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Pension Systems and their Influence on Fertiltiy and Growth

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Abstract

This paper studies the implications of different public pension systems on fertility and economic growth. Employing a three period overlapping generations endogenous growth model we compare the different impacts of pay-as-you-go-, fully funded- and informal pension systems. The novelty of our work lies in the formulation of altruism that is assumed to be one sided (descending) for economies represented by a public pension system and two sided (descending and ascending) for economies with informal pension systems. Through the incorporation of a mixed procreation motive we can study the case of fully crowded out intrafamilial transfers inside a public pension system model while still capturing fertility endogenously.

We show that the introduction of public pension systems to a developing economy reduce fertility and stimulate economic growth. Through a comparison of the different public pension systems we highlight that a fully funded pension system results in higher economic growth compared to a pay-as-you-go one despite higher fertility because the growth enhancing effect of the higher capital stock is dominant. This suggests that observed fertility and growth differences between the US and Europe can partly be explained by the different types of pension systems.

JEL classification: H55, J13, O41

Key words: Public and informal pension systems, endogenous growth, fertility

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1 Introduction

Developing economies are usually facing a whole bundle of obstacles on their way to development. Overpopulation, political instability and insecure property rights are only some of the problems. Our work tries to contribute to the topic by analyzing the secondary goal of pensions, the influence on fertility and economic growth.

While different works are dealing with growth and population effects of pay-as-you-go pension systems (Zhang (1995) and Boldrin, De Nardi, Jones (2005)) the first part of our study is focusing on the impact of funded and unfunded public pension system introduction to a developing economy where informal contributions finance pension benefits.

Holzmann (2005) observes that only 1/5th of the working population in Sub-Saharan Africa is covered by a public pension system and that the biggest part of the contributions is used for the inefficient and bureaucratic organizational structure of the systems. This is supporting our idea of solely informally financed developing country pension systems. In this context the paper by Zhang and Zhang (1995) is closest to our work. They show that the introduction of a pay-as-you-go public pension system compared to a fertility dependent one leads to positive growth effects because fertility decreases and savings increase. In contrast to their approach our work assumes, that the existence of a public pension system fully crowds out the old age security motive. Therefore we exclude the voluntary, non mandatory intergenerational transfers from adult children to their parents if a working public pension system is present. While this does not change the qualitative behavior of the model, it reduces the positive growth effect because the negative effect of pay-as-you-go contributions on capital accumulation through decreased savings can not be offset by decreased gifts.

Our assumption of fully crowded out gifts in the light of public pension systems is following Caldwells theory of intergenerational wealth flows (Caldwell (1982)) which defines two different types of societies. The "modern" society covers the case of developed countries which are in our work defined by a working public pension system. In these economies children are not expected to contribute to the retirement income of the parents because private savings and public pension payments are high enough to offer a sufficient level of retirement consumption. Therefore the public pension system takes over the role of the private intrafamilial intergenerational transfers and reduces procreation benefits which leads to lower fertility rates. Although private gifts are in reality of course present for developed economies, we argue that in these societies they occur only occasionally and households do not consider them in their optimization. In other words adults do not expect their own retirement income to be dependent on the number of own children. The second so called "traditional" society is characterized by high fertility since children directly contribute to the parental retirement budget. Translating this to our framework means that private intrafamilial gifts take place, or in other words a developing society values the old age security motive of fertility.

Within the pension system and endogenous fertility literature two motives of having children are prominent. The first motive captures the fact that individuals are expected to procreate because they expect their children to contribute to their retirement budget. Due to its insurance character this motive is known as the old age security motive of fertility (Leibenstein (1957)). The second so called consumption good motive of fertility states that parents simply enjoy the fact to have a successor and see children as a durable consumption good (Dasgupta (1993) and Zhang (1995)). Here children are treated as offering utility only by their existence. Our work picks up the idea of a mixed fertility motive first introduced by Wigger (1999), where both the insurance and consumption good motive determine fertility. Since pension benefits which are financed through a public pension system are independent on individual fertility, models determining fertility endogenously have to include altruistic intrafamilial transfers in the form of gifts from adult children to their parents (Bental (1989), Zhang and Nishimura (1993)). We can abstract from these gifts since the inclusion of the consumption good motive enables us to treat fertility endogenously also for the public pension system cases where the old age security motive is not existent because private intergenerational gifts are fully crowded out.

While our two model societies are expected to differ in their valuation of the old age security motive, we expect both to incorporate the consumption good motive because procreation is a basic need for human beings. This is creating the view of children as a durable consumption good.

Our work deals with the two societies by formulating corresponding scenarios inside a Diamond type overlapping generations model where the engine of growth is formed through labor productivity. While the first scenario describes the situation of a developing economy with an informal pension system and derives corresponding fertility and economic growth the second and third scenario examine the cases of pay-as-you-go and fully funded public pension systems. The comparison of the different outcomes enables us to show the impacts of different pension system introductions to a developing economy. Furthermore we analyze the role of different pension systems on fertility and growth level differences observed for the US and Europe.

Since the theoretical examination leaves certain questions unanswered we calibrate the model for observed average OECD and Sub-Saharan African total fertility and per capita production growth data.

2 Model

The basic framework of our study forms a Diamond type OLG model. The economy is populated by finitely living agents belonging to three generations. Each individual lives for three periods: childhood, adulthood and retirement. During childhood individuals consume θW_t, where θ is the fraction of working time needed to rear one child. While child costs are usually split in time and good costs (Morand (1996)) without loss of generality we abstract from this formulation. During adulthood households decide about fertility n_t, adult consumption c_t, and future retirement consumption c_{t+1}. The population dynamics for the productive adult population are described by $N_{t+1} = N_t n_t$. Retired people only consume and have no influence on household optimization. Our model does not include bequests. Retired agents are therefore assumed to consume their whole savings plus pension benefits during their third period of life.

Following Zhang (1995) and Doepke, De La Croix (2003) individuals preferences include a descending altruistic part capturing the consumption good motive of fertility. This approach can be seen as a modification of the Barro and Becker (1989) dynastic utility function. In contrast to their idea that adults incorporate the whole utility of their offsprings in the utility function we assume that parents value only the number of children. In other words we exclude the dynastic component of the Barro-Becker descending altruism.

Next to the consumption good motive of fertility we additionally model the old age security motive of fertility by incorporating ascending altruism. Inspired by Caldwells intergenerational flow theory we define different scenarios to capture the fact that ascending altruism is only present for countries without a mandatory public pension system. Following Morand (1996) the ascending altruism of individual's preferences is captured through gifts from the own adult children to their parents during retirement. These gifts only take place if a minimum retirement consumption level is not reached and can be viewed as free private intergenerational transfers. The ascending altruistic part of preferences is therefore captured in the composition of pension payments π_{t+1}.

We assume that individuals utility is represented by the following logarithmic additive separable function:

$$V_t = \log(c_t) + \beta \log(c_{t+1}) + \gamma \log(n_t) \tag{1}$$

Utility is dependent on adult consumption c_t, retirement consumption c_{t+1}, discount factor β, descending altruism factor γ and the number of children n_t. We assume $\beta, \gamma < 1$.

The household budget constraint is represented by adult age consumption c_t and old age consumption c_{t+1} . Adult consumption is dependent on wage, child rearing cost θn_t, pension contributions $W_t \tau$ and savings s_t. Pension contributions are formulated as an income tax τ. We additionally assume perfect foresight implying that individuals exactly know the future gross interest rate R_{t+1} at which savings are interested. Old age consumption is financed through interested savings and pension benefits. Notice that the pension system is assumed to be always budget balanced.

$$c_t = W_t(1 - \theta n_t - \tau) - s_t \tag{2}$$

$$c_{t+1} = s_t R_{t+1} + \pi_{t+1} \tag{3}$$

2.1 Informal pension arrangement

In this section the situation of a developing country is modeled. While there are a lot of differences between developing and developed countries in reality we are only focusing on the variations of retirement income composition. Developing countries are mainly represented by a not existent or unreliable public pension system that only covers a small part of the population resulting in retirement budget that is below a minimum[1]. This implies that savings alone are not enough to finance a sufficient level of retirement consumption. For this

[1] The World Bank."Averting the Old Age Security Crises": page 63 and 192.

reason we assume children to take over the role of the public pension system and finance the pensions of their parents through private contributions resulting in ascending altruism. Caldwell motivated developing societies high fertility levels exactly by these informal intrafamilial retirement age income contributions. Different empirical facts support our assumption. Due to the World Bank 70% of the old throughout the world rely exclusively on informal pension arrangements. International pension coverage data mentioned in the World Bank report additionally shows that low public pension system coverage rates correlate with a high percentage of inhabitants being supported by their own family while for high coverage rates the opposite is true[2]. Further evidence from developing country surveys (Arnold et. al. (1975), Kagitvibasi (1982)) also indicates that old age security certainly is a fertility motive in developing countries[3].

Although private intrafamilial transfers are usually freely chosen by the family members we abstract from heterogeneity in contributions because developing country contributions are often socially mandatory. This means that individuals are forced to contribute by the threat of punishment which can take the form of exclusion from social village life. Therefore we assume that the contribution rate τ is not a decision variable but socially determined and constant over time.

2.1.1 Production

The economy is populated by one representative firm that uses the production factors capital K_t and effective labour $A_t L_t$ to produce a single homogeneous good at time t. A_t determines labour productivity at time t which is assumed to be driven through a Romer type positive spillover. In equilibrium labor demand L_t equals labor supply which is determined by the adult population N_t. The aggregate production function is determined by:

$$Y_t = F(K_t, A_t N_t) \tag{4}$$

Following Grossman and Yanagawa (1993) the technological spillover is dependent on the fraction of capital per worker and the parameter m which is a positive technology parameter controlling for the influence of capital intensity on labor productivity. The lower m the higher is the productivity of labour.

[2]Nigeria and Kenya show the highest percentage (95% and 88%) of population over 60 covered by family transfers, while public pension coverage rate for these countries is almost zero. The World Bank."Averting the Old Age Security Crises": page 57 table 2.1.

[3]40% of respondents to the surveys indicated that old age help is a very important motive for having children.

$$A_t = \frac{K_t}{mN_t} \tag{5}$$

Now define capital per effective unit of labour with k_t. From (5) we follow that k_t is constant and that capital per unit of labour $\widehat{k}_t$ grows at the rate of A_t.

$$k_t = \frac{K_t}{A_t N_t} = m \tag{6}$$

$$\widehat{k}_t = A_t m \tag{7}$$

Profit maximization of the firm implies that production factors are paid by their marginal products.

$$F'_L(K_t, A_t N_t) = W_t = [f(k_t) - f'(k_t)k_t]\, A_t \tag{8}$$

$$F'_K(K_t, A_t N_t) = f'(k_t) = R_t \tag{9}$$

Since firm profits are distributed to capital owners, cleared capital markets imply that the return on savings is equal to the marginal product of capital. Equation (8) and (9) imply that capital and labor markets are cleared. Due to Walras' law capital market and labour market clearing together also imply a cleared goods market.

Now use capital and labor market clearing conditions together with the fact that capital per efficient unit of labor is constant over time (see equation (6)) to state that gross interest rate R_t and wage per efficient unit $w_t = \frac{W_t}{A_t}$ are constant.

$$R_t = f'(m) = R \tag{10}$$

$$w_t = [f(m) - f'(m)m] = w \tag{11}$$

From the labor market clearing condition we can furthermore see that wage is growing with the level of labor productivity A_t. This enables us to describe economic growth by the growth rate of technological spill over g.

$$\frac{\widehat{k}_{t+1}}{\widehat{k}_t} = \frac{W_{t+1}}{W_t} = \frac{A_{t+1}}{A_t} \equiv g \tag{12}$$

2.1.2 Households

Because retirement consumption before gifts is assumed to be below a sufficient level, ascending altruistic transfers in the form of children contributing to their parents retirement budget take place. These gifts can be seen as a kind of private intrafamilial pension system. Gifts offered to the parents are measured through our pension benefit variable π_{t+1}.

A balanced budget pension system demands that benefits equal contributions at every point in time. This is implying that the number of own children n_t times the part of children's adult income offered as a gift to their parents τW_{t+1} has to equal the pension benefits π_{t+1}.

$$\pi_{t+1} = \tau n_t W_{t+1} \tag{13}$$

(1), (2), (3) and (13) describe the problem of a representative developing country household.

$$\begin{aligned} V_t &= \log(c_t) + \beta \log(c_{t+1}) + \gamma \log(n_t) \\ c_t &= W_t(1 - \theta n_t - \tau) - s_t \\ c_{t+1} &= s_t R_{t+1} + \tau n_t W_{t+1} \end{aligned}$$

The utility function captures the consumption good value of children, the retirement budget constraint reflects the insurance value. Households choose fertility and savings due to the first order conditions:

$$c_t = \frac{c_{t+1}}{\beta R_{t+1}} \tag{14}$$

$$\frac{\theta W_t}{c_t} = \frac{\gamma}{n_t} + \frac{\beta \tau W_{t+1}}{c_{t+1}} \tag{15}$$

Adults can decide whether to spend their money in the first or second period. The optimal decision of splitting overall consumption between the two periods is represented by equation (14), which states that marginal utility of adult consumption has to equal marginal utility of retirement consumption. An increase in interest rates or a higher discount factor imply that consumption today will be skipped for consumption tomorrow. At an optimum marginal benefit of actual consumption has to equal discounted marginal benefit of future consumption.

Equation (15) deals with cost and benefit of having a child. It states that at an optimum the marginal cost of child rearing must equal the present value of marginal benefit gained through the birth of a child. Marginal benefit of having a child (the right hand side of equation (15)) consists of two parts representing our idea of modelling a mixed procreation motive. The first part ($\frac{\gamma}{n_t}$) is reflecting the consumption good value while the second part ($\frac{\beta\tau W_{t+1}}{c_{t+1}}$), measuring the present value of marginal benefit of child investments arising in period $t+1$, is capturing the security value.

Solving the two equations for fertility and savings, followed by algebraic reformulation leads to optimal household decisions (16) and (17).

$$s_t = \frac{(1-\tau)W_t((\beta+\gamma)\tau W_{t+1} - \beta\theta R_{t+1}W_t)}{(1+\beta+\gamma)(\tau W_{t+1} - \theta R_{t+1}W_t)} \tag{16}$$

$$n_t = \frac{\gamma R_{t+1}(1-\tau)W_t}{(1+\beta+\gamma)(\theta R_{t+1}W_t - \tau W_{t+1})} \tag{17}$$

Our assumption of homogeneous agents implies that aggregate savings can not be negative. Use this fact together with optimal savings (16) to follow that net present marginal value of child investment used as savings instead of being spent on child rearing (left hand side of (18)) has to be equal or higher than marginal benefit that arises by having a child (right hand side of (18)).

$$\theta R_{t+1}W_t \geqslant \tau(1+\frac{\gamma}{\beta})W_{t+1} \tag{18}$$

Through the use of $W_{t+1} = gW_t$ and equation (10) we can rewrite (18) in constant terms:

$$\theta R \geqslant \tau(1+\frac{\gamma}{\beta})g \tag{19}$$

Equation (19) implies that the benefits of the two different types of intertemporal transfers of income from consumption in period t to consumption in period $t+1$(savings and fertility) are weighted against each other. Households would optimally choose zero or negative savings if child investment would pay equal or more than saving investment. If the left hand side is smaller than the right hand side agents would be willing to borrow money since child investment returns would compensate the interest cost of such loans. In these cases third period consumption would solely be financed through interested fertility investments. The equality sign holds if households are indifferent between the two investment opportunities. Equation (19) again highlights the twofold value of children in our model. While τg represents the insurance value, $\frac{\gamma}{\beta}\tau g$ represents the consumption value.

Besides non-negativity of aggregate savings we also have to secure that optimal fertility can not become negative or infinite. Therefore the child rearing cost interested in the capital market in the form of savings have to be higher than the benefit parents get from the insurance motive of having children alone. If this would not be the case parents would take loans to finance infinitely many children since the interest on child investments (insurance plus consumption good value) is higher than on savings. This would again imply negative aggregate savings which are not possible in our model.

$$\theta R > \tau g$$

Combine both conditions and assume that aggregate savings are positive to show that the only case where fertility and savings are well behaved (positive and finite) is where:

$$\theta R > (1 + \frac{\gamma}{\beta})\tau g$$

Now rearrange this condition to show that our model implies a maximum level of pension contribution tax rate τ.

$$\tau^{\max} = \frac{\theta R}{(1 + \frac{\gamma}{\beta})g}$$

2.1.3 Capital Market

Capital market equilibrium demands that future capital is equal to actual aggregate savings plus depreciated capital. Since in our model only old people, who do not leave any bequests and totally use up their savings are holding capital, capital market equilibrium is described by:

$$K_{t+1} = s_t N_t$$

2.1.4 Equilibrium Analysis

Production- and household optimization together with the capital market equilibrium close the model by defining a competitive equilibrium with intergenerational transfers. From equation (12) we already know that per capita output growth is solely defined by labor productivity growth g. Use (5) and the capital market equilibrium to reproduce the in the endogenous growth theory literature well known feature (Grossmann, Yanagawa (1993)) of growth enhancing savings and growth diminishing fertility.

$$g = \frac{A_{t+1}}{A_t} = \frac{\frac{K_{t+1}}{mN_{t+1}}}{\frac{K_t}{mN_t}} = \frac{K_{t+1}N_t}{K_t N_{t+1}} = \frac{s_t}{n_t A_t m}$$

(5), (12) and the optimal household solutions for savings and fertility lead to:

$$g = \frac{(\beta\theta R_{t+1} - (\beta+\gamma)\tau g)w_t}{\gamma R_{t+1} m}$$

Solve for g and use the fact that R_t and w_t are constant to show, that growth is also constant.

$$g = \frac{\beta\theta R w}{\beta\tau w + \gamma\tau w + m\gamma R} \tag{20}$$

Besides constant per capita production growth the equilibrium is described by constant fertility.

$$n = \frac{\gamma R(1-\tau)}{(1+\beta+\gamma)(\theta R - \tau g)}$$

Use the already obtained value for g to fully solve for fertility.

$$n = \frac{(1-\tau)(w(\beta+\gamma)\tau + m\gamma R)}{(1+\beta+\gamma)\theta(Rm + \tau w)} \tag{21}$$

Because our equilibrium describes a situation where fertility is constant and consumption is growing at a constant rate g it describes the situation of a balanced growth path.

Now we are in the position to analyze the impact of the intrafamilial pension system contribution rate τ on the equilibrium values. From equation (20) we follow that pension contributions financed through gifts from children to the old are growth diminishing.

$$\frac{\partial g}{\partial \tau} < 0 \tag{22}$$

Proposition 1 *Informally financed pension system contributions in the form of gifts from adult children to their parents lead to decreasing per capita production growth.*

To understand the underlying dynamics the growth determining variables savings and fertility have to be examined. Use optimal savings, $W_{t+1} = gW_t$ and equation (20) to get:

$$s_t = \frac{m\beta(1-\tau)RW_t}{(1+\beta+\gamma)(w\tau+mR)}$$

Now derive savings with respect to τ to see that positive pension contributions $\tau > 0$ lead to decreasing savings. Pension contributions financed through gifts are therefore crowding out savings which is clearly growth diminishing.

$$\frac{\partial s_t}{\partial \tau} = -\frac{m\beta R(w+mR)W_t}{(1+\beta+\gamma)(w\tau+mR)^2}$$

$$\frac{\partial s}{\partial \tau} < 0$$

Proposition 2 *Informal, gift based pension contributions are crowding out savings.*

While the effect on savings is simple and easy to show, the effect on fertility is more complex.

$$\begin{aligned} \frac{\partial n}{\partial \tau} &= \frac{(\beta+\gamma)(1-\tau)w}{(1+\beta+\gamma)\theta(Rm+\tau w)} - \frac{(1-\tau)w(w(\beta+\gamma)\tau+m\gamma R)}{(1+\beta+\gamma)\theta(Rm+\tau w)^2} \\ &\quad - \frac{w(\beta+\gamma)\tau+m\gamma R}{(1+\beta+\gamma)\theta(Rm+\tau w)} \\ &= \frac{m(\beta-2\beta\tau-2\gamma\tau)Rw - m^2\gamma R^2 - (\beta+\gamma)w^2\tau^2}{(1+\beta+\gamma)\theta(Rm+\tau w)^2} \end{aligned} \tag{23}$$

The derivation of fertility with respect to τ shows, that informally financed contributions can lead to higher or lower fertility depending on the strength of the underlying effects on adult and retirement budget. While the effect on adult budget is clearly fertility decreasing because adult budget goes down, the effect on retirement budget is twofold. The retirement period effects can be summarized by the change of the insurance value of a child. While increasing pension contributions clearly increase the insurance value if contributions are lump sum this is not the case for our framework of wage dependent contributions where growth is determining future adult income. Here increasing pension contributions similar to the lump sum case increase the base of payments but also decrease their interest since children's adult income is lower due to reduced growth (see equation 22). The overall retirement budget effect on fertility can be negative or positive. Because the retirement budget effect is dependent on the values of $\beta, \tau, R, w, m, \gamma, \theta$, the overall fertility effect captured by equation (23) is also variable dependent.

Proposition 3 *Depending on whether* $m(\beta - 2\beta\tau - 2\gamma\tau)Rw - m^2\gamma R^2 - (\beta + \gamma)w^2\tau^2$ *is bigger or smaller than* 0 *an informal pension system leads to increasing or decreasing fertility.*

Independent of variable values, $\tau > 0$ leads to lower economic growth (22). Therefore the case where $m(\beta - 2\beta\tau - 2\gamma\tau)Rw - m^2\gamma R^2 - (\beta + \gamma)w^2\tau^2 < 0$ (informal pension system decreases fertility) implies that the growth decreasing effect of lower savings is dominant. If $m(\beta - 2\beta\tau - 2\gamma\tau)Rw - m^2\gamma R^2 - (\beta + \gamma)w^2\tau^2 > 0$, fertility- and savings effect are both growth increasing (fertility decreases and savings increase).

2.2 Pay-as-you-go public pension system

In this sub chapter we focus on fertility and growth implications caused by a pay-as-you-go pension system. In reality children support their parents for different reasons. On the one hand ascending transfers can be motivated by altruism, taking place only because parents are in need. On the other hand transfers can be part of an intergenerational exchange incorporating a connection between transfers and bequests. Since we exclude bequest from the analysis our model only captures the altruistic transfer motive. If in this framework the state steps in and introduces a public pension system, implying that parents get a guaranteed certain minimum wage, donors (adult children) no longer see the need to provide transfers. A public pension system therefore completely crowds out transfers (gifts) from adult children to their parents. In contrast to the developing economy scenario, pension benefits are now not dependent on own fertility decisions n_t but on average fertility of the whole economy $\bar{n}_t$. Furthermore pensions are also independent of the future adult income of the own child. In a public pension system the average future income $\bar{W}_{t+1}$ of children instead of W_{t+1} enters the pension benefit formula. The reflection of the transfer crowding out effect through public pension systems is the main difference between this part of our work and the paper by Wigger (1999). A balanced budget pay-as-you-go public pension system demands:

$$\pi_{t+1} = \tau \bar{n}_t \bar{W}_{t+1}$$

While production sector and capital market stay the same the described change in the pension system funding changes the household optimization problem.

2.2.1 Households

The crowding out of private intergenerational transfers through a public pension system has a big influence on the value of a child. Pension benefits are now independent of own fertility decisions and agents do not incorporate the old age security motive of fertility in their fertility decisions. This change is represented in the retirement budget constraint c_{t+1}. Notice that τ is now a policy decision variable instead of a socially determined rate.

$$\begin{aligned} V_t &= \log(c_t) + \beta \log(c_{t+1}) + \gamma \log(n_t) \\ c_t &= W_t(1 - \theta n_t - \tau) - s_t \\ c_{t+1} &= s_t R_{t+1} + \tau \bar{n}_t \bar{W}_{t+1} \end{aligned}$$

Optimization leads to following first order conditions:

$$\frac{1}{c_t} = \frac{\beta R_{t+1}}{c_{t+1}}$$

$$\frac{\theta W_t}{c_t} = \frac{\gamma}{n_t}$$

While the first equation handling the optimal split between present and future consumption is the same as in the informal pension contribution scenario, the second equation dealing with cost and benefit of having children changes. This fact is due to the change in marginal benefit of having children which now only reflects the consumption good motive. The insurance motive of fertility becomes obsolete.

Because our model economy assumes homogeneous agents one can set $\bar{n}_t$ and $\bar{W}_{t+1}$ equal to n_t and W_{t+1} after the optimization. Solving the two equations for fertility and savings gives us the following optimal household decisions:

$$s_t = \frac{(1-\tau)W_t(\gamma \tau W_{t+1} - \beta \theta R_{t+1} W_t)}{\tau \gamma W_{t+1} - (1 + \beta + \gamma)\theta R_{t+1} W_t}$$

$$n_t = \frac{\gamma R_{t+1}(1-\tau)W_t}{(1+\beta+\gamma)\theta R_{t+1}W_t - \gamma\tau W_{t+1}}$$

Like in the previous case negative aggregate savings are not possible implying that the marginal opportunity cost of having a child $\beta\theta R$ have to be higher or equal to the marginal benefit of procreation $\gamma\tau g$. Otherwise saving decisions would become zero or negative and the whole adult income would be used only for consumption, child rearing and pension contributions.

$$\beta\theta R \geqslant \gamma\tau g \tag{24}$$

The condition for positive and finite fertility $(1+\beta+\gamma)\theta R > \gamma\tau g$ is like in the previous model weaker and included in the condition for non-negativity of s_t. Notice that we again assume positive savings, implying that we abstract from the equality sign in (24). Well behaved savings and fertility demand:

$$\beta\theta R > \gamma\tau g$$

This allows us to define the maximum limit of pension contribution tax rate τ for the pa-as-you-go scenario which is higher than the maximum level in the case of the informal pension system. τ can not be higher than the fraction $\frac{\beta\theta R}{\gamma g}$. Otherwise investment in savings would pay less than investment in fertility and the non-negativity assumption of aggregate savings would not hold.

$$\tau^{\max} = \frac{\beta\theta R}{\gamma g}$$

2.2.2 Equilibrium Analysis

To calculate per capita production growth use the relationship between savings, fertility and labor productivity ($g = \frac{s_t}{n_t A_t m}$) resulting from capital market equilibrium and input prices . Now close the model by including the agent's optimal savings and fertility decisions.

$$g = \left(\frac{\beta\theta}{\gamma} - \frac{g\tau}{R_{t+1}}\right)\frac{W_t}{A_t m}$$

Solve for g and show with the help of (10) and (11) that the growth rate g is constant over time.

$$g = \frac{\beta\theta R w}{\gamma(mR + \tau w)} \tag{25}$$

Use the result for g to obtain optimal fertility. This shows that our equilibrium again describes the situation of a balanced growth path, because also fertility is constant.

$$n = \frac{\gamma(1-\tau)(Rm + \tau w)}{\theta((1+\beta+\gamma)mR + (1+\gamma)\tau w)} \tag{26}$$

Now we are in the position to analyze the impact of the pension system on growth. Use the first derivative of equation (25) with respect to τ to show that a pay-as-you-go pension system acts growth diminishing.

$$\frac{\partial g}{\partial \tau} < 0$$

Proposition 4 *A pay-as-you-go pension system decreases economic growth.*

The reason for this negative impact lies again in the behavior of fertility and savings. Use $W_t g = W_{t+1}$ and equation (25) to reformulate optimal savings.

$$s_t = \frac{m\beta R W_t(1-\tau)}{mR(1+\beta+\gamma) + \tau w(1+\gamma)}$$

Now derive s_t with respect to τ to see that a pay-as-you-go pension system acts savings reducing.

$$\frac{\partial s}{\partial \tau} = -\frac{m\beta R W_t(w(1+\gamma) + m(1+\beta+\gamma)R)}{(mR(1+\beta+\gamma) + \tau w(1+\gamma))^2}$$

$$\frac{\partial s}{\partial \tau} < 0$$

Proposition 5 *A pay-as-you-go pension system leads to lower savings since resources are intergenerationally redistributed from young to old. This crowds out private savings and reduces capital accumulation.*

Our assumption that a public pension system fully crowds out private intrafamilial gifts is the reason why the savings reducing effect of pay-as-you-go contributions can not be offset by reduced gifts like in Yoon, Talmain (2001).

To see the effect of a pay-as-you-go pension system on fertility, derive (26) with respect to τ.

$$\frac{\partial n}{\partial \tau} = -\frac{\gamma(m^2R^2(1+\beta+\gamma) + mRw(2\tau(1+\gamma) + \beta(2\tau-1)) + \tau^2 w^2(1+\gamma))}{\theta(mR(1+\beta+\gamma) + \tau w(1+\gamma))^2}$$

Similar to the informal pension system, a pay-as-you-go financed pension system has positive and negative fertility effects. It decreases the adult budget while the retirement budget can again increase or decrease depending on the strength of interest and base of pension payments effects. The possible decreasing retirement income effect again incorporates the link between economic growth and fertility. Because the pension system acts growth diminishing, the interest of pension contributions decreases while the base of pension paments is increasing.

The overall fertility effect is again dependent on the variable values of $R, m, w, \beta, \gamma, \tau$ and θ. The variable values decide whether the fertility increasing effect of higher pension payments base or the fertility decreasing effects of lower pension contribution interest payments and lower adult budget are dominant.

Proposition 6 *Depending on whether* $-\gamma(m^2(1+\beta+\gamma)R^2 + m(2(1+\gamma)\tau + \beta(2\tau-1))Rw + (1+\gamma)\tau^2 w^2)$ *is bigger or smaller than* 0*, a pay-as-you-go pension system leads to an increase or a decrease of fertility.*

Notice that if $-\gamma(m^2(1+\beta+\gamma)R^2 + m(2(1+\gamma)\tau + \beta(2\tau-1))Rw + (1+\gamma)\tau^2 w^2) < 0$ implying that $\frac{\partial n}{\partial \tau} < 0$ the savings reducing effect of a pay-as-you-go pension system is stronger than the fertility decreasing effect because the overall growth effect is always negative ($\frac{\partial g}{\partial \tau} < 0$).

2.3 Fully funded public pension system

Following the already stressed argument that intrafamilial gifts are not considered in the household optimization if a public pension systems is present, we assume fully crowded out private intergenerational transfers. Compared to the previous sub chapter only the retirement budget constraint and the capital market equilibrium change. Pension benefits are now financed through own contributions during adulthood which are invested in the capital market, paying the gross interest rate R_{t+1}. This clearly also changes the capital market equilibrium because the additional investments have to be considered. Notice that we again assume perfect foresight.

The balanced budget pension system constraint changes to:

$$\pi_{t+1} = \tau \bar{W}_t R_{t+1}$$

Capital market equilibrium is represented through:

$$K_{t+1} = N_t(s_t + \tau W_t)$$

2.3.1 Households

A fully funded pension system abstracts from the idea of intergenerational transfers. The system finances future pension benefits through own mandatory contributions which are invested in the capital market. Because no transfers from children to their parents are taking place fertility completely exits the retirement budget constraint.

$$\begin{aligned} V_t &= \log(c_t) + \beta \log(c_{t+1}) + \gamma \log(n_t) \\ c_t &= W_t(1 - \theta n_t - \tau) - s_t \\ c_{t+1} &= s_t R_{t+1} + \tau R_{t+1} \bar{W}_t \end{aligned}$$

Like in the previous scenarios the first order conditions again control the equalization between marginal benefit over time and between the two different investment opportunities savings and fertility.

$$\frac{1}{c_t} = \frac{\beta R_{t+1}}{c_{t+1}}$$

$$\frac{\theta W_t}{c_t} = \frac{\gamma}{n_t}$$

Solving the equations for s_t and n_t gives us the following optimal household decisions :

$$s_t = \frac{\beta W_t}{(1+\beta+\gamma)} - \tau W_t$$

$$n_t = \frac{\gamma}{(1+\beta+\gamma)\theta}$$

Aggregate savings are positive as long as $\beta > \tau(1+\beta+\gamma)$. Therefore the maximum pension contribution tax is determined by:

$$\tau^{\max} = \frac{\beta}{(1+\beta+\gamma)}$$

2.3.2 Equilibrium

Input prices and the new capital market equilibrium condition define economic growth g.

$$g = \frac{\widehat{k}_{t+1}}{\widehat{k}_t} = \frac{K_{t+1} N_t}{K_t N_{t+1}} = \frac{s_t + \tau W_t}{n_t A_t m}$$

Use optimal fertility and savings decision to show that g is again constant. This fact together with constant fertility defines a balanced growth path equilibrium.

$$g = \frac{\beta\theta w}{\gamma m}$$

Growth and fertility are independent of τ, implying that a fully funded pension system has no influence on their equilibrium values. The only effect of the funded pension system is the reduction of savings which is equivalent to the amount of pension contributions. Pension contributions, invested in the capital market exactly work like savings offsetting the impact of fully funded pension contributions on capital accumulation. Consumers anticipate additional future payments and therefore reduce savings exactly by the same amount reproducing the Ricardian equivalence theorem which states that economic growth is neutral towards fully funded pension contributions.

$$\frac{\partial n_t}{\partial \tau} = 0; \frac{\partial g}{\partial \tau} = 0$$

Proposition 7 *A fully funded pension system has no impact on economic growth and fertility.*

3 Public pension system implementation

This section highlights the impact of different types of pension system introductions on per capita production growth and fertility. For this reason we bring our already obtained results together and compare. In the first step an informally organized pension system is compared to a pay-as-you-go public pension system. Variables with indices inf and *pay* respectively indicate the informal and pay-as-you-go case. For a direct comparison of the results one has to assume that the part of income used for private intergenerational gifts τ of the informal system is equal to the pension contribution tax rate τ of the pay-as-you-go system. This implies an equal level of adult pension contributions for both pension systems. All other variables are assumed to be independent of the pension system type.

To analyze growth implications, one has to start by examining the effects on fertility and savings. Fertility of the two pension systems is represented by:

$$\begin{aligned} n^{\inf} &= \frac{(1-\tau)(w(\beta+\gamma)\tau + m\gamma R)}{\theta(1+\beta+\gamma)(Rm+\tau w)} \\ n^{pay} &= \frac{\gamma(1-\tau)(Rm+\tau w)}{\theta((1+\beta+\gamma)mR + (1+\gamma)\tau w)} \end{aligned}$$

Proposition 8 *An introduction of a pay-as-you-go pension system to an economy with informal pension system leads to lower population growth ($n_t^{\inf} > n_t^{pay}$).*

This is the case since the fertility increasing old age security motive is completely crowded out by the public pension system.

Proof. Rewrite optimal informal fertility to get:

$$n^{\inf} = \underbrace{\frac{1+\frac{\beta\tau w}{\gamma(Rm+\tau w)}}{1+\frac{\beta\tau w}{(1+\beta+\gamma)mR+(1+\gamma)\tau w}}}_{>1} \cdot \underbrace{\frac{\gamma(1-\tau)(Rm+\tau w)}{\theta((1+\beta+\gamma)mR+(1+\gamma)\tau w)}}_{n_t^{pay}}$$

Since the first term is bigger than 1 informal fertility is higher than pay-as-you-go fertility ($n^{\inf} > n^{pay}$). ∎

As we assumed for an economy without a public pension system own children are financing the pensions of their parents. Fertility decision have therefore a direct influence on retirement period consumption which will be reconsidered in the optimization process. A pay-as-you-go public pension system finances pensions through the average number of children. Therefore instead of own the average number of fertility enters the retirement budget constraint neglecting the security motive of fertility in the household optimization. In other words, economies with a pay-as-you-go public pension system are represented by households which do not expect own fertility decisions to have an influence on their pension benefits. Households living in an economy with informal pension system clearly do so because their pensions benefits are paid directly by their own children. This leads to the feature of our model that marginal benefits of procreation are decreasing if a public pension system is introduced because the security value of fertility cancels out.

Now compare savings $s_t^{\inf}$ and s_t^{pay} to see that the introduction of a pay-as-you-go pension system increases savings.

$$\begin{aligned} s_t^{\inf} &= \frac{m\beta RW_t(1-\tau)}{(1+\beta+\gamma)(w\tau+mR)} \\ s_t^{pay} &= \frac{m\beta RW_t(1-\tau)}{mR(1+\beta+\gamma)+\tau w(1+\gamma)} \end{aligned}$$

Proposition 9 *An introduction of a pay-as-you-go pension system to an economy with informal pension system acts savings increasing ($s_t^{\inf} < s_t^{pay}$).*

The positive change in savings is due to the fact that the public pension system reduces the crowding out effect of intergenerational transfers on savings. This is the case because the decreasing effect of the public system on the value of a child transfers income from procreation to savings.

After the examination of savings and fertility we are in the position to analyze impacts on economic growth. The informal and pay-as-you-go cases are represented by the following growth rates:

$$\begin{aligned} g^{\inf} &= \frac{\beta\theta Rw}{\beta\tau w+\gamma(mR+\tau w)} \\ g^{pay} &= \frac{\beta\theta Rw}{\gamma(mR+\tau w)} \end{aligned}$$

Proposition 10 *The introduction of a pay-as-you-go pension system to an economy with informal pension system increases economic growth since fertility decreases and savings increase ($g^{\inf} < g^{pay}$).*

The absence of private altruistic transfers from children to the old leads to positive impacts on both growth determining effects. Savings go up and fertility goes down. Our outcomes are closely connected to the results derived by Zhang and Zhang (1995) who show that a pay-as-you-go public pension system increases per capita output growth and reduces fertility compared to a fertility related security system. In contrast to Zhang and Zhang we do not only model ascending altruism from adult to old but also descending altruism from adult to the young by including the consumption good motive of fertility. This enables us to study the pay-as-you-go public pension system with endogenous fertility in the framework of fully crowded out private transfers. Because Zhang and Zhang are only modeling the security value of children they can not cover this case because the marginal benefit of procreation would become zero.Their model can be seen as covering the transition period between an informal system and a public pension system where private gifts are still positive nevertheless a public social security system is already present. Our model is focusing on the final period when the transition is already finished. The different periods could be reasoned by different levels of trust in the public pension system. People do not fully trust the public system during the adjustment period and therefore still support their parents with private gifts. The final period is characterized by zero gifts because the households have already adjusted their behavior. Our assumption of fully crowded out private gifts does not change the direction of the growth effect but changes its level. Pay-as-you-go pension system introduction leads in our model to lower future capital compared to an informal pension system and therefore to lower growth compared to Zhang and Zhang. This is the case because the Zhang and Zhang model compensates the negative effect of pension contributions on capital accumulation through a decrease of gifts while this is not possible in our model since we assume already fully crowded out gifts.

After clarifying the growth and fertility impacts caused by a pay-as-you-go public pension system introduction to an informal pension system economy we focus towards the impacts of the introduction of a fully funded (index ff) public pension system which is the most prominent alternative to a pay-as-you-go public pension system in reality. Use the results from the pervious chapters to describe informal and fully funded fertility:

$$\begin{aligned} n^{\text{inf}} &= \frac{(1-\tau)(w(\beta+\gamma)\tau + m\gamma R)}{\theta(1+\beta+\gamma)(Rm+\tau w)} \\ n^{ff} &= \frac{\gamma}{(1+\beta+\gamma)\theta} \end{aligned}$$

Proposition 11 *The level of pension system contribution tax τ decides about whether the introduction of a fully funded system to an economy without working social security leads to lower or higher population growth. While for $\tau < \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$ informal fertility is higher than fully funded fertility ($n^{\text{inf}} > n^{ff}$), $\tau > \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$ or $\frac{\beta}{\beta+\gamma} < \frac{Rm\gamma}{w(\beta+\gamma)}$ results in lower informal fertility than fully funded fertility ($n^{\text{inf}} < n^{ff}$). Informal and fully funded fertility are identical if $\tau = \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$.*

Proof. Rewrite informal fertility to get:

$$n^{\text{inf}} = \frac{\gamma}{\theta(1+\beta+\gamma)} \cdot \frac{(1-\tau)(w\beta\tau\frac{1}{\gamma} + w\tau + mR)}{(Rm + \tau w)}$$

For positive pension contributions three cases are observable:

- Case 1: if $\tau < \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$, the second term of n^{inf} is bigger than 1 and $n^{\text{inf}} > n^{ff}$.
- Case 2: if $\tau > \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$ or $\frac{\beta}{\beta+\gamma} < \frac{Rm\gamma}{w(\beta+\gamma)}$, the second term of n^{inf} is smaller than 1 and $n^{\text{inf}} < n^{ff}$.
- Case 3: if $\tau = \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$, the second term of n^{inf} cancels out and $n^{\text{inf}} = n^{ff}$.

■

The different cases are showing that the amount of income contributed to the pension system decides whether fertility is higher or lower. This is the case because informal fertility can decrease or increase depending on the variable values which decide about whether the decreasing effect on informal growth and available adult income or the increasing effect on pension payments base is stronger. The contrary effects are exactly offset if $\tau = \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$. In this case informal and fully funded fertility are equal and independent on contribution payments. If $\tau < \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$ the effect of lower informal growth and lower available adult income is weaker than the effect due to increasing pension payments base and informal fertility is higher than fully funded fertility. $\tau > \frac{\beta}{\beta+\gamma} - \frac{Rm\gamma}{w(\beta+\gamma)}$ implies exactly the opposite leading to lower informal fertility than fully funded fertility.

After the description of the growth determining fertility effect we focus towards the capital accumulation effect to fully understand the overall growth effect. A fully funded pension system invests the whole part of income reserved for retirement consumption in the capital market and therefore reaches the same capital stock than without a pension system. Capital holdings are clearly higher than in the informal case leading to a growth enhancing effect since in the Grossmann Yanagawa endogenous growth model growth is driven by labor productivity that is determined by capital intensity.

$$
\begin{aligned}
g^{\inf} &= \frac{\beta\theta R w}{\beta\tau w + \gamma\tau w + m\gamma R} \\
g^{ff} &= \frac{\beta\theta w}{\gamma m}
\end{aligned}
$$

$$g^{\inf} < g^{ff}$$

Proposition 12 *The introduction of a fully funded public pension system to an economy with informal social security system leads to higher economic growth.*

This implies that even for case 2 where fully funded fertility is higher, the growth increasing effect of higher capital accumulation is dominant.

Now we are in the position to state that countries aiming to increase per capita production growth should introduce a public pension system no matter whether the system is funded or unfunded. If the main goal is to decrease population growth only the pay-as-you-go pension system is useful for all contribution levels. To draw light on the question whether it is better to introduce a funded or unfunded system we now focus on the comparison of the two public pension systems.

Pay-as-you-go fertility and fully funded fertility are represented through:

$$
\begin{aligned}
n^{pay} &= \frac{\gamma(1-\tau)(Rm+\tau w)}{\theta((1+\beta+\gamma)mR + (1+\gamma)\tau w)} \\
n^{ff} &= \frac{\gamma}{(1+\beta+\gamma)\theta}
\end{aligned}
$$

Proposition 13 *The tax rate level τ decides about whether pay-as-you-go fertility is higher or lower than fully funded fertility. If $\tau < \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ fertility is higher in the pay-as-you-go system ($n^{pay} > n^{ff}$). If $\tau > \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ or $\frac{\beta}{(1+\beta+\gamma)} < \frac{Rm}{w}$ fertility is lower in the pay-as-you-go system ($n^{pay} < n^{ff}$). For the case where $\tau = \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ both systems lead to identical fertility decisions.*

Proof. Reformulate pay-as-you-go fertility to get:

$$n^{pay} = \frac{\gamma}{\theta(1+\beta+\gamma)} \cdot \frac{(1-\tau)(Rm+\tau w)}{mR + (1+\gamma)\tau w\frac{1}{(1+\beta+\gamma)}}$$

Now check if the second term on the right side is smaller, bigger or equal to 1. Therefore analyze if the nominator $(1-\tau)(Rm+\tau w)$ is bigger or smaller than the denominator $(mR + (1+\gamma)\tau w \frac{1}{(1+\beta+\gamma)})$.

- Case 1: if $\tau < \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ the second term is bigger than 1 and $n^{pay} > n^{ff}$.
- Case 2: if $\tau > \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ or $\frac{\beta}{(1+\beta+\gamma)} < \frac{Rm}{w}$ the second term is smaller than 1 and $n^{pay} < n^{ff}$.
- Case 3: if $\tau = \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ the second term is equal to 1 implying that $n^{pay} = n^{ff}$.

■

The three cases are again dependent on the strength of the different underlying pay-as-you-go contribution payment effects on fertility. Case 3 corresponds to the case where the fertility diminishing effect of lower growth and lower available adult income is offset by the fertility increasing effect of higher pension payments base. This is only possible if the overall insurance value of a child and therefore also the retirement budget increases with an increase in contribution payments. If $\tau > \frac{\beta}{(1+\beta+\gamma)} - \frac{Rm}{w}$ the negative growth and adult budget effects on fertility are higher than the positive effect through higher pension payments base implying that pay-as-you-go fertility is lower than fully funded fertility. Case 2 describes the opposite leading to higher pay-as-you-go fertility than fully funded fertility.

Now assume a Cobb-Douglas production function of the form:

$$F(A_tL_t, K_t) = K_t^{\alpha}(A_tL_t)^{1-\alpha}$$

Use the results for factor prices $w = m^{\alpha}(1-\alpha)$ and $R = \alpha m^{\alpha-1}$ to reformulate the threshold contribution level for the three above mentioned cases:

$$\tau \gtreqless \frac{\beta}{(1+\beta+\gamma)} - \frac{\alpha}{1-\alpha}$$

Because τ, β and γ are assumed to be positive and smaller than 1, case 1 is true implying that pay-as-you-go fertility is lower than fully funded fertility ($n^{pay} < n^{ff}$) if we set α equal to 1/3 which is standard in the literature.

Proposition 14 *If the production function is Cobb-Douglas and $\alpha = 1/3$ pay-as-you-go fertility is lower than fully funded fertility ($n^{pay} < n^{ff}$).*

While the general result for fertility comparison is case dependent, the result for growth comparison is not. Per capita production growth corresponding to the funded pension system is always higher than for the pay-as-you-go system.

$$
\begin{aligned}
g^{pay} &= \frac{\beta\theta R w}{\gamma(mR + \tau w)} \\
g^{ff} &= \frac{\beta\theta w}{\gamma m}
\end{aligned}
$$

Proposition 15 *A pay-as-you-go pension system leads to lower economic growth than a fully funded one ($g^{pay} < g^{ff}$).*

Our model therefore reproduces the classical result for models with exogenous fertility by Feldstein (1998). From the derivation of growth we know two growth effects. Capital accumulation is growth enhancing and fertility is growth diminishing. If fully funded growth is always higher than pay-as-you-go growth despite higher fully funded fertility for the Cobb-Douglas case with $\alpha = 1/3$, fully funded capital accumulation has to be higher than the one for a pay-as-you-go system. This is the case because the only possibility for the funded pension system to beat the growth increasing effect of lower pay-as-you-go fertility is to have an even stronger positive growth effect through higher future capital. We follow that savings plus pension contributions corresponding to a funded pension system are higher than savings for a pay-as-you-go pension system ($s_t^{ff} + \tau W_t > s_t^{pay}$) for the described Cobb-Douglas production function. To understand the result one has to examine the different effects on capital accumulation. While all pension contributions are always savings reducing because they transfer income to the future and reduce uncertainty, the type of the system decides about the impact on capital accumulation. Fully funded pension contributions exactly act like savings because they are invested in the capital market and therefore do not change capital accumulation. In contrast pay-as-you-go contributions which go directly from the adults to the old reduce future capital despite the fact that pay-as-you-go savings can be higher than fully funded. This is the case because contributions are not invested in the capital market and the savings reducing effect of pension contributions can not be offset.

The result that pay-as-you-go-growth is always lower than fully funded growth further implies that the growth enhancing effect of lower pay-as-you-go fertility can not compensate the growth decreasing effect of lower pay-as-you-go future capital. This is contrary to the findings of Yoon,Talmain (2001) who study exactly the same question similar to the already mentioned Zhang and Zhang model in a positive private transfer framework without descending altruism. The different result is again driven by our assumption of zero interfamilial intergenerational transfers which omits the growth increasing effect of gift reductions.

4 Numerical Example

The theoretical results obtained in the previous sections show that pension systems influence growth through impacts on fertility and capital accumulation. While the growth impacts of the different pension systems can clearly be ranked, the variable values of R, m, w, γ, β and τ decide about whether fully funded fertility is higher or smaller than informal fertility. To answer this question, we calibrate our model for an average Sub-Saharan as well as for an average OECD country inside a Cobb-Douglas production function economy.

We further use the calibrated model to produce new insights in the observed fertility and growth differences for the United States and Europe.

4.1 Calibration

The parameters are chosen such that the balanced growth path equilibrium matches the empirical features of an average OECD country with a pay-as-you-go pension system. Adult and retirement period have a length of 30 years, childhood of 15 years implying a life expectancy of 75 years. Due to empirical findings we set capital productivity α equal to 1/3. The discount factor β is assumed to be 0.99 per quarter of a year corresponding to the standard real-business-cycle literature. In our 30 years per adult period framework this corresponds to 0.99^{120}. Following Doepke and Croix (2003) child rearing cost, measured through the time parameter θ, corresponds to 15% of adult working time. Since we assumed that childhood only lasts for 15 years θ is set equal to 0.075. Pension contribution rate τ is chosen to be equal to the OECD average of 30%. This number together with the child rearing cost limits maximum fertility to 5.7 children per person. We further choose the descending altruism factor γ to be 0.142 and the technology parameter m controlling the influence of capital intensity equal to 0.0069 because these variable reproduce a steady state fertility rate at the reproduction level $n_t = 1$ and a steady state per capita output growth rate of 2% per year. The values of m and α are further implying an interest rate of 7.67%.

The chosen variable values reproduce our theoretical result that the pay-as-you-go system leads to lowest fertility. Additionally we show that for the observed contribution rate informal fertility is clearly higher than fully funded fertility. This is the case since the fertility increasing effects of higher pension payments base is dominating the fertility decreasing effect of reduced growth and adult budget. Only if pension contributions are unrealistically higher than 51.6% of adult income the fully funded system produces higher fertility than the informal one (see table 1). Since aggregate savings can not be negative these cases can be excluded and we follow that informal fertility is higher than fully funded fertility.

Table 1: Fertility dependence on τ	$\tau = 0.3$		$\tau = 0.516$		$\tau = 0.7$	
	n_t	g_t	n_t	g_t	n_t	g_t
Informal Pension System	1.65	1.01	1.31	0.69	0.88	0.54
Pay-as-you-go Pension System	1	1.81	0.71	1.43	0.45	1.21
Fully Funded Pension System	1.32	2.9	1.32	2.9	1.32	2.9

Graphical examination of the results (see figure 1) shows that informal fertility creates a hump shaped curve in a fertility and pension contribution rate plane. The behavior of the curve is reflecting the strength of the underlying effects which are dependent on the level of pension contributions τ. Hump shaped behavior can only be observed for the informal pension system where the old age security motive is still present. As the contribution payments per child increase the insurance motive becomes less important while the negative growth effect becomes stronger. At the fertility maximum the effects are offset. A further increase of τ leads to decreasing fertility. Despite the narrow scope of our simple analysis the comparison of fully funded and pay-as-you-go fertility suggests that fertility differences between the US and Europe can partly be explained by the different types and not only by the different contribution levels (Boldrin, De Nardi and Jones (2005)) of pension systems. The US, where pensions are mainly financed through a funded system show a Total Fertility Rate (TFR) of 2, while Europe, represented through mainly pay-as-you-go pension systems, shows a TFR of 1.4.

Figure 1:

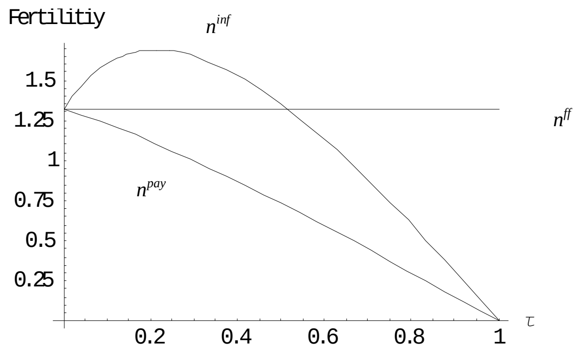
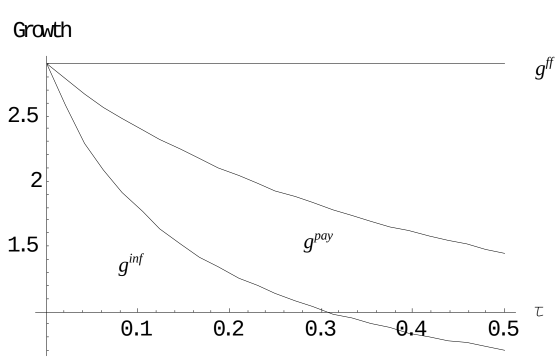


Figure 2 shows the growth diminishing effect of an increase in the pension contribution rate for an informal and a pay-as-you-go pension system. Like in the theoretical results, the pay-as-you-go growth level is always higher than the informal one since growth reducing fertility is lower and growth increasing savings are higher. If the contribution rate is too high the informal as well as the pay-as-you-go pension system could lead to negative growth. Fully funded growth is graphically represented by a horizontal line since it is independent on the pension contribution rate.

This suggests that also growth differences are dependent on the type of pension system. Higher US growth compared to European growth can therefore partly be explained by the regions differences in pension funding.

Figure 2:



The second numeric example is dealing with developing countries. Therefore the parameters are chosen such that the balanced growth path equilibrium matches the empirical features of an average Sub-Saharan country. Adult and retirement period have a length of 15 years, childhood of 10 years which is implying a life expectancy of 40 years. Capital productivity and the discount rate per quarter are equal to the OECD case ($\alpha = 1/3$ and $\beta = 0.99$). In our 15 years adulthood and retirement age framework the discount rate corresponds to the value $\beta = 0.99^{60}$. Child cost measured through the parameter θ are expected to be lower than for the OECD case since in informally organized societies children are looked after by a broader sense of the family which can even take the form of a village unity. Taking the above 7.5% of working time for OECD Countries into account we choose child raring cost for developing countries to be less than the OECD level and set θ equal to 0.042. This number leads together with the observed fertility rate of 2.75 [4] to a descending altruism factor γ equal to 0.117 which is only slightly smaller than the value for the OECD case. This is creating additional support for our child rearing cost choice since we can not see any reason why descending altruism representing the genetic imprint to procreate should be much different for developing countries. The parameterization of τ for the developing case is quite tricky since no data about social mandatory contribution is available. Therefore we again use the observed average benefits for OECD countries which are around 30% of working income and divide them through the steady state level of fertility to get $\tau = 0.11$. We implicitly assume that 30% of adult working income plus the own fruit of savings are high enough to finance a sufficient level of retirement consumption. We further use the growth rate of 0.6%[5] per year to set the technology parameter m, controlling the influence of capital intensity on labor productivity, equal to 0.012. The technology parameter which is governing the transition of capital intensity to labor productivity m is higher than the OECD one, implying the lower technological standard. Our numerical developing country example implies an interest rate of close to 13% what can partly be justified by existing risk prime.

Our variable values again result in lowest fertility for the fully funded system (see table 2). An unrealistically high contribution rate of $\tau = 73.5\%$ is needed to equal fertility levels for the informal and fully funded pension system. Positive aggregate savings again exclude these high levels of the contribution rate.

[4] World Population Data Sheet 2006.

[5] Sub-Saharan average for 1990-2004.

Our example therefore implies that the informal pension system leads to lower fertility than the fully funded one.

Table 2: Fertility dependence on τ	$\tau = 0.11$		$\tau = 73.5$		$\tau = 0.8$	
	n_t	g_t	n_t	g_t	n_t	g_t
Informal Pension System	2.75	1.09	1.68	0.26	1.3	0.24
Pay-as-you-go Pension System	1.59	2.01	0.55	0.99	0.42	0.95
Fully Funded Pension System	1.68	2.46	1.68	2.46	1.68	2.46

Graphical examination of the outcomes (see figure 3) shows that pay-as-you-go fertility and informal fertility, drawn in a fertility and pension contribution rate plane, create a hump shaped curve. Increasing pension contribution rates are leading to increasing fertility as long as the positive utility effect through higher retirement budget is dominant. At the maximum the increasing effects are offset by the decreasing growth and adult budget effects. From this level of τ onwards fertility is decreasing.

Figure 3:

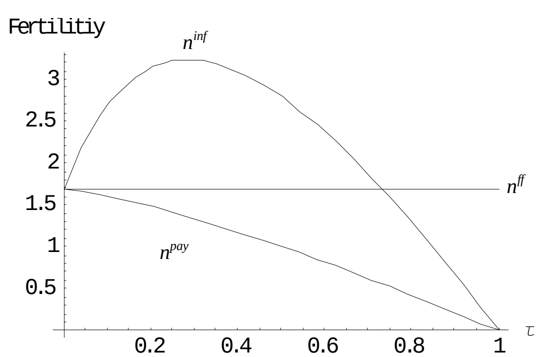
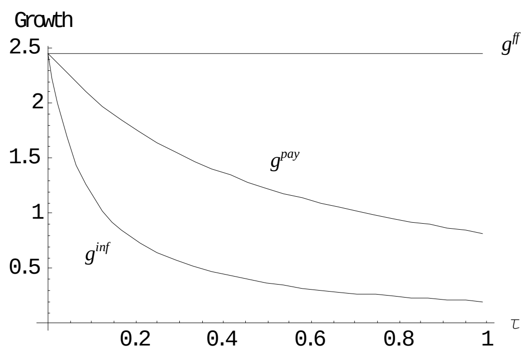


Figure 4 shows the growth diminishing effect of an increase in the pension contribution rate for an informal and a pay-as-you-go system. Like in the theoretical results prompted the pay-as-you-go growth level is always higher than the informal one since growth reducing fertility is lower and growth enhancing savings are higher. Fully funded growth is graphically represented by a horizontal line since it is independent on the pension contribution rate.

Now we are in the position to give a full description of pension system impacts to a developing economy. The fully funded Pension system clearly leads to highest economic growth while the pay-as-you-go one produces lowest fertility. Dependent on whether the reduction in fertility or the increase of per capita growth is the main task of the governmental program the pay-as-you-go or fully funded system should be introduced. Independent on this question any of the two described public pension systems lead compared to an informal pension system to a preferable outcome.

Figure 4:



5 Conclusion

In this paper we addressed two important questions concerning the influence of different pension systems on economic growth and fertility. The first deals with the consequences of public pension system introduction to a developing economy represented by an informal pension system, the second adresses which of the two public pension systems is preferable.

We show that no matter if the introduced public system is funded or pay-as-you-go the consequences on economic growth are positive. The theoretical results for a pay-as-you-go pension system introduction highlight that both growth determining effects, capital accumulation and fertility, are growth enhancing while two implied growth effects of a fully funded pension system introduction are not that clear. A fully funded pension system is increasing capital accumulation while fertility can be higher or lower depending on the level of pension contribution τ. The overall effect of a fully funded pension system introduction on economic growth is positive for all cases indicating that the growth enhancing effect of higher future capital is dominating the possible growth diminishing effect of higher fertility. The numerical example at the end of the study shows, that for realistic contribution levels the case of higher fertility can be excluded.

Within the debate about the impact of different public pension systems on growth, works incorporating endogenous determined fertility (Zhang and Zhang (1995), Yoon and Talmain (2001)) usually create the result that a pay-as-you-go public pension system implies higher growth than a fully funded one. Our work contributes to the topic by showing that the growth enhancing effect of a pay-as-you-go pension system is only driven by the inclusion of intrafamilial intergenerational gifts. The use of a mixed procreation motive (Wigger (1999)) allows us to study the influence of pension systems on growth and fertility in a framework of fully crowded out gifts. This creates the result that a fully funded pension system implies higher growth than a pay-as-you-go one reestablishing the conventional "exogenous fertility view" of a growth diminishing pay-as-you-go pension system (Feldstein and Samwick (1998)). The reult is implying that the growth increasing lower pay-as-you-go fertility can not outweight the growth decreasing effect of lower capital accumulation if no private interfamilial transfers take place. Developing countries which are especially crippled by high population growth can therefore be better off by introducing a pay-as-you-go instead of a fully funded pension system despite the corresponding lower economic growth.

The numerical example quantifies the impacts of the different pension systems and highlights that a part of the growth and fertility differences observed for the US and Europe can solely be explained by the different types of pension system even if the contribution rates are exactly the same.

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