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On the Persistence of Inequality in the Distribution of Personal Abilities and Income

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In this paper we discuss the impact of malnutrition on the distribution of abilities and income in a simple overlapping generations framework. Workers are distributed uniformly over a low-ability and a high-ability range. If workers earn below subsistence wages, the probability that their children will have low abilities is higher than with above subsistence wages due to the malnutrition resulting from low incomes. Using a nested Ethier production function we find that there is an optimal share of low-ability workers in the economy which maximizes output. Due to the intergenerational propagation of low abilities resulting from malnutrition, economies may however get trapped in sub-optimal equilibria with too large shares of low-ability workers.
Distributing food coupons financed by taxes of the parent generation to the offspring of these low-ability workers will increase the likelihood that they will be in the high-ability range, permanently increasing output for future generations. Using a numerical example, we show that this type of redistributive policy is welfare improving if the parent generation alive during the initiation of the policy is reimbursed for their loss in utility due to taxes.

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In this paper we discuss the impact of malnutrition on the distribution of abilities and income in a simple overlapping generations framework. Workers are distributed uniformly over a low-ability and a high-ability range. If workers earn below subsistence wages, the probability that their children will have low abilities is higher than with above subsistence wages due to the malnutrition resulting from low incomes. Using a nested Ethier production function we find that there is an optimal share of low-ability workers in the economy which maximizes output. Due to the intergenerational propagation of low abilities resulting from malnutrition, economies may however get trapped in sub-optimal equilibria with too large shares of low-ability workers. Distributing food coupons financed by taxes of the parent generation to the offspring of these low-ability workers will increase the likelihood that they will be in the high-ability range, permanently increasing output for future generations. Using a numerical example, we show that this type of redistributive policy is welfare improving if the parent generation alive during the initiation of the policy is reimbursed for their loss in utility due to taxes.

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1 Introduction

Just weeks ago, the Copenhagen Consensus 2004 identified hunger and malnutrition as one of the ten greatest challenges facing he world of today. Specifically, Behrmann et al.[1] discuss in their contribution in detail the extent of the problem from an economic point of view, arguing that apart from humanitarian reasons to alleviate the situation, potential productivity gains of the labor force would outweigh the cost of reducing malnutrition. The role of nutrition on labor productivity has been discussed before, e.g. by Leibenstein (1957); Mirlees, (1975); Bliss and Stern (1978); and especially by Dasgupta and Ray (1986, 1987) and Dasgupta (1991, 1993, 1997). In fact, Dasgupta (1993) suggests that malnutrition may lead to dynastic poverty traps. A recent example (Galor and Mayer, 2002) explores the further-reaching implications of malnutrition leading to lower ability to take advantage of (public) education, leading to the persistence of inequality if basic needs are not addressed. We want to discuss the impact of malnutrition on a much more basic level, paying particular attention to the persistence of inequality over generations brought about by low wages and malnutrition. As e.g. extended upon in Harper et al. (2003), malnutrition can have such detrimental effects that children born to undernourished women can remain small, underweight and even impaired in their cognitive abilities for life. Even with the current speed of improvement of nutritional standards, about 1 billion children will be impaired in their mental development in 2020 due to the compound effect of malnutrition of parents and children. Clearly, these children will have in turn a hard time to gain a wage that is sufficient to cover all basic needs, leading again to malnutrition and the same harmful consequences for their own children. In addition, even if children have later in life access to education and health care, they will still be affected by their previous actual and in utero malnutrition. Innate abilities and general health will be lower than they had been with adequate nutrition, such that schooling children and fighting diseases will be a lot less efficient than under normal conditions. This makes it especially hard to escape the intergenerational poverty trap described above.

The general idea of the paper is thus that an uneven distribution of income gives rise to an equally uneven distribution of (productive) abilities within a population, and hence to the underutilization of the ultimate productive potential of the population at large. From that perspective the redistribution of income may be a positive sum game. To illustrate the general idea outlined above, we present a simple overlapping generations model in which each member of a generation belongs to one of two different groups in society: a group of individuals distributed over a range of rela-

[1] Their Challenge Paper also contains a very thorough discussion of the latest literature in economics as well as nutritional science.

tively high abilities, and a group distributed over a relatively low ability range. Furthermore, each individual of the population goes through three separate stages in life. The first stage refers to infancy: the individual is inactive and depends on its parents for consumption. In the second stage, the individual works; the income is spent on the children's consumption as well as on own, present and future, consumption. In the third phase the individual, again inactive, will have to rely on the savings made during the active period. We derive the optimum levels of consumption for the individual's children and for itself both when adult and when old. We are thus able to derive the consumption of children relative to subsistence level consumption. Since we assume that below subsistence level consumption will increase the probability that these children will belong to the low-ability group in society, it follows that parents with below subsistence level income will have a relatively high probability of having low-ability offspring.

The production structure of the model can be characterized as a multi-level Ethier function (cf. Romer, 1990) in which capital is combined with a labor complex consisting of a high-skilled and a low-skilled labor sub-complex. These sub-complexes in turn are aggregates over the skill ranges associated with each labor-complex. We furthermore allow for asymmetries in the contribution by skill to output and corresponding asymmetries in marginal productivities by skill. For the same level of supply by ability it follows that high-skilled workers will earn a higher wage than people with low skills. Consequently, the probability that high-skilled people earn above subsistence level wages is higher than for people with low skills, and so will be the probability that their offspring will be in the high-ability range.

This setting may generate multiple equilibria with respect to the skill distribution in the population, two of the three potential equilibria are stable. Only coincidentally will one of these two equilibrium points however coincide with the utility maximizing distribution of the working population over the two ability ranges. If the economy is in a high-inequality steady state, redistributive policy will therefor be a welfare improvement. The reason why this is the case is that a higher proportion of people earning above subsistence level income is equivalent to a quality increase of the labor force that may offset the loss in potential output due to taxes. We thus illustrate how a more equal distribution of income via food subsidies may actually increase free disposable income for all, even if savings are decreased in the process. For some of the low-skilled this is due to an increase in the quality of innate abilities, while the rest of the unskilled benefit from their increased scarcity that leads to higher wages.

2 The Model

2.1 The basic equations

At each point in time, three generations live in a closed economy. The young are dependent on their parents for consumption, who also have to provide for their own consumption and save for old age. The savings of the parent generation determine the capital stock for the next period. Each of the three phases in life is assumed to take one unit of time. Utility is defined in terms of the parent generation:

$$U = c_a^{\alpha} \cdot c_o^{1-\alpha-\beta} \cdot c_y^{\beta} \tag{1}$$

where c_y is the consumption of their children, c_a is the consumption of the adults and c_o the consumption of the parents when old. Adults are optimizing utility taking into account their budget constraints:

$$w = c_y + c_a + s \tag{2}$$

$$c_o = s(1+r) \tag{3}$$

Setting up and solving a Lagrange function where w is the real wage rate, s are savings and r is the real interest rate leads to

$$c_a = w\alpha \tag{4}$$

$$c_y = w\beta \tag{5}$$

$$c_o = w(1+r)(1-\alpha-\beta) \tag{6}$$

$$s = w(1-\alpha-\beta) \tag{7}$$

The ability distribution in the population is modeled by distinguishing between two skill groups. The group of low-ability workers is uniformly distributed over a range of skills from 0 to $\bar{l}$. Similarly, the high-ability workers are uniformly distributed over a range starting at 0 and ending at $\bar{h}$. There are N_L low-ability and N_H high-ability workers in the total population N.

There is a spectrum of jobs that corresponds to the spectrum of skills outlined above. These jobs are part of a nested Ethier production function, which is organized as follows: at the upper level we assume a Cobb-Douglas function describing how output Y is produced using a function between capital K and effective labor services L.

$$Y = A \cdot K^{1-\zeta} L^{\zeta} \tag{8}$$

Total effective labor services L consist of a CES aggregate of low-ability labor L_L and high-ability labor L_H.

$$L = (L_L^{\zeta} + L_H^{\zeta})^{\frac{1}{\zeta}} \tag{9}$$

Low-ability labor and high-ability labor are in turn again a CES aggregate of the different abilities belonging to the low- and high-ability ranges:

$$L_L = \left(\int_{i=0}^{\bar{l}} c_i \cdot l_i^{\zeta} di \right)^{\frac{1}{\zeta}} \tag{10}$$

$$L_H = \left(\int_{j=0}^{\bar{h}} c_j \cdot l_j^{\zeta} dj \right)^{\frac{1}{\zeta}} \tag{11}$$

In equations (10) and (11) we have defined the direct contribution of each ability to its corresponding aggregate as:

$$c_{z,i} = c_{0,z} \cdot e^{\gamma_z \cdot i} \qquad 0 \leq i \leq \bar{z}, z = l, h \tag{12}$$

where c_0, $\gamma_{z,i}$ are constant positive numbers. Equation (12) has the effect of a marginal productivity of labor that rises with its ability level. It should be noted that for a positive value of γ, $c_{z,i}$ works as if it was skill-augmenting technical change. In fact, $c_{z,i}$ does not change over time, but differs among abilities. For the common employment levels by ability in each separate ability range we find:

$$l_i = \frac{N_L}{\bar{l}} \qquad \forall \qquad 0 \leq i \leq \bar{l}, \qquad h_j = \frac{N_H}{\bar{h}} \qquad \forall \qquad \bar{l} \leq j \leq \bar{h} \tag{13}$$

where N_L and N_H refer respectively to the shares of low- and high-ability workers in the population and are defined as follows:

$$N_L = \mu L \cdot N \tag{14}$$

$$N_H = (1 - \mu L) \cdot N \tag{15}$$

where μL is the share of people with relatively low abilities in the population. Wage rates are equal to the marginal product of each worker, thus the the wage for a low-ability worker with ability i is given by:

$$w_{L,i} = A \cdot c_{L,0} \cdot e^{i\gamma_L} \cdot K^{1-\zeta} \cdot \zeta \left(\frac{N\mu L}{\bar{l}} \right)^{-1+\zeta} \tag{16}$$

For a high-ability worker we find by analogy:

$$w_{H,j} = A \cdot c_{H,0} \cdot e^{j\gamma_H} \cdot K^{1-\zeta} \cdot \zeta \left(\frac{N(1 - \mu L)}{\bar{h}} \right)^{-1+\zeta} \tag{17}$$

2.2 Subsistence income and subsistence skills

We assume now that there is a real income level $\bar{w}$ such that an income below that level is not sufficient to provide the necessities of food and shelter to the children. Following the evidence presented in the introductory paragraphs, we assume that for people earning less than $\bar{w}$ this results in a higher probability that their offspring will have innate abilities that are in the low-ability range. This implies that the ability distribution for the adults of the next generation depends on the income distribution of the current adult generation.

It follows immediately from equations (12), (16) and (17) that there exists a subsistence ability level that earns the subsistence wage $\bar{w}$. This ability level can be obtained by finding i for which

$$\bar{w} = w_{z,i} = A \cdot c_{L,0} \cdot e^{i\gamma_L} \cdot K^{1-\zeta} \cdot \zeta \left(\frac{N\mu L}{\bar{l}} \right)^{-1+\zeta} \forall 0 \leq i \leq \bar{l} \qquad (18)$$

The corresponding value of i could be bigger than $\bar{l}$. In that case all low-ability workers would earn wages below the subsistence level. If i is in the low-ability range, then part of the low-ability population will earn an above subsistence level wage. The subsistence ability level for the high-ability group can be obtained in exactly the same way.

$$\bar{w} = w_{z,j} = A \cdot c_{H,0} \cdot e^{j\gamma_H} \cdot K^{1-\zeta} \cdot \zeta \left(\frac{N(1-\mu L)}{\bar{h}} \right)^{-1+\zeta} \forall 0 \leq i \leq \bar{h} \qquad (19)$$

If the corresponding ability level is below 0, then the entire high-ability sub-population earns above subsistence wages. If it is above $\bar{h}$, then the entire high-ability population has a wage below the subsistence level. We will refer to the respective subsistence ability levels as $i(\bar{w})$ and $j(\bar{w})$.[2] Consequently, the fraction of low-ability workers (which has to remain between 0 and 1) earning below subsistence wages can be expressed by

$$F_{\bar{w},L} = \max \left[0, \min \left[1, \frac{i(\bar{w})}{\bar{l}} \right] \right] \qquad (20)$$

and accordingly for high-ability workers.

$$F_{\bar{w},H} = \max \left[0, \min \left[1, \frac{j(\bar{w})}{\bar{h}} \right] \right] \qquad (21)$$

2.3 The propagation of innate abilities

Figure 1 depicts how the distribution of abilities within the population changes between generations. Going from left to right in the Figure mimics the succession of generations in time. We assume that each generation is exactly the same size. Generation t is divided into two main groups: those with relatively low abilities which are labeled L, and those with relatively high abilities labeled H. The adults in each ability group are subdivided into two classes: those who earn below subsistence wages, indicated by a minus

[2] $i(\bar{w})$ and $j(\bar{w})$ can be found by solving (18) and (19) for i and j respectively.

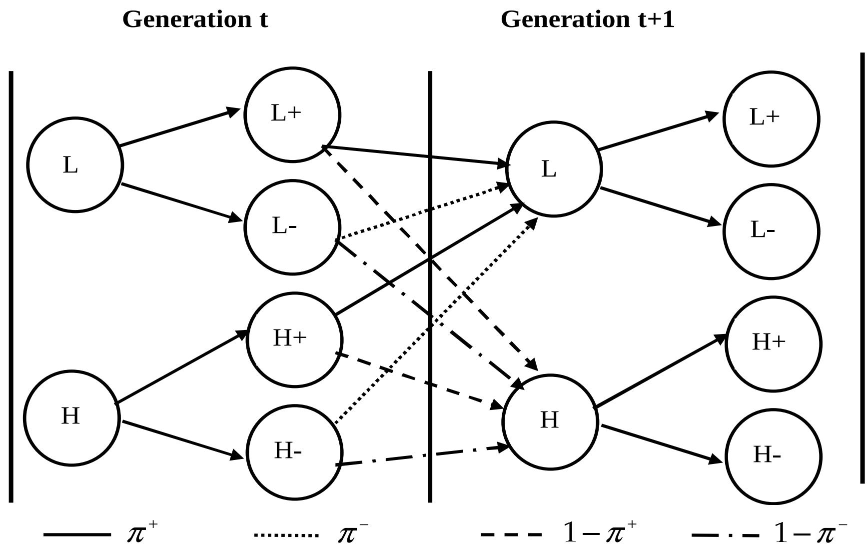

Figure 1: *The intergenerational propagation of abilities*

sign, and those earning above subsistence wages, indicated by a plus sign. The classes earning below subsistence wages will have offspring that has a relatively high probability of being in the next generation of low-ability adults. This probability is labeled Π^- . The classes earning above subsistence wages still have a probability $\Pi^+ < \Pi^-$ of having offspring that will end up as low-ability adults in the next generation. These probabilities can also be interpreted as the relative contributions of each class in generation i to the main ability groups in the next generation. Π^+ x 100 percent of the adults in the class of low-ability above subsistence wage earners in generation t, i.e. L+, will have offspring that will belong to the next generation's low-ability group of adults, whereas (1- Π^+) x 100 percent of the adults in L+ will have offspring that will belong to the next generation's high-ability adults. It should be noted that the sizes of the various sub-classes depend on the wage rates earned by the low-ability and high-ability workers. It should also be noted that once the class of low-ability workers is large enough, wages will be depressed to such an extent that the class L- grows at the expense of L+, thus giving rise to an even larger group L in the next generation (since $\Pi^+ < \Pi^-$, by assumption). This depresses wages for the low-ability adults in the next generation even more. However, the fact that we have assumed that the probability of the offspring of low-ability adults ending up in the high-ability group in the next generation is strictly larger than zero (but relatively small), ensures that the next generation will always contain some high-ability workers.

This implies that the size of the next generation of high-skilled workers will be given by:

$$N_{L,t+1} = (1 - \Pi^-) \cdot (F_{\bar{w},L} \cdot N_{L,t} + F_{\bar{w},H} \cdot N_{H,t})$$
$$+ (1 - \Pi^+) \cdot ((1 - F_{\bar{w},L}) \cdot N_{L,t} + (1 - F_{\bar{w},H}) \cdot N_{H,t}) \quad (22)$$

where the subscript t refers to a particular generation rather than ordinary time. For the number of low-ability workers in the next generation we find by analogy:

$$N_{H,t+1} = \Pi^- \cdot (F_{\bar{w},L} \cdot N_{L,t} + F_{\bar{w},H} \cdot N_{H,t})$$
$$+ \Pi^+ \cdot ((1 - F_{\bar{w},L}) \cdot N_{L,t} + (1 - F_{\bar{w},H}) \cdot N_{H,t}) \quad (23)$$

Given the values of the structural parameters of the model[3], we can depict the possible equilibria with respect to the skill distribution in Figure 1. Three possible equilibria arise, as given by the points of intersection of the

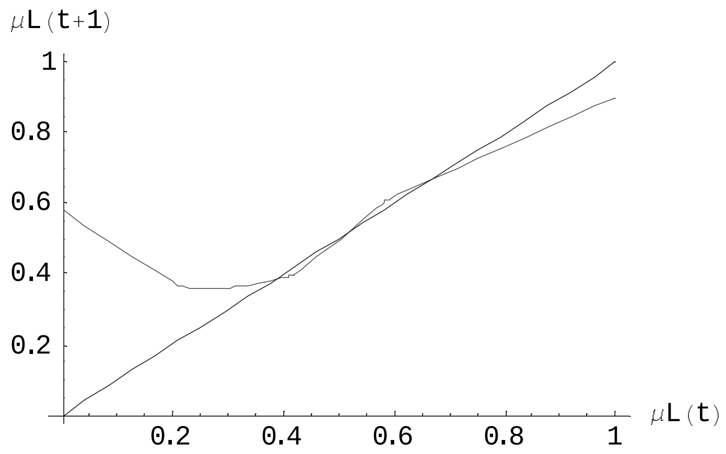

Figure 2: *Equilibrium distribution of low-ability workers*

'discontinuous' ability propagation curve (APC) and the 45^o line. Two of these equilibria are stable, and while the low-ability equilibrium, i.e. the one with the highest share of μL will always exist, the other two depend on the parameters chosen.

2.4 The share of low-ability workers and subsistence wages

Due to the Ethier production function including the "love of variety" argument with respect to the diversity of abilities, below subsistence wages may arise in the high-ability range without implying that all low-ability workers have to earn below subsistence wages, too. Figure (3)[4] shows the shares of

[3]The same parameter values as in the sensitivity analysis (section 3) were used, but the Π values adapted to induce the emergence of all three possible equilibria.

[4]See section 3 for the parameter values used.

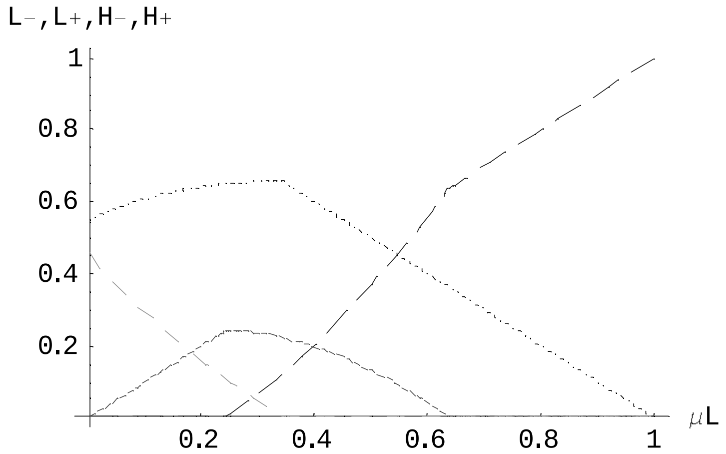

Figure 3: *Development of subsistence wage earners for different ability distributions*

low- and high-ability workers who earn either above or below subsistence wages. Specifically, for very low values of μL, no low-ability workers earn below subsistence wages (the low-ability workers L^- earning below subsistence wages are depicted by the black dashed line), while some workers in the high-ability range do (H^-, gray dashed line). This is due to the scarcity of low-ability workers that drives wages up and above the subsistence wage level. By contrast, high-ability workers are not scarce at all and some do thus earn below subsistence wages. Nevertheless, a larger share of high-ability workers still earns above subsistence wages (H^+, black dotted line). When moving towards a higher share of low-ability workers, some low-ability workers start to earn wages below the subsistence level as well, signalled by the rising black dashed line. The black solid line, depicting the share of low-ability workers earning above subsistence wages, starts to decrease at the same instance, until it reaches zero at around 0.65 in this case. Thus, all low-ability workers from there on will earn below subsistence wages. Since high-ability labor becomes increasingly scarce, the share of high-ability workers earning below subsistence level drops, until it reaches zero at around μL=0.35 in this example. These results suggest that output may not be optimal at the stable low-ability equilibrium found in the last section, since the lowest total share of below subsistence earning workers is found at a much lower level of low-ability individuals (in this numerical example, at around 0.3). The following section will therefor determine the utility maximizing distribution of abilities in the population.

2.5 The efficiency maximizing distribution of abilities

In order to judge which one of the equilibria is preferable in terms of produced output and, consequently, wages and utility, and whether one of them coincides with the actual optimal distribution of innate skills in the economy, it is necessary to calculate the effective labor supply EL and its average (AEL).

$$EL = (H+L)^{\frac{1}{\zeta}} \Rightarrow AEL = \frac{(H+L)^{\frac{1}{\zeta}}}{N} \tag{24}$$

Evaluating AEL using equations (9),(10) and (11) gives

$$AEL = \left(\frac{c_{H,0}(e^{\bar{h}\gamma_H}+1)\bar{h}^{-\zeta}(1-\mu L)^{\zeta}}{\gamma_H} + \frac{c_{L,0}(e^{\bar{l}\gamma_L}+1)\bar{l}^{-\zeta}\mu L^{\zeta}}{\gamma_L} \right)^{\frac{1}{\zeta}} \tag{25}$$

Using the same parameters as before, the path of average labor efficiency (AEL) and thus income per head for given K is shown in function of the share of low-ability workers. It is obvious that μL^*, the efficiency maximizing share of low-ability workers, does coincide with either one of the stable equilibria only by chance,[5]which leaves room for policy. Redistributive policy will

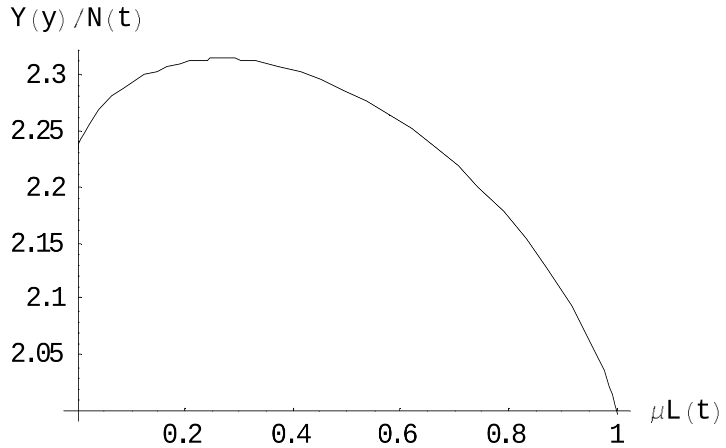

Figure 4: *Income per head in relation to the ability distribution*

make the equilibrium move to the left, which would imply higher total output if the output per head curve remained unchanged. In terms of Figure (5) further below, this would imply a move from C to D. However, the output curve will shift downwards since the wages of workers will be taxed, which will decrease total savings, and total real savings are equal to the capital stock by assumption.[6] If however the loss in income due to lower savings is smaller than the increase in income due to a more equal ability distribution,

[5]This result was generated using different sets of parameters.
[6]We disregard the depreciation of capital.

redistributive policy is welfare efficient. In terms of Figure (5), this is the case if the point F lies above point E, as it is actually depicted. The following section however first discusses the working of the model in order to find out how changes in the different variables and different tax rates actually affect the model, before moving on to an actual policy experiment in section 4.

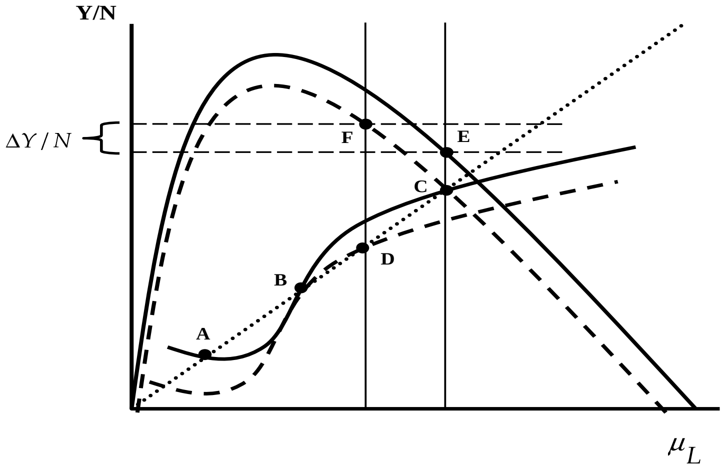

Figure 5: *The potential for welfare improvement due to redistributive policy*

3 Sensitivity analysis

Because a closed form solution for the model cannot readily be obtained, we illustrate the working of the model in two ways. In this section we first show how the main relations of the model are affected by changes in the various model parameters. There are two main relations in the model:

1. The relation between the ability distribution within the current generation of adults and that within the next generation of adults;

2. and the relation between output per head and the ability distribution within the current generation of adults.

In the next section, we then continue to illustrate the working of the model by implementing an income redistribution policy by means of the taxation of the wage income of adults and then recycling the tax proceeds in the form of consumption coupons for those who cannot offer their offspring at least subsistence level consumption from their after-tax income.

Because of the discontinuities present in the model, we use some numerical simulations to show how the model works. In order to obtain these numerical outcomes, we have used the following parameter values, next to the assumption that the entire population is scaled to one at all times.

Table 1: Model parameter values

Parameter	Value	Parameter	Value	Parameter	Value
$\bar{c_y}$	0.667	$\bar{h}$	1.000	γ_H	1.000
α	0.333	A	2.000	cl_0	1.000
β	0.333	ζ	0.900	ch_0	1.105
$\bar{l}$	1.000	γ_L	0.100	$\bar{w}$	2.000

In addition, Π^+, i.e. the probability that a well-nourished child will find itself in the low ability range is set equal to 0.05, and Π^-, i.e. the probability that a malnourished child will be in the low ability range is set equal to 0.7. The parameters for which we have performed the sensitivity analysis are $\bar{l}$, $\bar{h}$, γ_L, γ_H, cl_0, ch_0, $\bar{w}$, because we know only little about their impact on the intergenerational propagation of abilities a priori. However, all parameters, except the subsistence wage, influence the productivity of the corresponding ability classes directly and positively. In Figure 6 , where the dashed lines

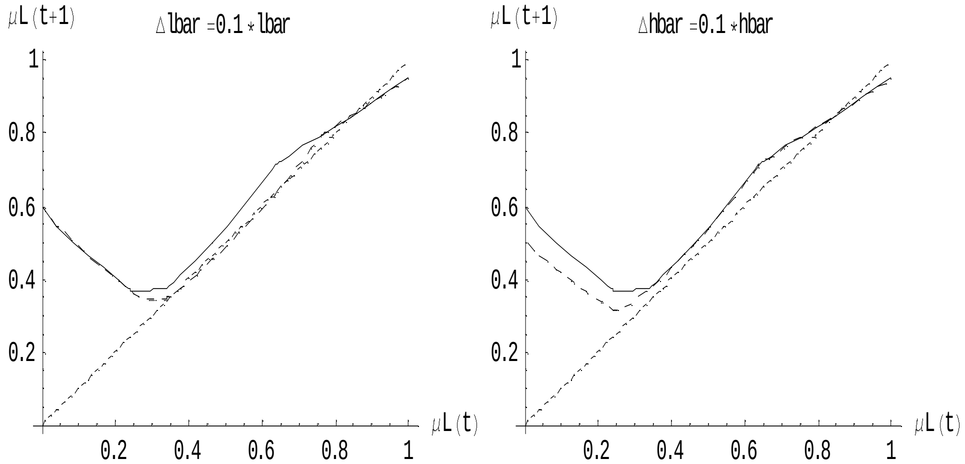

Figure 6: *(a)effect of a change in $\bar{l}$ (b) effect of a change in $\bar{h}$*

correspond with the new parameter values and the dotted line is a 45^o - line as usual, we have increased the ranges for the low-ability and high-ability classes by 10 percent respectively. This has the effect of increasing the average productivity of each class, since each class will be uniformly spread over a larger range, thus raising the average contribution of the classes as a whole to the labor complex. Looking at Figure 6, we find that an increase in $\bar{l}$ by 10 percent shifts the 'ability propagation curve' (or APC for short) downwards, especially for medium levels of the current share of low-ability workers in the population, i.e. μ_l. This downward shift is caused by the rise in low-ability wages that decreases the number of people earning below subsistence wages, and hence reduces the next generation of low-ability adults, ceteris paribus.

For low values of μ_l, hence high values of the share of high-ability adults in the current population, a rise in the wage rate of low-ability workers has little effect on the APC, since there are so few low-ability adults that they earned above subsistence wages anyway. That is precisely the reason why we only see an effect of a rise in $\bar{h}$ for low values of μ_l, hence high values of the share of high-ability workers and therefor relatively low (even below subsistence) wages for these workers. For the changes in the parameters γ_L and γ_H we can observe qualitatively the same results as in Figure 6, so these results are not reported in detail here. The results for cl_0 and ch_0 are distinctly different, though, as can be seen from Figure 7. The effects of a

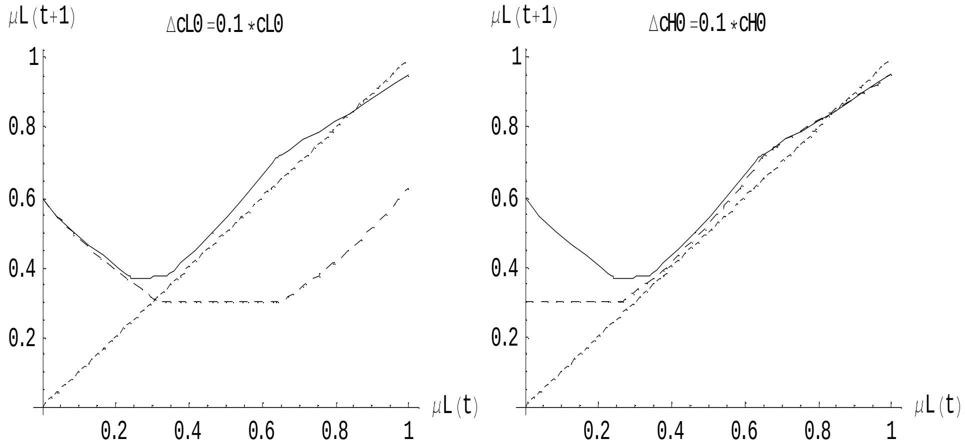

Figure 7: *(a)The effect of a change in* cl_0 *(b)The effect of a change in* ch_0

10 percent change in the distribution parameters are much larger than those of the change in the ability range. We see that the APC changes dramatically in the case of the low-ability workers, since with that change nearly all workers will start earning above subsistence wages, allowing the economy to move towards a high-ability composition of the population, thus enabling the economy to reap the positive productivity effects of such a move. It should be noted that even with a relatively large share of high-ability workers in the population, wages are generally above subsistence levels. Hence, the rise in the high-ability contribution coefficients does change the shape of the APC but only very little at the spot where it could actually make a difference for the ability composition of the population. Finally, we see that a fall in the subsistence wage by 10 percent (through a subsidy for instance), has very similar effects as a 10 percent rise in cl_0 , as shown in Figure 8. The only difference here is the horizontal part of the APC near the vertical axis, that is due to the fact that all workers, with both low and high abilities, now will earn enough to make sure their offspring has the best chances of ending up in the high-ability range. Still, 30 percent will end up in the low-ability range, since that is the minimum value that the share of low-ability workers

in the population can attain.[7]

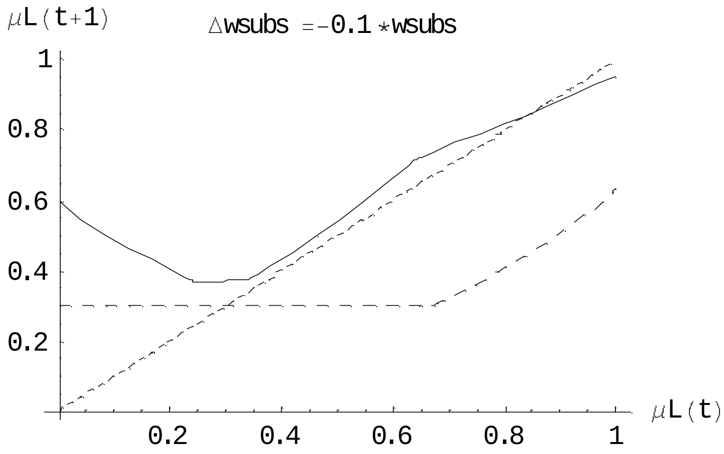

Figure 8: *The effect of a drop in the subsistence wage rate*

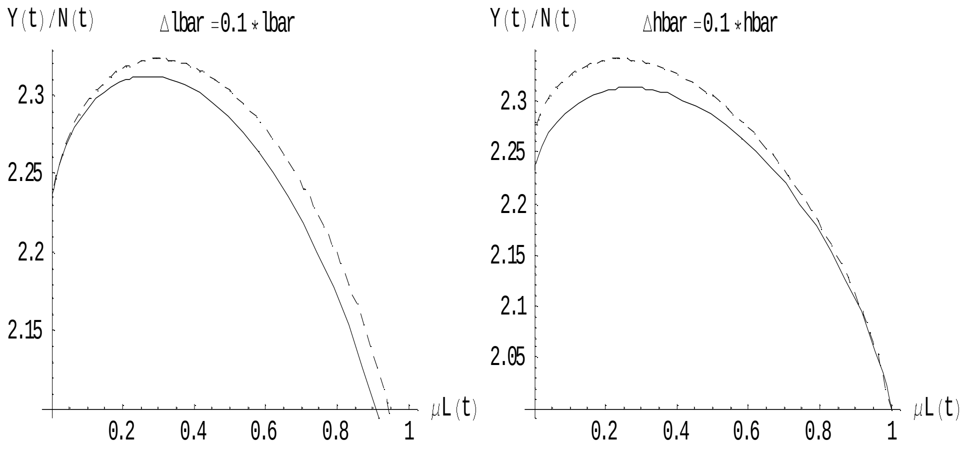

Figure 9: *(a)The effect of a change in $\bar{l}$ (b)The effect of a change in $\bar{l}$*

In Figure 9 we show how average product per head (further called APH) changes as the parameters are changed. Not surprisingly, the average product curve shifts upwards as indicated by the dashed line, and more so for high values of the share of low-ability workers in the population in Figure 9, part(a). Obviously the impact of an average productivity increase of low-ability workers is largest if there are many of those workers around. Mutatis mutandis this holds true for Figure 9,part(b) as well. In Figure 10 we show what happens if the distribution coefficients change, since, as before, the

[7]This follows immediately from Figure (1) and the values of Π^+, since in the case of wages being all above subsistence levels $L^- = H- = 0$ and we therefore find $\frac{L}{N} = \Pi^+$, where N measures the size of the population, i.e. in this case $N = L^+ + H^+$.

changes in the APH-curve due to changes in γ_L and γ_H are qualitatively very similar to those in Figure 9.

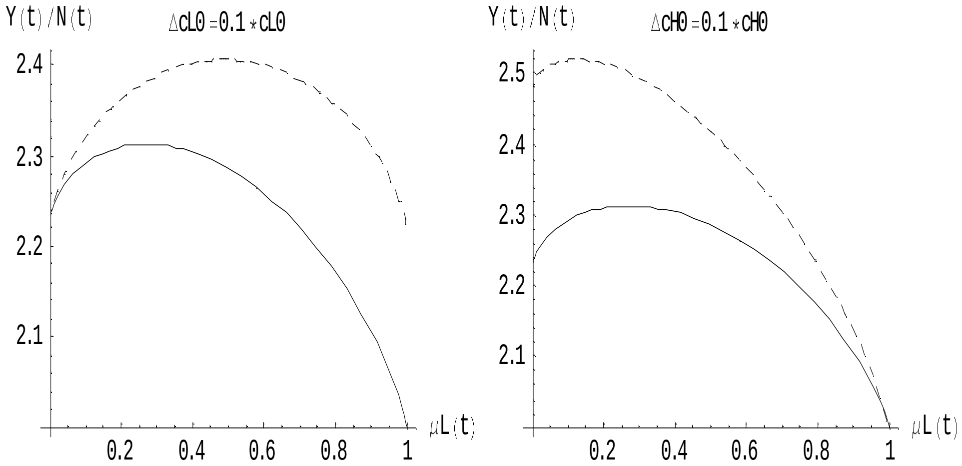

Figure 10: *Effect of a change in the (a)low ability and (b)high ability distribution coefficient*

In Figure 10 we see that a change in the distribution coefficients has fairly large and positive effects. The relative impact is largest for high values of μ_L in case of the low-ability distribution coefficients and for low values of μ_L in case of the high-ability distribution coefficients, for the reasons outlined above.

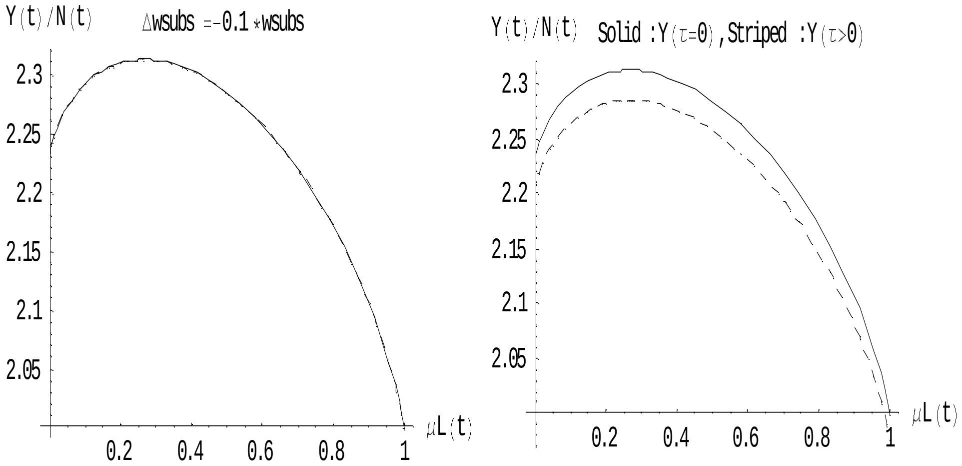

Figure 11: *(a)Effect of a change in the subsistence wage (b) Effect of taxes*

Finally, Figure 11, part (a) shows that nothing happens to the average product curve if we lower the subsistence wage by 10 percent. This is because we have assumed that only the younger generation is hit by malnutrition, and we did not specify any link between labor productivity at the individual level and having to lead a below subsistence life as such. We have done this for reasons of simplicity. Nonetheless, this implies that the positive general equilibrium productivity effects at the macro level will have a tendency to be underestimated. This Figure also suggests that it would be relatively simple to devise a policy that allows us to change the ability distribution of the population through income redistribution measures, since changing the subsistence wage itself does not change the APH-curve. However, if, as we plan to do, we finance the change in subsistence wages through wage taxes, this will negatively influence savings (cf. equation (7)) and hence the size of the capital stock for the next generation. This in turn will lead to a downward shift of the APH-curve, as shown in Figure 1, but that is also shown for this particular parameterization in Figure 11, part (b).

In the next section we will now describe how we have implemented an income redistribution policy that generates such a change in the ability distribution of the population that a welfare improvement for society as a whole can be realized even if savings do fall.

4 A Policy Experiment

4.1 Description of the policy experiment

The income redistribution experiment has the purpose of increasing the share of high-ability workers in the population, under the following constraints and principles:

1. The first generation of adults implementing the policy should not suffer from the policy. It will have to pay the taxes, but it should be compensated by the second generation when it is their turn to be productive;

2. The old in the first generation should not be taxed, since they cannot be compensated by the second generation as they will be no longer alive when the second generation becomes productive;

3. The second generation should compensate the first through a transfer of income to the old while generating a positive change in utility which exactly matches the negative change in utility experienced by the old when they were the adults of the previous generation. We will show that the upward movement along the APF-curve is in this case so large that the second generation of adults may also gain as compared to the no-policy situation;

4. The third generation and all subsequent generations will need to maintain the new ability distribution of the population as it resulted from

the policies executed by the first and the second generation. As we will show, there is no room to completely compensate the second generation of adults, since that entails ever-increasing wage taxes and ultimately the collapse of the economy;

5. The only 'rights' that the young have at any time is to be able to consume (at least) at subsistence level. A high income of their parents does not entitle them to a high income of their own, or compensation by somebody else if their income falls short of the income of their parents.

Under these principles we will show that the model we have specified enables a welfare improvement measured by total utility experienced by all generations, the source of which is the productivity improvement at the aggregate level that can be realized through a better match between abilities available in the population and the production technologies available to that population.

4.2 Policy Implementation

We implement the redistribution policy by levying a proportional wage tax on all wages, including those of below subsistence earners. Then the tax revenue is used to finance consumption coupons that are used to fill the gap between consumption of the young out of disposable (after-tax) wage income of their parents (cf. equation (5) where the wage rate should be replaced by disposable wages $(1-\tau)*w$, and where τ represents the proportional tax rate) and the subsistence level consumption. For the group of low- and high-ability workers we therefor have:

$$C_{A,i} = \max(0, \bar{c} - \beta(1-\tau)w_{A,i}) \qquad for A = L, H \tag{26}$$

where $C_{A,i}$ is the volume of consumption by the offspring of class i in ability group A financed using the consumption coupons. For reason of simplicity we assume that one coupon corresponds to one unit of consumption, hence $C_{A,i}$ is also the number of coupons to which class i of ability group A is entitled. Since the wage rate rises for increasing i, it follows that we can calculate the marginal class for which the after-tax wage is enough to finance the subsistence level consumption of their young without any coupons from the requirement $C_{A,i} = 0$. Equation 26 can also be used for the first generation to calculate the marginal subsistence class by setting $\tau = 0$, and then finding the i that makes $C_{A,i} = 0$ by substituting equation (16) or (17) for the low- and high-ability groups respectively. It turns out that for the parameters we have chosen, all low-ability workers earn below subsistence wages, and all high-ability workers earn above subsistence wages. Hence the generation of high-ability adults will surely lose from the implementation of the redistribution policy, and will therefor have to be compensated by the next generation of adults. But even though the low-ability adults all earn below subsistence level wages, it is well possible that the introduction of a proportional wage tax will make the relatively high-wage earners less well off than they would be in the no-tax situation. These workers would also need to be compensated when old. We will come back to this compensation issue in more detail below, but first we will outline how the tax rate necessary to cover all coupon expenses is calculated given the structure of the model outlined above. The total costs of consumption coupons to be covered by taxes can be obtained by straightforward integration of (26) over all sub-classes of low- and high-ability workers. We get:

$$TC = \int_0^{\bar{i}} (\bar{c_y} - \beta(1-\tau)w_{L,i})l_i di + \int_0^{\bar{j}} (\bar{c_y} - \beta(1-\tau)w_{H,j})h_j dj \qquad (27)$$

where $\bar{i}$ and $\bar{j}$ are the marginal disposable subsistence wage earning classes in the low-ability and high-ability groups of adults respectively. Moreover, l_i and h_j are the number of individuals in these low- and high-ability sub-classes respectively (cf. equation (13)) . At the same time, for a given tax rate τ , total tax income would be given by:

$$TT = \int_0^{\bar{l}} \tau \cdot w_{L,i} \cdot l_i \cdot di + \int_0^{\bar{h}} \tau \cdot w_{H,j} \cdot h_j \cdot dj \qquad (28)$$

and the tax rate that is necessary to cover the total cost of all coupons out of tax revenue can then be obtained by equating (27) and (28). An impression of the shape of the graphs behind (27) and (28) as a function of the share of low-ability adults in the population is provided in Figure 12.

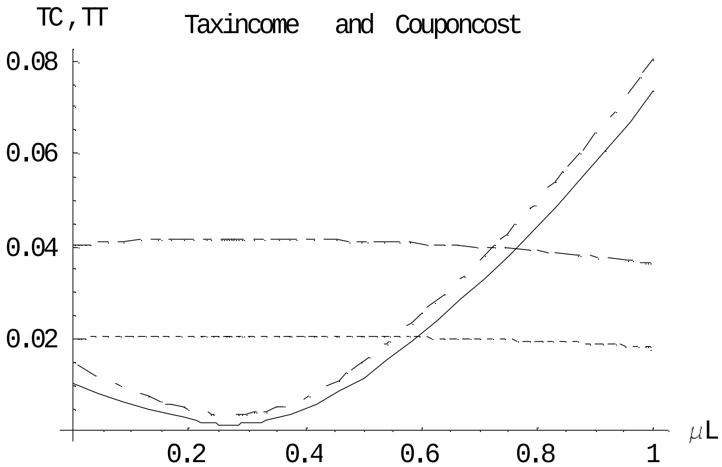

Figure 12: *Total tax revenue and total coupon cost for different tax rates*

The U-shaped curves in Figure 12 are the total coupon costs for a given ability distribution in the current population and for a given tax rate (that influences disposable wage income hence the need for coupons, ceteris paribus). The lower of the two curves is associated with a tax rate equal to 0.01, and the higher of the two with a tax rate equal to 0.02. The two curves that have a concave shape are the corresponding tax revenue curves. They have an inverted u-shape, because of the shape of the APH-curve that shows a maximum where the coupon costs show a minimum, and also tax revenue shows a maximum since tax revenue should be roughly proportional to average income per head. We can conclude two things. First of all, if the tax rate goes up, a larger proportion of low-ability workers can be supplied with coupons. At this point, one should recall that the parameters we have chosen lead to a point of intersection of the APC with the 45^o line at a value of $\mu L = 0.857$. This means that in this low-ability equilibrium only about 14 percent of all people are in the high-ability range. The minimum low-ability composition of the population is equal to $\mu L = 0.3$ (cf. footnote 8). This means that we should try to find a point of intersection between the TT and TC curves for a value of $\mu L = 0.857$. Since the point of intersection between both curves moves to the right if the tax rate increases, we can conclude that the tax rate that can finance all the coupons necessary to get from $\mu L = 0.857$ to $\mu L = 0.300$ must be higher than 0.02. In fact, numerical solution of TT=TC results in a tax rate equal to 0.0345. This tax rate will ensure that the next generation of adults will consist of a 30-70 percent mix of low- and high-ability workers respectively. Even at this tax rate, high-ability workers continue to earn a disposable wage that allows above subsistence level consumption for their offspring. However, the next generation of high-ability workers may not be so lucky, since there are many more of them. At the same time we would expect that since the number of low-ability workers has decreased dramatically in the next generation, at least some of them might now earn above subsistence level wages. We can check this by evaluating the marginal subsistence level wage classes in the groups of high- and low-ability workers in the next generation at the new 30-70 percent ability distribution of the second generation. It turns out that this change in the ability composition of the population does indeed make some of the high-ability workers earn below subsistence wages. At a zero tax rate the fraction of high-ability workers earning such a wage is 0.0628, whereas the corresponding fraction of low-ability workers earning below subsistence wages is now 0.2155, having dropped considerably. Indeed, in the old equilibrium, all low-ability workers earned below subsistence wages, amounting to about 86% of the population, while in the new equilibrium only about 11%[8] of the people earn below subsistence wages. In addition to this, average income per head has risen, which ensures that the coupon burden for the current generation should be much less than it has been for the previous generation of adults. Indeed, it is even the case that

[8] $0.0628 \cdot 70 + 0.2115 \cdot 30 = 10.741$

the adults from the previous generation that have suffered utility losses can be completely compensated for their losses by transferring tax revenue to them now that they are old. This is the subject of the next section.

4.3 Compensation of the parent generation

All high-ability adults of the first generation are 'losers', and perhaps also part of the low-ability workers. The marginal 'loser' is characterized by the fact that his utility in the situation with coupons would be exactly the same as his utility would be in the situation without coupons (and without taxes!). From equations (1)-(5), we can rewrite utility for an individual in ability group A and class i earning a disposable wage $(1-\tau)\cdot w_{A,i}$ for the case where he would obtain coupons (indicated by uc) and the normal case (indicated by u) as a function of the wage rate as follows:

$$u_{A,i} = \alpha^{\alpha} \cdot \beta^{\beta} \cdot (1-\alpha-\beta)^{1-\alpha-\beta} w_{A,i} \tag{29}$$

$$uc_{A,i} = \alpha^{\alpha} \cdot (\bar{c_y}^{\beta}) \cdot (1-\alpha-\beta)^{1-\alpha-\beta} \left((1-\tau) w^{c}_{A,i}\right)^{1-\beta} \tag{30}$$

We can solve (29) and (30) for the marginal loser by equating them and noting that $w_{A,i}$ depends positively on i. Moreover (29) and (30) should be evaluated for $\mu = 0.857$, since we want to identify the losers in the first generation adults. Before doing that, however, it should be noted that the $w_{A,i}$'s themselves will be different in the two different situations because of the impact of the tax on wages, hence the addition of a superscript c to denote the situation with coupons. So $w^{c}_{A,i}$ and $w_{A,i}$ actually represent different values in both equations. However, given the parameterization we have chosen, we can numerically determine the marginal losers for both ability groups of the adults of generation 1. It turns out that the marginal loser in the low-ability group is in class 0.749, i.e. about 25 percent of the low-ability workers will actually lose some utility when adults. The marginal loser in the high-ability group is in class 0 as expected, since all high-ability workers earned above subsistence wages in the no-tax situation. However, adding gains and losses together for all losers and winners using equations (29) and (30) we find that there is a net utility gain from the redistribution of income alone equal to 1.99%, that is before the positive productivity effects of the shift in the ability distribution within the population are even realized. Nonetheless, the losers will have to be compensated by an increase in their consumption in the next period when they are old in such a way that their compensated utility matches exactly the level of utility they would have had without the tax. One should realize that the marginal losers do not get any coupons, hence the requirements that compensated utility and no-tax utility should be the same results in the following equation for the compensated consumption level of the old in the second generation:

$$\frac{u^n}{u^0} = \left(\frac{c_a^n}{c_a^0}\right)^{\alpha} \left(\frac{c_y^n}{c_y^0}\right)^{\beta} \left(\frac{c_0^n}{c_0^0}\right)^{1-\alpha-\beta} = 1 \qquad \Leftrightarrow$$

$$\frac{c_y^n}{c_y^0} = \left(\left(\frac{c_a^n}{c_a^0}\right)^{\alpha} \left(\frac{c_y^n}{c_y^0}\right)^{\beta}\right)^{\frac{-1}{1-\alpha-\beta}} \tag{31}$$

where the superscripts n and o refer to the new situation (with the tax) and the old situation (without the tax), and where we have dropped all other superscripts. Equation (31) describes how much the new level of consumption of the old should be relative to the no-tax situation if they are to be compensated completely for their losses while being adults in the previous generation. Without compensation, the level of consumption of the losers when old would have been given by $c_0 = (1-\tau)\cdot w_n \cdot (1-\alpha-\beta)$, and providing an increase of this level of z percent, would then enable us to calculate the required level of z from (31) and the definitions of old and new levels of consumption in function of old and new wages and tax rates, since we immediately find when substituting ((4)-(6) and $c_0^n = (1+z)\cdot(1-\tau)\cdot w_n\cdot(1-\alpha-\beta)$ into (31):

$$1+z = \left(\frac{w^n(1-\tau)}{w^0}\right)^{\frac{-1}{1-\alpha-\beta}} = \left(\frac{u^T}{u^0}\right)^{\frac{-1}{1-\alpha-\beta}} \tag{32}$$

where u^T refers to utility for first generation adults in the situation with a tax and without compensation. Given the value of z that can be obtained for all the losers in the first generation belonging to the relevant sub-classes of the low-ability and high-ability adults, we can calculate similarly as before by aggregating over all losers how much compensation is needed for a given tax rate. This compensation then has to be added to the cost of the coupons needed to guarantee subsistence level consumption for the young generation associated with the 21% below subsistence wage earning low-ability workers and the 6% below subsistence wage earning high-ability workers. The tax rate necessary to cover the coupon cost in this case is only 0.000277, firstly since there are far less coupons to finance in the first place, and secondly because income per head has risen. The tax rate including the compensation of the losses of the first generation of adults is only 0.000434. Because this tax rate is slightly higher than the tax rate needed to finance the coupons, the number of below subsistence wage earners will rise slightly, too. This then means that passing this slightly higher tax burden to the next generation by a similar compensation as for the first generation, and so on, will lead to still higher demand for coupons, tax burdens, compensations, coupons etc. This implies that somebody will have to foot the bill, and we have chosen for the second generation to do that, the more so because as a whole the second generation is a net winner already. In principle, however, one could no doubt calculate a more equitable distribution of gains and losses between the second and the third generation, but that is not the purpose of this paper as such. We only wanted to show that a welfare improvement is possible, and that turns out to be the case, because the first generation gains 1.99 % in utility even before compensation: the low-ability workers gain 5.59 % units whereas the high-ability workers lose 3.54 %, but there are many more low-ability workers in the beginning than there are high-ability workers. Total utility of the entire population amounted to 0.706 units before the tax, and 0.720 units after the tax (without compensation yet) for the first generation of adults. When including the compensation scheme, total utility for the second generation of adults amounts to 0.763 units which exceeds total utility after the redistribution of income for the first generation by about 6 percent. For generations t+2 and later, utility will be even higher, since by assumption they do not have to take over the tax burden from generation t+1. Since the first generation t is completely compensated by generation t+1, while generation t+1 experiences higher utility than generation t even after compensating generation t for its initial loss in utility, whereas generations t+2 and later experience even higher utility than generation t+1 because of a lower tax burden, we conclude that this consumption coupon scheme leads to a long-term improvement of utility for the population at no real costs at all.

5 Concluding Remarks

In this paper we have shown that malnutrition due to low innate abilities and resulting low wages can lead to a low-ability, low-income equilibrium in which low abilities are passed on from parents to children via insufficient food intake. Redistributing income from "rich" individuals to "poor", as well as from the parent to the young generation, will lead to an increase in the quality of labor such that the policy can be welfare improving if the initial tax-paying generation is reimbursed for their utility loss during old age. Due to the static nature of the model, i.e. no technological progress, schooling or other sources of economic growth that could lift *all* wages above subsistence level, tax policy will have to persist in all future periods in order to ensure that the economy does not slip back into the poverty trap it emerged from.

It is worth noting that the increase in utility is possible although the tax policy will decrease savings. This increase in utility is due to the fact that investment in a higher share of people with relatively high abilities in the population may provide a higher rate of return than investment in physical capital. Obviously, this model served the purpose of highlighting the existence of a situation like this. The non-algebraic nature of the ability propagation curve in particular makes it impossible to analytically derive the parameter ranges for which this situation does occur. Therefor it is not possible to assess at this stage whether or not the parameters we have chosen come from a range of values that is 'reasonable' a priori. An extension of the model to find such ranges in a numerical way thus seems to be called for. Given the role of savings in our model, a further interesting extension would be to introduce economic growth into the model, in order to investigate in how far capital-driven growth and the proposed redistributive policy are compatible.

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