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Sequential Auctions with Synergies: The Paradox of Positive Synergies*

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Abstract

In multi-unit procurement auctions winning multiple contracts can lead to cost advantages due to synergies. In this paper we analyze the effects of the presence of such synergies on bidding behavior and thus auction outcomes in general. We find that the presence of synergies on the bidders' side induces more competitive bidding and therefore leads to lower expected payoffs for bidders and higher expected revenues for sellers. Thus, instead of benefiting from the presence of synergies, bidders suffer from it. Moreover it is found that serious bankruptcy problems can occur. In particular the negative welfare consequences caused by these bankruptcy problems are of major importance for auction design when synergies are present. Finally, the presence of synergies leads to a decreasing price trend and can therefore explain the declining price anomaly.

Keywords: Sequential Auctions, Procurement Auctions, Synergies, Bankruptcy, Afternoon Effect

JEL Classification: D44 (Auctions), H57 (Procurement)

1 Introduction

Many (procurement) auctions can be characterized by the following setting. A government auctions a large building project that takes several years to complete. A year later another large building project is auctioned, using the same auction format, by that government. The group of bidders are approximately identical for both auctions and the winner of the first contract participates in the second auction.

A distinguishing characteristic of procurement auctions is their sequential nature. Often similar projects are auctioned within a short period of time. Construction contracts, military

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procurement and the uncoordinated sequence of European spectrum auctions during 2000 and 2001 are examples of this sequentiality (Reiß and Schöndube, 2002). For these spectrum auctions possible synergies are obvious. For instance, there may be local advertisement and management economies of scale. Global synergies can occur in the development and management of the network and consumers' willingness to pay can increase due to network effects (Ausubel et al., 1997).

Often large-scale projects need to be divided into small pieces or subprojects and then procured sequentially. For instance, this can be due to current governments that may not commit resources that will become available to their successors. Also there can be significant scheduling problems in undertaking large-scale projects. The project as a whole can be too complex to auction at once or essential facilities cannot be shut down simultaneously. Finally there may be too few firms with sufficient resources to complete the project as a whole (Yildirim, 2004).

It is well known that in multi-unit procurement auctions winning multiple contracts can lead to cost advantages due to synergies. These synergies can be material, for instance owning specialized equipment, but also intangible like expertise. A consequence for the settings above is that bidders' valuations are stochastically dependent across the auctions and therefore participants have an exposure problem.

Synergies indeed influenced the outcomes of the sequence of European UMTS auctions. The auction held by the United Kingdom raised the highest per person revenue of all European auctions. Cramton (2002) argues that a reason for this is that it was the first in the sequence of UMTS auctions throughout Europe. Winners in the UK auction were well positioned for subsequent auctions and hence bidders could view it as a foot-in-the-door to Europe.

The literature on auctions is voluminous. Much interest in multi-unit auctions was generated by the spectrum auctions that were conducted all over the world. Most recent literature on sequential auctions has focussed on price trends. Weber (1983) shows that if bidders demand a single unit, the prices in a sequential auction of identical objects are a martingale; in expectation prices drift neither up nor down. However there is ample empirical evidence of declining price trends in sequential auctions, which is known as the declining price anomaly or afternoon effect.

Branco (1997) was the first to attribute declining prices to the presence of positive synergies. He transformed the model of Krishna and Rosenthal (1996) into a sequential auction of two identical objects. In each auction one local bidder participates and a global bidder participates in both auctions. Winning both objects increases the payoff for the bidder with positive constant.

Menezes and Monteiro (2003 and 2004) criticize this way of modeling synergies by noting that for low valuations the marginal synergy factor is infinite. The synergy between two highly valued objects is the same as between two worthless. Therefore they define the valuation for both objects as a function of the value of one. If this function is larger or smaller than two times the value of one object, there are respectively positive or negative synergies.

Jeitschko and Wolfstetter (2002) depart from these models by analyzing a sequential auction of two nonidentical objects. In their model two bidders are active in both rounds and a bidder's valuation for an object is either high or low. Economies and diseconomies of scale are modeled as respectively an increase or decrease in the probability of having a high valuation for the second object. In their appendix they also report results for this setting with uniformly distributed valuations between zero and one without synergy and between zero and two with synergy.

Hendricks and Porter's (1988) study of drainage lease auctions was the first empirical study to show that interdependencies among the values of objects affect the outcome of a sequential auction. Ausubel et al. (1997) show that synergies associated with winning multiple adjacent licenses in the United States spectrum auctions affected bidding strategies. Rusco and Walls (1999) find that in timber auctions spatial correlation of bids induces more aggressive bidding. De Silva (2005) finds the same for road construction auctions in Oklahoma. De Silva et al. (2005) show that in sequential construction auctions by the Oklahoma Department of Transportation previous winners are more likely to win in later auctions.

All these empirical studies show that the presence of synergies on the bidders' side changes a sequential auction substantially. So far, theoretical work has only focussed on price trends and not the consequences for bidders. In standard private value auctions it is not very interesting to analyze the consequences for bidders since they will never make losses. However in presence of positive synergies this is not guaranteed anymore. Therefore an analysis of the effect of positive synergies on bidders is needed.

In this paper we analyze the consequences for bidders and show that bidders not only dissipate the complete expected rent of having synergies but also forgo a part of their intrinsic share. Thus, instead of benefiting from the presence of synergies, the bidders suffer from it. Moreover it is found that serious bankruptcy problems can occur. In particular the negative welfare consequences caused by these bankruptcy problems are of major importance for auction design when synergies are present.

We consider a sequential auction of two stochastically equivalent objects. It is well-known that asymmetric first price auctions are inefficient and that a closed form expression of the bidding strategies is not always available (Krishna, 2002). Therefore we will focus on second price auctions. The bidders' valuations for each object are independent draws from the same distribution. These valuations are drawn just before the start of each round of the auction. We model positive synergies by multiplying the second round valuation with a constant—the synergy factor—larger than one.

In the next section the model is described. Section 3 gives the equilibrium outcomes of which the implications are discussed in section 4. Next the effect of changes in the number of bidders is analyzed in section 5. In section 6 we give an example for uniformly distributed valuations and section 7 contains the conclusion.

2 The Model

We consider a private value auction with $n \geq 2$ risk neutral bidders. Two objects are auctioned sequentially using the second price sealed bid format.[1] Bidders' valuations are distributed according to the differentiable cumulative distribution function $F(v)$ with associated density function $f(v)$ and without loss of generality restricted to the interval [0,1].[2] Valuations are individually uncorrelated and drawn independently from the same identical distribution at the start of each round.

Although no bidder knows his second round valuation during the first round, it is common knowledge that winning the first round increases this valuation by factor $s > 1$. This synergy factor only applies to the second round valuation v_2. Winning round 1 then increases the valuation in the second round from v_2 to $s\,v_2$, but does not have any effect on the first round valuation v_1.

After each round the bidders are informed whether or not they have won the round. In our setting the identity of the winner and whether the winning bid is made public or not are irrelevant. The first auction informs every bidder whether he or one of his opponents can benefit from synergies in the second auction. However, it does not convey any information on the actual valuations of bidders in the second auction. We rule out the possibility of a resale of the first object after the auction of the second.

When positive synergies are present in a sequential auction of two identical objects, the losers of the first auction have no reason to participate in the second auction. In the literature it is often assumed that, without participation costs, they still participate since they are indifferent. With non-identical objects these bidders can still win the second auction and thus do have an incentive to participate.

Our manner of modeling positive synergies is comparable to the way Black and De Meza (1992) model negative synergies. In contrast to them we allow the valuations for both objects to be uncorrelated. Attributing cost reductions only to the second project makes sense considering time lags between projects. For instance, expertise is created during the first project and gives only benefits during the second. This makes the resale of the first project after the auction of the second project impossible.

We also ensure there is a relationship between the increase in valuation due to synergies and the intrinsic value of the second object. Although the synergy factor is known a priori, the actual gain due to synergies depends on the realized second round valuation. Thus one expects to complete a project at only a percentage of the costs if one has expertise. The exact cost reduction then depends on the actual costs of the second project.

Our model resembles Engelbrecht-Wiggans' (1994) in having bidders' valuations drawn

[1] Although most applications are procurement settings, we analyze standard 'highest bid wins' auctions for expositional ease and without loss of generality.

[2] Most of the results do not rely on the compactness of the set from which the valuations are drawn and can be shown to be satisfied when values are drawn from the noncompact interval $[0, \infty)$.

independently across objects and only be known at the start of each round. Especially when there is a time gap between two following auctions, this is applicable. Then, the exact valuation for the second object is not known at the time the first auction takes place. In contrast to Engelbrecht-Wiggans we assume independence of valuations across bidders and do not restrict to unit-demands in this paper.

The difference between our model and learning-by-doing models is that in our case the cost reductions for future periods do not depend on the relatedness of the actions taken in both periods. In learning-by-doing settings, the cost reductions are larger the closer the future action is to previous and do not depend on the actions of other players. The cost reduction in our model depends on the outcomes of the auction and thus also the actions of the other bidders. Moreover there is no relationship between the benefits from synergies and the closeness of the first and second round bid.

There is an intuitive relationship between our setting and switching cost models. In those settings there is also fiercer competition in early rounds to get an advantage in subsequent rounds. Although switching costs can lead to a decrease in oligopolists' profits, typically firms are found to gain from switching costs (Farrell and Klemperer, 2005). However, we will show that in our setting bidders' profits are always lower when positive synergies are present.

3 The Equilibrium and its Behavioral Implications

We write b_{kn} and v_{kn} for respectively the bid and valuation in round k of bidder n. In the second round the winner of the first auction is denoted by w and the bidders ℓ, i refer to the $n-1$ bidders that did not win the first round.

With $\bar{\pi}_{1i}$ we denote the expected instantaneous payoff of round 1 for bidder i, prior to the realization of the valuations for this round. The expected instantaneous payoff of round 2 for bidder i, $\bar{\pi}_{2i}$, is prior to the realization of the valuations for this round but given the outcome of round 1. Finally, the expected price of round k, $\bar{p}_k$, is of course also prior to the realization of the valuations for this round and thus the seller's expected revenue of that round.

Proposition 3.1. *For a sequential second price sealed bid auction with independent, individually uncorrelated valuations, n risk-neutral bidders, two objects and a synergy factor s, the bidding strategies given by*

$$b^*_{1i} = v_{1i} + \Delta \quad \textit{with} \quad \Delta = \bar{\pi}_{2w} - \bar{\pi}_{2\ell}$$

and

$$b^*_{2i} = \begin{cases} v_{2i} & \textit{if round 1 is lost;} \\ s\, v_{2i} & \textit{if round 1 is won.} \end{cases}$$

constitute a symmetric linear equilibrium in weakly dominant strategies.

Proof *1. The bid strategies given by b^*_{2i} in the proposition constitute a symmetric linear equilibrium in weakly dominant strategies for the round 2 auction.*
Bidder w's actual valuation for the second object is $s\, v_{2w}$. Setting $v'_{2w} = s\, v_{2w}$ and then following the normal analysis for a second price auction shows that bidding v'_{2w} is a weakly dominant strategy. For all $n-1$ bidders of type ℓ there are no synergies possible in the second round. Therefore the analysis is the same as for an auction without synergies.
2. Given the second round bidding behavior, the bid strategies given by b^*_{1i} constitute a symmetric linear equilibrium for the round 1 auction.
The actual value of winning round 1 is not only the value of the first object but also the change in the expected instantaneous payoff of round 2 that results from winning round 1. Then it readily follows that bidding this is a weakly dominant strategy. □

In the first round all bidders are symmetric and bid their valuations plus Δ. The computation of the expected instantaneous payoff follows immediately from the notation denoted above.

$$\bar{\pi}_{1i} = \int_0^1 v\, F^{n-1}(v)\, f(v)\, \mathrm{d}v - (n-1) \int_0^1 (v+\Delta)\, F^{n-2}(v)\, f(v)\, (1 - F(v))\, \mathrm{d}v.$$

From the equilibrium bidding strategies it follows that in the second round all bidders bid the synergy-adjusted value. That is, the first round losers bid their second round valuation and the winner bids his second round valuation multiplied by the synergy factor. The positive synergy factor induces the first round winner to increase his bid and subsequently increase his probability to win. In fact, if the first round winner's second round valuation is above the reciprocal of the synergy factor, his bidding will not leave any opportunity at all for a first round loser to win.

The expected instantaneous second round payoff for the winner of the first round is given by

$$\bar{\pi}_{2w} = \int_0^1 s\, v\, F^{n-1}(s\, v)\, f(v)\, \mathrm{d}v - (n-1) \int_0^1 v\, F^{n-2}(v)\, f(v)\, (1 - F(v/s))\, \mathrm{d}v$$

and for a loser by

$$\begin{aligned}\bar{\pi}_{2\ell} = & \int_0^1 v\, F^{n-2}(v)\, F(v/s)\, f(v)\, \mathrm{d}v - \int_0^1 s\, v\, F^{n-2}(s\, v)\, f(v)\, (1 - F(s\, v))\, \mathrm{d}v \\ & -(n-2) \int_0^1 v\, F^{n-3}(v)\, F(v/s)\, f(v)\, (1 - F(v))\, \mathrm{d}v.\end{aligned}$$

Bidder w wins the second auction for sure if and only if $v_{2w} > \frac{1}{s}$. This can be observed in the first term of $\bar{\pi}_{2w}$ and the second term of $\bar{\pi}_{2\ell}$ by noting that $F^{n-1}(s\, v) = 1$ and $1 - F(s\, v) = 0$ respectively if $v > \frac{1}{s}$.

The bidder of type w wins the auction if his synergy-adjusted bid is higher than that of all the other bidders and the price he has to pay is always equal to the highest bid among the $n-1$ bidders of type ℓ. A bidder of type ℓ only wins if his valuation is larger than that of the other losers and larger than the bid of bidder w. For the expected instantaneous second round payoff of a type ℓ bidder there are two possibilities to consider; one of the $n-2$ remaining type ℓ bidders has the second highest bid (third term) or bidder w has the second highest bid (second term). In case bidder w has the second highest bid, it must hold that $v_{2w} < \frac{1}{s}$.

A bidder's ex-ante expected total payoff, thus before the first round valuation is determined, of the auction sequence as a whole is given by

$$\bar{\mu}_i = \bar{\pi}_{1i} + \tfrac{1}{n}\,\bar{\pi}_{2w} + \tfrac{n-1}{n}\,\bar{\pi}_{2\ell}.$$

Ex-ante bidders are symmetric and thus bidder i wins the first round with probability $\frac{1}{n}$.

In the first round all bidders are identical, hence the expected price for the first round equals the expected second highest valuation plus delta. The expected price for the second round is the sum of the expected payments made by each of the $n-1$ bidders of type ℓ and the single bidder of type w. The expected prices are then

$$\bar{p}_1 = n(n-1)\int_0^1 (v+\Delta)\,F^{n-2}(v)\,f(v)\,(1-F(v))\,\mathrm{d}v$$

and

$$\begin{aligned}\bar{p}_2 = (n-1)&\int_0^1 v\,F^{n-2}(v)\,f(v)\,(1-F(v/s))\,\mathrm{d}v\\ &+(n-1)\Big\{\int_0^1 s\,v\,F^{n-2}(s\,v)\,f(v)\,(1-F(s\,v))\,\mathrm{d}v\\ &\qquad+(n-2)\int_0^1 v\,F^{n-3}(v)\,F(v/s)\,f(v)\,(1-F(v))\,\mathrm{d}v\Big\}.\end{aligned}$$

The seller's ex-ante expected revenue of the auction sequence as a whole is given by

$$\bar{R} = \bar{p}_1 + \bar{p}_2.$$

Lemma 3.2. *$\bar{\pi}_{2\ell}$ is decreasing in the synergy factor and converges to zero if the synergy factor approaches infinity.*

Proof Suppose $s' > s$. Consider the expected instantaneous payoff of a loser ℓ, i when the synergy factor is s, but restricted to the area where he would win given the synergy factor is s'. This is the area where $v_{2\ell,i'} < v_{2\ell,i}$ for all $i' \neq i$ and $s'\,v_{2w} < v_{2\ell,i}$ and this payoff is given by

$$\begin{aligned}\bar{\pi}_{2\ell,i}|_{\text{rest.}}(s) = &\int_0^1 v\,F^{n-2}(v)\,F(v/s')\,f(v)\,\mathrm{d}v - \int_0^1 s\,v\,F^{n-2}(s'\,v)\,f(v)\,(1-F(s'\,v))\,\mathrm{d}v\\ &-(n-2)\int_0^1 v\,F^{n-3}(v)\,F(v/s')\,f(v)\,(1-F(v))\,\mathrm{d}v.\end{aligned}$$

Comparing this restricted payoff to the payoff with s' gives

$$\bar{\pi}_{2\ell,i}|_{\text{rest.}}(s) > \bar{\pi}_{2\ell,i}(s') \quad\Longleftrightarrow\quad \int_0^1 (s'-s)\,v\,F^{n-2}(s'\,v)\,f(v)\,(1-F(s'\,v))\,\mathrm{d}v > 0.$$

which holds by definition.

Of course, the expected instantaneous payoff to the loser over the unrestricted area is larger than over the restricted area since he never bids above his valuation: $\bar{\pi}_{2\ell,i}(s) > \bar{\pi}_{2\ell,i}|_{\text{rest.}}(s)$. Hence, $\bar{\pi}_{2\ell,i}(s') < \bar{\pi}_{2\ell,i}(s)$. This holds for any pair of s' and s such that $s' > s$ and consequently $\bar{\pi}_{2\ell,i}$ is strictly decreasing in the synergy factor.

Since $\bar{\pi}_{2\ell}$ is bounded from below by zero and the only term being positive (the first term) converges to zero when s approaches infinity, $\bar{\pi}_{2\ell}$ converges to zero when s approaches infinity: $\lim_{s\to\infty} \bar{\pi}_{2\ell} = 0$. □

Lemma 3.3. *$\bar{\pi}_{2w}$ is increasing in the synergy factor and diverges to infinity if the synergy factor approaches infinity.*

Proof Suppose $s' > s$. Consider the expected instantaneous payoff of the winner when the synergy factor is s', but restricted to the area where he would win given the synergy factor is s. For this area it holds that $v_{2\ell,i} < s\, v_{2w}$ for all i and this restricted payoff is given by

$$\bar{\pi}_{2w}|_{\text{rest.}}(s') = \int_0^1 s'\, v\, F^{n-1}(s\, v)\, f(v)\, \mathrm{d}v - (n-1)\int_0^1 v\, F^{n-2}(v)\, f(v)\, (1 - F(v/s))\, \mathrm{d}v.$$

Comparing $\bar{\pi}_{2w}|_{\text{rest.}}(s')$ and $\bar{\pi}_{2w}(s)$ gives

$$\bar{\pi}_{2w}|_{\text{rest.}}(s') > \bar{\pi}_{2w}(s) \quad\Longleftrightarrow\quad \int_0^1 (s' - s)\, v\, F^{n-1}(s\, v)\, f(v)\, \mathrm{d}v > 0.$$

which holds by definition. The expected instantaneous payoff of the winner over the unrestricted area is larger than over the restricted area: $\bar{\pi}_{2w}(s') > \bar{\pi}_{2w}|_{\text{rest.}}(s')$. Thus, $\bar{\pi}_{2w}(s') > \bar{\pi}_{2w}(s)$. This holds for any pair of s' and s such that $s' > s$ and therefore $\bar{\pi}_{2w}$ is strictly increasing in the synergy factor.

It is readily seen that the first term of π_{2w} diverges to infinity and the second term is finite (for all values of s this term is bounded from below by $-(n-1)$). Thus $\bar{\pi}_{2w}$ diverges to infinity when s approaches infinity: $\lim_{s\to\infty} \bar{\pi}_{2w} = \infty$. □

For $s = 1$ our sequential auction coincides with two one-shot auctions and thus $\bar{\pi}_{2w} = \bar{\pi}_{2\ell}$. This knowledge combined with the previous two lemmas implies that $\bar{\pi}_{2w} > \bar{\pi}_{2\ell}$ for all synergy factors larger than 1. Therefore, the value of Δ defined in the proposition is positive and increasing for increasing values of the synergies factor. In the limit, the value of Δ diverges. According to the equilibrium bidding strategies from the proposition, in the first round all bidders upgrade their one-stage bid with the option value Δ. This indicates that the first period bidding behavior becomes more competitive for increasing values of the synergy factor.

Theorem 3.4. *The presence of positive synergies leads to more aggressive bidding in the first round of the auction. Moreover, bidding behavior becomes fiercer for increasing values of the synergy factor.*

4 Welfare Implications

In this section the welfare implications of the presence of synergies in the dynamic auction setting are discussed. First, the consequences of synergies for the bidders' ex-ante expected payoffs are analyzed. Next, the revenues to the auctioneer are discussed. Finally, efficiency and general welfare consequences are studied.

4.1 Ex-ante Expected Total Payoff

Without synergies the expected instantaneous payoff of each round is given by

$$\bar{\pi} = \int_0^1 v\, F^{n-1}(v)\, f(v)\, \mathrm{d}v - (n-1) \int_0^1 v\, F^{n-2}(v)\, f(v)\, (1 - F(v))\, \mathrm{d}v.$$

The difference between the expected first round payoff in the situation with and without synergies is then that in the first situation the winner pays an amount Δ in addition to the price of the latter situation. Namely, the winner has to pay the second highest first round valuation plus Δ. This means that the synergy has no effect on the probability to win, nor the winners revenue, but on the price only. So, the expected first round payoff is lower in case synergies are present. In the presence of synergies, bidders invest in this first round to be eligible for synergies. Moreover, the expected first round payoff is decreasing in the synergy factor, since Δ is increasing in the synergy factor. Apparently, the larger these possible synergies are, the higher the sacrifice bidders are willing to make to get those synergies.

The uncertainty concerning the benefits from synergies leads to an exposure problem in this sequential auction. Bidders bid above their valuation in the first auction and consequently it is possible that the instantaneous payoff of the first round is negative. The winner of the first auction may not win the second auction or win it but still not recover the first round loss. Then the total payoff of the sequential auction is negative and therefore we have the following theorem.

Theorem 4.1. *The equilibrium bidding strategy can result in bankruptcy.*

We can even show that the expected first round instantaneous payoff can become negative. In the paragraph preceding Theorem 3.4 it is shown that Δ is increasing in s and diverges if s approaches infinity. Since, each bidder's valuation is restricted to the interval between zero and one, there must exist some $\bar{s}$ such that for every $s > \bar{s}$ it holds that $\bar{\pi}_{1i} < 0$.

Theorem 4.2. *The ex-ante expected total payoff for bidders will be lower with than without synergies. Moreover, the ex-ante expected total payoff is decreasing in the synergy factor.*

Proof The first round winner pays besides the price he would pay if there was only one round, the difference in the expected payoff of the second round between winning and losing the first round. This means that the ex-ante expected total payoff of the auction sequence as a whole equals the expected payoff of a single auction without synergies plus the expected payoff in the second round when the first round has been lost:

$$\bar{\mu}_i = \bar{\pi} - \tfrac{1}{n}(\bar{\pi}_{2w} - \bar{\pi}_{2\ell}) + \tfrac{1}{n}\bar{\pi}_{2w} + \tfrac{n-1}{n}\bar{\pi}_{2\ell} = \bar{\pi} + \bar{\pi}_{2\ell}.$$

From Lemma 3.2 we know that this is less than $2\bar{\pi}$ and decreasing for increasing values of the synergy factor. □

The ex-ante expected total payoff is decreasing in the synergy factor. This means that the larger the possible benefit from synergies becomes, the smaller the expected total payoff of the bidders will be. The ex-ante expected total payoff of the bidders converges to the expected payoff of a single auction without synergies. Since the ex-ante expected total payoff is always larger than the possible expected utility of a single round, bidders will always participate in both rounds.

It is well known that (part of) a possible rent is dissipated during the competition for that rent. Not only the possible rent is completely dissipated here, bidders also forgo part of their original share. When the synergies are large, bidders have only half of the ex-ante expected total payoff they would have if there were no synergies. Instead of benefiting from the presence of synergies bidders suffer from it. In this setting positive synergies form a paradox.

4.2 Prices

When there are no synergies the expected price of each round is given by

$$\bar{p} = n(n-1)\int_0^1 v\, F^{n-2}(v)\, f(v)\,(1-F(v))\,\mathrm{d}v.$$

The expected prices of both the first and second auction are higher when synergies are present. Namely, from Lemma 3.2 and 3.3 we know that Δ is positive and therefore the expected price of the first round is higher. In the second round the $n-1$ losers bid as if there were no synergies and the winner bids higher than in an auction without synergies (except when $v = 0$). Consequently the expected price in the second round must be higher in an auction with synergies than in one without synergies. This leads to the following theorem.

Theorem 4.3. *The expected prices in both rounds are increasing in the synergy factor. Hence, also the expected revenue of the seller is increasing in the synergy factor.*

Proof The first round expected price equals the sum of the expected price in an auction without synergies and the discount factor: $\bar{p}_1 = \bar{p} + \Delta$. The first term is clearly independent of s and from Lemma 3.2 and 3.3 we know that the second term is increasing in s. Hence, the first round expected price is increasing in s.

The price in the second round equal the second highest synergy-adjusted value. For all realization of valuations the resulting price with a larger synergy factor will be at least as large as the resulting price with a lower synergy factor. With positive the probability the resulting price will even be strictly larger. Hence, the expected second round price is larger when synergies are larger, or equivalently, the expected second round price is increasing in the synergy factor.

All properties on the expected revenue follow easily as being the sum of expected prices of the two rounds. □

Theorem 4.2 shows that the increase in the revenue of the seller is not only due to the increased surplus that is divided. The seller also gains part of the share bidders originally had. Thus the gain from synergies for the seller is more than the value of the synergies itself. Making bidders aware of synergy possibilities can therefore be lucrative for the seller.

The seller gains from the synergies that the bidders can have. In government procurement it would therefore be profitable for the auctioneer to announce future projects well in advance. Transparency enables bidders in current auctions to form expectations about possible synergies which leads to higher prices in all auctions.

Theorem 4.1 shows that a transparent policy can also have negative welfare consequences. The winner of the first auction might run out of credit if he does not win the second project. Bankruptcies have a disastrous effect on welfare.

All bidders increase their first round bids with the aim of obtaining an advantage for the second round. The only change in the bids of second auction is the increased bid of the participant of type w. Therefore it can be that the expected first round price is higher than the expected second round price.

Corollary 4.4. *The declining price anomaly or afternoon effect will be observed if*

$$\bar{p}_1 > \bar{p}_2 \qquad \Longleftrightarrow \qquad \int_0^1 v\, f(v)\, G(v)\, \mathrm{d}v > 0$$

where

$$G(v) = (n-1)\,[s\,F^{n-1}(s\,v) - n\,F^{n-1}(v) + (n-1)\,F^{n-2}(v)\,F(v/s)]$$
$$-(n-2)\,[s\,F^{n-2}(s\,v) - (n-1)\,F^{n-2}(v) + (n-2)\,F^{n-3}(v)\,F(v/s)]$$

and, of course, $F(s\,v) = 1$ *for each* $v \in [1/s, 1]$.

When there are two bidders the inequality in Corollary 4.4 is always satisfied. Thus declining prices follow immediately from our model when there are only two participants. Whether declining prices will be observed for more bidders depends on the synergy factor and the distribution function.[3] However, for any distribution function declining prices are found if the synergy facto is sufficiently large.

Theorem 4.5. *For any* $n \geq 2$ *there exists some* $\bar{s}$ *such that for any* $s > \bar{s}$ *a declining price trend will be observed.*

[3]Simulations show that for any $F(v) = v^a$ with $a > 0$ declining prices will be observed for any $n > 2$.

Proof From Lemma 3.2 and Lemma 3.3 it follows that in the limit $\bar{p}_1$ will diverge. However $\bar{p}_2 \in [0, 1]$ for any synergy factor. □

The finding that the presence of positive synergies can explain the afternoon effect supports Branco (1997), Jeitschko and Wolfstetter[4] (2002) and Menezes and Monteiro (2003 and 2004).

As noted by Chanel and Vincent (1999), one should be careful with attributing empirical observations of the afternoon effect to theoretical predictions. We assume bidders demand more than a single unit and the objects are nonidentical. Therefore our model should not be considered to account for observations of the declining price anomaly by, for instance, Milgrom and Weber (1982) for transponder leases, Thiel and Petry (1995) in stamp auctions, Ashenfelter and Genesove (1992) for condominiums or Pesando and Shum (1996) for Picasso prints. For all these observations unit-demands are very credible and products are identical.

Our model can give insights in Beggs and Graddy's (1997) observation of declining prices in auctions for impressionist and modern paintings. Collectors might be present in these auctions and for them the value of the missing paintings increases with the collection. Therefore synergies can play a role in these auctions.

The variations in per person revenues of the European UMTS auctions can be partly attributed to differences in auction designs (Klemperer, 2002) and differences in the number of bidders. We offer an additional insight and support Cramton (2002) by showing that indeed synergies can have a profound influence on the outcomes of sequential auctions. In the European spectrum auctions Deutsche Telekom, Hutchison and Vodafone won licenses in almost all countries and benefits from synergies are obvious. When the specifications of the Belgium spectrum auction was officially announced, the United Kingdom, the Netherlands and Germany had already conducted their auctions. Thus bidders did not know their exact valuations for all auctions ex-ante.

4.3 Efficiency

An auction is efficient if the objects are allocated to the persons that value it the most ex-post (Krishna, 2002). In the first auction, the winner is the bidder with the highest valuation for the object. The winner of the second auction is also the bidder with the highest valuation, although this might be due to the positive synergies. Both auctions are therefore efficient and consequently the sequential auction as a whole is efficient.

With hindsight it might be that welfare could have been improved by having a different bidder win the first round. There are then two types of social inefficiencies that may occur. First, two different bidders win an item whereas it would have been better that a single bidder (not the winner of the first item) would win the two items. Second, a bidder wins both items whereas it would have been better had a different bidder won the two items. However, since the second round valuations are not known during the first round the sequential auction is

[4] The results in their appendix coincides with ours for the parameters: $F(v) = v$, $n = 2$ and $s = 2$.

still efficient.

Under the first type of inefficiency, bidder i has the highest valuation in the first round and thus wins that round. However, his synergy-adjusted second round valuation is not sufficient to win the second auction and therefore it might be that a higher social welfare could have been achieved had a different bidder won the first round. This bidder could be the winner of the second auction or a bidder who originally does not win any round. Thus, this type of social inefficiency occurs if there exist bidders i and k ($i \neq k$), such that $v_{1i} > v_{1j}$ for all $j \neq i$, $v_{2k} > s\, v_{2i}$ and $v_{2k} > v_{2j}$ for all $j \neq i, k$, but there exists a bidder $j \neq i$ such that $v_{1j} + s\, v_{2j} > v_{1i} + v_{2k}$.

The second type of social inefficiency can occur if the second round valuation of bidder i is lower than the valuation of at least one other bidder j, but still wins the second round due to the synergies. Suppose a bidder's first round valuation is marginally below that of bidder i but his synergy-adjusted second round valuation would more than offset this. Then the total surplus would be higher if this bidder instead of bidder i wins the first round. Thus, this type of inefficiency occurs if there exists a bidder i, such that $v_{1i} > v_{1j}$ for all $j \neq i$ and $s\, v_{2i} > v_{2j}$ for all $j \neq i$, but there exists a bidder $j \neq i$ such that $v_{1j} + s\, v_{2j} > v_{1i} + s\, v_{2i}$.

Without synergies these types of inefficiencies cannot occur. Due to synergies the valuations are stochastically dependent across auction rounds and only then the lack of information can give rise to an efficient but socially non-optimal ex-post allocation. When the synergies become large, the probability of a non-optimal ex-post allocation converges to $\frac{n-1}{n}$. For larger synergies the allocation is only optimal if the bidder that draws the highest second round valuation has also won the first round.

5 Number of Bidders

In a second price sealed bid auction without synergies the expected instantaneous payoff of a bidder is decreasing in the number of bidders and converges to zero. In our setting with synergies the expected instantaneous second round payoff of the bidder of type w is decreasing in the number of bidders but does not converge to zero, since the first round winner will win the second round for sure if and only if his valuation is above $\frac{1}{s}$ (see appendix). The expected instantaneous second round payoff of a bidder of type ℓ is decreasing in the number of bidders and converges to zero.[5] From the convergence of $\bar{\pi}_{2w}$ and $\bar{\pi}_{2\ell}$ it follows that Δ converges to

$$\int_{1/s}^{1} (s\, v - 1)\, f(v)\, \mathrm{d}\, v$$

when the number of bidders approaches infinity. This is the expected instantaneous payoff of bidder w given $v_{2w} \in [\frac{1}{s}, 1]$ and the expected price equals 1.

The expected instantaneous first round payoff equals that of a second price sealed bid auction without synergies minus $\frac{1}{n}\Delta$. As mentioned before the expected instantaneous payoff

[5] See the appendix for the proof.

of a second price sealed bid auction without synergies is decreasing in the number of bidders. However $\frac{1}{n}\Delta$ is also likely to be decreasing in the number of bidders,[6] thus there are two opposing forces at work. It depends on the precise parameter-setting which of these forces dominates. For instance if $F(v) = v^{\frac{1}{2}}$ and $s = \frac{21}{10}$, $\bar{\pi}_{1i}$ increases if the number of bidders goes from two to three but decreases if we go from four to five bidders.

The ex-ante expected total payoff equals the expected instantaneous payoff of an auction without synergies plus the expected instantaneous second round payoff of a first round loser. Both components are decreasing in the number of bidders and converge to zero.

6 Example: Uniform Distribution

In case all values are randomly chosen according to a uniform distribution, the symmetric linear equilibrium from Proposition 3.1 gives the following specification:

$$b^*_{1i} = v_{1i} + \left(\tfrac{1}{2}\, s - \tfrac{n-1}{n} + \tfrac{1}{2}\, \tfrac{n-2}{n}\, \tfrac{1}{s}\right) \qquad \bar{\pi}_{1i} = \tfrac{1}{n(n+1)} - \tfrac{1}{n}\left(\tfrac{1}{2}\, s - \tfrac{n-1}{n} + \tfrac{1}{2}\, \tfrac{n-2}{n}\, \tfrac{1}{s}\right)$$

$$b^*_{2i} = \begin{cases} v_{2i} & \text{if round 1 is lost} \\ s\, v_{2i} & \text{if round 1 is won} \end{cases} \qquad \bar{\pi}_{2i} = \begin{cases} \tfrac{1}{n(n+1)}\, \tfrac{1}{s} & \text{if round 1 is lost} \\ \tfrac{1}{2}\, s - \tfrac{n-1}{n} + \tfrac{1}{2}\, \tfrac{n-1}{n+1}\, \tfrac{1}{s} & \text{if round 1 is won} \end{cases}$$

$$\bar{p}_1 = \tfrac{n-1}{n+1} + \left(\tfrac{1}{2}\, s - \tfrac{n-1}{n} + \tfrac{1}{2}\, \tfrac{n-2}{n}\, \tfrac{1}{s}\right) \qquad \bar{\mu}_i = \tfrac{1}{n(n+1)} + \tfrac{1}{n(n+1)}\, \tfrac{1}{s}$$

$$\bar{p}_2 = \tfrac{n-1}{n} - \tfrac{n-1}{n(n+1)}\, \tfrac{1}{s} \qquad \bar{R} = \tfrac{1}{2}\, s + \tfrac{n-1}{n+1} + \tfrac{1}{2}\, \tfrac{n-3}{n+1}\, \tfrac{1}{s}.$$

One can easily verify for the specification above that the first round bid is increasing in s and decreasing in n, that the expected instantaneous second round payoff is increasing in s if the first round is won and decreasing in s if the first round is lost, that the expected instantaneous second round payoff is decreasing in n regardless whether the first round has been won or not, that the expected instantaneous first round payoff is decreasing in s, and that the ex-ante expected total payoff is decreasing in s.

As was already mentioned in Section 5, an increase in the number of bidders results in two opposing forces on the expected instantaneous first round payoff. The first term equals the expected payoff of a single auction without synergies and is decreasing in n. The part between the brackets is the difference in the expected instantaneous second round payoffs between having won and having lost the first auction and is also decreasing in n. The total effect of an increase in n on the expected instantaneous first round payoff depends on which of the two forces dominates. For the uniform distribution it is easily verified that the first force dominates and thus the expected instantaneous first round payoff is always decreasing in n.

Both the expected first and second round price, and hence the auction revenue, are increasing in s. The derivative of the expected first round price towards n is given by

$$\frac{\mathrm{d}\bar{p}_1}{\mathrm{d}n} = \frac{2}{(n+1)^2} - \left(1 - \frac{1}{s}\right)\frac{1}{n^2}.$$

[6] Simulations show that for any $F(v) = v^a$ with $a > 0$, Δ is strictly decreasing in the number of bidders.

This derivative can be negative as well as positive. To that end, take for instance $s = 9$ and $n = 2$ to find that the derivative is zero. An decrease in s makes the derivative positive, an increase negative. The effect of an increase in n on the expected first round price is then indeterminate. The effect on the expected second round price is definitely positive.

Theorem 4.3 showed that the expected revenue is increasing in s. Here it is also easily seen that the expected revenue is increasing in n. This implies that although the expected first round price can be decreasing, for the uniform distribution the increase of the expected second round price dominates.

The expected first round price is larger than the expected second round price if and only if

$$s < \tfrac{(n-1)(n+2)}{n(n+1)} - \tfrac{2}{n(n+1)} \quad \text{or} \quad s > \tfrac{(n-1)(n+2)}{n(n+1)} + \tfrac{2}{n(n+1)} = 1.$$

As the latter condition is trivially satisfied, we find that for uniformly distributed valuations the declining price anomaly is observed.

7 Conclusion

In this paper we did not only analyze the effect of positive synergies on prices, but also on the bidders' payoffs. Although synergies on the bidders' side appear beneficial for them, we prove this is not necessarily the case. The equilibrium bidding strategies result in such a fierce competition that bidders lose all the benefits from synergies and part of their intrinsic expected payoff. The seller is the only one who benefits from positive synergies. He captures all the gains from synergies and also part of the share bidders had without synergies. The larger the possible benefits due to synergies are, the smaller the expected payoff of the bidders will be. Thus in this setting positive synergies on the bidders' side form a paradox.

The presence of positive synergies can have severe negative welfare consequences. Although the revenue of a sequential auction is higher for the seller, the winner of the first auction can go bankrupt. By announcing projects well in advance governments can increase their procurement auction revenue, but also the number of bankruptcies. Consequently transparent procurement policy is a two-edged sword. If there is sufficient competition, a government might consider excluding previous winners from subsequent auctions in order to avoid bankruptcy problems.

Finally, we find support for the claim that positive synergies can explain the declining price anomaly. When there are only two bidders, prices will always decline. For more bidders, there always exists a synergy factor such that prices will be declining. Simulations show that for a rich class of distribution functions prices will always be declining for two or more bidders.

We only analyzed a sequential second price auction with two objects. Adding more rounds raises the question of how to model the synergies. If the synergies are reinforcing, the model will only be more complicated. Then bidders will also take into consideration the benefits they can have in later rounds when bidding in current. The qualitative results will therefore be the same. Analyzing an asymmetric first price auction with n bidders is complicated, since a closed form expression for the bidding strategies does not exist.

References

1. Ashenfelter, O. and D. Genesove (1992). Testing for price anomalies in real-estate auctions. *American Economic Review*, 82 (2), 501-505.

2. Ausubel, L., P. Cramton, R.P. McAfee, and J. McMillan (1997). Synergies in wireless telephony: Evidence from the broadband PCS auctions. *Journal of Economics and Management Strategy*, 6 (3), 497-527.

3. Beggs, A. and K. Graddy (1997). Declining values and the afternoon effect: Evidence from art auctions, *RAND Journal of Economics*, 28 (3), 544-565.

4. Black, J. and D. De Meza (1992). Systematic price differences between successive auctions are no anomaly. *Journal of Economics and Management Strategy*, 1 (4), 607-628.

5. Branco F. (1997). Sequential auctions with synergies: An example. *Economics Letters*. 54 (2), 159-163.

6. Chanel, O. and S. Vincent (1999). The declining price effect in sequential auctions: What theory does not predict. Working Paper. University of Copenhagen.

7. Cramton, P. (2002). Spectrum auctions. In: Cave, M., S. Majumdar, and I. Vogelsang *Handbook of telecommunications economics*. Amsterdam: Elsevier 605-639.

8. De Silva, D.G. (2005). Synergies in recurring procurement auctions: An empirical investigation. *Economic Inquiry*, 43 (1), 55-66.

9. De Silva, D.G., T.D. Jeitschko, and G. Kosmopoulou (2005). Stochastic synergies in sequential auctions. *International Journal of Industrial Organization*, 23 (3-4), 183-201.

10. Engelbrecht-Wiggans, R. (1994). Sequential auctions of stochastically equivalent objects. *Economics Letters*, 44 (1-2), 87-90.

11. Farrell, J. and R. Klemperer (2005). Coordination and lock-In: Competition with switching costs and network effects. In: Armstrong, M. and R.H. Porter *Handbook of Industrial Organization, Volume 3*. Elsevier Science. New York.

12. Hendricks, K. and R. Porter (1988). An empirical study of an auctions with asymmetric information. *The American Economic Review*, 78 (5), 865-883.

13. Jeitschko, T.D. and E. Wolfstetter (2002). Scale economies and the dynamics of recurring auctions. *Economic Inquiry*, 40 (3), 403-414.

14. Klemperer, P. (2002). How (not) to run auctions: The European 3G telecom auctions. *European Economic Review*, 46 (4-5), 829-845.

15. Krishna, V. and R. Rosenthal (1996). Simultaneous auctions with synergies. *Games and Economic Behavior*, 17 (1), 1-31.

16. Krishna, V. (2002). *Auction Theory.* Academic Press. San Diego CA.

17. Menezes, F. and P. Monteiro (2003). Synergies and price trends in sequential auctions. *Review of Economic Design*, 8 (1), 85-98.

18. Menezes, F. and P. Monteiro (2004). Auctions with synergies and asymmetric buyers. *Economics Letters*, 85 (2), 287-294.

19. Milgrom, P.R. and R.J. Weber (1982). A theory of auctions and competitive bidding II. Working Paper. Northwestern University.

20. Pesando, J. and P. Shum (1996). Price anomalies at auction: Evidence from the markets for modern prints. In: Ginsburgh, V.A. and P.-M. Menger *Economics of the Arts: Selected Essays.* Amsterdam: Elsevier.

21. Reiß, J.P. and J.R. Schöndube (2002). On participation in sequential procurement auctions. Working Paper. University of Magdeburg.

22. Rusco, F. and D. Walls (1999). Competition in a repeated spatial auction market with an application to timber sales. *Journal of Regional Science*, 39 (3), 449-465.

23. Thiel, S. and G. Petry (1995). Bidding behavior in second-price auctions: Rare stamp sales: 1923-1937. *Applied Economics*, 27 (1), 11-16.

24. Weber, R.J. (1983). Multiple-objects auctions. In: Engelbrecht-Wiggans, R., M. Shubik, and R.M. Stark *Auctions, bidding and contracting: Uses and theory.* New York: New York University Press 165-194.

25. Yildirim, H. (2004). Piecewise procurement of a large-scale project. *International Journal of Industrial Organization*, 22, 1349-1375.

A Proof of $\lim_{n\to\infty} \pi_{2w} = \int_{1/s}^{1} (s\,v - 1)\, f(v)\, \mathrm{d}\,v$

First notice that

$$(n-1)\int_0^1 v\,F^{n-2}(v)\,f(v)\,(1-F(v/s))\,\mathrm{d}v = (n-1)\int_0^1 v\,F^{n-2}(v)\,f(v)\int_{v/s}^1 f(x)\,\mathrm{d}x\,\mathrm{d}v$$
$$= (n-1)\int_0^1\int_{v/s}^1 v\,F^{n-2}(v)\,f(v)\,f(x)\,\mathrm{d}x\,\mathrm{d}v \overset{3}{=} (n-1)\int_0^1\int_0^{s\,x} v\,F^{n-2}(v)\,f(v)\,f(x)\,\mathrm{d}v\,\mathrm{d}x$$
$$\overset{4}{=} (n-1)\int_0^1\int_0^{s\,v} x\,F^{n-2}(x)\,f(x)\,f(v)\,\mathrm{d}x\,\mathrm{d}v = \int_0^1\int_0^{s\,v} x\,\tfrac{\mathrm{d}F^{n-1}(x)}{\mathrm{d}x}\,\mathrm{d}x\,f(v)\,\mathrm{d}v$$
$$= \int_0^1\int_0^{s\,v} x\,\mathrm{d}F^{n-1}(x)\,f(v)\,\mathrm{d}v \overset{7}{=} \int_0^1 \Big[s\,v\,F^{n-1}(s\,v) - \int_0^{s\,v} F^{n-1}(x)\,\mathrm{d}x\Big]\,f(v)\,\mathrm{d}v$$
$$= \int_0^1 s\,v\,F^{n-1}(s\,v)\,f(v)\,\mathrm{d}v - \int_0^1\int_0^{s\,v} F^{n-1}(x)\,\mathrm{d}x\,f(v)\,\mathrm{d}v.$$

In the third equality the order of integration is changed. For this to be correct one has to note that $f(v) = 0$ when v is larger than 1. The fourth equation is just a switch in the use of variables: x becomes v, and v becomes x. The seventh equation comes from integrations by parts.

Then,

$$\bar{\pi}_{2w} = \int_0^1 s\,v\,F^{n-1}(s\,v)\,f(v)\,\mathrm{d}v - (n-1)\int_0^1 v\,F^{n-2}(v)\,f(v)\,(1-F(v/s))\,\mathrm{d}v$$
$$= \int_0^1 s\,v\,F^{n-1}(s\,v)\,f(v)\,\mathrm{d}v - \Big[\int_0^1 s\,v\,F^{n-1}(s\,v)\,f(v)\,\mathrm{d}v - \int_0^1\int_0^{s\,v} F^{n-1}(x)\,\mathrm{d}x\,f(v)\,\mathrm{d}v\Big]$$
$$= \int_0^1\int_0^{s\,v} F^{n-1}(x)\,\mathrm{d}x\,f(v)\,\mathrm{d}v$$
$$= \int_0^1 \Big\{\int_0^1 F^{n-1}(x)\,\mathrm{d}x + \int_1^{s\,v} F^{n-1}(x)\,\mathrm{d}x\Big\} f(v)\,\mathrm{d}v$$
$$= \int_0^1\int_0^1 F^{n-1}(x)\,\mathrm{d}x\,f(v)\,\mathrm{d}v + \int_0^1 (s\,v-1)\,f(v)\,\mathrm{d}v$$

We see that $\bar{\pi}_{2w}$ is decreasing in n and converges to $\int_{1/s}^1 (s\,v-1)\,f(v)\,\mathrm{d}v$ when n approaches infinity.

B Proof of $\lim_{n\to\infty} \pi_{2\ell} = 0$

First note that

$$\bar{\pi} = \int_0^1 v\,F^{n-1}(v)\,f(v)\,\mathrm{d}v - (n-1)\int_0^1 v\,F^{n-2}(v)\,f(v)\,(1-F(v))\,\mathrm{d}v$$
$$= \int_0^1\int_0^v F^{n-1}(x)\,\mathrm{d}x\,f(v)\,\mathrm{d}v.$$

Rewriting the first expression using the same steps as in Appendix A leads to the second expression. From the latter expression the well-known result follows that $\bar{\pi}$ converges to zero when n approaches infinity. Combining this with Lemma 3.3 and the fact that $\bar{\pi}_{2\ell}$ cannot be negative, it must be that $\bar{\pi}_{2\ell}$ converges to zero when n approaches infinity

Now we will also show that $\bar{\pi}_{2\ell}$ is decreasing in n. Consider an auction with n bidders. Suppose now we take bidder ℓ, i and add a $(n+1)$th bidder whose bid is only accepted if this bid is lower than that of bidder ℓ, i. In that case the $(n+1)$th bidder will only increase the expected price bidder ℓ, i has to pay. It then immediately follows that adding a normal $(n+1)$th bidder will decrease the expected payoff of bidder ℓ, i.