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Random walks and cointegration relationships in international parity conditions between Germany and USA for the Bretton-Woods period.[1]

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Abstract

Juselius (1995), MacDonald (2000), Juselius and MacDonald (2000) provided an explanation to some basic issues in international monetary economics concerning the validity of parity conditions. This paper instead restricts the analysis to the years between 1957 and 1969 and the countries under scrutiny are Germany and USA. Results can be easily compared with the Post Bretton Woods analysis by Juselius and MacDonald (2000). The main result of this paper is that important cointegration relationships found for the Post Bretton-Woods by Juselius and MacDonald (2000) essentially hold for the Bretton-Woods period as well, albeit the two periods were characterized by distinct exchange rate regimes and a different regulation in capital markets.

Keywords: *ppp*, *uip*, Fisher parity, Bretton-Woods regime.
JEL codes: E31, E43, F31, F32.

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[1] This paper benefited from in-depth comments and helpful advises by Katarina Juselius and Annetta Binotti. Responsibility for any mistakes and misapprehensions rests solely upon me.

1 Introduction

In some recent papers, Juselius (1995), Juselius and MacDonald (2000), Bevilacqua and Daraio (2001) and Bevilacqua (2006) an explanation to some basic issues in international monetary economics concerning the validity of parity conditions, namely the Purchasing Power Parity (*ppp*) and Uncovered Interest rate Parity (*uip*), has been provided . The idea behind these papers is that *ppp* and *uip*, just because of their inherent seemingly puzzling non stationary nature, may produce cointegration relations. In other words, once the stationarity hypothesis about *ppp* and *uip* is relaxed and both the *ppp* and *uip* are treated as non stationary variables, a long run relation between *ppp* and *uip* is found stationary. Thus, an important long run interaction between the goods, via the *ppp*, and the capital markets, via the *uip*, is evinced.

So far, the empirical analysis has focused on few countries such as Germany, USA and Japan and refers to the recent float period after the end of the Bretton-Woods regime[1]. The choice of the sample period, the Post Bretton-Woods period, was there justified to avoid mixing different regimes where economic relations might be found valid and meaningful in one period and unreasonable for another. As Lucas put forward in his *critique* (Lucas 1976), aggregate relationships may in fact change when the structure of the economy or the nature of policy changes. Shifts in policy can change economic relationships as long as changes in policy affect expectations and expectations affect economic relationships.

This paper restricts the analysis to the years between 1957 and 1969, which is before the Bretton-Woods collapse and some years after the rounding off of the WWII post-war recovery and the end of the Korean war. This period was characterized by the successful operation of a pegged exchange rate regime, although the system was slowly but steadily going into crisis already in the last years of 1960s when the US involvement in the Vietnam war became markedly priced for the deploying of a massive military force in 1968, which created unsustainable imbalances in payments culminated in August 1971 with the resolution by USA to suspend the convertibility of the US dollar in gold on which the Bretton-Woods system was based.

[1]Krugman and Obstfeld (1994) define the Bretton Woods regime as 'the system set up by the Bretton Woods agreement" in 1944 "called for fixed exchange rates against the U.S. dollar and an unvarying dollar price of gold ... . Member countries held their official international reserves largely in the form of gold or dollar assets and had the right to sell dollars to the Federal Reserve for gold at the official price. The system was thus a gold exchange standard with the dollar as its reserve currency'.

Albeit the fixed exchange rate regime was the prevailing monetary operating system of the period and in general of the past, it is relevant to talk over a fixed exchange system not only for historical reasons. Even the current world of floating exchange rates is known to be indeed at best 'dirty' or 'managed'. Not rarely, central banks of whatever country intervene to avert the complete adjustment of the exchange rate towards its equilibrium value. Central banks often practise foreign exchange transactions when they feel dissatisfied with the changes in the exchange rates. Many developing countries, such as China, still maintain fixed parities with one currency, while other central banks operate so often in the foreign exchange market that, although they formally operate in a flexible exchange rate regime, they manage the float quite aggressively anyway. Studying the experience from the past when fixed rates were the rule can be rewarding to analyze the present and future prospects for some countries to partake flexible exchange rates and others to join a monetary union.

This paper is organized as follows. In Section 2 international parity conditions are defined. In Section 3, the choice of the variables and the data set is discussed. Some preliminary visual analysis of the variables and the parity conditions is provided. Section 4 explains the statistical model we use to test parities. In Section 5 parity conditions are tested using a model with a minimal number of variables with production price indices. Section 6 tests parity conditions with consumer price indices. In Section 7 the model includes both the production and consumer price indices. Using the moving average (MA) representation, the weakly exogenous variables and the long run impacts of shocks are also discussed in Sections 5, 6 and 7. Section 8 concludes and summarizes the main results.

2 International parities conditions

2.1 The absolute *ppp*

As for the *ppp* theory, while in freely floating exchange rate regime the market tends to maintain the real exchange rate value of currencies so that changes of the nominal exchange rate compensate inflation differentials between countries, in a fixed exchange rate regime if the domestic inflation rate is greater (lower) that of the other countries, the real exchange rate relative appreciates (depreciates) and, in order to keep the parity condition, countries need to have both the same prices and inflation rates.

While the parity condition excludes the possibility of real changes in exchange rates, as a matter of facts, when domestic currency appreciates (depreciates) in real terms, spending outside the country is furthered (discouraged) while foreign spending in the nation are discouraged (furthered). In general, in a fixed exchange rate regime, spending in the higher inflation nations is discouraged while is furthered in the countries with lower inflation. This comes down to say that changes in real exchange rate induce changes in the trade balance and the current accounts of the trading partners and while in a freely floating regime changes in prices may be offset by changes in the exchange rates, in a fixed exchange rate regime inflation spreads have a direct impact on the balance of payments. In a fixed exchange rate regime, countries will feel more compelled to keep the inflation rate equal or below to that their most important trading partners, otherwise they would experience a continual drain on foreign exchange reserves. Thus, inflation stability by means of monetary and fiscal policy may become a necessary target to keep in balance the payments accounts when exchange rates are fixed. Fixed exchange rates may induce monetary and fiscal discipline to keep up in close check inflationary pressures.

Formally, the absolute *ppp* is defined as $p_t - p_t^* - s_t = 0$, with p_t the log of domestic prices (in this case the German producer price level index), p_t^* the log of the foreign price level (the producer US price level index) and s_t the log of the spot exchange rate (home currency price of a unit of foreign currency).

The *ppp* states that, once converted to a common currency, the price levels in the two countries equalize. If the *ppp* holds empirically we expect that $p_t - p_t^* - s_t \sim I(0)$, where $I(0)$ stands for an integrated process of order zero (that is a stationary process). If exchange rates are fixed, that is $\Delta s_t = 0$, the *ppp* condition becomes $p_t - p_t^* \sim I(0)$ in the case the *ppp* condition holds, which may happen either in the case prices were stationary or non stationary but cointegrating. As the Bretton-Woods system was also planned to be mild inflationary to overcome the deflation periods typical of the Gold Standard, prices should not be stationary and the parity condition could only hold in the case of cointegrating prices.

2.2 The *uip*

The condition of *uip*, is defined as $E_t\Delta_t s_{t+l} - i_t^l + i_t^{l*} = 0$ with i_t^l a long term bond yield with maturity $t+l$ and E_t the conditional expectations operator on the basis of time-t information set. The *uip* states that, in the capital market, the interest rate differential between the two countries is equal to the expected change in the spot exchange rates.

Hence, once converted to a common currency, the interest rates in the two countries should be equal otherwise investors would have the incentive to move capitals from the country where the interest rate is lower to the country where the interest rate is higher. In short, the *uip* is an arbitrage relation that describes an equilibrium in the capital markets. According to the *uip* theory, countries that have a high interest rates should have depreciating currencies, otherwise a simple trading rule could allow to earn profits most of the time.

If the *uip* holds empirically we expect that $E_t\Delta_l s_{t+l} - i_t^l + i_t^{l*} \sim I(0)$, otherwise $E_t\Delta_l s_{t+l} - i_t^l + i_t^{l*} \sim I(1)$. Since during the Bretton-Woods regime the exchange rates were fixed against the US dollar, the expected change of the exchange rates could not be anything but equal to zero $E_t\Delta_l s_{t+l} = 0$[2]. Hence, the *uip* would reduce to $-i_t^l + i_t^{l*} = 0$. If the *uip* holds empirically, we would expect that $-i_t^l + i_t^{l*} \sim I(0)$, otherwise $-i_t^l + i_t^{l*} \sim I(1)$.

Empirical studies have shown that the *uip* theory is not confirmed by the data and a trading rule could be even devised to earn profits in the past. The recent experience of floating exchange rates has shown that most of industrialized countries with high interest rates usually had appreciating rather than depreciating currencies and the expectations implied by the *uip* were systematically miscalculated (Colombo and Lossani 2000). Juselius (1995) and Juselius and MacDonald (2000) maintain that empirical tests by other authors (Cumby and Obstfeld 1981) have confirmed that the *uip*, similarly to *ppp*, is a non stationary relation such as $E_t\Delta_l s_{t+l} - i_t^l + i_t^{l*} \sim I(1)$.

As during the Bretton-Woods regime exchange rates were pegged, the *uip* would have hold only if nominal interest rates were cointegrated. However, during the Bretton-Woods period

[2] There were changes in the exchange rates but all of very modest magnitude if compared to the recent floating experience. However the Bretton Woods regime was not impeccable in assuring that the expected change of exchange rate was zero. Indeed one criticism was that the Bretton Woods system spurred speculations. The experience of the German mark in 1969 is very informative in this respect: speculative movements of Dollars changed in Marks swarmed during 1969 and the German monetary authority was forced not to intervene anymore in the fall of 1969 leaving the exchange rate free to float letting the bubble to burst untill the market cleared.

a spread among interest rates, partly due to controls on capital accounts, was evident. While restrictions on payments for current transactions the payments were gradually removed promoting economic growth, capital markets, which were presumed a source of potential disturbance to exchange rate stability subverting a condition for economic growth, were heavily regulated by means of interest rate ceilings, credit controls and entry restrictions. Thus, investors were not free to move capitals from the country where the interest rate was lower to the country where the interest rate was higher and we have no reason to realistically expect that the *uip* would hold.

2.3 Combining the *ppp* with the *uip*

In this paper further evidence that *ppp* and *uip* as such do not find empirical support will be presented, however the aim here is to check whether a linear combination of the two parities are able to generate a stationary relation as they were found in Juselius (1995), Juselius and MacDonald (2000 and 2003) for the Post Bretton-Woods period.

The starting point, is that both *ppp* and *uip* are non stationary.

Given the definition of *ppp*, taking the first differences and the expected change of exchange rate[3] $E_t\Delta_l p_{t+l} - E_t\Delta_l p^*_{t+l} - E_t\Delta_l s_{t+l} \sim I(1)$. Since for the Bretton-Woods period $E_t\Delta_l s_{t+l} = 0$, $E_t\Delta_l p_{t+l} - E_t\Delta_l p^*_{t+l} \sim I(1)$. If $-i^l_t + i^{l*}_t \sim I(1)$, $(E_t\Delta_l p_{t+l} - E_t\Delta_l p^*_{t+l}) - (i^l_t - i^{l*}_t) \sim I(0)$ or $I(1)$ in the case that expected real interest rates cointegrate or else.

If inflation rate is non stationary, rational expectations in prices may be simply modeled by $(\Delta p_t - \Delta p^*_t) = E_t(\Delta_l p_{t+l} - \Delta_l p^*_{t+l}) + v_t$ and $(\Delta p_t - \Delta p^*_t) - v_t - (i^l_t - i^{l*}_t) \sim I(0)$ or $I(1)$ with v_t unpredictable *i.i.d.* shock. Since by definition $v_t \sim I(0)$, $(\Delta p_t - \Delta p^*_t) - (i^l_t - i^{l*}_t)$, thus real interest rates, $\sim I(0)$ or $I(1)$. Testing this last relation is equivalent to test whether or not the *uip* is stationary. Testing $(\Delta p_t - \Delta p^*_t) - (i^l_t - i^{l*}_t) + \omega ppp \sim I(0)$ is equivalent to test whether or not the *uip* and *ppp* produce a long run relationship. The parameter ω might be interpreted as the responsiveness of the capital movements that enter in the capital and financial account to *uip*. A small value of the parameter ω may imply a large responsiveness of capital movements to the net interest rate differential. Due to the restrictions and the heavy regulation for capital movements typical of the Bretton-Woods period, we expect a slower responsiveness of capital

[3]Note that the finite difference and expectations are mathematically linear operators.

movements to the net interest rate differential, then a higher value of the parameter ω for the Bretton-Woods period than the recent floating exchange rate experience.

3 Choice of the variables, data set and a visual analysis

3.1 Choice of the variables and data set

The variables that enter in equation the last relation are, the home price index p_t, the foreign price index p_t^*, the home interest rate i_t, the foreign interest rate i^*, the spot exchange rate s.

The focus of this paper is on two countries, Germany and USA, and a subset of the Bretton-Woods period (1957-1969).

The choice of the countries and the sample period may be so justified:

- Macroeconomic relations may change when the structure of the economy or policy regimes change as pointed out in the *Lucas critique*. Therefore it may be worth to divide the sample in regime periods, and conduct one specific analysis for the post war era and another for the Post Bretton-Woods period. It should help to prevent parameter instability which may be caused by structural changes.

- The analysis is limited to the period January 1957- August 1969, which is before the Bretton-Woods collapse and some years after the rounding off of the WWII post-war recovery and the end of the Korean war in 1953. The period was characterized by a rather successful operation of a pegged exchange rate regime, although the system was slowly but steadily going into crisis already in the last years of 1960s when the US involvement in the Vietnam war became markedly priced for the deploying of a massive military force in 1968, which created unsustainable imbalances in payments culminated in August 1971 with the resolution by USA to suspend the convertibility of the US dollar in gold on which the Bretton-Woods system was based. The ending date in August 1969 was also chosen because the Germany authority in Sept. 1969 decided the German mark to freely float in response to speculative attacks in the currency market and too many dummies would have otherwise necessary to take into account of all these historical facts.

- The two countries, Germany and USA, are certainly two 'big' countries if considered during the last thirty years. In the last 25-30 years, a change in policy in one of the two countries would have probably affected the other country. However in the immediate post war and the following period 1957-1969 the financial hegemony of USA in Europe was clear and manifest in the renounce of war compensations and the establishment of the European Recovery Program, the Marshall plan for the reconstruction of Europe. The amount of American investments in Europe was in the order of tens of billions of dollars. Production in Western Europe was so successfully recovered in the beginning of 1950s that those goods that were once made in USA started to be made in Europe by American companies and imported in USA (Kenen 1967). In this historical context of dependency of Germany and Western Europe on USA, shocks occurred in the US economy were likely propagated to Europe rather than the other way round. In other words, statistical analysis of data should show Germany and USA to be respectively a 'small' and a 'big' country.

This analysis takes also up other issues concerning the categories of price indices and interest rates that could be used. Often it is opted for the CPI but other price indices can also be chosen such as for instance unit labor costs etc.; in this paper only PPI are chosen.

There is not such a right *ppp* measure and there is not a right measure for the *uip* either. Which interest rate should be picked out? Often the long interest rate is chosen, in this paper both are considered.

In the analysis that follows Germany and USA will be considered respectively 'home' and 'foreign' countries. An asterisk will denote the 'foreign' country. The database consists of the producer price indices p_t and p_t^*, long bond yield (10 years) i_t^l and $i_t^{l^*}$, the Frankfurt interbank offered rate[4] i_t^s, the three month US Treasury bill rate $i_t^{s^*}$ and the spot exchange rate USdollar/Deutschemark s_t. The Frankfurt interbank offered rate i_t^s was chosen as we could not have the three month Treasury bill rate for Germany. The particular type of data used for the short term interest rate for Germany show a recurrent anomalous cyclicality between September and November of every year that we cannot explain. Using seasonal and other dummies in the model we hope to have partially cleaned a bit the data but great caution should be paid in

[4] It is an interbank interest rate. More precisely, it refers to an interest rate, determined at the Frankfurt Banking Centre, at which banks may invest Deutschmark deposits with other banks for a period of 3 to 6 months, in the form of fixed or time deposits.

interpreting the results of the analysis when we include the short term interest rates. As short term interest rate can be steered by the central bank, the German short term interest rate we used should reflect the respective discount rate, as it seems to do except for the above-mentioned months.

This database was extracted from Datastream but data on prices with four decimals were provided by Prof. Juselius. Data are monthly, not seasonally adjusted.[5] The starting date of our sample is January 1957 for p_t, p_t^*, i_t^l, $i_t^{l^*}$, s_t, and December 1959 for i_t^s[6] and $i_t^{s^*}$. Prices and exchange rate are taken in natural logs, the yearly interest rates in percentage and divided by 12 to obtain monthly rates.[7]

3.2 Visualizing data

The visual inspection is a preliminary first step aimed to grasp a rough idea of the dynamic properties of time series but only statistical testing will provide more conclusive results. The graphs of the time series of all the variables relevant for this paper are shown both in levels and differences.

Prices and Inflation rates

The producer price indices for Germany and USA (see Fig. 1, LGEPPI time series, call LGEPPI the log of the producer price index in Germany; in Fig. 2 see LUSPPI where LUSPPI is the log of the US producer price index) show moderate inflation rates with periods of slight price decline and an upswing early in 1968 in USA. This rise of inflation was probably due to the rapid rise in real incomes and the approaching to full employment spurred by the military involvement in Vietnam. The joint effect of fixed exchange rates and relatively high US inflation rates making US goods less competitive pulling down a hitherto sound current account.

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[5] In this paper the effects from seasonality are removed using centered seasonal dummies that sum to zero over each year (see Johansen 1995 p. 84 for further details). Motivations for using not seasonally adjusted time series in cointegration analysis are found in Johansen 1995.

[6] In Germany, only money market rates (three-month funds) reported by Frankfurt banks are available from December 1959. The data are calculated from bid and ask rates reported by the major market participants at Frankfurt Bankplatz.

[7] Note that the exchange rate itself is a price of a currency with respect another at a given time. Not so the interest rate which is indeed a variation over time, in this respect more similar to the inflation rate.

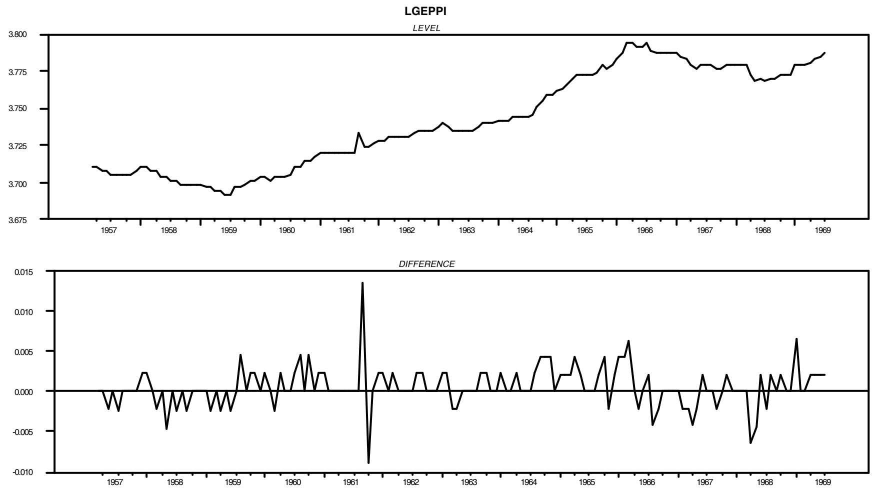

Fig. 1: The log PPI index in Germany.

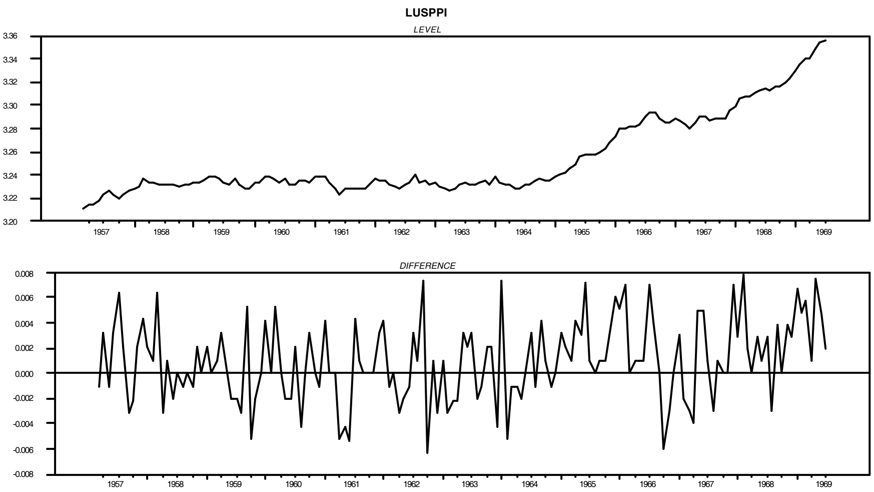

Fig. 2: The log PPI index in USA.

Just viewing at these graphs it is very hard to guess whether or not inflation rates could hide some $I(1)$ structure. The only reliable way to discern it is testing. Prices, which were found $I(2)$ for the recent float period, might contain a significant $I(2)$ component that should be statistically taken into account using a truly $I(2)$ procedure, or alternatively inflation rates should be analyzed in a $I(1)$ framework. However, it will be shown that for the Bretton-Woods period prices could actually be possibly modeled within a $I(1)$ framework. In this paper the

$I(0)$ inflation rates will be analyzed within a $I(1)$ framework and results will be compared with the results for the Post Bretton-Woods period.

Fig. 3 (call GEUSPPR the spread of the log of the producer price index between Germany and USA) shows relations $(p_t - p_t^*)$ (upper panel) and $(\Delta p_t - \Delta p_t^*)$ (lower panel), that is inflation rates in the two countries were quite different with periods in which nominal prices in either the two countries became higher or lower for extended periods. As the period was characterized by pegged exchange rates, Fig. 3 is indicative of the absolute and relative *ppp* except when an official change in the exchange rate occurred.

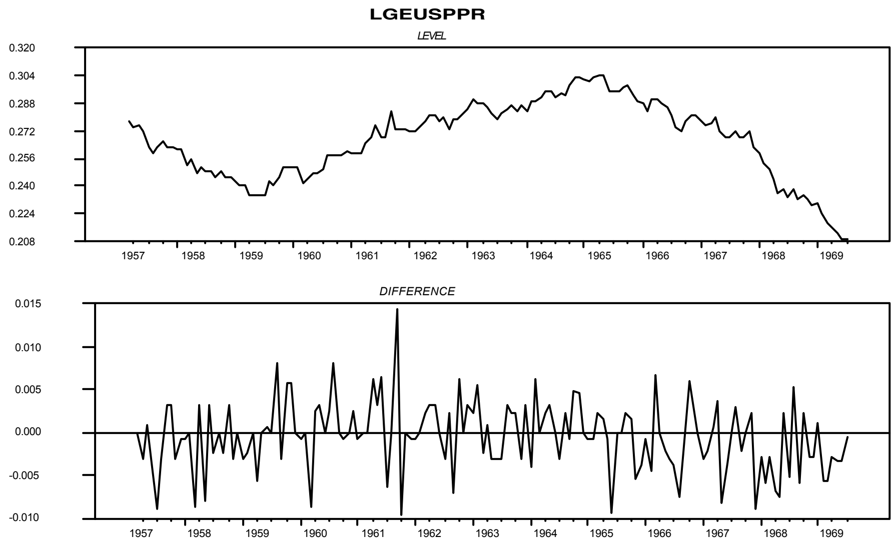

Fig. 3: The producer price spread between Germany and US.

Exchange rates and *ppp*

The Bretton-Woods regime, the system of adjustable peg, was established to assure maximal stability for the exchange rates, to facilitate fluctuations only when they were necessary and to eschew the competitive devaluations that characterized the 30s. During the Bretton-Woods period there were important variations in exchange rates[8]. During the Bretton-Woods period, Governments were free to vary the exchange rate versus the US Dollar up to 1% without the approval of the IMF. Larger fluctuations were thus possible but needed to be approved by

[8]For instance England devaluated the pound versus the US dollar from 4.03 to 2.8 in 1949 and to 2.4 in 1967.

the IMF depending on the country had to face fundamental imbalances in the international accounts such as currents accounts. As a matter of fact Governments considered exchange rates devaluations as a remedy for extreme cases, variations instead of being a routine, were considered very important and as the admission of guilt. Governments preferred to renounce to the economic development to keep the equilibrium the current accounts, through a strict control in internal demand by means of restrictive fiscal policies. This was a typical criticism moved to the American policy of that time, although it was able to keep in control inflation rate.

In conformity with the IMF 'International Financial Statistics', the DEM/USD rate was set at 4.2 between 1949 and February 1961. Between March 1961 and October 1969 it was set at 4.0 DEM/USD. Until the German Mark was allowed to float in May 1971, the rate was set at 3.66 DEM/USD[9]. Even if looking at graph there seem to be a long term trend (see Fig. 4 LDMUSD time series, we call LDMUSD the log of exchange rate of the German Mark against the US Dollar), the variation in exchange rates was generally really small especially with respect to the recent floating period. The evaluation of the German Mark vs. the US dollar occurred in 1961 was just about 5% of the value. From Fig. 4 it is however difficult to understand the degree of integration of exchange rates. In theory, they should be $I(0)$ because in principle they were fixed once the evaluation in 1961 is taken into account by means of a shift dummy.

[9]The evaluation of the German mark in Sept. 1969 was due to a speculation that had spurred an inflow in Germany of many billions of Dollars changed in marks, as a future evaluation of the mark versus the US dollar was expected. The German authority decided in Sept. 1969 not to intervene leaving the exchange rate free to float for one month up the market cleared to the level of 3.66 marks per dollar.

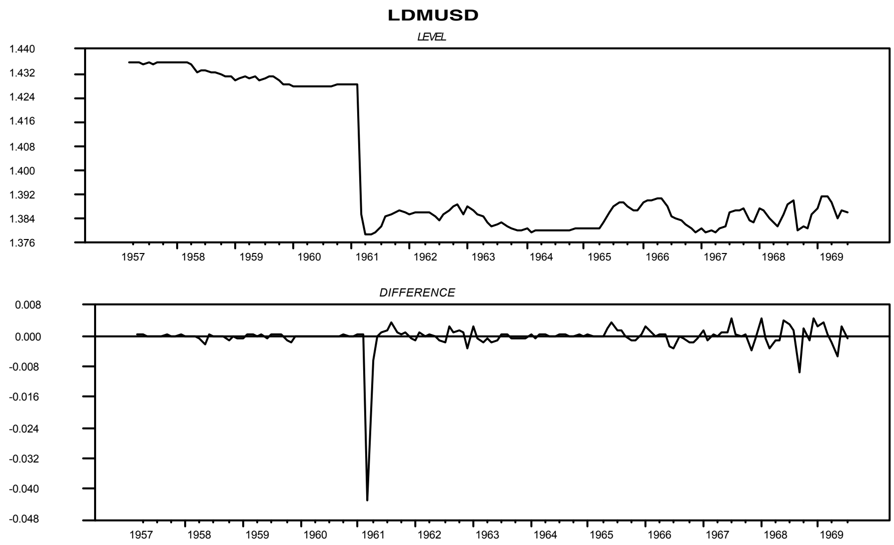

Fig. 4: The log of exchange rate.

Fig. 5 (call PPPP the *ppp* calculated with the producer prices) shows that *ppp* is non stationary and mimics the price spread shown in Fig. 3 and the shift in the exchange rate in Fig. 4. To test a relation such as $(\Delta p_t - \Delta p_t^*) - (i_t^l - i_t^{l*}) + \omega ppp \sim I(0)$ seems necessary to include a shift dummy for the evaluation of the German Mark vs. the US Dollar.

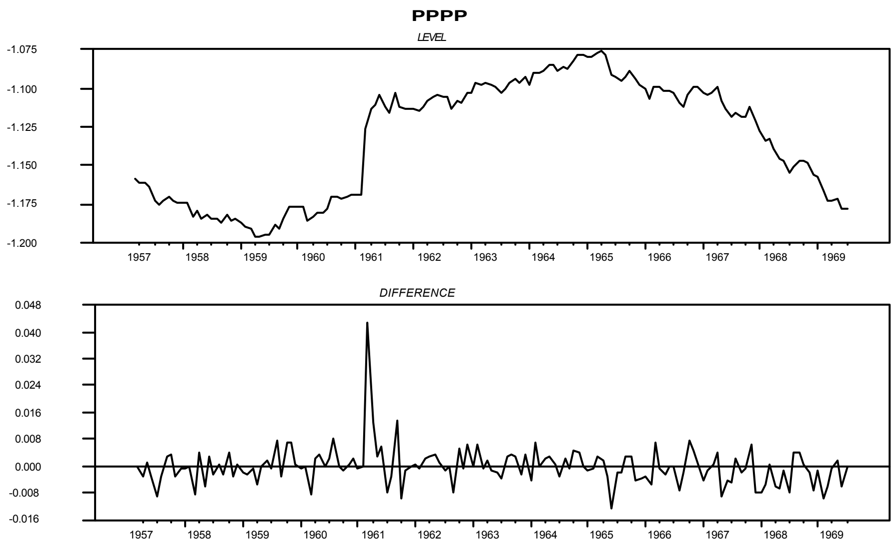

Fig. 5: Purchasing Power Parity (based on producer prices).

Interest rates and spread

Consider first the spread of interest rates (see Fig 6, we call BONDSP the spread of the long term interest rates in the two countries). The spread looks non stationary $I(1)$ so interest rates should be $I(1)$ without cointegrating. It can be noticed that in the last years of 1960s the spread has become smaller as the nominal interest rate in US was becoming higher and higher. That period in USA was characterized by a relatively high inflation which worsened the US current account. The major factor positively contributing to the balance of payments became rather unexpectedly the financial and capital account. Net capital flows became suddenly positive as an effect of the implementation of the Foreign Credit Restraint Program in the beginning of 1968 and the high interest rates serviceable to finance the Vietnam war (Fig. 8 and 10) compared to European countries such as Germany (Fig. 7 and 9). Fig. 9 should warn to use with great caution the Frankfurt Interbank Offered rate as a proxy for a short term interest rate for Germany. The data used for the short term interest rate for Germany shows a recurrent anomalous cyclicality between September and November that we cannot explain. Using seasonal and other dummies in the model we hope to have partially cleaned a bit the data but great caution should be paid in interpreting the results of the analysis when we include the German short term interest rates. As short term interest rate can be steered by the central bank, the German short term interest rate we used should in theory reflect the respective discount rate (Fig. 11), as it seems to do to some extent (there are similar peaks in 1960, 1966 and the upswing 1969) but they only do it very roughly and except for the above-mentioned months.

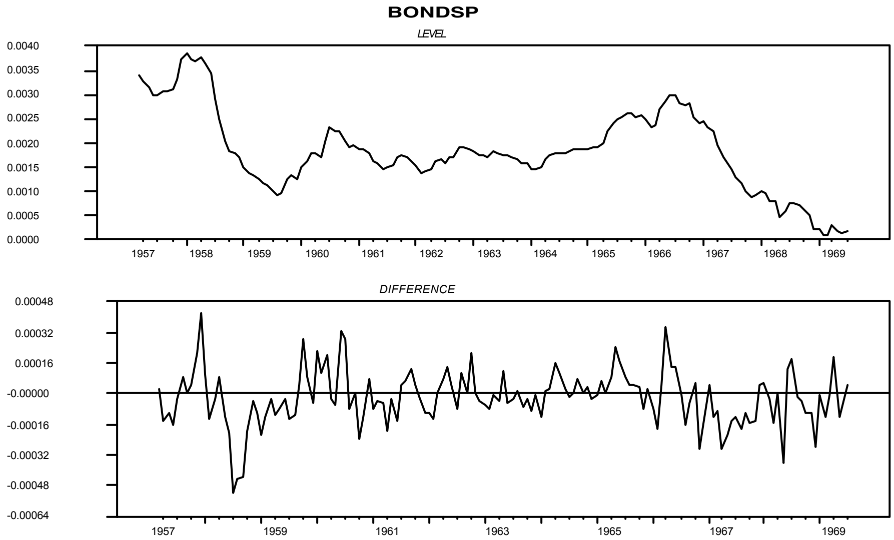

Fig. 6: The bond rate spread.

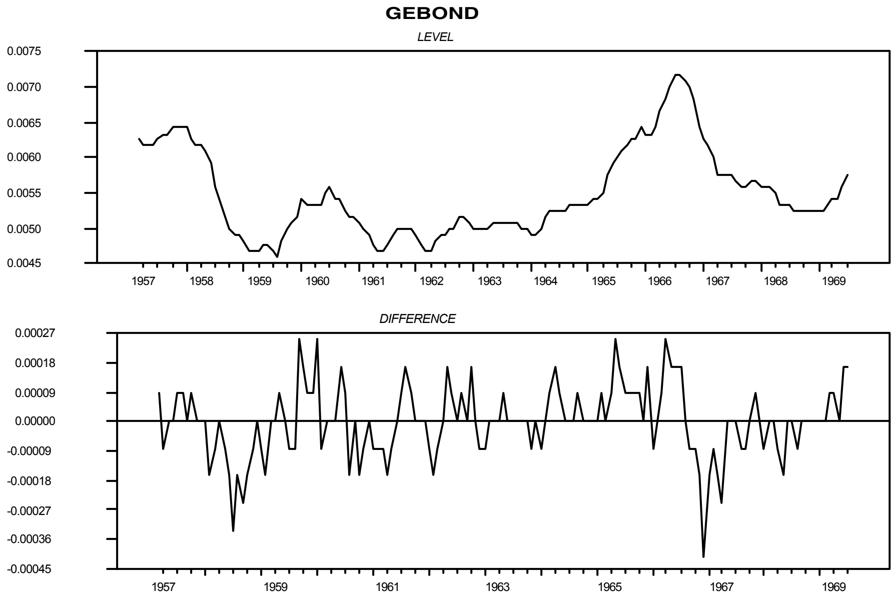

Fig. 7: The long term interest rate in Germany.

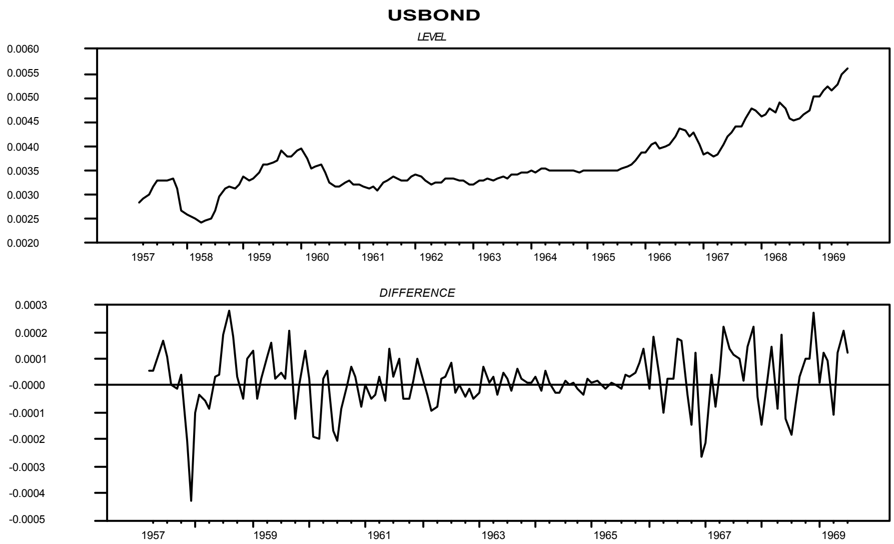

Fig. 8: The long term interest rate in the US.

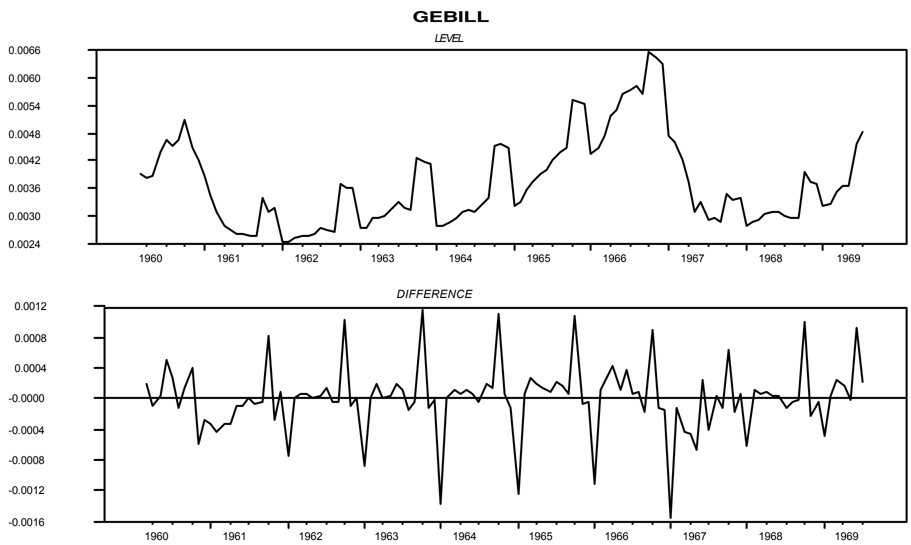

Fig. 9: The short term interest rate in Germany.

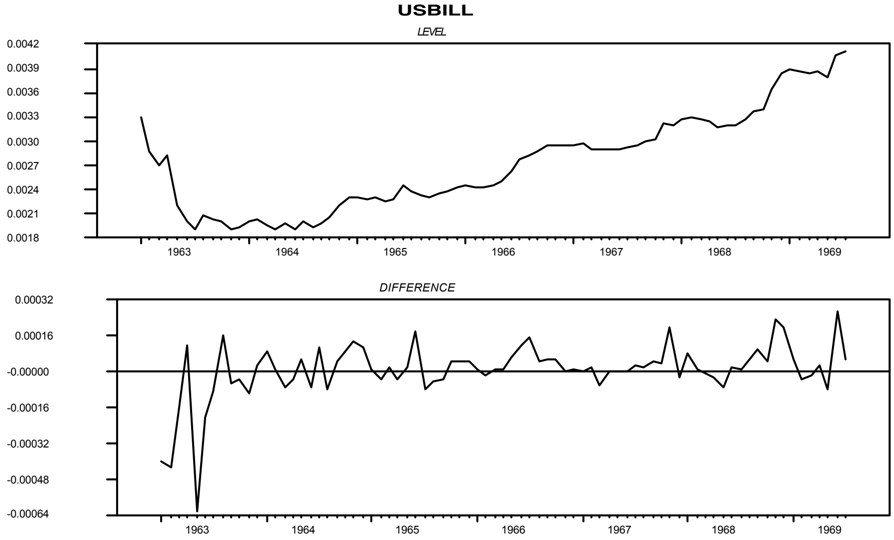

Fig. 10: The short term interest rate in the US.

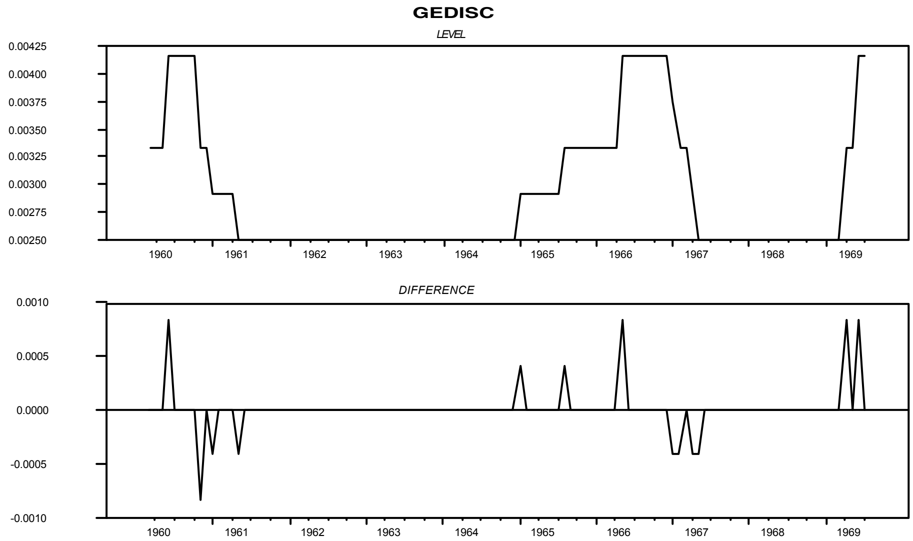

Fig. 11: The discount rate in Germany.

Summarizing, from a simple visual inspection of the data, it appears that:

- $p_t \sim I(2)$ or $I(1)$, $p_t^* \sim I(2)$ or $I(1)$, $(p_t - p_t^*) \sim I(2)$, $I(1)$ or $I(0)$ where p_t are producer price indices.
- $\Delta p_t \sim I(1)$ or $I(0)$, $\Delta p_t^* \sim I(1)$ or $I(0)$.

- $i^l \sim I(1)$, $i^{l^*} \sim I(1)$, $(i^l - i^{l^*}) \sim I(1)$, and $i^s \sim I(1)$, $i^{s^*} \sim I(1)$.

We anticipate that testing will show that prices are $I(1)$ and testing a relation such as $(\Delta p_t - \Delta p_t^*) - (i_t^l - i_t^{l*}) + \omega ppp \sim I(0)$ can be either done considering a system formed either by $(p_t, p_t^*, i_t^l, i_t^{l*}, s)$ or $(\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, ppp)$. In both cases the economic variables that enter are at most $I(1)$ and testing with the $I(1)$ procedure, the so called 'Johansen procedure', is in both cases appropriate[10]. We preferred to analyze the system with the vector of variables $\left(\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, ppp\right)$ to easily compare the results with a work of mine (Bevilacqua 2006) where the same set of variables were chosen. However a system with a vector of variables $(p_t, p_t^*, i_t^l, i_t^{l*}, s)$ could also be chosen.

4 The $I(1)$ model

The $I(1)$ model can be formulated in two equivalent forms: the vector autoregressive model VAR and the vector moving average representation VMA. While the VAR model enables us to single out the long run relations in the data, the VMA representation is useful for the analysis of the common trends that have generated the data (Juselius 1995).

4.1 The VAR representation and the long run relations

The VAR model formulated in the error correction form is $\Delta x_t = \Gamma_1 \Delta x_{t-1} + ... + \Gamma_{k-1} \Delta x_{t-k+1} + \Pi x_{t-1} + \alpha\beta_0 + \alpha\beta_1 DS_t + \gamma_1 DS + \Psi_0 Dp_t + \Psi_1 Dtr_t + \Psi_2 Dq_t + \varepsilon_t$ with $\varepsilon_t \sim N_p(0, \Sigma)$, $t = 1, ..., T$

where $p = 5$ (or 7 for the extended model that includes short run interest rates) is the dimension of the VAR model, $x_t' = \left[\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, ppp_t\right]$ (or $x_t' = \left[\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, i_t^s, i_t^{s*}, ppp_t\right]$), $x_t' \sim I(1)$, k is the lag length ($k = 3$ in our case), DS_t is a vector of mean shift dummy variables which accounts for a mean in Δx_t and cumulates to a broken trend in x_t serving to capture regime shifts, Dp_t a vector of deterministic components with permanent effect such as intervention dummies, Dtr_t a vector of transitory shock dummy variables, Dq_t centered seasonal dummies which sum to zero in samples comprising complete years, Γ_1,..., Γ_{k-1}, Ψ matrices of freely varying parameters and $\Pi = \alpha\beta'$ where α and β are $p \times r$ matrices of full rank, r is the

[10]The $I(1)$ procedure can be applied only to the variables that are 'at most' $I(1)$. This means that not all the individual variables x_t have to be $I(1)$. They can be also $I(0)$, but not more than $I(1)$.

rank of the Π matrix, and $\beta' x_t$ is stationary, i.e. the stationary relations among non stationary variables. β_0 and γ are parameters. The constant is restricted to lie in the cointegration space and the shift dummy was decomposed in two new vectors to allow one of them to lie in the cointegration space. This model does not allow for linear trends in the data and in the cointegration relations as no reason for their existence was suggested by economic theory[11], but then again it takes into account of transitory shocks, permanent interventions and regime shifts grounded on historical facts. The rank of the Π matrix is fundamental since it is equal to the number of stationary relations between the levels of the variables, i.e. the number of long run steady states towards which the process starts adjusting when it has been pushed away from the equilibrium (Hansen and Juselius 2000).

4.2 The VMA representation

The VMA representation is used to analyze the common trends that have generated the data, i.e. the pushing forces from equilibrium that create the non stationary property in the data.

The VMA representation is $x_t = C \sum_{i=1}^{t-1} \varepsilon_i + C \sum_{i=1}^{t-1} (\alpha\delta_0 + \delta_1) DS_i + C \sum_{i=1}^{t-1} \Psi_0 Dp_i + C \sum_{i=1}^{t-1} \Psi_1 Dtr_i + +C^* (L) (\varepsilon_t + (\alpha\delta_0 + \delta_1) DS_t + \Psi_0 Dp_t + \Psi_1 Dtr_t + \alpha\beta_0 + \gamma_0) + X_0$

where $C = \beta_\perp \left(\alpha'_\perp \left(I - \sum_{1}^{k-1} \Gamma_i \right) \beta_\perp \right)^{-1} \alpha'_\perp$, $\alpha_\perp$ and $\beta_\perp$ are $(p-r) \times (p-r)$ matrices orthogonal to α and β, C matrix is of reduced rank of order $(p-r)$ and X_0 the initial values. $C^* (L)$ is an infinite polynomial in the lag operator L. The component $C \sum_{i=1}^{t-1} \varepsilon_i$ represents stochastic trends of the process, the $C \sum_{i=1}^{t-1} (\alpha\delta_0 + \delta_1) DS_i$ captures a broken trend in x_t while $C \sum_{i=1}^{t-1} \Psi_0 Dp_i$ and $C \sum_{i=1}^{t-1} \Psi_1 Dtr_i$ are a shift in the level of x_t and a temporary change in x_t respectively. The C matrix is also of great importance as, although the number of common trends can be guessed sometimes by means of economic considerations, the rank of the C matrix may be informative about the stochastic trends that are in the process. The rank of the C matrix is equal to the number of stochastic trends that push economic variables away from steady states. The VMA representation is of valuable help since it shows how common trends affect all the variables in the system.

[11]Conversely if the system with the vector $(p_t, p_t^*, i_t^l, i_t^{l*}, s)$ were chosen, a linear trend in the data would be necessary to take into account the upward drifting in prices.

4.3 'General to specific' and 'specific to general' approach

We adopt a 'general to specific' principle in statistical modelling and a 'specific to general' approach in the choice of variables. By imposing restrictions on the VAR such as reduced rank restrictions, zero parameter restrictions and other parameter restrictions, the idea is to arrive to a parsimonious model with economically interpretable coefficients (Juselius and MacDonald 2000).

The vector x_t is composed by five variables but it may be extended to seven if interest rates of different maturity are also considered. It had rather better to begin to analyze small models since for each added variable we have $(2p+1)*k$ new parameters in the system. Of course when the sample is small (less than 100 observations for instance, like quarterly macroeconomic models) it is often impossible to estimate the model because the number of parameters to estimate is greater than the number of observations. As we have only 148 observations and we try to estimate directly also system with seven variables and few lags necessary to remove significant autocorrelations in the residuals. However it may be not advantageous estimate it directly. Reducing at minimum the number of variables often helps in identifying the cointegration relations and cointegration relations remain valid in a more extended model. This property is called 'invariance' of cointegration relations in extended sets. If cointegration is found within a small set of variables, the same cointegration relations should be valid within any larger set of variables. The gradual expansion of the information set facilitates a sensitivity analysis of the results associated with the 'ceteris paribus' assumption. This strategy is known as 'specific to general' approach in the choice of variables (see Hendry and Juselius 2000, Juselius and MacDonald 2000). We first analyze the small model ($x_t' = [\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, ppp_t]$) excluding short term interest rates before analyzing the extended model with all the seven variables ($x_t' = [\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, i_t^s, i_t^{s*}, ppp_t]$).

4.4 Deterministic components

Since the asymptotic distribution of the test for cointegration depends on the assumptions made on the deterministic components, namely dummies and constant term, its choice may be crucial for inference. Without going into the details about the issues relating to the deterministic components in the cointegrated model, we need to make a sensible choice of the deterministic components in our $I(1)$ model.

We decided to set no trends both in the data and in the cointegration relations. There is no reason that is economically justified to expect trends in $\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, ppp_t$. The VAR, thus, was estimated with a constant restricted to the cointegration space. The only deterministic components, except the dummies allowed in our model in the data and a shift dummy allowed to lie in the cointegration space, were the intercepts in the cointegration relations.

Dummies

The likelihood-based inference methods on cointegration are derived upon the gaussian likelihood but the asymptotic properties of the methods depend on the *i.i.d.* assumption of the errors (Johansen 1995 p. 29). Thus the fact that the residuals are not distributed normally is not so important. Simulation studies have in fact shown that some assumptions are more important for the properties of the estimates than normality in the residuals. Generally if we reject the normality hypothesis (which is the null hypothesis of a test for normality) we should check the skewness and the kurtosis to see whether the residuals are well-behaved. If we do not include any dummy we would get highly bad-behaved residuals especially for which regards skewness, and all the inference would result heavily distorted. To secure valid statistical inference we need to take into account for shocks that fall outside the normality confidence level. We set a dummy variable whenever the residual was larger than $|3.5\sigma_\varepsilon|$. We have used three types of dummies, transitory, permanent and a shift dummy to capture a relevant evaluation of the German Mark vs. the US Dollar occurred in March 1961.

Since the asymptotic distribution for the cointegration test depends on the assumptions made on the deterministic components (dummies and the constant term), its choice may be crucial for inference. Without going into the details about the issues relating to the deterministic components in the cointegrated model, a sensible choice of the deterministic components in our $I(1)$ model is needed.

5 The 'small' model

We needed the following dummy variables for the small model:

$$Dt_t = \left[\; DS61.03, \Delta DS61.03, D57.12, D60.06, Di61.09, D62.09, D66.12 \; \right]$$

where:

$Dixx.yy$ is 1 at $19xx.yy_t$, -1 at $19xx.yy_{t+1}$ and 0 otherwise measuring a transitory shock.

$Dxx.yy$ is 1 at $19xx.yy_t$ and 0 otherwise measuring a permanent intervention shock.

$DS61.03$ is 1 since March 1961 and zero otherwise. $DS61.03$ takes into account an important official change in exchange rate of the German Mark vs. the US Dollar.

We tested whether these dummies were significant and hence necessary. All of them were significant for at least one of the variables (see Tab. 1 for the t-values for transitory and permanent dummies):

Tab. 1: t-values of transitory and permanent dummies

	$D5712$	$D6006$	$Di6109$	$D6209$	$D6612$
Δp_t	1.20	−0.08	**9.16**	−0.36	0.51
Δp_t^*	1.37	−1.32	−0.24	**2.56**	−0.94
i_t^l	0.39	1.58	0.94	−0.85	**−4.22**
i_t^{l*}	**−4.70**	**−2.54**	0.31	−0.02	**−4.48**
ppp_t	−0.76	1.21	**−4.57**	**−2.33**	0.98

the shift dummy is modeled in the VAR model like an exogenous variable. The differences of the exogenous variables, in this case the shift dummy, were significant with a maximal t-value of 12.45. The component of the shift dummy that enters in the cointegration space, as will be shown also later, was found significant with t-value of 3.00 in our final choice for the restricted cointegration space.

5.1 Lag length and misspecification tests

Probably the most important requirement for unbiased results is that estimated residuals show no serial correlation. If serial correlation is found adding one lag may be sufficient to remove it. Changing the number of lags may require a change in the dummies. The dummies above were based on a VAR model with two lags.

To provide an overall picture of the adequacy of the model we report some univariate and multivariate misspecification tests in Tab. 2.

Tab. 2: Misspecification tests

Multivariate tests					
Residual autocorr. $LM(1)$	$\chi^2\,(25)$	=	32.4	$p-val.$	0.15
Residual autocorr. $LM(4)$	$\chi^2\,(25)$	=	24.3	$p-val.$	0.50
Normality	$\chi^2\,(10)$	=	37.1	$p-val.$	**0.00**
Univariate tests	$\Delta^2 p_t$	$\Delta^2 p_t^*$	Δi_t^l	$\Delta i_t^{l^*}$	Δppp_t
ARCH(2)	1.05	1.18	0.68	3.55	0.84
JB(2)	4.52	1.44	2.64	3.77	1.28
Skewness	0.14	0.13	0.11	−0.38	−0.13
Ex. Kurtosis	0.67	−0.45	0.45	0.22	0.22
$\hat{\sigma}_\varepsilon \times 0.01$	0.17	0.26	0.01	0.01	0.00
R^2	0.74	0.62	0.53	0.38	0.63

Looking at Tab. 2 it seems that there are not any problems with autocorrelations of first and fourth order since $LM(1)$ and $LM(4)$ test statistics suggest that the null hypothesis for zero autocorrelation cannot be rejected. Normality is rejected as often happens, and the rejection was not mainly due to an excess of kurtosis rather than skewness. The Jarque-Bera test statistics (distributed like a $\chi^2(2)$, the $\chi^2(2)$ at 5% significance level has a critical value of 5.99) suggests that the rejection from normality was not due to excess of kurtosis. However skewness seems rather ordinary with the worst value of -0.38 for the long term interest rate. The $ARCH(2)$ (also distributed like a $\chi^2(2)$) statistic shows that significant heteroskedasticity for any variables was not found. The R^2 measures the improvement in the explanatory power of the model compared to a random walk hypothesis. The model is able to explain more about changes in inflation rates than changes in interest rates and purchasing parity.

To support that the model is reasonably well specified Fig. 12-16 are provided. Fig. 12-16 give four plots for each endogenous variable: the actual and the fitted values, the standardized residuals, a histogram of the standardized residuals with the histogram of the standardized Normal distribution as background and the correlograms for lag 1 to $T/4$. Fig. 12-16 show that the standardized residuals are reasonably well behaved thanks to the selection of dummies and lags.

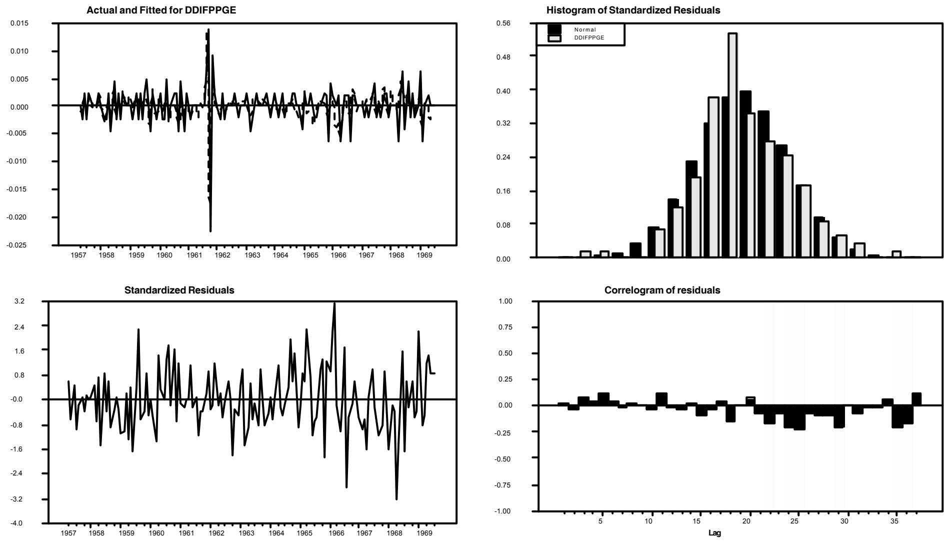

Fig. 12: Estimated residuals in the German inflation.

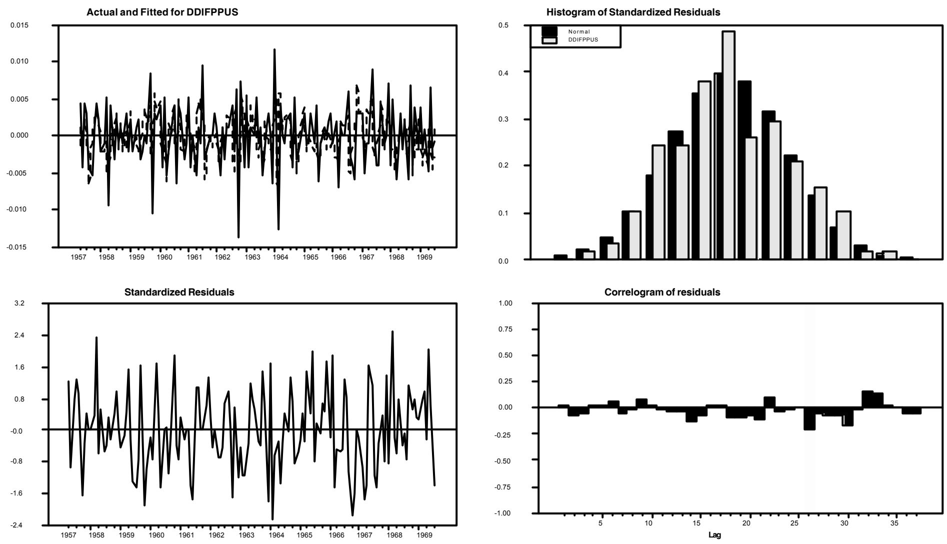

Fig. 13: Estimated residuals in the US inflation.

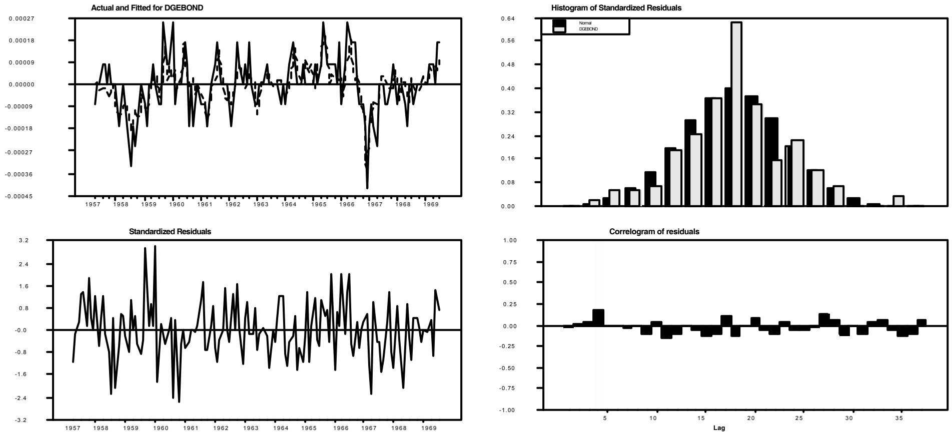

Fig. 14: Estimated residuals in the German bond rate.

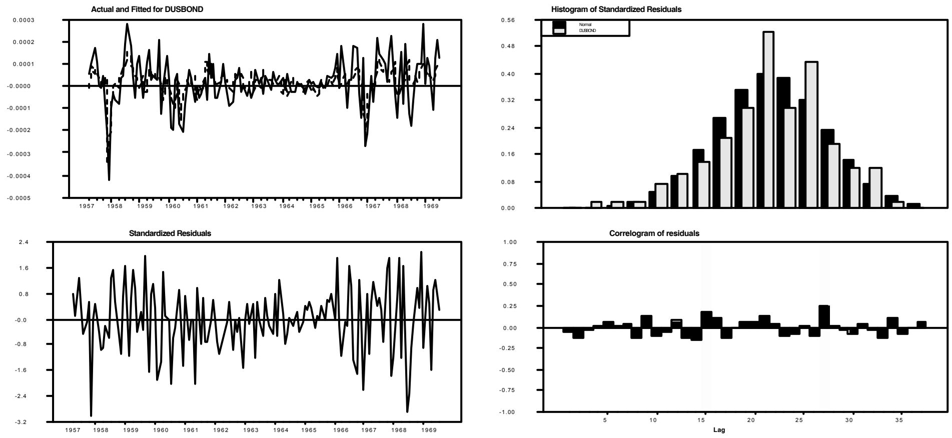

Fig. 15: Estimated residuals in the US bond rate.

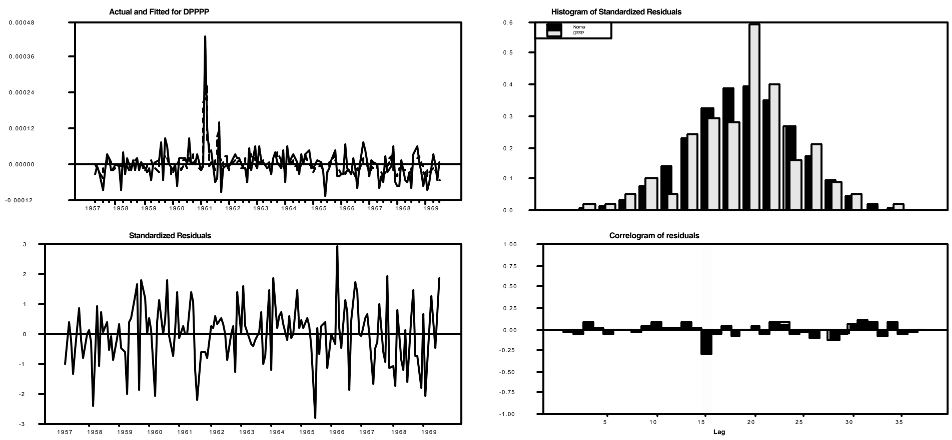

Fig. 16: Estimated residuals in the *ppp*.

5.2 Determination of the cointegration rank

The Eigenvalues of the Π matrix are reported in Tab. 3. We notice that only two eigenvalues are quite close to zero. How many of them are significantly different from zero? This question is fundamental since the rank of the Π matrix is equal to p less the number of zero eigenvalues.

If we could set two eigenvalues to zero, it would mean that the rank is equal to $5 - 2 = 3$, i.e. there would be three linearly independent stationary relations.

To discriminate among zero eigenvalues from non-zero eigenvalues, i.e. to calculate the cointegration rank, we use the *Trace* test. Tab. 3 shows that the null hypothesis of the *Trace* test, $r \leq 3$ against $r > 4$ cannot be rejected at 10% significance level.

Because the asymptotic distributions of these statistics can be rather bad approximations to the true small sample distributions we calculate in Tab. 4 the five largest roots of the companion matrix of Π to help us in the choice of the cointegration rank. Either in case the model is unrestricted, or the rank of Π is set to 2 or 3, there are 2 roots that are equal or very close to one. Since the number of roots of the companion matrix of Π is complementary to the rank of the Π, since $p = 5$, $r = 3$ and $p - r$ are roots of the companion matrix set to one, $r = 3$, in accordance with the *Trace* test, is our choice.

TAB. 3: EIGENVALUES OF THE Π MATRIX AND RANK TESTS

Eigenvalues of the Π matrix	0.48	0.36	0.12	0.07	0.04
r	0	1	2	3	4
Trace test	**197.5**	**101.6**	**36.1**	16.9	6.1
Trace 90	71.7	49.9	31.9	17.8	7.5

TAB. 4: THE EIGENVALUES OF THE COMPANION MATRIX

Modulus of 5 largest roots					
Unrestricted model	1.02	0.95	0.89	0.55	0.30
$r = 3$	1.00	1.00	0.88	0.48	0.48
$r = 2$	1.00	1.00	1.00	0.54	0.39

Once restricted the cointegration rank $r = 3$, and normalized the first eigenvector by Δp_t, the second by Δp_t^*, the third by i we obtained the estimated α, β and Π with the respective t-values (Tab. 5, 6 and 7). The ppp_t was multiplied by 0.01 to avoid to show very small but significant estimates. A smaller (bigger) parameter for ppp may point out that the flow of financial capitals is bigger (smaller) to changes in *uip*, once that proportionality between *ppp* and the current account and between the *uip* and the capital account are ascertained. While free trade in goods and services was considered to promote economic growth, short term capital flows were still seen as a potential source of disturbance to exchange rate stability undermining conditions for stable growth. Financial markets were heavily regulated and characterized by the existence of capital controls. It is thus legitimate to expect that the flow of financial capitals was smaller to changes in *uip* during the Bretton-Woods period rather than the recent floating exchange rate period. A higher parameter for the *ppp* should be thus found for the Bretton-Woods period.

TAB. 5: BETA TRANSPOSED

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t	$DS61.03$	*constant*
β_1'	1.000	−1.796	0.756	0.068	−1.644	0.001	−0.019
β_2'	−6.412	1.000	−1.177	−1.013	−0.976	0.001	0.003
β_3'	0.108	−0.084	1.000	−1.420	−4.377	0.003	−0.042

Based on the estimated α coefficients we note that:

1) the first relation is significantly adjusting in the US inflation rate.

2) the second relation is significantly adjusting in the German inflation and interest rates.

3) the third relation is significantly adjusting in the German inflation and interest rates and possibly the US interest rate.

We note that the rows correspondent to Δppp_t in Tab. 6 are not significant. The row corresponding to Δi_t^{l*} is boundary significant just for one value. This implies that the equations for Δi_t^{l*} and Δppp_t might not contain information about the long run parameters β, i.e. i_t^{l*} and ppp_t are weakly exogenous.

Tab. 6: ALPHA, T-VALUES FOR ALPHA

	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
$\Delta^2 p_t$	0.114 -1.58	**0.122** 6.59	**-0.436** -2.00
$\Delta^2 p_t^*$	**0.559** 5.07	0.049 1.75	-0.240 -0.72
Δi_t^l	0.002 0.53	**-0.003** 4.09	**-0.030** -3.22
Δi_t^{l*}	0.001 0.38	-0.000 -0.28	**0.023** 2.18
Δppp_t	0.002 -1.19	0.000 1.20	-0.000 -0.08

In the Π matrix (Tab. 7), the rows give the estimates of the combined effect of the three cointegration relation. The inflation rates and the German interest rate are equilibrium error correcting while the ppp_t is not. The t-values for i_t^{l*} are borderline.

Tab. 7: Π matrix and t-values

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t	$DS61.03$	$constant$
$\Delta^2 p_t$	**-0.714** -5.07	-0.046 0.35	**-0.493** -2.18	0.504 1.62	1.603 1.66	-0.001 -1.60	0.017 1.77
$\Delta^2 p_t^*$	0.217 1.01	**-0.934** -4.63	0.125 0.36	0.328 0.69	0.081 0.55	-0.000 -0.14	-0.001 -0.04
Δi_t^l	**0.019** 3.18	-0.004 -0.64	**-0.025** -2.59	**0.046** 3.47	**0.133** 3.21	**-0.000** -3.29	**0.001** 3.08
Δi_t^{l*}	0.005 0.80	**$-$0.005** -0.71	**$-$0.024** 2.22	**-0.033** -2.15	**-0.103** -2.20	**0.000** 2.19	**-0.001** -2.21
Δppp_t	-0.001 -0.41	-0.003 -0.98	0.000 0.09	0.000 0.03	-0.002 -0.09	0.000 0.08	-0.000 -0.09

The long run weak exogeneity test is formulated as a zero row in α and the null hypothesis is that the variable is weakly exogenous. If the null hypothesis is accepted, the variable pushes the system without being pushed and can be considered a driving force in the system. We notice that i_t^{l*} and ppp_t turned out to be weakly exogenous with $p - values$ of 0.44 and 0.47 respectively (Tab. 8). This is consistent with our choice of the rank $r = 3$. A joint test for weak exogeneity, restricting the α parameters for the US bond rate and ppp is accepted with a $p - value = 0.45$ in conformity with the rank restriction $r = 3$.

Tab. 8: Test for Weak Exogeneity

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t	$\chi^2(\nu)$
Long run weak exogeneity	38.3	24.0	18.6	**2.70**	**2.55**	$\chi^2(2)$=5.99

5.3 Single cointegration hypothesis

Looking for cointegration relations means to search for stationary linear combinations of the variables x_t. Single cointegration tests test whether a restricted relation can be accepted leaving the other relation unrestricted. If the hypothetical relations exists empirically, this procedure maximizes the chance to find them (Juselius and MacDonald 2000).

$\mathcal{H}_1$ to $\mathcal{H}_5$ (Tab. 9) are hypothesis on single variables. Inflation rates for both countries turned to be stationary with high $p-values$. Relative *ppp* is logically accepted as it turns out to be a linear combination of two stationary inflation rates. Stationarity in inflation rates implies that: *i*) prices are most likely $I(1)$; *ii*) the *ppp* could only be satisfied only in the case of cointegration between prices. This shows that the Bretton-Woods system planned to be mild inflationary proved to guarantee stability in inflation rates. Interest rates and *ppp* turned out to be non stationary.

$\mathcal{H}_6$ is a hypothesis on a pair of variables, the relative interest rates. Cointegration between US and German long term interest rates is excluded. Needless to say that Fisher parity cannot be supported as inflation rates are $I(0)$ while interest rates are $I(1)$. Real interest rates are found not stationary contrary to what is implied in most macroeconomic models. This same result was also found for the Post Bretton-Woods period (Juselius and MacDonald 2003, Bevilacqua 2006). $\mathcal{H}_6$ can be interpreted also as a hypothesis on the *uip* parity: since during the Bretton-Woods regime the exchange rates were fixed against the US dollar, the expected change of the exchange rates could not be anything but equal to zero $E_t\Delta_l s_{t+l}=0$. The *uip* reduces to $-i_t^l+i_t^{l*}=0$ and if the *uip* holds empirically, $-i_t^l+i_t^{l*}\sim I(0)$, otherwise $-i_t^l+i_t^{l*}\sim I(1)$. Evidence shown in Tab. 9 points out the *uip* does not hold even during the Bretton-Woods period.

$\mathcal{H}_7$ is a combination of $\mathcal{H}_6$, the *uip* condition, with *ppp*. Testing $\mathcal{H}_7$ is the equivalent of testing our fundamental relation. It is interesting to note that $\mathcal{H}_7$ can be interpreted in many ways.

$\mathcal{H}_7$ can be primarily interpreted like a linear long-run relationship between *ppp* and *uip*:

$$i_t^l - i_t^{l*} = \omega ppp_t$$
$$-uip_t = \omega ppp_t$$

which also shows the log of real exchange rate (which is *ppp*) to be proportional to the spread between the nominal interest rates in the two countries.

Since inflation rates are found stationary, any linear combination of $\mathcal{H}_1$, $\mathcal{H}_2$ and $\mathcal{H}_7$, such as the following equation that was found to hold for the Post Bretton-Woods period, should be found stationary for a pegged exchange rate regime:

$$(-\Delta p_t + \Delta p_t^*) + (i_t^l - i_t^{l*}) = \omega ppp_t$$

Rearranging, the log of real exchange rate has to be proportional to the spread between the real interest rates in the two countries for both the periods:

$$(i_t^l - \Delta p_t) - (i_t^{l*} - \Delta p_t^*) = \omega ppp_t$$

Dividing up the *ppp* into prices and the nominal exchange rate, we come up with a relation, which might be interpreted like an equation for the determinants of the exchange rate, that shows the nominal exchange rate in function of the spread of prices and the spread of real interest rates:

$$s_t = (p_t - p_t^*) - \frac{1}{\omega}(i_t^l - \Delta p_t) + \frac{1}{\omega}(i_t^{l*} - \Delta p_t^*)$$

which is a relation similar to the equation (6) in MacDonald (2000) that was derived with few assumptions directly from the balance of payments and could be thought as a very general representation of an equilibrium exchange rate in that it satisfied balance of payments equilibrium under floating exchange rates (MacDonald 2000).

The same relation should also hold under fixed exchange rates. Since inflation rates are stationary and exchange rates constant, hence stationary, simplifying prices and interest rates should produce a stationary relation such as $(p_t - p_t^*) - \left(\frac{1}{\omega} i_t^l - \frac{1}{\omega} i_t^{l*}\right) \sim I(0)$, which shows that any spread in prices should be compensated with a proportional spread in interest rates to keep the exchange rates constant. If domestic prices are higher (lower) than foreign prices, import (exports) will exceed exports (imports) worsening (improving) the current account (if the Marshall-Lerner condition holds). This pushes up (down) the demand and hence the value of the foreign currency. To obtain constant exchange rates, domestic interest rates have to be higher (lower) than foreign interest rates making more (less) attractive financial investments in the country, improving (worsening) the financial and capital account, pushing up (down) the demand and hence the value of domestic currency. Alternatively, only in a fixed or in a dirty floating exchange rate regime, another way to obtain constant exchange rates is worsening (improving) the official reserves: any sell (buy) of foreign for domestic currency means an increase of supply (demand) for foreign currency and demand (supply) for domestic currency pushing up (down) the value of domestic currency. When official reserves decrease (increase) money supply decreases (increases) pushing up (down) interest rates. The fact that the coefficients in the relation $(p_t - p_t^*) - \left(\frac{1}{\omega} i_t^l - \frac{1}{\omega} i_t^{l*}\right) \sim I(0)$ could be found different in magnitude, but not in the signs, might be ascribed to the fact that prices are not actual prices but simply indices referred to a base year.

$\mathcal{H}_7$ is accepted with a $p - value = 0.45$ meaning that relation $uip_t + \omega ppp_t \sim I(0)$ is empirically valid with $\omega = 3.499$. Lower values for ω ($\omega \sim 1$), were found by Juselius and MacDonald (2000 and 2003) and in Bevilacqua (2006). This parameter change could be due to the removal of capital flows restrictions occurred during the collapse of the Bretton-Woods regime. A lower (greater) value of ω might be just the symptom of the greater (lower) speed of capital flows to *uip* imbalances. As the Bretton-Woods system was intentionally planned to be without integrated capital markets for different reasons, such as to minimize spillover effects from financial crisis in other countries, a lower value of ω for the Post Bretton-Woods period was expected.

All this shows a remarkable robustness of the validity of the relation $uip_t + \omega ppp_t \sim I(0)$ found by Juselius and MacDonald to changes in price indices, different shift dummies and even different exchange rate systems. Compared to the Post Bretton-Woods regime we can only report that fixed exchange rates guaranteed stationary inflation rates and the constraints to capital flows could be responsible for a higher value of the parameter ω.

Tab. 9: Cointegration relations

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t	$DS6103$	$constant$	$\chi^2(\nu)$	$p-val$
$\mathcal{H}_1$	1	0	0	0	0	0.000	-0.001	0.44 (3)	**0.80**
$\mathcal{H}_2$	0	1	0	0	0	-0.000	-0.000	1.22 (3)	**0.54**
$\mathcal{H}_3$	0	0	1	0	0	-0.000	-0.005	7.85 (3)	0.02
$\mathcal{H}_4$	0	0	0	1	0	0.000	-0.003	9.58 (3)	0.01
$\mathcal{H}_5$	0	0	0	0	1	-0.001	0.009	7.85 (3)	0.01
$\mathcal{H}_6$	0	0	1	-1	0	-0.001	-0.001	10.68 (3)	0.00
$\mathcal{H}_7$	0	0	1	-1	-3.499	0.003	-0.035	0.58 (2)	**0.45**

The ppp term has been divided by 100

5.4 Fully specified cointegrating relations

We are now ready to test jointly $\mathcal{H}_7$, which shows a cointegration relationship between *uip* and *ppp*, with $\mathcal{H}_1$ and $\mathcal{H}_2$, which shows the stationarity of inflation rates. What we test is a vector space, the cointegration space, and any linear combination of the cointegration vector belong to the same space, which, if tested, should be accepted with the same $p-value$. Thus testing jointly $\Delta p_t \sim I(0)$, $\Delta p_t^* \sim I(0)$ and $-i_t^l + i_t^{l*} + \omega ppp_t \sim I(0)$ is just equivalent to test jointly $\Delta p_t \sim I(0)$, $\Delta p_t^* \sim I(0)$ and $\Delta p_t - \Delta p_t^* - i_t^l + i_t^{l*} + \omega ppp_t \sim I(0)$, the last being also the same fundamental and statistical significant equation we tested for the Post Bretton-Woods period.

The test statistic $\chi^2(7)$ was found equal to 3.64 with a $p-value$ of 0.82. The first relation is given by:

$$\Delta p_t \sim I(0) \tag{1}$$

The second relation is:

$$\Delta p_t^* \sim I(0) \tag{2}$$

The third relation is:

$$-i_t^l + i_t^{l*} + 0.0375 ppp_t + 0.003 DS6103 \sim I(0) \tag{3}$$

As these three relationships form a vector space, the relation $(i_t^{l*} - \Delta p_t^*) = (i_t^l - \Delta p_t) - 0.0375ppp_t + 0.003DS6103$, which is a linear combination of the former relationships, belongs to the same cointegration space. This last relationship is exactly (except for the value of the parameter ω and the choice of the shift dummy) the same fundamental relationship which was found significant for the recent floating exchange rate period.

In Tab. 10, a structural representation of the cointegration space is finally given. The adjustment coefficients and t-values are reported. What is noticeable is that none of the adjustment parameters referring to interest rates and *ppp* are significant while there are significant values for inflation rates and the German bond rate, suggesting that US interest rate and *ppp* are not adjusting to the two steady state relations as we would expect from weakly exogenous variables. A boundary significant value for α was found for the US bond rate. Testing joint weakly exogeneity for the US interest rate and the *ppp* the cointegration space was still accepted with a $p - value = 0.60$. Thus, there is evidence that the US interest rate and *ppp* are the driving forces of the system.

TAB. 10: A STRUCTURAL REPRESENTATION OF THE COINTEGRATION SPACE

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$		$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
Δp_t	**1**	0	0	$\Delta^2 p_t$	**−0.702** −5.02	−0.049 −0.38	**−0.493** −2.03
Δp_t^*	0	**1**	0	$\Delta^2 p_t^*$	0.140 0.65	**−0.838** −4.21	−0.250 −0.67
i_t^l	0	0	**−1**	Δi_t^l	**0.018** 3.02	−0.002 −0.29	**−0.028** −2.63
i_t^{l*}	0	0	**1**	Δi_t^{l*}	0.005 0.79	−0.005 −0.76	**−0.030** −2.54
ppp_t^1	0	0	**3.752** 5.73	Δppp_t	−0.000 −0.12	−0.003 −1.28	0.002 0.50
$DS6103$	0	0	**−0.003** −3.00				
constant	−0.001	−0.001	0.038				

The ppp term has been divided by 100

We report in Fig 17 the result of recursive estimation for testing the constancy of the cointegration space. The value *1* corresponds to a test with 5% significance level. It appears that the restricted model shows some β constancy as the test supports the hypothesis of parameter constancy (see the lower line which corresponds to the restricted cointegration space) for most of the period, although there is a peak beyond the critical value in 1966 that we were not able to remove choosing different sets of dummies.

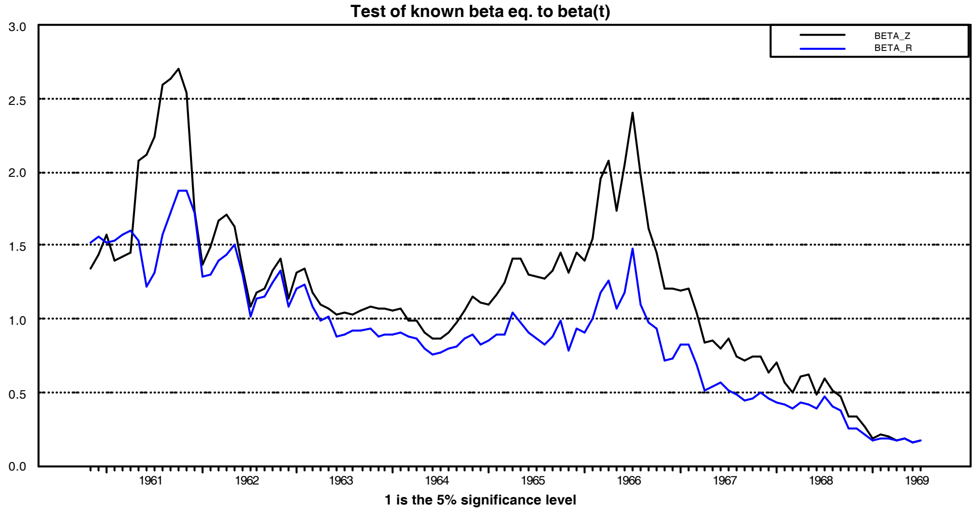

Fig. 17: Cointegration space constancy test.

5.5 Common trends

We report the VMA (common trends) representation for two different cases: (i) based on the rank restricted VAR model for $r = 3$, (ii) based on (i) but after having fully specified cointegrating relations with weak exogeneity of i_t^{l*} and ppp_t imposed on α.

The estimates of the C matrix in Tab. 11 measure the total impact of permanent shocks to each of the variables on all other variables. A row of the C matrix gives an indication of which variables have been particularly important for the stochastic trend behavior of the variable in the row.

Tab. 11: The estimates of the long run impact matrix C

C	$\sum \hat{\varepsilon}_{\Delta p_t}$	$\sum \hat{\varepsilon}_{\Delta p_t^*}$	$\sum \hat{\varepsilon}_{i_t^l}$	$\sum \hat{\varepsilon}_{i_t^{l*}}$	$\sum \hat{\varepsilon}_{ppp_t}$	$\sum \hat{\varepsilon}_{i_t^{l*}}$	$\sum \hat{\varepsilon}_{ppp_t}$
Δp_t	−0.005 −1.42	0.003 1.28	**-0.146** −2.33	**-0.255** −4.60	−0.221 −0.88	–	–
Δp_t^*	0.008 1.69	−0.001 −0.44	0.180 1.98	**0.366** 4.55	−0.569 −1.56	–	–
i_t^l	0.017 1.12	−0.013 −1.36	**0.616** 2.10	**1.025** 3.94	1.834 −1.56	**1.019** 3.93	**3.348** 2.52
i_t^{l*}	−0.020 −1.45	0.000 0.03	0.371 1.40	**0.834** 3.55	**-2.474** −2.33	**1.262** 5.60	−0.831 −0.72
ppp_t	−0.003 −0.53	−0.003 −0.88	0.013 0.13	−0.050 −0.54	**1.227** 2.96	−0.073 −0.84	**1.251** 2.83

We note that cumulative shocks to inflation rates, which were found to be and modeled as stationary variables in the restricted model, have no significant long run impact on any other variable in the unrestricted VAR model. Estimated cumulative shocks to the German long term interest rate assume boundary $t - values$ for the German inflation. Cumulative shocks to long term interest rates and to *ppp* are found often highly significant. Restricting the model with respect to both the β and α parameters, i.e. imposing inflation rates stationarity, cointegration between *ppp* and *uip* and weak exogeneity for both the US long term interest rate and *ppp*, we found that: cumulative shocks to the US long term interest rate and *ppp* have a significant impact on the German long term interest rate. This result emphasizes the evidence that the German long term interest rate was pushed from the USA.

Given the results from Tab. 11, the restricted VMA representation may be simplified as:

$$\begin{bmatrix} \Delta p_t \\ \Delta p_t^* \\ i_t^l \\ i_t^{l*} \\ ppp_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ c_{31} & c_{32} \\ c_{41} & 0 \\ 0 & c_{53} \end{bmatrix} \begin{bmatrix} \sum \varepsilon_{i_t^{l*}} \\ \sum \varepsilon_{ppp_t} \end{bmatrix} + \begin{matrix} \text{stationary and} \\ \text{deterministic} \\ \text{components} \end{matrix}$$

6 The 'extended model'

The 'extended model' includes the US Treasury Bill rate and the Frankfurt Interbank Offered rate that should be more closely linked to the monetary policy than long term interest rates as bond rates with a maturity of ten years. Given its monopoly over the creation of base money, the central bank can determine the official interest rate and exert a dominant influence on money market conditions steering money market interest rates having an impact on short term interest rates (ECB 2004), in this case the Frankfurt Interbank Offered rate. Conversely, the impact of money market rate changes on interest rates at long maturities (e.g. government bond yields) is less direct as these rates depend to a large extent on market expectations for long term growth and inflation trends (ECB 2004). In general, changes in the central bank's official rates do not normally affect long term rates unless they lead to a change in market expectations on long term economic trends (ECB 2004). Extending the small model including short term interest rates, we can test whether short term interest rates shocks normally do not lead to changes in long term interest rates as the ECB maintains unlike the standard expectations model of the term structure for which short rates drive long rates.

The analysis of the extended model for the Bretton-Woods period is subject to two major limitations:

- The number of observation is rather limited, especially for a system with 7 variables.
- The particular type of data used for the short term interest rate for Germany with its recurrent anomalous cyclicality between September and November should warn to use the Frankfurt Interbank Offered rate as a good proxy for a short term interest rate for Germany. However it was the only data available we could use as a proxy of the German short term interest rate. We are aware that other alternative series for the German short term interest rate could well allow to obtain far more precise results.

We needed the following dummy variables for the extended model:

$$\left[\ DS61.03, \Delta DS61.03, D60.06, Di61.09, D66.12, D67.07\ \right]$$

We tested whether these dummies were significant, and hence necessary and we found that all of them were highly significant for at least one of the variables (not shown here).

6.1 Lag length and misspecification tests

Two lags and the set of dummies shown above were sufficient to remove first and fourth order autocorrelation.

To provide an overall picture of the adequacy of the model we report some univariate and multivariate misspecification tests in Tab. 12. A significant test statistic is given in bold font (the $\chi^2(2)$, at 5% significance level has a critical value of 5.99).

Tab. 12: Misspecification tests

Multivariate tests							
Residual autocorr. $LM(1)$	$\chi^2\,(49)$	$=$	45.8	$p-val.$	$=$	0.60	
Residual autocorr. $LM(4)$	$\chi^2\,(49)$	$=$	44.2	$p-val.$	$=$	0.67	
Normality	$\chi^2\,(14)$	$=$	33.3	$p-val.$	$=$	**0.00**	
Univariate tests	$\Delta^2 p_t$	$\Delta^2 p_t^*$	Δi_t^l	Δi_t^{l*}	Δi_t^s	Δi_t^{s*}	Δppp_t
ARCH(2)	2.1	1.5	0.5	**7.2**	0.6	1.8	1.7
JB(2)	1.7	1.3	0.5	0.4	**8.5**	1.0	1.0
Skewness	0.05	0.25	−0.08	−0.11	0.44	0.07	−0.18
Ex. Kurtosis	0.35	−0.18	0.06	−0.01	1.28	0.52	0.09
$\hat{\sigma}_\varepsilon\ \times 0.01$	0.17	0.25	0.01	0.01	0.02	0.01	0.00
R^2	0.77	0.66	0.63	0.41	0.86	0.73	0.71

The $ARCH(2)$ (distributed like a $\chi^2(2)$) statistic shows that there no significant heteroskedasticity for all but the US long term interest rate. However, cointegration estimates are not very sensitive to $ARCH$ structures (Gonzalo 1994, Rahbek *et Al* 2002), so we are not forced to use to a VAR model that model also $ARCH$ effects.

Normality was probably rejected because of significant heteroskedasticity and excess of kurtosis, as according to the $JB(2)$ (also distributed like a $\chi^2(2)$) statistic the rejection was not due to an excess skewness rather than kurtosis. The R^2 measurements for the improvement in the explanatory power of the model compared to a random walk hypothesis are reported.

To support that the model is rather adequately specified Fig. 18-24 are provided. Fig. 18-24 show that the standardized residuals are well behaved thanks to a proper choice of dummies and lags.

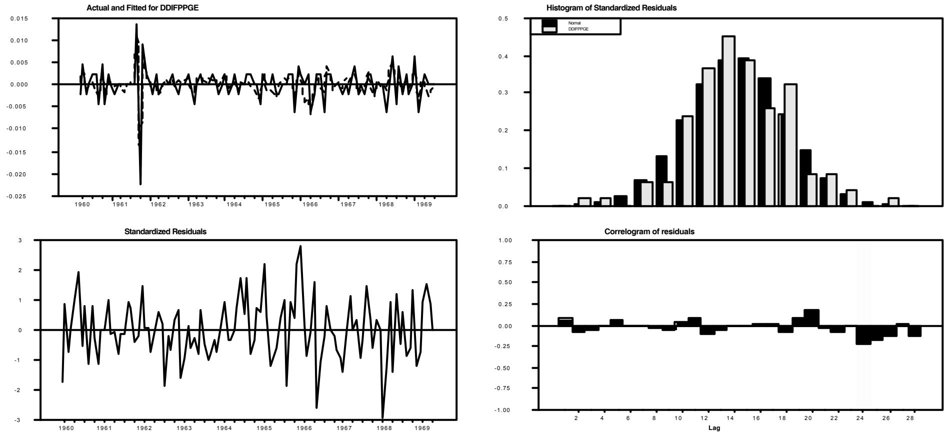

Fig. 18: The estimated residuals of German inflation.

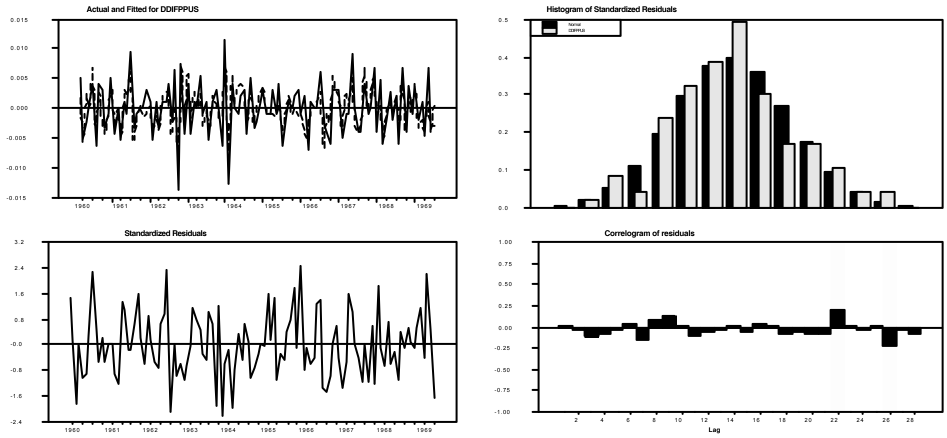

Fig. 19: The estimated residuals of US inflation.

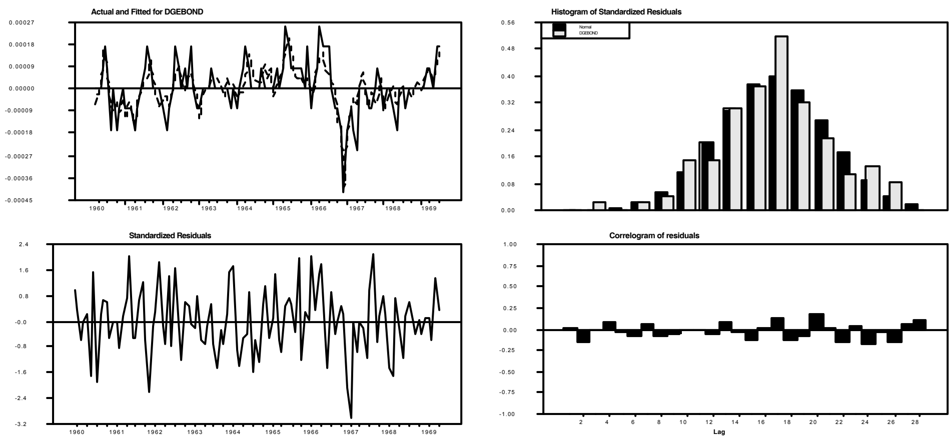

Fig. 20: The estimated residuals of the German bond rate.

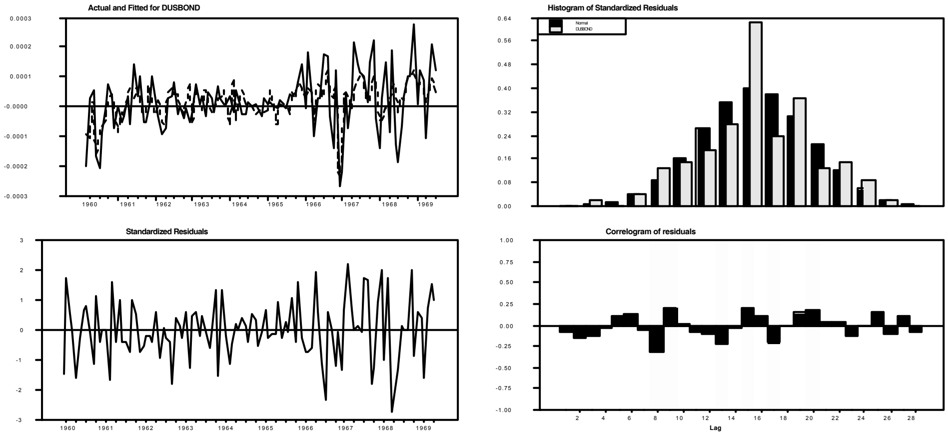

Fig. 21: The estimated residuals of the US bond rate.

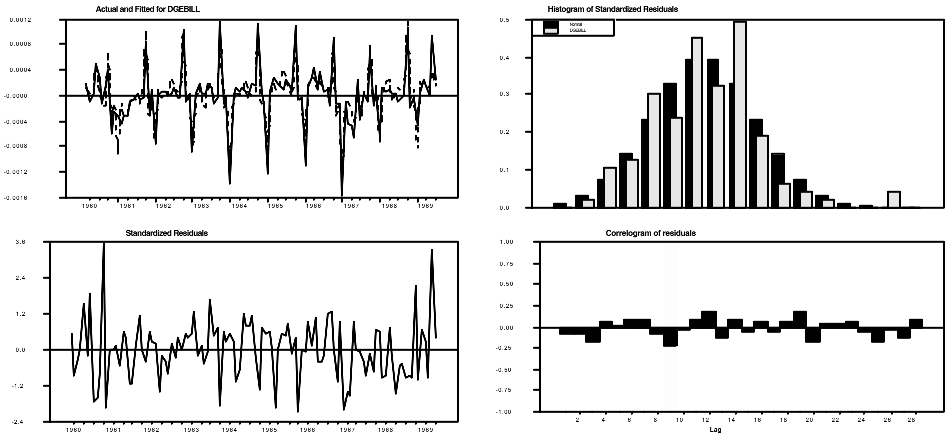

Fig. 22: The estimated residuals of the German short term interest rate.

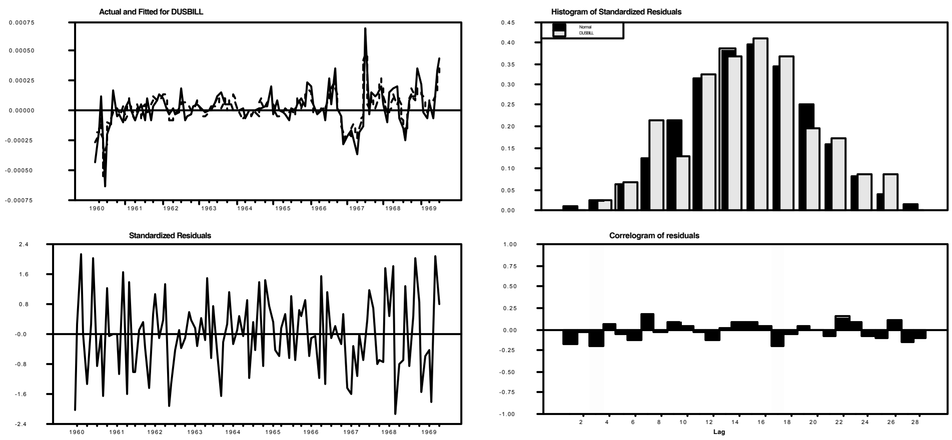

Fig. 23: The estimated residuals of the US treasury bill rates.

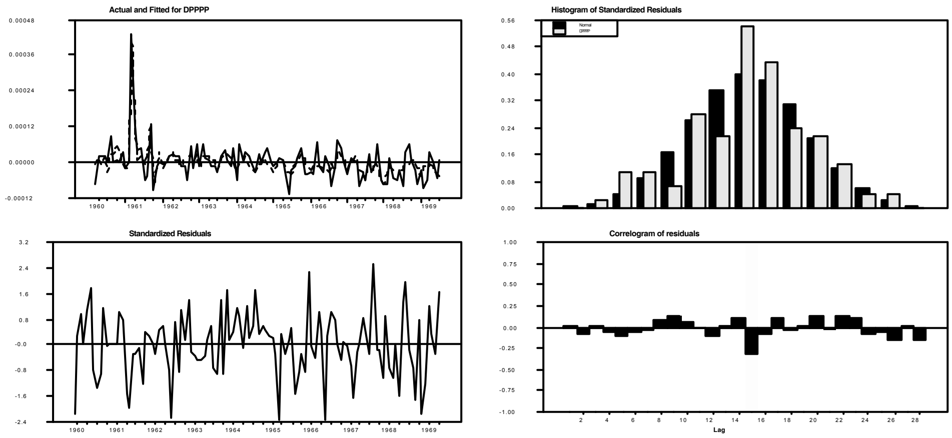

Fig. 24: The estimated residuals of the PPP.

6.2 Determination of the cointegration rank

The Eigenvalues of the Π matrix are reported in Tab. 13. We notice that at least three eigenvalues are quite close to zero.

Tab. 13 shows that the null hypothesis of the *Trace* test, $r \leq 3$ against $r > 4$ cannot be rejected at 10% significance level, but it is very close to be rejected.

TAB. 13: EIGENVALUES OF THE Π MATRIX AND RANK TESTS (EXTENDED MODEL)

Eigenvalues of the Π matrix	0.53	0.47	0.23	0.16	0.11	0.10	0.03
r	0	1	2	3	4	5	6
Trace test	**234.21**	**148.96**	**77.47**	47.98	28.65	15.77	3.85
Trace 90	126.71	97.17	71.66	49.91	31.88	17.79	7.50

As the asymptotic distributions of the trace test statistics can be rather bad approximations to the true sample distributions and should be used with caution in particular in the case of special dummy variables (Hansen and Juselius 1995) such as the shift dummy we have used we check the eigenvalues of the companion matrix (Tab. 14). Keeping the restrictions of the cointegrating vectors in the small model and calculating the rank with $r = 3$ we obtain a fifth root of 0.86 which is a rather high remaining root and even higher than the case $r = 4$. The analysis of eigenvalues of the companion matrix supports the hypothesis the rank restriction $r = 4$. We choose $r = 4$, implying that the inclusion of the short term interest rates have introduced only one additional stochastic trend. This means that the short term interest rates can be jointly cointegrated or cointegrated by with the remaining variables of the system.

Tab. 14: the eigenvalues of the companion matrix

Modulus of 5 largest roots					
$r = 3$	1.00	1.00	1.00	1.00	0.86
$r = 4$	1.00	1.00	1.00	0.82	0.82

6.3 Single cointegration hypothesis

An advantage of the 'specific to general' approach is that we can in principle keep unchanged the two cointegration relations found for the small model in the extended model. The impact of the two new variables, the short term interest rates, should involve an additional cointegrating relation. To have some idea about the new cointegration relation we first estimate the partially restricted long run structure keeping the three cointegration relation unchanged ($\mathcal{H}_1, \mathcal{H}_2$ and $\mathcal{H}_{13}$) but leaving unrestricted the fourth one and the shift dummy in all the cointegration vectors. The hypothesis was accepted with a $p - value$ of 0.26. This fourth cointegrating relation could contain information about the spread between long and short interest rates in the two countries or about the spread of real interest rates (Tab. 15). Before making further parameter restrictions we test the single cointegration hypothesis for the extended model to see which relations hold both for the small and the extended model and to find the fourth cointegration vector that will allow to form a restricted but significant (both in statistical and economic terms) cointegration space.

Tab. 15: The third unrestricted cointegrating relation

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	i_t^s	$i_t^{s^*}$	ppp_t	$DS61.03$	$const.$	$\chi^2(\nu)$	p-val
$\mathcal{H}$	−0.000	−0.000	−1.421	1.000	1.528	−1.134	−0.718	0.002	−0.006	10.04	0.26

$\mathcal{H}_1$ to $\mathcal{H}_7$ (Tab. 16) are hypothesis on single variables. Inflation rates for both countries turned to be stationary with high $p - values$ ($\mathcal{H}_1$and $\mathcal{H}_2$). Relative *ppp* is logically accepted as it turns out to be a linear combination of two stationary inflation rates. Stationarity in inflation rates implies that: *i*) prices are most likely $I(1)$; *ii*) the *ppp* could only be satisfied only in the case of cointegration between prices. This shows that the Bretton-Woods system planned to be mild inflationary proved to guarantee stability in inflation rates. Both the short and long term interest rates and *ppp* turned out to be non stationary.

$\mathcal{H}_8$ to $\mathcal{H}_{11}$ are hypothesis on a pair of variables. $\mathcal{H}_8$ and $\mathcal{H}_9$ are hypothesis on the relative interest rates. Cointegration between US and German interest rates is excluded. $\mathcal{H}_8$ and $\mathcal{H}_9$ can be interpreted also as a hypothesis on the *uip* parity: since during the Bretton-Woods regime the exchange rates were fixed against the US dollar, the expected change of the exchange rates could not be anything but equal to zero $E_t\Delta_m s_{t+m} = 0$. The *uip* reduces to $-i_t + i_t^* = 0$ and if the *uip* holds empirically, $-i_t + i_t^* \sim I(0)$, otherwise $-i_t + i_t^* \sim I(1)$. Evidence shown in Tab. 9 points out the *uip* does not hold during the Bretton-Woods period. $\mathcal{H}_{10}$ and $\mathcal{H}_{11}$ are rejected hypothesis on the spread between interest rates.

$\mathcal{H}_{12}$ and $\mathcal{H}_{13}$ combine $\mathcal{H}_8$ with $\mathcal{H}_9$, i.e. the spread among interest rates between the two countries. $\mathcal{H}_{13}$ can also be seen as a combination of the term spreads ($\mathcal{H}_{10}$ and $\mathcal{H}_{11}$). Both $\mathcal{H}_{12}$ and $\mathcal{H}_{13}$ are accepted with rather high $p-value$. $\mathcal{H}_{13}$ is the restricted third cointegration relation we were trying to find. It can be interpreted in many ways:

- As inflation rates are found stationary ($\mathcal{H}_1$and $\mathcal{H}_2$), $\mathcal{H}_{13}$ can be seen as:

$$\left(i_t^l - i_t^{l*}\right) - \left(i_t^s - i_t^{s*}\right) = -\left(\Delta p_t - \Delta p_t^*\right) \tag{4}$$

which shows that when the spread between actual domestic and foreign inflation is stationary, then the spread between domestic and foreign yield gap would also has to be stationary.

- Alternatively $\mathcal{H}_{13}$ may be interpreted as:

$$\left(i_t^{s*} - \Delta p_t^*\right) = \left(i_t^s - \Delta p_t\right) - \left(i_t^l - i_t^{l*}\right)$$

which shows the short term real interest rate parity as a stationary relation if the long term bond spread would be stationary $\mathcal{H}_{13}$ is accepted with a $p-value$ of 0.66.

This very same relation was found in the Post Bretton-Woods period (Bevilacqua 2006). What changes is the degree of integration of the inflation rates, not the relationships. Simplifying for the inflation rates the last two relationships reduce to:

$$\left(i_t^l - i_t^{l*}\right) - \left(i_t^s - i_t^{s*}\right) \sim I\left(0\right)$$

which shows that when the spread between domestic and foreign yield gap would also has to be stationary and

$$i_t^{s*} = i_t^s + \left(i_t^l - i_t^{l*}\right) \sim I\left(0\right)$$

which shows the nominal short term interest rate parity as a stationary relation whenever the nominal long term bond spread were stationary. These last equations could not be supported during the Post Bretton-Woods period because inflation rates for that period were neither stationary or cointegrating.

$\mathcal{H}_{14}$ combines the *uip* condition shown in $\mathcal{H}_8$ with the *ppp* producing a stationary relation accepted with a $p-value$ of 0.16. $\mathcal{H}_{14}$ corresponds to $\mathcal{H}_7$ tested in the small model. All the considerations concerning $\mathcal{H}_7$ in the small model apply to $\mathcal{H}_{14}$ in this extended model.

Tab. 16: Cointegration relations

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	i_t^s	i_t^{s*}	ppp_t	$DS61.03$	$constant$	$\chi^2(\nu)$	$p-val$
$\mathcal{H}_1$	1	0	0	0	0	0	0	0.002	-0.002	2.01 (3)	**0.57**
$\mathcal{H}_2$	0	1	0	0	0	0	0	0.001	-0.001	2.81 (3)	**0.42**
$\mathcal{H}_3$	0	0	1	0	0	0	0	0.007	-0.009	10.06 (3)	0.02
$\mathcal{H}_4$	0	0	0	1	0	0	0	0.001	-0.004	7.20 (3)	0.07
$\mathcal{H}_5$	0	0	0	0	1	0	0	0.003	-0.006	9.40 (3)	0.02
$\mathcal{H}_6$	0	0	0	0	0	1	0	0.001	-0.003	6.95 (3)	0.07
$\mathcal{H}_7$	0	0	0	0	0	0	1	-0.001	0.010	9.08 (3)	0.03
$\mathcal{H}_8$	0	0	1	-1	0	0	0	-0.003	-0.000	9.91 (3)	0.02
$\mathcal{H}_9$	0	0	0	0	1	-1	0	-0.001	-0.000	9.30 (3)	0.03
$\mathcal{H}_{10}$	0	0	1	0	-1	0	0	-0.001	-0.001	6.50 (3)	0.09
$\mathcal{H}_{11}$	0	0	0	1	0	-1	0	-0.001	-0.000	7.15 (3)	0.07
$\mathcal{H}_{12}$	0	0	1	-1	-1.013	1.013	0	-0.001	-0.000	1.59 (2)	**0.45**
$\mathcal{H}_{13}$	0	0	1	-1	-1	1	0	-0.001	-0.000	1.60 (3)	**0.66**
$\mathcal{H}_{14}$	0	0	1	-1	0	0	-4.078	0.003	-0.040	3.61 (2)	**0.16**

The ppp term has been divided by 100

6.4 Fully specified cointegrating relations

In Tab. 17 a structural representation of the cointegration space is finally given. The fully specified cointegrating relations were tested with the LR test procedure in Johansen and Juselius (1994) and accepted with a $p - value$ of 0.42.

The adjustment coefficients are also reported. There is only one adjustment parameters boundary significant for the US long term interest rates, suggesting it might be a weakly exogenous variables that pushes the system while some of the adjustment parameters referring to *ppp* are significant meaning that the weak exogeneity for *ppp* is less evident in the extended than in the small model. Restricting to zero the adjustment parameters for US long term interest rate the hypothesis is still accepted with a $p - value$ of 0.38.

Tab. 17: A structural representation of the cointegration space (extended model)

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$		$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$
Δp_t	1	0	0	0	$\Delta^2 p_t$	**−0.769** −4.6	−0.068 −0.4	−0.630 −1.5	−0.590 −1.1
Δp_t^*	0	1	0	0	$\Delta^2 p_t^*$	−0.123 0.5	**−0.864** −4.0	−0.189 −0.3	**2.560** 3.4
i_t^l	0	0	1	1	Δi_t^l	**0.019** 3.2	−0.002 −0.4	−0.020 −1.3	0.003 0.1
i_t^{l*}	0	0	−1	−1	Δi_t^{l*}	0.003 0.4	−0.006 −0.9	**0.044** 2.5	0.041 1.8
i_t^s	0	0	0	−1	Δi_t^s	0.015 1.0	0.026 1.8	0.007 0.2	**0.154** 3.0
i_t^{s*}	0	0	0	1	Δi_t^{s*}	**−0.019** 2.4	**0.025** 3.3	0.005 0.3	−0.012 −0.5
ppp_t^1	0	0	**−3.263** −7.33	0	Δppp_t	−0.001 −0.4	−0.003 −1.1	0.004 0.6	**−0.031** −3.2
$DS61.03$	**0.002** 9.55	**0.001** 4.65	**0.002** 6.30	**−0.001** −5.29					
constant	−0.002	−0.000	−0.033	−0.000					

The ppp term has been divided by 100

We report in Fig 25 the result of recursive estimation for testing the constancy of the cointegration space. The value *1* corresponds to a test with 5% significance level. It appears that the restricted model shows some β constancy as the test supports the hypothesis of parameter constancy for almost all the period we investigated (see the lower line which corresponds to the restricted cointegration space).

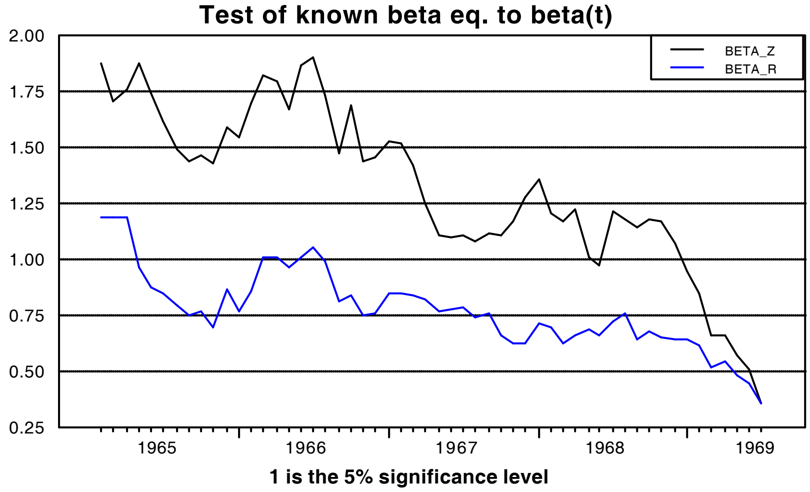

Fig. 25: Cointegration space constancy test.

6.5 Common trends

We report the VMA (common trends) representation for two different cases: (i) based on the rank restricted VAR model for $r = 4$ and having fully specified cointegrating relations (ii) based on (i) but imposing weak exogeneity of i_t^{l*} imposed on α.

The estimates of the C matrix in Tab. 18 measure the total impact of permanent shocks to each of the variables on all other variables. A row of the C matrix gives an indication of which variables have been particularly important for the stochastic trend behavior of the variable in the row.

Tab. 18: The estimates of the long run impact matrix C

C	$\sum \hat{\varepsilon}_{\Delta p_t}$	$\sum \hat{\varepsilon}_{\Delta p^*_t}$	$\sum \hat{\varepsilon}_{i^l_t}$	$\sum \hat{\varepsilon}_{i^{l*}_t}$	$\sum \hat{\varepsilon}_{i^s_t}$	$\sum \hat{\varepsilon}_{i^{s*}_t}$	$\sum \hat{\varepsilon}_{ppp_t}$	$\sum \hat{\varepsilon}_{i^{l*}_t}$
Δp_t	0	0	0	0	0	0	0	0
Δp^*_t	0	0	0	0	0	0	0	0
i^l_t	0.030 1.95	−0.012 −1.25	**0.894** 2.19	**0.767** 3.11	0.096 0.47	−0.160 −0.63	0.058 0.05	**1.129** 3.30
i^{l*}_t	0.009 0.45	−0.002 −0.17	1.090 1.93	**0.817** 2.39	−0.514 −1.80	0.557 1.58	−1.951 −1.13	**1.470** 4.59
i^s_t	0.006 0.22	0.024 1.44	1.245 1.79	0.330 0.79	0.136 0.39	**1.193** 2.77	2.552 1.21	1.051 1.72
i^{s*}_t	−0.015 −0.48	0.033 1.73	1.441 1.76	0.379 0.77	−0.473 −1.14	**1.910** 3.74	0.543 0.22	**1.392** 2.36
ppp_t	0.006 0.92	−0.003 −0.70	−0.060 −0.33	−0.015 −0.14	**0.187** 2.03	−0.220 −1.94	0.616 1.11	−0.122 −1.114

From the C matrix we note that:

- Cumulative shocks to inflation rates, which were found to be and modeled as stationary variables in the restricted model, have obviously no significant long run impact on any other variable in the unrestricted VAR model.
- Cumulative shocks to the US short and long term interest rates are found significant.
- Cumulative shocks to the US long term interest rate have a significant impact on the German long term interest rate.
- Cumulative shocks to the US short term interest rate have a significant impact on the German short term interest rate.
- Cumulative shocks to the German interest rated do not have significant effects on the other variables of the system.

  This result emphasizes the evidence that the German long term interest rate was pushed from the USA. Imposing weak exogeneity for the US long term interest rate we find that: cumulative shocks to the US long term interest rate have a significant impact on the German long term and the US short term interest rates. However the evidence that the US short term interest rate was driven by the long term interest rate is less evident than it was found in other studies referring to the Post Bretton-Woods period.

7 Conclusions

The main result of this paper is that important cointegration relationships found to hold for the Post Bretton-Woods essentially hold for the Bretton-Woods period as well. We think that this is a remarkable result because the two periods were characterized by distinct exchange rate regimes and a different regulation in capital markets.

It appears that the relationships found to hold for the Bretton-Woods period are a particular case of the relationships that hold for the Post Bretton-Woods period. In both periods a linear long-run relationship between *ppp* and *uip*, namely $uip_t + \omega ppp_t \sim I(0)$ holds, so that $\Delta p_t - \Delta p_t^* - i_t^l + i_t^{l*} + \omega ppp_t \sim I(0)$. However the pegged exchange rate system seemed to ensure stationary inflation rates so that the simplified stationary relation $-i_t^l + i_t^{l*} + \omega ppp_t \sim I(0)$ also holds for the Bretton-Woods period.

Similarly the relationships $\left(i_t^l - i_t^{l*}\right) - \left(i_t^s - i_t^{s*}\right) = \left(\Delta p_t - \Delta p_t^*\right)$, which shows that when the spread between actual domestic and foreign inflation is stationary then the spread between domestic and foreign yield gap would also has to be stationary, and $\left(i_t^{s*} - \Delta p_t^*\right) = \left(i_t^s - \Delta p_t\right) + \left(i_t^l - i_t^{l*}\right)$, which shows the short term real interest rate parity as a stationary relation if the long term bond spread would be stationary, hold for both periods. However because of the stationary inflation rates in the Bretton-Woods period, simplifying, the two relationships reduce to $\left(i_t^l - i_t^{l*}\right) - \left(i_t^s - i_t^{s*}\right) \sim I(0)$, which shows that when the spread between domestic and foreign yield gap would also has to be stationary, and $i_t^{s*} = i_t^s + \left(i_t^l - i_t^{l*}\right) \sim I(0)$, which shows the nominal short term interest rate parity as a stationary relation whenever the nominal long term bond spread were stationary.

Different values of the parameter ω between the two periods were estimated. We maintain that the parameter ω might be interpreted as the responsiveness of the capital movements that enter in the capital and financial account to *uip*. A small value of the parameter ω may imply a large responsiveness of capital movements to the net interest rate differential. Due to the restrictions and the heavy regulation for capital movements typical of the Bretton-Woods period, we expected a slower responsiveness of capital movements to the net interest rate differential, then a higher value of the parameter ω for the Bretton-Woods period than the recent floating exchange rate experience. In fact, it was found that ω was between 2 and 6 times greater during the Bretton-Woods than the Post Bretton-Woods period.

We maintained that statistical analysis of data should have shown Germany and USA to be respectively a 'small' and a 'big' country. In this respect we found that cumulative shocks to the US long term interest rate have a significant impact on the German long term interest rate, cumulative shocks to the US short term interest rate have a significant impact on the German short term interest rate and cumulative shocks to the German interest rated do not have significant effects on the other variables of the system. This result emphasizes the evidence that the German long term interest rate was indeed pushed from the USA. Imposing weak exogeneity for the US long term interest rate we find that: cumulative shocks to the US long term interest rate have a significant impact on the German long term and the US short term interest rates. However the evidence that the US short term interest rate was driven by the long term interest rate is less evident than it was found in other studies referring to the Post Bretton-Woods period.

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