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**An efficiency analysis of banking systems: a comparison of European and United States large commercial banks using different functional forms.**

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*April 2003*

**Abstract:**

*This paper aims at investigating the efficiency of European and U.S. commercial banks. Scale and scope economies indicators, as well as a measurement of X-efficiency are derived from three cost functions: Fourier flexible form, translog and Box-Cox. This allows checking the stability and the robustness of the evidence across the different specifications. Our results over the period 1995-98 show that overall the average cost curve is relatively flat with some evidence of scale efficiency gains. More puzzling are the results on the presence of scope economies.*

**JEL classification:** G21; C23; C52; D24.

**Keywords:** European and U.S. commercial banks, cost efficiency, scale and scope economies, Translog and Fourier flexible cost function, Box-Cox.

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## **1. Introduction**

This paper aims at analyzing the efficiency of European and U.S. commercial banks over the period 1995-98. We employ here a broad definition of efficiency, which covers scale and scope economies, as well as cost efficiency.

The importance of such a work is in the provision of evidence as to whether significant variations in efficiency emerged after the consolidation process occurred within the United States during the 80s, and if some gains could be derived by the restructuring process in the European banking industries.

Regarding the European banking industry many factors have contributed to increase competition among financial institutions in the last few years. The first important factor is deregulation, promoted by the Second European Directive on Banking and Financial Services, which leads banks to compete not only in the domestic markets but potentially all over the world. Second, European Monetary Union affects the level of competition in the banking sector of countries adopting the Euro. Moreover, technological advances and deregulation have favored a process of despecialization, allowing banks to lend at any maturity, and reducing the differences among sectors.

Banks reacted to the increased European competition with an intense process of restructuring and growth leading the banking sector to experience an unprecedented level of consolidation through mergers and acquisitions operations among large financial institutions, very similar to that which occurred in the U.S. banking industry in the 1980s.

The consolidation process aims at reaping profitability, reducing cost inefficiency, increasing market power, and exploiting scale and scope economies.

So far, the empirical literature concerned with the U.S. experience (Berger et al., 1993; Clark, 1996; Clark and Speaker, 1994; Evanoff and Israilevich, 1991; Gilbert, 1984; Humphrey, 1990; Mester, 1987; Mitchell and Onvural, 1996) shows that overall the average cost curve is relatively flat with some evidence of scale efficiency gains for small banks. Results on scope economies are even more controversial since the literature provides little consensus on the existence and the extent of product mix efficiency (Berger and Humphrey, 1991 and 1994). The lesson would be that the only way to lower cost in banking is to improve the X-efficiency rather than focus on cross-border mergers and acquisitions (Allen and Rai, 1996).

The much smaller number of cost studies on output banking efficiency for Europe shows that the average cost curve tends to be U-shaped and, to a lesser extent, scope economies

also exist. The European banking industry is interesting not only for its differences with the U.S. experience but also for the implications of financial markets integration policies.

Empirical papers on the European experience have mainly focused on cost functions using data from a single bank or a single country (Altunbas et al., 1997; Athanassopoulos, 1998; Berg et al., 1993; Drake and Howcroft, 1994; Drake and Simper, 2002; Glass and McKillop, 1992; Parisio, 1992; Simper, 1999; Zardkoohi and Kolaris, 1994).

Cross-countries analysis of efficiency and scale and scope economies in Europe refer to a pre-integration period (e.g. Altunbas and Molyneux, 1996; Vander-Vennet, 1996). Only a few studies provide, to our knowledge, comparison to recent data to analyze the effects of the opening up of the banking domestic markets (Cavallo and Rossi, 2001; Vander-Vennet, 2002).

The main innovations of our paper are:

- a) Providing panel data evidence over the period 1995-98 on output and X-inefficiency for both European and U.S. commercial banks. This enables a comparison across different banking models employing recent data for commercial banks in 15 European countries and the United States. Specifically, we have a first block of 338 commercial banks belonging to the fifteen European countries run as a whole by using country fixed effects and a second one built up with a sample of 279 U.S. commercial banks. Given the differences in the factors market we make comparisons between them by building up specific models for the U.S. and European banking system.
- b) We depart from the empirical literature on this topic by representing the production function using three different cost function specifications: *a)* the widely used translog functional form; *b)* the relatively under-used flexible Fourier functional form (Gallant, 1981); and *c)* the Box-Cox cost function specification. A comparison of scale and scope economies scores deriving from these different specifications allows us to identify any mis-specification arising from the translog form and the robustness of the evidence provided.
- c) Our results for the U.S are in line with the evidence provided by the previous literature with some evidence in favor of slight scale economies. Results on scope economies are more controversial regarding the existence and the extent of product mix efficiency. The evidence for the European countries shows that overall the average cost curve is relatively flat with some evidence of scale efficiency gains for small banks. More puzzling are the results on the presence

of scope economies, which to some extent could be motivated by the consolidation and restructuring process in the banking industry.

## **2. Methodology and data**

### *2.1 Definition of a bank cost function*

The concept of efficiency has been widely analyzed in the literature (Fried et al., 1993; Coelli, et al., 1998). A production function is efficient, in the Pareto and Hoopmans sense, when it represents the maximum output attainable from each input level, or the minimum level of each input leaving the output unchanged. As is well known from the theory of duality (Diewert, 1974; Shephard, 1953 and 1970) under given conditions (exogenous prices and optimal behavior of the producer) the cost function is dual to the production function and gives an alternative and equivalent description of the technology of the producing unit (Jorgenson, 1986).

In modeling the cost function of multiproduct firms such as banks, we deal with the problem of defining the appropriate specification. Despite the large body of literature on banks efficiency there is no general consensus on how to define inputs and outputs of multi-product financial firms. The two main issues are related to the role of deposits and whether inputs and outputs should be measured in physical or monetary units. The following five are the most used approaches in literature.

The *production approach*, being more concerned with the technical efficiency of financial institutions, defines the bank activity as production of services. Deposits are counted as output and interests paid on deposits are not included in bank total costs (Ferrier and Lovell, 1990). According to this approach input and output are measured in physical quantity (number of accounts, transactions processed, etc.).

The *intermediation approach* views banks as institutions that collect and allocate funds in loans and other assets; deposits are included among the inputs and interests in the total costs.

The *asset approach* is a variant of the intermediation approach where liabilities are considered as inputs and assets as output.

The *value added approach* identifies any balance sheet item as output if it absorbs a relevant share of capital and labor, otherwise it is considered as an input or non relevant output; according to this approach deposits are considered as an output since they imply the creation of value added.

Finally the *user cost approach* assumes that it is the net contribution to the bank revenue that defines inputs and outputs; in this case deposits are counted as outputs.

The choice of a particular approach and consequently the definition used for the inputs and outputs are likely to affect the results of the efficiency estimates (Favero and Papi, 1995; Hunter and

Timme, 1995; Resti, 1997). The researcher's choice is often a pragmatic compromise between theoretical considerations and data availability.

## 2.2 Description of the variables and data

In modeling the cost functions of European and U.S. commercial banks we employ here two different approaches. In the case of European commercial banks we employ the *modified production approach*<sup>1</sup> as in Berger and Humphrey (1991) and Bauer et al. (1993). Under this approach, the interests paid on deposits are counted as input, while the volume of deposits is considered to be an output, on the assumption that it is able to approximate the amount of services provided to customers. Following this approach, we shape the cost function for European banks using three outputs: deposits, loans and services, all expressed as dollar amounts. The deposits variable comprises all funds raised from retail. The loans variable includes all forms of performing and non-performing loans to customers. The services variable is constructed as the total value of services income<sup>2</sup>.

The price of labor, capital and deposits are the three input variables considered in the cost function. The total costs associated with these inputs are, respectively: total personnel expenses, non-staff expenses and the total interest on deposits. The labor price is calculated as total personnel cost divided by the number of employees. The capital price is obtained by dividing the cost of capital (operative cost associated with capital expenses) by fixed assets net of depreciation<sup>3</sup>.

Finally, the deposit price is computed by dividing the total interest expenses by the total amount of deposits. Total costs are obtained as the sum of operating costs and interest expenses.

Differently from some previous studies (Hunter et al., 1990; Mitchell and Onvural, 1996), in setting the cost function for U.S. banks we employ the *value added approach*. Deposits, loans and services are counted as outputs, labor and capital are inputs.

The choice of adopting a slightly different cost function specification for U.S. banks has been driven by the fact that so far the cost of depositing for U.S. banks has been quite negligible compared to European banks. This analysis has also been supported by the evidence obtained on

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<sup>1</sup> Since the specification used may affect the efficiency results, we also test an alternative specification based on the *value added approach*. The Hausman test shows that the specification we adopted in this analysis is more appropriate to better fit the data.

<sup>2</sup> It comprises fee-based income, net revenues from security and currency trading.

<sup>3</sup> In order to adjust the book value of fixed capital to account for distortions - due to the fact that fixed capital may have been recorded in different periods, and revalued because of tax laws or mergers - we use an adjusted value computed as the fitted value of a fixed effects panel estimation (overall  $R^2 = 83\%$ ) in logarithms, where, the book value fixed asset is regressed on a constant term (9.34;  $t = 7.7$ ), the size (deposit and loans to customers) (0.008;  $t = 13$ ) and the number of employees (21.31;  $t = 7.63$ )<sup>3</sup> (coefficients and t-statistics in parenthesis). Obviously the use of branches instead of the

tests performed on the different alternative specifications of the cost function: the value added approach seems to better fit our data from U.S. commercial banks<sup>4</sup>.

We focus on the period 1995-98 to analyze the effect of the deregulation and the increased competition in the European banking industry. Moreover, as pointed out by Vander-Vennet (2002), during these years all the European countries faced a positive business cycle period and were trying to match the Maastrich convergence criteria. Finally, also the U.S. economy experienced a positive business cycle with high growth pace over the period 1995-98.

We use two separate balanced panels: one, consisting of 1352 observations, refers to a sample of banks belonging to the 15 members of the European Union, and the second refers to a sample of U.S. banks and consists of 1116 observations. All the data come from balance sheet and profit and loss accounts provided by *Bankscope* (BVD-IBCA Ltd) an international database, which provides data on financial institutions. Data are expressed in monetary values in U.S. dollars at 1995 prices and are adjusted for the PPP.

Although our analysis is based on data only from large commercial banks we test for robustness of results over the sample. To do this we divide our data-set into three sub-groups - large, medium and small - by selecting for each country, banks belonging to the highest, the middle and the lowest decile of the asset size distribution resulting from the balance sheet.

### **3. Estimation methodology**

The first step of our analysis consists in modeling the bank cost function. We will then calculate the X-efficiency scores using the distributional free approach, and finally the scale and scope economies.

A few recent studies on banking efficiency concentrate on the comparison between profit and cost efficiency (e.g. Berger and Mester, 1997; Vander-Vennet, 2002). A distinction between these two problems arises when markets are not perfect. In this paper we employ the cost function approach assuming that the European Union integration and the more competitive environment introduced by the II European Directive for the Banking industry bring markets closer to perfect competition. A competitive market can also be assumed for the U.S. banking industry.

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number of employees would have been more appropriate (see also Resti, 1997) in the fit of fixed capital. However in our data base the number of branches is not available.

<sup>4</sup> Preliminary estimates give a non significant t statistic for the deposit price, and its omission does not alter the result. Moreover the Hausman test supports the specification we adopt.

### 3.1 Cost function specifications

In modeling the cost function we use three different functional forms: (i) the translog cost function (TL), (ii) the Box-Cox specification and (iii) the Fourier Flexible form (FF). The choice of using different specifications is particularly felt here in order to derive robust conclusions when dealing with comparisons between countries.

(i) As is known, the TL is one of the most widely used functional form in the empirical literature on bank efficiency. It presents the well-known advantages of being a flexible form - in the sense that it imposes few restrictions on the underlying cost structure and hence (by duality theory) on the production technology - and of including, as a particular case, the Cobb-Douglas specification. Furthermore, since the TL is the most used form in modeling the cost function of financial institutions, our findings can easily be compared with previous studies.

Following this specification the  $s$ -th firm total cost can be written as follows:

$$\ln TC_s = \alpha_0 + \sum_i \beta_i \ln y_i + \sum_k \gamma_k \ln p_k + (\frac{1}{2}) \sum_i \beta_i (\ln y_i)^2 + \sum_i \sum_j \beta_{ij} |i < j| \ln y_i \ln y_j + (\frac{1}{2}) \sum_k \gamma_{kk} (\ln p_k)^2 + \sum_k \sum_l \gamma_{kl} |k < l| \ln p_k \ln p_l + \sum_k \sum_i \varphi_{ki} |k < i| \ln p_k \ln y_i \quad (1)$$

$i, j = 1, 2, 3;$

$k, l = 1, 2, 3$  for Europe;

$k, l = 1, 2$  for the U.S.

where  $TC$  is the total cost,  $y_i$  is the  $i$ -th output and  $p_k$  is the price of the  $k$ -th input.

The cost function has been estimated imposing the linear homogeneity conditions and cost exhaustion, obtained by normalizing total cost ( $TC$ ), the price of labor and the price of deposits by the price of capital<sup>5</sup>. Moreover, we impose the symmetry conditions ( $\beta_{ij} = \beta_{ji} \forall i, j$  and  $\gamma_{kl} = \gamma_{lk} \forall k, l$ ) and the linear homogeneity restrictions<sup>6</sup>.

$$\sum_{k=1}^3 \gamma_k = 1; \sum_{k=1}^3 \gamma_{kl} = 0, \text{ for all } l; \quad \sum_{k=1}^3 \varphi_{ki} = 0, \text{ for all } i.$$

In order to improve the quality of the TL approximation, the logs of outputs and prices are all expressed as differences from the sample mean ( $\ln y_{is} - \ln[1/n \sum_{s=1}^n y_{is}]$ ,  $\ln p_{ks} - \ln[1/n \sum_{s=1}^n p_{ks}]$ , where  $n$  is the sample size), as in Resti, 1997.

<sup>5</sup> The traditional share equations, are the alternative way to estimate the cost function, where the parameters of the TL derive from the simultaneous estimation of the cost function and the input-share equations obtained from the Shepard's Lemma (Shephard, 1970) and partially differentiating the TL function with respect to each factor price  $p_k$ . In our study we drop the share equations. This also avoids the problems discussed in Bauer (1990) and Cebenoyan et al. (1993) which arise from using share equations to measure X-inefficiency.

<sup>6</sup> A likelihood-ratio test shows that the restrictions on the parameters fit well with the data.



Although the TL is the most used functional form in banking efficiency studies, it presents two main pitfalls: (a) the estimated values of product specific scale economies and scope economies are often unreliable, since they require the calculation of the cost function at zero output level<sup>7</sup>. (b) As pointed out in White (1980) and Mitchell and Onvural (1996) the TL estimates do not necessarily correspond to the second order Taylor approximation of the underlying function at an expansion point.

In order to deal with the first problem we use the generalized translog function, better known as Box-Cox specification, while to address the second argument we use the Fourier Flexible (FF) function.

ii) The usefulness of Box-Cox in this setting is twofold as it allows for the possibility to consider zero values as arguments of the function, and secondly it represents an alternative instrument to test the robustness of our results. Following Caves et al. (1980), and Fuss and Waverman (1981) we adopt the following Box-Cox specification<sup>8</sup>.

$$\ln TC_s = \alpha_0 + \sum_i \beta_i (y_i^\lambda - 1)/\lambda + \sum_k \gamma_k \ln p_k + (1/2) \sum_i \beta_{ii} [(y_i^\lambda - 1)/\lambda]^2 + \sum_i \sum_j \beta_{ij} |i < j| (y_i^\lambda - 1)(y_j^\lambda - 1)/\lambda^2 + (1/2) \sum_k \beta_{kk} (\ln p_k)^2 + \sum_k \sum_l \gamma_{kl} |k < l| \ln p_k \ln p_l + \sum_k \sum_i \varphi_{ki} |k < i| \ln p_k (y_i^\lambda - 1)/\lambda; \quad (2)$$

$$\lim (y_i^\lambda - 1)/\lambda = \ln y_i \lambda \rightarrow 0$$

where  $\lambda$  is the Box-Cox filter (parameter) whose positive value confers on this specification the proprieties of the translog as  $\lambda$  tends to zero<sup>9</sup>.

As far as the applicability of the Box-Cox is concerned, the literature has pointed to the lack of coherence with the theory for the non-homogeneity in input prices as demonstrated in Shaffer (1994). Despite the correctness of the remark<sup>10</sup> and the solution found through the modified Box-Cox function, the problem is posed in wrong terms since the crucial point is the arbitrariness of the

<sup>7</sup> Empirical literature addresses the problem by using a positive number instead that zero output, such as 10% of the mean outputs (e.g. Kim, 1986).

<sup>8</sup> See Christensen et al. (1973) and Brown et al. (1979) for the multi-product version. Lau (1974) introduced the squared specification as a second order approximation.

<sup>9</sup> Spitzer (1982) and Zarembka (1987) consider an estimation with this transformation also for the dependent variable.

<sup>10</sup> Considering for simplicity, only the transformed terms:

$$\ln(C/p_1) = \theta'_0 + \sum_{j \neq 1} \theta_j \frac{(p_j/p_1)^{\lambda_j} - 1}{\lambda_j}$$

where:  $\theta'_0 = \theta_0 + \theta_1$  is the usual Box-Cox function normalised by an input price, and the following expression

$$\ln(C/p_1) = p_1 \sum_{j \neq 1} \theta_j \frac{(p_j/p_1)^{\lambda_j} - 1}{\lambda_j}$$

approximation made by means of any quadratic forms. However, also neglecting the latter aspect one can easily skip the problem of the homogeneity in prices, by recurring to a functional form in which the transformation applies only to outputs terms. Outputs (expressed in logs), in fact, are difficult to be addressed when dealing with scale and scope efficiency. Then the price homogeneity may be imposed on the coefficients following the Cobb-Douglas hypothesis.

The Box-Cox specification does not mitigate the mentioned critique about the use of a second order approximation that opens the way to the third approach based on the more complete and sophisticated Fourier flexible form.

(iii) The FF, combines the standard TL, nested in the FF, with the non-parametric Fourier form *i.e.* the trigonometric terms. As pointed out by Berger and De Young (1997) because the FF includes trigonometric transformations of the variables, it can globally approximate the underlying cost function over the entire range of data. This theoretical improvement has been proved to give a better fit of the data than the TL (McAllister and McManus, 1993; and Mitchell and Onvural, 1996, Berger and Mester, 1997).

FF:

$$\begin{aligned}
\ln TC_s = & \alpha_0 + \sum_i \beta_i \ln y_i + \sum_k \gamma_k \ln p_k + (\frac{1}{2}) \sum_i \beta_{ii} (\ln y_i)^2 + \sum_i \sum_j \beta_{ij} |i < j| \ln y_i \ln y_j + (\frac{1}{2}) \sum_k \gamma_{kk} \\
& (\ln p_k)^2 + \sum_k \sum_l \gamma_{kl} |k < l| \ln p_k \ln p_l + \sum_k \sum_i \phi_{ki} |k < i| \ln p_k \ln y_i + \\
& \sum_i a_i \cos(y_i) + \sum_i b_i \sin(y_i) + \sum_k c_k \cos(p_k) + \sum_k d_k \sin(p_k) + \\
& \sum_{ij} e_{ij} [\cos(y_i) + \cos(y_j)] + \sum_{ij} f_{ij} [\sin(y_i) + \sin(y_j)] + \sum_{ij} g_{ij} [\cos(y_i) - \cos(y_j)] + \\
& \sum_{ij} h_{ij} [\sin(y_i) - \sin(y_j)] + \sum_{kl} i_{kl} [\cos(p_k) + \cos(p_l)] + \\
& \sum_{kl} l_{kl} [\sin(p_k) + \sin(p_l)] + \sum_{kl} m_{kl} [\cos(p_k) - \cos(p_l)] + \sum_{kl} n_{kl} [\sin(p_k) - \sin(p_l)]
\end{aligned}
\tag{3}$$

In the FF specification the trigonometric addends have been truncated coherently with our sample size<sup>11</sup>. It is also possible to check that the way of building such a trigonometric part is analogous to the logarithmic one except for the terms whose combined effect is evaluated by means of additions and subtractions instead of multiplication as in the TL.

The trigonometric part represents the non-parametric size of the function that confers it with the nice propriety to correctly infer on the coefficients of the underlying true cost function. In other

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is the modified Box-Cox. In both cases the sum of the elasticities is constrained by the form adopted to 0 and 1 respectively (demonstration available upon request). Note that there is no way to switch from the first function to the other one by fixing opportune values for  $p_0 = 0$  and  $\lambda_0 = -p_1$ , as the resulting parameter  $\theta_0^? = \theta_0 + \theta_0/p_1$  is not constant.  
<sup>11</sup> The truncation point has been chosen according to the rule of thumb expounded in Eastwood and Gallant (1991) that the number of parameters should be set equal to the number of the observation raised to the power of the two-thirds in order to obtain consistent and asymptotically normal estimates. However, as suggested in Gallant (1981), the effective

words the use of the FF confers a substantial gain to the analysis for the fact that it minimizes the distance from the true function in the Sobolev sense. This allows our analysis the possibility to work with asymptotically correct nominal sizes of the rejection region for statistical tests (Gallant, 1982) and enables us to estimate the true function with an average prediction bias arbitrarily small as the number of terms in the Fourier expansion increases (Gallant, 1981). Such a fact attributes a sort of non parametric property to this flexible functional form, that is not present in the other two functions considered (TL and Box-Cox). Moreover, special care must be addressed to the choice of the rescaling form for the trigonometric terms in order to coherently fix their argument in the  $0-2\pi$  range. The choice here has fallen on the simple criterion of the ratio to the sample mean as suggested in Mitchell and Onvural (1996) after having verified the pertinence with the mentioned interval. Given the local approximation set up of the Box-Cox and TL the corresponding results must be assessed with particular care for the possible bias in the indicators derived.

It goes without saying that restrictions imposed on TL also hold for the other functional forms.

### 3.2 *The measurement of efficiency*

Following Berger (1993) we employ here the distribution-free model for computing the cost efficiency levels<sup>12</sup>. The advantage of this model is that it avoids the strong distributional assumptions of stochastic frontier<sup>13</sup>. According to this approach the distribution free inefficiency is based on the distance between the estimated cost function and the  $s$ -th effective bank cost in the sample ( $s=1,\dots,N$ ), assuming that over time the random part of the error term is negligible with only the error term caused by inefficiency remaining. Hence a straightforward measure of inefficiency can be denoted as:

$$\text{inefficiency} = \exp(\min(\ln u_s) - \ln u_s) \quad (4)$$

where  $u_s$  is the residual vector after having averaged over time and  $\min(\ln u_s)$  is the least inefficient bank in the sample.

### 3.3 *Scale and scope economies*

In order to measure how changes in bank output affect cost we use the three estimated cost equations to construct indicators of scale and scope economies. We use the well-known measures:

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number of the coefficients is corrected by reducing the number of the regressors to cope with the possible multicollinearity.

<sup>12</sup> For discussion on this approach see Allen and Rai (1996), Ashton (1998), Baltagi (1995), De Young (1997), Drake and Weyman-Jones (1996).

<sup>13</sup> See Aigner et. al. (1977), Bauer (1990), Berger (1993), Battese and Coelli (1995).

(i) the ray-scale economies; (ii) the specific scale economies; (iii) scope economies; (iv) specific scope economies.

(i) For a multiproduct firm, ray scale economies (*RSCE*) are measured by the inverse of the elasticity ( $\eta_{TC/y}$ ) of cost with respect to output taken along a ray that holds output mix constant (Baumol et al., 1982):

$$RSCE = 1/(\eta_{TC/y}) = \left[ \sum_i (\partial \ln TC / \ln y_i) \right]^{-1} \quad (5)$$

For the normalized translog cost function specified as above the economies of scale are expressed as:

$$RSCE = \left[ \sum_i \left( \beta_i + \sum_j \beta_{ij} \ln y_j + \sum_k \varphi_{ik} \ln p_k \right) \right]^{-1} \quad (6)$$

At the point of approximation, where  $y_i = p_i = 1$  this expression reduces to:  $(\sum_i \beta_i)^{-1}$ .

Production presents *increasing*, *constant* or *decreasing* returns to scale depending on the RSCE to be respectively *greater*, *equal* or *less* than the unity.

(ii) In the multiproduct case, it is also possible to calculate the product specific scale economies ( $SL_i$ ), which measures the difference in cost incurred by the firm when producing the given level of output  $i$  as opposed to producing a zero level, holding the other outputs fixed (Kim, 1986).

$SL_i$  are defined as (Baumol et al., 1982):

$$SL_i = \frac{IC_i / TC}{y_i \partial TC / \partial y_i} = \frac{IC_i / TC}{\eta_{TC/y}} \quad (7)$$

where  $IC_i$  is the incremental cost of the  $i$ -th product defined as  $TC - TC(y_{i=0}, p_k)$  where the subtrahend is the total cost excluded the  $i$ -th output.

The estimation of product specific economies of scale requires the calculation of the cost function at zero output levels. As suggested by Kim (1986) the empirical problems in the application of these measures when dealing with logs can be solved using a reference point, such as 10% of the sample mean outputs. Following this approach, Kim (1986) derives the following expression for product specific returns to scale at the approximation point:

$$SL_i = [\exp\{\alpha_0\} - \exp\{\alpha_0 - \beta_i (\ln \varepsilon) + \beta_i / 2 (\ln \varepsilon)^2\}] / \beta_i \exp\{\alpha_0\} \quad (8)$$

where  $\varepsilon = 0.1$  is used in place of zero output.

(iii) Economies of scope exist when the total costs of a firm producing more than one output are lower than the sum of the costs of producing each output separately.

In the case of a firm producing three outputs, the degree of economies of scope is:

$$Scope = [(\sum_i TC_i(y_i, p_k) - TC)/TC] =$$

$$[\sum_i \exp\{- (\ln \varepsilon) \sum_{j \neq i} \beta_j + 1/2 (\ln \varepsilon)^2 \sum_{j \neq i} \sum_{v \neq i} \beta_{jv}\} - \exp\{\alpha_0\}]/\exp\{\alpha_0\}$$

$i, j, v = 1, 2, 3$

where  $TC_i(y_i, p_k)$  is the cost of producing  $y_i$  on a stand-alone.

A value of this indicator greater/smaller than zero indicates respectively the presence of economies/diseconomies of scope.

(iv) Product specific scope economies ( $Pscope_i$ ) arise when the costs of the joint production of the  $i$ -th output with the existing combination of the other outputs is lower than the cost of producing that output separately. At the point of approximation  $Pscope_i$  is given by:

$$Pscope_i = [(TC_i(y_i, p_k) + TC(y_{i=0}, p_k) - TC)/TC] =$$

$$[\exp\{-\beta_i \ln \varepsilon + \beta_i/2 (\ln \varepsilon)^2\} + \exp\{-\ln \varepsilon \sum_{j \neq i} \beta_j + (1/2 (\ln \varepsilon)^2) \sum_{j \neq i} \sum_{v \neq i} \beta_{jv}\} - \exp\{\alpha_0\}]/\exp\{\alpha_0\}$$

$i, j, v = 1, 2, 3$

As regards the indicators relative to the Box-Cox functional form, they are derived from the specifications (2).

(i) Scale economies:

$$RSCE = 1/\sum_i \beta_i$$

(ii) Specific scale economies:

$$SL_i = [\exp\{\alpha_0\} - \exp\{\alpha_0 - \beta_i/\lambda + \beta_i/2 \lambda^2\}]/\beta_i \exp\{\alpha_0\}$$

(iii) Scope economies:

$$Scope = [\sum_i \exp\{- (1/\lambda) \sum_{j \neq i} \beta_j + (1/2 \lambda^2) \sum_{j \neq i} \sum_{v \neq i} \beta_{jv}\} - \exp\{\alpha_0\}]/\exp\{\alpha_0\}$$

$i, j, v = 1, 2, 3$

(iv) Specific scope economies:

$$Pscope_i = \exp\{-\beta_i/\lambda + \beta_i/2 \lambda^2\} + \exp\{- (1/\lambda) \sum_{j \neq i} \beta_j + (1/2 \lambda^2) \sum_{j \neq i} \sum_{v \neq i} \beta_{jv}\} - \exp\{\alpha_0\}]/\exp\{\alpha_0\}$$

$i, j, v = 1, 2, 3$

Given the length of the presentation for the FF scale and scope indicators we refer to Appendix A. In the case of FF scale and scope analysis only the out of the mean sample indicators are reported, in view of the robustness of the FF on the entire range of data (Gallant, 1981).

#### 4. Empirical findings

In our empirical analysis we present the evidence from the TL, FF and Box-Cox cost function specifications both for European and for U.S. commercial banks.

In the case of European banks we run a common frontier. This analysis, on the one hand, allows to compare performances of banks across countries. On the other hand, it does not allow to determine whether divergence in inefficiency is due to differences in the technology used or to environmental conditions<sup>14</sup>. We face the latter criticism using only one type of financial institution: the commercial banks. For these banks the assumption of employing the same technology could be supported by considering that commercial banks provide all over Europe the same financial services and activities. Moreover, we employ dummies for each of the 15 European countries in order to capture differences across countries.

##### 4.1 Cost functions estimations

In this section we present both the results of the cost functions estimations for European and U.S. commercial banks.

In Table 1 we show the evidence from the FF, TL and Box-Cox specifications for European banks. Overall the results obtained are consistent over the different specifications used.

As regards the TL estimations we can point out the following results.

##### **[insert Table 1]**

1) It is noteworthy that all the outputs and input price coefficients present the expected positive sign and are significant.

2) The elasticities of production costs to unit staff ( $\gamma_{p1}=0.324$ ) and to deposit price ( $\gamma_{p2}=0.297$ ) are smaller than the elasticity to the capital price ( $0.379=1-\gamma_{p1}-\gamma_{p2}$ ) for the linear homogeneity conditions imposed on the TL cost function. This means that banks can control more personnel and deposit expenses than capital expenses when prices rise. A plausible explanation is that, at least in the short run, it seems more difficult to cut capital expenses, especially in the field of

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<sup>14</sup> To account for these effects, Dietsch and Lozano Vivas (2000) impose a cross-equation equality restriction on the parameters of each country's cost frontier in order to obtain results which are not influenced by the country's

information technology, less so far labor costs. Similar evidence has been found by Resti (1997) for a sample of Italian banks and is quite consistent with the ongoing tendency in Europe of restructuring.

The elasticity to deposit price shows the importance of interest on deposits in the European market. Moreover, the positive and significant deposits price coefficient supports the choice of considering such price as an input.

3) Among outputs, deposits ( $\beta_{y1} = 0.568$ ) are more cost absorbing than loans and services. This result at a first look could appear surprising if one considers that the lending activity is expected to be more cost absorbing than the deposit management, because of high costs connected with the monitoring and collection of non performing loans. However, as pointed out also by Resti (1997), who found the same evidence on a sample of Italian banks, deposits may imply a larger network of branches which in turn increases operating costs.

As we expected, services are the least cost absorbing output ( $\beta_{y3} = 0.065$ ) since they are less dependent on the firm's physical capital. Moreover, they have not been the most important source of income for banks in Europe. The sharp reduction of the bank's main profit source (interest income), which has occurred in the last few years, is forcing banks to find new income alternatives to deal in. In this respect, services may become one of the non-interest income sources, thus implying a larger impact on the cost function.

As far as the FF and Box-Cox are concerned we find some small differences when compared with TL estimates. The result that deposits are the most cost absorbing output is largely confirmed (0.587 for FF and 0.934 for Box-Cox). Moreover there is also clear evidence on services (0.183 for FF and 0.178 for Box-Cox) which result to be the least cost absorbing output with respect to loans (0.205 for FF and 0.372 for Box-Cox) and deposits.

The coefficients for deposits and loans are largely overvalued with the Box-Cox specification when compared with the TL and the FF specifications. The FF and the Box-Cox provide almost the same elasticity for services, which is significantly higher than that obtained with the TL.

Considering the input prices, both the FF and the Box-Cox estimates produce lower elasticity coefficients for capital price (0.248 for FF and 0.19 for the Box-Cox).

Finally, the common evidence across the three cost function specifications is that deposits are the most cost absorbing output and services the least cost absorbing one; among the input prices, capital shows the highest elasticity.

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technology. They add country-specific environmental variables to the cost function specification to measure the impact of those variables on the differences among country inefficiencies.

This evidence describes European commercial banks as characterized by high cost elasticity to deposits which pay high interest rates and increase operative costs associated with the network of branches.

4) As far as the countries effect is concerned the UK, Denmark and Ireland lay on the European average<sup>15</sup>. We find that the FF specification supports fixed effects better than the other two functional forms.

Table 2 presents the evidence for the U.S. commercial banks obtained from the three different functional form estimates. As pointed out above (par. 2.2), in the case of U.S. commercial banks we model the cost functions following the value added approach. The choice of adopting a different cost function - in which we do not include the price of deposits among the inputs - is supported by the test on the specification, and by the fact that the cost of deposits for U.S. banks is quite negligible.

**[insert Table 2]**

In the case of U.S. commercial banks the evidence provided by our analysis seems quite consistent across the different specifications employed: coefficients present almost the same values, and are strongly significant. Only the Box-Cox estimates for loans and services tend to be slightly overvalued compared to the TL and FF figures. Overall, the evidence suggests considering results reliable.

As expected the outcome underlines a large difference between European and U.S. commercial banks.

As opposed to the European case, in all specifications the most cost absorbing output is services, with the elasticity for deposits and loans always lower.

The level and the quality of services provided to customers, more than deposits, have been an important source of income for U.S. commercial banks. Therefore it seems plausible to expect higher cost connected to the production of this item, which requires high investment in financial system technology (software, telecommunication, etc.) and human capital.

Looking at the input prices, surprisingly and contrarily to what one might expect, given also the flexibility of the labor market, the elasticity of production cost to unit staff is much higher than the elasticity to capital price. Such a result has also been confirmed by the other two functional forms. This evidence would imply that firms can control more easily capital, cutting down its demand, rather than labor, when prices rise.

The puzzling evidence has to be addressed with caution since we are considering a period of exceptional growth for the U.S. economy (driven also by the new economy and information

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<sup>15</sup> For these countries the dummies are not significantly different from zero.



technology). High profit expectations and growing demand for factors, which characterized the period, may make constraints less binding<sup>16</sup>. This is also consistent with the high level of inefficiency obtained from our analysis.

Finally, the analysis for Europe and the U.S. shows that the three functional forms employed performed quite well producing in most cases consistent evidence. However, tests performed on the nested TL *versus* FF functional forms show, tenuous evidence in favor of FF for Europe and a stronger one for the U.S.<sup>17</sup>. This empirical outcome finds support in White (1980), Gallant (1981) and Mitchell and Onvural (1996) which show the superiority of the FF with the respect to TL estimations.

#### 4.2 *Distribution free cost efficiency results.*

The distribution free cost inefficiency estimates, in Table 3, point out a significant level of inefficiency both for European and U.S. commercial banks. This evidence is consistent with the analysis in Bauer (1990), Allen and Rai (1996), Cavallo and Rossi (2001) and Vander-Vennet (2002), which also find significant level of inefficiency in the banking industry. Moreover, our results are strengthened by the inefficiency scorers we obtained by using the stochastic frontier approach<sup>18</sup>.

While Box-Cox tends to overestimate the efficiency scores, both TL and FF cost functions show similar outcomes (see Table 3).

#### **[insert Table 3]**

For Europe, we find an average inefficiency level of 32 per cent with the TL, 36 per cent with the FF, and 20 per cent with the Box-Cox specification. While the magnitude of inefficiency dispersion between TL (reporting a deviation of 0.11 with a minimum of 15 per cent inefficiency) and the FF (standard deviation of 0.093 and a minimum value of 17 per cent) is quite similar, the distribution of Box-Cox cost inefficiency shows lower inefficiency levels (20 per cent) with a standard deviation of 0.091 and a minimum of 6 per cent.

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<sup>16</sup> This evidence may be also supported by the fact that under fixed term contracts, which are the standard in the flexible structure of the U.S. labour market, workers in financial institutions benefit from large compensation payments if fired.

<sup>17</sup> We reject the  $H_0$  hypothesis on the base of the Ramsey-Reset test according to which model has no omitted variables at a level of confidence of 5%:  $Pvalue = 0.010$  for Europe,  $Pvalue = 0.12$ , for the U.S.

<sup>18</sup> Berger and Mester (1997) also find that the efficiency level with the stochastic and the free distribution approach are similar. Our stochastic frontier estimates, obtained by Frontier 4.1 (Coelli, 96), are available upon request.

The analysis by bank dimension shows that for both European and U.S. commercial banks there is not a consistent size effect, as also found by Vander-Vennet (2002) for a sample of European banks. This evidence shows that our findings are robust over the size samples used<sup>19</sup>.

In particular, while FF presents almost the same score across sizes, TL and Box-Cox estimates show that efficiency tends to slightly decrease as the bank size increases. The cost inefficiency results for U.S. commercial banks suggest that the Box-Cox specification, as in the European case, overestimates the efficiency scores.

#### 4.3 *Scale and scope economies findings*

In this section we present the evidence on scale and scope economies indicators obtained from the different cost functions for European and U.S. commercial banks. The aim here is to provide new evidence on bank output efficiency by using different functional forms (TL, FF and Box-Cox), and to check the robustness of the results over the specifications employed.

The estimations are computed both *out of the mean sample* and *at the mean sample*. Obviously the former analysis allows us to infer on the indicators obtained.

The evidence computed *out of the mean sample* for European Commercial banks (Table 4) can be summarized as follows.

#### **[insert Table 4]**

- (i) Constant scale economies ( $RSCE$ ), product specific diseconomies of scale ( $SL_i$ ), diseconomies of scope ( $Scope$ ) and specific scope ( $Pscope_i$ ) diseconomies are detected in our study<sup>20</sup>.
- (ii) Since the results provided seem to be sensitive to the different functional forms employed, output efficiency scores should be interpreted with caution. Box-Cox provides evidence in favor of increasing returns to scale with the exception of large banks. On the contrary, TL scale economies are roughly constant both overall and at size level as in the FF case.
- (iii) Bank size classification - small, medium and large - does not significantly affect scale and scope economies indicators in the TL and FF. Almost all the scale indicators - both global and specific - obtained with these two specifications exhibit a quite stable pattern through the different bank sizes with a slight tendency for loans to increase as the dimension of a bank becomes larger. On the contrary, the Box-Cox scale economies indicators appear to be

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<sup>19</sup> As mentioned in par. 2.2, in our analysis we consider data only from large commercial banks. We split the sample in three classes according to the asset size distribution. Consequently, for small banks we mean the smallest in our sample.

<sup>20</sup> While some of our results are consistent with Vander-Vennet (2002), different evidence has been provided in Cavallo and Rossi (2001) over a wider typology of financial institutions for the period 1992-97 in six European countries.

sensitive to the bank size, suggesting that the returns to scale are more pronounced for small banks. Looking at the Box-Cox product specific scale indicators the production of deposits seems to generate smaller diseconomies of scale as the size of the bank increases.

- (iv) Product specific diseconomies of scales are detected in all the specifications, with the exception of deposits and loans in the TL specification.
- (v) Diseconomies of scope are obtained by all functional forms. The presence of negative scope economies - lower than one in absolute values - indicates that the joint production of the three outputs - deposits, loans and services - is inefficient. Looking at the evidence from the three specifications, in the case of scope indicators the most significant results are those obtained with FF specification followed by the TL, which appear to fit the data poorly with respect to services scope economies. The Box-Cox specification seems to produce poorer scope indicators, although the sign, the magnitude and size effect, are consistent with the other two specifications. The evidence shows higher absolute values of scope diseconomies as the size of the firm decreases. This result may suggest that large size banks present less diseconomies of scope than small banks, suggesting that small financial institutions are less efficient in the joint multi output production.

Due to the second order approximation of the TL and the Box-Cox, their fit, in the *out of the mean sample* data, may be poorer compared to the FF (Gallant, 1981). Therefore, TL and Box-Cox scale and scope indicators have also been computed at the mean sample (Table 5).

**[insert Table 5]**

While TL scale and scope estimates in the approximation point produce consistent evidence with previous analysis (out of the mean sample), the Box-Cox indicators seem to deviate from the *out of the mean sample* results: they undervalue the overall scale economies and slightly overvalue the scope indicators.

As far as U.S. *out of the mean sample* indicators are concerned (Table 6), we can summarize the following evidence.

**[insert Table 6]**

- (i) Results show the presence of overall scale economies, being more pronounced for small and medium banks. Large banks present slight diseconomies of scale (FF, Box-Cox) and almost constant return to scale (TL). This evidence may suggest that a broader firm may incur higher physical capital and innovation costs, which do not suffice to exploit scale economies, given the U shaped average cost function.
- (ii) Product specific diseconomies of scale are detected by the analysis. Only in the case of the TL estimates we find evidence in favor of increasing specific scale economies

for loans and services. Therefore, at the product specific scale economies level, the most consistent evidence across the different specifications, is the presence of product specific diseconomies of scale for deposits, which is significantly smaller than unity (with the exception for large banks which present product specific increasing economies of scale). This evidence suggests that the production of deposits, due to the fact that it requires a wide network of branches, may cause a more than proportional rise in total costs<sup>21</sup>.

- (iii) The tendency of product specific scale indicators to grow with bank size shows that U.S. large commercial banks, seem to be more efficient or less inefficient in the production of specific output than in the overall case.
- (iv) As far as the scope indicators are concerned, also for U.S. banks we find negative values of scope economies indicators over the different specifications.
- (v) Based on the Ramsey test, which shows that FF for U.S. banks approximates the true function better than the other functional forms, we tend to rely more upon the FF scale and scope indicators. Moreover, the indicators obtained from FF present always higher  $t$  values.

As regards the indicators computed *at the mean sample*, the evidence almost confirms the results obtained by the *out of the mean sample* estimates: for TL, presence of overall constant returns to scale are detected, and increasing product scale economies are found for the three outputs (Table 7).

#### [insert Table 7]

For the Box-Cox estimates we obtain, consistently with previous analysis, specific diseconomies of scale and, differently from the *out of the mean sample* estimates, overall diseconomies of scale. Given the evidence on the overall scale economies, the Box-Cox approximation point estimates, tend to deviate more than the TL from the *out of the mean sample* analysis.

Finally, the evidence from the *out of* and *at the mean sample* estimations support the view that both scale and scope economies, are larger in U.S. than in European banks. An acceptable reason for this is probably the higher level of technology included in the productive structure of U.S. banks, as shown in Hunter and Timme (1991), and the restructuring process occurred previously than in Europe.

## 5. Conclusions

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<sup>21</sup> Note that this result does not contradict the low elasticity values for deposits, as specific economies of scale

The econometric analysis performed through the different specifications enables us to reach some conclusions regarding the cost structure of U.S. and European commercial banks.

The use of the Fourier, translog and Box-Cox functions helped us in evaluating the robustness of our results through the several specifications proposed. Although the evidence on the cost function parameters presents some consistent values over the different functional forms, tests performed on the specifications are in favor of the Fourier which, turns up to better fit our data.

Results for the two banking systems are coherent with the evidence that European commercial banks are traditionally more oriented to deposits and retails, while U.S. commercial banks are also focused on the services, given also the impact of the innovation in the field of information technology.

Consistently we find for Europe that deposits are the most cost absorbing output, since they may require a broad network of branches which causes an increase in costs. On the contrary, services turn out to be the least cost absorbing output as they traditionally had never been the core bank activity in Europe.

For U.S. commercial banks services are found to be the most cost absorbing output, since they may involve high investments in information technology and human capital.

As far as the input prices are concerned, European commercial banks turn out to control more personnel and deposits expenses than capital when prices rise. This evidence is quite consistent with the ongoing tendency in Europe to restructuring through mergers and acquisitions. The need for restructuring is also consistent with significant level of inefficiency detected by the cost efficiency estimates.

Looking at the input prices for U.S. commercial banks, surprisingly and contrarily to what one might expect, production costs are more sensitive to unit staff than to capital price. This evidence would imply that firms can control more easily capital, cutting down its demand, rather than the demand for labor, when prices rise.

The puzzling evidence has to be considered with caution since we are looking at a period (1995-1998) of exceptional growth and low levels of unemployment for the U.S. economy. High profit expectations and the consequent growing demand for labor, which characterized the period, may make constraints less binding. This also turns out to be consistent with the high level of inefficiency obtained from our analysis.

Looking at scale economies indicators, we find increasing global scale economies for the U.S. and less pronounced evidence for Europe. This may be due to the higher information technology

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indicators are based on the definition of incremental cost which is posed in discrete sense.

embedded in the production function of U.S. banks which obviously has an impact on the productivity of the system.

The findings for scale economies in Europe, however have to be interpreted with caution, since indicators obtained seem to be sensitive to the different functional forms employed. Box-Cox provides evidence in favor of increasing return to scale economies with the exception of large banks. On the contrary, TL scale economies are roughly constant both overall and at size level as in the FF case.

As far as the scope indicators are concerned we find evidence of diseconomies of scope, detected by all the functional forms, both in Europe and in the U.S. Such evidence, which is consistent with the significant level of inefficiency detected, is likely to be associated to the consolidation process for Europe and to the less binding constraints for U.S., given the positive expectations and the high pace growth during the period considered.

**Table 1. Cost functions estimates – European commercial banks**

Variables	parameters	FF		TL		Box-Cox	
		Coef.	z	Coef.	z	Coef.	z
<i>ITC</i>							
$ly_1$ (deposits)	$\beta_{y1}$	0.587	13.190	0.568	11.260	$bxy_1$	0.934 11.120
$ly_2$ (loans)	$\beta_{y2}$	0.205	5.92	0.258	6.390	$bxy_2$	0.372 5.050
$ly_3$ (services)	$\beta_{y3}$	0.183	6.63	0.065	2.140	$bxy_3$	0.178 4.460
$lp_1$ (labor price)	$\gamma_{p1}$	0.248	20.120	0.324	23.510	$lp_1$	0.190 11.66
$lp_2$ (deposits price)	$\gamma_{p2}$	0.362	22.75	0.297	20.62	$lp_2$	0.414 17.69
$ly_1^2$	$\beta_{y1}^2$	0.149	5.190	0.090	2.590	$bxy_1^2$	0.041 0.330
$ly_2^2$	$\beta_{y2}^2$	0.023	2.420	0.047	4.99	$bxy_2^2$	-0.050 -0.830
$ly_3^2$	$\beta_{y3}^2$	0.029	3.72	0.015	1.92	$bxy_3^2$	-0.089 -1.980
$lp_1^2$	$\gamma_{p1}^2$	-0.061	-6.150	-0.055	-4.670	$lp_1^2$	-0.164 -14.690
$lp_2^2$	$\gamma_{p2}^2$	0.135	13.490	0.102	11.98	$lp_2^2$	0.067 0.208
$ly_1y_2$	$\beta_{y1y2}$	-0.045	-3.500	-0.021	-1.320	$bxy_1y_2$	-0.151 -1.980
$ly_1y_3$	$\beta_{y1y3}$	-0.080	-6.450	-0.048	-3.670	$bxy_1y_3$	-0.080 -1.290
$ly_2y_3$	$\beta_{y2y3}$	0.030	4.410	0.008	1.140	$bxy_2y_3$	0.101 2.520
$lp_1p_2$	$\gamma_{p1p2}$	-0.040	-4.810	-0.016	-1.750	$lp_1p_2$	0.025 2.060
$ly_1p_1$	$\varphi_{y1p1}$	-0.042	-2.290	-0.054	-2.640	$bxy_1p_1$	0.047 0.087
$ly_2p_1$	$\varphi_{y2p1}$	-0.004	-0.310	-0.005	-0.370	$bxy_2p_1$	-0.015 -0.630
$ly_3p_1$	$\varphi_{y3p1}$	0.024	2.340	0.014	1.370	$bxy_3p_1$	0.018 1.260
$ly_1p_2$	$\varphi_{y1p2}$	0.081	5.470	0.066	4.390	$bxy_1p_2$	0.087 2.320
$ly_2p_2$	$\varphi_{y2p2}$	-0.041	-4.060	-0.028	-2.600	$bxy_2p_2$	-0.071 -2.290
$ly_3p_2$	$\varphi_{y3p2}$	-0.046	-5.630	-0.036	-4.510	$bxy_3p_2$	-0.087 -3.890
$sumcosy_1y_2$	$e_{sumcosy_1y_2}$	0.031	0.106				
$sumsiny_1y_2$	$f_{sumsiny_1y_2}$	0.019	1.200				
$Difcosy_1y_3$	$g_{difcosy_1y_3}$	-0.006	-0.200				
$Difcosy_2y_3$	$g_{difcosy_2y_3}$	0.007	0.270				
$difsiny_1y_3$	$h_{difsiny_1y_3}$	-0.029	-0.950				
$difsiny_2y_3$	$h_{difsiny_2y_3}$	0.018	0.042				
$sumcosp_1p_2$	$i_{sumcosp_1p_2}$	0.177	0.212				
$sinp_1p_2$	$l_{sumsinp_1p_2}$	0.038	1.150				
$Difcosp_1p_2$	$m_{difcosp_1p_2}$	0.142	0.189				
$difsinp_1p_2$	$n_{difsinp_1p_2}$	0.015	0.520				
<i>Austria</i>	$D[Austria]$	-0.254	-2.590	-0.306	-1.530	$D[Austria]$	-0.379 -2.480
<i>Belgium</i>	$D[Belgium]$	-0.320	-3.590	-0.432	-2.420	$D[Belgium]$	-0.473 -3.410
<i>Denmark</i>	$D[Denmark]$	0.061	0.067	-0.084	-0.450	$D[Denmark]$	-0.085 -0.600
<i>France</i>	$D[France]$	-0.284	-3.590	-0.248	-1.570	$D[France]$	-0.300 -2.430
<i>Germany</i>	$D[Germany]$	-0.403	-5.050	-0.456	-2.850	$D[Germany]$	-0.436 -3.500
<i>Greece</i>	$D[Greece]$	0.719	7.560	0.790	4.240	$D[Greece]$	0.765 5.170
<i>Ireland</i>	$D[Ireland]$	0.071	0.074	-0.122	-0.660	$D[Ireland]$	-0.067 -0.460
<i>Italy</i>	$D[Italy]$	0.138	1.69	0.177	1.080	$D[Italy]$	-0.006 -0.050
<i>Luxemburg</i>	$D[Luxemburg]$	-0.288	-3.520	-0.258	-1.590	$D[Luxemburg]$	-0.268 -2.100
<i>Holland</i>	$D[Holland]$	-0.300	-3.530	-0.262	-1.530	$D[Holland]$	-0.204 -1.540
<i>Portugal</i>	$D[Portugal]$	0.273	3.300	0.330	1.97	$D[Portugal]$	0.302 2.340
<i>Spain</i>	$D[Spain]$	0.279	3.160	0.365	2.050	$D[Spain]$	0.214 1.560
<i>Sweden</i>	$D[Sweden]$	-0.297	-2.790	-0.663	-3.110	$D[Sweden]$	-0.531 -3.140
<i>UK</i>	$D[UK]$	-0.033	-0.410	-0.001	-0.010	$D[UK]$	-0.113 -0.900
<i>Cons</i>	$\alpha_0$	14.59	133.90	15.01	94.58	$\alpha_0$	15.013 121.42
R2		0.9851		0.9762		R2	0.9670
Obs=1352							

A random-effects GLS regression is used in all specifications. The prefix l stands for logs; sum and dif represent respectively the sum and difference between trigonometric operators. The total cost, the price of labor ( $p_1$ ) and the price of deposits ( $p_2$ ) are normalized to the price of capital. Mixed products and squares of inputs and outputs represent the second order terms of the flexible form.

**Table 2. Cost functions estimates - United States commercial banks**

variables	parameters	FF		TL		BoxCox		
		Coef.	z	Coef.	z	variables	Coef.	z
<i>ITC</i>								
$ly_1$ (deposits)	$\beta_{y1}$	0.268	6.360	0.289	8.050	$bxy_1$	0.276	4.420
$ly_2$ (loans)	$\beta_{y2}$	0.248	5.430	0.238	5.970	$bxy_2$	0.449	7.800
$ly_3$ (services)	$\beta_{y3}$	0.425	14.76	0.417	15.290	$bxy_3$	0.570	13.460
$lp_1$ (labor price)	$\gamma_{p1}$	0.797	36.59	0.828	40.420	$lp_1$	0.820	27.650
$ly_1^2$	$\beta_{y1}^2$	0.025	1.520	0.024	1.530	$bxy_1^2$	-0.348	-3.090
$ly_2^2$	$\beta_{y2}^2$	-0.004	-0.250	0.002	0.170	$bxy_2^2$	-0.614	-5.040
$ly_3^2$	$\beta_{y3}^2$	0.050	3.440	0.037	2.570	$bxy_3^2$	-0.140	-4.120
$lp_1^2$	$\gamma_{p1}^2$	-0.037	-2.020	-0.031	-1.740	$lp_1^2$	-0.064	-2.810
$ly_1y_2$	$\beta_{y1y2}$	0.057	0.178	0.050	3.300	$bxy_1y_2$	0.490	4.510
$ly_1y_3$	$\beta_{y1y3}$	-0.041	-2.080	-0.025	-1.300	$bxy_1y_3$	-0.047	-1.060
$ly_2y_3$	$\beta_{y2y3}$	-0.043	-3.130	-0.047	-3.510	$bxy_2y_3$	-0.014	-0.330
$ly_1p_1$	$\varphi_{y1p1}$	0.036	0.108	0.044	2.370	$bxy_1p_1$	0.120	2.530
$ly_2p_1$	$\varphi_{y2p1}$	-0.094	-5.610	-0.080	-4.790	$bxy_2p_1$	-0.206	-4.500
$ly_3p_1$	$\varphi_{y3p1}$	0.002	0.090	-0.020	-1.110	$bxy_3p_1$	-0.001	-0.020
$cosp_1$	$c_{\cos p1}$	0.016	0.068					
$sinp_1$	$d_{\sin p1}$	-0.084	-4.780					
$sumcosy_1y_2$	$e_{sumcosy1y2}$	0.019	0.380					
$sumcosy_1y_3$	$e_{sumcosy1y3}$	-0.034	-0.690					
$sumsiny_1y_2$	$f_{sumsiny1y2}$	-0.019	-0.780					
$difcosy_1y_3$	$g_{difcosy2y3}$	-0.090	-1.720					
$difcosy_1y_2$	$g_{difcosy2y2}$	0.125	0.126					
$difsiny_2y_3$	$h_{difsiny2y3}$	0.083	2.260					
<i>cons</i>	$\alpha_0$	11.73	249.93	11.64	464.680	$\alpha_0$	11.56	300.410
R2		0.9752		0.9648			0.9612	
Obs=1116								

A random-effects GLS regression is used in all specifications. The prefixes l stands for logs; sum and dif represent respectively the sum and difference between trigonometric operators. The total cost and the price of labor ( $p_1$ ) are normalized to the price of capital.



**Table 3. Distribution free cost inefficiency estimates – European and U. S. commercial banks**

	<b>FF</b>	<b>TL</b>	<b>BOX-COX</b>
<b>EUROPE</b>			
Total	0.36 (0.093) <i>t</i> = 3,87	0.32 (0.111) <i>t</i> = 2,90	0.21 (0.091) <i>t</i> = 2,33
Small	0.36 (0.0778) <i>t</i> = 5,0	0.35 (0.101) <i>t</i> = 3,5	0.267 (0.085) <i>t</i> = 3,21
Medium	0.36 (0.111) <i>t</i> = 3,20	0.31 (0.112) <i>t</i> = 3,10	0.17 (0.087) <i>t</i> = 2,13
Large	0.35 (0.063) <i>t</i> = 5,83	0.30 ( 0.112) <i>t</i> = 3,0	0.21 ( 0.076) <i>t</i> = 3,0
<b>U.S.</b>			
Total	0.38 (0.141) <i>t</i> = 2,70	0.37 (0.136) <i>t</i> = 2,70	0.17 ( 0.084) <i>t</i> = 2,12
Small	0.37 (0.129) <i>t</i> = 3,08	0.35 (0.123) <i>t</i> = 2,90	0.19 ( 0.100) <i>t</i> = 1,90
Medium	0.38 (0.157) <i>t</i> = 2,53	0.37 (0.152) <i>t</i> = 2,46	0.15 ( 0.072) <i>t</i> = 2,14
Large	0.39 (0.118) <i>t</i> = 3,54	0.37 ( 0.112) <i>t</i> = 3,36	0.18 ( 0.078) <i>t</i> = 2,67

In parenthesis we report the standard deviation values.

**Table 4. Scale and scope economies indicators from the Fourier, TL and Box-Cox specifications for European commercial banks**

	FF		TL		Box-Cox	
	<i>RSCE</i>	<i>t</i>	<i>RSCE</i>	<i>t</i>	<i>RSCE</i>	<i>t</i>
Overall	1,00	9,87	1,08	11,45	1,34	2,52
Small	1,00	22,47	1,11	12,34	2,14	10,61
Medium	1,00	27,87	1,08	14,56	1,35	3,74
Large	1,00	10,01	1,03	9,06	0,80	3,30
	<i>SL<sub>1</sub> (deposits)</i>	<i>t</i>	<i>SL<sub>1</sub></i>	<i>t</i>	<i>SL<sub>1</sub></i>	<i>t</i>
	0,69	4,7	1,49	6,61	0,39	1,10
Small	0,64	4,79	1,57	8,26	0,09	2,48
Medium	0,7	5,43	1,47	7,86	0,32	1,88
Large	0,69	3,27	1,45	4,95	0,84	2,38
	<i>SL<sub>2</sub> (loans)</i>	<i>t</i>	<i>SL<sub>2</sub></i>	<i>t</i>	<i>SL<sub>2</sub></i>	<i>t</i>
	0,17	2,29	1,63	5,23	0,11	0,92
Small	0,17	2,53	1,54	5,13	0,05	1,70
Medium	0,16	2,45	1,72	5,76	0,10	1,60
Large	0,19	1,67	1,92	13,97	0,19	1,31
	<i>SL<sub>3</sub> (services)</i>	<i>t</i>	<i>SL<sub>3</sub></i>	<i>t</i>	<i>SL<sub>3</sub></i>	<i>t</i>
	0,11	1,67	0,84	0,23	0,03	0,71
Small	0,15	2,77	0,40	1,49	0,02	1,36
Medium	0,1	1,9	0,72	1,24	0,03	0,99
Large	0,07	0,98	1,32	1,67	0,08	0,71
	<i>Scope</i>	<i>t</i>	<i>Scope</i>	<i>t</i>	<i>Scope</i>	<i>t</i>
	-0,87	-3,07	-0,88	-10,30	0,05	0,11
Small	-0,97	-79,01	-0,92	-61,88	0,49	3,15
Medium	-0,95	-30,17	-0,91	-21,45	-0,03	-0,10
Large	-0,64	-1,34	-0,80	-6,56	-0,26	-0,44
	<i>Pscope<sub>1</sub> (deposits)</i>	<i>t</i>	<i>Pscope<sub>1</sub></i>	<i>t</i>	<i>Pscope<sub>1</sub></i>	<i>t</i>
	-0,87	-2,82	-0,89	-8,65	-0,04	-0,12
Small	-0,99	-16,5	-0,95	-51,26	0,20	2,47
Medium	-0,96	-28,6	-0,92	-19,80	-0,12	-0,69
Large	-0,59	-1,18	-0,79	-5,64	-0,15	-0,23
	<i>Pscope<sub>2</sub> (loans)</i>	<i>t</i>	<i>Pscope<sub>2</sub></i>	<i>t</i>	<i>Pscope<sub>2</sub></i>	<i>t</i>
	-0,94	-20,88	-0,89	-23,95	-0,17	-0,72
Small	-0,91	-40,47	-0,85	-40,91	0,09	1,07
Medium	-0,93	-21,12	-0,89	-45,33	-0,19	-1,21
Large	-0,97	-22,85	-0,94	-48,82	-0,40	-1,74
	<i>Pscope<sub>3</sub> (services)</i>	<i>t</i>	<i>Pscope<sub>3</sub></i>	<i>t</i>	<i>Pscope<sub>3</sub></i>	<i>t</i>
	-0,64	-2,49	-0,27	-0,54	-0,08	-0,31
Small	-0,79	-10,53	-0,59	-6,67	0,20	2,31
Medium	-0,7	-5,35	-0,42	-2,39	-0,07	-0,43
Large	-0,39	-1,12	-0,29	-0,43	-0,41	-2,46

RSCE: ray scale economies;  $SL_1$   $SL_2$   $SL_3$ : product specific return to scale. Production presents increasing, constant and decreasing return to scale depending on the scale indicators (RSCE,  $SL_1$   $SL_2$   $SL_3$ ) being greater equal and less than unity. Indicators are computed *out of the mean sample*.

**Table 5. Scale and scope economies indicators computed at the mean sample data for European commercial banks**

	<b>TL</b>	<b>Box-Cox</b>
Mean <i>RSCE</i>	1,12	0,67
Mean <i>SL</i> <sub>1</sub> ( <i>deposits</i> )	0,99	0,76
Mean <i>SL</i> <sub>2</sub> ( <i>loans</i> )	1,12	0,23
Mean <i>SL</i> <sub>3</sub> ( <i>services</i> )	1,05	0,09
Mean <i>Scope</i>	-0,92	-0,61
Mean <i>Pscope</i> <sub>1</sub> ( <i>deposits</i> )	-0,92	-0,53
Mean <i>Pscope</i> <sub>2</sub> ( <i>loans</i> )	-0,87	-0,55
Mean <i>Pscope</i> <sub>3</sub> ( <i>services</i> )	-0,40	-0,47

**Table 6. Scale and scope economies indicators from the Fourier, TL and Box-Cox specifications for U.S. commercial banks**

USA	FF		TL		Box-Cox	
	<i>RSCE</i>	<i>t</i>	<i>RSCE</i>	<i>t</i>	<i>RSCE</i>	<i>t</i>
	1,2	5,6	1,13	18,19	1,55	2,44
Small	1,2	18,5	1,15	21,40	2,30	6,24
Medium	1,2	6,9	1,12	16,71	1,32	3,33
Large	0,9	2,4	1,10	23,36	0,89	2,33
	<i>SL<sub>1</sub> (deposits)</i>	<i>t</i>	<i>SL<sub>1</sub></i>	<i>t</i>	<i>SL<sub>1</sub></i>	<i>t</i>
	0,3	2,2	0,92	1,71	0,26	0,66
Small	0,2	4,2	0,23	1,37	0,07	2,99
Medium	0,3	3,2	0,93	2,59	0,19	1,48
Large	0,5	1,6	1,68	14,69	1,03	1,66
	<i>SL<sub>2</sub> (loans)</i>	<i>t</i>	<i>SL<sub>2</sub></i>	<i>t</i>	<i>SL<sub>2</sub></i>	<i>t</i>
	0,2	10,0	1,66	5,14	0,17	1,67
Small	0,1	2,1	1,66	5,70	0,11	2,38
Medium	0,1	1,4	1,63	3,81	0,21	1,85
Large	0,5	1,5	1,94	80,06	0,32	1,17
	<i>SL<sub>3</sub> (services)</i>	<i>t</i>	<i>SL<sub>3</sub></i>	<i>t</i>	<i>SL<sub>3</sub></i>	<i>t</i>
	0,4	4,3	1,61	1,93	0,21	1,41
Small	0,4	7,3	1,26	1,23	0,10	1,39
Medium	0,4	4,5	1,50	1,89	0,27	1,81
Large	0,4	1,6	2,45	6,36	0,38	2,33
	<i>Scope</i>	<i>t</i>	<i>Scope</i>	<i>t</i>	<i>Scope</i>	<i>t</i>
	-0,9	-23,3	-0,95	-23,41	-0,77	-5,44
Small	-0,9	-16,9	-0,91	-20,00	-0,62	-9,65
Medium	-1,0	-58,5	-0,96	-48,31	-0,82	-9,05
Large	-1,0	-34,9	-0,97	-91,13	-0,97	-65,99
	<i>Pscope<sub>1</sub> (deposits)</i>	<i>t</i>	<i>Pscope<sub>1</sub></i>	<i>t</i>	<i>Pscope<sub>1</sub></i>	<i>t</i>
	-0,9	-5,6	-0,21	-0,72	-0,69	-4,08
Small	-0,8	-3,6	0,07	0,54	-0,52	-10,94
Medium	-0,9	-9,5	-0,25	-1,35	-0,71	-6,31
Large	-1,0	-54,2	-0,67	-5,26	-0,97	-24,36
	<i>Pscope<sub>2</sub> (loans)</i>	<i>t</i>	<i>Pscope<sub>2</sub></i>	<i>t</i>	<i>Pscope<sub>2</sub></i>	<i>t</i>
	-1,0	-44,0	-0,50	-2,98	-0,79	-6,36
Small	-1,0	-53,5	-0,38	-2,04	-0,66	-14,28
Medium	-1,0	-98,9	-0,56	-4,35	-0,82	-10,24
Large	-1,0	-27,7	-0,56	-5,39	-0,97	-24,21
	<i>Pscope<sub>3</sub> (services)</i>	<i>t</i>	<i>Pscope<sub>3</sub></i>	<i>t</i>	<i>Pscope<sub>3</sub></i>	<i>t</i>
	-1,0	-36,9	0,07	0,11	-0,56	-2,85
Small	-1,0	-149,4	0,69	1,19	-0,39	-4,69
Medium	-1,0	-104,9	-0,14	-0,35	-0,61	-4,43
Large	-0,9	-26,2	-0,54	-6,01	-0,78	-2,93

RSCE: ray scale economies;  $SL_1$   $SL_2$   $SL_3$ : product specific return to scale. Production presents increasing, constant and decreasing return to scale depending on the scale indicators (RSCE,  $SL_1$   $SL_2$   $SL_3$ ) being greater equal and less than unity. Indicators are computed *out of the mean sample*.

**Table 7. Scale and scope economies indicators computed at the mean sample data for U.S. banks**

	<b>TL</b>	<b>Box-Cox</b>
Mean RSCE	1,05	0,77
Mean $SL_1$ (deposits)	1,43	0,24
Mean $SL_2$ (loans)	1,74	0,43
Mean $SL_3$ (services)	1,28	0,46
Mean $Scope$	-0,98	-0,94
Mean $Pscope_1$ (deposits)	-0,88	-0,85
Mean $Pscope_2$ (loans)	-0,88	-0,94
Mean $Pscope_3$ (services)	-0,93	-0,78

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## APPENDIX A

### *Scale and scope economies indicators for the Fourier flexible functional form*

The calculations of the scale and scope economies indicators for FF are shown in this appendix. The indicators here are based on the European banks cost function from which the U.S. case can be easily derived.

Symbols are the same as in the paper and they label the correspondent coefficients.  $\eta(y_i)$  stands for the elasticity of the total cost with respect to the  $i$ -th output.

*Overall Scale economies:*

$$\begin{aligned} \eta(y_1) = & \beta_{y_1} + \beta_{y_1}^2 * l_{y_1} + \beta_{y_1 y_2} * l_{y_2} + \beta_{y_1 y_3} * l_{y_3} + \varphi_{y_1 p_1} * l_{p_1} + \varphi_{y_1 p_2} * l_{p_2} \\ & + \exp(l_{y_1}) * (e_{\text{sumcosy}1y_2} * (-\sin y_1) + f_{\text{sumsiny}1y_2} * \cos y_1 + g_{\text{difcosy}1y_3} * (-\sin y_1) + \\ & h_{\text{difsiny}1y_3} * \cos y_1); \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} \eta(y_2) = & \beta_{y_2} + \beta_{y_2}^2 * l_{y_2} + \beta_{y_1 y_2} * l_{y_1} + \beta_{y_2 y_3} * l_{y_3} + \varphi_{y_2 p_1} * l_{p_1} + \varphi_{y_2 p_2} * l_{p_2} \\ & + \exp(l_{y_2}) * (e_{\text{sumcosy}1y_2} * (-\sin y_2) + f_{\text{sumsiny}1y_2} * \cos y_2 + g_{\text{difcosy}2y_3} * \sin y_2 \\ & + h_{\text{difsiny}2y_3} * (-\cos y_2)); \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \eta(y_3) = & \beta_{y_3} + \beta_{y_3}^2 * l_{y_3} + \beta_{y_1 y_3} * l_{y_1} + \beta_{y_2 y_3} * l_{y_2} + \varphi_{y_3 p_1} * l_{p_1} + \varphi_{y_3 p_2} * l_{p_2} \\ & + \exp(l_{y_3}) * (g_{\text{difcosy}1y_3} * \sin y_3 + g_{\text{difcosy}2y_3} * \sin y_3 + h_{\text{difsiny}1y_3} * \\ & (-\cos y_3) + h_{\text{difsiny}2y_3} * (-\cos y_3)); \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} RSCE = 1/\sum_i \eta(y_i) = & 1/(\beta_{y_1} + \beta_{y_2} + \beta_{y_3} + \beta_{y_1}^2 * l_{y_1} + \beta_{y_1 y_2} * (l_{y_1} + l_{y_2}) + \\ & \beta_{y_1 y_3} * (l_{y_1} + l_{y_3}) + \beta_{y_2}^2 * l_{y_2} + \beta_{y_2 y_3} * (l_{y_2} + l_{y_3}) + \beta_{y_3}^2 * l_{y_3} + \\ & \varphi_{y_1 p_1} * l_{p_1} + \varphi_{y_1 p_2} * l_{p_2} + \varphi_{y_2 p_1} * l_{p_1} + \\ & \varphi_{y_2 p_2} * l_{p_2} + \varphi_{y_3 p_1} * l_{p_1} + \varphi_{y_3 p_2} * l_{p_2} \\ & + \exp(l_{y_1}) * (e_{\text{sumcosy}1y_2} * (-\sin y_1) + f_{\text{sumsiny}1y_2} * \cos y_1 + g_{\text{difcosy}1y_3} * (-\sin y_1) + \\ & h_{\text{difsiny}1y_3} * \cos y_1) + \exp(l_{y_2}) * (e_{\text{sumcosy}1y_2} * (-\sin y_2) + \\ & f_{\text{sumsiny}1y_2} * \cos y_2 + g_{\text{difcosy}2y_3} * \sin y_2 + h_{\text{difsiny}2y_3} * (-\cos y_2)) \\ & + \exp(l_{y_3}) * (g_{\text{difcosy}1y_3} * \sin y_3 + g_{\text{difcosy}2y_3} * \sin y_3 + \\ & h_{\text{difsiny}1y_3} * (-\cos y_3) + h_{\text{difsiny}2y_3} * (-\cos y_3)); \end{aligned} \quad (\text{A.4})$$

*Specific Scale Economies:*

$$TC = \exp(\alpha_0 + \beta_{y1} * l_{y1} + \beta_{y2} * l_{y2} + \beta_{y3} * l_{y3} + \gamma_{p1} * l_{p1} + \gamma_{p2} * l_{p2} + \beta_{y1}^2 * l_{y1}^2 + \beta_{y2}^2 * l_{y2}^2 + \beta_{y3}^2 * l_{y3}^2 + \gamma_{p1}^2 * l_{p1}^2 + \gamma_{p2}^2 * l_{p2}^2 + \beta_{y1y2} * l_{y1}l_{y2} + \beta_{y1y3} * l_{y1}l_{y3} + \beta_{y2y2} * l_{y2}l_{y3} + \gamma_{p1p2} * l_{p1}l_{p2} + \varphi_{y1p1} * l_{y1}l_{p1} + \varphi_{y2p1} * l_{y2}l_{p1} + \varphi_{y3p1} * l_{y3}l_{p1} + \varphi_{y1p2} * l_{y1}l_{p2} + \varphi_{y2p2} * l_{y2}l_{p2} + \varphi_{y3p2} * l_{y3}l_{p2} + D[Austria] * Austria + D[Belgium] * Belgium + D[Denmark] * Denmark + D[France] * France + D[Germany] * Germany + D[Greece] * Greece + D[Ireland] * Ireland + D[Italy] * Italy + D[Luxembourg] * Luxembourg + D[Holland] * Holland + D[Portugal] * Portugal + D[Spain] * Spain + D[Sweden] * Sweden + D[UK] * UK + f_{sumsiny1y2} * sumsiny1y2 + e_{sumcosy1y2} * sumcosy1y2 + h_{difsiny1y3} * difsiny1y3 + h_{difsiny2y3} * difsiny2y3 + g_{difcosy1y3} * difcosy1y3 + g_{difcosy2y3} * difcosy2y3 + l_{sumsinp1p2} * sumsinp1p2 + i_{sumcosp1p2} * sumcosp1p2 + n_{difsinp1p2} * difsinp1p2 + m_{difcosp1p2} * difcosp1p2);$$

$$TC(y_{1=0}, p_k) = \exp(\alpha_0 + \beta_{y1} * \ln(\varepsilon) + \beta_{y2} * l_{y2} + \beta_{y3} * l_{y3} + \gamma_{p1} * l_{p1} + \gamma_{p2} * l_{p2} + \beta_{y1}^2 * (\ln(\varepsilon))^2 + \beta_{y2}^2 * l_{y2}^2 + \beta_{y3}^2 * l_{y3}^2 + \gamma_{p1}^2 * l_{p1}^2 + \gamma_{p2}^2 * l_{p2}^2 + \beta_{y1y2} * l_{y2} \ln(\varepsilon) + \beta_{y1y3} * l_{y3} \ln(\varepsilon) + \beta_{y2y2} * l_{y2}l_{y3} + \gamma_{p1p2} * l_{p1}l_{p2} + \varphi_{y1p1} * l_{p1} * \ln(\varepsilon) + \varphi_{y2p1} * l_{y2}l_{p1} + \varphi_{y3p1} * l_{y3}l_{p1} + \varphi_{y1p2} * \ln(\varepsilon)l_{p2} + \varphi_{y2p2} * l_{y2}l_{p2} + \varphi_{y3p2} * l_{y3}l_{p2} + D[Austria] * Austria + D[Belgium] * Belgium + D[Denmark] * Denmark + D[France] * France + D[Germany] * Germany + D[Greece] * Greece + D[Ireland] * Ireland + D[Italy] * Italy + D[Luxembourg] * Luxembourg + D[Holland] * Holland + D[Portugal] * Portugal + D[Spain] * Spain + D[Sweden] * Sweden + D[UK] * UK + f_{sumsiny1y2} * siny2 + e_{sumcosy1y2} * (1 + cosy2) + h_{difsiny1y3} * (-siny3) + h_{difsiny2y3} * difsiny2y3 + g_{difcosy1y3} * (1 - cosy3) + g_{difcosy2y3} * difcosy2y3 + l_{sumsinp1p2} * sumsinp1p2 + i_{sumcosp1p2} * sumcosp1p2 + n_{difsinp1p2} * difsinp1p2 + m_{difcosp1p2} * difcosp1p2);$$

$$SL_1 = (TC - TC(y_{1=0}, p_k)) / TC * (1 / \eta(y_1));$$

$$TC(y_{2=0}, p_k) = \exp(\alpha_0 + \beta_{y_1} * y_1 + \beta_{y_2} * \ln(\varepsilon) + \beta_{y_3} * y_3 + \gamma_{p_1} * p_1 + \gamma_{p_2} * p_2 + \beta_{y_1}^2 * y_1^2 + \beta_{y_2}^2 * \ln(\varepsilon)^2 + \beta_{y_3}^2 * y_3^2 + \gamma_{p_1}^2 * p_1^2 + \gamma_{p_2}^2 * p_2^2 + \beta_{y_1 y_2} * y_1 * \ln(\varepsilon) + \beta_{y_1 y_3} * y_1 * y_3 + \beta_{y_2 y_2} * \ln(\varepsilon) * y_3 + \gamma_{p_1 p_2} * p_1 * p_2 + \varphi_{y_1 p_1} * y_1 * p_1 + \varphi_{y_2 p_1} * \ln(\varepsilon) * p_1 + \varphi_{y_3 p_1} * y_3 * p_1 + \varphi_{y_1 p_2} * y_1 * p_2 + \varphi_{y_2 p_2} * \ln(\varepsilon) * p_2 + \varphi_{y_3 p_2} * y_3 * p_2 + f_{\text{sumsin}y_1 y_2} * \text{sin}y_1 + e_{\text{sumcos}y_1 y_2} * (1 + \text{cos}y_1) + h_{\text{difsiny}y_1 y_3} * \text{difsiny}y_1 y_3 + h_{\text{difsiny}y_2 y_3} * (-\text{sin}y_3) + g_{\text{difcos}y_1 y_3} * \text{difcos}y_1 y_3 + g_{\text{difcos}y_2 y_3} * (1 - \text{cos}y_3) + l_{\text{sumsin}p_1 p_2} * \text{sumsin}p_1 p_2 + i_{\text{sumcos}p_1 p_2} * \text{sumcos}p_1 p_2 + n_{\text{difsinp}p_1 p_2} * \text{difsinp}p_1 p_2 + m_{\text{difcos}p_1 p_2} * \text{difcos}p_1 p_2);$$

$$D[Austria] * Austria + D[Belgium] * Belgium + D[Denmark] * Denmark + D[France] * France + D[Germany] * Germany + D[Greece] * Greece + D[Ireland] * Ireland + D[Italy] * Italy + D[Luxembourg] * Luxembourg + D[Holland] * Holland + D[Portugal] * Portugal + D[Spain] * Spain + D[Sweden] * Sweden + D[UK] * UK$$

$$SL_2 = (TC - TC(y_{2=0}, p_k)) / TC * (1 / \eta(y_2));$$

$$TC(y_{3=0}, p_k) = \exp(\alpha_0 + \beta_{y_1} * y_1 + \beta_{y_2} * y_2 + \beta_{y_3} * \ln(\varepsilon) + \gamma_{p_1} * p_1 + \gamma_{p_2} * p_2 + \beta_{y_1}^2 * y_1^2 + \beta_{y_2}^2 * y_2^2 + \beta_{y_3}^2 * \ln(\varepsilon)^2 + \gamma_{p_1}^2 * p_1^2 + \gamma_{p_2}^2 * p_2^2 + \beta_{y_1 y_2} * y_1 * y_2 + \beta_{y_1 y_3} * y_1 * \ln(\varepsilon) + \beta_{y_2 y_2} * y_2 * \ln(\varepsilon) + \gamma_{p_1 p_2} * p_1 * p_2 + \varphi_{y_1 p_1} * y_1 * p_1 + \varphi_{y_2 p_1} * y_2 * p_1 + \varphi_{y_3 p_1} * \ln(\varepsilon) * p_1 + \varphi_{y_1 p_2} * y_1 * p_2 + \varphi_{y_2 p_2} * y_2 * p_2 + \varphi_{y_3 p_2} * \ln(\varepsilon) * p_2 + f_{\text{sumsin}y_1 y_2} * \text{sumsin}y_1 y_2 + e_{\text{sumcos}y_1 y_2} * \text{sumcos}y_1 y_2 + h_{\text{difsiny}y_1 y_3} * \text{sin}y_1 + h_{\text{difsiny}y_2 y_3} * \text{sin}y_2 + g_{\text{difcos}y_1 y_3} * (\text{cos}y_1 - 1) + g_{\text{difcos}y_2 y_3} * (\text{cos}y_2 - 1) + l_{\text{sumsin}p_1 p_2} * \text{sumsin}p_1 p_2 + i_{\text{sumcos}p_1 p_2} * \text{sumcos}p_1 p_2 + n_{\text{difsinp}p_1 p_2} * \text{difsinp}p_1 p_2 + m_{\text{difcos}p_1 p_2} * \text{difcos}p_1 p_2);$$

$$D[Austria] * Austria + D[Belgium] * Belgium + D[Denmark] * Denmark + D[France] * France + D[Germany] * Germany + D[Greece] * Greece + D[Ireland] * Ireland + D[Italy] * Italy + D[Luxembourg] * Luxembourg + D[Holland] * Holland + D[Portugal] * Portugal + D[Spain] * Spain + D[Sweden] * Sweden + D[UK] * UK$$

$$SL_3 = (TC - TC(y_3=0, p_k))/TC * (1/\eta(y_3)); \quad (A.11)$$

Scope economies:

$$TC_1(y_1, p_k) = \exp(\alpha_0 + \beta_{y_1} * y_1 + \beta_{y_2} * \ln(\varepsilon) + \beta_{y_3} * \ln(\varepsilon) + \gamma_{p_1} * p_1 + \gamma_{p_2} * p_2 + \beta_{y_1}^2 * y_1^2 + \beta_{y_2}^2 * \ln(\varepsilon)^2 + \beta_{y_3}^2 * \ln(\varepsilon)^2 + \gamma_{p_1}^2 * p_1^2 + \gamma_{p_2}^2 * p_2^2 + \beta_{y_1 y_2} * y_1 \ln(\varepsilon) + \beta_{y_1 y_3} * y_1 \ln(\varepsilon) + \beta_{y_2 y_2} * \ln(\varepsilon)^2 + \gamma_{p_1 p_2} * p_1 p_2 + \varphi_{y_1 p_1} * y_1 p_1 + \varphi_{y_2 p_1} * \ln(\varepsilon) p_1 + \varphi_{y_3 p_1} * \ln(\varepsilon) p_1 + \varphi_{y_1 p_2} * y_1 p_2 + \varphi_{y_2 p_2} * \ln(\varepsilon) p_2 + \varphi_{y_3 p_2} * \ln(\varepsilon) p_2 + D[Austria] * Austria + D[Belgium] * Belgium + D[Denmark] * Denmark + D[France] * France + D[Germany] * Germany + D[Greece] * Greece + D[Ireland] * Ireland + D[Italy] * Italy + D[Luxembourg] * Luxembourg + D[Holland] * Holland + D[Portugal] * Portugal + D[Spain] * Spain + D[Sweden] * Sweden + D[UK] * UK + f_{sumsiny_1 y_2} * siny_1 + e_{sumcosy_1 y_2} * (\cos y_1 + 1) + h_{difsiny_1 y_3} * siny_1 + g_{difcosy_1 y_3} * (\cos y_1 - 1) + l_{sumsin p_1 p_2} * \sumsin p_1 p_2 + i_{sumcos p_1 p_2} * \sumcosp_1 p_2 + n_{difsinsin p_1 p_2} * difsinsin p_1 p_2 + m_{difcos p_1 p_2} * difcosp_1 p_2); \quad (A.12)$$

$$TC_2(y_2, p_k) = \exp(\alpha_0 + \beta_{y_1} * \ln(\varepsilon) + \beta_{y_2} * y_2 + \beta_{y_3} * \ln(\varepsilon) + \gamma_{p_1} * p_1 + \gamma_{p_2} * p_2 + \beta_{y_1}^2 * \ln(\varepsilon)^2 + \beta_{y_2}^2 * y_2^2 + \beta_{y_3}^2 * \ln(\varepsilon)^2 + \gamma_{p_1}^2 * p_1^2 + \gamma_{p_2}^2 * p_2^2 + \beta_{y_1 y_2} * \ln(\varepsilon) y_2 + \beta_{y_1 y_3} * \ln(\varepsilon)^2 + \beta_{y_2 y_2} * y_2 \ln(\varepsilon) + \gamma_{p_1 p_2} * p_1 p_2 + \varphi_{y_1 p_1} * \ln(\varepsilon) p_1 + \varphi_{y_2 p_1} * y_2 p_1 + \varphi_{y_3 p_1} * \ln(\varepsilon) p_1 + \varphi_{y_1 p_2} * \ln(\varepsilon) p_2 + \varphi_{y_2 p_2} * y_2 p_2 + \varphi_{y_3 p_2} * \ln(\varepsilon) p_2 + D[Austria] * Austria + D[Belgium] * Belgium + D[Denmark] * Denmark + D[France] * France + D[Germany] * Germany + D[Greece] * Greece + D[Ireland] * Ireland + D[Italy] * Italy + D[Luxembourg] * Luxembourg + D[Holland] * Holland + D[Portugal] * Portugal + D[Spain] * Spain + D[Sweden] * Sweden + D[UK] * UK + f_{sumsiny_1 y_2} * (-siny_2) + e_{sumcosy_1 y_2} * (\cos y_2 + 1) + h_{difsiny_2 y_3} * siny_2 + g_{difcosy_2 y_3} * (\cos y_2 - 1) + l_{sumsin p_1 p_2} * \sumsin p_1 p_2 + i_{sumcos p_1 p_2} * \sumcosp_1 p_2 + n_{difsinsin p_1 p_2} * difsinsin p_1 p_2); \quad (A.13)$$

$$TC_2(y_2, p_k) = \exp(\alpha_0 + \beta_{y_1} * \ln(\varepsilon) + \beta_{y_2} * y_2 + \beta_{y_3} * \ln(\varepsilon) + \gamma_{p_1} * p_1 + \gamma_{p_2} * p_2 + \beta_{y_1}^2 * \ln(\varepsilon)^2 + \beta_{y_2}^2 * y_2^2 + \beta_{y_3}^2 * \ln(\varepsilon)^2 + \gamma_{p_1}^2 * p_1^2 + \gamma_{p_2}^2 * p_2^2 + \beta_{y_1 y_2} * \ln(\varepsilon) y_2 + \beta_{y_1 y_3} * \ln(\varepsilon)^2 + \beta_{y_2 y_2} * y_2 \ln(\varepsilon) + \gamma_{p_1 p_2} * p_1 p_2 + \varphi_{y_1 p_1} * \ln(\varepsilon) p_1 + \varphi_{y_2 p_1} * y_2 p_1 + \varphi_{y_3 p_1} * \ln(\varepsilon) p_1 + \varphi_{y_1 p_2} * \ln(\varepsilon) p_2 + \varphi_{y_2 p_2} * y_2 p_2 + \varphi_{y_3 p_2} * \ln(\varepsilon) p_2 + D[Austria] * Austria + D[Belgium] * Belgium + D[Denmark] * Denmark + D[France] * France + D[Germany] * Germany + D[Greece] * Greece + D[Ireland] * Ireland + D[Italy] * Italy + D[Luxembourg] * Luxembourg + D[Holland] * Holland + D[Portugal] * Portugal + D[Spain] * Spain + D[Sweden] * Sweden + D[UK] * UK + f_{sumsiny_1 y_2} * (-siny_2) + e_{sumcosy_1 y_2} * (\cos y_2 + 1) + h_{difsiny_2 y_3} * siny_2 + g_{difcosy_2 y_3} * (\cos y_2 - 1) + l_{sumsin p_1 p_2} * \sumsin p_1 p_2 + i_{sumcos p_1 p_2} * \sumcosp_1 p_2 + n_{difsinsin p_1 p_2} * difsinsin p_1 p_2); \quad (A.13)$$

$$TC_2(y_2, p_k) = \exp(\alpha_0 + \beta_{y_1} * \ln(\varepsilon) + \beta_{y_2} * y_2 + \beta_{y_3} * \ln(\varepsilon) + \gamma_{p_1} * p_1 + \gamma_{p_2} * p_2 + \beta_{y_1}^2 * \ln(\varepsilon)^2 + \beta_{y_2}^2 * y_2^2 + \beta_{y_3}^2 * \ln(\varepsilon)^2 + \gamma_{p_1}^2 * p_1^2 + \gamma_{p_2}^2 * p_2^2 + \beta_{y_1 y_2} * \ln(\varepsilon) y_2 + \beta_{y_1 y_3} * \ln(\varepsilon)^2 + \beta_{y_2 y_2} * y_2 \ln(\varepsilon) + \gamma_{p_1 p_2} * p_1 p_2 + \varphi_{y_1 p_1} * \ln(\varepsilon) p_1 + \varphi_{y_2 p_1} * y_2 p_1 + \varphi_{y_3 p_1} * \ln(\varepsilon) p_1 + \varphi_{y_1 p_2} * \ln(\varepsilon) p_2 + \varphi_{y_2 p_2} * y_2 p_2 + \varphi_{y_3 p_2} * \ln(\varepsilon) p_2 + D[Austria] * Austria + D[Belgium] * Belgium + D[Denmark] * Denmark + D[France] * France + D[Germany] * Germany + D[Greece] * Greece + D[Ireland] * Ireland + D[Italy] * Italy + D[Luxembourg] * Luxembourg + D[Holland] * Holland + D[Portugal] * Portugal + D[Spain] * Spain + D[Sweden] * Sweden + D[UK] * UK + f_{sumsiny_1 y_2} * (-siny_2) + e_{sumcosy_1 y_2} * (\cos y_2 + 1) + h_{difsiny_2 y_3} * siny_2 + g_{difcosy_2 y_3} * (\cos y_2 - 1) + l_{sumsin p_1 p_2} * \sumsin p_1 p_2 + i_{sumcos p_1 p_2} * \sumcosp_1 p_2 + n_{difsinsin p_1 p_2} * difsinsin p_1 p_2); \quad (A.13)$$

$$+ m_{\text{difcos } p_1 p_2} * \text{difcos } p_1 p_2);$$

$$TC_3(y_3, p_k) = \exp(\alpha_0 + \beta_{y_1} * \ln(\varepsilon) + \beta_{y_2} * \ln(\varepsilon) + \beta_{y_3} * y_3 + \gamma_{p_1} * p_1 + \gamma_{p_2} * p_2 + \beta_{y_1}^2 * \ln(\varepsilon)^2 + \beta_{y_2}^2 * \ln(\varepsilon)^2 + \beta_{y_3}^2 * y_3^2 + \gamma_{p_1}^2 * p_1^2 + \gamma_{p_2}^2 * p_2^2 + \beta_{y_1 y_2} * \ln(\varepsilon)^2 + \beta_{y_1 y_3} * \ln(\varepsilon) y_3 + \beta_{y_2 y_2} * \ln(\varepsilon) y_3 + \gamma_{p_1 p_2} * p_1 p_2 + \varphi_{y_1 p_1} * \ln(\varepsilon) p_1 + \varphi_{y_2 p_1} * \ln(\varepsilon) p_1 + \varphi_{y_3 p_1} * y_3 p_1 + \varphi_{y_1 p_2} * \ln(\varepsilon) p_2 + \varphi_{y_2 p_2} * \ln(\varepsilon) p_2 + \varphi_{y_3 p_2} * y_3 p_2 + D[Austria] * Austria + D[Belgium] * Belgium + D[Denmark] * Denmark + D[France] * France + D[Germany] * Germany + D[Greece] * Greece + D[Ireland] * Ireland + D[Italy] * Italy + D[Luxembourg] * Luxembourg + D[Holland] * Holland + D[Portugal] * Portugal + D[Spain] * Spain + D[Sweden] * Sweden + D[UK] * UK + e_{\text{sumcos } y_1 y_2} * 2 + h_{\text{difsiny } y_3} * (-\text{siny}_3) + h_{\text{difsiny } 2 y_3} * (-\text{siny}_3) + g_{\text{difcos } y_1 y_3} * (1 - \text{cos } y_3) + g_{\text{difcos } 2 y_3} * (1 - \text{cos } y_3) + l_{\text{sumsin } p_1 p_2} * \text{sumsin } p_1 p_2 + i_{\text{sumcos } p_1 p_2} * \text{sumcos } p_1 p_2 + n_{\text{difsiny } p_1 p_2} * \text{difsiny } p_1 p_2 + m_{\text{difcos } p_1 p_2} * \text{difcos } p_1 p_2);$$

$$P_{\text{scope}_1} = (TC_1(y_1, p_k) + TC(y_1=0, p_k) - TC) / TC; \quad (\text{A.15})$$

$$P_{\text{scope}_2} = (TC_2(y_2, p_k) + TC(y_2=0, p_k) - TC) / TC; \quad (\text{A.16})$$

$$P_{\text{scope}_3} = (TC_3(y_3, p_k) + TC(y_3=0, p_k) - TC) / TC; \quad (\text{A.17})$$

Overall Scope Economies:

$$\text{Scope} = (\sum_i TC_i(y_i, p_k) - TC) / TC; \quad (\text{A.18})$$

## Appendix B

In the tables below we report the scale and scope economies indicators for banks in the 15 European countries.

**Table B.1. Scale and scope economies indicators from the FF specification.**

FFF	<i>Rsce</i>	<i>z</i>	<i>SL</i> <sub>1</sub>	<i>z</i>	<i>SL</i> <sub>2</sub>	<i>z</i>	<i>SL</i> <sub>3</sub>	<i>z</i>	<i>Scope</i>	<i>z</i>	<i>Pscope</i> <sub>1</sub>	<i>z</i>	<i>Pscope</i> <sub>2</sub>	<i>z</i>	<i>Pscope</i> <sub>3</sub>	<i>z</i>
<b>EUROPA</b>	1,00	9,87	0,69	4,7	0,17	2,29	0,11	1,67	-0,87	-3,07	-0,87	-2,82	-0,94	-20,88	-0,64	-2,49
<b>Austria</b>	1,02	29,45	0,7	12,93	0,14	4,21	0,09	2,04	-0,92	-15,85	-0,93	-14,45	-0,93	-28,82	-0,67	-4,8
<b>Belgium</b>	1,00	20,14	0,74	5,07	0,14	3,02	0,09	1,47	-0,9	-7,03	-0,9	-6,43	-0,93	-28,64	-0,63	-2,97
<b>Denmark</b>	1,03	14,14	0,63	6,32	0,16	3,4	0,14	2,64	-0,95	-29,25	-0,96	-22	-0,92	-8,24	-0,77	-7,4
<b>Finland</b>	1,01	52,57	0,62	5,88	0,2	3,24	0,11	1,77	-0,94	-26,76	-0,94	-26,4	-0,96	-30,08	-0,69	-5,36
<b>France</b>	0,99	15,64	0,69	5,12	0,18	2,6	0,11	1,82	-0,82	-1,87	-0,83	-1,87	-0,93	-19,22	-0,61	-1,79
<b>Germany</b>	1,00	7,22	0,65	4,79	0,19	2,44	0,13	2,33	-0,88	-3,26	-0,88	-2,71	-0,94	-30,17	-0,7	-3,03
<b>Greek</b>	1,03	40,01	0,61	4,26	0,19	2,63	0,13	1,47	-0,92	-10,02	-0,91	-8,97	-0,95	-39,71	-0,69	-3,39
<b>Ireland</b>	0,97	18,68	0,7	6,52	0,17	3,68	0,12	2,47	-0,95	-51,58	-0,97	-39,72	-0,92	-27,7	-0,75	-10,91
<b>Italy</b>	1,00	11,86	0,64	5,89	0,2	3,37	0,11	1,73	-0,84	-3,39	-0,84	-3,33	-0,96	-37,58	-0,64	-2,56
<b>Luxembourg</b>	1,00	31,25	0,81	6,36	0,11	2,19	0,06	1,24	-0,93	-18,86	-0,94	-17,52	-0,92	-35,85	-0,57	-3,39
<b>Holland</b>	1,30	10,68	0,71	4,07	0,16	1,86	0,1	1,74	-0,8	-1,86	-0,81	-1,85	-0,92	-10,45	-0,61	-1,92
<b>Portugal</b>	1,03	28,85	0,68	5,69	0,16	2,77	0,09	1,46	-0,92	-12,74	-0,91	-11,41	-0,95	-30,44	-0,64	-4,1
<b>Spain</b>	1,03	9,64	0,7	4,99	0,16	2,45	0,09	1,56	-0,66	-1,43	-0,66	-1,42	-0,96	-40,94	-0,44	-1,1
<b>Sweden</b>	1,03	9,33	0,6	3	0,19	2,77	0,12	1,22	-0,84	-6,74	-0,78	-6,05	-0,96	-33,27	-0,67	-3,34
<b>UK</b>	1,00	5,79	0,64	4,15	0,21	2,15	0,11	1,82	-0,92	-7,66	-0,88	-3,18	-0,95	-27,29	-0,69	-3,65
<b>Size</b>																
<b>Small</b>	1,00	22,47	0,64	4,79	0,17	2,53	0,15	2,77	-0,97	-79,01	-0,99	-16,5	-0,91	-40,47	-0,79	-10,53
<b>Medium</b>	1,00	27,87	0,7	5,43	0,16	2,45	0,1	1,9	-0,95	-30,17	-0,96	-28,6	-0,93	-21,12	-0,7	-5,35
<b>Large</b>	1,00	10,01	0,69	3,27	0,19	1,67	0,07	0,98	-0,64	-1,34	-0,59	-1,18	-0,97	-22,85	-0,39	-1,12



**Table B.2. Scale and scope economies indicators from the TL specification.**

Translog	<i>R</i> <sub>sce</sub>	<i>z</i>	<i>SL</i> <sub>1</sub>	<i>z</i>	<i>SL</i> <sub>2</sub>	<i>z</i>	<i>SL</i> <sub>3</sub>	<i>z</i>	<i>Scope</i>	<i>z</i>	<i>P</i> <sub>scope1</sub>	<i>z</i>	<i>P</i> <sub>scope2</sub>	<i>z</i>	<i>P</i> <sub>scope3</sub>	<i>z</i>
<b>EUROPA</b>	1,08	11,45	1,49	6,61	1,63	5,23	0,84	0,23	-0,88	-10,30	-0,89	-8,65	-0,89	-23,95	-0,27	-0,54
<b>Austria</b>	1,08	12,65	1,46	11,52	1,90	38,54	0,82	0,22	-0,89	-23,64	-0,90	-19,59	-0,90	-28,75	-0,29	-0,99
<b>Belgium</b>	1,12	8,53	1,51	4,55	1,81	11,05	1,16	1,56	-0,89	-16,38	-0,89	-11,83	-0,89	-24,80	-0,31	-0,86
<b>Denmark</b>	1,12	9,70	1,65	5,29	1,72	9,17	0,62	1,12	-0,91	-29,98	-0,92	-20,27	-0,88	-26,61	-0,49	-2,54
<b>Finland</b>	1,03	15,89	1,54	7,95	1,01	2,50	0,90	1,34	-0,91	-22,37	-0,92	-21,19	-0,91	-24,86	-0,34	-1,42
<b>France</b>	1,07	11,48	1,47	8,27	1,60	4,79	0,65	1,04	-0,87	-7,29	-0,88	-7,01	-0,89	-24,84	-0,24	-0,37
<b>Germany</b>	1,06	12,28	1,51	8,43	1,59	5,73	0,64	1,46	-0,90	-11,03	-0,91	-8,50	-0,88	-23,17	-0,36	-0,74
<b>Greek</b>	1,10	22,04	1,58	7,56	1,73	7,55	0,83	0,90	-0,89	-12,42	-0,90	-12,00	-0,90	-31,21	-0,35	-0,92
<b>Ireland</b>	1,02	20,84	1,48	8,95	1,59	5,78	0,82	0,23	-0,92	-46,35	-0,94	-34,53	-0,88	-21,71	-0,48	-3,06
<b>Italy</b>	1,08	18,00	1,57	8,63	1,58	3,40	0,67	1,63	-0,87	-9,77	-0,88	-9,30	-0,91	-23,02	-0,17	-0,31
<b>Luxembourg</b>	1,10	15,10	1,34	9,26	1,59	4,15	1,12	1,29	-0,88	-17,55	-0,89	-15,71	-0,88	-34,68	-0,30	-1,26
<b>Holland</b>	1,04	15,58	1,44	6,77	1,65	6,72	0,78	2,06	-0,88	-9,39	-0,89	-8,95	-0,90	-23,41	-0,18	-0,26
<b>Portugal</b>	1,08	16,38	1,51	5,88	1,73	6,48	0,69	1,07	-0,89	-17,99	-0,89	-17,80	-0,91	-37,98	-0,26	-0,88
<b>Spain</b>	1,07	19,80	1,46	7,37	1,86	12,70	0,61	1,04	-0,81	-5,33	-0,81	-5,28	-0,92	-33,63	0,14	0,18
<b>Sweden</b>	1,18	5,03	1,83	2,79	1,67	5,97	1,70	2,88	-0,86	-13,21	-0,82	-10,14	-0,94	-29,82	-0,15	-0,35
<b>UK</b>	1,07	8,98	1,53	7,56	1,71	6,75	1,12	1,46	-0,91	-22,04	-0,91	-7,52	-0,90	-20,88	-0,34	-0,89
<b>Size</b>																
<b>Small</b>	1,11	12,34	1,57	8,26	1,54	5,13	0,40	1,49	-0,92	-61,88	-0,95	-51,26	-0,85	-40,91	-0,59	-6,67
<b>Medium</b>	1,08	14,56	1,47	7,86	1,72	5,76	0,72	1,24	-0,91	-21,45	-0,92	-19,80	-0,89	-45,33	-0,42	-2,39
<b>Large</b>	1,03	9,06	1,45	4,95	1,92	13,97	1,32	1,67	-0,80	-6,56	-0,79	-5,64	-0,94	-48,82	-0,29	-0,43

**Table B.3. Scale and scope economies indicators from the Box-Cox specification.**

BoxCox	<i>R</i> <sub>sce</sub>	<i>z</i>	<i>SL</i> <sub>1</sub>	<i>z</i>	<i>SL</i> <sub>2</sub>	<i>z</i>	<i>SL</i> <sub>3</sub>	<i>z</i>	<i>Scope</i>	<i>z</i>	<i>P</i> <sub>scope1</sub>	<i>z</i>	<i>P</i> <sub>scope2</sub>	<i>z</i>	<i>P</i> <sub>scope3</sub>	<i>z</i>
<b>EUROPA</b>	1,34	2,52	0,39	1,10	0,11	0,92	0,03	0,71	0,05	0,11	-0,04	-0,12	-0,17	-0,72	-0,08	-0,31
<b>Austria</b>	1,26	2,75	0,40	1,32	0,12	1,48	0,02	1,46	-0,03	-0,08	-0,13	-0,50	-0,22	-0,89	-0,08	-0,33
<b>Belgium</b>	1,28	2,16	0,62	1,00	0,05	1,77	0,02	1,70	0,12	0,34	-0,01	-0,02	-0,10	-0,46	-0,07	-0,26
<b>Denmark</b>	1,43	2,09	0,39	0,82	0,08	1,43	0,04	0,93	0,10	0,23	-0,05	-0,16	-0,13	-0,51	-0,03	-0,12
<b>Finland</b>	0,92	1,95	0,47	2,06	0,24	1,25	0,09	0,77	-0,36	-0,81	-0,31	-1,19	-0,42	-1,50	-0,31	-1,02
<b>France</b>	1,48	2,86	0,33	0,95	0,10	1,36	0,02	1,34	0,11	0,30	-0,01	-0,04	-0,13	-0,65	-0,01	-0,04
<b>Germany</b>	1,53	2,98	0,28	0,87	0,13	0,65	0,03	1,10	0,15	0,36	0,04	0,09	-0,12	-0,53	-0,03	-0,12
<b>Greek</b>	1,29	2,17	0,45	1,18	0,12	1,28	0,05	0,51	-0,09	-0,23	-0,15	-0,60	-0,23	-1,03	-0,14	-0,55
<b>Ireland</b>	1,49	2,94	0,25	1,26	0,12	1,48	0,02	0,73	0,08	0,21	-0,05	-0,21	-0,15	-0,65	-0,01	-0,04
<b>Italy</b>	1,18	2,54	0,44	1,46	0,12	1,40	0,04	1,25	0,02	0,04	0,01	0,02	-0,24	-0,99	-0,21	-0,79
<b>Luxembourg</b>	1,37	2,76	0,41	1,38	0,05	1,41	0,01	1,10	0,13	0,37	-0,04	-0,16	-0,10	-0,49	0,03	0,18
<b>Holland</b>	1,50	3,03	0,33	1,19	0,11	1,22	0,02	1,16	0,07	0,17	-0,02	-0,04	-0,16	-0,79	-0,06	-0,27
<b>Portugal</b>	1,08	2,33	0,55	1,73	0,14	1,21	0,04	0,50	-0,23	-0,68	-0,25	-1,21	-0,32	-1,58	-0,21	-0,92
<b>Spain</b>	1,03	2,85	0,67	1,76	0,13	1,26	0,04	0,86	-0,10	-0,16	-0,09	-0,16	-0,30	-1,48	-0,24	-1,43
<b>Sweden</b>	0,81	2,19	0,89	1,71	0,21	0,91	0,12	1,42	-0,24	-0,39	-0,17	-0,30	-0,29	-0,65	-0,51	-1,78
<b>UK</b>	1,27	2,38	0,30	1,31	0,15	1,00	0,05	0,97	-0,01	-0,01	-0,07	-0,19	-0,20	-0,71	-0,11	-0,36
<b>Size</b>																
<b>Small</b>	2,14	10,61	0,09	2,48	0,05	1,70	0,02	1,36	0,49	3,15	0,20	2,47	0,09	1,07	0,20	2,31
<b>Medium</b>	1,35	3,74	0,32	1,88	0,10	1,60	0,03	0,99	-0,03	-0,10	-0,12	-0,69	-0,19	-1,21	-0,07	-0,43
<b>Large</b>	0,80	3,30	0,84	2,38	0,19	1,31	0,08	0,71	-0,26	-0,44	-0,15	-0,23	-0,40	-1,74	-0,41	-2,46