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On Bundling in Insurance Markets*

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Abstract. This paper analyzes the welfare consequences of bundling different risks in one insurance contract in markets where adverse selection is important. This question is addressed in the context of a competitive insurance model a la Rothschild and Stiglitz (1976) with two sources of risk. Accordingly, there are four possible types of individuals and many incentive compatibility constraints to be considered. We show that the effect of bundling on these incentive compatibility constraints is such that bundling always yields a welfare improvement, and this result only holds when all four types have strictly positive shares in the population. Due to the competition between insurance companies, these benefits accrue to consumers who potentially have fewer contracts to choose from, but benefit from the better sorting possibilities due to bundling.

Key Words: insurance, adverse selection, multiple-risks, bundling.

JEL Classification: G22, D82.

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1. Introduction

Bundling of different insurances is typically observed across insurance markets. Bundling can take two different forms. In the first form, a single insurance contract covers many different types of risk and each risk cannot be insured against in a separate contract. For example, many health or travel insurances cover a wide variety of accidents that may happen and it is not possible to insure oneself against only one of these accidents occurring. Typically, one cannot take a travel insurance against the loss of a photo camera only. Similarly, most basic health insurance contracts offer coverage against many possible diseases and it is impossible to break up such a contract and insure only against, say, heart diseases.

In the second form, insurance companies offer discounts to consumers who take out multiple separately sold insurance. For example, if a consumer takes a health insurance, a car insurance, and a travel insurance from the same insurance company, a certain discount is provided. In this paper, we focus on the welfare consequences of the first more extreme form of bundling contracts. One possible explanation for this type of bundling of contracts to exist is related to the multiplication of the transaction costs handling all these different contracts separately. We will provide, however, an alternative explanation by abstracting completely away from these costs.

A bundled insurance contract offers insurance against multiple risks at a certain premium, possibly with a deductible, which depends on a risk or combination of risks that has actually materialized. Apart from the obvious fact that the deductibles on individual risks and the premium for the bundle may be different from the deductibles and the sum of the premiums on individual contracts, two potentially important effects arise from bundling different insurance contracts. First, under a bundled contract, the deductible that applies if multiple risks materialize does not need to be equal to the sum of the deductibles that apply when the individual risks materialize. For example, a bundled health insurance contract may stipulate that the first visit to a general practitioner (family doctor) and a first visit to a physiotherapist are not covered, unless the general practitioner prescribes going to the physiotherapist. Second, with single-risk contracts, consumers have generally more choice as they can combine different single-risk insurance contracts and the set of insurance contracts offered for each individual risk can potentially be as large as the set of bundled multi-risk contracts. This is because insurance companies may be tempted to offer different single-risk contracts to individuals who differ only with respect to another risk dimension. As we will see, this is especially important when companies assess the profitability of offering non-equilibrium insurance contracts. Thus, in insurance markets bundling has important implications for the incentive compatibility constraints that are binding in equilibrium.

There is a large literature on the welfare effects of bundling in ordinary commodity markets. This literature (see, *e.g.*, the seminal paper of Adams and Yellen, 1976) considers markets where firms have some form of market power and in fact most literature considers whether a monopolist can leverage its market power in one market to gain market power in another, related market. The general consensus seems to be that bundling in situations where at least one market is perfectly competitive does not yield any benefits to the seller (at least if there are no cost advantages of joint production) as one product is always sold at marginal cost (*cf.*, Whinston, 1990).

In this paper, we study bundling in insurance markets. To underline the difference with ordinary markets we consider perfectly competitive markets and build on the seminal research by Rothschild and Stiglitz (1977). Unlike bundling in product markets, bundling in perfectly competitive insurance markets is feasible because of the effects bundling may have on the incentive compatibility constraints mentioned above. In fact, we will show that in general, bundling always leads to a Pareto-superior allocation compared to the no bundling case. Because of the assumption of perfect competition, it is immediate that consumers benefit from the bundling of insurance products.

The model we use is based on the Rothschild and Stiglitz framework. We consider two possible risky events and for each event, consumers can be of two possible types, high-risk and low-risk. This means that in principle there are four different types of individuals in our model, a type which has high-risks of both events occurring, a type which has low-risks of both events occurring, and two types which are low-risk for one event and are high-risk for the other event. An insurance contract is a tuple consisting of a premium and three deductibles (possibly equal to zero): a deductible if only the first event occurs, a deductible if only the second event occurs, and a deductible if both events occur. In this setting, we analyze the equilibrium contracts when insurance companies offer bundled contracts and when they offer only single-risk contracts.

We arrive at the following results. A first result says that individuals that have high risk of both events occurring get full insurance in any equilibrium. This follows from standard considerations based on Rothschild and Stiglitz, where the high-risk individual also receives full insurance. Second, if there are just three types of individuals in the population and no individual has a low probability of both events occurring, then equilibria where only single-risk contracts offered can perform equally well as equilibria where bundled contracts are offered. This result may be of independent interest, but should especially be seen as a stepping stone to the last result that says that when all four types are present, equilibria with single-risk contracts always perform strictly worse than equilibria with multi-risk contracts.

As far as we know, only one other paper has investigated the issue of bundling in insurance markets under adverse selection,[1] namely Fluet and Pannequin (1997). They consider a world like ours with two possible risky events and two risk types per event, but restrict the analysis to the special situation where there are only two types of individuals in the population. Under positive correlation, an individual that is a high-risk type for one event is also a high-risk type for the second event, while under negative correlation, an individual that is a high-risk type for one event is a low-risk type for the second event. Their main result says that under bundling in the equilibrium under negative correlation, the low-risk type with respect to a particular source of risk does not necessarily obtain only partial coverage against that particular risk. Because of the restriction to two types, Fluet and Pannequin (1997) have only one relevant incentive compatibility constraint to satisfy. The main point of the present paper is that bundling affects the nature of the incentive compatibility constraints in such a way that insurance companies can screen more efficiently and that this results in the existence of more efficient equilibria. This is impossible in the simpler framework of Fluet and Pannequin (1997) where only two types exist.

The rest of the paper is organized as follows. Section 2 presents the model and provides the equilibrium definitions under both single-risk contracts and bundled multi-risk contracts. The equilibrium analysis and welfare comparisons are given in section 3. Section 4 concludes and the Appendix contains two lemmas that are used in the proofs of propositions.

2. The Model

We consider a population of risk averse individuals who possess the same state independent strictly concave and increasing utility function $u(m)$. Individuals are endowed with some income level, which we normalize to 1, and are subject to two uncorrelated risks, or binary

[1] A recent paper by Laux (2004) considers bundling in insurance markets from a moral hazard perspective. This perspective may be more relevant to the multiline insurance contracts one recently may find in business-to-business relationships (see, also Shimpi 2001).

lotteries, $r \in R \equiv \{1,2\}$. With respect to each risk *r* we follow Rothschild and Stiglitz (1976) and assume that individuals come into two types. An individual of type $i_r \in I_r \equiv \{L, H\}$ is characterized by a probability of an accident $q_r^{i_r}$, $0 < q_r^L < q_r^H < 1$, in which case he incurs a loss of $e_r \in (0,1)$, $e_1 + e_2 < 1$. Therefore, with respect to both risks, individuals come into four types, which we denote by *i*:

$$i \equiv i_1 i_2 \in I \equiv \{LL, HL, LH, HH\},$$

so that the first symbol i_1 refers to an individual's type with respect to risk one, and the second symbol i_2 refers to his type with respect to risk two.

Depending on which of the two risky events occur one of four states of the world materializes, which we denote by *s*:

$$s \equiv (s_1, s_2) \in S \equiv \{(0,0), (1,0), (0,1), (1,1)\}.$$

Here, $s_r = 1$ corresponds to a state in which risk *r* results in an accident, and $s_r = 0$ corresponds to a state in which risk *r* does not result in an accident. Individuals' loss in state *s* is denoted by e_s. Thus, we use subscripts to denote risks and states, and superscripts to denote individuals' types with respect to risks.

Due to statistical independence, an individual of type $i \in I$ ends up in state $(0,0)$ with probability $q^i_{(0,0)} = (1 - q_1^{i_1})(1 - q_2^{i_2})$, in state $(0,1)$ with probability $q^i_{(0,1)} = (1 - q_1^{i_1})q_2^{i_2}$, in state $(1,0)$ with probability $q^i_{(1,0)} = q_1^{i_1}(1 - q_2^{i_2})$, and in state $(1,1)$ with probability $q^i_{(1,1)} = q_1^{i_1} q_2^{i_2}$. Table 1 below presents accidental probabilities for all four types and expenditures in all four states of the world.

State	**States' probabilities for types**				**Losses**
s	q_s^{HH}	q_s^{HL}	q_s^{LH}	q_s^{LL}	e_s
$(0,0)$	$(1-q_1^H)(1-q_2^H)$	$(1-q_1^H)(1-q_2^L)$	$(1-q_1^L)(1-q_2^H)$	$(1-q_1^L)(1-q_2^L)$	0
$(0,1)$	$(1-q_1^H)q_2^H$	$(1-q_1^H)q_2^L$	$(1-q_1^L)q_2^H$	$(1-q_1^L)q_2^L$	e_2
$(1,0)$	$q_1^H(1-q_2^H)$	$q_1^H(1-q_2^L)$	$q_1^L(1-q_2^H)$	$q_1^L(1-q_2^L)$	e_1
$(1,1)$	$q_1^H q_2^H$	$q_1^H q_2^L$	$q_1^L q_2^H$	$q_1^L q_2^L$	$e_1 + e_2$

Table 1. Accidental probabilities and losses of all types in different states.

Accidental probabilities q_s^i of an individual are exogenously fixed and private information of the individual. We denote the *ex-ante* shares of individuals' types in the whole population by α^i, where $\alpha^i > 0$ and $\sum_{i \in I} \alpha^i = 1$. It is worth to note that although risks are statistically independent, we do not assume the independence of individuals' types with respect to two risks, *i.e.*, we impose no further restrictions on the values of α^i.

The supply side of the market consists of a number of competing risk-neutral and profit-maximizing insurance companies. These companies are not able (or, not allowed) to discriminate between different types of individuals. Each insurer offers an insurance contract[2] $\Theta \equiv \left(P, D_{(1,0)}, D_{(0,1)}, D_{(1,1)}\right)$ which consists of a premium $P \in (0,1)$ that individuals pay upfront, and deductibles D_s such that in case of an accident, *i.e.*, in a state $s \in S \setminus (0,0)$, an insured individual receives his loss net of the deductible D_s from the insurance company. Table 2 summarizes individual *i*'s wealth and utility levels in all states with and without insurance.

State	**Probability**	**Wealth with**		**Utility with**	
s	q_s^i	no insurance	insurance	no insurance	insurance
$(0,0)$	$q_{(0,0)}^i$	1	$1-P$	$u(1)$	$u(1-P)$
$(0,1)$	$q_{(0,1)}^i$	$1-e_2$	$1-P-D_{(0,1)}$	$u(1-e_2)$	$u(1-P-D_{(0,1)})$
$(1,0)$	$q_{(1,0)}^i$	$1-e_1$	$1-P-D_{(1,0)}$	$u(1-e_1)$	$u(1-P-D_{(1,0)})$
$(1,1)$	$q_{(1,1)}^i$	$1-e_1-e_2$	$1-P-D_{(1,1)}$	$u(1-e_1-e_2)$	$u(1-P-D_{(1,1)})$

Table 2. Accidental probabilities, wealth and utility of type i individual in different states.

Depending on the contexts, we either restrict insurance companies to sell insurance contracts against each risk separately and independently, or allow them to offer multiple-risk insurance contracts. In the latter case, the (multi-risk) contract takes its general form:

$$\Theta \equiv \left(P, D_{(1,0)}, D_{(0,1)}, D_{(1,1)}\right).$$

By $\Theta^0 \equiv (0, e_1, e_2, e_1 + e_2)$ we denote an artificial contract, which provides no insurance against both risks. In what follows, we implicitly assume that Θ^0 is always offered.

In the former, single-risk case, a contract against risk *r* will be $\Theta_r = (P_r, D_r)$, where P_r is the price and D_r is the deductible in case risk *r* results in a loss. For comparison reasons, we will also refer to a single-risk contract Θ_r as follows:

$$\Theta_1 \equiv (P_1, D_1, e_2, D_1 + e_2) \text{ and } \Theta_2 \equiv (P_2, e_1, D_2, e_1 + D_2).$$

By $\Theta^S = \langle \Theta_1, \Theta_2 \rangle$ we denote a collection of two single-risk contracts, which insures an individual against both risks:

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[2] We do not need to restrict insurance companies to offer only one contract. As in the classical Bertrand (1883) price competition model, two competing multi-contract insurance companies will ensure that the considered insurance market is perfectly competitive.

$$\Theta^S \equiv (P_1 + P_2, D_1, D_2, D_1 + D_2).$$

Hence, one important difference between single-risk and multi-risk contracts is that the latter do not need to satisfy the additivity constraint $D_{(1,0)} + D_{(0,1)} = D_{(1,1)}$.

Formally defining $D_{(0,0)} \equiv 0$ allows us to write the *ex-ante* expected utility of an individual of type *i* who buys a (single-risk or multi-risk) contract in the following compact form:

$$U^i(\Theta) = \sum_s q_s^i u(1 - P - D_s).$$

An insurance company that sells a contract Θ to an individual of type *i* gets an expected profit (here after simply denoted by profit) of

$$\pi^i(\Theta) = P - \sum_s q_s^i (e_s - D_s).$$

Average per-consumer profit from contract Θ depends on the shares α_Θ^i of all four types amongst those who buy Θ:

$$\pi(\Theta) \equiv \sum_i \alpha_\Theta^i \pi^i(\Theta) = P - \sum_{i,s} \alpha_\Theta^i q_s^i (e_s - D_s).$$

Let Σ^M be the set of all feasible multi-risk insurance contracts. The formal definition of a multi-risk competitive Nash equilibrium, *i.e.*, the competitive Nash equilibrium when firms are allowed to offer multi-risk insurance contracts, is as follows.

Definition 1. A multi-risk competitive Nash equilibrium is a subset of insurance contracts $\Psi^M \subset \Sigma^M$ present in the market satisfying the following conditions:

a) Each individual chooses an insurance contract that maximizes his expected utility, *i.e.*, each type *i* chooses a contract $\Theta^{i,M} \in \arg\max_{\Theta \in \Psi^M} U^i(\Theta)$, for all $i \in I$.

b) Each equilibrium contract offered by insurance companies is bought by at least one individual, *i.e.*, $\Psi^M = \{\Theta^0\} \cup \bigcup_{i \in I} \{\Theta^{i,M}\}$.

c) Each equilibrium contract yields nonnegative profit to an insurer, *i.e.*, $\pi(\Theta^{i,M}) \geq 0$ for all $\Theta^M \in \Psi^M$.

d) No insurance company can benefit by unilaterally offering a different insurance contract, *i.e.*, if there exists a contract $\Theta^* \in \Sigma^M \setminus \Psi^M$ such that $U^i(\Theta^*) > U^i(\Theta^{i,M})$ for some $i \in I$ then $\pi(\Theta^*) \leq 0$.

Parts (a), (c) and (d) are part of any standard definition of equilibrium. In part (a), if an individual prefers to remain uninsured, he chooses the contract Θ^0 which is a part of any equilibrium as stated in part (b). Part (b) is introduced in order to get rid of a multiplicity of uninteresting equilibria, in which some very unfavorable insurance contacts are offered but not sold. We denote individuals' equilibrium utilities by $W^{i,M} \equiv U^i\left(\Theta^{i,M}\right)$.

In a single-risk environment, Definition 1 has to be modified. First, individuals must choose pairs of single-risk insurance contracts. Second, an equilibrium must be immunized to deviations where a firm deviates in one insurance market only, and to deviations where a firm deviates in both insurance markets. Let $\Sigma^S \subset \Sigma^M$ be the set of all feasible single-risk insurance contracts. The formal definition of a single-risk competitive Nash equilibrium, *i.e.*, the competitive Nash equilibrium when firms are only allowed to offer single-risk insurance contracts, is as follows.

Definition 2. A single-risk competitive Nash equilibrium is a subset of single-risk insurance contracts $\Psi^S \subset \Sigma^S$ present in the market satisfying the following conditions:

a) Each individual chooses a pair of insurance contracts that maximizes his expected utility, *i.e.*, each type *i* chooses a pair of contracts $\Theta^{i,S} \equiv \left\langle \Theta_1^i, \Theta_2^i \right\rangle \in \arg \max_{\Theta_1^i, \Theta_2^i \in \Psi^S} U^i\left(\left\langle \Theta_1^i, \Theta_2^i \right\rangle\right)$, for all $i \in I$.

b) Each equilibrium contract offered by insurance companies is bought by at least one individual, *i.e.*, $\Psi^S = \left\{\Theta^0\right\} \cup \bigcup_{\substack{i \in I \\ r \in R}} \left\{\Theta_r^i\right\}$.

c) Each equilibrium contract yields nonnegative profit to an insurer, *i.e.*, $\pi\left(\Theta_r^i\right) \geq 0$ for all $\Theta_r^i \in \Psi^S$.

d) No insurance company can benefit by unilaterally offering a different insurance contract, *i.e.*, if there exists a contract $\Theta_{r^*}^* \in \Sigma^S \setminus \Psi^S$ against risk $r^* \in R$ such that $U^i\left(\left\langle \Theta_{r^*}^*, \widetilde{\Theta}_{R \setminus r^*} \right\rangle\right) > U^i\left(\Theta^{i,S}\right)$ for some $i \in I$ and some contract $\widetilde{\Theta}_{R \setminus r^*} \in \Psi^S$ against the complementary risk $R \setminus r^*$, then it must be that $\pi\left(\Theta_{r^*}^*\right) \leq 0$.

e) No insurance company can benefit by unilaterally offering a pair of insurance contracts, *i.e.*, if there exist two contracts Θ_1^*, $\Theta_2^* \in \Sigma^S \setminus \Psi^S$ such that $U^i\left(\left\langle \Theta_1^*, \Theta_2^* \right\rangle\right) > U^i\left(\Theta^{i,S}\right)$ for some $i \in I$, then it must be that $\pi\left(\Theta_1^*\right) + \pi\left(\Theta_2^*\right) \leq 0$.

In equilibrium, an individual of type *i* chooses a contract Θ_1^i against risk 1 and a contract Θ_2^i against risk 2. In part (a) of Definition 2 we allow that, for example, $\Theta_1^{HL} \neq \Theta_1^{HH}$, *i.e.*, that different types choose different contracts against risk $r = 1$ even if they are of the same type i_1 with respect to that risk. This implies that the set of single-risk equilibrium insurance contracts can be larger (up to 8 contracts in total, 4 contracts for each risk) than the set of multi-risk equilibrium insurance contracts can be (up to 4 contracts in total). Thus, individuals have potentially more choices in a single-risk equilibrium than in a multi-risk equilibrium. This is the second important difference between multi-risk and single-risk insurance contracts.

We require in part (c) that each single-risk contract makes no losses. In part (d), we explicitly state that if one company deviates from the equilibrium by offering a contract $\Theta_{r^*}^*$ against risk r^*, individuals are allowed to combine this contract with any other existing (equilibrium) contract $\Theta_{R \setminus r^*}^*$ against the other risk. Thus, a *single contract* $\Theta_{r^*}^*$ results in *multiple pairs* of contracts from which consumers may choose. In part (e), we consider multi-contract deviations, in which a deviating insurance company offers two contracts Θ_1^* and Θ_2^* simultaneously.

Standard arguments rule out any *pooling* contract to be a Nash equilibrium. For pairs of single-risk contracts $\Theta^S = (\Theta_1, \Theta_2)$, the argument is given by Rothschild and Stiglitz (1976).[3] For multi-risk contracts, a similar argument generically holds true: for any (partially) pooling contract, there exists a contract that marginally differs from it in its price and only one deductible in such a way that only the type with the lowest expected loss prefers the latter new contract. This makes the former pooling contract unprofitable and, at the same time, ensures that the deviation yields strictly positive profit.

On the other hand, a *separating* Nash equilibrium $\Psi_{sep} = \{\Theta^{LL}, \Theta^{HL}, \Theta^{LH}, \Theta^{HH}\}$, which involves four contracts, one for each type, may not exist if there exists a *profitable* pooling contract that provides a higher utility level to either of the types. In what follows, we always assume that the shares of types in the population are such that pooling contracts are always inferior to separating contracts, in both single-risk and multi-risk settings. Due to the same reasoning as in Rothschild and Stiglitz (1976), this will be the case when the share of the most risky type in any given pooling contract is sufficiently large (close to one). If this is the case,

[3] The argument, however, does not forbid equilibria in which, *e.g.*, types $i \in \{HL, HH\}$ buy the same contract Θ_1^H. Thus, the term "pooling" in this context refers to a pair of single-risk contracts.

the same inequality as the incentive compatibility constraint will guarantee that the less risky types will not be attracted by such a pooling contract.[4] Similar to the multi-risk case, we denote individuals' equilibrium utilities by $W^{i,S} \equiv U^i\left(\Theta^{i,S}\right)$.

3. Analysis

We begin the analysis of the model by showing that the type *HH* in a competitive equilibrium gets full insurance against both risks in both single-risk and multi-risk settings.

Proposition 1. In a competitive equilibrium, type *HH* gets full insurance in both single-risk and multi-risk settings, *i.e.*, $\Theta^{HH,M} = \Theta^{HH,S} = \left(P^{HH},0,0,0\right)$ with $P^{HH} = e_1 q_1^H + e_2 q_2^H$.

Proof. The proof follows from the fact that *HH* is the most risky and, therefore, $\Theta^{HH,M}$ must provide the highest possible utility. If it were not the case, offering more insurance at fair price to type *HH* than in contract $\Theta^{HH,M}$ would have been a profitable deviation. Hence, it must be that $\Theta^{HH,M} = \arg\max_{\Theta\in\Sigma^M} U^{HH}(\Theta)$, subject to the profitability condition

$$0 \le \pi\left(\Theta^{HH,M}\right) = \pi^{HH}\left(\Theta^{HH,M}\right) = P - \sum_s q_s^{HH}\left(e_s - D_s\right).$$

Solving the optimization problem yields that the zero-profit condition binds, *i.e.*, $P = \sum_s q_s^{HH}\left(e_s - D_s\right)$, and that $\Theta^{HH,M} = \left(P^{HH},0,0,0\right)$ where $P^{HH} = e_1 q_1^H + e_2 q_2^H$.

It is seen that $\Theta^{HH,M}$ satisfies the additivity constraint, *i.e.*, it can be written as $\Theta^{HH,M} = \left\langle \Theta_1^{H,S}, \Theta_2^{H,S} \right\rangle$, where $\Theta_r^{H,S} = \left(P_r^{H,S}, D_r^{H,S}\right) = \left(e_r q_r^H, 0\right)$. Therefore, $\Theta^{HH,M}$ can be implemented as a pair of single-risk contracts, *i.e.*, $\Theta^{HH,S} = \Theta^{HH,M}$, where $\Theta_r^H = \left(e_r q_r^H, 0\right)$. ■

In accordance with Proposition 1, the most risky type *HH* gets full insurance, as in Rothschild and Stiglitz (1976). This implies that no other contract can provide full insurance at a lower price than the contract $\Theta^{HH,M}$. Therefore, any other type of individual will not receive full insurance under any circumstance, and will receive at most partial insurance.[5]

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[4] Formally, we assume that ratios α^{LL}/α^{LH}, α^{LL}/α^{HL}, α^{LH}/α^{HH}, and α^{HL}/α^{HH} are sufficiently small.

[5] Due to the restriction to two types, a type different from type *HH* might receive full insurance in Fluet and Pannequin (1997). This, however, only happens under what they call negative correlation, when the type *HH* does not exist in the population, violating our assumption that $\alpha^{HH} > 0$.

In order to find the equilibrium contract $\Theta^{i,M}$ for a type $i \in \{LL, HL, LH\}$, one must solve the following optimization problem:

$$\Theta^{i,M} = \arg\max_{\Theta \in \Sigma^M} U^i(\Theta),$$

subject to three incentive compatibility constraints

$$U^i(\Theta^{i,M}) \geq U^i(\Theta^{j,M}), \ j \in I \setminus i,$$

and one zero-profit constraint

$$P = \sum_s q_s^i (e_s - D_s).$$

The solutions to the three optimization problems for all $i \in \{LL, HL, LH\}$, together with $\Theta^{HH,M}$ is the only candidate for the multi-risk competitive Nash equilibrium. In order to simplify notation, we will refer to the incentive compatibility constraint (ICC) $U^i(\Theta^{i,M}) \geq U^i(\Theta^{j,M})$ as to either $i \triangleright j$ or $i \triangleright \Theta^j$. In order to avoid confusion, we will also refer to $i \triangleright j$ as to $i \overset{S}{\triangleright} j$ in single-risk settings, and as to $i \overset{M}{\triangleright} j$ in multi-risk settings.

In equilibrium, depending on the values of parameters different ICC's will bind. In order to further characterize a separating Nash equilibrium Ψ_{sep}, and to analyze which ICC's bind, we temporary assume that $\alpha^{LL} = 0$ so that there are no individuals of type *LL* in the population and the contract Θ^{LL} does not need to be offered. To distinguish the case $\alpha^{LL} = 0$ from the general case $\alpha^{LL} > 0$, we use underlined notation for the $\alpha^{LL} = 0$ case.

When $\alpha^{LL} = 0$, a multi-risk competitive Nash equilibrium $\underline{\Psi}_{sep}^M$ is unique and can easily be derived. The analysis of $\underline{\Psi}_{sep}^M$ shows that for a large class of utility functions, single-risk contracts perform equally well from the social welfare point of view as multi-risk contracts do. Relaxing this temporary assumption allows us to understand better which ICC's bind, and why multi-risk contracts are strictly welfare-superior to single-risk contracts in the presence of *LL* type. Thus, our assumption $\alpha^{LL} = 0$ also serves didactical purposes.

Equilibrium without LL-type

When only types *HH*, *HL*, and *LH* are present in the population, a multi-risk competitive Nash equilibrium consists of 3 contracts, $\underline{\Psi}_{sep}^M \equiv \{\underline{\Theta}^{HL,M}, \underline{\Theta}^{LH,M}, \underline{\Theta}^{HH,M}\}$, which satisfy in total six incentive compatibility constraints. In the following proposition, we show that in $\underline{\Psi}_{sep}^M$, only two of them are binding.

Proposition 2. Let $\alpha^{LL} = 0$. Then, in a multi-risk competitive equilibrium, type *HH* gets full insurance against both risks, *i.e.*, $\underline{\Theta}^{HH,M} = \Theta^{HH,M}$. Types *HL* and *LH* get full insurance against risks 1 and 2 respectively, *i.e.*,

$$\underline{\Theta}^{HL,M} = \left(\underline{P}^{HL}, 0, \underline{D}^{HL}, \underline{D}^{HL}\right) \text{ and } \underline{\Theta}^{LH,M} = \left(\underline{P}^{LH}, \underline{D}^{LH}, 0, \underline{D}^{LH}\right).$$

ICC's $HL \triangleright LH$, $LH \triangleright HL$, $HL \triangleright HH$, and $LH \triangleright HH$ do not bind, whereas ICC's $HH \triangleright LH$ and $HH \triangleright HL$ do bind thereby determining $\underline{D}^{LH}$ and $\underline{D}^{HL}$.

Proof. First of all, Proposition 1 holds for $\alpha^{LL} = 0$ and, therefore, $\underline{\Theta}^{HH} = \Theta^{HH}$.

Second, we temporarily drop ICC's $HL \triangleright LH$, $LH \triangleright HL$, and $HL \triangleright HH$. Lemma 1 in Appendix proves that maximizing $U^{HL}\left(\Theta^{HL}\right)$ subject only to ICC $HH \triangleright HL$ and to the zero-profit constraint $P = \sum_s q_s^{HL}\left(e_s - D_s\right)$ yields that both constraints bind and determine a corner solution in which $D_{(0,1)}^{HL} = D_{(1,1)}^{HL}$, $D_{(1,0)}^{HL} = 0$, and $P^{HL} = q_1^H e_1 + q_2^L\left(e_2 - D_{(0,1)}^{HL}\right)$. Similarly, $HH \triangleright LH$ binds, $D_{(0,1)}^{LH} = 0$, $D_{(1,0)}^{LH} = D_{(1,1)}^{LH}$, and $P^{LH} = q_1^L\left(e_1 - D_{(1,0)}^{LH}\right) + q_2^H e_2$.

Finally, Lemma 2 in Appendix proves that the contracts $\Theta^{HL} = \left(P^{HL}, 0, D_{(0,1)}^{HL}, D_{(1,1)}^{HL}\right)$ and $\Theta^{LH} = \left(P^{LH}, D_{(1,0)}^{LH}, 0, D_{(1,1)}^{LH}\right)$ from Lemma 1 satisfy ICC's $HL \triangleright LH$, $LH \triangleright HL$, $HL \triangleright HH$, and $LH \triangleright HH$ as strict inequalities. Hence, $\underline{D}^{HL} = D_{(0,1)}^{HL}$, $\underline{D}^{LH} = D_{(1,0)}^{LH}$, $\underline{P}^{HL} = P^{HL}$, and $\underline{P}^{LH} = P^{LH}$ define contracts $\underline{\Theta}^{HL,M} = \left(\underline{P}^{HL}, 0, \underline{D}^{HL}, \underline{D}^{HL}\right)$ and $\underline{\Theta}^{LH,M} = \left(\underline{P}^{LH}, \underline{D}^{LH}, 0, \underline{D}^{LH}\right)$ which satisfy all necessary equilibrium conditions. ■

This Proposition can easily be interpreted if we note that equilibrium multi-risk contracts $\underline{\Theta}^{HL,M}$ and $\underline{\Theta}^{LH,M}$ can be represented by a pair of single-risk contracts:

$$\underline{\Theta}^{LH,M} = \left\langle \underline{\Theta}_1^L, \Theta_2^H \right\rangle \text{ and } \underline{\Theta}^{HL,M} = \left\langle \Theta_1^H, \underline{\Theta}_2^L \right\rangle,$$

where $\left\{\underline{\Theta}_r^L, \Theta_r^H\right\}$ is the set of competitive equilibrium contracts against risk *r* from Rothschild and Stiglitz (1976). Thus, if type *LL* is absent in the multi-risk environment, Proposition 2 basically says that the competitive equilibrium takes the additive form, in which individuals get insurance against each risk in accordance with their types with respect to that risk, provided this equilibrium exists.

As all three multi-risk equilibrium contracts can be implemented as pairs of single-risk contracts, *i.e.*, $\underline{\Theta}^{HH,M} = \left\langle \underline{\Theta}_1^H, \Theta_2^H \right\rangle$, $\underline{\Theta}^{LH,M} = \left\langle \underline{\Theta}_1^L, \Theta_2^H \right\rangle$, and $\underline{\Theta}^{HL,M} = \left\langle \Theta_1^H, \underline{\Theta}_2^L \right\rangle$, they can be replicated by the following set of single-risk contracts: $\left\{\underline{\Theta}_1^H, \underline{\Theta}_1^L, \underline{\Theta}_2^H, \underline{\Theta}_2^L\right\}$. However, this set of single-risk contracts is a single-risk competitive Nash equilibrium only if no type $i \in \{LL, HL, LH\}$ prefers the pair of contracts $\left\langle \underline{\Theta}_1^L, \underline{\Theta}_2^L \right\rangle$, which we formally denote by $\underline{\Theta}^{LL}$: $\underline{\Theta}^{LL} \equiv \left\langle \underline{\Theta}_1^L, \underline{\Theta}_2^L \right\rangle$. In other words, even though type *LL* is absent in the population, nothing prevents individuals in the single-risk environment from buying a pair of contracts $\underline{\Theta}_1^L$ and $\underline{\Theta}_2^L$.

Formally, for a given utility function, a set of single-risk contracts $\left\{\underline{\Theta}_1^H, \underline{\Theta}_1^L, \underline{\Theta}_2^H, \underline{\Theta}_2^L\right\}$ is a single-risk competitive equilibrium, *i.e.*, $\underline{\Psi}_{sep}^S = \left\{\underline{\Theta}_1^H, \underline{\Theta}_1^L, \underline{\Theta}_2^H, \underline{\Theta}_2^L\right\}$, if and only if three ICC's $HH \triangleright \underline{\Theta}^{LL}$, $HL \triangleright \underline{\Theta}^{LL}$, and $LH \triangleright \underline{\Theta}^{LL}$ are satisfied. Otherwise, at least one of the single-risk contracts in $\left\langle \underline{\Theta}_1^L, \underline{\Theta}_2^L \right\rangle$, or even both, has to be adjusted by increasing the corresponding deductible $\underline{D}^{HL}$ and/or $\underline{D}^{LH}$. In the latter case, at least one of the types *HL* and *LH* is strictly worse-off under single-risk contracts than under multi-risk contracts.

In the light of the discussion above, we extend the notation and explicitly write the utility function as an argument in any notation, *e.g.*, $\underline{\Psi}_{sep}^S(u)$, $\underline{\Theta}_1^L(u)$, $\underline{D}^{LH}(u)$ *etc*. A natural question that arises is whether $\underline{\Psi}_{sep}^S(u) = \left\{\underline{\Theta}_1^H(u), \underline{\Theta}_1^L(u), \underline{\Theta}_2^H(u), \underline{\Theta}_2^L(u)\right\}$ for a given utility function *u* so that single-risk and multi-risk insurance contracts yield the same utility levels, or $\underline{\Psi}_{sep}^S(u) \neq \left\{\underline{\Theta}_1^H(u), \underline{\Theta}_1^L(u), \underline{\Theta}_2^H(u), \underline{\Theta}_2^L(u)\right\}$ so that single-risk insurance contracts perform strictly worse than multi-risk insurance contracts.

In Proposition 3 we provide a partial answer to this question. Let us denote by

$$\underline{m}(u) = 1 - \max\left(e_1 q_1^H + \underline{P}_2^L + \underline{D}_2^L, e_2 q_2^H + \underline{P}_1^L + \underline{D}_1^L\right)$$

the lowest wealth that individuals may end up with in a multi-risk equilibrium in the worst state of the world, and by

$$\underline{m}^{LL}(u) = 1 - \left(\underline{P}_1^L + \underline{D}_1^L + \underline{P}_2^L + \underline{D}_2^L\right)$$

the lowest wealth that individuals may end up with in single-risk setting when they buy a pair of contracts $\underline{\Theta}^{LL} \equiv \left\langle \underline{\Theta}_1^L, \underline{\Theta}_2^L \right\rangle$. Then, Proposition 3 basically says that an arbitrary utility function *u* can be changed to $\tilde{u}$ for the lowest income levels such that, first, multi-risk equilibria for *u* and $\tilde{u}$ coincide, and, second, a single-risk equilibrium and a multi-risk equilibrium for $\tilde{u}$ also coincide. Thus, without the *LL* types being present in the population, equilibria under single-risk contracts may perform equally well as equilibria under multi-risk contracts.

Proposition 3. Let $\alpha^{LL} = 0$. Then, for an arbitrary strictly concave and increasing utility function u there exists also strictly concave and increasing utility function $\tilde{u}$ so that:

a) $\tilde{u}(m) = u(m)$ for $m \in [\underline{m}(u), 1]$ so that $\underline{\Psi}^{M}_{sep}(\tilde{u}) = \underline{\Psi}^{M}_{sep}(u)$;

b) $\underline{\Psi}^{S}_{sep}(\tilde{u}) = \{\underline{\Theta}^{H}_{1}(u), \underline{\Theta}^{L}_{1}(u), \underline{\Theta}^{H}_{2}(u), \underline{\Theta}^{L}_{2}(u)\}$.

Proof. In accordance with Lemma 1, prices $\underline{P}^{L}_{r}$ and deductibles $\underline{D}^{L}_{r}$ are determined by the shape of the utility function $u(m)$ over the range [$\underline{m}$, 1], and are independent of the shape of $u(m)$ for $m \in (0, \underline{m})$. Hence, any legitimate utility function $\tilde{u}$, which coincides with u over the range $m \in [\underline{m}(u), 1]$, results in the same multi-risk equilibrium set: $\underline{\Psi}^{M}_{sep}(\tilde{u}) = \underline{\Psi}^{M}_{sep}(u)$. Consequently, we can drop utility function in the notion of multi-risk contracts $\underline{\Theta}^{i_r}_{r}(u)$.

Suppose now that the set $\{\underline{\Theta}^{H}_{1}, \underline{\Theta}^{L}_{1}, \underline{\Theta}^{H}_{2}, \underline{\Theta}^{L}_{2}\}$ of contracts is offered in single-risk setting. If an individual of type $i \in \{LL, HL, LH\}$ buys a pair of contracts $\underline{\Theta}^{LL} \equiv \langle \underline{\Theta}^{L}_{1}, \underline{\Theta}^{L}_{2} \rangle$, with probability $q^{i}_{(1,1)}$ he will suffer from both accidents, and his wealth will be equal to $\underline{m}^{LL}(u)$. The ICC $i \triangleright LL$ can be written as $G^{i} \geq 0$, where

$$
\begin{aligned}
G^{i}(u) &\equiv U^{i}\left(\underline{\Theta}^{i}(u)\right) - U^{i}\left(\underline{\Theta}^{LL}(u)\right) \\
&= q^{i}_{(0,0)}\left(u\left(1 - \sum_{r} \underline{P}^{i_r}_{r}\right) - u\left(1 - \sum_{r} \underline{P}^{L}_{r}\right)\right) + q^{i}_{(1,0)}\left(u\left(1 - \sum_{r} \underline{P}^{i_r}_{r} - \underline{D}^{i_1}_{1}\right) - u\left(1 - \sum_{r} \underline{P}^{L}_{r} - \underline{D}^{L}_{1}\right)\right) + \\
&\quad + q^{i}_{(0,1)}\left(u\left(1 - \sum_{r} \underline{P}^{i_r}_{r} - \underline{D}^{i_2}_{2}\right) - u\left(1 - \sum_{r} \underline{P}^{L}_{r} - \underline{D}^{L}_{2}\right)\right) + q^{i}_{(1,1)}\left(u\left(1 - \sum_{r} \left(\underline{P}^{i_r}_{r} + \underline{D}^{i_r}_{r}\right)\right) - u\left(\underline{m}^{LL}(u)\right)\right)
\end{aligned}
$$

Hence, outside the range $m \in [\underline{m}(u), 1]$, only the value $u\left(\underline{m}^{LL}(u)\right)$ determines whether the ICC $i \triangleright LL$ is satisfied or not. Hence, a sufficiently low value of $\tilde{u}\left(\underline{m}^{LL}(u)\right)$ guarantees that $G^{i}(u) > 0$, and that no types buy a pair of contracts $\underline{\Theta}^{LL} \equiv \langle \underline{\Theta}^{L}_{1}, \underline{\Theta}^{L}_{2} \rangle$. ■

Proposition 3 exploits the fact that the shape of the utility function at the lowest level of wealth, *i.e.*, $\underline{m}^{LL}(u)$, does not affect multi-risk equilibrium contracts but does effect single-risk incentive compatibility constraints. It says that any utility function defined over the range $m \in [\underline{m}(u), 1]$ can be extended over the range $m \in (0, \underline{m}(u))$ in such a way that the multi-risk equilibrium set of contracts is also a single-risk equilibrium set. A contraposition of Proposition 3 also holds: there are utility functions, which are defined over the range

$m \in [\underline{m}(u),1]$, for which any (increasing and concave) extension $\tilde{u}$ over the range $m \in (0, \underline{m}(u))$ yields the result of Proposition 3, namely that single-risk and multi-risk insurance contracts perform equally well.

Equilibrium with LL-type

The main reason why in the absence of *LL*-type single-risk contracts may perform equally well as multi-risk contracts is that for a given utility function, none of the ICC's $HH \triangleright \langle \underline{\Theta}_1^L, \underline{\Theta}_2^L \rangle$, $HL \triangleright \langle \underline{\Theta}_1^L, \underline{\Theta}_2^L \rangle$, and $LH \triangleright \langle \underline{\Theta}_1^L, \underline{\Theta}_2^L \rangle$ bind. If, to the contrary, one of these ICC's is violated, multi-risk contracts are strictly welfare superior to single-risk contracts. In Proposition 4 below we show that in the presence of *LL*-types, multi-risk contracts are always (generically) welfare superior to single-risk contracts.

Proposition 4. Let $\alpha^{LL} > 0$. For any generic utility function *u* the multi-risk competitive Nash equilibrium contracts are strictly Pareto-superior to the single-risk competitive Nash equilibrium contracts, *i.e.*, $W^{i,M} \geq W^{i,S}$ for all $i \in \{HL, LH, HH, LL\}$ and at least one of the inequalities is strict.

Proof. Suppose that contracts $\Theta^{HH,M} = (\Theta_1^H, \Theta_2^H)$, $\Theta^{LH,M} = (\underline{\Theta}_1^L, \Theta_2^H)$ and $\Theta^{HL,M} = (\Theta_1^H, \underline{\Theta}_2^L)$ are offered. Let us consider the contract $\underline{\Theta}^{LL} = (\underline{\Theta}_1^L, \underline{\Theta}_2^L)$ and the associated with it ICC's $HH \triangleright LL$, $HL \triangleright LL$, and $LH \triangleright LL$. Generically, neither of these constraints hold as equality; some of them will not be binding, *i.e.*, $G^i > 0$, and all the others will be violated, *i.e.*, $G^i < 0$ (see proof of Proposition 3). We consider these two mutually exclusive cases.

a) Let $G^i > 0$ for $i \in \{HL, LH, HH\}$. This implies that there exist a contract Θ^* which (i) violates none of constraints $i \triangleright LL$ for $i \in \{HL, LH, HH\}$, and (ii) is such that $U^{LL}(\Theta^*) > U^{LL}(\underline{\Theta}^{LL})$. In this case, insurance companies in multi-risk environment are able to offer the superior contract Θ^* to type *LL* so that all ICC's are satisfied. In single-risk environment, to the contrary, any improvement of the contract $\underline{\Theta}^{LL} = (\underline{\Theta}_1^L, \underline{\Theta}_2^L)$ requires an improvement of at least one of its single-risk components, which is not possible because $HH \triangleright LH$ and $HH \triangleright HL$ already bind. Choosing $\Theta^* \in \arg\max U^{LL}(\Theta)$ subject to all relevant ICC's yields an equilibrium contract $\Theta^{LL,M}$

which strictly Pareto-dominates $\underline{\Theta}^{LL} = \left(\underline{\Theta}_1^L, \underline{\Theta}_2^L\right)$. As $W^{LL,S} \le U^{LL}\left(\underline{\Theta}^{LL}\right)$, it follows that $W^{LL,M} > W^{LL,S}$.

b) Let $G^j < 0$ for some $j \in \{HL, LH, HH\}$. If this is the case, insurance companies in single-risk environment cannot provide contracts $\underline{\Theta}_1^L$ and $\underline{\Theta}_2^L$ because they are not incentive compatible. Consequently, just like in the case $\alpha^{LL} = 0$, one of the inequalities $W^{LH,M} \ge W^{LH,S}$ and $W^{HL,M} \ge W^{HL,S}$ must be strict. The only Pareto-condition that needs to be shown is that $W^{LL,M} \ge W^{LL,S}$. This condition follows from the former two inequalities because the constraint $i \overset{M}{\rhd} LL$ in the multi-risk setting, which is $W^{i,M} \ge U^i\left(\Theta^{LL,M}\right)$, is less restrictive than the constraints $i \overset{S}{\rhd} LL$ in the single-risk setting, which is $W^{i,S} \ge U^i\left(\Theta^{LL,S}\right)$, for all $i \in \{HL, LH, HH\}$. Consequently, it must be that $U^{LL}\left(\Theta^{LL,M}\right) \ge U^{LL}\left(\Theta^{LL,S}\right)$. ■

In accordance with Proposition 4, at least one of the types $i \in \{HL, LH, HH\}$ is strictly better off under multi-risk contracts than under single-risk contracts. Which type gets a superior contract depends on the specific utility function and other model parameters.

4. Conclusion

In this paper, we have analyzed the welfare consequences of bundling different risks in one insurance contract. The model we have developed to analyze this question is an extension of the competitive insurance model of Rothschild and Stiglitz (1976) to two sources of risk. Accordingly, there are four possible types and many incentive compatibility constraints to be considered. The main effect of bundling in insurance contracts is on these incentive compatibility constraints. We have shown that these effects are such that if all four possible types are present in the population, bundling always yields welfare improvements. Due to the competition between insurance companies, these benefits accrue to consumers who potentially have fewer contracts to choose from, but benefit from the better sorting possibilities due to bundling.

Our results can be easily generalized to the case of more than two types for each risk. Because multi-risk contracts are always strictly welfare-superior to single-risk contracts in case of four types, it is clear that the incentive compatibility constraints in case of more than four generic types are weaker for multi-risk contracts than for single-risk contracts. Consequently, bundling always yields welfare improvements. Another possible generalization is to assume more than two sources of risk. Under this assumption, our results continue to hold because of the following reasoning. Suppose that there are three sources of risk, and two types with respect to each risk. We have shown that bundling risks 2 and 3 is always strictly welfare improving. With respect to the combined outcome of these two risks, individuals come into four types, and the model can now be reformulated as if there were two risks: risk 1 with two types, and another risk (combination of risks 2 and 3) with four types. In the light of the previous generalization for more than two types, bundling all three risks is strictly welfare improving as well. Lastly, the results remain intact even when different risks are correlated. The exact prices and deductibles in equilibrium will certainly be affected by the correlation. Nevertheless, all our propositions will continue to hold.

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Appendix

Lemma 1. A contract Θ^{HL}, which maximizes $U^{HL}(\Theta^{HL})$ subject to ICC $HH \triangleright HL$ and the zero-profit constraint $P = \sum_s q_s^{HL}(e_s - D_s)$, is a corner solution such that both constraints bind, and that $D_{(0,1)}^{HL} = D_{(1,1)}^{HL}$, $D_{(1,0)}^{HL} = 0$, and $P^{HL} = q_1^H e_1 + q_2^L(e_2 - D_{(0,1)}^{HL})$.

Proof of Lemma 1. The Lagrangian function for the maximization problem is

$$L = \sum_s q_s^{HL} u(1 - P^{HL} - D_s^{HL}) - \lambda\left(\sum_s q_s^{HL}(e_s - D_s^{HL}) - P^{HL}\right) - \mu\left(\sum_s q_s^{HH} u(1 - P^{HL} - D_s^{HL}) - W^{HH}\right),$$

and the first-order conditions are

$$\begin{cases} 0 = \dfrac{\partial L}{\partial P^{HL}} = \lambda - \sum\limits_s (q_s^{HL} - \mu q_s^{HH}) u'(1 - P^{HL} - D_s^{HL}) \\ 0 = \dfrac{\partial L}{\partial D_s^{HL}} = \lambda q_s^{HL} + (q_s^{HL} - \mu q_s^{HH}) u'(1 - P^{HL} - D_s^{HL}), s \in S \setminus (0,0) \end{cases}$$

Because it cannot be generically the case that $q_s^{HL} = \mu q_s^{HH}$ for all *s*, it follows that $\lambda \neq 0$. Consequently, $q_s^{HL} \neq \mu q_s^{HH}$ for any *s*. Taking into account the definition of q_s^i allows us to rewrite the above equations as follows:

$$\begin{cases} u'(1 - P^{HL}) = u'(1 - P^{HL} - D_{(1,0)}^{HL}) = \dfrac{\lambda(1 - q_2^L)}{(1 - q_2^L) - \mu(1 - q_2^H) q_{(0,0)}^{HH}} \\ u'(1 - P^{HL} - D_{(0,1)}^{HL}) = u'(1 - P^{HL} - D_{(1,1)}^{HL}) = \dfrac{\lambda q_2^L}{q_2^L - \mu q_2^H} \end{cases}$$

Hence, $D_{(1,0)}^{HL} = 0$ and $D_{(0,1)}^{HL} = D_{(1,1)}^{HL}$ due to $u' > 0$.

In order to show that $\mu \neq 0$ we assume, to the contrary, that $\mu = 0$. This implies $D_{(0,1)}^{HL} = D_{(1,1)}^{HL} = D_{(1,0)}^{HL} = 0$ and, consequently, $HH \triangleright HL$ violates. Thus, $\mu \neq 0$, which implies that $HH \triangleright HL$ binds and, together with the zero-profit condition, it determines $D_{(0,1)}^{HL}$ and P^{HL} as a corner solution in the following system of two equations:

$$\begin{cases} (1 - q_2^H) u(1 - P^{HL}) + q_2^H u(1 - P^{HL} - D_{(0,1)}^{HL}) = u(1 - e_1 q_1^H - e_2 q_2^H) \\ P^{HL} = e_1 q_1^H + (e_2 - D_{(0,1)}^{HL}) q_2^H \end{cases}$$

Interchanging risks 1 and 2 yields similar expressions for the contract Θ^{LH}. ■

Lemma 2. Contracts Θ^{HL} and Θ^{LH} derived in Lemma 1 satisfy ICC's $HL \triangleright LH$, $LH \triangleright HL$, $HL \triangleright HH$, and $LH \triangleright HH$ as strict inequalities.

Proof of Lemma 2. Let define functions $G^{LH}\left(q_1^L\right)$ and $F^{LH}\left(q_1^L\right)$ of the exogenous parameter q_1^L as follows:

$$G^{LH}\left(q_1^L\right) \equiv U^{LH}\left(\Theta^{LH}\right) - U^{LH}\left(\Theta^{HL}\right), \text{ and}$$

$$F^{LH}\left(q_1^L\right) \equiv U^{LH}\left(\Theta^{LH}\right) - U^{LH}\left(\Theta^{HH}\right).$$

By the definitions of ICC's, we need to show that $G^{LH}\left(q_1^L\right) > 0$ and $F^{LH}\left(q_1^L\right) > 0$ for all $q_1^L \in \left(0, q_1^H\right)$.

It is easy to see that $G^{LH}\left(q_1^H\right) = 0$:

$$G^{LH}\left(q_1^H\right) = U^{HH}\left(\Theta^{HH}\right) - U^{HH}\left(\Theta^{HL}\right) = 0$$

because, in accordance with Lemma 1, ICC $HH \triangleright HL$ binds. Contracts $\Theta^{HL} = \left(P^{HL}, 0, D^{HL}, D^{HL}\right)$ and $\Theta^{LH} = \left(P^{LH}, D^{LH}, 0, D^{LH}\right)$ are determined by

$$\begin{cases} \left(1 - q_2^H\right)u\left(1 - P^{HL}\right) + q_2^H u\left(1 - P^{HL} - D^{HL}\right) = u\left(1 - e_1 q_1^H - e_2 q_2^H\right) \\ P^{HL} = e_1 q_1^H + \left(e_2 - D^{HL}\right)q_2^L \end{cases}, \text{ and}$$

$$\begin{cases} \left(1 - q_1^H\right)u\left(1 - P^{LH}\right) + q_1^H u\left(1 - P^{LH} - D^{LH}\right) = u\left(1 - e_1 q_1^H - e_2 q_2^H\right) \\ P^{LH} = e_2 q_2^H + \left(e_1 - D^{LH}\right)q_1^L \end{cases}.$$

Considering these equations as implicit functions $P^{LH}\left(q_1^L\right)$, $D^{LH}\left(q_1^L\right)$, $P^{HL}\left(q_1^L\right)$, and $D^{HL}\left(q_1^L\right)$, one can get the following equalities:

$$\frac{dP^{HL}}{dq_1^L} = \frac{dD^{HL}}{dq_1^L} = 0, \; D^{LH}\left(q_1^H\right) = 0, \; P^{LH}\left(q_1^H\right) = P^{HH} = e_2 q_2^H + e_1 q_1^H, \text{ and}$$

$$\begin{cases} \dfrac{dD^{LH}}{dq_1^L} = -\dfrac{q_1^H u'\left(1 - P^{LH} - D^{LH}\right) + \left(1 - q_1^H\right)u'\left(1 - P^{LH}\right)}{q_1^H\left(1 - q_1^L\right)u'\left(1 - P^{LH} - D^{LH}\right) - q_1^L\left(1 - q_1^H\right)u'\left(1 - P^{LH}\right)}\left(e_1 - D^{LH}\right) \\ \dfrac{dP^{LH}}{dq_1^L} = \dfrac{q_1^H u'\left(1 - P^{LH} - D^{LH}\right)}{q_1^H\left(1 - q_1^L\right)u'\left(1 - P^{LH} - D^{LH}\right) - q_1^L\left(1 - q_1^H\right)u'\left(1 - P^{LH}\right)}\left(e_1 - D^{LH}\right) \end{cases}$$

This allows us to write dG^{LH}/dq_1^L as follows:

$$\begin{aligned} \frac{dG^{LH}}{dq_1^L} &= \frac{d}{dq_1^L}\left(\left(1 - q_1^L\right)u\left(1 - P^{LH}\right) + q_1^L u\left(1 - P^{LH} - D^{LH}\right) - \left(1 - q_2^L\right)u\left(1 - P^{HL}\right) - q_2^L u\left(1 - P^{HL} - D^{HL}\right)\right) \\ &= -\left(u\left(1 - P^{LH}\right) - u\left(1 - P^{LH} - D^{LH}\right)\right) - \\ &\quad - \frac{\left(q_1^H - q_1^L\right)u'\left(1 - P^{LH} - D^{LH}\right)u'\left(1 - P^{LH}\right)}{q_1^H\left(1 - q_1^L\right)u'\left(1 - P^{LH} - D^{LH}\right) - q_1^L\left(1 - q_1^H\right)u'\left(1 - P^{LH}\right)}\left(e_1 - D^{LH}\right) \end{aligned}$$

As $D^{LH} \in (0, e_1)$ for all $q_1^L \in (0, q_1^H)$, it follows that $dG^{LH} / dq_1^L < 0$, and, consequently, $G^{LH}(q_1^L) > 0$ for all $q_1^L \in (0, q_1^H)$.

Finally, as $U^{LH}(\Theta^{HH}) = U^{HH}(\Theta^{HH})$ and is independent of dq_1^L, it follows that $dF^{LH} / dq_1^L = dG^{LH} / dq_1^L < 0$, and, consequently, $F^{LH}(q_1^L) > 0$ for all $q_1^L \in (0, q_1^H)$. ■