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RM/08/023

JEL code: C72, D78

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The Maximal Domain for the Revelation Principle when Preferences are Menu Dependent*

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April 16, 2008

Abstract

We extend the domain of preferences to include menu-dependent preferences and characterize the maximal subset of this domain in which the revelation principle holds. Minimax-regret preference is shown to be outside this subset.

Keywords: Revelation Principle; Menu-dependent Preferences; Minimax-regret

JEL: C72; D78

1 Introduction

The revelation principle is the foundation of the theory of mechanism design (see, for example, Dasgupta, Hammond and Maskin (1979), Myerson (1979)). Applied to an environment of incomplete information, it states that for any Bayesian-Nash equilibrium of any mechanism there exists an outcome-equivalent Bayesian-Nash equilibrium of a direct mechanism in which all players report their respective types truthfully. Thus, the revelation principle greatly simplifies the search for "optimal" mechanisms; we only need to search in the set of incentive compatible direct mechanisms.

The notion of Bayesian-Nash equilibrium assumes that players' preferences satisfy the von Neumann-Morgenstern axioms and thus, are represented by expected-utility

*I thank Jean-Jacques Herings, Flip Klijn, Takashi Kunimoto, Ronald Peeters, Roberto Serrano and Markus Walzl for helpful suggestions and discussions.

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functions (von Neumann and Morgenstern (1944)). As nonstandard preferences become standard in the literature, it is pertinent to ask whether the revelation principle holds if players are not expected-utility maximizers.[1] In this paper, we extend the domain of preferences to include menu-dependent preferences, i.e., preference of a player can depend on the set of available alternatives (which is called a menu).[2] We characterize the maximal subset of this domain in which the revelation principle holds – this requires a modified notion of the equilibrium. This subset is the set of preference relations that satisfy what we call *weak contraction consistency*.

The setup is that of incomplete information in which a state is a realization of a type profile. The set of states is commonly known; however, each player privately knows only her type. An alternative is a Savage act that specifies an outcome for every possible realization of the state. We assume that for any menu, each type of each player has a complete preference relation over the Savages acts that are elements of that menu. The preference relation over two Savage acts can be different in different menus that contain these two acts; thus, the preferences can be menu dependent.

We provide two results. First, if the preferences of all types of all players satisfy weak contraction consistency, then the revelation principle holds. Second, if the preference of any type of any player does not satisfy weak contraction consistency, then there exists a preference profile such that the revelation principle does not hold. In this sense, the set of preferences that satisfy weak contraction consistency for all types of all players is the maximal domain in which the revelation principle holds. We also show that it is not possible to strengthen the second result; thus, if the preference of a type of a player does not satisfy weak contraction consistency, then the revelation principle can hold for some preference profile.

Weak contraction consistency is weakening of contraction consistency.[3] Contraction consistency requires that for *any* menu, a maximal element in that menu remains maximal after *any* contraction of the menu around that element (i.e., when the maximal element is available in the menu after the contraction). Weak contrac-

[1] The literature on implementation theory has also incorporated boundedly rational players; see, for example, Hurwicz (1986), Eliaz (2002) and Cabrales and Serrano (2007).

[2] Sen (1993, 1994) argues for the need to incorporate menu dependence in standard rational choice theory. Also see Sen (1997) for a formal analysis. Menu dependence of choice has been well documented in experimental studies; see, for example, Huber, Payne and Puto (1982), Simonson and Tversky (1992).

[3] Contraction consistency was originally introduced by Chernoff (1954) and has alternatively been termed Property α by Sen (1971). In its original formulation, contraction consistency is a property of the choice function. We adapt it to the primitive of our model, a preference relation.

tion consistency weakens contraction consistency at two levels: first, the property must be satisfied by only a class of menus – instead of *any* menu – and secondly, it requires that a maximal element in a menu remains maximal after only a unique contraction – instead of *any* contraction – of the menu around that element (see Definition 2.1 for details).

Minimax regret is a menu-dependent preference relation that has been often studied in the literature starting with Savage (1951). According to minimax regret, each type of a player chooses the alternative that minimizes her maximum regret. Regret of choosing an alternative in a state is defined as the difference between the payoff that is obtained and the maximum payoff that could have been attained in that state; since the latter depends on the set of alternatives available in that state, the relation is menu dependent. Maximum regret of choosing an alternative is the maximum of these differences over all states of the world. We provide an example in which the minimax-regret preference relation does not satisfy weak contraction consistency and consequently, the revelation principle does not hold.[4]

We also provide an example in which the preference of each type of a player is menu dependent due to extremeness aversion (Simonson and Tversky (1992)) and satisfies weak contraction consistency.

In presenting our results, we assume that a type of a player has a preference relation over each menu, which is a set of Savage acts. A Savage act is an *ex-ante lottery* that specifies an outcome for each realization of *all* players' types. However, a type of a player plays only in the *interim stage* (when she knows her type) and thus, one might argue, she should "care" about *interim lotteries* that specify an outcome for each realization of *other* player's types instead of *ex-ante lotteries*. In the Appendix, we show that our results do not change with this alternative formulation.

The paper is organized as follows. We explain the model and collect all the results and examples in Section 2. Section 3 provides a brief conclusion. In the Appendix, we show that our results hold even if we assume that each type of each player "cares" about interim lotteries.

2 Model and Results

Let N be the set of players. A type of player i is denoted by t_i and the set of types of player i is T_i. Each player privately knows only her type. Let $T = \prod_{i \in N} T_i$ be

[4]It is known that the minimax-regret preference relation does not satisfy contraction consistency. See Chernoff (1954) for an example.

the commonly known type space and t be a type profile. t_{-i} is a type profile of all players other than i and T_{-i} is the set of such profiles.

A is the set of outcomes. ΔA denotes the set of probability measures on A. A *Savage act* is a function $f : T \to \Delta A$. Let $\mathcal{F}$ be the set of all Savage acts. A *menu* F is a subset of $\mathcal{F}$.

Each type t_i of each player i has a complete preference relation $\succeq_{t_i}^F$ over each menu F. Let $\succ_{t_i}^F$ and $\sim_{t_i}^F$ be, respectively, the strict preference and indifference relations derived from $\succeq_{t_i}^F$. Let $\succeq_{t_i} = (\succeq_{t_i}^F)_{F\subseteq\mathcal{F}}$ and $\succeq_i = (\succeq_{t_i})_{t_i\in T_i}$.

N, T and A are fixed throughout the paper. We call $\mathcal{E} = (\succeq_i)_{i\in N}$ to be the *environment*.

Given a menu F, let $\xi_i^F : T_i \to F$ be an arbitrary function and denote the set of such functions by Ξ_i^F. Let $\psi_i : T_i \to T_i$ be an arbitrary function and denote the set of such functions by Ψ_i.

For any $F \subseteq \mathcal{F}$, $\xi_i^F \in \Xi_i^F$ and $\psi_i \in \Psi_i$, define the Savage act $h_{\psi_i}^{\xi_i^F}$ as follows:

$$h_{\psi_i}^{\xi_i^F}(t_i, t_{-i}) \equiv \xi_i^F(t_i)(\psi_i(t_i), t_{-i}), \forall (t_i, t_{-i}) \in T.$$

Thus, the outcome of the Savage act $h_{\psi_i}^{\xi_i^F}$ in state (t_i, t_{-i}) is equal to the outcome of the Savage act $\xi_i^F(t_i)$ in state $(\psi_i(t_i), t_{-i})$. Let $F(\xi_i^F) = \left\{h_{\psi_i}^{\xi_i^F} | \psi_i \in \Psi_i\right\}$.

For any $f \in \mathcal{F}$ and $\psi_i \in \Psi_i$, define the Savage act f_{ψ_i} as follows:

$$f_{\psi_i}(t_i, t_{-i}) \equiv f(\psi_i(t_i), t_{-i}), \forall (t_i, t_{-i}) \in T.$$

Let $F(f, i) = \{f_{\psi_i} | \psi_i \in \Psi_i\}$.

Definition 2.1. We say that $\succeq_{t_i}$ satisfies *weak contraction consistency* (henceforth *WCC*) if the following holds: $\forall F \subseteq \mathcal{F}$ such that $F = \bigcup_{\xi_i^F \in \Xi_i^F} F(\xi_i^F)$ and $\forall f \in F$,

$$f \succeq_{t_i}^F f', \ \forall f' \in F \implies f \succeq_{t_i}^{F(f,i)} f_{\psi_i}, \ \forall \psi_i \in \Psi_i.$$

For any $F \subseteq \mathcal{F}$ and $f \in F$, if $\xi_i^F(t_i) = f, \forall t_i \in T_i$, then $h_{\psi_i}^{\xi_i^F} = f_{\psi_i}, \forall \psi_i \in \Psi_i$ and thus, $F(\xi_i^F) = F(f, i)$. So, if $F = \bigcup_{\xi_i^F \in \Xi_i^F} F(\xi_i^F)$, then $F(f, i) \subseteq F$. Thus, we see that *WCC* is a weakening of contraction consistency at two levels: first, the property must be satisfied by only a class of menus (menus F which are equal to $\bigcup_{\xi_i^F \in \Xi_i^F} F(\xi_i^F)$) and second, it only requires that a maximal element f remains maximal after a unique contraction of the menu around f (which is $F(f, i)$).

A *mechanism* $\Gamma = ((M_i)_{i\in N}, g)$ defines the set of messages M_i available to each player and the outcome $g : \prod_i M_i \to \Delta A$ associated with each message profile. Let Σ_i be the set of strategies $\sigma_i : T_i \to M_i$ of player i.[5] Then, $\sigma = (\sigma_i)_{i \in N}$ is a strategy profile. Note that $g(\sigma)$ is a Savage act. Let $\Lambda^\Gamma = [N, T, g, (\Sigma_i)_{i\in N}, \mathcal{E}]$ be the game induced by the mechanism Γ in the environment $\mathcal{E}$.

A *direct mechanism* is a mechanism such that $M_i = T_i, \forall i \in N$. We identify a direct mechanism $((T_i)_{i\in N}, f)$ by its outcome function f, which is a Savage act. Let Λ^f be the game induced by the direct mechanism in $\mathcal{E}$. Note that Ψ_i is the set of strategies of player i in Λ^f. Let ψ_i^* be the truthful strategy, i.e., $\psi_i^*(t_i) = t_i, \forall t_i \in T_i$.

Now we incorporate the fact that the preferences can be menu dependent in the definition of the equilibrium. For any profile of the other players' strategies σ_{-i}, let $F(g, \sigma_{-i}) = \{g(\sigma_i, \sigma_{-i}) | \sigma_i \in \Sigma_i\}$. $F(g, \sigma_{-i})$ is the menu of Savage acts that is available to player i when other players are playing σ_{-i}.

Definition 2.2. A strategy profile σ^* is an *equilibrium* of Λ^Γ if $\forall i \in N$and $\forall t_i \in T_i$,

$$g(\sigma^*) \succeq_{t_i}^{F(g,\sigma^*_{-i})} g(\sigma_i, \sigma^*_{-i}), \forall \sigma_i \in \Sigma_i.$$

Thus, σ^* is an equilibrium of Λ^Γ if the Savage act generated by σ^* is maximal for each type of each player in the menu available to that player when other players play according to σ^*.

Fix an environment $\mathcal{E}$. The revelation principle states that for every mechanism Γ and for every equilibrium outcome of Λ^Γ, there exists a direct mechanism f which induces a game Λ^f with an outcome-equivalent equilibrium in which all players report their type truthfully. Note that $(\psi_i^*)_{i\in N}$ is an equilibrium in Λ^f if and only if $\forall i \in N$ and $\forall t_i \in T_i$, $f \succeq_{t_i}^{F(f,i)} f_{\psi_i}$, $\forall \psi_i \in \Psi_i$.

The next theorem characterizes the maximal domain in which the revelation principle holds.

Theorem 2.3.

1. *If $\mathcal{E}$ is such that $\succeq_{t_i}$ satisfies WCC for all $t_i \in T_i$ and $i \in N$, then the revelation principle holds.*

2. *If $\succeq_{t_i'}$ does not satisfy WCC, then there exists a $\mathcal{E} = ((\succeq_{t_i'}, (\succeq_{t_i})_{t_i \neq t_i'}), (\succeq_j)_{j\neq i})$ such that the revelation principle does not hold in $\mathcal{E}$.*

Proof.

1. Suppose $\mathcal{E}$ is such that $\succeq_{t_i}$ satisfies WCC for all $t_i \in T_i$ and $i \in N$. Let Γ be an

[5] We restrict ourselves to pure strategies.

arbitrary mechanism and σ^* be an equilibrium of Λ^Γ. Then $\forall i \in N$ and $\forall t_i \in T_i$,

$$g(\sigma^*) \succeq_{t_i}^{F(g,\sigma_{-i}^*)} g(\sigma_i, \sigma_{-i}^*), \forall \sigma_i \in \Sigma_i. \tag{1}$$

Let $F_i = F(g, \sigma_{-i}^*)$. First, we argue that $F_i = \bigcup_{\xi_i^{F_i} \in \Xi_i^{F_i}} F(\xi_i^{F_i})$.

Pick a $f' \in F_i$. Consider $\xi_i'^{F_i} \in \Xi_i^{F_i}$ such that $\xi_i'^{F_i}(t_i) = f', \forall t_i \in T_i$. Then $f' = f'_{\psi_i^*} = h_{\psi_i^*}^{\xi_i'^{F_i}}$ and thus, $f' \in F(\xi_i'^{F_i})$. So, $F_i \subseteq \bigcup_{\xi_i^{F_i} \in \Xi_i^{F_i}} F(\xi_i^{F_i})$.

Next, pick a $f' \in \bigcup_{\xi_i^{F_i} \in \Xi_i^{F_i}} F(\xi_i^{F_i})$. Then $f' \in F(\xi_i^{F_i})$ for some $\xi_i^{F_i} \in \Xi_i^{F_i}$. Therefore,

$$\begin{aligned} f' &= h_{\psi_i}^{\xi_i^{F_i}} \text{ for some } \psi_i \in \Psi_i \text{ and } \xi_i^{F_i} \in \Xi_i^{F_i} \\ \implies f'(t) &= \xi_i^{F_i}(t_i)(\psi_i(t_i), t_{-i}), \forall t \in T \\ &= g(\sigma_i^{t_i}(\psi_i(t_i)), \sigma_{-i}^*(t_{-i})), \text{ for some } \sigma_i^{t_i} \in \Sigma_i, \forall t \in T \\ &= g(\sigma_i'(t_i), \sigma_{-i}^*(t_{-i})), \forall t \in T, \end{aligned}$$

where $\sigma_i' \in \Sigma_i$ is such that $\sigma_i'(t_i) = \sigma_i^{t_i}(\psi_i(t_i)), \forall t_i \in T_i$. Thus, $f' \in F_i$ and so $\bigcup_{\xi_i^{F_i} \in \Xi_i^{F_i}} F(\xi_i^{F_i}) \subseteq F_i$.

Now, let $f = g(\sigma^*) \in F_i$. Therefore, $\forall i \in N$ and $\forall t_i \in T_i$,

$$\begin{aligned} (1) \iff & f \succeq_{t_i}^{F_i} f', \ \forall f' \in F_i \\ \implies & f \succeq_{t_i}^{F(f,i)} f_{\psi_i}, \forall \psi_i \in \Psi_i, \end{aligned}$$

since $F_i = \bigcup_{\xi_i^{F_i} \in \Xi_i^{F_i}} F(\xi_i^{F_i})$, $f \in F_i$ and $\succeq_{t_i}$ satisfies WCC. Thus, $(\psi_i^*)_{i \in N}$ is an equilibrium of Λ^f and the outcome of this equilibrium is equal to $g(\sigma^*)$.

2. Suppose $\succeq_{t_i'}$ does not satisfy WCC. Then there exist a $F \subseteq \mathcal{F}$ such that $F = \bigcup_{\xi_i^F \in \Xi_i^F} F(\xi_i^F)$ and a $f \in F$ such that $f \succeq_{t_i'}^F f'$, $\forall f' \in F$ but $f_{\psi_i'} \succ_{t_i'}^{F(f,i)} f$, for some $\psi_i' \in \Psi_i$.

For all $t_i \neq t_i'$, pick $\succeq_{t_i}$ such that,

$$f \succeq_{t_i}^F f', \forall f' \in F. \tag{2}$$

For all $j \neq i$, pick $\succeq_j$ such that $\forall t_j \in T_j$,

$$f \succeq_{t_j}^{F(f,j)} f_{\psi_j}, \forall \psi_j \in \Psi_j. \tag{3}$$

Fix $\mathcal{E} = ((\succeq_{t_i'}, (\succeq_{t_i})_{t_i \neq t_i'}), (\succeq_j)_{j \neq i})$ to be the environment.

Define Γ to be such that $M_i = F \times T_i$, $M_j = T_j, \forall j \neq i$, and

$$g((f', t_i), t_{-i}) = f'(t_i, t_{-i}), \forall (f', t_i) \in F \times T_i, t_{-i} \in T_{-i}.$$

Thus, $\Sigma_i = \Xi_i^F \times \Psi_i$ and $\Sigma_j = \Psi_j, \forall j \neq i$.

We argue that σ^* such that

$$\begin{aligned} &\sigma_j^*(t_j) = t_j, \forall t_j \in T_j, j \neq i, \text{ and} \\ &\sigma_i^*(t_i) = (f, t_i), \forall t_i \in T_i \end{aligned}$$

is an equilibrium of Λ^Γ. Note that $g(\sigma^*) = f$.

$F(g, \sigma_{-j}^*) = F(f, j), \forall j \neq i$. This is because $\Sigma_j = \Psi_j$, all players other than j announce their type truthfully in σ_{-j}^* and player i announces f in σ_i^*. It follows from (3) that $\forall j \neq i$ and $\forall t_j \in T_j$, $g(\sigma^*) \succeq_{t_j}^{F(g,\sigma_{-j}^*)} g(\sigma_j, \sigma_{-j}^*), \forall \sigma_j \in \Sigma_j$.

Now we show that $F(g, \sigma_{-i}^*) = \bigcup_{\xi_i^F \in \Xi_i^F} F(\xi_i^F)$. This is because

$$\begin{aligned} &f' \in F(g, \sigma_{-i}^*) \\ \iff &f'(t) = g(\sigma_i(t_i), \sigma_{-i}^*(t_{-i})), \forall t \in T, \text{ for some } \sigma_i \in \Sigma_i \\ \iff &f'(t) = \xi_i^F(t_i)(\psi_i(t_i), t_{-i}), \forall t \in T, \text{for some } \xi_i^F \in \Xi_i^F \text{ and } \psi_i \in \Psi_i \\ \iff &f' \in \bigcup_{\xi_i^F \in \Xi_i^F} F(\xi_i^F) \end{aligned}$$

Thus, $F(g, \sigma_{-i}^*) = \bigcup_{\xi_i^F \in \Xi_i^F} F(\xi_i^F) = F$. It follows from the hypothesis and (2) that $\forall t_i \in T_i$,

$$f \succeq_{t_i}^F f', \ \forall f' \in F \iff g(\sigma^*) \succeq_{t_i}^{F(g,\sigma_{-i}^*)} g(\sigma_i, \sigma_{-i}^*), \forall \sigma_i \in \Sigma_i.$$

Hence, σ^* is an equilibrium of Λ^Γ and $g(\sigma^*) = f$. If the revelation principle holds, then $(\psi_i^*)_{i \in N}$ must be an equilibrium of Λ^f. However, this is impossible since $f_{\psi_i'} \succ_{t_i'}^{F(f,i)} f$. $\square$

Remark 2.4. If $\succeq_{t_i}$ is menu independent (i.e., there exists a complete relation $\succeq_{t_i}'$ over $\mathcal{F}$ such that $\forall f, f' \in \mathcal{F}$ and $\forall F \subseteq \mathcal{F}$ with $f, f' \in F$, we have $f \succeq_{t_i}^F f' \iff f \succeq_{t_i}' f'$), then $\succeq_{t_i}$ satisfies WCC. Thus, in particular, if $\succeq_i$ is an expected-utility preference then each $\succeq_{t_i}$ is menu independent and so it satisfies WCC.

Remark 2.5. We cannot strengthen part 2 of Theorem 2.3 because even if $\succeq_{t_i'}$ does not satisfy WCC, there exists a $\mathcal{E} = ((\succeq_{t_i'}, (\succeq_{t_i})_{t_i \neq t_i'}), (\succeq_j)_{j \neq i})$ such that the revelation principle holds in $\mathcal{E}$. To see this, let $\mathcal{F}'$ be the set of all Savage acts f such that there exist a $F \subseteq \mathcal{F}$ with $F = \bigcup_{\xi_i^F \in \Xi_i^F} F(\xi_i^F)$, $f \in F$ and $f \succeq_{t_i'}^F f'$, $\forall f' \in F$ but $f_{\psi_i'} \succ_{t_i'}^{F(f,i)} f$ for some $\psi_i' \in \Psi_i$. Now, $\mathcal{F}' \neq \emptyset$ since $\succeq_{t_i'}$ does not satisfy WCC.

We know that player i has at least two types since for some $f \in \mathcal{F}'$, we have $f_{\psi_i'} \succ_{t_i'}^{F(f,i)} f$. Pick $t_i'' \in T_i$ such that $t_i'' \neq t_i'$. Let $\succeq_{t_i''}$ be menu independent and such that $\forall f, f' \in \mathcal{F}$,

$$
\begin{aligned}
f \sim_{t_i''} f' \quad & \text{if } f, f' \notin \mathcal{F}' \text{ or if } f, f' \in \mathcal{F}' \\
f \succ_{t_i''} f' \quad & \text{if } f \notin \mathcal{F} \text{ and } f' \in \mathcal{F} \\
f' \succ_{t_i''} f \quad & \text{if } f \in \mathcal{F} \text{ and } f' \notin \mathcal{F}.
\end{aligned}
$$

For all $t_i \neq t_i', t_i''$, pick $\succeq_{t_i}$ such that it is menu independent. For all $j \neq i$ and $\forall t_j \in T_j$, pick $\succeq_{t_j}$ such that it is menu independent. We show that the revelation principle holds in this environment.

Pick any $f \in \mathcal{F}'$. Consider $\psi_i'' \in \Psi_i$ such that $\psi_i''(t_i) = t_i'', \forall t_i \in T_i$. Clearly, $f_{\psi_i''} \notin \mathcal{F}'$ since $F(f_{\psi_i''}, i) = \{f_{\psi_i''}\}$. Therefore, $f_{\psi_i''} \succ_{t_i''} f$. There does not exist any mechanism Γ such that σ^*, where $g(\sigma^*) = f$, is an equilibrium of Λ^Γ; otherwise,

$$
\begin{aligned}
& g(\sigma^*) \succeq_{t_i''} g(\sigma_i, \sigma_{-i}^*), \forall \sigma_i \in \Sigma_i \\
\Longrightarrow\ & g(\sigma^*) \succeq_{t_i''} g(\sigma_i^*(\psi_i), \sigma_{-i}^*), \forall \psi_i \in \Psi_i \\
\Longrightarrow\ & f \succeq_{t_i''} f_{\psi_i''}, \text{ since } g(\sigma^*) = f \text{ and } g(\sigma_i^*(\psi_i''), \sigma_{-i}^*) = f_{\psi_i''}; \text{ a contradiction.}
\end{aligned}
$$

Now, pick any $f' \notin \mathcal{F}'$. If there exists a mechanism Γ' such that σ'^*, where $g'(\sigma'^*) = f'$, is an equilibrium of $\Lambda^{\Gamma'}$, then $\forall j \in N$ and $\forall t_j \in T_j$,

$$
\begin{aligned}
& g'(\sigma'^*) \succeq_{t_j}^{F(g', \sigma_{-j}'^*)} g'(\sigma_j', \sigma_{-j}'^*), \forall \sigma_j' \in \Sigma_j' \\
\Longrightarrow\ & g'(\sigma'^*) \succeq_{t_j}^{F(g', \sigma_{-j}'^*)} g'(\sigma_j'^*(\psi_j), \sigma_{-j}'^*), \forall \psi_j \in \Psi_j \\
\Longrightarrow\ & f' \succeq_{t_j}^{F(g', \sigma_{-j}'^*)} f_{\psi_j}', \forall \psi_j \in \Psi_j, \text{ since } g'(\sigma'^*) = f' \text{ and } g'(\sigma_j'^*(\psi_j), \sigma_{-j}'^*) = f_{\psi_j}' \\
\Longrightarrow\ & f' \succeq_{t_j}^{F(f', j)} f_{\psi_j}', \forall \psi_j \in \Psi_j,
\end{aligned}
$$

where the last implication follows for t_i' since $f' \notin \mathcal{F}'$ (see the proof of part 1 of Theorem 2.3 to argue that $F(g', \sigma_{-i}'^*) \equiv F_i = \bigcup_{\xi_i^{F_i} \in \Xi_i^{F_i}} F(\xi_i^{F_i})$) and for all other types of all players since their preferences are menu independent. Thus, the revelation principle holds in this environment.

Next, we provide an example in which the minimax-regret preference relation does not satisfy WCC and consequently, the revelation principle does not hold.

Example 2.6. We say that $\succeq_i$ is a *minimax-regret preference* if there exists a $u_i : A \times T \to \Re$ such that $\forall t_i \in T_i$, $\forall f, f' \in \mathcal{F}$ and $\forall F \subseteq \mathcal{F}$ with $f, f' \in F$, we have

$$\begin{aligned} & f \succeq^F_{t_i} f' \\ \iff & \sup_{t_{-i} \in T_{-i}} \left[\sup_{f'' \in F} \int_A u_i(a, t_i, t_{-i}) df''(t_i, t_{-i}) - \int_A u_i(a, t_i, t_{-i}) df(t_i, t_{-i}) \right] \\ & \leq \sup_{t_{-i} \in T_{-i}} \left[\sup_{f'' \in F} \int_A u_i(a, t_i, t_{-i}) df''(t_i, t_{-i}) - \int_A u_i(a, t_i, t_{-i}) df'(t_i, t_{-i}) \right] \end{aligned}$$

Minimax-regret preference need not satisfy WCC. Consider the following example. Suppose $N = \{1, 2\}$, $T_1 = \{t_1^1, t_1^2\}$, $T_2 = \{t_2^1, t_2^2\}$ and $A = \{a, b, c\}$. Player 1 has the minimax-regret preference with $u_1(a, t_1^1, t_2) = 0$, $u_1(b, t_1^1, t_2) = 1$ and $u_1(c, t_1^1, t_2) = 2, \forall t_2 \in T_2$ and $u_1(a, t_1^2, t_2) = u_1(b, t_1^2, t_2) = 1$ and $u_1(c, t_1^2, t_2) = 0, \forall t_2 \in T_2$.

Let ψ_1^1 be such that $\psi_1^1(t_1^1) = \psi_1^1(t_1^2) = t_1^2$; ψ_1^2 be such that $\psi_1^2(t_1^1) = \psi_1^2(t_1^2) = t_1^1$; and ψ_1^3 be such that $\psi_1^3(t_1^1) = t_1^2$ and $\psi_1^3(t_1^2) = t_1^1$.

Consider $F(f, 1)$ as follows:

	t_2^1	t_2^2
t_1^1	a	b
t_1^2	b	b

f

	t_2^1	t_2^2
t_1^1	b	b
t_1^2	b	b

$f_{\psi_1^1}$

	t_2^1	t_2^2
t_1^1	a	b
t_1^2	a	b

$f_{\psi_1^2}$

	t_2^1	t_2^2
t_1^1	b	b
t_1^2	a	b

$f_{\psi_1^3}$

In the menu $F(f, 1)$, we have

$$\max_{t_2 \in T_2} \left[\max_{f'' \in F(f,1)} u_1(f''(t_1^1, t_2), (t_1^1, t_2)) - u_1(f(t_1^1, t_2), (t_1^1, t_2)) \right] = 1$$
$$\max_{t_2 \in T_2} \left[\max_{f'' \in F(f,1)} u_1(f''(t_1^1, t_2), (t_1^1, t_2)) - u_1(f_{\psi_1^1}(t_1^1, t_2), (t_1^1, t_2)) \right] = 0$$

Therefore, $f_{\psi_1^1} \succ^{F(f,1)}_{t_1^1} f$.

Define the following Savage acts:

	t_2^1	t_2^2
t_1^1	a	c
t_1^2	a	c

f_1

	t_2^1	t_2^2
t_1^1	a	c
t_1^2	a	b

f_2

	t_2^1	t_2^2
t_1^1	a	c
t_1^2	b	b

f_3

	t_2^1	t_2^2
t_1^1	a	b
t_1^2	a	c

f_4

	t_2^1	t_2^2
t_1^1	b	b
t_1^2	a	c

f_5

Define the menu $F = F(f, 1) \bigcup \{f_1, f_2, f_3, f_4, f_5\}$. Pick a $f' \in \bigcup_{\xi_1^F \in \Xi_1^F} F(\xi_1^F)$. Graphically, f' has two rows and each row of f' can equal any one of the following:

a	b

b	b

a	c

Therefore, $\bigcup_{\xi_1^F \in \Xi_1^F} F(\xi_1^F)$ contains exactly 9 Savage acts and these are the ones that define F. Thus, $F = \bigcup_{\xi_1^F \in \Xi_1^F} F(\xi_1^F)$.

However, $\forall f' \in F$, we have

$$\max_{t_2 \in T} \left[\max_{f'' \in F} u_1(f''(t_1^1, t_2), (t_1^1, t_2)) - u_1(f'(t_1^1, t_2), (t_1^1, t_2)) \right] = 1$$

Therefore, $f \succeq_{t_1^1}^F f', \forall f' \in F$ but $f_{\psi_1^1} \succ_{t_1^1}^{F(f,1)} f$, which is a violation of WCC.

Similarly, we have $f \succeq_{t_1^2}^F f', \forall f' \in F$. Now, using the construction in the proof of part 2 of Theorem 2.3 and appropriately defining the preference of player 2 (e.g., player 2 also has minimax-regret preference with $u_2(a, t_1, t_2^n) = u_2(b, t_1, t_2^n), \forall t_1 \in T_1, \forall n = 1, 2$), it is easy to show that the revelation principle fails in this example.

Finally, we provide an example in which the preference of each type a player is menu dependent and it satisfies WCC.

Example 2.7. Suppose $N = \{1, 2\}$, $T_1 = \{t_1^1, t_1^2\}$, $T_2 = \{t_2\}$ and $A = \{a, b\}$. Then $\Delta A = \{(p, 1-p) \in \Re_+^2\}$, where p is the probability of outcome a; i.e., ΔA is the unit-simplex in $\Re_+^2$. Let $t^1 = (t_1^1, t_2)$ and $t^2 = (t_1^2, t_2)$. Thus, $T = \{t^1, t^2\}$.

For any Savage act f and any state t, let $f^a(t)$ be the probability of outcome a in state t. Thus, geometrically, a Savage act $f = [f(t^1), f(t^2)]$ is a pair of points $f(t^1) = (f^a(t^1), 1 - f^a(t^1))$ and $f(t^2) = (f^a(t^2), 1 - f^a(t^2))$ on the unit-simplex in $\Re_+^2$.

For any menu F, let $\bar{p}^F(t^n), n = 1, 2$, be defined as follows:

$$\bar{p}^F(t^n) = \frac{1}{2} \left(\inf_{f \in F} f^a(t^n) + \sup_{f \in F} f^a(t^n) \right)$$

$\bar{p}^F(t^n)$ is thus the "mid-point" of the menu in state t^n.

For all $n = 1, 2$, the preference $\succeq_{t_1^n}^F$ is defined as follows: for all $f, \tilde{f} \in F$, we have

$$f \succeq_{t_1^n}^F \tilde{f} \iff |f^a(t^n) - \bar{p}^F(t^n)| \leq |\tilde{f}^a(t^n) - \bar{p}^F(t^n)|$$

Each type of player 1 thus displays *extremeness aversion*, which Tversky and Simonson (1993, p. 1183) describe as follows, "In some situations, [...] decision makers may evaluate options in terms of their advantages and disadvantages, defined relative to each other [...] As a consequence, options with extreme values within an offered set will be relatively less attractive than options with intermediate values." Similarly, in the example, the further away a Savage act is from the "mid-point" of the menu in state t^n, the less it is "liked" by type t_1^n of player 1.

By definition, $\succeq_{t_1^n}$ is menu dependent. Let f be any Savage act. It is straightforward to show that $\bar{p}^{F(f,1)}(t^n) = \frac{1}{2}(f^a(t^1) + f^a(t^2))$. Therefore, for any $\tilde{f} \in F(f,1)$, we have

$$|\tilde{f}^a(t^n) - \bar{p}^{F(f,1)}(t^n)| = \frac{1}{2}|f^a(t^1) - f^a(t^2)|.$$

Hence, for any Savage act f, type t_1^n of player 1 is indifferent between all Savage acts in the menu $F(f,1)$. Therefore, $\succeq_{t_1^n}$ satisfies WCC for all $n = 1, 2$.

3 Conclusion

We extended the domain of preferences to include menu-dependent preferences and characterized the maximal subset of this domain in which the revelation principle holds. The condition that characterizes this maximal domain is what we called weak contraction consistency for all types of all players. We argued that weak contraction consistency is a weaker version of the well-known contraction consistency. Although weak contraction consistency is a weak condition, we showed that an important menu-dependent preference relation, minimax-regret preference, lies outside the maximal domain. We also gave an example in which the preference of each type of a player is menu dependent and satisfies weak contraction consistency.

4 Appendix

A type of a player plays only in the *interim stage*. Therefore, one might argue that a type of a player should "care" about *interim lotteries* that specify an outcome for each realization of *other* player's types instead of Savage acts which are *ex-ante lotteries*. In that case, we need to make three changes to our model: first, we must assume that each type of each player has a preference relation over each *interim menu*, which is a set of *interim lotteries*; second, we must define an equilibrium concept, say *interim equilibrium*, that incorporates the *interim menu* that is available to each type of each player; and third, we must restate the revelation principle as follows: for every *interim equilibrium* of any mechanism, there exists a a direct mechanism with an outcome-equivalent *interim equilibrium* in which all types of all players report their respective types truthfully. Here, we show that from any such preference relation over *interim menus* we can derive a preference relation over menus such that the set of *interim equilibria* of any mechanism is the same as the set of equilibria (as defined in Section 2) of that mechanism. Then it is easy to show that all the results of Section 2 hold if this derived preference relation over menus satisfies their respective premises.

An *interim lottery* for type t_i of player i is a function $l_{t_i} : T_{-i} \to \Delta A$ and let $\mathcal{L}_{t_i}$ be the set of such functions. An *interim menu* for type t_i, denoted by L_{t_i}, is a subset of $\mathcal{L}_{t_i}$. Now, let's suppose that each type t_i of each player i has a complete preference relation $\tilde{\succeq}_{t_i}^{L_{t_i}}$ over each $L_{t_i} \subseteq \mathcal{L}_{t_i}$. Let $\tilde{\succeq}_{t_i} = (\tilde{\succeq}_{t_i}^{L_{t_i}})_{L_{t_i} \subseteq \mathcal{L}_{t_i}}$ and $\tilde{\succeq}_i = (\tilde{\succeq}_{t_i})_{t_i \in T_i}$.

Pick any mechanism Γ and consider the game $\tilde{\Lambda}^\Gamma = [N, T, g, (\Sigma_i, \tilde{\succeq}_i)_{i \in N}]$ induced by the mechanism. For any strategy profile σ, the interim menu available to type t_i of player i is $L_{t_i}(\sigma_{-i}) = \{g(m_i, \sigma_{-i}) | m_i \in M_i\}$. We say that a strategy profile σ^* is an *interim equilibrium* of $\tilde{\Lambda}^\Gamma$ if $\forall i \in N$ and $\forall t_i \in T_i$,

$$g(\sigma_i^*(t_i), \sigma_{-i}^*) \tilde{\succeq}_{t_i}^{L_{t_i}(\sigma_{-i}^*)} g(m_i, \sigma_{-i}^*), \forall m_i \in M_i.$$

First, we derive a complete preference relation $\succeq_{t_i} = (\succeq_{t_i}^F)_{F \subseteq \mathcal{F}}$ from $\tilde{\succeq}_{t_i}$. For any Savage act f, define $f|_{t_i} : T_{-i} \to \Delta A$ as $f|_{t_i}(t_{-i}) = f(t_i, t_{-i}), \forall t_{-i} \in T_{-i}$. For any menu $F \subseteq \mathcal{F}$, define $F|_{t_i} = \{f|_{t_i} | f \in F\}$. By definition, $f|_{t_i} \in \mathcal{L}_{t_i}$ and $F|_{t_i} \subseteq \mathcal{L}_{t_i}$. Finally, define $\succeq_{t_i}^F$ as follows: $\forall f, f' \in F$, let $f \succeq_{t_i}^F f' \iff f|_{t_i} \tilde{\succeq}_{t_i}^{F|_{t_i}} f'|_{t_i}$. The preference relation $\succeq_{t_i}^F$ is complete because $\tilde{\succeq}_{t_i}^{F|_{t_i}}$ is complete. Let $\succeq_i = (\succeq_{t_i})_{t_i \in T_i}$.

Next, consider the mechanism Γ. We argue that a strategy profile σ^* is an equilibrium of $\Lambda^\Gamma = [N, T, g, (\Sigma_i, \succeq_i)_{i \in N}]$ if and only if σ^* is an interim equilibrium of $\tilde{\Lambda}^\Gamma = [N, T, g, (\Sigma_i, \tilde{\succeq}_i)_{i \in N}]$. Notice that $L_{t_i}(\sigma_{-i}) = F(g, \sigma_{-i})|_{t_i}$. Therefore, $\forall i \in N$ and $\forall t_i \in T_i$,

$$\begin{aligned}
& g(\sigma_i^*(t_i), \sigma_{-i}^*) \tilde{\succeq}_{t_i}^{L_{t_i}(\sigma_{-i}^*)} g(m_i, \sigma_{-i}^*), \forall m_i \in M_i \\
\iff & g(\sigma^*)|_{t_i} \tilde{\succeq}_{t_i}^{F(g, \sigma_{-i}^*)|_{t_i}} g(\sigma_i, \sigma_{-i}^*)|_{t_i}, \forall \sigma_i \in \Sigma_i \\
\iff & g(\sigma^*) \succeq_{t_i}^{F(g, \sigma_{-i}^*)} g(\sigma_i, \sigma_{-i}^*), \forall \sigma_i \in \Sigma_i.
\end{aligned}$$

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