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Abstract

We analyze imperfect competition in dynamic environments where firms use rivalrous but nonexcludable industry-specific capital that is provided exogenously. Capital depreciation depends on utilization, so firms influence the evolution of the capital equipment through more or less intensive supply in the final-goods market. Strategic incentives stem from, (i) a dynamic externality, arising due to the non-excludability of the capital stock, leading firms to compete for its use (rivalry), and, (ii) a market externality, leading to the classic Cournot-type supply competition. Comparing alternative market structures, we isolate the effect of these externalities on strategies and industry growth.

Key Words: Cournot competition, oligopolistic non-cooperative dynamic games

JEL classification: D43, D92, L13, O12, Q20

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1. Introduction

The role of capital deepening on economic growth is usually studied through highly aggregated growth models. In these models, typically focusing on the macroeconomy, perfect competition is the prevailing market structure. Yet, little attention has been paid to the forces and incentives behind capital deepening and growth in smaller markets, like industries.

In a significant number of industries, firms sell their final products in imperfectly competitive markets. Classic Cournot-type oligopolistic incentives arise in such industries. Yet, whenever specific capital is useful for production, in addition to its supply strategy, each firm's intertemporal capital allocation is a strategic lever. By accumulating capital, a firm can retain a high productive capacity, and boost its oligopolistic competitiveness and profit margin over time.

We focus on industries with production that relies on the use of specific types of capital, infrastructure or specific equipment, which is provided exogenously. Capital in our analysis is nonexcludable, it can be used at no cost by all firms, and also rivalrous, subject to a first-come first-served rule or to congestion costs. Examples of publicly provided infrastructure are airports, harbors, roads, pipe-lines, transmission grids, railroads, or telecommunications lines. Governments determine and control the full provision and the growth rate of the available public infrastructure. Typically, governments follow rules for financing its maintenance or growth.[1]

Another key feature of capital in our study is that it depreciates with utilization. Yet, the evolution of specific capital, and, consequently the long-run growth of industries that use it, depend on the production, i.e., aggregate supply decisions of firms; more intensive use increases the cost of its maintenance or speeds up depreciation. Often, governments impose

[1] For example, a government may use a constant fraction of the fiscal budget every year in order to finance a certain type of infrastructure.

special taxes on firms for the use of roads or ports, in order to slow down the depreciation of these structures.

Another important example of specific capital is human resources in basic research. Basic research activities, usually taking place in universities, target inventions. Research for marketable innovations, on the other hand, is usually financed by firms. Industries may take human resources out of universities, out of basic research activities, in order to direct them to market-oriented innovations. For example, the high attrition rates in computer-science graduate programs or engineering, can be explained by the effort of companies to tempt students to work on innovations concerning their IT products, by offering higher salaries. The time of researchers is rival and also wasted from producing inventions, a critical determinant for the growth rate of revolutionary new knowledge.

The classic Cournot quantity competition among firms stems from the fact that the presence of a firm in the market constitutes a *market externality* for its market competitors. Unlike perfectly competitive environments with price-taking behavior, the supply strategies of competitors directly enter the revenue function of a firm, a fundamental part of the firm's profit maximization objective.[2]

Yet, the nonexcludability and rivalry of capital allows more than one firms to utilize capital from the same source, with all firms contributing to the depreciation of the stock. From the viewpoint of each firm, competitors diminish future production possibilities. This creates a commons problem, an additional competitive element to this of Cournot supply competition. The presence of a firm in the common pool for capital utilization, constitutes a *dynamic externality* for its competitors for capital utilization.

Our framework also fits, directly, industries with production based on natural resources.

[2] Several recent papers still deal with the issue of existence and uniqueness of Cournot-Nash equilibrium in static frameworks. See, for example, Gaudet and Salant (1991), Novshek (1984a), (1984b) and (1985).

The typical commons problems that have been studied in the literature are not limited, however, to these applications.[3] They are transparent in other industries, as in the above examples, that utilize capital from outside providers and the capital is rivalrous and non-excludable.

In order to examine the link between the market structure and industry growth, the key, and our focus, is to uncover the impact of each externality, the market externality and the dynamic externality, on the supply strategies of firms. The type of non-cooperative dynamic games that we study, involve infinite-horizon strategic optimal control.[4] To our knowledge, so far, only the impact of the dynamic externality has been studied in the literature. So, introducing the market externality is a crucial contribution of this paper.

The nature of the two externalities brings significant technical complexities into the optimal control problem faced by each firm in such non-cooperative games. The presence of these externalities in the dynamic game makes the strategies of competitors part of the structural maximization problem of a firm. In order that the problem of each firm be well-defined, the primitives of the model should imply that the equilibrium strategies possess convenient functional properties. So, we model the economic environment so as to obtain linear equilibrium strategies that make the analysis tractable. In order to accomplish this, we use homogeneous or isoelastic functions to capture consumer demand and the primitives of firms. Such parametric functions are the most commonly used in both theoretical and applied analysis of dynamic problems.

To meet our goal, which is to study the *economic impact* of the two externalities on

[3] See, for example, Mirman (1979), Levhari and Mirman (1980), who were the first to analyze the commons problem using noncooperative dynamic games, and the following work by Amir (1989), Sundaram (1989), Benhabib and Radner (1992), Dutta and Sundaram (1992) and (1993), and Sorger (2004) who apply their analysis to natural-resource games, focusing mostly on the commons problem.
[4] Other terms for the class of dynamic games we examine is "Markovian games," or, as in Cohen and Michel (1988), "memoryless feedback Nash games."

industry growth, our parametric model is composed of a set of primitives that encompasses alternative market structures, which are compared analytically. First, we characterize the strategy of a dynamic monopolist. This is the benchmark case, where the output market is supplied by a monopolist who is also alone in influencing the evolution of the capital stock. Second, we consider the case of two monopolists utilizing capital from the same provider. In other words, there are two markets, each supplied by a monopolist. We compare the results in this market structure to the results of the pure monopolist case and find that the presence of the dynamic externality leads the two monopolistic firms to supply more in each period, compared to the pure monopolist. But more production in each period means more capital utilization, hence more capital depreciation. Therefore, the dynamic externality reduces capital growth, a 'tragedy of the commons' result, as in Levhari and Mirman (1980).

Third, when the two firms (both influencing the capital stock) also compete in the same market, both the dynamic and the market externality are present. Compared to the benchmark monopoly, we find that the aggregate supply of firms in each period is always higher. But compared to the market structure of two monopolists with common capital utilization, the impact depends on the model's primitives. We find that below a threshold level for the demand elasticity (depending only on the number of firms in the market), the aggregate supply of firms increases even more as compared to the structure of two monopolists. When the demand elasticity is higher than that threshold level, the aggregate supply of two monopolists can be higher than the aggregate supply of the duopolists, depending on the values of the other primitives. These primitives are: a relatively low convexity of the cost function, relatively low growth opportunities for capital, relatively high interest rates, and a relatively weak endogenous depreciation technology.

Last, to relax the assumption of non-excludable capital. We study two firms selling in the same market, but utilizing capital from separate exogenous providers. We show that this problem is very complex, as it involves two capital stocks and strategic considerations about the evolution of both stocks by both firms. Yet, we characterize, analytically, the symmetric strategies of firms within our parametric framework, which turn out to be linear with respect to the two capital stocks.

The presence of the two externalities, the dynamic and the market externality, in dynamic oligopolistic markets was first studied by Mirman (1979). Although Mirman (1979) does not present an analysis of the impact of the two externalities on strategies, the two elements are pointed out in necessary equilibrium conditions. Moreover, Mirman (1979) explores problems that can arise in dynamic oligopoly models under usual assumptions on the objective of each firm and on the dynamic constraints, assumptions that would lead to tractable decision rules in a standard optimal control problem. In particular, in the dynamic oligopoly case supply strategies may not, in general, be continuous functions. However, continuous differentiability of supply strategies of all firms is an important property for determining equilibrium strategies.

Mirman (1979) shows how linear demand functions lead to either a corner solution or an interior solution that is exactly the same as the static solution. It is clear from this analysis of Mirman (1979) that the linear demand model is not appropriate for addressing the issues raised in this paper. Thus, our parametric framework does not involve linear demand functions, but isoelastic ones. Koulovatianos and Mirman (2003b) study the link between market structure and industry growth when firms pursue cost-reducing knowledge accumulation through R&D investment using an alternative model specification, but they point out the same strategic elements behind firm behavior, namely the importance of the dynamic and the market externality.

There is little theoretical work in the literature dealing with the dynamic problem of firms interacting both in the market and for the utilization of capital. On the other hand, Ericson and Pakes (1995) show the importance of Markov-perfect dynamics in an imperfectly competitive environment for empirical work. Vedenov and Miranda (2001) and Pakes and McGuire (2001) discuss numerical procedures for oligopoly games with accumulation of some state variable. Both studies suggest ways of overcoming the several technical difficulties.

In section 2, we present the general formulation of three alternative market structures with nonexcludable rivalrous capital, pointing out some technical complexities for securing equilibrium existence and also for characterizing strategies. In section 3, we outline our parametric model and, through comparing equilibrium supply strategies among three alternative market structures, we reveal the role of the dynamic externality and the market externality in explaining the dependence of industry growth on the market structure. In section 4 we examine a setup where firms utilize capital from their own, exclusive provider, showing that the supply strategies in our parametric framework are linear.

2. Economic environment and alternative market structures

Time is discrete with an infinite horizon, $t = 0, 1, ...$ The inverse-demand function,

$$p_t = D(q_t) \ ,$$

characterizes consumer demand for the final good, q, in each period. Specific capital, denoted as k, is necessary for production. In particular, the production of q units of the final good requires that $\psi(q)$ units of capital are consumed by utilization, with $\psi'(q) > 0$. In the case where capital is some form of infrastructure, capital depreciates endogenously. Capital utilization wears equipment out, or it leaves less time for its maintenance.[5] In the case

[5] This idea of capital utilization is also studied by Greenwood et. al. (1988) in a general-equilibrium framework.

where capital is a natural resource (renewable or not) and units of this natural resource are necessary as raw material for producing the final good, it is straightforward that producing a certain level of output requires the *consumption* of part of this stock.

Capital obeys an exogenous rule of renewal, depending on previous period's stock, namely,

$$k_{t+1} = f(k_t) \ ,$$

with $f'(k) > 0$. When N firms exploit the same capital stock in order for each of them to produce a quantity, $q_{i,t}$ in period t, with $i \in \{1, ..., N\}$, the law of motion of capital is,

$$k_{t+1} = f(k_t) - \sum_{i=1}^{N} \psi(q_{i,t}) \ . \tag{1}$$

In our model, firms do not have the explicit option to add to the capital stock.[6] In order to produce output, a firm also needs to hire labor. The cost of hired labor, l, in each period is given by,

$$c_t = c(l_t) \ , \tag{2}$$

where $c' > 0$. The capital stock can have a positive effect on production capacity. More capital stock can augment the productivity of labor, giving the possibility to reduce production costs per output unit. In the case of infrastructure, more infrastructure may mean less congestion during productive activities. When capital is a natural resource, abundance of the resource may reduce search costs or costs of extraction of a unit of the resource. The production technology is given by,

$$q_t = F(k_t, l_t) \ , \tag{3}$$

[6] We present this extension in Koulovatianos and Mirman (2003b), where, as we mentioned in the introduction, the stock of capital is knowledge and firms make explicit investments out of their profits in order to achieve cost reducing innovations.

with $F_2 > 0$ and $F_1, F_{12} \geq 0$.[7] Applying the implicit function theorem, the production function given by (3) implies that, for a particular level of available capital, k, in order to produce a certain quantity of final good, q, at least

$$l_t = L(k_t, q_t) \tag{4}$$

units of labor need to be hired (where $L_1 \leq 0$, and $L_2 > 0$). Substituting the least necessary labor for producing q given by (4), into (2), the cost for producing a certain level of the final good, given a specific level of capital is,

$$c_t = c(L(k_t, q_t)) \equiv C(k_t, q_t) \ , \tag{5}$$

with $C_2 > 0$ and $C_1 \leq 0$.

2.1 The Dynamic Monopoly

The objective of the dynamic monopoly, our benchmark market structure, is to determine a supply-quantity decision rule as a function of the available capital, $q = Q(k)$, so that it maximizes its life-time profits,

$$\sum_{t=0}^{\infty} \delta^t \left[D(q_t) q_t - C(k_t, q_t) \right] \ , \tag{6}$$

given $k_0 > 0$ and with $\delta \equiv \frac{1}{1+r}$, the profit discount factor, determined by an exogenous constant interest rate $r > 0$.

The problem of the monopolist can be written in a Bellman-equation form,

$$V_M(k) = \max_{q \geq 0} \left\{ D(q) q - C(k, q) + \delta V_M(f(k) - \psi(q)) \right\} \ . \tag{7}$$

[7] Throughout the paper, whenever functions are multivariate, i.e. with $n > 1$ variables, we use numbered subscripts in order to denote the partial derivative with respect to the i-th argument of a function, where $i \in \{1, ..., n\}$.

Using the first-order condition implied by (7), and the envelope theorem to derive $V_M'\left(\widehat{k}\right)$, the marginal lifetime profits implied by the stock of capital that the firm leaves available for next period, yields the Euler equation,

$$\frac{D(q)+D'(q)\,q-C_2(k,q)}{\psi'(q)}=\delta\left\{-C_1\left(\widehat{k},\widehat{q}\right)+\frac{D(\widehat{q})+D'(\widehat{q})\,\widehat{q}-C_2\left(\widehat{k},\widehat{q}\right)}{\psi'(\widehat{q})}f'\left(\widehat{k}\right)\right\}\ ,\quad (8)$$

where $\widehat{q}$ is the output strategy of the firm in the subsequent period. A static monopoly would set the right-hand side of equation (8) to zero. On the other hand, the dynamic monopoly takes into account the influence that its current supply has on the evolution of capital in the future. Supplying more in the current period reduces the capital stock in the future, so, the firm's cost per unit of output increases. The Euler equation (8) serves as the benchmark equation for understanding the strategic elements that appear in other market structures, when the dynamic and the market externality are introduced.

2.2 Two monopolists utilizing capital from the same provider

In this subsection we look at two identical firms A and B, each selling in its own market as a monopolist, facing the same inverse demand function, having the same cost function, and utilizing capital from the same provider. So, capital evolves according to,

$$k_{t+1}=f(k_t)-\psi(q_{A,t})-\psi(q_{B,t})\ . \quad (9)$$

Compared to the monopoly problem of the previous subsection, the two monopolistic firms have a direct capital-accumulation interaction. We say that the presence of both firms using the same source of capital gives rise to a *dynamic externality*.[8]

[8] The term 'dynamic externality' was first introduced by Mirman (1979). Levhari and Mirman (1980) provide another model that offers an explicit analysis of the dynamic externality. It is important to stress that the corresponding form of our equation (9) in the Levhari and Mirman (1980) setup is,

$$k_{t+1}=f(k_t-\psi(k_{A,t})-\psi(k_{B,t}))\ ,$$

We denote the value function of the two monopolistic firms with a direct capital-accumulation interaction as $V_{A,m}$ and $V_{B,m}$.[9] Due to the symmetry of the setup we can focus on the problem of firm A without loss of generality. The problem of firm A in a Bellman-equation form is given by,

$$\begin{aligned} V_{A,m}(k) = & \max_{q_A \geq 0} \{ D(q_A) q_A - C(k, q_A) q_A + \\ & + \delta V_{A,m}(f(k) - \psi(q_A) - \psi(Q_{B,m}(k))) \} , \end{aligned} \tag{10}$$

where $Q_{B,m}(k)$ is the supply strategy as a function of the capital stock of firm B. The problem of firm B is given by the same Bellman equation as in (10), with the roles of A and B switched.

At this point, we emphasize that the Bellman equation given by (10) has a particularly complex element: the presence of the other firm's strategy, $Q_{B,m}(k)$, in the objective of firm A. With $Q_{B,m}(k)$ in the objective function of the firm it is difficult to find conditions that imply the concavity of $V_{A,m}$, the existence of equilibrium or useful properties of strategies, such as continuity and monotonicity.[10]

Levhari and Mirman (1980) use specific parametric forms in order to tackle the technical complexities arising from the difficulty in characterizing $Q_{B,m}(k)$, and, in particular, their functional forms imply that $Q_{B,m}(k)$ is a linear function in k.[11] In our parametric framework, $V_{A,m}$ is concave throughout the whole domain of k, so we proceed under the convention that

which means that capital exloitation takes place at the beginning of each period. On the contrary, in our setup, capital exploitation takes place at the end of each period. This different timing of actions helps us to present our analysis more easily, but there is no influence on the qualitative results.

[9] We denote the setup of two monopolists exploiting capital from the same source using the subscript 'm' distinguishing it from the benchmark monopoly model of the previous subsection, which goes with the subscript 'M.'

[10] Mirman (1979) presents examples of 'classic' or seemingly 'innocent' (at least in the single monopolist case) functional forms used to capture the fundamentals of this two-monopolist setup, where the value function $V_{A,m}$ is not concave, or not continuous, and the strategy $Q_{A,m}$ is not continuous or monotonic.

[11] It is easy to show that with our timing of exploitation (exploitation occurs at the end of each period), the strategy $Q_{B,m}(k)$ is linear in $f(k)$, which is another way of tackling the technical problems of this setup.

every desirable property is present.

Using the first-order condition implied by (10), and the envelope theorem to derive $V'_{A,m}\left(\widehat{k}\right)$, the marginal lifetime profits implied by the stock of capital that the firm leaves strategically available for next period, after taking into account the response of its competitor, we arrive at the necessary condition,

$$\frac{D\left(q_A\right)+D'\left(q_A\right)q_A-C_2\left(k,q_A\right)}{\psi'\left(q_A\right)}=\delta\left\{-C_1\left(\widehat{k},\widehat{q}_A\right)+\frac{D\left(\widehat{q}_A\right)+D'\left(\widehat{q}_A\right)\widehat{q}_A-C_2\left(\widehat{k},\widehat{q}_A\right)}{\psi'\left(\widehat{q}_A\right)}\times\right.$$
$$\left.\times\left[f'\left(\widehat{k}\right)-\psi'\left(Q_B\left(\widehat{k}\right)\right)Q'_B\left(\widehat{k}\right)\right]\right\}, \tag{11}$$

where $\widehat{q}_A$ is the output strategy of firm A in the next period. In Levhari, Michener and Mirman (1981) a simple method is used to show that, under quite general conditions on f, the tendency of this setup is to lead to overexploitation compared to the benchmark monopoly model (which can also be considered as a cooperative solution).

2.3 Duopoly with firms utilizing capital from the same provider

When firms interact both in the market and also compete for the exploitation of capital from the same provider, the competitive motives become very rich. Indeed, not only are both the static market (Cournot) externality and the dynamic externality present, but, due to the dynamic nature of the competitive environment, they interact. The dynamic externality implies an *increase in present output, before the competitor exploits the capital stock in the future.* Yet, due to the dependence of the cost function on capital, and with cost being a key part of the profit margin of firms, by influencing next period's capital stock, *the firm influences the optimal supply strategy in the next period, i.e., it can influence the market (Cournot) externality in the future.*

These interactions appear more clearly in the necessary optimality conditions of the firms. We denote the value function of the duopolistic firms with a direct capital-accumulation interaction as $V_{A,d}$ and $V_{B,d}$. Due to the symmetry of the firms, again, we can focus on the problem of firm A without loss of generality. The problem of firm A in a Bellman-equation form is given by,

$$V_{A,d}(k) = \max_{q_A \geq 0} \left\{ D\left(q_A + Q_B(k)\right) q_A - C(k, q_A) + \delta V_{A,d}\left(f(k) - \psi(q_A) - \psi(Q_B(k))\right) \right\} , \tag{12}$$

where $Q_B(k)$ is the supply strategy of firm B. The problem of firm B is given by the Bellman equation as in (12), except that A and B are switched.

Using the first-order condition implied by (12), and the envelope theorem to derive $V'_{A,d}\left(\widehat{k}\right)$, we reach the necessary condition,

$$\begin{aligned} \frac{D\left(q_A + Q_B(k)\right) + D'\left(q_A + Q_B(k)\right) q_A - C_2(k, q_A)}{\psi'(q_A)} = \delta \Bigg\{ & D'\left(\widehat{q}_A + Q_B(k)\right) \widehat{q}_A Q'_B\left(\widehat{k}\right) - \\ & -C_1\left(\widehat{k}, \widehat{q}_A\right) + \frac{D\left(\widehat{q}_A + Q_B\left(\widehat{k}\right)\right) + D'\left(\widehat{q}_A + Q_B\left(\widehat{k}\right)\right) \widehat{q}_A - C_2\left(\widehat{k}, \widehat{q}_A\right)}{\psi'\left(\widehat{q}_A\right)} \times \\ & \times \left[f'\left(\widehat{k}\right) - \psi'\left(Q_B\left(\widehat{k}\right)\right) Q'_B\left(\widehat{k}\right) \right] \Bigg\} \end{aligned} \tag{13}$$

Note that, as in the previous section, the dynamic externality is embodied in the term, $\psi'\left(Q_B\left(\widehat{k}\right)\right) Q'_B\left(\widehat{k}\right)$, appearing at the end of the right hand side of equation (13). However, both the market externality and the dynamic externality appear in various other places of (13). For example, the term $D\left(\widehat{q}_A + Q_B\left(\widehat{k}\right)\right) + D'\left(\widehat{q}_A + Q_B\left(\widehat{k}\right)\right) \widehat{q}_A - C_2\left(\widehat{k}, \widehat{q}_A\right)$, of (13), the marginal profit of firm A in the next period, contains the strategy $Q_B(k)$ wherever next period's capital, $\widehat{k}$, appears, through the dynamic condition (9). Moreover, the market externality appears, with the term $Q_B\left(\widehat{k}\right)$, on next period's marginal profit of firm A. More interestingly, as the term $D'\left(\widehat{q}_A + Q_B\left(k\right)\right)\widehat{q}_A Q_B'\left(\widehat{k}\right)$ of (13) reveals, firm A takes into account the impact of the change in next period's supply strategy of the competitor, $Q_B\left(\widehat{k}\right)$, on next period's marginal revenue of A, due to a change in the level of future capital.

3. A parametric model encompassing all three market structures

Central in our model is the tradeoff that industrial firms face between exploiting capital today and saving capital for the future, given that part of the specific capital is consumed during the production process. This tradeoff, however, depends on the alternative imperfect-competition environments. As has been shown, several strategic incentives arise due to the interaction of the firms with each other in the market for the final good as well as in the process that determines future capital deepening. These incentives are fundamental to understanding the link between market structure and industry dynamics.

We have shown that there are two externalities present when two firms supply in the same market and utilize capital from the same provider, a dynamic externality and a market externality. However, we have not been able to characterize the economic impact of these externalities on growth, since the model we have described so far is too general to study in this setting. In this section, we develop a *common* parametric framework that encompasses *all three* market structures that were analyzed in the previous section.

Crucial about the presence of these externalities in the models that we analyze is that *strategies of competitors* become part of the *structural* maximization problem of a firm. Thus, in order that the problem of each firm be well-defined, and, above all, that equilibrium strategies exist, we present a model in which the *primitives* imply linearity of the *equilibrium strategies.* In particular, our focus is on using *homogeneous or isoelastic functions* to capture the primitives of firms, capital growth possibilities, and consumer demand. We apply some parametric restrictions that yield linear decision rules of the form $Q(k) = \omega f(k)$.

Let the inverse demand function in the market for the final product, q, be,

$$D(q) = q^{-\frac{1}{\eta}} , \quad \text{with } \eta > 1 , \tag{14}$$

and

$$f(k) = \left(\alpha k^{1-\frac{1}{\eta}} + \phi\right)^{\frac{\eta}{\eta-1}} \qquad \text{with } \phi \geq 0 , \tag{15}$$

i.e. the intertemporal production function of capital is a CES function. Endogenous depreciation is captured by the function,

$$\psi(q) = \theta q , \qquad \text{with } \theta \in (0, 1] \ .$$

In other words, the depreciation of capital is proportional to the supply of the final good in each period. The final-good production function is,

$$q = F(k, l) = \left(\alpha k^{1-\frac{1}{\eta}} + \phi\right)^{\frac{\zeta\eta}{\eta-1}} l^{\upsilon} , \qquad \text{with } \zeta \in (0, 1) \text{ and } \upsilon > 0 , \tag{16}$$

so,

$$L(k, q) = \left(\alpha k^{1-\frac{1}{\eta}} + \phi\right)^{-\frac{\zeta\eta}{\upsilon(\eta-1)}} q^{\frac{1}{\upsilon}} \ .$$

The labor-cost function is,

$$c(l) = \nu l^{\xi} , \quad \text{with } \nu \in (0, 1) \text{ and } \xi > 0 \ .$$

Therefore, given that each firm has free utilization rights over capital, the cost function of a firm is given by,

$$c(L(k, q)) = C(k, q) = \nu \left(\alpha k^{1-\frac{1}{\eta}} + \phi\right)^{-\frac{\xi\zeta\eta}{\upsilon(\eta-1)}} q^{\frac{\xi}{\upsilon}} \ . \tag{17}$$

3.1 Parameter restrictions and scope of analysis

To simplify notation, let

$$\beta \equiv \frac{\xi\zeta}{\upsilon} \ ,$$

and

$$\rho \equiv \frac{\xi}{\upsilon} \ .$$

Using these two definitions, from (17) and (16) the cost function becomes,

$$C(k,q) = \nu \left[f(k)\right]^{-\beta} q^{\rho} \ . \tag{18}$$

In order to obtain strategies of the form $Q(k) = \omega f(k)$, for all firms and for all market structures we set,

$$\rho - \beta = 1 - \frac{1}{\eta} \ , \tag{19}$$

and

$$\rho > 1 - \frac{1}{2\eta} \ . \tag{20}$$

The first parameter restriction, given by (19), implies linear strategies, while the second constraint, (20), yields a unique equilibrium in all market structures.

The parameters, α, ϕ, η, θ, υ, ξ, ϕ, give enough degrees of freedom for studying the empirical link between the market structure and the growth rate in industries. As we show below, none of the strategies is influenced by the value of parameter ϕ. Yet, different values for parameter ϕ imply different dynamics, and the selection of ϕ is important for addressing different economic questions.

Specifically,

(i) Set $\phi = 0$ and $\alpha \in \left[1, \frac{1}{\delta}\right]$. In this case, the intertemporal production function of capital is $f(k) = \alpha^{\frac{\eta}{\eta-1}} k = Zk$, a growth-theory ingredient that can lead to perpetual growth for the market if $\alpha \in \left(1, \frac{1}{\delta}\right]$.[12] With $\alpha > 1$, this setup is appropriate for addressing the question of the link market structure and industry growth in industries depending on growing publicly provided forms of infrastructure, like airports, harbors, roads, pipe-lines, transmission grids, railroads, telecommunications lines, and also for addressing the trade-off between how markets allocate human resources into basic research for inventions versus research for innovations. In case $\alpha = 1$, the model is appropriate for the study of markets trading a non-renewable resource.

(ii) Set $\phi > 0$ and $\alpha \in (0, 1)$. In this case, the production function of capital is a function of current capital and a constant ϕ. The elasticity of substitution between current capital and the constant factor is the same as the elasticity of demand. There are two reasons for this assumption. First, it provides linear analytic solutions. The analytical simplicity of this framework allows us to derive the comparative statics (or dynamics) of the model. Second, with $\phi > 0$, the model has a zero-growth-rate steady state.[13] This setup is also appropriate for studying markets for goods depending on renewable natural resources.

3.2 Equilibrium in the three market structures

For the cases of, (i) the benchmark monopolist (carrying the subscript "M"), (ii) two monopolists utilizing capital from the same provider (subscript "m"), and (iii) duopolists utilizing capital from the same provider (subscript "d"), the common element is that the state space of all these three games is one-dimensional, namely there is one state variable, k. Moreover,

[12] We place the upper bound $\frac{1}{\delta}$ on parameter α in order to guarantee the boundedness of the value function of each firm.

[13] Moreover, for empirical applications of our model, the function given by (15) has three parameters, α, η and ϕ, giving enough degrees of freedom for treating data through data-mining approaches.

the equilibrium strategies for these three games are of the form $Q(k) = \omega f(k)$. For these reasons, we can accommodate the calculation of the strategies in all three market structures in a single presentation.

Let N_μ be the number of firms in the same market, with $N_\mu \in \{1, 2\}$, and also let N_κ be the number of firms utilizing capital from the same source, with $N_\kappa \in \{1, 2\}$.[14] Moreover, for simplicity,

$$y_t \equiv \left(\alpha k_t^{1-\frac{1}{\eta}} + \phi\right)^{\frac{\eta}{\eta-1}} \qquad t = 0, 1, ... \tag{21}$$

The maximization problem of firm $j \in \{1, ..., N_\mu\} \cap \{1, ..., N_\kappa\}$ is given by,

$$\max_{\{(q_{j,t}, k_{t+1})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \left[\left(\sum_{i=1}^{N_\mu} q_{i,t} \right)^{-\frac{1}{\eta}} q_{j,t} - \nu y_t^{-\beta} q_{j,t}^{\rho} \right] \tag{P1}$$

subject to,

$$k_{t+1} = y_t - \theta \sum_{i=1}^{N_\kappa} q_{i,t} \tag{22}$$

given $k_0 > 0$, and with parameters β and ρ obeying the restrictions given by (19) and (20).

We solve problem (P1) in two steps. First, we use the symmetric strategies,

$$q_{i,t} = \omega y_t \ , \tag{23}$$

with ω treated as an undetermined coefficient and we calculate the value function given the functional form, (23). Then, we form the Bellman equation, derive the optimality conditions of firm j, show that, indeed, the equilibrium strategies fall in the class of the functional form given by (23). We further show that ω is such that the maximization problems of all players are well-defined, and that this solution is unique. With these results at hand, we take the second step, to characterize and compare the strategies of different market structures.

[14] So, for the benchmark monopoly it is $N_\mu = N_\kappa = 1$, for the two monopolists with a common capital provider it is $N_\mu = 1$ and $N_\kappa = 2$, whereas for the duopolists with a common capital provider it is $N_\mu = N_\kappa = 2$.

3.2.1 Calculation of the value function

Using the strategies given by (23) with ω as an undetermined coefficient, applying (19), and substituting y_t from (21), yields the profits of the firm in period t,

$$\pi(k_t) = \left(N_\mu^{-\frac{1}{\eta}}\omega^{1-\frac{1}{\eta}} - \nu\omega^\rho\right)\left(\alpha k_t^{1-\frac{1}{\eta}} + \phi\right) \ . \tag{24}$$

Substituting (23) in the constraint of (P1), yields,

$$k_{t+1} = (1-\theta N_\kappa \omega)\, y_t \ ,$$

or, using (21),

$$k_{t+1}^{1-\frac{1}{\eta}} = \alpha(1-\theta N_\kappa\omega)^{1-\frac{1}{\eta}} k_t^{1-\frac{1}{\eta}} + \phi(1-\theta N_\kappa\omega)^{1-\frac{1}{\eta}} \ . \tag{25}$$

This is a linear first-order difference equation in the variable $k^{1-\frac{1}{\eta}}$, with solution,

$$k_t^{1-\frac{1}{\eta}} = \left[\alpha(1-\theta N_\kappa\omega)^{1-\frac{1}{\eta}}\right]^t \left[k_0^{1-\frac{1}{\eta}} - \frac{\phi(1-\theta N_\kappa\omega)^{1-\frac{1}{\eta}}}{1-\alpha(1-\theta N_\kappa\omega)^{1-\frac{1}{\eta}}}\right] + \frac{\phi(1-\theta N_\kappa\omega)^{1-\frac{1}{\eta}}}{1-\alpha(1-\theta N_\kappa\omega)^{1-\frac{1}{\eta}}} \ .$$

Substituting this last equation into (24) and the result back into the objective of problem (P1), the value function of firm $j \in \{1, ..., N_\mu\} \cap \{1, ..., N_\kappa\}$ is,

$$V(k_0) = \frac{\alpha\left(N_\mu^{-\frac{1}{\eta}}\omega^{1-\frac{1}{\eta}} - \nu\omega^\rho\right)}{1-\alpha\delta(1-\theta N_\kappa\omega)^{1-\frac{1}{\eta}}} k_0^{1-\frac{1}{\eta}} + b \ , \tag{26}$$

where b is a constant.

3.2.2 The strategies

The Bellman equation of firm $j \in \{1, ..., N_\mu\} \cap \{1, ..., N_\kappa\}$ is given by,

$$V(k) = \max_{q_j \geq 0} \left\{ \left(\sum_{i=1}^{N_\mu} q_i\right)^{-\frac{1}{\eta}} q_j - \nu y^{-\beta} q_j^\rho + \frac{\alpha\delta\left(N_\mu^{-\frac{1}{\eta}}\omega^{1-\frac{1}{\eta}} - \nu\omega^\rho\right)}{1-\alpha\delta(1-\theta N_\kappa\omega)^{1-\frac{1}{\eta}}} \left(y - \theta\sum_{i=1}^{N_\kappa} q_i\right)^{1-\frac{1}{\eta}} + \delta b \right\} \ ,$$

with first-order condition,

$$\left(\sum_{i=1}^{N_\mu} q_i\right)^{-\frac{1}{\eta}} \left(1 - \frac{q_j}{\eta \sum_{i=1}^{N_\mu} q_i}\right) - \rho\nu y^{-\beta} q_j^{\rho-1} = \frac{\alpha\delta\theta\left(1-\frac{1}{\eta}\right)\left(N_\mu^{-\frac{1}{\eta}}\omega^{1-\frac{1}{\eta}} - \nu\omega^\rho\right)}{1-\alpha\delta\left(1-\theta N_\kappa\omega\right)^{1-\frac{1}{\eta}}} \left(y - \theta\sum_{i=1}^{N_\kappa} q_i\right)^{-\frac{1}{\eta}} ,$$

or,

$$\left(\sum_{i=1}^{N_\mu} \frac{q_i}{y}\right)^{-\frac{1}{\eta}} \left(1 - \frac{\frac{q_j}{y}}{\eta \sum_{i=1}^{N_\mu} \frac{q_i}{y}}\right) - \rho\nu y^{\rho-\beta-1+\frac{1}{\eta}} \left(\frac{q_j}{y}\right)^{\rho-1} =$$

$$= \frac{\alpha\delta\theta\left(1-\frac{1}{\eta}\right)\left(N_\mu^{-\frac{1}{\eta}}\omega^{1-\frac{1}{\eta}} - \nu\omega^\rho\right)}{1-\alpha\delta\left(1-\theta N_\kappa\omega\right)^{1-\frac{1}{\eta}}} \left(1 - \theta\sum_{i=1}^{N_\kappa} \frac{q_i}{y}\right)^{-\frac{1}{\eta}} . \tag{27}$$

Thus, applying condition (19), the nature of strategies given by (23) is cross-validated. The symmetry of firms implies that all strategies are the same, so substituting (23) into (27) and re-arranging terms, the condition that gives parameter ω is,

$$\frac{\left(N_\mu\omega\right)^{-\frac{1}{\eta}}\left(1-\frac{1}{\eta N_\mu}\right) - \rho\nu\omega^{\rho-1}}{\left(N_\mu\omega\right)^{-\frac{1}{\eta}} - \nu\omega^{\rho-1}} = \frac{\alpha\delta\theta\left(1-\frac{1}{\eta}\right)\omega}{\left(1-\theta N_\kappa\omega\right)^{\frac{1}{\eta}} - \alpha\delta\left(1-\theta N_\kappa\omega\right)} . \tag{28}$$

3.2.3 Admissibility and uniqueness of strategies

The strategies of firms are admissible if the maximization problem of each firm is well-defined. In our setup, the key to admissibility is that $\theta N_\kappa\omega \in [0,1)$. To show this, we set,

$$\chi \equiv \theta N_\kappa \omega .$$

Using this transformation in (28), yields,

$$g(\chi) \equiv \frac{\left(\theta N_\kappa\right)^\beta N_\mu^{-\frac{1}{\eta}}\left(1-\frac{1}{\eta N_\mu}\right)\chi^{-\beta} - \rho\nu}{\left(\theta N_\kappa\right)^\beta N_\mu^{-\frac{1}{\eta}}\chi^{-\beta} - \nu} = \frac{\frac{\alpha\delta}{N_\kappa}\left(1-\frac{1}{\eta}\right)\chi}{\left(1-\chi\right)^{\frac{1}{\eta}} - \alpha\delta\left(1-\chi\right)} \equiv h(\chi) , \tag{29}$$

after having used (19) as well.

In (29), we have named the left-hand side $g(\chi)$, and the right-hand side $h(\chi)$. The properties of $g(\chi)$ for $\chi \in [0,1]$ are,

$$g(0) = 1 - \frac{1}{\eta N_\mu} \ , \quad \text{and} \quad g'(\chi) < 0 \ , \tag{30}$$

where $g'(\chi) < 0$ rests upon our parameter restriction (20).[15] The properties of $h(\chi)$ when $\chi \in [0,1]$ are,

$$h(0) = 0 \ , \quad h(1) = \infty \ , \quad \text{and} \quad h'(\chi) > 0 \ , \tag{31}$$

where $h'(\chi) > 0$ for all $\chi \in [0,1]$ if $\frac{\alpha\delta}{N_\kappa} < 1$, whereas if $\frac{\alpha\delta}{N_\kappa} = 1$, it is, $h'(1) = 0$ and $h'(\chi) > 0$,

[15]In particular,

$$g'(\chi) = \frac{\beta\nu\left(1 - \frac{1}{\eta N_\mu} - \rho\right)(\theta N_\kappa)^{-\beta} N_\mu{}^{-\frac{1}{\eta}} \chi^{-\beta-1}}{\left[(\theta N_\kappa)^{-\beta} N_\mu{}^{-\frac{1}{\eta}} \chi^{-\beta} - \nu\right]^2} \ ,$$

which is strictly negative for all $\chi \in [0,1]$ if (20) holds. In the special case where,

$$\nu \leq \frac{(\theta N_\kappa)^\beta N_\mu{}^{-\frac{1}{\eta}}}{\rho}\left(1 - \frac{1}{\eta N_\mu}\right) \ ,$$

a condition that guarantees that the marginal profits are positive, and, thus, profits in each period are also positive, for all $\chi \in [0,1]$, with,

$$g(1) = \frac{(\theta N_\kappa)^\beta N_\mu{}^{-\frac{1}{\eta}}\left(1 - \frac{1}{\eta N_\mu}\right) - \rho\nu}{(\theta N_\kappa)^\beta N_\mu{}^{-\frac{1}{\eta}} - \nu} \ .$$

In the case where $\nu \leq (\theta N_\kappa)^\beta N_\mu{}^{-\frac{1}{\eta}}$, a condition guaranteeing that profits in each period are non-negative for all $\chi \in [0,1]$, $g(\chi)$ is continuously differentiable on $[0,1]$. For $\nu > (\theta N_\kappa)^\beta N_\mu{}^{-\frac{1}{\eta}}$, $g(\chi)$ is continuously differentiable on $\left[0\,,\, \theta N_\kappa \nu^{-\frac{1}{\beta}} N_\mu^{-\frac{1}{\beta\eta}}\right) \cup \left(\theta N_\kappa \nu^{-\frac{1}{\beta}} N_\mu^{-\frac{1}{\beta\eta}}\,,\, 1\right]$. For all parametric setups, the interesting feature of $g(\chi)$ is that it is continuously differentiable with $g'(\chi) < 0$ on the region of $[0,1]$ where marginal profits (and thus, each period's profits) are positive.

for all $\chi \in (0,1]$.[16]

Figure 1 summarizes graphically the properties given by (30) and (31), and depicts why the equilibrium strategies, denoted by ω^*, *in all three market structures*, are both admissible (specifically, $\theta N_\kappa \omega^* \in (0,1)$), and *unique* for the region of non-negative marginal profits, and thus, for the region where each period's profits are strictly positive. Note that for some values of the parameters, explained in footnote 15, the set of values of χ for which marginal profits are negative can be empty.

3.3 The impact of the dynamic externality

In order to assess the impact of the dynamic externality on firm strategies, and, in particular, whether the 'tragedy of the commons' dominates, we compare the benchmark monopoly case, denoted as "M," with the case of two monopolies utilizing capital from the same provider, denoted as "m." This comparison is given by the following proposition.

Proposition 1 *The aggregate exploitation rate of firms in market structure "m" is always higher than the exploitation rate of the benchmark monopoly, "M," namely,* $\chi_m^* > \chi_M^*$.

Proof

Using the fact that $N_\mu = 1$ in both cases, we can express the right-hand side of (29) as a function of N_κ as,

$$G(\chi, N_\kappa) \equiv N_\kappa \frac{(\theta N_\kappa)^\beta \left(1-\frac{1}{\eta}\right)\chi^{-\beta} - \rho\nu}{(\theta N_\kappa)^\beta \chi^{-\beta} - \nu} = \frac{\alpha\delta\left(1-\frac{1}{\eta}\right)\chi}{(1-\chi)^{\frac{1}{\eta}} - \alpha\delta(1-\chi)} \equiv H(\chi)\ . \qquad (32)$$

[16]Specifically,

$$h'(\chi) = \left(1-\frac{1}{\eta}\right)\frac{\alpha\delta}{N_\kappa}\frac{\frac{1-\left(1-\frac{1}{\eta}\right)\chi}{(1-\chi)^{1-\frac{1}{\eta}}} - \alpha\delta}{\left[(1-\chi)^{\frac{1}{\eta}} - \alpha\delta(1-\chi)\right]^2}\ .$$

Noticing that $1-\left(1-\frac{1}{\eta}\right)\chi \geq (1-\chi)^{1-\frac{1}{\eta}}$ for all $\chi \in [0,1]$ with equality if and only if $\chi = 0$, the fact that $\frac{\alpha\delta}{N_\kappa} \leq 1$ implies that $h'(\chi) > 0$ for all $\chi \in (0,1]$.

Since,

$$G_2(\chi, N_\kappa) = \frac{(\theta N_\kappa)^\beta \left(1-\frac{1}{\eta}\right)\chi^{-\beta} - \rho\nu}{(\theta N_\kappa)^\beta \chi^{-\beta} - \nu} + \frac{\beta\nu\theta^\beta N_\kappa^\beta \left[\rho - \left(1-\frac{1}{\eta}\right)\right]}{\left[(\theta N_\kappa)^\beta \chi^{-\beta} - \nu\right]^2},$$

our parameter restriction (20) implies that $G_2(\chi, N_\kappa) > 0$. Therefore, the equilibrium strategies in the two cases are captured by Figure 2, which proves the proposition.□

The conclusion from Proposition 1 is that, as in Levhari and Mirman (1980), the dynamic externality leads to the dominance of the 'tragedy of the commons,' irrespective of the parameters affecting the demand function, the cost function, growth possibilities given by $f(k)$, and the endogenous capital depreciation technology.

3.4 The impact of the market externality in addition to the dynamic externality

The starting point for studying the impact of the market externality in addition to the dynamic externality, is the comparison of the market structure of the benchmark monopoly, denoted as "M," with the market structure of a duopoly with both firms, again, utilizing capital from the same provider, denoted as "d."

3.4.1 Comparison of the benchmark monopoly with a duopoly utilizing capital from the same provider

We express condition (29) somewhat differently, re-arranging the position of parameter ρ and setting the right-hand side as,

$$X(\chi) = \frac{\frac{\alpha\delta}{\rho}\left(1-\frac{1}{\eta}\right)\chi}{(1-\chi)^{\frac{1}{\eta}} - \alpha\delta(1-\chi)} \tag{33}$$

which is common for both cases that we examine, whereas, the two left-hand sides for each market structure become,

$$\Gamma_M(\chi) = \frac{\frac{\theta^\beta\left(1-\frac{1}{\eta}\right)}{\rho}\chi^{-\beta} - \nu}{\theta^\beta\chi^{-\beta} - \nu}, \tag{34}$$

and

$$\Gamma_d(\chi) = 2\frac{\frac{2^{\rho-1}\theta^{\beta}\left(1-\frac{1}{2\eta}\right)}{\rho}\chi^{-\beta} - \nu}{2^{\rho-1}\theta^{\beta}\chi^{-\beta} - \nu} \ , \tag{35}$$

where the parameter constraint (19) has been used. Since the right-hand side of (29), given by (33) is common across the two market structures "M" and "d," the key is to compare $\Gamma_d(\chi)$ with $\Gamma_M(\chi)$.

To simplify notation,

$$\chi^{-\beta} \equiv z \ ,$$

$$a_M \equiv \frac{\theta^{\beta}\left(1-\frac{1}{\eta}\right)}{\rho} \ ,$$

$$b_M \equiv \theta^{\beta} \ ,$$

$$c \equiv \frac{2^{\rho-1}\theta^{\beta}\left(1-\frac{1}{2\eta}\right)}{\rho} \ ,$$

and,

$$d \equiv 2^{\rho-1}\theta^{\beta} \ .$$

Thus, (34) and (35) imply,

$$\Gamma_M(\chi) < \Gamma_d(\chi) \Leftrightarrow \frac{a_M z - \nu}{b_M z - \nu} < 2\frac{cz - \nu}{dz - \nu} \ . \tag{36}$$

We pay attention to strategies that lead to positive profits in both setups. From condition (27) it is implied that the value function is positive if the marginal profit in each period is also positive, namely if,

$$z > \max\left\{\frac{\nu}{a_M} \ , \ \frac{\nu}{c}\right\} \ . \tag{37}$$

Note that, recalling Figure 1, equation (29) implies that, in equilibrium, profits are always positive, namely condition (37) holds in equilibrium.

Proposition 2 *The aggregate exploitation rate of firms in market structure "d" is always higher than the exploitation rate of the benchmark monopoly, "M," (namely, $\chi_d^* > \chi_M^*$).*

Proof

We prove that (36) always holds. For all z complying with (37), both $\Gamma_M(\chi)$ and $\Gamma_d(\chi)$ are strictly positive, so it suffices to show that the *stronger* condition,

$$\frac{a_M z - \nu}{b_M z - \nu} < \frac{cz - \nu}{dz - \nu} \ , \tag{38}$$

always holds, i.e.,

$$\frac{a_M z - \nu}{b_M z - \nu} < \frac{cz - \nu}{dz - \nu} \Leftrightarrow z > \nu \frac{b_M + c - a_M - d}{cb_M - da_M} \ . \tag{39}$$

It is straightforward to see that under, (20),

$$b_M > a_M \ . \tag{40}$$

Moreover, for any values of ρ complying with (20),

$$c > a_M \ , \tag{41}$$

which implies that $\max\left\{\frac{\nu}{a_M}, \frac{\nu}{c}\right\} = \frac{\nu}{a_M}$.[17] In equilibrium it must be that $z > \frac{\nu}{a_M}$. Noticing that

$$\frac{\nu}{a_M} > \nu \frac{b_M + c - a_M - d}{cb_M - da_M} \Leftrightarrow (c - a_M)(b_M - a_M) > 0 \ ,$$

which always holds, due to (40) and (41), completes the proof.□

The result of Proposition 2 is depicted graphically by Figure 3. So, we have proved that, for all parameters of the model, the dynamic externality and the market externality, *together*, lead to overexploitation of capital compared to the benchmark monopoly case.

[17]Condition (41) holds even for the lower bound of ρ, namely for $1 - \frac{1}{2\eta}$, i.e., $2^{-\frac{1}{2\eta}} - \left(1 - \frac{1}{\eta}\right) > 0$, for all $\eta > 1$.

3.4.2 Comparison between strategies of two monopolists utilizing capital from the same provider with the duopoly

For completing the comparison among all market structures, in this section we compare the market structure of two monopolists utilizing capital from the same provider, denoted as "m," with the market structure of a duopoly with both firms, again, utilizing capital from the same provider, denoted as "d."

We use condition (29), rearranging terms, and noting that $N_\kappa = 2$ in both cases. This makes the right-hand side of the new version of (29),

$$\Xi(\chi) = \frac{\frac{\alpha\delta}{2\rho}\left(1-\frac{1}{\eta}\right)\chi}{(1-\chi)^{\frac{1}{\eta}} - \alpha\delta(1-\chi)} , \tag{42}$$

for both cases that we examine, whereas,

$$\Lambda_m(\chi) = \frac{\frac{(2\theta)^\beta\left(1-\frac{1}{\eta}\right)}{\rho}\chi^{-\beta} - \nu}{(2\theta)^\beta\chi^{-\beta} - \nu} , \tag{43}$$

and

$$\Lambda_d(\chi) = \frac{\frac{2^{-\frac{1}{\eta}}(2\theta)^\beta\left(1-\frac{1}{2\eta}\right)}{\rho}\chi^{-\beta} - \nu}{2^{-\frac{1}{\eta}}(2\theta)^\beta\chi^{-\beta} - \nu} . \tag{44}$$

The two definitions, (43) and (44) imply that,

$$\Lambda_M(\chi) < \Lambda_d(\chi) \Leftrightarrow \frac{a_m z - \nu}{b_m z - \nu} < \frac{cz - \nu}{dz - \nu} , \tag{45}$$

where

$$a_m \equiv \frac{(2\theta)^\beta\left(1-\frac{1}{\eta}\right)}{\rho} ,$$

and

$$b_m \equiv (2\theta)^\beta ,$$

whereas the constants c and d are defined above. Again, we pay attention to strategies that lead to positive profits in both setups. Therefore,

$$z > \max\left\{\frac{\nu}{a_m} , \frac{\nu}{c}\right\} . \tag{46}$$

The role of the demand elasticity In a static framework, the lower the demand elasticity, η, the lower the supply of a monopolist. In our dynamic framework, this means that monopolists would consume the capital stock at a lower rate and reach a higher steady state. In the following proposition we point out a cutoff level for the demand elasticity, below which it is *always* the case that the monopolisties grow more than the duopoly.

Low demand elasticity

Proposition 3 *If,*

$$2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right) \geq 1-\frac{1}{\eta} \Leftrightarrow \eta \leq 2.73 \ ,$$

the aggregate exploitation rate of firms in market structure "d" is always higher than the exploitation rate of the benchmark monopoly, "m," (namely, $\chi_d^* > \chi_m^*$).

Proof

Under (46), rearranging the terms of (45),

$$\Lambda_M(\chi) < \Lambda_d(\chi) \Leftrightarrow \frac{dz-\nu}{b_m z-\nu} < \frac{cz-\nu}{a_m z-\nu} \ . \tag{47}$$

Using (19), $d = 2^{-\frac{1}{\eta}}(2\theta)^{\beta} < b_m$, which implies that,

$$\frac{dz-\nu}{b_m z-\nu} < 1 \ ,$$

for all z satisfying (46). Using (19) again,

$$c \geq a_m \Leftrightarrow 2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right) \geq 1-\frac{1}{\eta} \Leftrightarrow \eta \leq 2.73 \ ,$$

yielding,

$$1 \leq \frac{cz-\nu}{a_m z-\nu} \ ,$$

proving that (45) holds in this case. The equilibrium exploitation rate of firms, under the restriction that $\eta \leq 2.73$, is captured by Figure 4.□

The cutoff level for demand elasticity is different, depending on N_μ. Indeed, in our model, the only factor influencing the threshold level of demand elasticity for which the market structure "d" to more capital utilization, is the number of firms in the market, N_μ.

Steady States for a low demand elasticity ($\eta \leq 2.73$) When $\eta \leq 2.73$ and $\phi > 0$, the steady state growth of capital is zero. With propositions 1 and 3, the steady-state levels of capital for the three market structures is given by figure 5, which shows that,

$$k_M^{ss} > k_m^{ss} > k_d^{ss} \ .$$

Also for the case that $\phi = 0$, growth in the case of a duopoly with both firms utilizing capital from the same provider is lower than in the case of two monopolistic firms utilizing capital from the same provider.

Higher demand elasticity ($\eta > 2.73$) For the case where $\eta > 2.73 \Leftrightarrow c < a_m$, under restriction (46), we can solve inequality (45) for z, to see that

$$\Lambda_m(\chi) < \Lambda_d(\chi) \Leftrightarrow z > \nu \frac{b_m + c - a_m - d}{cb_m - da_m} \equiv \bar{z} = \nu (2\theta)^\beta \left[2\eta(\rho - 1)\left(2^{\frac{1}{\eta}} - 1\right) + 1\right] > 0 \ . \tag{48}$$

Substituting $\bar{z} = (\bar{\chi})^{-\beta}$ from (48) into (43) and (44),

$$\Lambda_m(\bar{\chi}) = \Lambda_d(\bar{\chi}) = \frac{d - c}{b - a} = \frac{1}{\rho} \frac{2^{-\frac{1}{\eta}}\left(1 - \frac{1}{2\eta}\right) - \left(1 - \frac{1}{\eta}\right)}{1 - 2^{-\frac{1}{\eta}}} \ . \tag{49}$$

The results expressed by (48) and (49), reveal a notable property of $\bar{\chi}$, the point where the functions $\Lambda_m(\chi)$ and $\Lambda_d(\chi)$ meet (while for all $\chi < \bar{\chi}$, $\Lambda_m(\chi) < \Lambda_d(\chi)$): $\bar{\chi}$ depends on η, ρ, θ and ν. On the other hand, the level $\Lambda_m(\bar{\chi}) = \Lambda_d(\bar{\chi})$ depends *only* on η and ρ. These observations prove very useful for motivating and understanding the comparisons that follow. When $\eta \leq 2.73$, the meeting point of $\Lambda_m(\chi)$ and $\Lambda_d(\chi)$ implies negative values for these functions. Therefore, the equilibrium strategy rates χ_d^*, χ_m^* are less than $\bar{\chi}$, since for any $\chi > \bar{\chi}$ it is impossible for $\Lambda_m(\chi)$ or $\Lambda_d(\chi)$ to meet with $\Xi(\chi)$, since $\Xi(\chi)$ is positive for all $\chi \in (0,1]$. This insight is depicted in Figure 4.

In the case $\eta > 2.73$, the possibility of a monopoly having higher supply is open, and difficult to characterize analytically. Thus, we present a numerical example, in Figure 6. The parameter values we use are $\eta = 5$ (> 2.73), $\delta = .96$ (reflecting an interest rate of about 4%), $\alpha = .3$, $\rho = 1$, $\theta = .6$, $\nu = .9$, while β is derived using condition (19), i.e. it is equal to .2 in this example. Figure 6 shows that for these parameter values, $\chi_d^* < \chi_m^*$, and, given Propositions 1 and 2, $k_m^{ss} < k_d^{ss} < k_M^{ss}$.

Since a complete analytical account of the factors leading to strategies where $\chi_d^* < \chi_m^*$ is cumbersome, in what follows, we restrict our numerical example to the parameter values used above, but relax each parameter, *one by one*, showing the impact of each economic primitive on strategies.

The impact of a 'more convex' cost function (higher ρ) A key determinant of natural monopolies is that their production function exhibits increasing returns to scale, or, alternatively, a 'slightly convex' or even a concave cost function. In our numerical example, we retain all parameter values at the same level, but we increase the value of parameter ρ, by setting $\rho = 7$. As can be seen in Figure 7.a, this discourages the two monopolists from supplying more, and the implied strategies have the property $\chi_d^* > \chi_m^*$, once again.

The impact of higher growth possibilities of the capital stock (higher α) or of a lower interest rate (higher δ) The insights gained from (49) and (48), that the levels of $\Lambda_m(\bar{\chi}) = \Lambda_d(\bar{\chi})$ do not depend on any parameters other than ρ and η, and that the cutoff point $\bar{\chi}$ depends only on ρ, η, θ and ν, lead us to observe that whenever other factors, influencing the function $\Xi(\chi)$, change, then the result of Figure 6, that $\chi_d^* < \chi_m^*$, can be reversed. For example, whenever, α increases, the growth possibilities of the capital stock increase. This pushes the function $\Xi(\chi)$ upwards, and can lead both equilibrium strategies, χ_d^* and χ_m^*, to levels below the cutoff $\bar{\chi}$, while the level $\Lambda_m(\bar{\chi}) = \Lambda_d(\bar{\chi})$ remains the same, which implies that $\chi_d^* > \chi_m^*$. Such a case is shown in Figure 7.b, where $\alpha = .7$, with all other parameter values set at the levels of the example of Figure 6. Note that a higher δ, namely a lower interest rate (the opportunity cost of a firm), leads to the same result as increasing α: to lower the levels of χ_d^* and χ_m^*, with a potential to drive them below the cutoff level $\bar{\chi}$.

The impact of lower cost (lower ν) or of a faster depreciation technology (higher θ) As is implied by (48), a lower cost parameter ν, or a faster depreciation technology, a higher θ, both drive the cutoff point $\bar{\chi}$ upwards, while $\Lambda_m(\bar{\chi}) = \Lambda_d(\bar{\chi})$ stay at the same level. At the same time, due to (49), $\Xi(\chi)$ remains unaffected. In Figure 7.c we have set $\nu = .8$, with all other parameter values set at the levels of the example of Figure 6. In Figure 7.d, we have set $\theta = 1$, with all other parameter values, again, set at the numerical benchmark of Figure 6. In both Figures, 7.c and 7.d, the result is that $\chi_d^* > \chi_m^*$, unlike the implication of Figure 6.

To summarize, Proposition 1 states that the dynamic externality leads to more aggregate capital utilization, a 'tragedy of the commons' result. Proposition 2 shows that the impact of the dynamic and the market externality *combined* is more aggregate capital utilization. Proposition 3 states that adding the market externality 'on top of' an already existing dynamic externality, leads to more aggregate capital utilization if the elasticity of demand is sufficiently low with the threshold level of demand elasticity depending only on the number of firms in the market. When the elasticity of demand is above this threshold level, it is possible that adding the market externality to the dynamic externality, leads to *less* aggregate capital utilization, depending on a combined contribution of: (a) relatively low convexity of the cost function (sufficiently low ρ), (b) relatively low growth opportunities for capital (a sufficiently low α), (c) relatively high interest rates (a sufficiently low δ), (d) relatively high cost of labor (a sufficiently high ν), and, (e) relatively weak endogenous depreciation technology (a sufficiently low θ).

4. Isolating the market externality: excludability - duopoly with firms exploiting capital from different providers

A third departure from the benchmark monopoly case is the market structure of two firms, A and B, with each firm having exclusive expropriation rights on their own, separate capital stock, but selling their products in the same market. Since nonexcludability of capital is not the case any more, we must distinguish between two stocks of capital, k_A and k_B, and assume that the initial capital stocks are equal, i.e., $k_{A,0} = k_{B,0} > 0$. This market structure eliminates the dynamic externality and allows us to study the isolated effects of the market externality.

The capital stocks evolve according to,

$$k_{A,t+1} = f\left(k_{A,t}\right) - \psi\left(q_{A,t}\right) \ , \tag{50}$$

$$k_{B,t+1} = f\left(k_{B,t}\right) - \psi\left(q_{B,t}\right) \ . \tag{51}$$

We use the *superscript* 'D' for this market structure and we denote the value function of the duopolistic firms as $V^{A,D}$ and $V^{B,D}$.[18] These value functions depend on both capital stocks, (k_A, k_B). Due to the symmetry of the problem, again, we can focus on the problem of firm A, without loss of generality. The problem of the firm A in a Bellman-equation form is given

[18]Because all value functions and firm strategies in this section are bivariate, we use symbols "A, D" and "B, D" as superscripts, in order to allow for partial derivatives to be denoted as subscripts. Despite this slight notational discrepancy with the previous sections, this simplifies the exposition.

by,

$$V^{A,D}(k_A,k_B) = \max_{q_A\geq 0}\left\{D\left(q_A+Q^{B,D}(k_A,k_B)\right)q_A - C(k_A,q_A) + \right.$$
$$\left. +\delta V^{A,D}\left(f(k_A)-\psi(q_A)\ ,\ f(k_B)-\psi\left(Q^{B,D}(k_A,k_B)\right)\right)\right\}\ , \quad (52)$$

where $Q^{B,D}(k_A,k_B)$ is the supply strategy of firm B. The problem of firm B is given by switching A and B in the Bellman equation (52).

The first-order conditions implied by (52) are,

$$\frac{D\left(q_A+Q^{B,D}(k_A,k_B)\right)+D'\left(q_A+Q^{B,D}(k_A,k_B)\right)q_A-C_2(k_A,q_A)}{\psi'(q_A)} = \delta V_1^{A,D}\left(\widehat{k}_A,\widehat{k}_B\right)\ ,$$

with $\widehat{k}_A$ and $\widehat{k}_B$ being next period's capital stocks. Applying the envelope theorem on (52) yields,

$$V_1^{A,D}(k_A,k_B) = D'\left(q_A+Q^{B,D}(k_A,k_B)\right)q_A Q_1^{B,D}(k_A,k_B) - C_1(k_A,q_A) +$$
$$+\delta\left[V_1^{A,D}\left(\widehat{k}_A,\widehat{k}_B\right)f'(k_A) - V_2^{A,D}\left(\widehat{k}_A,\widehat{k}_B\right)\psi'\left(Q^{B,D}(k_A,k_B)\right)Q_1^{B,D}(k_A,k_B)\right]\ .$$

Combining the last two equations yields the necessary condition,

$$\frac{D\left(q_A+Q^{B,D}(k_A,k_B)\right)+D_1\left(q_A+Q^{B,D}(k_A,k_B)\right)q_A-C_2(k_A,q_A)}{\psi'(q_A)} =$$
$$= \delta\left\{-C_1\left(\widehat{k}_A,\widehat{q}_A\right)+\left[\frac{D\left(\widehat{q}_A+Q^{B,D}\left(\widehat{k}_A,\widehat{k}_B\right)\right)}{\psi'(\widehat{q}_A)}+\right.\right.$$
$$\left.+\frac{D'\left(\widehat{q}_A+Q^{B,D}\left(\widehat{k}_A,\widehat{k}_B\right)\right)\widehat{q}_A - C_2\left(\widehat{k}_A,\widehat{q}_A\right)}{\psi'(\widehat{q}_A)}\right]f'\left(\widehat{k}_A\right)+$$
$$\left.+\left[D'\left(\widehat{q}_A+Q^{B,D}\left(\widehat{k}_A,\widehat{k}_B\right)\right)\widehat{q}_A-\delta V_2^{A,D}\left(\widehat{\widehat{k}}_A,\widehat{\widehat{k}}_B\right)\psi'\left(Q^{B,D}\left(\widehat{k}_A,\widehat{k}_B\right)\right)\right]Q_1^{B,D}\left(\widehat{k}_A,\widehat{k}_B\right)\right\}\ , \quad (53)$$

where $\widehat{\widehat{k}}$ is the capital stock two periods ahead. The necessary optimal condition of firm B is given by the same equation as (53), except that A and B are switched.

Compared to the benchmark monopoly model, the two new elements introduced in the current setup are, (i) the presence of the other firm in the market contemporaneously, the classic Cournot-competition motive, and, (ii) the fact that each firm takes into account that changing its own capital stock may stimulate a response by its competitor that could trigger a decrease in future revenues. A key complication that is revealed by (53), is that the partial derivative of the strategy of firm B with respect to the capital stock of firm A, namely $Q_1^{B,D}\left(\widehat{k}_A, \widehat{k}_B\right)$, is essential for firm A to solve its problem.

In the parametric setup that we examine, we point out a way of tackling the problem, although we do not perform analytical comparisons with the benchmark monopoly case. It is very important to see, from (53), that the comparison with the benchmark monopoly is not immediate or simple, and that the ways the market externality can influence the dynamics require exploration.[19]

4.1 Firm strategies in the parametric framework

In this section we show that for the special case of,

$$\phi = 0 \ ,$$

i.e. with,

$$f\left(k_A\right) = \alpha^{\frac{\eta}{\eta-1}} k_A \qquad \text{and} \qquad f\left(k_B\right) = \alpha^{\frac{\eta}{\eta-1}} k_B \ ,$$

our example can lead to linearly homogeneous supply strategies with respect to (k_A, k_B). Our conjecture is motivated by the observation that linearly homogeneous strategies $Q^{A,D}\left(k_A, k_B\right)$,

[19] We are not aware of studies that deal with the model of this subsection.

$Q^{B,D}(k_A, k_B)$ work in the static case for the symmetric equilibrium. Setting,

$$Z = \alpha^{\frac{\eta}{\eta-1}} ,$$

in order to simplify notation, the static problem of firm A is,

$$\max_{q_A \geq 0} (q_A + q_B)^{-\frac{1}{\eta}} q_A - \nu (Z k_A)^{-\beta} q_A^{\rho}$$

with the necessary condition,

$$(q_A + q_B)^{-\frac{1}{\eta}} \left(1 - \frac{1}{\eta} \frac{q_A}{q_A + q_B}\right) = \rho \nu Z^{\beta} k_A^{-\beta} q_A^{\rho - 1} .$$

Considering the equivalent expression for firm B, and working out algebraically the two equations, we arrive at the useful presentation of the Cournot-Nash equilibrium system, with

$$\frac{q_B + \left(1 - \frac{1}{\eta}\right) q_A}{q_A + \left(1 - \frac{1}{\eta}\right) q_B} \left(\frac{q_B}{q_A}\right)^{\rho - 1} = \left(\frac{k_B}{k_A}\right)^{\beta} , \tag{54}$$

which comes from dividing both sides of the necessary optimality conditions of the two firms and,

$$\left(2 - \frac{1}{\eta}\right) (q_A + q_B)^{-\frac{1}{\eta}} = \rho \nu Z^{\beta} \left(k_A^{-\beta} q_A^{\rho - 1} + k_B^{-\beta} q_B^{\rho - 1}\right) , \tag{55}$$

which results from adding up both sides of the two conditions. It is quite complicated to solve the system given by (54) and (55) analytically for q_A and q_B, especially when $k_A \neq k_B$.[20] Yet, it is easy to prove that the equilibrium strategies, $Q^A(k_A, k_B)$ and $Q^B(k_A, k_B)$, that solve (54) and (55), under (19) are linearly homogeneous when $k_A = k_B$. This observation

[20] Under the stronger parametric restriction that $\rho = 1$ and $\beta = \frac{1}{\eta}$, which is the restriction used in a previous version of this paper (see Koulovatianos and Mirman (2003a)), the general analytical solution to the static problem with $k_A \neq k_B$, is,

$$q_A = Z\eta \left(\frac{2 - \frac{1}{\eta}}{\nu}\right)^{\eta} \frac{k_B^{-\frac{1}{\eta}} - \left(1 - \frac{1}{\eta}\right) k_A^{-\frac{1}{\eta}}}{\left(k_A^{-\frac{1}{\eta}} + k_B^{-\frac{1}{\eta}}\right)^{\eta + 1}} ,$$

enables us to calculate the partial derivative $Q^B(k_A, k_B)$ from equations (54) and (55). Using Euler's theorem,

$$Q^A(k_A, k_B) = Q_1^A(k_A, k_B)\, k_A + Q_2^A(k_A, k_B)\, k_B \ ,$$

or

$$Q_2^A(k_A, k_B) = \frac{Q^A(k_A, k_B)}{k_B} - Q_1^A(k_A, k_B)\, \frac{k_A}{k_B} \ .$$

We also know that $Q_1^A(k_A, k_B)$ and $Q_2^A(k_A, k_B)$ are homogeneous functions of degree 0. When $k_A = k_B$, this condition holds throughout the entire equilibrium path in the dynamic case due to symmetry, and the two partial derivatives are constants, say, $Q_1^A(k_A, k_B) = \omega_1$ and $Q_2^A(k_A, k_B) = \omega_2$, which makes the strategies linear. Hence the dynamic strategies can be written as,

$$Q^{A,D}(k_A, k_B) = \omega_1 k_A + \omega_2 k_B \ , \tag{56}$$

and,

$$Q^{B,D}(k_A, k_B) = \omega_1 k_B + \omega_2 k_A \ . \tag{57}$$

Therefore, (50) and (51) become,

$$\hat{k}_A = Z k_A - \theta\left(\omega_1 k_A + \omega_2 k_B\right) \ , \tag{58}$$

and

$$\hat{k}_B = Z k_B - \theta\left(\omega_1 k_B + \omega_2 k_A\right) \ . \tag{59}$$

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and,

$$q_B = Z\eta \left(\frac{2 - \frac{1}{\eta}}{\nu}\right)^{\eta} \frac{k_A^{-\frac{1}{\eta}} - \left(1 - \frac{1}{\eta}\right) k_B^{-\frac{1}{\eta}}}{\left(k_A^{-\frac{1}{\eta}} + k_B^{-\frac{1}{\eta}}\right)^{\eta+1}} \ ,$$

which is also a quite complicated pair of formulas to work with in a dynamic setup (for example, a guess to generalize for moving to a two-period model and for continuing onwards).

Using (56), (57), (58) and (59), we construct the value function of firm A,

$$V^{A,D}(k_A,k_B)=\frac{(\omega_1+\omega_2)^{-\frac{1}{\eta}}(k_A+k_B)^{-\frac{1}{\eta}}(\omega_1k_A+\omega_2k_B)-\nu(Zk_A)^{-\beta}(\omega_1k_A+\omega_2k_B)^{\rho}}{1-\beta[Z-\theta(\omega_1+\omega_2)]^{1-\frac{1}{\eta}}}. \tag{60}$$

The Bellman equation of firm A is,

$$\begin{aligned}V^{A,D}(k_A,k_B)=&\max_{q_A\geq 0}\Bigg\{\left(q_A+Q^{B,D}(k_A,k_B)\right)^{-\frac{1}{\eta}}q_A-\nu k_A^{-\beta}q_A^{\rho}+\\&+\delta\frac{(\omega_1+\omega_2)^{-\frac{1}{\eta}}\left(Zk_A-\theta q_A+Zk_B-\theta Q^{B,D}(k_A,k_B)\right)^{-\frac{1}{\eta}}}{1-\beta[Z-\theta(\omega_1+\omega_2)]^{1-\frac{1}{\eta}}}\times\\&\times\left[\omega_1(Zk_A-\theta q_A)+\omega_2\left(Zk_B-\theta Q^{B,D}(k_A,k_B)\right)\right]-\\&-\delta\frac{-\nu[Z(Zk_A-\theta q_A)]^{-\beta}\left[\omega_1(Zk_A-\theta q_A)+\omega_2\left(Zk_B-\theta Q^{B,D}(k_A,k_B)\right)\right]^{\rho}}{1-\beta[Z-\theta(\omega_1+\omega_2)]^{1-\frac{1}{\eta}}}\Bigg\}.\end{aligned}$$

Taking the first-order condition and, then, imposing the parameter constraint (19) and symmetry, namely, $\hat{k}_A=\hat{k}_B=[Z-\theta(\omega_1+\omega_2)]k$ and $q_A=q_B=(\omega_1+\omega_2)k$, we arrive at the following equation involving ω_1 and ω_2,

$$\begin{aligned}&2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)(\omega_1+\omega_2)^{-\frac{1}{\eta}}-\rho\nu Z^{-\beta}(\omega_1+\omega_2)^{\rho-1}=\\&=\frac{\delta\theta\left\{2^{-\frac{1}{\eta}}\left(\omega_1-\frac{\omega_1+\omega_2}{2\eta}\right)+\nu Z^{-\beta}\left[\rho(\omega_1+\omega_2)^{\rho-1}-\beta(\omega_1+\omega_2)^{\rho}\right]\right\}}{[Z-\theta(\omega_1+\omega_2)]^{\frac{1}{\eta}}-\beta[Z-\theta(\omega_1+\omega_2)]}.\end{aligned}\tag{61}$$

Turning now to the necessary condition given by (53), using the value function, (60), the strategies (56), (57), (58) and (59), and, imposing the parameter constraint (19) and symmetry, namely, $\hat{k}_A=\hat{k}_B=[Z-\theta(\omega_1+\omega_2)]k$ and $q_A=q_B=(\omega_1+\omega_2)k$, we arrive at the second equation involving ω_1 and ω_2,

$$\begin{aligned}&2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)(\omega_1+\omega_2)^{-\frac{1}{\eta}}-\rho\nu Z^{-\beta}(\omega_1+\omega_2)^{\rho-1}=\\&=\delta[Z-\theta(\omega_1+\omega_2)]^{-\frac{1}{\eta}}\Bigg\{\beta\theta\nu(\omega_1+\omega_2)^{\rho}+\end{aligned}$$

$$+2^{-\frac{1}{\eta}}\left(1-\frac{1}{2\eta}\right)(\omega_1+\omega_2)^{-\frac{1}{\eta}}-\rho\nu Z^{-\beta}(\omega_1+\omega_2)^{\rho-1}-$$

$$-\frac{\theta(\omega_1+\omega_2)^{-\frac{1}{\eta}}}{2\eta}-$$

$$\left.-\frac{\delta\theta^2\omega_2\left[2^{-\frac{1}{\eta}}\left(\omega_1-\frac{\omega_1+\omega_2}{2\eta}\right)-\rho\nu\omega_2 Z^{-\beta}(\omega_1+\omega_2)^{\rho-1}\right]}{\left[Z-\theta(\omega_1+\omega_2)\right]^{\frac{1}{\eta}}-\beta\left[Z-\theta(\omega_1+\omega_2)\right]}\right\}. \qquad (62)$$

Thus, *equations (61) and (62) reconfirm that the strategies,* $Q^{A,D}(k_A,k_B)$ *and* $Q^{B,D}(k_A,k_B)$ *are, indeed, linear.*

The calculation method we suggest combines insights from (a) the necessary condition with asymmetric stocks, (53), and, (b) from the necessary condition resulting from the Bellman equation, (60), where the symmetry has already been imposed. The necessary condition (53), with ex-ante asymmetric stocks (but ex-post symmetry, when calculating ω_1 and ω_2), and the value function (that has ex-ante the symmetry imposed), give the same information in equilibrium. So, there are two different but equivalent equations with two degrees of freedom: ω_1 and ω_2. These ω's can be calculated from this 2x2 system.

This is possible to do in symmetric games (with the same primitives and with the same initial capital stocks of firms), where a carefully chosen parametric structure yields linear strategies. First, the symmetry of the stocks implies a key feature: both capital stocks grow at the same rate. Without balanced growth, the dynamics of prices would be cumbersome or impossible to tackle analytically. Second, compared to the market structures examined in the previous sections of this paper, the only additional information needed in order to calculate the equilibrium strategies, is the partial derivative $Q_1^{B,D}(k_A,k_B)$, as (53) reveals. The 0-degree homogeneity of partial derivatives when strategies are linearly homogeneous, and the balanced growth of the two firms in symmetric equilibrium imply that the partial derivative $Q_1^{B,D}(k_A,k_B)=\omega_2$, i.e., $Q_1^{B,D}(k_A,k_B)$ is constant. With the method we suggest in this section we shed light on the difficult problem of calculating the equilibrium strategies in such a market structure.

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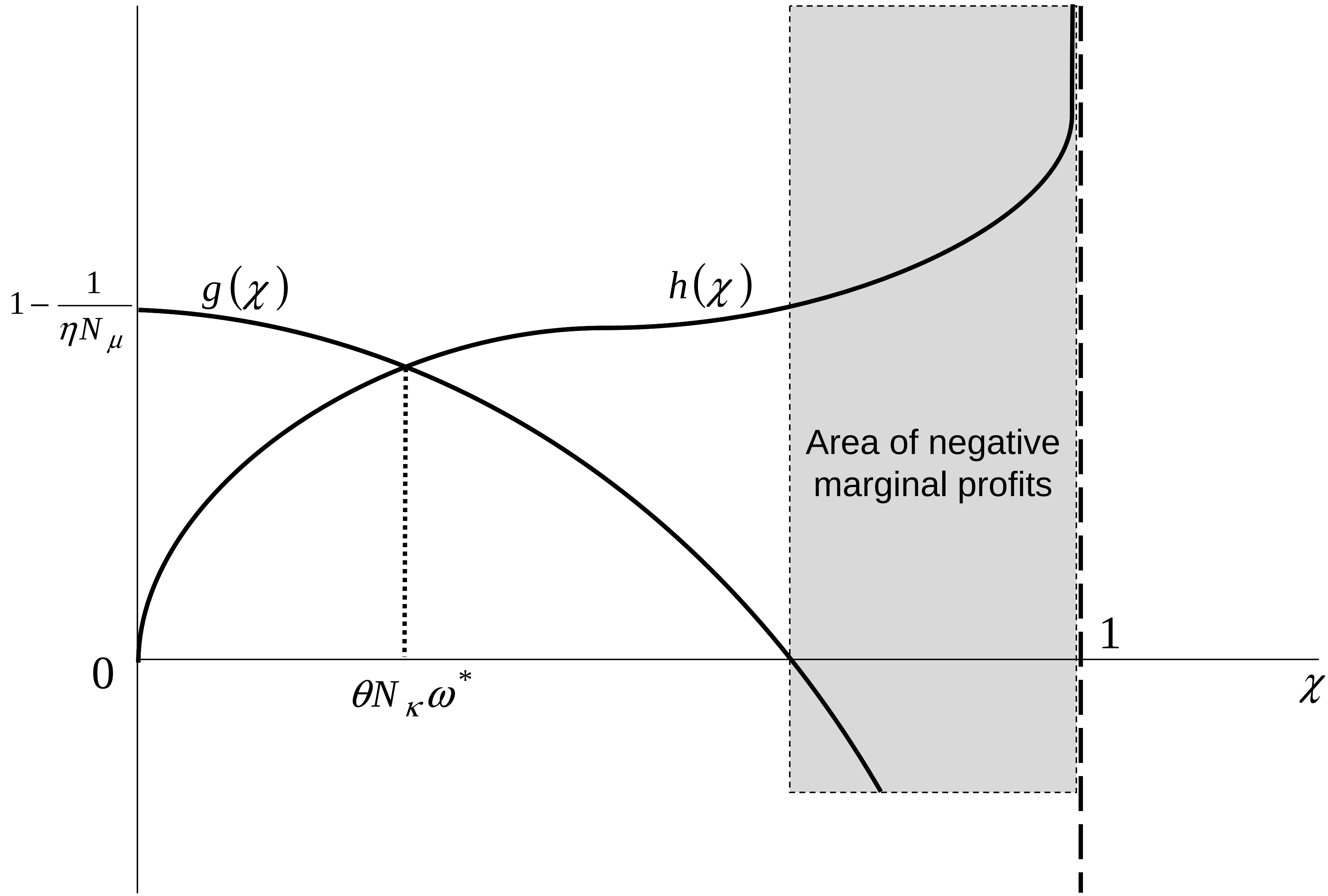

Figure 1 Equilibrium strategies

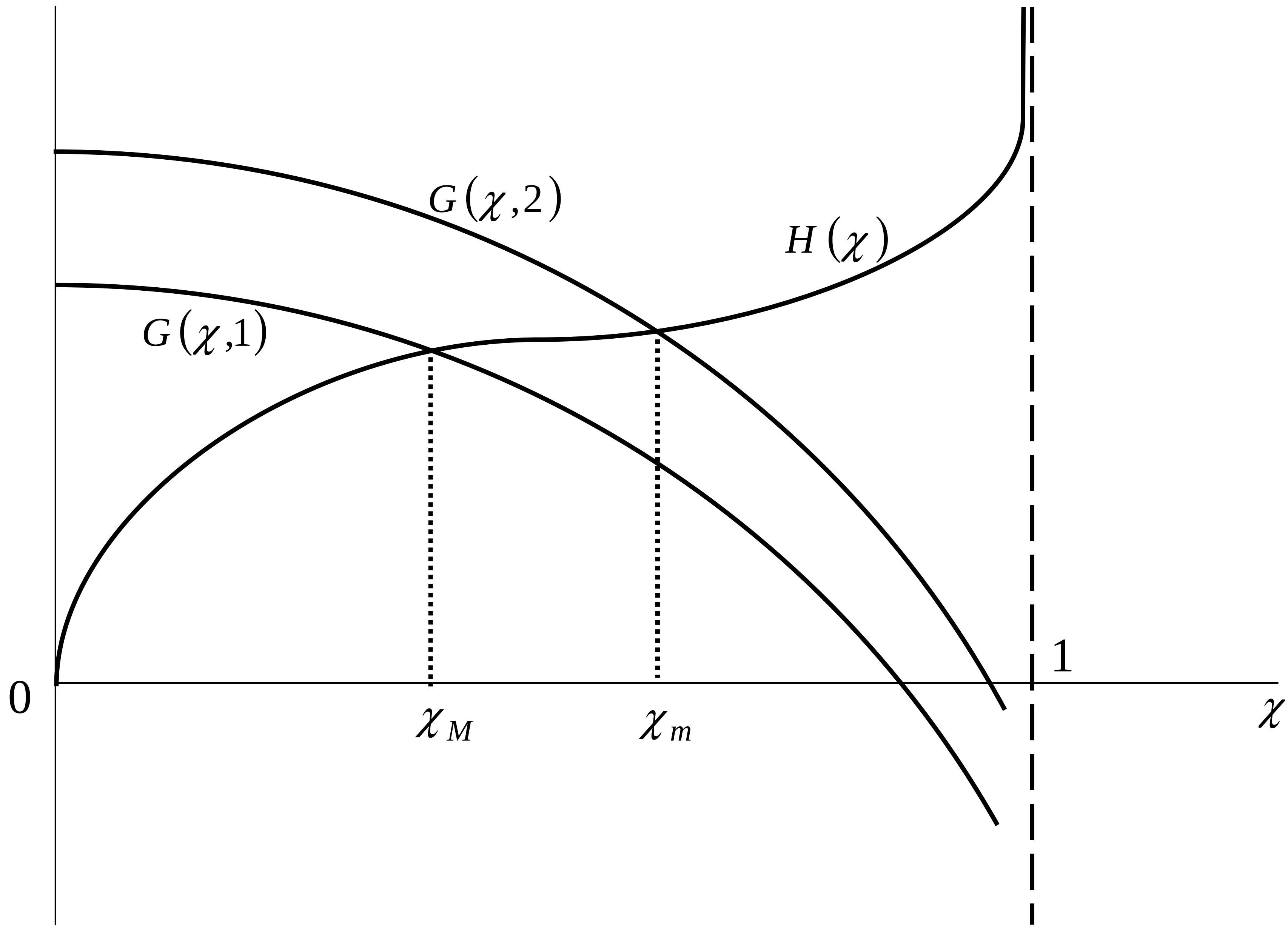

Figure 2 Impact of the dynamic externality on strategies

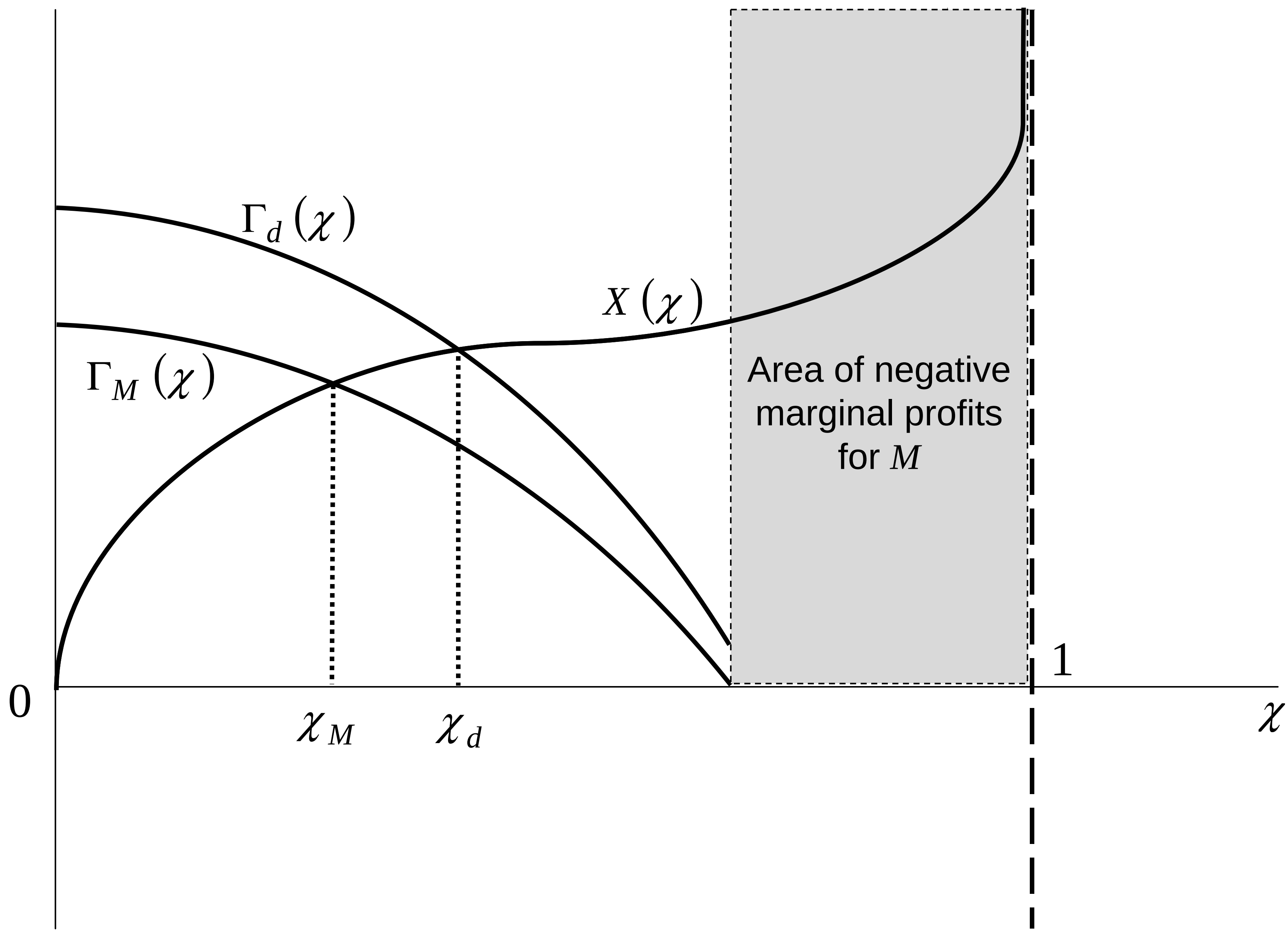

Figure 3 Comparison of χ_M and χ_d

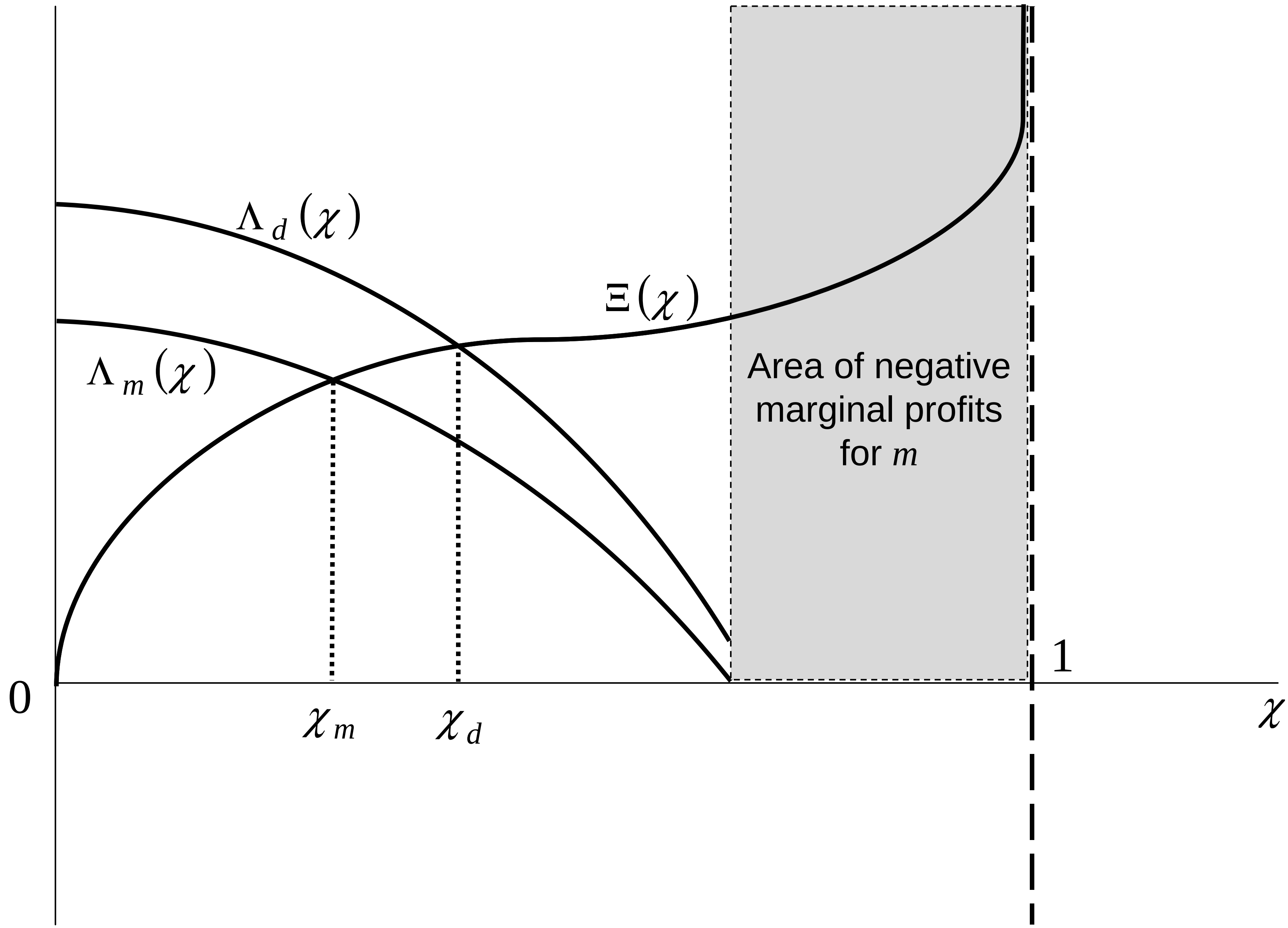

Figure 4 Comparison of χ_m and χ_d when $\eta \leq 2.73$

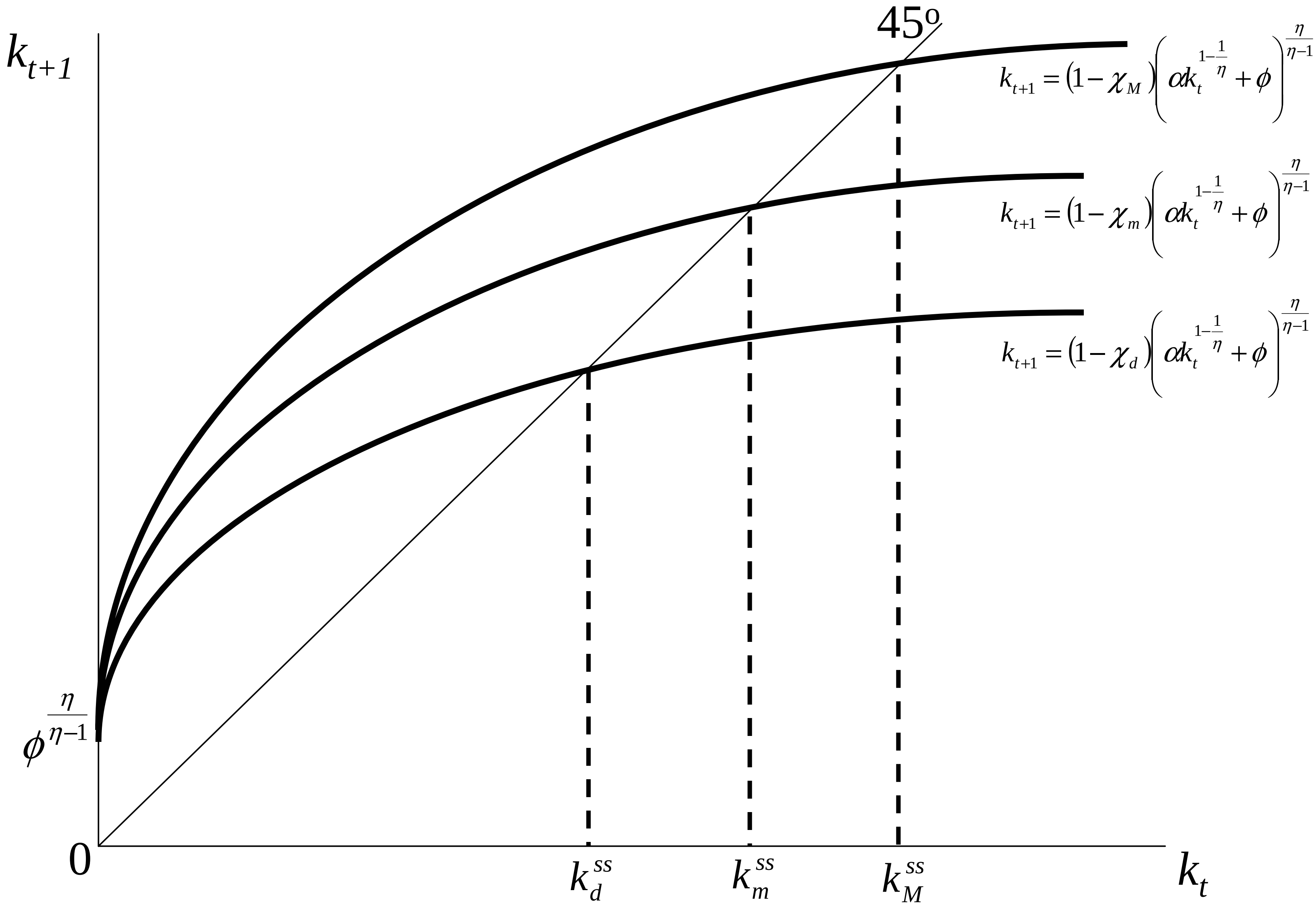

Figure 5 Comparison of steady states for different market structures: $\eta \leq 2.73$

Figure 6 - Example of a case where $\chi_m > \chi_d$

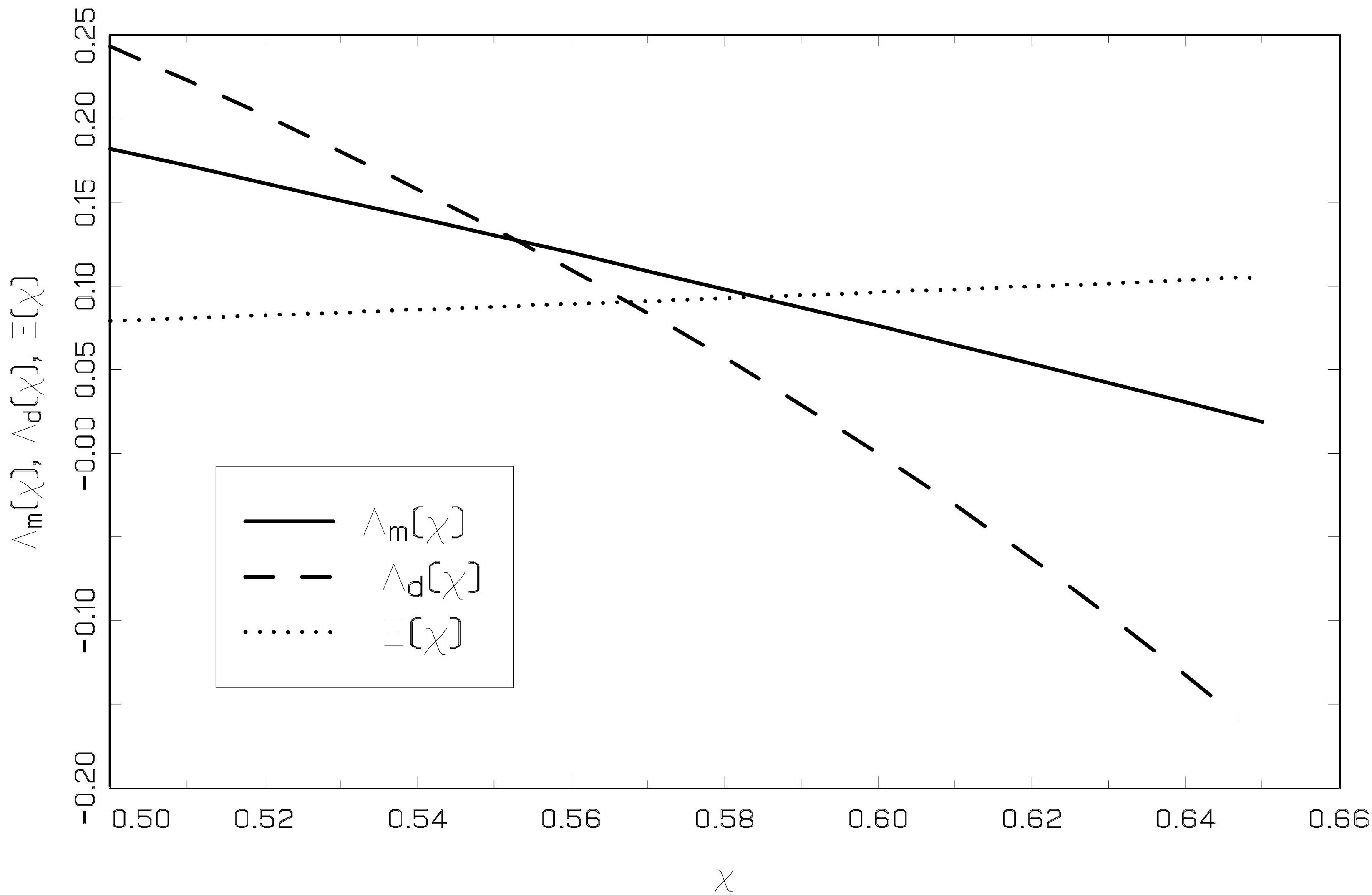

Figure 7.a – Higher ρ

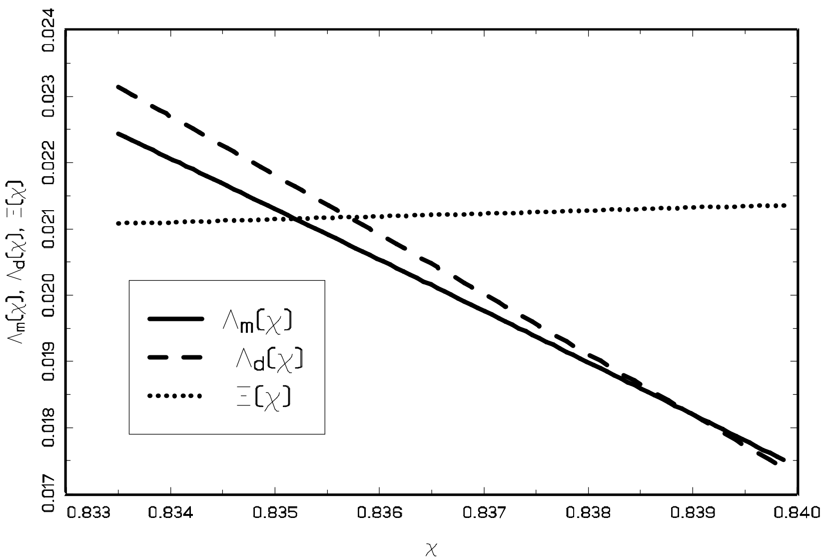

Figure 7.b – Higher $\alpha \subset\delta\supset$

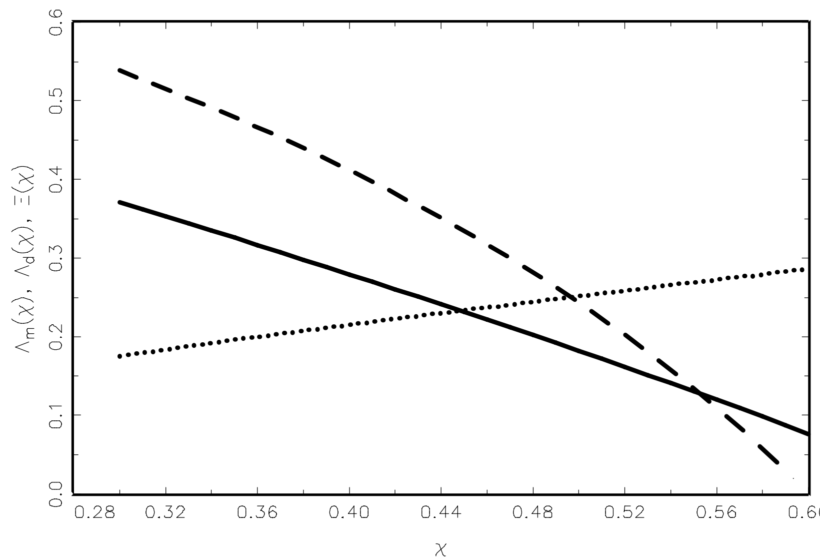

Figure 7.c – Lower ν

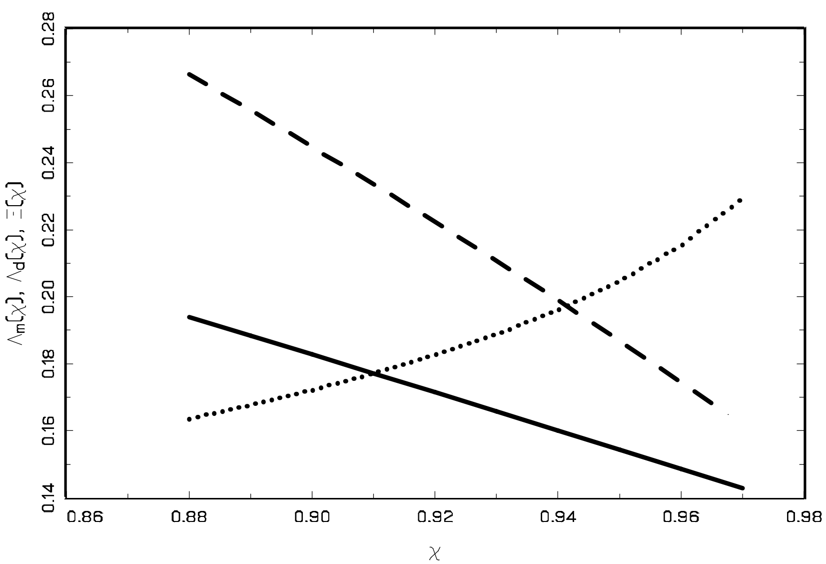

Figure 7.d – Higher Θ