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Public Capital and Income Distribution: a Marriage of Hicks & Newman-Read*

Yoseph Yilma Getachew

United Nations University - Maastricht Economic and social Research and training centre on Innovation and Technology
Keizer Karelplein 19, 6211 TC Maastricht, The Netherlands
Tel: (31) (43) 388 4400, Fax: (31) (43) 388 4499, e-mail: info@merit.unu.edu, URL: http://www.merit.unu.edu

Public Capital and Income Distribution: a Marriage of Hicks & Newman-Read *

Yoseph Yilma Getachew

UNU-MERIT, Maastricht University, The Netherlands

Abstract
This paper examines how publicly provided inputs could affect income distribution. By applying the Newman-Read production function- a generalized Cobb-Douglas production function- to Hicks's idea of the determinant of factor share, such usually complex dynamics remain analytically tractable. The paper shows that whether public capital has an effect on income distribution dynamics depends on its elasticity of substitution to private capital. If the elasticity of substitution of public capital to private capital is greater than unity, then an investment in public capital increases its relative income share and, hence, decreases the private capital share. In such a case, public capital would have a positive impact on income distribution dynamics.
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1 Introduction

Does public capital have effect in income distribution? Many argue informally that under certain conditions, in particular if it is targeted at lower income social groups, public capital may reduce inequality.[1] On the other hand, it may aggravate it if only the rich few have access to it. The question remains: how precisely is public capital linked to income distribution dynamics, especially, when it is provided on a non-discriminatory basis? Put differently, how does the income of two individuals who are heterogenous in terms of their initial wealth but similar otherwise, –one is rich and the other is poor–, be affected differently from using a public good in their production functions? In this paper, we propose theoretical answers to these questions.

We begin with the general question of what determines income distribution dynamics, especially, when both private and public inputs are involved in production. Income distribution evolves according to relative *private* factor income shares. When there are differences in initial endowment among households who are otherwise similar, the dynamics of income distribution depends on the degree to which households are able to exploit their relative initial advantage. The presence of any other (public good-type) inputs (e.g. infrastructure) in production have no effect on income distribution dynamics *unless* it alters the relative private factor income shares.[2]

Hicks (1932) argues that elasticity of substitution is the only determinant factor for changes in relative factor shares.[3] If the elasticity of substitution of a factor is greater than unity, then an increase in the supply of that factor more rapid than that of the others will increase its relative income share. Of course, if the elasticity of substitution of the factor is less than unity, then the relative share of the factor decreases. If it is equal to unity (the case of Cobb-Douglas), changes in supplies of factors do not have an effect on relative factors shares.

Hicks's argument provides useful hints for those seeking (informal) solutions to the above problems. Whether public capital has an effect on income distribution dynamics depends on its elasticity of substitution with regard to private capital. If the elasticity of substitution of public

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[1] See, for instance, The World Bank (1994), Songco (2002) and Brenneman and Kerf (2002).

[2] For a detailed discussion, refer Getachew (2008). See also the example below.

[3] In the beginning of 1930s, Sir John Hicks (1932) marked an important advance on identifying the determinant of the distribution of income. In his book, "On The Theory of Wages," Hicks dedicated a chapter, (with an appendix), on analyzing the resultant configuration of relative factor share. He developed a very important concept, which is useful till this time, what he called it "elasticity of substitutions."

capital to private capital, in a production function, is greater than unity, an investment in public capital increases its relative income share, and decreases the private capital income share. Consequently, public capital would have a positive impact on income distribution dynamics. However, if its elasticity of substitution is less than unity, then public capital increases the private capital income share, and hence, public capital would aggravate income inequality. Of course, if the elasticity of substitution is unity, infrastructure investment is neutral to the distribution of income.

We formalize these ideas using a variable elasticity of substitution (VES) production function, which is analytically tractable, and, at the same time, allows some flexibility in the parameters. Note that for a change in factor supply in a production function to change the structure of income distribution, first of all, the elasticity of substitution must be different from unity. However, the standard Cobb-Douglas (CD) production function has no such property. In the CD function, the elasticity of substitution is equal to unity and hence the factor shares are fixed. The analytical tractability of this popular production function comes with the cost of stringent restrictions on factor shares, which makes the production function unsuited for income distribution analysis.[4]

Analysed within the framework of a CD production function, income distribution dynamics are wholly independent of the level or change of the infrastructure inputs used in the production function.[5] For example, suppose that the individual production function is the standard CD function $y_t = A\left(k_t\right)^{\alpha}\left(X_t\right)^{\theta}$ where k_t is private capital and X_t is public capital. Assume further that private capital, which initially differs among individuals, is distributed lognormally, i.e., $\ln k_t\tilde{}N(\mu_t,\, \sigma_t^2)$. Then, an individual's saving at $t+1$ is $k_{t+1} = sy_t = sA\left(k_t\right)^{\alpha}\left(X_t\right)^{\theta}$, where s is an exogenous saving rate. Income distribution at $t+1$ is given by, a long story cut short, $var(\ln k_{t+1}) = \sigma_{t+1}^2 = \alpha^2\sigma_t^2$.[6] Therefore, in this economy, what matters for income distribution dynamics is neither X_t nor its output elasticity θ but the *predetermined* private capital income share α.

Therefore, the production function that is used for the study is the

[4]Although Solow (1957; 1960) argue a Cobb-Douglas production function may do fairly well in tracking observed changes in production, he states in a condition, "as long as no deep distributive meaning is read into the results."

[5]An exception to this case is presented in Getachew (2008). Getachew (2008) models public capital, in a CD function, as rival congestible input where its importance varies among households. In that case, in the absence of a perfect credit market, public capital could relax resource constraints of the poor, and brings a disproportional positive impact on the income of the poor, which in turn goes to reducing income inequality.

[6]See Appendix A for the aggregation.

less common *generalized* Cobb-Douglas production function, which is developed by Newman and Read (1961). The Newman-Read production function is a VES production function, *but for a certain value of its parameter it contains the standard CD function.* As is discussed above, the standard CD production function $y_t = A(h_t)^{\alpha}(G_t)^{\theta}$ imposes strict restrictions on the factor shares and on the elasticity of substitution. Particularly, the relative factor shares, –denoted by α and θ–, are fixed and the elasticity of substitution is equal to unity. On the other hand, the constant elasticity of substitutions (CES) production function, which is the other standard production function, has a relatively less stringent restrictions on the parameters but may not provide an analytical solution for the problems we pose due to difficulties that may arise during aggregation.

The Newman-Read generalized CD function is both flexible in the values of the parameters and analytically tractable in regard to income distribution studies. Moreover, the production function reduces to the standard CD function for a certain value of its parameter. Newman and Read (1961) developed the generalized CD production function specifically to address the need for less stringent restrictions on factor income shares. They consider a case where the income share of a factor remains invariant to changes in that input itself, with the other factor held constant, but varies with changes of the other factor. In this case, the production function becomes a VES production function where the factor shares vary alongside changes in factors supply.

In the next sections, we present a model of a two-sector economy where one of the sectors uses the Newman and Read (1961) production function. In the model, we suppose an economy, populated by heterogenous agents, consists of two production sectors: human capital accumulation and goods production sector. In the former, human capital is generated using inputs from public and private resources while production technology is characterized by the standard CD function. Moreover, production in the goods sector takes place using both private and public resources, but the production function that is used this time is the generalized CD function of Newman and Read (1961).

Within this setting, we show that a change in the supply of public capital in the goods production sector could affect income distribution dynamics, at least in the short run, although no additional specification are imposed to vary the benefits that accrue from using public capital among different households. As mentioned earlier, the effect of public capital on income distribution dynamics could be negative or positive depending on the elasticity of substitution of public capital to private capital. If the elasticity of substitutions is greater (lesser) than unity, then public capital decreases (increases) income inequality. In the long run, in the context of a growing economy, the elasticity of substitution is assumed to tend toward unity and, hence, the generalized CD function reduces to the standard CD function.[7] The model thus turns into a standard AK type endogenous growth model in the long term.

Some functions of public capital may also be worth mentioning. In growth literature public capital is usually treated as a factor that enhances productivity and complements the accumulation of private inputs. However, there is no doubt that public capital could also function as a substitutable factor that could relax households' resource constraints„ particularly for poor households', in the context of imperfect credit markets.[8] Of course, whether the elasticity of substitution between public and private capital is greater or lesser than one depends on the type of public capital made available.[9] But, in general, one of the functions of public capital is to provide opportunities and resources to individuals that they could not otherwise afford. This role of public capital may suggest that it could have a higher elasticity of substitution than private capital as a general proposition. As Hicks noted one of the things upon which elasticity of substitution of factors depends is "...the mere extension of the use of instruments and methods of production from firms where they were previously employed to firms which could not previously afford them" (Hicks 1932, p.120). Moreover, it is possible to mention some instances where public capital has clearly a large elasticity of substitutions, particularly in developing countries. Public education and public health infrastructure, for instance, may be considered good substitutions of private education and private health infrastructure.

The strand of literature related to the present study deals with the relationship between income inequality and public capital. Recently, a growing number of empirical studies try to assess the impact of infrastructure on income inequality. For instance, Calderón and Serven (2004), Calderón and Chong (2004) and Lopez (2004) find that infrastructure (water, sanitary, electricity, etc.) reduce income inequality and enhance economic growth at the same time. World Bank (2003) and Estache (2003) argue infrastructure has a positive and *disproportionate* impact on growth. OECD (2006) reports that "infrastructure is important for pro-poor growth". Whereas, Ferreira (1995) argues

[7] Empirical studies show factor shares, particularly that of private inputs (such as labor), show a large movement in the short run whereas they become stable in the long run (e.g. Acemoglu 2003; Revankar 1971).

[8] See, for instance, World Bank (1994) and Romp and de haan (2005; 2007).

[9] And, this is a question which is more of an empirical.

theoretically, in a model with quite a complex setup, public capital is important for income distribution dynamics. Getachew (2008) attempts to analytically capture their relationship using the popular standard CD function (refer footnote 4).

Another strand of literature related to the present study deals with the dynamics of income inequality and growth within an imperfect credit markets (e.g. Loury (1981), Galor and Zeira (1993), Banerjee and Newman (1993), Piketty (1997), Aghion and Bolton (1997), Aghion and Howit (1998), Aghion, Caroli, and García-Peñalosa (1999), and Benabou (1996; 2000; 2002)). These literatures in general show that in the face of capital market imperfection, income inequality has a definite negative effect on growth. This is because when the credit markets are imperfect, relatively more high-return investment opportunities would be forgone by resource poor households in inegalitarian society than egalitarian ones.

The paper is organized as follows. In section 2 we set up the model, with a brief discussion on the property of the Newman-Read production function. Section 3 is all about the evolution of income distribution and public capital. Section 4 discusses the long run (equilibrium) properties of the model, and section 5 concludes.

2 The Model

2.1 Households

Assume a continuum of heterogenous households, in overlapping generations, $i \in [0, 1]$. Each household i consists of an adult of generation t and a child of generation $t+1$. Population size is thus constant and normalized to be one. Let, at the beginning initial period, each household i of the initial generation be endowed with an initial human capital h_0 and a public infrastructure G_0 which is shared with others. Assume further that the distribution of initial human capital $\Gamma(h_0)$ takes a lognormal form, $\ln h_0 \tilde{} N(\mu_0, \sigma_0^2)$.

Agents care about their consumption level and the human capital stock of their children. When young, they accumulate human capital using both private and public resources. Once they become adults, they use their human capital for final goods production. Government taxes income with two fixed flat rate taxes, ψ and τ, in order to finance public capital, denoted by G_t and M_t, in the goods production and human capital accumulation sectors, respectively.[10] During their active periods, individuals allocate after tax income between current consumption c_t

[10]Note that in the paper, small and capital letters are used to denote individual and aggregate (average) variables, respectively.

and saving e_t that will be used to educate their children. The latter is incorporated in individuals' utility functions as the "joy of giving".

In a logarithmic preference, the utility of an individual is defined as

$$\ln c_t + \beta \ln h_{t+1} \tag{1}$$

subject to

$$c_t + e_t = (1 - \tau - \psi) y_t \tag{2}$$

where y_t is income of an individual at time t.

The human capital accumulation function of the offspring h_{t+1} is a function of public M_t and parental investment e_t. The accumulation function takes the standard CD form, in this sector, with constant returns to scale in factors. Thus, for an individual who is born at time t, the human capital at $t+1$ is given by

$$h_{t+1} = B \left(M_t\right)^{1-\eta} \left(e_t\right)^{\eta} \tag{3}$$

2.2 The Firm

There is an infinite number of competitive small firms. Households own the firms.[11] Production at the firm level occurs using both private and public capital, in a Newman-Read production function setting. Thus, the income of an agent of generation t, in the Newman-Read production function, is defined as

$$y_t = A \left(h_t\right)^{\alpha} \left(G_t\right)^{1-\alpha} \exp\left(\lambda_t \ln h_t \ln G_t\right) \tag{4}$$

where y_t and G_t represent output, and public capital, respectively. λ_t is a parameter, whose sign is found to be an important determinant of the behavior of the elasticity of substitution. We will discus its property in detail later on.[12]

The aggregate production function Y_t is computed, simply, by aggregating (4), $Y_t \equiv \int_0^1 y_t \Gamma(h_t)$,

$$Y_t = A \left(G_t\right)^{1-\alpha} \left(H_t\right)^{(\alpha + \lambda_t \ln G_t)} \exp \frac{\sigma_t^2}{2} \left(\alpha + \lambda_t \ln G_t\right) \left(\alpha + \lambda_t \ln G_t - 1\right) \tag{5}$$

[11]This shuts off the input and the credit markets.

[12]Note that the production function, in (4), reduces to the standard CD function if λ_t equal to zero, $\lambda_t = 0$.

where H_t is the aggregate (average) human capital, which is defined, $H_t \equiv \int_0^1 h_t \Gamma(h_t)$. $\Gamma(h_t)$ is the distribution of wealth at time t, which is assumed to take a lognormal distribution, i.e., $\ln h_t \tilde{} N(\mu_t, \sigma_t^2)$.[13]

2.2.1 Properties of the Newman-Read Production Function

Although its use is relatively rare in the literature of economic growth, the Newman-Read production function offers a flexible analytical framework for income distribution studies. Empirical studies reveal factor shares show large fluctuations in the short run whereas they show no trend in the long run (e.g. Acemoglu 2003).[14] The Newman-Read generalized CD production function contains a framework that provides a satisfactory approximation to this reality, in particular contrast to the popular CD function. The less popular production function provides a framework for analyzing the short run fluctuations in factor income shares whereas the standard CD function does not.[15] The latter is more appropriate for analyzing long run trends. Whereas, the Newman-Read production function is a general form of the standard CD function that could reduce to it when necessary(e.g., when analyzing long run trends).

According to Solow (1957), a particular functional form adopted for a production function is a matter of no great consequence as far as it posses a positive partial derivative and the right curvature. For certain values of its parameters, the Newman-Read production function, in (4), satisfies important properties of the neoclassical production function, i.e., it has a positive marginal productivity and a concave curvature:

The first and the second derivative of the production function in (4) are given by

$$\frac{\partial y_t}{\partial h_t} = \frac{y_t}{h_t}(\alpha + \lambda_t \ln G_t)$$
$$\frac{\partial y}{\partial G_t} = \frac{y}{G_t}(1 - \alpha + \lambda_t \ln h_t)$$

and

[13] While aggregating (4), we use the fact that

$$\int_0^1 \left((h_t)^\alpha \exp\left(\lambda_t \ln h_t \ln G_t\right)\right) \Gamma(h_t) = (H_t)^{(\alpha+\lambda_t \ln G_t)} \exp \frac{\sigma_t^2}{2} \left(\alpha + \lambda_t \ln G_t\right) \left(\alpha + \lambda_t \ln G_t - 1\right)$$

(See Appendix A).

[14] Acemoglu states factor shares of labor show a large movement in the short run whereas they remain stable in the long run. See Figure 1 and 2, which he plots for United State and France, respectively.

[15] Recall that elasticity of substitution equals to unity and hence factor shares are fixed in the popular CD function.

$$\frac{\partial y_t}{\partial h_t \partial h_t} = \frac{y_t}{(h_t)^2} (\alpha + \lambda_t \ln G_t) ((\alpha + \lambda_t \ln G_t) - 1)$$
$$\frac{\partial y_t}{\partial G_t \partial G_t} = \frac{y_t}{(G_t)^2} (\lambda_t \ln h_t - \alpha) ((\lambda_t \ln h_t - \alpha) - 1)$$

Assumptions 2.2

1. $0 < \alpha,\, \eta < 1$; A, G_t,$h_t > 1$
2. $(\ln G_t)^{-1} > -\lambda_t (\alpha)^{-1}$; $(\ln h_t)^{-1} > \lambda_t (\alpha)^{-1}$ and $(\ln h_t)^{-1} > -\lambda_t (1-\alpha)^{-1}$
3. $\lim_{t\to\infty} \lambda_t = 0$, $|\lambda_t| < 1$ otherwise; where λ_t is sufficiently large.

Under Assumptions 2.2, the Newman-Read production function in (4) obeys the neoclassical rule in that it has positive marginal productivity and concave curvatures with respect to both private and public capital. Formally,

$$y_t > 0;\ \frac{\partial y_t}{\partial h_t} > 0; \frac{\partial y}{\partial G_t} > 0; \frac{\partial y_t}{\partial h_t \partial h_t} < 0 \text{ and } \frac{\partial y_t}{\partial G_t \partial G_t} < 0$$

Moreover, under the above assumptions,

$$\frac{\partial y_t}{\partial h_t \partial G_t} = \frac{\partial y_t}{\partial G_t \partial h_t} = \frac{y_t}{G_t h_t} ((1 - \alpha + \lambda_t \ln h_t)(\alpha + \lambda_t \ln G_t) + \lambda_t) > 0$$

which implies that public capital enhances the marginal productivity of private capital, and vice versa.

The elasticity of substitutions, of the production function, implied by (4), denoted by δ_{hG} and δ_{Gh}, for public and private capital, respectively, are[16]

$$\delta_t = \delta_{hG} = \delta_{Gh} \equiv \frac{\frac{\partial y_t}{\partial h_t}\frac{\partial y_t}{\partial G_t}}{y_t \frac{\partial y_t}{\partial G_t \partial h_t}}$$
$$= \frac{(\alpha + \lambda_t \ln G_t)(1 - \alpha + \lambda_t \ln h_t)}{(\alpha + \lambda_t \ln G_t)(1 - \alpha + \lambda_t \ln h_t) + \lambda_t} > 0 \tag{6}$$

Equation (6) implies that the sign of λ_t determines whether the elasticity of substitution between private and public capital is greater or lesser than, or, equal to unity. That is,

$$\text{If } \lambda_t \lesseqgtr 0, \text{ then } \delta_t \gtreqless 1 \tag{7}$$

[16] The original formula was presented by Hicks (1932), Appendix (iii).

2.3 Government

We assume government budget is balanced at all times:

$$I_t^g = \int_0^1 y_t \psi \Gamma(h_t) = Y_t \psi \tag{8}$$

$$M_t = \int_0^1 y_t \tau \Gamma(h_t) = Y_t \tau \tag{9}$$

Thus, the government collects fixed proportional taxes ψ and τ on output Y_t to finance public investment, while the accumulation of public capital in the goods production sector follows the rule,

$$G_{t+1} = I_t^g + G_t(1 - \varkappa^g) \tag{10}$$

where G_t, I_t^g and $\varkappa^g$ are the public capital stock, public investment and depreciation, respectively.

2.4 Competitive Equilibrium

According to the above description, an individual at period t solves the following problem, which is derived by substituting (2) and (3) into (1),

$$\underset{e_t}{Max} \ln(1 - \psi) y_t - e_t) + \beta \ln B \left(M_t\right)^{1-\eta} \left(e_t\right)^{\eta} \tag{11}$$

taking as given, ψ, τ, I_t^g M_t and G_t.

The first order condition gives

$$e_t = a(1 - \tau - \psi) y_t \tag{12}$$

where $a = \frac{\beta\eta}{1+\beta\eta}$; (12) shows the agent's optimal saving as the function of his income.

To derive individuals' human capital accumulation equation, which is associated to their optimal behavior, substitute (9) and (12) into (3), and using (4) and (5), to get,[17]

$$h_{t+1} = B\left(G_t\right)^{(1-\alpha)} \left(\tau A \left(H_t\right)^{(\alpha + \lambda_t \ln G_t)}\right)^{1-\eta} \left(h_t\right)^{\alpha\eta} \left(a(1 - \tau - \psi) A\right)^{\eta} \exp\left(\Xi_t\right) \tag{13}$$

where

$$\Xi_t \equiv \frac{\sigma_t^2}{2} \left(\left(\alpha + \lambda_t \ln G_t\right)\left(\alpha + \lambda_t \ln G_t - 1\right)(1 - \eta)\right) + \left(\eta \lambda_t \ln h_t \ln G_t\right)$$

[17] See Appendix B for the details of the derivation of (13).

According to (13), an individual's human capital accumulation is determined by the human capital of his parent h_t, and aggregate private and public capital stock and initial income distribution, H_t, G_t, and σ_t^2, respectively. Income inequality apparently has a negative impact on individuals' capital accumulation.

3 Public Capital and Income Distribution Dynamics

The following two difference equations that are derived from (13) characterize the evolution of capital accumulation and income distribution in the economy

$$\begin{aligned}\mu_{t+1} = E\left[\ln h_{t+1}\right] &= \alpha\eta\mu_t + \ln BA + \ln\left(\tau\right)^{1-\eta}\left(a(1-\tau-\psi)\right)^{\eta} \\ &+(1-\alpha)\ln G_t + \left(\left(\alpha + \lambda_t \ln G_t\right)(1-\eta)\right)\ln H_t + \Xi_t \end{aligned} \tag{14}$$

$$\sigma_{t+1}^2 \equiv Var\left[\ln h_{t+1}\right] = \eta^2\left(\alpha + \lambda_t \ln G_t\right)^2 \sigma_t^2 \tag{15}$$

Under assumptions 2.2, which implies $0 < \eta\left(\alpha + \lambda_t \ln G_t\right) < 1$, income inequality will decline through time and ultimately disappear:

$$\lim_{t\to\infty} \sigma^2 = 0$$

where σ^2 is equilibrium income distribution. However, the disappearance of income inequality in the steady state should not be confused with a stylized fact. The reason that income inequality disappears in the long run is that, in the model, initial wealth is the only source of heterogeneity between individuals; but agents are otherwise similar (in terms of ability, technology, etc.) Therefore, a diminishing return on net private accumulative factors, $\eta\left(\alpha + \lambda_t \ln G_t\right) < 1$, implies resource poor households are more productive than rich ones; consequently, it is inevitable that the poor will catch up with the rich in the long run.

The model thus captures only the short run effect of public capital. The role of public capital in this model influences the speed of convergence of income distribution. Depending on the sign of λ_t (and hence the size of δ_t), the public good G_t, in the final goods production sector, affects income distribution dynamics negatively or positively during the economy's transition to its long run equilibrium. Recall from equation (7) that the sign of λ_t determines the value of elasticity of substitution δ_t whether it is greater, less than, or equal to unity, which in turn determines what effect the public capital G_t could have on income distribution dynamics.

If $\lambda_t = 0$, then $\delta_t = 1$,–the Cobb-Douglas case–, there is no relationship between public capital and income distribution. But, if $\lambda_t > 0$, then $\delta_t < 1$, investment in public capital G_t aggravates income disparities, at least in the short run. The reason is, if $\delta_t < 1$, then, an increase in the public capital increases the relative income share of private capital, which ultimately disproportionally benefits the rich who hold much of the production resources. On the other hand, if $\lambda_t < 0$, then $\delta_t > 1$, an increase in the public good G_t decreases the relative income share of private capital. Consequently, income inequality decreases as poor household are able to relax some of their resource constraints through factor substitution.

4 Dynamics of Other Macroeconomic Variables

To determine the dynamics of other macro variables, first aggregate (13) to obtain the equation that characterizes the evolution of aggregate human capital,

$$\begin{aligned} H_{t+1} = & BA\left(G_t\right)^{(1-\alpha)} a^{\eta}(1-\tau-\psi)^{\eta}\left(\tau\right)^{1-\eta}\left(H_t\right)^{(\alpha+\lambda_t \ln G_t)} \\ & \exp\left(\Phi_t\left(\eta^2-\eta+1\right)\right) \end{aligned} \tag{16}$$

where[18]

$$\Phi_t = \frac{\sigma_t^2}{2}\left(\left(\alpha+\lambda \ln G_t\right)\left(\alpha+\lambda \ln G_t - 1\right)\right)$$

Then, derive the dynamic equation for the public capital in the goods production sector by substituting (8) into (10), using (5), and assuming a complete depreciation ($\varkappa^g = 1$)

$$G_{t+1} = \psi A\left(G_t\right)^{1-\alpha}\left(H_t\right)^{(\alpha+\lambda \ln G_t)} \exp\left(\Phi_t\right) \tag{17}$$

At equilibrium $\lim_{t\to\infty} \sigma^2 = 0$, aggregate capital ratio converges to a constant value, using (16) and (17),

$$\frac{H}{G} = \frac{Ba^{\eta}(1-\tau-\psi)^{\eta}\left(\tau\right)^{(1-\eta)}}{\psi} \tag{18}$$

[18] While aggregating (13), we use the fact that

$$\int_0^1 \left(\left(h_t\right)^{\alpha\eta} \exp\left(\eta\lambda_t \ln h_t \ln G_t\right)\right)\Gamma(h_t) = \left(H_t\right)^{\eta(\alpha+\lambda_t \ln G_t)} \exp\left(\eta^2\Phi_t\right)$$

(see appendix A).

From, equation (18), it is apparent that in the long run, the economy behaves as a standard AK_t model. Aggregate variables will be in a balanced growth path, where H, G and Y grow at the same rate.[19]

The long run growth rate γ of the economy can easily be computed from the equilibrium value of (16),

$$\begin{aligned}\gamma &= \ln H_{t+1} - \ln H_t \\ &= \ln B + \ln A + \eta \ln a + \eta \ln(1-\tau-\psi) + (1-\eta)\ln \tau + (\alpha - 1)\ln \frac{H_t}{G_t}\end{aligned}$$

and then substituting (18) into the above equation,

$$\gamma = \ln A + \alpha \ln B + \alpha\eta \ln a + \alpha\eta \ln(1-\tau-\psi) + \alpha(1-\eta)\ln\tau + (1-\alpha)\ln\psi \tag{19}$$

As of the literatures in public capital and economic growth, both taxes relate positively but non-linearly to long term growth (e.g., Barro 1990). The growth maximizing taxes, ψ^* and τ^* are given by

$$\psi^* = (1-\alpha) \tag{20}$$
$$\tau^* = (1-\eta)\,\alpha \tag{21}$$

The growth maximizing tax for public capital in the goods production sector ψ^* is equal to the share of public capital in the sector (similar to Barro's (1990) finding) while the growth maximizing tax for public capital in human capital accumulation sector τ^* is equal to the share of the public capital in that sector times the long run output elasticity of human capital.

5 Conclusion

It is well understood that public capital is important for economic growth. But, is public capital important for income inequality? It is intuitive that public capital may reduce inequality, particularly if it is targeted at lower income groups. Nonetheless, public capital may aggravate income inequality if only the rich few have access to it. But, as we have shown here, even if it is provided in a non-discriminatory basis, public capital remains important for income distribution dynamics.

The effect of public capital on income distribution is strongly linked to its elasticity of substitution to private capital. If the elasticity of

[19] Let the variables without time subscript (H, G, Y and σ^2) denote steady state values.

substitution of a given type of public input is greater than unity, then it might have a positive and disproportionate impact on the income of the poor. Particularly, if the credit market is imperfect, the provision of public capital enjoying high elasticity of substitution to private capital, even on a non-discriminatory basis, might help the poor more by relaxing some of their resource constraints. This, in turn, results in an improvement in the distribution of income of the economy.

A Aggregation

With the assumption of a lognormal distribution for individual's human capital, $\ln h_t \tilde{} N(\mu_t, \sigma_t^2)$, we have the following relation

$$\begin{aligned} \ln E[h_t] &= E[\ln h_t] + \frac{\sigma_t^2}{2} \\ &\Longleftrightarrow E[\ln h_t] = E[\ln h_t] - \frac{\sigma_t^2}{2} \equiv \ln H_t - \frac{\sigma_t^2}{2} \end{aligned} \tag{A1}$$

since $E[h_t] = \int_0^1 h_t \Gamma(h_t) \equiv H_t$

The fact that $\int_0^1 \ln y_t(h_t) \Gamma(h_t) = E(\ln y_t)$, where $\Gamma(h_t)$ is the distribution function of (the random variable) h_t. Then,

$$\begin{aligned} Y_t &\equiv E[y_t] = E\left[A (h_t)^\alpha (G_t)^{1-\alpha} \exp(\lambda_t \ln h_t \ln G_t)\right] \\ &= A (G_t)^{1-\alpha} E[(h_t)^\alpha \exp(\lambda_t \ln h_t \ln G_t)] \end{aligned}$$

We derive, in equation (5),

$$Y_t = A (G_t)^{1-\alpha} (H_t)^{(\alpha + \lambda_t \ln G_t)} \exp \frac{\sigma_t^2}{2} (\alpha + \lambda_t \ln G_t)(\alpha + \lambda_t \ln G_t - 1) \tag{5}$$

by using the above facts.

That is, if h_t is a lognormal distribution then $(h_t)^\alpha \exp(\lambda_t \ln h_t \ln G_t)$ is also a lognormal distribution since $\ln(h_t^\alpha \exp(\lambda_t) \ln h_t \ln G_t)) = (\alpha + \lambda_t \ln G_t) \ln h_t$ is normal. Then, according to (A1),

$$\begin{aligned} &\ln E[(h_t)^\alpha \exp(\lambda_t \ln h_t \ln G_t)] \\ &= E[\ln (h_t)^\alpha + (\lambda_t \ln h_t \ln G_t)] + \frac{1}{2} var[\ln (h_t)^\alpha + (\lambda_t \ln h_t \ln G_t)] \\ &= E[(\alpha + \lambda_t \ln G_t) \ln h_t] + \frac{1}{2} var[(\alpha + \lambda_t \ln G_t) \ln h_t] \\ &= (\alpha + \lambda_t \ln G_t) \left(\ln H_t - \frac{\sigma_t^2}{2} \right) + (\alpha + \lambda_t \ln G_t)^2 \frac{\sigma_t^2}{2} \\ &= (\alpha + \lambda_t \ln G_t) \ln H_t + (\alpha + \lambda_t \ln G_t)(\alpha + \lambda_t \ln G_t - 1) \frac{\sigma_t^2}{2} \end{aligned}$$

Then,

$$\begin{aligned} &E[(h_t)^\alpha \exp(\lambda_t \ln h_t \ln G_t)] \\ &= (H_t)^{(\alpha + \lambda_t \ln G_t)} \exp \frac{\sigma_t^2}{2} (\alpha + \lambda_t \ln G_t)(\alpha + \lambda_t \ln G_t - 1) \end{aligned} \tag{A2}$$

To derive

$$
\begin{aligned}
& E\left[(h_t)^{\alpha\eta}\exp\left(\eta\lambda_t \ln h_t \ln G_t\right)\right] \\
&= (H_t)^{\eta(\alpha+\lambda_t \ln G_t)} \exp\left(\frac{\sigma_t^2}{2}\eta^2\left(\alpha+\lambda_t \ln G_t\right)\left(\alpha+\lambda_t \ln G_t - 1\right)\right)
\end{aligned}
$$

in (16), follow similar steps as above.

B Dynamics of Individual Human Capital

To derive the individual's human capital accumulation equation, substitute (9) and (12), into (3), to get

$$h_{t+1} = B\left(\tau Y_t\right)^{1-\eta}\left(a(1-\tau-\psi)y_t\right)^{\eta}$$

Then, substitute (4) and (5) into the above equation

$$
\begin{aligned}
h_{t+1} &= B\left(\tau A\left(G_t\right)^{1-\alpha}\left(H_t\right)^{(\alpha+\lambda_t \ln G_t)} \exp\frac{\sigma_t^2}{2}\left(\alpha+\lambda_t \ln G_t\right)\left(\alpha+\lambda_t \ln G_t - 1\right)\right)^{1-\eta} \\
&\quad (h_t)^{\alpha\eta}\left(a(1-\tau-\psi)A\left(G_t\right)^{1-\alpha}\right)^{\eta}\exp\left(\eta\lambda_t \ln h_t \ln G_t\right) \\
&= B\left(G_t\right)^{(1-\alpha)(1-\eta)+(1-\alpha)\eta}\left(\tau A\left(H_t\right)^{(\alpha+\lambda_t \ln G_t)}\right)^{1-\eta}(h_t)^{\alpha\eta}\left(a(1-\tau-\psi)A\right)^{\eta} \\
&\quad \exp\left(\frac{\sigma_t^2}{2}\left(\left(\alpha+\lambda_t \ln G_t\right)\left(\alpha+\lambda_t \ln G_t - 1\right)(1-\eta)\right)+\left(\eta\lambda_t \ln h_t \ln G_t\right)\right)
\end{aligned}
$$

After rearranging, we get equation (13) of Section 2.4,

$$h_{t+1} = B\left(G_t\right)^{(1-\alpha)}\left(\tau A\left(H_t\right)^{(\alpha+\lambda_t \ln G_t)}\right)^{1-\eta}(h_t)^{\alpha\eta}\left(a(1-\tau-\psi)A\right)^{\eta}\exp(\Xi_t) \tag{13}$$

where

$$\Xi_t \equiv \frac{\sigma_t^2}{2}\left(\left(\alpha+\lambda_t \ln G_t\right)\left(\alpha+\lambda_t \ln G_t - 1\right)(1-\eta)\right)+\left(\eta\lambda_t \ln h_t \ln G_t\right)$$

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