t)(-a+b\theta h_t)}{aRw+b(\beta-1)(\sigma-1)\tau B-bRw\theta h_t} & \text{if } h_t \geqslant \overline{h}^P \end{cases}$$

The new results highlight that for income levels below the pay-as-you-go subsistence threshold $\underline{h}^P$ fertility in the pay-as-you-go pension system is now lower than fertility in the informal pension system. Retirement consumption is still fixed at $\underline{c}$. The utility generated by an informal pension system economy is therefore higher than for a pay-as-you-go pension system economy. Income levels supporting a post-malthusian state are still lower for the pay-as-you-go pension system economy but the difference is decreasing if bureaucracy costs are increasing. This is due to the fact that lower B drives down life time income translating into higher income increases needed to equal marginal utility of retirement consumption and fertility. Income levels above $\underline{h}^P$ lead to pay-as-you-go retirement consumption above the subsistence level. From a certain income level onward the effect on retirement consumption becomes strong enough to compensate for lower fertility and the pay-as-you-go pension system becomes again utility maximizing.

Proposition 9 *A traditional, informal pension system is optimal if income levels are very low or bureaucracy costs are large.*

By including $B < 1$ our model covers income cases where an informally financed pension system is dominating a publicly financed pay-as-you-go scheme. This underlines that country specific conditions have to be considered in deciding whether the introduction of a public pension system is a viable development device.

5 Conclusion

Through the comparison of a pay-as-you-go and an informal pension system we show that the type of pension system has an impact on economic development and population growth. The introduction of a public pension system breaks the link between individual fertility and pension benefits that can be observed for traditional societies. Marginal benefit of procreation is therefore lower for the pay-as-you-go pension system, leading to lower demand for children. Reflecting on the quantity/quality trade off, pay-as-you-go education investments are higher and start at lower income levels. This is the reason why economic take-off to a high long-run equilibrium per capita income is supported by lower income levels for a pay-as-you-go pension system economy. Next to the lower poverty trap threshold, education investments under the pay-as-you-go pension scheme which exceed informal ones, imply larger per capita income for the high steady state equilibrium.

A switch from an informal- to a pay-as-you-go pension system leads to decreasing marginal utility of fertility, increasing high steady state equilibrium income and a possibility to escape a poverty trap. Nevertheless, if bureaucracy costs are considered, the described positive effects of a pay-as-you-go pension system can be accompanied by a reduction in utility for very low income or high corruption cases. Countries experiencing a "Malthusian stage" of their economy with human capital levels below the pay-as-you-go subsistence threshold are worse off if a public pension system is introduced. This result is supported by even higher income levels if bureaucracy costs are soaring.

In addition, we show that developed countries facing a sharp decrease in fertility can weaken this effect by introducing a fertility related clause in their pension scheme. While such a policy could absorb part of the negative demographic trend it comes at a cost of reduced long run equilibrium per capita income.

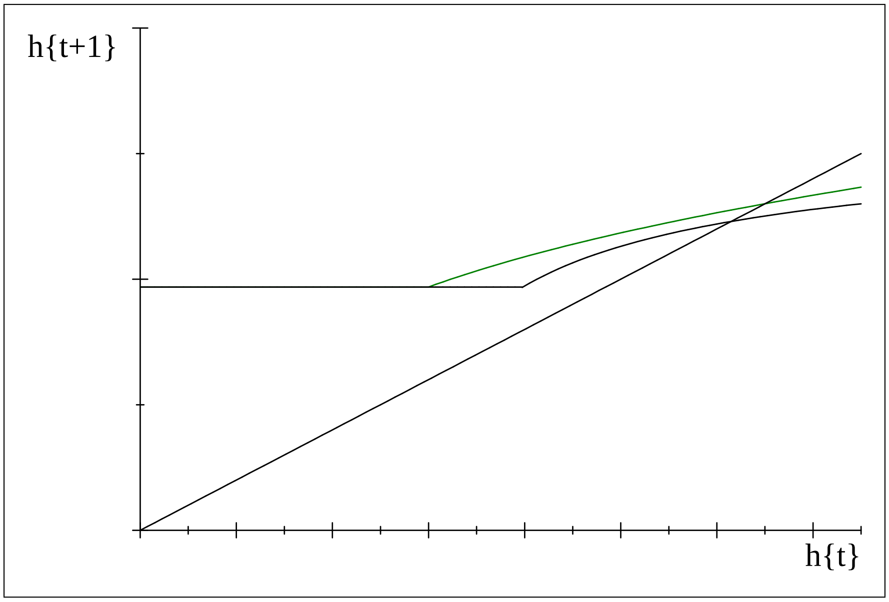
Our results further underline that a pay-as-you-go pension system needs a lower income level to escape the "Malthusian stage" of an economy because investments, contrary to fertility, are more competitive. This allows us to conclude that pension systems appear to play a vital role in the timing of demographic transition. The divergence of pension systems for developed and developing countries can therefore partly explain the observed regional differences in the behavior of population dynamics.

Acknowledgements

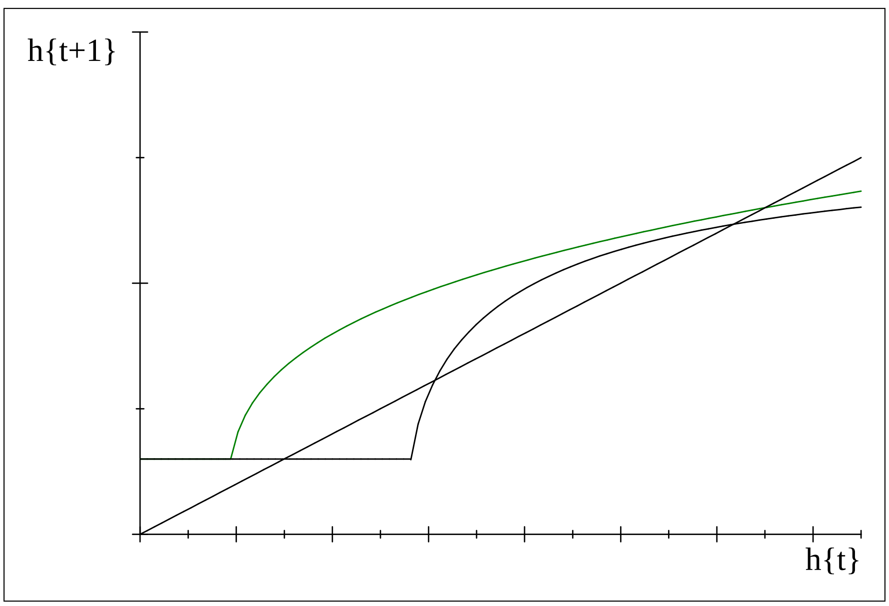
I would like to thank Gerhard Sorger and Alexia Fürnkranz-Prskawetz for their support, ideas and helpful comments. The paper further benefited from comments by participants at the *EERTEC* at Florence, the 6th Workshop on Pensions and Savings at University Paris Dauphine and fruitful discussions with Michael Weichselbaumer and Ana B. Ania.

Appendix A

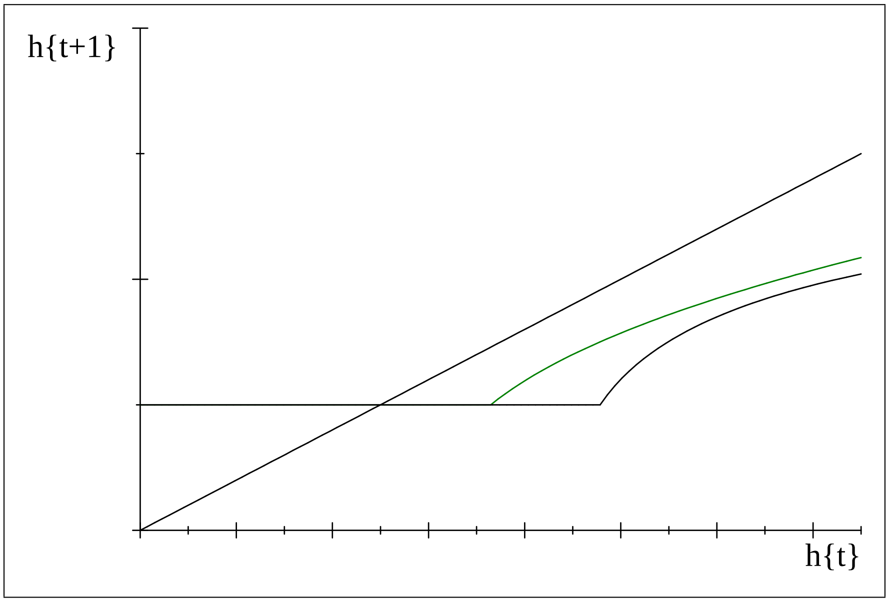
$\eta a^{\sigma} > \overline{h}^{I}$:



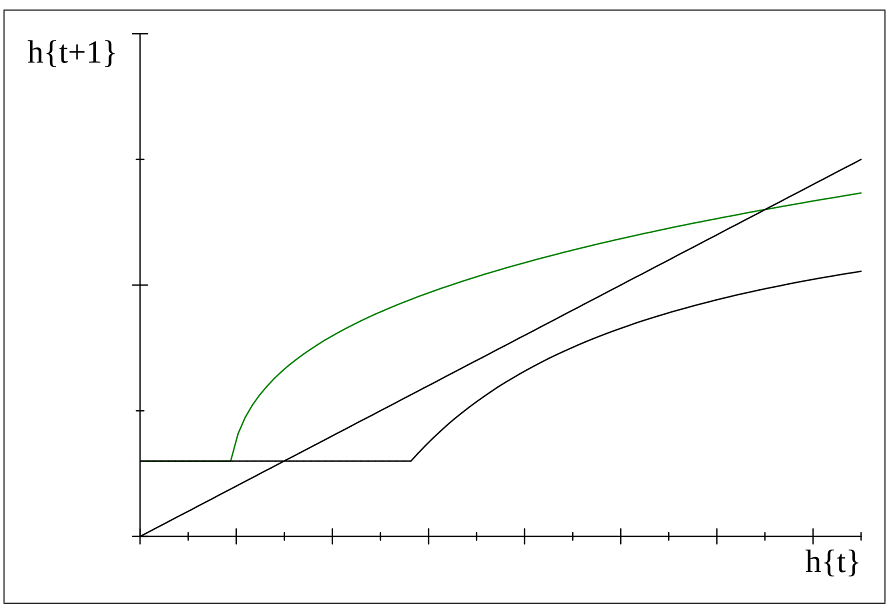
$\overline{h}^{P} < \eta a^{\sigma} < \overline{h}^{I}; L(h^{'}) > R(h^{'})$:



$\eta a^{\sigma} < \overline{h}^{P}; L(h^{'}) < R(h^{'}):$



$\overline{h}^{P} < \eta a^{\sigma} < \overline{h}^{I}; L(h^{'}) < R(h^{'}):$



References

[1] Azariadis, C. and Drazen, A. 1990. "Threshold Externalities in Economic Development". *The Quarterly Journal of Economics*, MIT Press 105(2): 501-26.

[2] Becker, G. and Barro, R. 1988. "A Reformulation of the Economic Theory of Fertility". *Quarterly Journal of Economics* 103: 1-25.

[3] Becker, G., Murphy, K. and Tamura, R. 1990. "Human Capital, Fertility, and Economic Growth". *Journal of Political Economy*, University of Chicago Press 98(5): 12-37.

[4] Boldrin, M., De Nardi, M. and Jones, L. 2005. "Fertility and Social Security". *NBER Working Paper Series* 11146.

[5] Brezis, E., Krugman, P. and Tsiddon, D. 1993."Leapfrogging in International Competition: A Theory of Cycles in National Technological Leadership". *American Economic Review* 83(5): 1211-19.

[6] Caldwell, J. 1982. *Theory of Fertility Decline.* London: Academic Press.

[7] De la Croix, D. and Doepke, M. 2003. "Inequality and Growth: Why Fertility Matters". *American Economic Review* 93(4): 1091-1113.

[8] P. Diamond, P. 1982. "Aggregate Demand Management in Search Equilibrium". *Journal of Political Economy* 90(5): 881-94.

[9] Dyson, T. and Murphy M. "The Onset of Fertility Transition". *Population and Development Review* 11(3): 399-440.

[10] Galor, O. and Weil, D. 2000. "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond". *American Economic Review* 90(4): 806-28.

[11] Groezen, B., Leers, T. and Mejdam, L. 2003. "Social security and endogenous fertility: pensions and child allowances as Siamese twins". *Journal of Public Economics* 87: 233-251.

[12] Holler, J. 2007. "Pension Systems and their Influence on Fertility and Growth". WP University of Vienna Nr.704.

[13] Kremer, M.1993. "Population Growth and Technological Change One Million B.C. to 1990". *The Quarterly Journal of Economics* 108(3):681-716.

[14] Lee, R. 2003. "The Demographic Transition: Three Centuries of Fundamental Change". *The Journal of Economic Perspectives* 17(4): 167-190.

[15] Lucas, R. 2002. *Lectures on Economic Growth.* Cambridge, Massachusetts, London: Harvard University Press.

[16] Lucas, R. 1999. "The Industrial Revolution: Past and Future". *Mimeo, Department of Economics, University of Chicago*, IL.

[17] Morand, O. F. 1999. "Endogenous Fertility, Income Distribution, and Growth". *Journal of Economic Growth* 4: 331-349.

[18] Sinn, H.-W. 2004. "The pay-as-you-go pension system as fertility insurance and an enforcement device". *Journal of Public Economics* 88: 1335-1357.

[19] Tabata, K. 2003. "Inverted U-shaped fertility dynamics, the poverty trap and growth". *Economic Letters* 81: 241-248.

[20] Tabata, K. and Kazuhiro, Y. 2006. "Sectorial sift, inverted U-shaped fertility dynamics, and growth". *Economics Bulletin* 10 (5): 1-7.

[21] Wigger, B. 1999. "Pay-as-you-go financed public pensions in a model of endogenous growth and fertility". *Journal of Population Economics* 12: 625-640.

[22] World Bank. 1991. *Averting the Old Age Crisis. World Bank Policy Research Paper.* New York: Oxford University Press.

[23] World Bank. 2005. *World Development Indicators 2005.* http://devdata.worldbank.org/wdi2005. (*10.1.2008).*

[24] Zhang, J. and Zhang J. 1995. "The Effects of Social Security on Population and Output Growth". *Southern Journal of Economics.* 62(2): 440-450.