# Learning About Climate Sensitivity

# From the Instrumental Temperature Record[+]

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**The debate over the magnitude of anthropogenically induced climate change has raged for over a century (1-6). Today considerable uncertainty remains about the magnitude of greenhouse-gas-induced climate change, particularly the climate sensitivity – the equilibrium change in global-mean surface temperature per unit of radiative forcing. The rapidity at which uncertainty in the climate sensitivity is resolved has significant policy implications. If resolution is expected soon, deferring action until the picture is clearer may be prudent. If uncertainty is likely to be resolved only slowly, then action today on the basis of expected costs and damages may be the wisest course. Here we use a Bayesian learning model, the instrumental temperature record, and IPCC scenarios of future emissions of greenhouse gases and $SO_2$ to estimate the time required to reduce the uncertainty in the climate sensitivity. We find that more than half a century is required to be 95% confident that the true value of the climate sensitivity lies within ±20% of the estimated value. Further, accelerated control of greenhouse-gas emissions significantly slows this rate of learning, while control of $SO_2$ emissions accelerates it.**

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It might seem that it is impossible to shed any light on future resolution of uncertainty; after all, who can know what future research on climate will yield? Our approach is to assume that the process of learning about climate sensitivity that has occurred during the last 100 years will continue to be driven by the instrumental temperature record. Given this assumption, we can use standard statistical methods to quantify the current uncertainty and the rate at which it can be expected to be resolved.

One approach to estimating the climate sensitivity, $\lambda$, is to use a simple physical model to simulate the instrumental temperature record in a best-fit, maximum-likelihood sense (7). One such simple physical model is (8, 9):

$$T_t = T_{t-1} + \frac{1}{\alpha}\left[ (F_t + F_{SO_4} S_t) - \frac{T_{t-1} - \Gamma}{\lambda} \right] - K(T_{t-1} - O_{t-1}) \qquad (1a)$$

$$O_t = O_{t-1} + L(T_{t-1} - O_{t-1}) \qquad (1b)$$

where $T_t$ is the annual global temperature (°C) difference of the upper ocean in year t from the 1961-1990 average temperature, taken to be synonymous with the surface-air temperature difference, and $T_0 = O_0 = \Gamma$ is the initial temperature difference in some initial year; $O_t$ is the corresponding temperature difference for the deep ocean; $F_t$ is the radiative forcing by greenhouse gases (GHGs), including tropospheric ozone; $S_t$ is the emission rate of sulfur dioxide ($SO_2$), normalized by its value in 1990 (75 TgS/yr), which is converted to sulfate aerosol in the atmosphere; $F_{SO_4}$ is the radiative forcing by sulfate aerosols in 1990; $\alpha$ is the heat capacity of the upper ocean; and K and L equal the coefficient of heat transfer between the upper and deep ocean divided by their respective heat capacities. Equation (1) can be written as the following statistical model (10),

$$T_t = \frac{\Gamma}{\alpha\lambda} + \beta_1 T_{t-1} + \beta_2 F_t + \beta_3 S_t + K O_{t-1} + \varepsilon_t, \qquad (2a)$$

$$O_t = O_{t-1} + L\,(T_{t-1} - O_{t-1}), \tag{2b}$$

where $\varepsilon_t$ is an error term of mean zero. Equation (2) could be estimated from observed records of $T_t$ and $O_t$, from which $\lambda = \beta_2 / (1 - \beta_1 - K)$ and $F_{SO_4} = \beta_3 / \beta_2$. However, the absence of an observational record for the deep-ocean temperature, $O_t$, makes it impossible to statistically estimate all of the coefficients. Because it is unlikely that $O_t$ has changed very much over our sample (1856-1995), we make the assumption that $O_t$ is constant and thus roll $KO_{t-1}$ into the constant term. Equation (2a) thus reduces to:

$$T_t = \beta_0 + \beta_1 T_{t-1} + \beta_2 F_t + \beta_3 S_t + \varepsilon_t = \beta X_t + \varepsilon_t, \tag{3}$$

where $\beta = (\beta_0, \beta_1, \beta_2, \beta_3)$ and $X_t = (1, T_{t-1}, F_t, S_t)^T$. Equation (3) can be estimated from the historic record. We perform ordinary least squares (OLS) estimation of Eq. (3), using the instrumental temperature record (1856–1995; 140 observations) (11) and the historical GHG forcing and $SO_2$ emissions (Fig. 1). Results are summarized in Table 1**.** Also shown in the table is the implied value of $\lambda$. Since we were unable to estimate K in Eq.(2), due to an absence of deep ocean temperature observations, we have used a value of K for computing $\lambda$ drawn from our much more complex energy-balance-climate/upwelling-diffusion-ocean (EBC/UDO) model (7,12-20).

The more familiar $T_{2X} = \lambda\ F_{2X}$ (with $F_{2X}$ = 4.39 $Wm^{-2}$) is the equilibrium temperature rise from a doubling of GHG concentration from preindustrial levels (Table 1). Its value of 2.3$^0$C with a standard error of 0.7$^0$C is not inconsistent with IPCC estimates. However, as we shall see later, the time to reach equilibrium from a GHG shock, as implied by the coefficients in Table 1, is unrealistically rapid.

We now define learning, the resolution of uncertainty. We can never be perfectly certain of our estimate of the climate sensitivity. Statistically, at any point in time we will only have a certain level of confidence in our estimate. We use the 95% confidence level from statistics as our criterion for having learned. We suppose uncertainty is resolved at the time in the future when, for a standard 95% level of confidence, the true climate sensitivity $\lambda$ will first lie between $(1-c)\hat{\lambda}$ and $(1+c)\hat{\lambda}$, where $\hat{\lambda}$ is the then-estimated climate sensitivity and c defines any given confidence interval. Note that uncertainty in the true value of $\lambda$ does not mean that we are unable to reject the hypothesis that $\lambda$ is zero.

To be more precise, we are interested in the first point in time at which we fail to reject the hypothesis, $H_0$ at the 95% level, where $H_0 = \{(1-c)\hat{\lambda} < \lambda < (1+c)\hat{\lambda}\}$. For example, when with 95% probability will the true climate sensitivity first lie within ±20% (c = 0.2) of the estimated sensitivity? We use Bayes Rule (which uses information optimally and thus is the fastest way to resolve uncertainty from the temperature record) to estimate $\beta$ and then compute the median time to achieve a particular confidence level for several scenarios of future GHG concentrations and $SO_2$ emissions, denoted High, Med, and Low, corresponding to their levels (Fig. 1). Med (IS92a) is the IPCC business-as-usual case with expansion in both $F_t$ and $S_t$. Low (IS92c) is the most aggressive control scenario, both for GHGs and $SO_2$. The High (IS92e) case has the largest increase in $F_t$ and $S_t$.

We take a series of draws from the distribution of our OLS estimate of $\beta$ and the distribution of $\varepsilon$ in the present (1995). Each draw $(\beta_{1994}, \varepsilon)_j$ together with the assumed future $F_t$ and $S_t$ is sufficient to generate from Eq. (3) a sample century (1996-2095) of temperatures $(T_t)_j$. Letting $\hat{\beta}_t$ be the OLS estimate of $\beta$ based on data through year t > 1995 for a particular trajectory, Bayes Rule (21) defines how $\hat{\beta}_t$ will evolve over time as a new observation $(Y_t, X_t)$ is added to the data set, assuming $\varepsilon$ is normally distributed with mean zero and variance $1/\rho$:

$$\hat{\beta}_t = [\hat{P}_{t-1} + \rho(X_t X_t')]^{-1}[\hat{P}_{t-1}\hat{\beta}_{t-1} + \rho Y_t X_t] \quad (6a)$$

$$\hat{P}_t = \hat{P}_{t-1} + \rho\left(X_t X_t'\right) \quad . \quad (6b)$$

Here $\hat{P}_t$ is the precision of the estimate $\hat{\beta}_t$ of $\beta$, and prime denotes the transpose. For a particular value of c and $(\beta_{1994}, \varepsilon)$, the minimum value of t for which $\mathrm{Prob}\left\{(1-c)\hat{\lambda} < \lambda < (1+c)\hat{\lambda}\right\} \geq 0.95$, $\tau_j$, is then determined numerically from the sampling distribution for $\hat{\lambda}$ which is generated by Monte Carlo method (1000 draws) from the distribution of the OLS estimate of $\beta$.

Let $\tau^+(c)$ be the median value of $\tau_j$ taken over 1000 trajectories. $\tau^+(c)$ is the time at which we can expect the true climate sensitivity to be within a prespecified confidence interval (±c%). Figure 2a shows $\tau^+(c)$ for each of the three scenarios, that is, the median year when we are 95% sure that the true value of climate sensitivity lies within a specified range (±c%) of the estimated value. The results are striking: reducing climate sensitivity from its current uncertainty of approximately ±50% to ±20% will take a long time.

We see from Figure 2 that learning that the value of climate sensitivity lies within ±20% of the estimated value takes nearly a century with the High or Med scenarios. This time is cut to approximately 30 years for the low scenario. The High scenario has high levels of GHGs which increase the temperature signal, but also high $SO_2$ emissions which reduce the signal. The net effect is that the signal is weaker for "High" than for "Low."

Figure 2b decouples the effects of GHG concentrations from sulfate aerosols by fixing $SO_2$ emissions at the levels associated with Med, the mid-range emission scenario. Figure 2b shows that controlling GHG emissions slows down the rate of learning about climate sensitivity. The most rapid learning occurs when we have the largest signal from GHGs: over 50 years to achieve a ±20% confidence band. Aggressive control makes the climate change trend more difficult to see within the noisy temperature record. The learning model has difficulty discerning between random warm and cold years and the warm years due to emission of GHGs. Figure 2c shows the result for GHG concentrations set at their level in the Med scenario, and varying $SO_2$ emission rates. Here aggressive control of $SO_2$ increases the upward trend in temperature and thus makes the climate change more visible within the noisy temperature record.

As mentioned earlier, one troubling feature of Eq. (3) as estimated with the instrumental record is that the response from an instantaneous and permanent increase in GHG forcing is surprisingly, and unrealistically, rapid. Since the effect on the temperature n years after a hypothetical shock occurs is $\beta_1^n T_0$, where $T_0$ is the temperature at the time the shock occurred, the closer $\beta_1$ is to unity, the slower the response to a forcing shock. Because of this, we have also estimated Eq. (3) assuming a value of $\beta_1$ that is consistent with the physical model; in fact, we use a value obtained from our more detailed EBC/UDO model. We obtain $\beta_1$ estimates by simulating the effects of a $CO_2$ doubling for 245 years (equal in time to 1856-2100), using the EBC/UDO model, for three different assumed temperature sensitivities, $T_{2x}$, and then use the generated data to estimate the appropriate coefficient values in Eq. (1) with $O_t$ neglected. For $T_{2x}$ = 1.5°C, 2.5°C, and 4.5°C, the results imply $\beta_1$ = 0.87941, 0.92277 and 0.95167, respectively. Consequently, we have also estimated Eq. (3) assuming a fixed value for $\beta_1$ of 0.9, and then estimated the equation again assuming $\beta_1 = 0.94$.

Table 2 shows how the learning times vary with the assumed value of $\beta_1$. The table clearly demonstrates that increasing the value of $\beta_1$ only serves to slow down learning about climate sensitivity. This is as would be expected since fixing $\beta_1$ leads to a poorer fit of Eq. (3) to the instrumental record, and thus more error. This logically increases the amount of time necessary to reduce the error in the estimate of climate sensitivity.

In conclusion, using a simplified model of learning we have shown that achieving some confidence regarding the value of climate sensitivity may take many decades. Although additional factors could be included in our model (such as volcanoes, the sun, and regional temperature variations), the results suggest a very slow resolution to the question of the magnitude of climate sensitivity. This notwithstanding, we have shown that accelerated control of GHGs and $SO_2$ emissions will have opposing effects on the rate at which we learn about climate sensitivity, the former slowing learning and the latter accelerating it. The policy implication is that if we wait until uncertainty is resolved before controlling emissions of greenhouse gases, we may wait a very long time.

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**Figure Captions**

**Figure 1.** (a) Global average temperature relative to 1961-1990 ($^0$C); (b) Radiative forcing due to greenhouse gases (GHGs) (22), and (c) $SO_2$ emission (22) normalized by its 1990 value of 75 TgS/yr.

**Figure 2** (a) Median year $\tau^+(c)$ at which we can be 95% sure that the true climate sensitivity is within ±c% of the estimated climate sensitivity, under emissions assumptions associated with the Low, Med and High scenarios. (b) as in (a), except with the $SO_2$ emission scenario fixed at the Med(IS92a) scenario; (c) as in (a), except with the $CO_2$ emission scenario fixed at the Med (IS92a) scenario. Parameter estimates as in Table 1.

**Table 1.** Ordinary Least Squares Estimates of Parameters in Equation (3)

| Quantity | Estimate | Standard Error |
|---|---|---|
| $\beta_0$ | -0.2209 | 0.0428 |
| $\beta_1$ | 0.5589 | 0.0712 |
| $\beta_2$ | 0.2295 | 0.0751 |
| $\beta_3$ | -0.2517 | 0.1455 |
| $\varepsilon$ | NA | 0.1008 |
| $\lambda$ | 0.5350 | 0.1592 |
| $T_{2x}$ | 2.3486 | 0.6988 |

**NB: 1.** $\lambda \equiv \beta_2 / (1 - \beta_1 - K)$ computed using K = 0.012 from complex energy-balance-climate/upwelling-diffusion-ocean (EBC/UDO) model (7,12-20).

2. $T_{2x} \equiv \lambda F_{2x}$, with $F_{2x} = 4.39 W m^{-2}$.

**Table 2.** Estimated Learning Times for Three Models.

| c | $\beta_1$ = 0.5589* | | | $\beta_1$ = 0.9 | | | $\beta_1$ = 0.94 | | |
|---|---|---|---|---|---|---|---|---|---|
| % | IS92e | IS92a | IS92c | IS92e | IS92a | IS92c | IS92e | IS92a | IS92c |
| 5 | >2100 | >2100 | >2100 | >2100 | >2100 | >2100 | >2100 | >2100 | >2100 |
| 10 | >2100 | >2100 | 2077 | >2100 | >2100 | >2100 | >2100 | >2100 | >2100 |
| 15 | 2096 | 2085 | 2056 | >2100 | >2100 | >2100 | >2100 | >2100 | >2100 |
| 20 | 2089 | 2073 | 2033 | >2100 | >2100 | >2100 | >2100 | >2100 | >2100 |
| 30 | 2067 | 2039 | 2011 | >2100 | >2100 | 2081 | >2100 | >2100 | >2100 |
| 50 | 2001 | 2001 | 2000 | >2100 | 2081 | 2044 | >2100 | >2100 | >2100 |

*Results from Eq. (3) with all parameters estimated. Other cases involve $\beta_1$ fixed at indicated value during estimation of Eq. (3).

NB:

>2100 states that learning is resolved at some time after the year 2100, due to a lack of estimates of greenhouse gas and sulfate forcing post-2100.

Learning time is defined as the first year in which we expect to be 95% confident that the true climate sensitivity is within ±c% of the estimated climate sensitivity.

Figure 1:

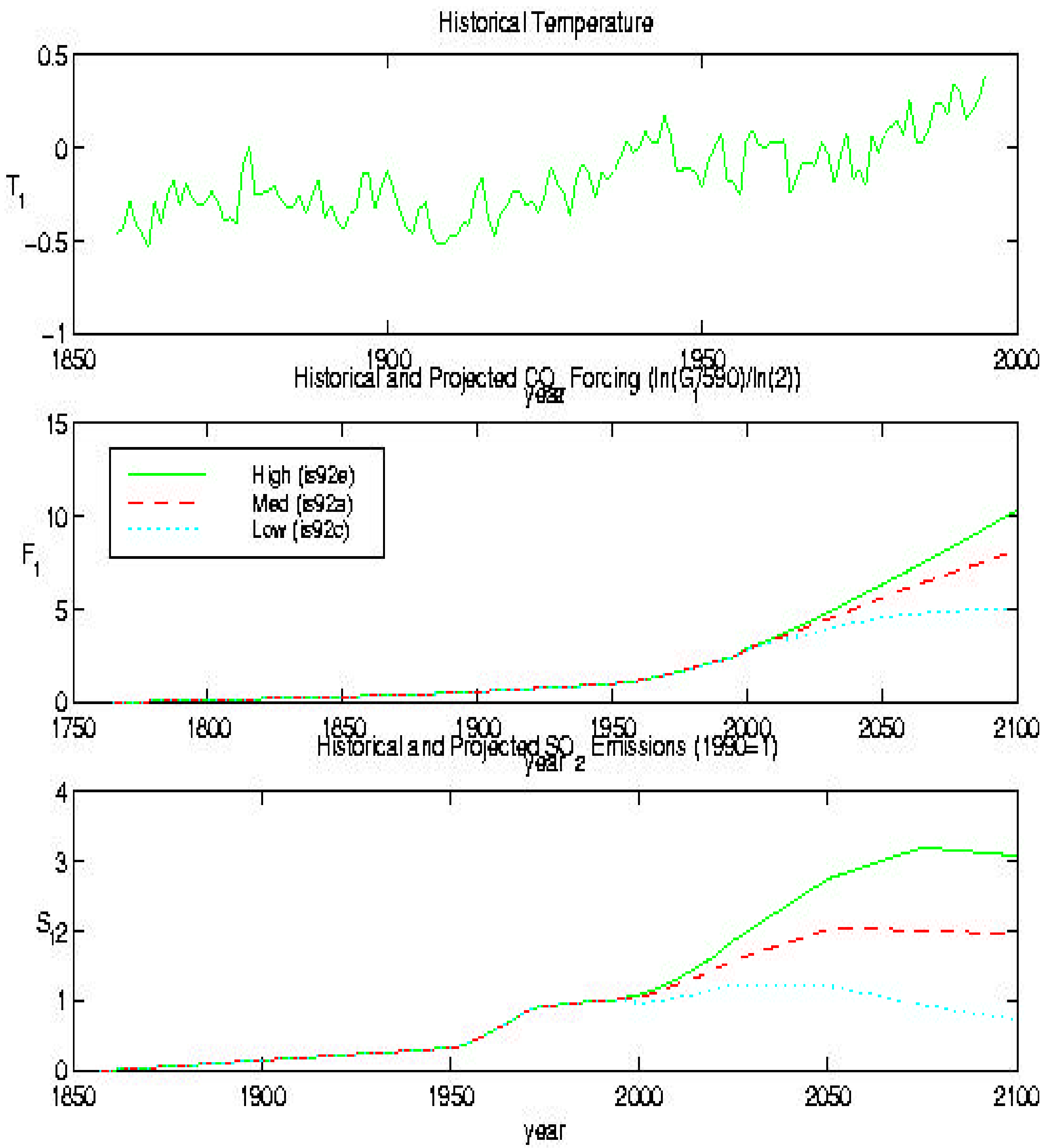
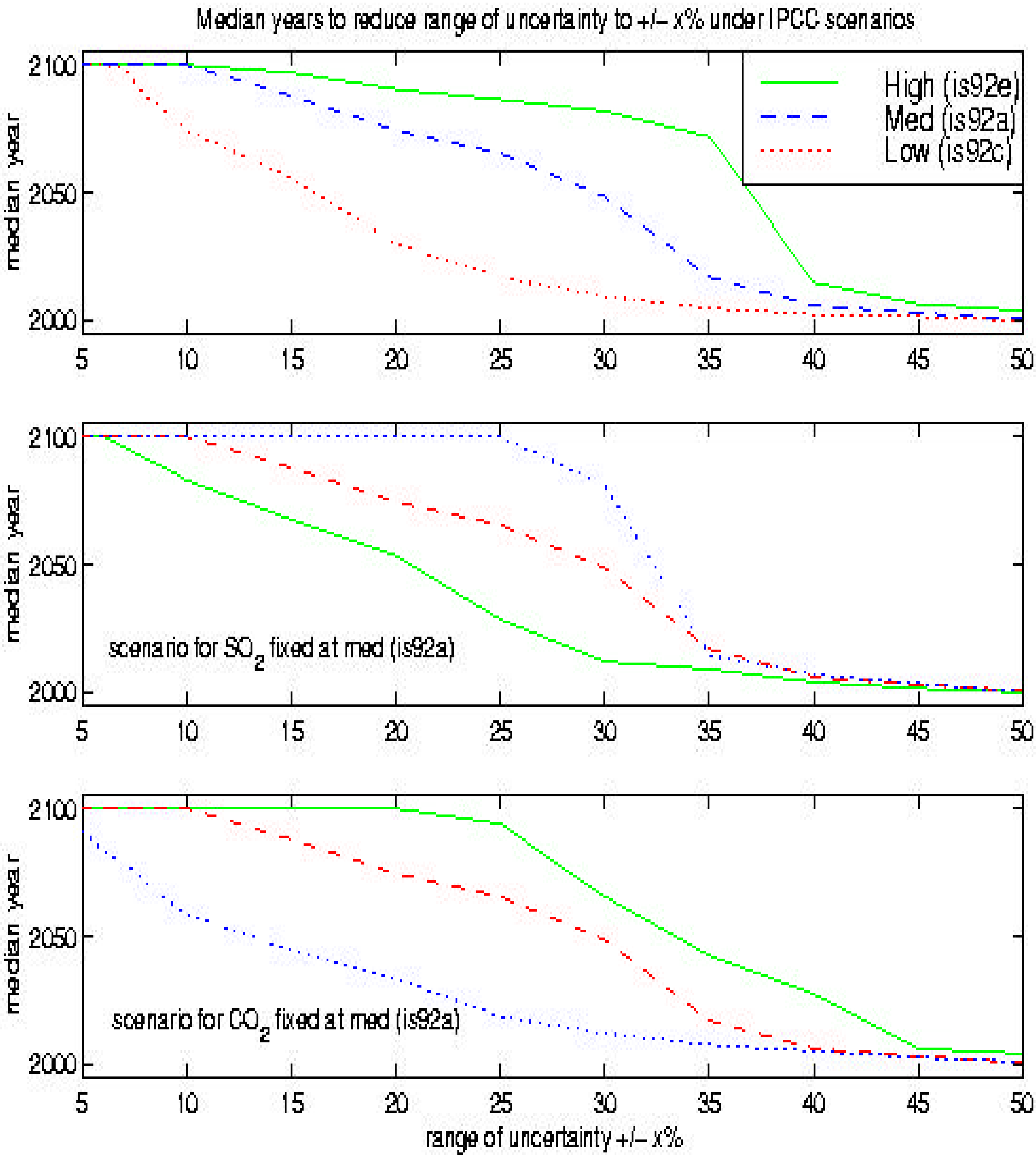

Historical Temperature
T1
year
Historical and Projected CO2 Forcing (ln(G1/590)/ln(2))
F1
High (is92e)
Med (is92a)
Low (is92c)
year
Historical and Projected SO2 Emissions (1990=1)
S12
year

Figure 2:



Median years to reduce range of uncertainty to +/- x% under IPCC scenarios
median year
2100
2050
2000
5
10
15
20
25
30
35
40
45
50
High (is92e)
Med (is92a)
Low (is92c)
scenario for SO2 fixed at med (is92a)
scenario for CO2 fixed at med (is92a)
range of uncertainty +/- x%