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Corporate Control and Multiple Large Shareholders

Abstract

Many firms have more than one blockholder, but finance theory suggests that one blockholder should be sufficient to bestow all benefits on a firm that arise from concentrated ownership. This paper identifies a reason why more blockholders may arise endogenously. We consider a setting where multiple shareholders have endogenous conflicts of interest depending on the size of their stake. Such conflicts arise because larger shareholders tend to be less well diversified and would therefore prefer the firm to pursue more conservative investment policies. When the investment policy is determined by a shareholder vote, a single blockholder may be able to choose an investment policy that is far away from the dispersed shareholders' preferred policy. Anticipating this outcome reduces the price at which shares trade. A second blockholder (or more) can mitigate the conflict by shifting the voting outcome more towards the dispersed shareholders' preferred investment policy and this raises the share price. The paper derives conditions under which there are blockholder equilibria.The model shows how different ownership structures affect firm value and the degree of underpricing in an IPO.

1 Introduction

Finance theory has long recognized the important role that a large shareholder may play in corporate governance. Due to his size such a shareholder may be less subject to the well known free-rider problem among shareholders[1] and thus act as an important source of discipline for a manager.[2] On the other hand, optimal diversification implies that no investor should want to invest too high a proportion of his wealth in a single firm: the only differences in shareholdings should come from different wealth levels and different degrees of risk aversion. This suggests that the equilibrium ownership structure is a compromise between these two trade offs: larger size comes at the cost of diversification while smaller size leads to inefficiencies due to free riding.

Indeed, what is observed empirically is neither dispersed ownership in the firm nor one large shareholder. The presence of a second large shareholder (or *blockholder*) is well documented, and seems to be linked strongly to the voting power associated with a larger share ownership. Bloch and Hege (2001) note that in eight out of nine largest stock markets in the European Union, the median size of the second largest voting block in large publicly listed companies exceeds five percent (results of the European Corporate Governance Network). The voting power of the second blockholder varies depending on the firm characteristics, but usually a second blockholder is almost always present in every firm across country, sector and type. Porta, Lopez-De-Silanes, and Shleifer (1999) find that 25% of the firms in various country have at least two blockholders while Laeven and Levine (2008) finds that 34% (12%) of listed Western European firms have more than one (two) large owners where large owners are considered shareholders with more than 10%. Even US firms which are often cited as example of dispersed ownership have blockholders: 90% of the S&P500 firms have shareholders with more than 5% participation and more generally in US firms 40% of the equity is owned by blockholders (Holderness, forthcoming).

The trade-offs noted above suggest too that firms which have blockholders are those where the trade offs are especially severe: often ownership structure is related to the characteristics of the industry e.g. Carlin and Mayer (2000; 2003) present evidence that multiple large investors are present in companies which invest in high risk projects, while when uncertainty is low, single investors are more common.

These stylized facts raise some natural questions: If all the benefits of size can be reaped by a single large shareholder, what causes the emergence of multiple large shareholders in a firm? Can the same reasons that cause one large shareholder to be beneficial also explain the emergence of multiple large shareholders? What is the nature of strategic interactions between multiple large shareholders and what can we say about the types of firms that would have multiple blockholders? This paper attempts to answer some of these questions. We provide a positive theory explaining the emergence

[1] See Grossman and Hart (1980).

[2] See e.g. Stiglitz (1985), Shleifer and Vishny (1986), Holmstrom and Tirole (1993), Admati, Pfleiderer, and Zechner (1994), Burkart, Gromb, and Panunzi (1997).

of multiple blockholders.

Our formal model analyzes the problem of an initial owner who needs to raise capital to finance a project, and who must decide whether to undertake costly monitoring to raise the value of the firm. The initial owner faces a moral hazard problem: he can only commit to monitor (and hence a higher value of the firm) if his own stake in the firm is sufficiently high. He raises capital through issuing shares.[3] Once the ownership structure is established, shareholders vote on the riskiness of the investment project that the firm subsequently undertakes. One way to think of this is as a choice that shareholders face between managers who come with different reputations for risk taking. Investors face a trade off between holding the optimal portfolio and having little influence on the firm's decisions, or buying more shares, holding a suboptimal portfolio, but influencing the firm's decisions.

Once investment decisions are made however, investors have different preference for the risk/return characteristic of the project, depending on the fraction of shares they own. Investors, who hold the firm's shares for portfolio reasons (*liquidity shareholders*), would prefer a higher return even at the expense of higher risk. The initial owner, who holds a larger fraction (due to the commitment problem) would prefer a lower return project in order to bear less risk: there is a conflict of interest between shareholders who differ in terms of their shareholdings.[4] This is what causes some investors to buy bigger blocks, to guarantee that the risk profile of the firm is closer to their optimum. Paradoxically, of course, when they do buy a larger fraction of shares, their preferences become closer to those of the initial large shareholder! This puts a natural limit on the size of a single blockholder: no outside investor would ever want to own more shares than the initial owner. Hence a potential second (or third or fourth..) blockholder with fewer shares than the initial owner votes for a level of risk and return that lies between the choice of liquidity shareholders and the choice of the initial owner. Hence, having larger shares than liquidity shareholders means that he can influence the voting outcome and having less shares than the initial owner means that can mitigate the conflicts of interests between the initial owner and outside investors. He shifts the decision towards a higher return outcome at the cost of holding a suboptimal portfolio. We can think of this in the following way: the (ex-ante identical) underlying preferences of each outside investor on the optimal risk-return choice are single peaked. The benefits of owning more shares is that he can affect the voting decision to make it closer to his underlying preferences while the cost to him is the loss in diversification. So blockholders emerge in an equilibrium whenever these benefits are bigger than the costs. By thus mitigating the conflicts of interests, blockholders make the liquidity shareholders unambiguously better off.

Our main results are that blockholder ownership structures exist when monitoring costs are not too high nor too low. This can be interpreted as saying that a blockholder ownership structure should

[3]In the BBC program "Dragon's Den", we see many instances of such entrepreneurs who are willing to give up some control of their company in exchange for funding, but are reluctant to cede full control.

[4]The broader message of our paper is that blockholders emerge as a response to the way the firm aggregates preferences of shareholders: see e.g. the ongoing discussion on the objective of the firm - Tirole (2001) and **?**.

be more common in industries which are less mature, where innovation (or where the moral hazard problem) exists but is less important, where there are capital constraints on investors (larger firms) and in countries with higher minority shareholder protection. Since in our set up, blockholders must be active, our interpretation of institutional investors fits more with hedge funds rather than mutual funds. Finally, we show how the ownership structure correlates with firm value and risk profiles (increasing number of blockholders raise both) of firms.

In line with our results, empirical studies find that the second blockholder (or a blockholder who acts as an antagonist to the first blockholder) might be value enhancing. For example, Laeven and Levine (2008) find that Western European widely held firms have the highest Tobin's Q while firms with blockholders have a higher Tobin's Q than firms with only one big shareholder. Lehmann and Weigand (2000) find that a second large shareholder improves the profitability of listed companies in Germany. According to Volpin (2002) in Italy, when blockholders form syndicates the firm market value is higher than when there is a single blockholder. In Europe and Asia, higher dividends are positively related to the number of blockholders (Faccio, Lang, and Young, 2001). Maury and Pajuste (2005) have shown that when there are 2 blockholders with similar interests, the existence of a third blockholder increases the firm value. In Spain the number of blockholders is positively related to a better performance of private firms (Gutierrez and Tribo, 2004).

Our model predicts that firms with multiple blockholders should be characterized by higher risk/return. Carlin and Mayer (2000; 2003) confirm this prediction. Firms with more dispersed ownership tend to invest in higher risky projects, like R&D and skill intensive activities.

Our model confirms the findings of Laeven and Levine (2008) on size and ownership structure. In small firms we see one main blockholder; while in large corporations complex ownership structures are more likely to arise.

Intriguingly, our model also predicts that when multiple blockholders arise, liquidity shareholders pay a price that is lower than what they are willing to pay. Hence, there is underpricing. This is consistent with the the findings of Stoughton and Zechner (1998) and DeMarzo and Urosevic (2006) where IPO underpricing guarantees the participation of large investors who can monitor and hence be value enhancing. However, in their paper control considerations are absent and hence the role of multiple large investors is not analyzed. The connection between ownership and underpricing is confirmed by the empirical studies of Brennan and Franks (1997), Fernando, Krishnamurthy, and Spindt (2003) and Goergen and Renneboog (2002)).

Finally, the paper contributes to the literature on voting. Typically, the models on voting do not endogenize the individual preferences, the price of votes and hence the voting power of an agent (Dhillon, 2005). When applying voting theories to corporate governance issues, the firm value (and hence share prices) and shareholders decision are closely related. Furthermore, an investor can decide how many shares to buy and their voting decision changes depending on the block he chooses to buy.

The price, being set by the initial owner, becomes an endogenous variable that affects and is affected by the existence of a second blockholder and the voting outcome.

The paper is organized as follows. Section 2 discusses the related literature, Section 3 outlines the model. In section 4 we describe the equilibrium requirements. Section 5 describes all the possible equilibria. In section 6 we derive the empirical implications of the model. Finally, section 7 concludes. All the proofs are in the Appendix.

2 Related Literature

The related theoretical literature focuses on the role of blockholders as a tool to discipline the manager or to take value enhancing actions but do not focus on the conflict on interest we are interested in.[5]

Some exceptions to this are Zwiebel (1995) which suggests that multiple blockholders emerge as the optimal response to a situation where there are divisible private benefits of control; coalitions of small blockholders control the firm and this enables large investors to be part of these strategic coalitions in a large number of firms. In contrast, Bennedsen and Wolfenzon (2000) show that the initial owner in a privately held firm has incentives to chooses an ownership structure that commits him to more efficient decisions. The preferences of the blockholders are driven by the trade off between firm value and divisible private benefits of control. In equilibrium, the minimal coalition needed in order to limit the sharing of the private benefits emerges. Gomes and Novaes (2001) focus on firm's decision when blockholders have veto power. Here, again, the initial owner chooses the ownership structure and there are some divisible benefits of control. The trade off the initial owner faces is between selling the shares at a low price to investors and holding control or setting a higher price and sharing the private benefits. An ownership structure with multiple blockholders might arise. The veto power of the blockholders can determine either a paralysis of the decision process or a more efficient outcome. Noe (2002), Edmans and Manso (2008) look at the role of blockholders as a tool to discipline the manager through the threat of exit as an information tool. Bloch and Hege (2001) argue that the existence of two blockholders reduces the appropriation of private benefits of control. When competing for the acceptance of a proposal, two existing blockholders choose projects with limited private benefits of control in order to attract the votes of their minority shareholders. However they focus on the contest between two blockholders leaving open the question why two blockholders arise in the first place.

All these approaches essentially view the emergence of multiple blockholders (i) as a choice by the initial owner whereby he can commit himself to higher firm value and (ii) focus on some exogenously given factor such as capital constraints or private benefits of control. (iii) only allow equity as the method of financing. Hence they assume away the problems of strategic interaction between block-

[5]See Holmstrom and Tirole (1993), Admati, Pfleiderer, and Zechner (1994) Burkart, Gromb, and Panunzi (1997), Pagano and Röell (1998), Bolton and Von Thadden (1998), Maug (1998), Katz and Owen (2000).

holders by letting the initial owner choose the ownership structure directly or do not endogenize the emergence of the blockholders.

In contrast, our paper (i) lets the initial owner choose the ownership structure indirectly through pricing of shares: the model could work as well with the initial owner choosing the ownership structure directly when there is a resale market for shares. It is therefore more general. Second, we explicitly allows strategic interactions between blockholders. This is an important consideration as free-riding considerations between blockholders can be very important when we move from single to multiple blockholders. (ii) We endogenize the degree of private benefits as the risk/return choice. This allows us both to highlight how private benefits can differ depending on share participation and to avoid assumptions over the divisibility of private benefits of control. (iii) We allow the initial owner to borrow as well.

Our paper is also related to Admati, Pfleiderer, and Zechner (1994) and DeMarzo and Urosevic (2006) who analyze the trade off between risk sharing and monitoring: risk sharing is worse with concentration in ownership but monitoring is more efficient. A single large blockholder emerges in response to a large gain from monitoring since he loses out on risk sharing (diversification).[6] Our paper is not about this trade off, but rather on the aggregation problems inherent when there are conflicts of interest. These conflicts happen to be on risk sharing objectives of the firm.

3 The Model

The initial sole owner of a firm needs a minimum amount of capital, K, to finance an expansion. He is willing to invest w_E in the firm. The remaining amount $K - w_E$ needs to be raised by issuing equity– either privately or by taking the firm public. The initial owner can raise more capital than is strictly needed for the expansion, i.e. $I \geq K$ and use it to diversify risk. In this case the extra capital is invested in the risk free asset, which is the only other asset in the economy. Initially we assume that $I = K$; later in section 5.3 we show that this assumption is without loss of generality.

In exchange for capital K, he offers an aggregate fraction $1 - \alpha_E$ of future profits to the new investors, keeping α_E for himself. A higher α_E implies a higher fraction of profits for the initial owner and a higher control of the firm; while the lower α_E is, the higher is his dilution and the lower is his control. When $\alpha_E = 0$, the initial owner does not hold any shares in the firm, i.e. he exits from the firm.

The set of (potential) outside investors is denoted M and the number of such investors is also (to save on notation) denoted M. We assume that the set M is partitioned into two types of investors

[6]As one blockholder cannot commit to a particular level of monitoring before trading, he/she can increase firm value increasing his/her participation and thus monitoring. The effect of the value increase is reflected in the higher price of the newly acquired shares, but not on those already owned. The incentive of the blockholder to monitor derives from the gain through the increase of value of the initially owned shares.

– those who are *active* shareholders, denoted M_A, and those who are *passive* shareholders, denoted M_P. Active shareholders are those who anticipate that their vote is going to have an impact on the decisions of the firm while passive shareholders act competitively and take the decisions as given, ignoring their own potential influence.

For simplicity, we assume the following utility function for the initial owner and all investors:

$$U_j = -\frac{1}{\gamma}e^{-\gamma Y_j} \tag{1}$$

where $j = \{i, E\}$, i refers to the generic outside investor, E refers to the initial owner, γ is the parameter of risk aversion and $Y_j = f(w_j)$ is the final wealth when a fraction w_j of the wealth is invested in the project. This function has constant absolute risk aversion so that the level of the investors wealth does not matter for his investment decisions. We assume that initial owner and investors have the same wealth of 1[7] and they decide how to divide their wealth between the firm's shares and a risk free asset. We assume no financial constraints so both the initial owner and the investors can borrow money at the risk free rate or go short in the firm shares, i.e. w_j is not bounded. Hence when raising capital the initial owner decides not only on the ownership structure but also the composition of his portfolio between debt and equity. The higher the debt the higher the risk exposure but the higher the control he has. We normalize the gross return of the risk free asset to 1. Hence the certainty equivalent representation of the utility function (1) is:

$$(1 - w_j) \cdot 1 + w_j E\left[Y(w_j)\right] - \frac{\gamma}{2} w_j^2 V\left[Y_j(w_j)\right] \tag{2}$$

where $E\left[Y(w_j)\right]$ and $V\left[Y_j(w_j)\right]$ are respectively the expected value and the variance of the final wealth.

As mentioned in Section **??**, we need to introduce some friction in the model to give the owner a reason to be less diversified. We do this through a commitment problem: monitoring the manager increases firm value by $f(m)$ but has a fixed cost $c(m) = \bar{m}K$. The owner cannot commit to monitor unless he has a significant share of the firm's returns. When outside investors observe a low share of the initial owner, this is reflected in a lower firm value and hence a lower share price. The variable $m \in \{0, 1\}$ captures the monitoring decision of the initial owner. In particular we assume, as in Maug (1998), that if the initial owner monitors, $f(m) = 1$ per unit of K at a monetary cost of $c(m) = \bar{m} < 1$ per unit of K.[8] If no monitoring is exerted, $f(m) = 0$ and $c(m) = 0$. We assume that the cost of monitoring is not divisible among shareholders.[9]

[7] Since the motivation of this paper is to show that blockholders are sub-optimally diversified, we are interested in the *fraction* of their wealth that is invested in the firm. Introducing heterogeneity in wealth levels would not change the results unless attitudes to risk depend on wealth. In this case, for sure, the identity of blockholders would be easier to predict: more wealthy investors who are relatively less risk averse are more likely to be blockholders.

[8] If $\bar{m} > 1$ the initial owner would never choose to monitor.

[9] This assumption strengthens rather than weakens our results, as the main driving force as to why blockholders emerge in our model is not the sharing of monitoring costs, but rather the conflict of interest between the initial

The issue we are interested in is the control over important operating decisions of the firm. We model this as a choice between different projects which differ in their risk return profile or as a choice between managers in the market who come with different reputations for risk: Each project (or manager) available has a value which is normally distributed with mean $E\left(R(X), m\right)$ and standard deviation $\sigma(X)$. $X \in [0, \bar{X}]$ is the possible frontier of risk-return profiles among which the shareholders can choose.

The conflict of interest between shareholders is captured in a simple way through the uni-dimensional linear efficiency frontier of possible projects as explained above. Hence, $E(R(X), m) = \bar{R}X + f(m)$ and $\sigma^2(X) = \sigma^2 X^2$. The decisions of shareholders are aggregated through a voting mechanism where each voter votes on possible choices of X. $X = 0$ is associated with expected returns of 1 (the same as the risk free asset) and 0 variance while $X = \bar{X}$ represents the maximum expected return ($\bar{R}\bar{X}$) and maximum variance ($\sigma^2 \bar{X}^2$). If no monitoring is carried out and $X = 0$ the firm has a negative NPV as it has an expected return lower than the risk free asset. The conflict of interest arises because the initial owner when monitoring holds a sub-optimally diversified portfolio and will vote for a low X, while outside investors who can choose an optimally diversified portfolio, will prefer a high X.

In addition, we assume that $\bar{X} > \frac{\bar{R}}{\gamma\sigma^2}$. Recall that $\bar{X}$ represents the upper bound on the risk return choice. A high $\bar{X}$ represents a greater choice of projects, if it is too small, then we may be artificially causing the choice of investors and initial owner to come very close, hence removing any potential conflict of interest.

Assume first that X is fixed. We first derive the utility function of the initial owner and outside investors. Investors get a share of the returns that is proportional to their contribution w_i, i.e. $\alpha_i = \frac{w_i}{K - w_E}(1 - \alpha_E)$ (assuming full subscription). Hence the investors' utility function is:

$$\frac{w_i}{K - w_E}(1 - \alpha_E)\left(\bar{R}X + f(m)\right) - \frac{\gamma}{2}\sigma^2 X^2 \left(\frac{w_i}{K - w_E}(1 - \alpha_E)\right)^2 + (1 - w_i) \cdot 1 \tag{3}$$

The last part of this expression represents the share of wealth invested in the risk free asset, the opportunity cost of investing in the firm. The first part represents the share of the returns from investing in the firm and the second part represents the dis-utility from the risk of investing in the firm.

Equation (3) can be re-written as:

$$EU_i = \alpha_i \left(\bar{R}X + f(m)\right) - \frac{K - w_E}{1 - \alpha_E}\alpha_i - \frac{\gamma}{2}\sigma^2 X^2 \alpha_i^2 + 1 \tag{4}$$

Note that $\frac{K - w_E}{1 - \alpha_E}$ can be interpreted as the price per share. Hence the first part is the expected return from investing in the firm, the second part is the price paid and the third is the dis-utility from investing in a risky asset. Investor i will maximize (4) by choice of α_i, given $\alpha_E, K - w_E, X, f(m)$.

owner and other potential investors. If in addition, investors can share in monitoring there is an additional reason for blockholders to emerge.

Investors and the initial owner are identical in their risk aversion and endowment and in their possibility to diversify. The only asymmetry between them is that the initial owner has all the bargaining power in setting the share price (α_E and $K - w_E$) and in choosing the level of monitoring.[10]

The initial owner's exponential utility function can be re-written in terms of certainty equivalent as:

$$EU_E = \alpha_E(\bar{R}X + f(m)) - \frac{\gamma}{2}\sigma^2 X^2 \alpha_E^2 + 1 - w_E - c(m) \tag{5}$$

The initial owner chooses α_E, w_E and m to maximize (5) subject to the constraint that he needs to raise the capital, i.e. $K - w_E \leq \sum_i w_i$, or equivalently $1 - \alpha_E \leq \sum_{i=1}^{N} \alpha_i$ where $\sum_i \alpha_i$ is the sum of the total shares demanded and the supply is given by $1 - \alpha_E$. In addition he needs to satisfy $\alpha_E + \sum_i \alpha_i = 1$.[11]

3.1 The Extensive Form Game

The timing of the game is as follows: in period 0 the initial owner decides how much money he is going to invest from his own wealth w_E and how many shares he tenders, α_E. In case he wants to invest more than his initial wealth he raises debt ($w_E > 1$) at the risk free rate. These choices in period 0 are equivalent to the initial owner announcing the share price and the amount of shares tendered. In period 1 investors decide how many shares to buy, having observed the share price. A passive investor chooses an amount of shares which optimizes her portfolio taking the voting decision as given, i.e., disregarding the effect her own votes can have on the voting outcome.[12]

An active investor, therefore, will anticipate the effect that her vote can have on the voting outcome and her demand for shares will consider the potential effect of ownership structure on X. To make things simple, we will assume that passive investors *never* vote and active investors *always* vote. Notice that the demand of passive investors cannot be distinguished from those of active investors who strategically decide to hold the same level of shares (which are optimal for diversification purposes). If there is under-subscription, then the project cannot go ahead. If there is oversubscription, we assume that this is a stable situation only when no investor who gets shares is willing to sell them at a price lower than the maximum price that an excluded investor is willing to pay.[13]

[10] Hence, as in most IPO models (for a review see Jenkinson and Ljungqvist (2001) and Welch and Ritter (2002)), the initial owner acts like a monopolist or Stackelberg Leader.

[11] Note that we allow $w_E < 0$ which means that the initial owner can sell shares in return for cash from the other shareholders in order to invest in the risk free asset. $w_i < 0$ on the other hand means that investors are going short in shares, although we see later that this never occurs in equilibrium. Finally when $w_j > 1$ with $j \in \{E, i\}$ it means that the players borrow money (at the risk free rate) in order to invest in the firm.

[12] A more micro founded approach is to assume that there is a cost of voting which determines how many people vote in equilibrium. We could incorporate this but at the risk of adding to already heavy notation and complexity in the model.

[13] We do not explicitly model the secondary market in shares but we capture some of the spirit of the secondary market by imposing this particular additional requirement of Nash equilibrium.

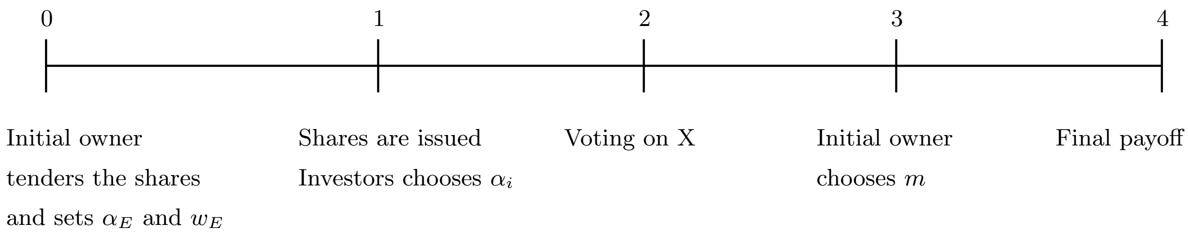

Figure 1: The Time Structure

In period 2 the shareholders of the firm – the initial owner and the investors who bought the shares – have to take a decision on X. At this stage, the ownership structure is common knowledge. The decision X is taken through voting. We assume one-share-one-vote and majority voting rule. We show later that once the ownership structure is fixed, preferences on X are single peaked so that the Condorcet winner[14] in the space $[0, \bar{X}]$ is the median shareholders choice. Voting is costless. However since only a subset of investors are assumed to vote the Condorcet winner must be chosen from among the ideal points of the voting subset only. In period 3 the initial owner decides whether to monitor inducing an increase in the expected return from the project or not to monitor and have a lower expected return.

Finally at period 4 the outcome is realized. The time structure of the game is sketched in Fig.1.

We assume that the financial market is efficient, that the number of investors in the market is sufficiently big so that there is never a problem of excess supply and investors are rational: they are willing to buy shares as long as they make non-negative risk adjusted net return. However, the firm value (i.e. expected returns) depend on both the initial owner's monitoring and on the decision, X, that is taken once the ownership structure is determined.

Hence, in our model there is both an issue of incentives and of control. Ideally, the initial owner and the investors would like to have as few shares as possible and choose maximum return, even though it comes with high risk. However, the incentive problems associated with monitoring imply that the initial owner faces a trade off between increasing the value of the firm and diversification. In the event that the initial owner resolves this trade off by holding a less diversified portfolio, his choice of X conditional on his shareholdings is likely to be biased towards lower X. The first best choice for the investors, on the other hand, is to hold fewer shares, diversify optimally, and have a high return (high X). Hence there is an endogenously generated conflict of interest. Notice that outside investors are identical and hence have the same preferences: the only *ex-ante* conflict on risk- return is between the initial owner on the one side and other investors on the other. *Ex-post* conflicts depend on equilibrium ownership structure.

As a result, active investors too face a trade-off: holding a sufficiently big block means control on

[14]The Condorcet Winner is the project that wins against every other project in pair-wise majority voting.

the voting decisions, but means that diversification is less than optimal. When holding a block an investor incurs a cost that is increasing in the price $\frac{K-w_E}{1-\alpha_E}$ and in the risk exposure.

We can now derive the ideal point $X_j(\alpha_j) \in [0, \bar{X}]$ for any investor j. We can then determine the payoff functions of players given the ownership structure defined as the vector of shares owned by investors: $\vec{\alpha} = (\alpha_E, \alpha_1, ..., \alpha_k)$ where k is the number of active investors who hold shares in the firm.

Lemma 1 *The preferred choice of X given α_j, for any shareholder $j \in \{i, E\}$, denoted X_j is uniquely defined by:*

$$X_j = \min\left[\frac{\bar{R}}{\gamma\sigma^2\alpha_j}, \bar{X}\right] \tag{6}$$

The choice of X depends only on the investor's shareholdings α_j and not on the decision to monitor. Observe that $\frac{\bar{R}}{\gamma\sigma^2\alpha_j}$ is a one-to one function of α_j. Hence we can define $\bar{\alpha} \equiv \frac{\bar{R}}{\gamma\sigma^2\bar{X}}$ as the fraction of shares $\bar{\alpha}$ such that $X_j(\bar{\alpha}) = \bar{X}$.

It follows from Lemma 1 above that once $\vec{\alpha}$ is fixed, preferences of investors and the initial owner on X are single peaked. Hence the Condorcet Winner on the set $[0, \bar{X}]$ is well-defined and is given by the preferences of the median shareholder.[15] Denote $X_{med}(\vec{\alpha})$ as the median X when the ownership structure is $\vec{\alpha}$. To save on notation, we suppress the argument $\vec{\alpha}$. For convenience we will denote the median shareholdings as α_{med}.

Now we derive the payoff functions of the initial owner and each investor (respectively) as follows:

$$EU_E = \alpha_E\left(\bar{R}X_{med} + f(m)\right) - \frac{\gamma}{2}\sigma^2 X_{med}^2\alpha_E^2 + 1 - w_E - c\,(m) \tag{7}$$

– this is maximized by choice of α_E, w_E, m subject to full subscription of shares.

For the investors:

$$U_i = \alpha_i\left(\bar{R}X_{med} + f\,(m)\right) - \frac{K - w_E}{1-\alpha_E}\alpha_i - \frac{\gamma}{2}\sigma^2 X_{med}^2\alpha_i^2 + 1 \tag{8}$$

– this is maximized by choice of α_i. Notice that α_E determines both the price paid and (potentially) X_{med} through the share ownership structure. Hence the indirect utility function for active investors depends on α_E, as well as the anticipated m and $\vec{\alpha}$ given α_E and $K - w_E$. The passive investors' indirect utility depends also on α_E, but here X is taken as given.

Pure strategies of the owner are 3-tuples $(w_E, \alpha_E, m(\alpha_E, X_{med}))$ together with a function from α_E to a voting decision over X. Pure strategies of investors are functions from $(\alpha_E, K - w_E)$ to a shareholding α_i and a voting decision over X.[16] This describes an extensive form game, Γ where the set of players are the initial owner and other active investors, the pure strategies and payoffs are as above.

[15] Consider the frequency distribution of shares of *initial owner and active investors only* on the set X. The median X is the unique X_j such that exactly half the shares are on either side of it. Since it is common knowledge that passive investors never vote the α_{med} is defined only on the basis of shares of initial owner and active investors.

[16] Since there is pairwise voting, voting is assumed to be sincere, we rule out strategic agenda setting issues.

4 Equilibrium definition

We define investors who endogenously hold the optimally diversified portfolio as *liquidity shareholders* and the shares held by these investors as $\alpha_{l,j}$ when X_j is the vote outcome. Out of these shareholders some are active and go and vote, and some are passive.

An investor is a *blockholder* rather than a liquidity shareholder if in equilibrium he holds a sub-optimal portfolio: $\alpha_i > \alpha_{l,j}$ where α_i is the shareholding of blockholder 1 and X_j is the anticipated vote outcome.

Our notion of equilibrium is subgame perfect equilibrium of the game described in Fig. 1. Note that because in many potential equilibria more than one investor need to buy shares for the initial owner to find it worthwhile to start the firm, there is always a No Trade equilibrium. In this equilibrium, no investor buys any shares anticipating that no other investor will buy shares. Below we provide a definition for equilibria with positive trade.

Equilibrium is a monitoring level, $m \in \{0, 1\}$, a fraction α_E^* of shares held by the initial owner, fraction of wealth invested w_E^*, a decision X_{med} and an allocation of shares among investors, $\vec{\alpha}^*$ such that:

1. α_E^* and w_E^* maximize the utility of the owner given the anticipated demand, the anticipated monitoring level, m, and the anticipated ownership structure $\vec{\alpha}(K - w_E, \alpha_E)$.

2. Each active investor chooses α_i to maximize her utility given $K - w_E^*, \alpha_E^*$, the anticipated m and the anticipated α_{-i}.

3. Each passive investor chooses α_i to maximize her utility given $K - w_E^*, \alpha_E^*$ and the anticipated m and X_{med}.

4. In equilibrium there must be full subscription. There can be excess demand in equilibrium as long as no investor who owns shares is willing to sell them at a price lower than the maximum willingness to pay of the excluded investors.

5. The monitoring decision must be optimal for the owner.

6. Expectations are rational.

We will distinguish between monitoring and no monitoring equilibria as well: (No) monitoring equilibria are those where the initial owner is assumed to choose (not) to monitor in the last stage, and anticipating that, he chooses the optimal α_E in period 0.

We solve the problem in two stages: first assume the monitoring decision and then find the equilibrium value. Then compare the two to see which one the initial owner prefers. We are interested in equilibria which are characterized by a conflict of interest among shareholders: such equilibria are of two types: (1) the initial owner is the sole blockholder and is in control of the voting outcome, i.e.

$X = X_E$. We call this the *Initial Owner Equilibrium.* (2) the initial owner and a subset of the active investors are blockholders. When there are $n+1$ blockholders, including the initial owner we call it an *n-blockholder equilibrium.*[17] In case of blockholder equilibria we focus only on *symmetric* equilibria, where each blockholder holds exactly the same shares. However we allow asymmetry among active investors on the decision to be a blockholder. When neither the initial owner nor other investors are blockholders we will call it a *Liquidity Shareholder Equilibrium.* When there is no conflict among shareholders we call it *No Conflicts Equilibrium.*

As we see later, the model admits multiple blockholder equilibria. We consider only equilibria where the fraction of active investors among liquidity investors is fixed and known at $\lambda = \frac{M_A}{M}$. λ captures the anticipated fraction of small shareholders who take part in the voting decisions of the firm. In general we expect λ to be "small" reflecting the proportion of active investors in the market for shares of the firm.

5 Equilibria

Now we look for the subgame perfect equilibria of this game. We solve the game by backward induction. We first look for the conditions under which the initial owner has the incentive to monitor. We then analyze the subgame following the pricing and issuing of shares, called the *ownership subgame* as it is at this stage that outside investors make their decisions on shares, having observed (α_E, w_E) (see Section 5.1). Finally we solve for the subgame perfect equilibria in Section 5.2. Before that however, we need a few preliminary lemmas.

Lemma 2 *The initial owner monitors iff $\alpha_E \geq \bar{m}$.*

The initial owner can commit to monitor only when he owns enough shares (as e.g. in Shleifer and Vishny (1986)). Observe that the decision to monitor is independent of the voting outcome.

Given the investors beliefs on $X_{med} = X_j$ we can determine the demand for shares by passive investors:

Lemma 3 *Let X_j be the belief on the voting outcome where $j = \{E, i\}$ and $K - w_E$ the capital demanded. Then liquidity shareholders demand:*

$$\alpha_{l,j} = \frac{X_j \bar{R} + f(m) - \frac{K - w_E}{1 - \alpha_E}}{\gamma X_j^2 \sigma^2} \tag{9}$$

Lemma 3 shows that the fraction of shares chosen by the passive investors depends on their beliefs on the voting outcome X_j. When they believe the voting outcome is going to be low risk/return, i.e. low X_j, the optimal portfolio they choose has a higher fraction of shares and vice versa. Observe that

[17] For expositional simplicity, we refer only to investors (and not the initial owner) as blockholders although, strictly speaking, the initial owner is the "first" blockholder.

the value function of liquidity investors, $U_{l,j}$ is always bigger than 1 since there is no restriction on $\alpha_{l,j}$: if the utility from holding the firms' shares is less than the risk free asset, then $\alpha_{l,j}$ could be negative, i.e. investors can go short on the shares. We show later that the constraint on full subscription by the initial owner implies that in equilibrium $\alpha_{l,j} > 0$.

The participation constraint of liquidity investors, $U_{l,j} \geq 1$, also implies that the maximum price they are willing to pay, $\frac{K-w_E}{1-\alpha_E}$, is higher, the higher is the risk/return profile of the firm, i.e. the higher is X_{med}.

Lemma 4 *Liquidity shareholders are better off than blockholders regardless of the monitoring decision.*

Liquidity investors hold the fraction of shares that provides optimal diversification given the anticipated voting outcome, X_j. Any other shareholding gives lower utility.

The next lemma shows that it turns out that if the share price is sufficiently high,[18] then the optimal liquidity shareholding is always smaller than $\alpha_{med} = \alpha_j$.

Lemma 5 *Assume* $\frac{K-w_E}{1-\alpha_E} > f(m)$, *then* $\alpha_{l,j} \leq \alpha_j$.

5.1 The Ownership Subgame

In this section we analyse the *ownership subgame* i.e. the second stage of the game where outside investors decide how many shares to buy after observing the pair $(\alpha_E, w_E) \in S \equiv [0,1] \times (-\infty, +\infty)$. This is the crucial and most complicated part of the analysis where ownership structure is decided conditional on the observed price. The complication arises from the fact that both α_E and w_E are chosen from a continuum, hence there are an infinity of possible subgames. As we consider symmetric blockholder equilibria, w.l.o.g. let α_1 be the shares of the representative outside blockholder. Let U_1^{nBH} denote the value function of a blockholder in an ownership structure which admits n such blockholders with shares $\alpha_1, ..., \alpha_n$. Since we consider only symmetric blockholder equilibria, $\alpha_1 = \alpha_2 ... = \alpha_n$ so we may think of 1 as the representative blockholder. $U_{l,1}$ denotes the value function of a liquidity shareholder as before.

Definition 1 *An Equilibrium Ownership Structure (EOS) corresponding to a pair* (α_E, w_E) *is an equilibrium of the subgame beginning at the information set* (α_E, w_E). *In particular the following must be satisfied in equilibrium: (1)* $U_1^{nBH} \geq 1$*(if there are any blockholders in equilibrium) and* $U_{l,j} \geq 1$ *(if there are any liquidity shareholders in equilibrium). We call this the Participation Constraint or PC, (2) No active investor wants to unilaterally increase or decrease his shares. We call this the Incentive Constraint or IC. (3) Passive investors maximize their utility conditional on the anticipated* $X = X_{med}$. *(4) no investor who owns shares is willing to sell them at a price lower than the maximum price that any excluded investors is willing to pay.*

[18] Later we show that in equilibrium this always happens as the initial owner maximizes the share price in order to reduce dilution and the loss of control.

The equilibrium concept is that of Nash between active investors, and passive investors act to maximize their utility given their beliefs on X, which must be the right beliefs so $X = X_{med}$. Note that in our model, the initial owner does not directly determine the ownership structure in equilibrium, he can only influence it through the choice of share price. We believe that this structure is much more plausible: given that there is a secondary market for shares, in practice, the initial owner does not really determine ownership structure directly.

Definition 2 *A symmetric n-Blockholder ownership structure is one where there are $n > 0$ active investors with shares α_1 each and $N_A \geq 0$ active investors with shares $\alpha_{l,1}$ such that $\alpha_1 > \alpha_{l,1} > 0$, and $X_{med} = X_1$.*

We will call an EOS that satisfies Definition 2 an n- Blockholder EOS or n BH EOS. Denote the size of the shareholdings of other *active* investors excluding investor 1 as α_{-1}. So $\alpha_{-1} = (n-1)\alpha_1 + N_A\alpha_{l,1}$.

Lemma 6 *Suppose $\alpha_E > 0$ and $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$. If a n-Blockholder ownership structure is an EOS then (1) $\alpha_{-1} > 0$; (2) $\alpha_1 + \alpha_{-1} \geq \alpha_E > \alpha_1$; (3) $\alpha_E \geq N_A\alpha_{l,1}$. $X_1 > X_E$ in any such equilibrium and $N_A\alpha_{l,1} + n\alpha_1 \geq \alpha_E$.*

Corollary 1 *Assume full subscription of shares. If an n-blockholder EOS exists, $\alpha_E > 0$ and $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$ each blockholder holds*

$$\alpha_1 = \frac{\alpha_E (1+\lambda) - \lambda}{(1-\lambda)\, n} \tag{10}$$

This corollary follows from point (2) and (3) in Lemma 6 and the fact that the utility of a blockholder is decreasing in α_1. In particular from point (2) and when the price condition of Lemma 5 is satisfied, it follows that $\alpha_{l,1} < \alpha_1 < \alpha_E$. Hence, when blockholders emerge in equilibrium, they mitigate the conflicts of interest between the initial owner and the liquidity shareholders. If, to the contrary, the share price is too low, investors will demand all the shares tendered. We show later that this cannot be an equilibrium as the initial owner will always maximize the share price.

We now define the *first best* choice for investor i as the optimal α_i assuming that in the second stage agent i acts as a dictator in the choice of X.

Lemma 7 *Assume $\frac{K-w_E}{1-\alpha_E} > f(m)$, the first best choice of the investors is $X = \bar{X}$ and $\alpha_i = \bar{\alpha}_l$.*

This lemma helps to clarify the conflict of interest between investors and the initial owner. Outside investors prefer to diversify maximally, i.e. buy the smallest amount of shares and choose the project with maximum returns. When the initial owner does not have a monitoring constraint, he would also prefer the same diversification as we show later (Proposition 1). However the monitoring constraint forces him to have a different preference ex-post (after shares are allocated) than outside investors. Given the constraint on his shareholdings ($\alpha_E \geq \bar{m}$), he prefers lower return/risk projects while outside investors prefer higher risk/return.

The ownership structure can be of four types based on the who is the median shareholder (as discussed above in Section 4): (1) The *Initial Owner EOS*, where $X_{med} = X_E < \bar{X}$ and all investors are liquidity shareholders . (2) *Liquidity Shareholder EOS* where active liquidity investors (who hold a perfectly diversified portfolio) are in control of the firm, $X_{med} = \bar{X}$. (3) *No Conflicts EOS* where $X_{med} = X_E = \bar{X}$, hence there are no conflicts of interest between the initial owner and outside investors and all investors are liquidity shareholders. (4) An *n-Blockholder EOS*, where $X_{med} = X_1$ and n active investors hold a non-perfectly diversified portfolio. The first three types are all ownership structures with no blockholders. However we distinguish between them, because of the differences in the median shareholder (and X_{med}). In the first case, the initial owner always gets his preferred X, in the second the outside investors get their most preferred X. In the third, there is no divergence between the two. In what follows we describe the EOS for all pairs (α_E, w_E).

Let η_j be the fraction of shares corresponding to one share and hence define $\epsilon = \frac{\gamma X_j^2 \sigma^2}{\eta_j}$. In general we consider this amount very small.

Here are the definitions of some symbols we are going to use for the rest of the paper. Let:

$$\hat{\alpha}(n) \equiv \max[\hat{\alpha}_1(n), \hat{\alpha}_2(n)] \tag{11}$$

$$\hat{\alpha}_1(n) \equiv \frac{2\lambda}{2(1+\lambda) - n(1-\lambda)} \tag{12}$$

$$\hat{\alpha}_2(n) \equiv \frac{2\bar{\alpha}(1-\lambda)n + \lambda - \sqrt{\lambda^2 + 4\bar{\alpha}(1-\lambda)n\left(\bar{\alpha}(1-\lambda)n - 1 - 2\bar{\alpha}(1+\lambda)\right)}}{2\,(1+\lambda)} \tag{13}$$

$$\check{\alpha}(n) \equiv \frac{n(1-\lambda)\bar{\alpha} + \lambda}{1+\lambda} \tag{14}$$

$$\tilde{\alpha}(n) \equiv \frac{\lambda}{(1-\lambda)\left(1 + n\frac{\epsilon}{\bar{R}}\right)} \tag{15}$$

$$\underline{w}_E^E(\alpha_E) \equiv K - \left(\frac{\bar{R}^2}{\gamma\sigma^2\alpha_E} + f(m)\right)(1-\alpha_E) + \epsilon \tag{16}$$

$$\underline{w}_E^n(\alpha_E) \equiv K - \left(\bar{R}X_1 + f(m) - \frac{\gamma}{2}X_1^2\sigma^2\alpha_1\right)(1-\alpha_E) \tag{17}$$

$$\underline{w}_E^{LS}(\alpha_E) \equiv K - \left(\bar{R}\bar{X} + f(m)\right)(1-\alpha_E) + \epsilon \tag{18}$$

$$\underline{\underline{w}}_E(\alpha_E) \equiv K - f(m)(1-\alpha_E) \tag{19}$$

Suppose α_E is fixed. Then $\underline{w}_E^E(\alpha_E)$ is the minimum w_E that the initial owner needs to invest in the firm in order to guarantee that liquidity shareholders' participation constraint is satisfied and $X < \bar{X}$. $\underline{w}_E^n(\alpha_E)$ on the other hand, is the analogous expression if a blockholder participates. $\underline{w}_E^{LS}(\alpha_E)$ is the analogous expression to $\underline{w}_E^E$ when $X = \bar{X}$: it is the minimum w_E that the initial owner needs to invest in the firm to satisfy the participation constraint of liquidity shareholders when $X = \bar{X}$. $w_E \leq \underline{\underline{w}}_E(\alpha_E)$ guarantees $f(m) > \frac{K - w_E}{1-\alpha_E}$. Note that when $m = 1$ this condition implies that the price of the risky asset is lower than that of the risk free one. Later we show this is always the case in equilibrium. $\hat{\alpha}(n)$ is the value of α_E such that if $\alpha_E \leq \hat{\alpha}(n)$ then $\underline{w}_E^E \leq \underline{w}_E^n$. In particular if $\alpha_E \leq \hat{\alpha}_1(n)$, $\underline{w}_E^E \leq \underline{w}_E^n$ where $X_1 < \bar{X}$, while $\alpha_E \leq \hat{\alpha}_2(n)$, $\underline{w}_E^E \leq \underline{w}_E^n$ where $X_1 = \bar{X}$. $\check{\alpha}(n)$ is the value of α_E such that when $\alpha_E \leq \check{\alpha}(n)$, $\alpha_1 = \bar{\alpha}$. Finally $\tilde{\alpha}$ is the value of α_E such that when $\alpha_E \leq \tilde{\alpha}(n)$ a single extra active liquidity shareholder can become pivotal in the voting decision and change it from X_E to $\bar{X}$

Notice that in any EOS the expressions (11) – (19) are functions of $\alpha_E, \lambda, \bar{\alpha}$ as X_1 and α_1 are given respectively by (1) and (10).

Observe that for certain combinations of (α_E, w_E) there always exists a no trade equilibrium where no investors participate in the share issue. Define

$$\underline{w}_E^{NT} = \begin{cases} \underline{w}_E^E & \text{if } \alpha_E > \max\left[\frac{1}{2}, \bar{\alpha}\right] \\ \underline{w}_E^1 & \text{if } \bar{\alpha} \leq \alpha_E \leq \frac{1}{2} \\ \underline{w}_E^{LS} & \text{if } \alpha_E \leq \bar{\alpha} \end{cases}.$$

This is the minimum amount the initial owner needs to invest in order to guarantee that at least one investor is willing to buy share, this is the minimum needed to rule out the No trade equilibrium. This is what the next lemma shows.

Lemma 8 *There always exists a No trade EOS if $w_E < \underline{w}_E^{NT}$.*

Lemma 9 *There exists an Initial Owner EOS, with $X_{med} = X_E < \bar{X}$ for any pair (α_E, w_E), satisfying the following conditions:*

$$\alpha_E \in \left(\max\left(\bar{\alpha}, \min\left[\frac{1}{2}, \max\left[\hat{\alpha}(1), \tilde{\alpha}(n)\right]\right]\right), 1\right] \tag{20}$$

$$w_E \in \left[\underline{w}_E^E(\alpha_E), \min\left[\underline{w}_E^1(\alpha_E), \underline{\underline{w}}_E(\alpha_E)\right]\right) \tag{21}$$

When the initial owner retains a high fraction of the shares, (condition (20)) and the price is low enough to guarantee that investors are willing to buy $(w_E \geq \underline{w}_E^E)$ an Initial Owner EOS might arise. If the price is higher investors are better off just investing all their wealth in the risk-free asset. If the initial owner sets too low a price $(w_E \geq \underline{w}_E^1)$ then holding blocks of shares is less costly and hence an investor might be willing to unilaterally holds more shares and be undiversified but be able to choose a higher return project via the voting decision. Note that condition $\alpha_E < \hat{\alpha}(1)$ ensures that $\underline{w}_E^E < \underline{w}_E^1$. Moreover, if the price is too low, $(w_E > \underline{\underline{w}}_E)$, investors demand all the shares tendered. In order to guarantee that no investor unilaterally deviates from being a liquidity shareholder, we need to impose some conditions on α_E. First of all we need to ensure that no investor prefers to hold a block and change the vote outcome rather than hold the optimal portfolio and let the control be in the hands of the initial owner. This is done by setting $\alpha_E < \hat{\alpha}(1)$ and $w_E < \underline{w}_E^1$. Alternatively a passive liquidity investor, who never votes, might be willing to sell the shares to an active investor who would still hold a perfectly diversified portfolio but could become pivotal simply by going to vote. This cannot be an EOS as condition (4) of the equilibrium conditions would be violated. In order to guarantee that this does not occur we set $\alpha_E < \check{\alpha}(1)$. All these conditions on α_E become irrelevant if the initial owner has the majority of the shares as in such a case the investors cannot gain control over the vote outcome. These 3 conditions together set the upper bound on α_E (see condition (20)). Finally note that in order to guarantee conflicts of interests between investors and initial owner, we set $\alpha_E > \bar{\alpha}$.

The proof of Lemma 9 in the Appendix shows that the interval (21) is not empty when the initial owner monitors, i.e when $m = 1$, $X_1 < \bar{X}$ and $\alpha_E > \hat{\alpha}(1)$. This will be important when we look for the different equilibria.

Lemma 10 *There exists an n-Blockholder EOS, with $X_{med} = X_1 < \bar{X}$, for any pair (α_E, w_E), satisfying the following conditions:*

$$\alpha_E \in \left(\max\left[\check{\alpha}(n)\right], \min\left[\hat{\alpha}_1(n), \frac{1}{2}\right]\right] \tag{22}$$

$$w_E \in [\underline{w}_E^n(\alpha_E), \underline{\underline{w}}_E(\alpha_E)) \tag{23}$$

where $1 \leq n \leq M_A$[19] .

When the initial owner tenders enough shares at a sufficiently low price ($w_E \geq \underline{w}_E^N$) some outside investors are willing to buy a block, that is to hold an undiversified portfolio, in order to be able to determine the vote outcome and hence guarantee a higher return. The upper bound on w_E guarantees, as before, that one single investor is not willing to buy all the shares tendered. In particular when $m = 1$ this condition guarantees that the price of the risky asset is higher than that of the risk free one otherwise investors demand all the shares tendered.

Like in a (discrete) public goods provision problem, the blockholders contribute to the public good provision (i.e. moving the decision on the project closer to the most preferred point) because given the other shareholders contributions, it is a Nash equilibrium for them to contribute as long as the value of the public good to them is sufficiently high. This translates into the condition that α_E is sufficiently low: when $\alpha_E \geq \frac{1}{2}$, the value of holding larger blocks is zero, since the initial owner always holds the majority. However, for lower α_E the incentives to hold larger blocks increases. This is because, in the first place, as α_E decreases fewer shares are needed in order to gain control over X and hence the cost of holding a block is lower. Second, because fewer shares are needed a larger shift in X can be achieved, i.e., $X_E - X_1$ is greater and hence the gain in terms of increase in expected return of becoming blockholder is greater. Hence there exists a threshold, $\hat{\alpha}_1(n)$, such that when $\alpha_E \leq \hat{\alpha}_1(n)$ the utility of being a blockholder is higher than being a liquidity shareholder when the initial owner is in control. On the other hand, when the initial owner's share α_E is very low, $\alpha_E < \check{\alpha}(n)$, a trade between a passive and an active investor is sufficient to cause an active investor to affect the vote outcome without holding a block. Hence, there will be no EOS involving blocks.

Unlike the usual public goods contribution game, however, when blockholders buy a larger block of shares, their preferences over X are closer to the initial owner. This is why the presence of blockholders mitigates, but does not remove, the conflict of interest between the entrepreneur and the outside investors.

[19] Observe that $\hat{\alpha}(n) > \frac{\lambda}{1+\lambda} > 0$ for all $n \geq 1$. Moreover when $n \geq 2$, $\hat{\alpha}(n) \geq \frac{1}{2}$. So $\frac{1}{2} = \min\left(\hat{\alpha}(n), \frac{1}{2}\right)$ for $n \geq 2$. Finally the interval of α_E is always positive when $n \geq \frac{2((1+\lambda)\bar{\alpha}-\lambda)}{(1-\lambda)\bar{\alpha}}$ and in such a case $\max\left[\bar{\alpha}, \tilde{\alpha}(n)\right] = \bar{\alpha}$.

As α_E decreases, the blockholders need to hold fewer and fewer shares in order to be able to change the vote outcome. The shares needed to blockholders to shift decision tconverge to $\bar{\alpha}$ and their choice of X converges to $\bar{X}$. This is shown in the following Corollary. Observe that although the first best is chosen, the blockholders still hold a non-perfectly diversified portfolio. At the same time if the initial owner retains fewer shares, $\alpha_E \leq \bar{\alpha}$, his risk exposure is limited and he chooses very risky projects. Hence there are no conflicts among shareholders. In both these cases As before, if α_E is smaller than $\bar{\alpha}$ then there are no conflicts and no blocks in an EOS. $\alpha_E \geq \tilde{\alpha}(1)$ guarantees that no outside investor has the incentive to offer to buy the shares from a passive investor at a higher price than the one offered by the initial owner and be able to shift the decision to $\bar{X}$. Finally $\alpha_E \leq \hat{\alpha}_2(n)$ guarantees that n blockholders have not the incentive to deviate and choose to be liquidity shareholder and let X shifts to X_E.

Corollary 2 *There exists an n-Blockholder EOS, with $X_{med} = X_1 = \bar{X}$, for any pair (α_E, w_E), satisfying the following conditions:*

$$\alpha_E \in \left(\max\left[\bar{\alpha}, \tilde{\alpha}(n)\right], \min\left[\check{\alpha}(n), \hat{\alpha}_2(n), \frac{1}{2}\right]\right] \quad (24)$$

$$w_E \in [\underline{w}_E^n(\alpha_E), \underline{\underline{w}}_E(\alpha_E)) \quad (25)$$

where $1 \leq n \leq M_A$.

Note that both Lemma 10 and Corollary 2 we do not have a uniqueness result: it is possible that there is an Initial Owner EOS for the same parameters. However when $n = 1$, we can show that there is no Initial Owner EOS for these parameters: one investor is willing to deviate unilaterally from being a liquidity shareholder and hold a block. Hence in this case the Initial Owner EOS cannot be a Nash equilibrium.

We present this case of a one blockholder equilibrium as Corollary.

Corollary 3 *There exists a 1-Blockholder EOS, with $X_{med} = X_1$, for any pair (α_E, w_E), satisfying the following conditions:*

$$\alpha_E \in (\max\left[\bar{\alpha}, \tilde{\alpha}(n)\right], \hat{\alpha}_E(1)] \quad (26)$$

$$w_E(\alpha_E) \in [\underline{w}_E^1(\alpha_E), \underline{\underline{w}}_E(\alpha_E)) \quad (27)$$

Moreover, there does not exist an Initial Owner EOS for any w_E when α_E is in interval (26).

When the fraction of shares retained by the initial owner is sufficiently small, a 1-Blockholder EOS can exist. In such a case one investor has the incentive to hold a large block rather than holding an optimal portfolio. This is because he needs a smaller block to both win the vote (using the vote of the active liquidity investors) and to have a large shift in X. This Corollary is at the base of our main result (see the Section 5.2.4), since for this specific combination of α_E and w_E (see conditions (26) and (27)) only Blockholder equilibria are possible.

Let $\tilde{\alpha}(n)$ the smallest solution for α_E of the equation $\alpha_1 = \bar{\alpha}_l$ and

The following lemma and corollaries demonstrate the EOS that arise in the interval $\alpha_E \in [0, \tilde{\alpha}_E(n)]$. Let:

Lemma 11 *Let $n \in [1, M_A - N_A]$. There exists a Liquidity Shareholder EOS, with $X_{med} = \bar{X}$ with $N_A + n$ active investors for any pair (α_E, w_E), if:*

$$\alpha_E \in \left(\bar{\alpha}, \min\left(\tilde{\alpha}(n), \frac{1}{2}\right)\right] \tag{28}$$

$$w_E(\alpha_E) \geq \underline{w}_E^{LS}(\alpha_E) \tag{29}$$

When α_E is sufficiently small, $\alpha_E \in [\bar{\alpha}, \tilde{\alpha}(n)]$, there are sufficiently many active liquidity shareholders so that $\alpha_{med} = \bar{\alpha}_l$ and there will be no blocks in the EOS. In particular we can have two cases: one where the λ fraction of the liquidity shareholder is sufficient to change the vote outcome (i.e. $n = 0$) and the other there there are n active investors in addition to the fraction λ (N_A) active liquidity investors who vote and ensure that the outcome is $\bar{X}$. Since α_E is low enough, they do not need to be sub-optimally diversified to achieve this.

Finally we need to analyze the case when the initial owner retains very few shares or when he sells the firm.

Corollary 4 *There exists a No Conflicts EOS, with $X_{med} = X_E = \bar{X}$ iff $\alpha_E \in (0, \bar{\alpha}]$, $w_E \geq \underline{w}_E^{LS}(\alpha_E)$.*

When the initial owner retains very few shares, i.e. $\alpha_E \in (0, \bar{\alpha}]$, his preferred return is $X_E = \bar{X}$. Hence, no conflicts of interest arise and there is no incentive for any investor to hold a block.

Corollary 5 *There exists a Liquidity Shareholder EOS, with $X_{med} = \bar{X}$, if $\alpha_E = 0$, and $w_E \geq \underline{w}_E^{LS}(\alpha_E)$.*

When the initial owner sells the firm, $\alpha_E = 0$, the liquidity shareholders are in control and they can choose their preferred return, $X_{med} = \bar{X}$.

Fig. 2 shows graphically the different EOS which can arise when $n = 1$. From the graph we can see that the 1 Blockholder EOS emerges for intermediate values of α_E and may even involve negative values of w_E, that is the initial owner does not need to invest any money from his own wealth, but he is rather compensated for the entrepreneurial idea.

We now use these lemmas when we describe the EOS that support a particular type of subgame perfect equilibrium. The first such equilibrium we characterize is the no-monitoring equilibrium: Suppose the initial owner decides not to monitor in the last stage. We show that he always chooses $\alpha_E = 0$ and the ownership structure that emerges is a Liquidity Shareholder Ownership Structure: the initial owner either sells the firm or does not invest in it. Our interest in the no monitoring payoff stems from the fact that in any monitoring equilibrium, the owner would have to get a higher payoff from monitoring in equilibrium than non-monitoring.

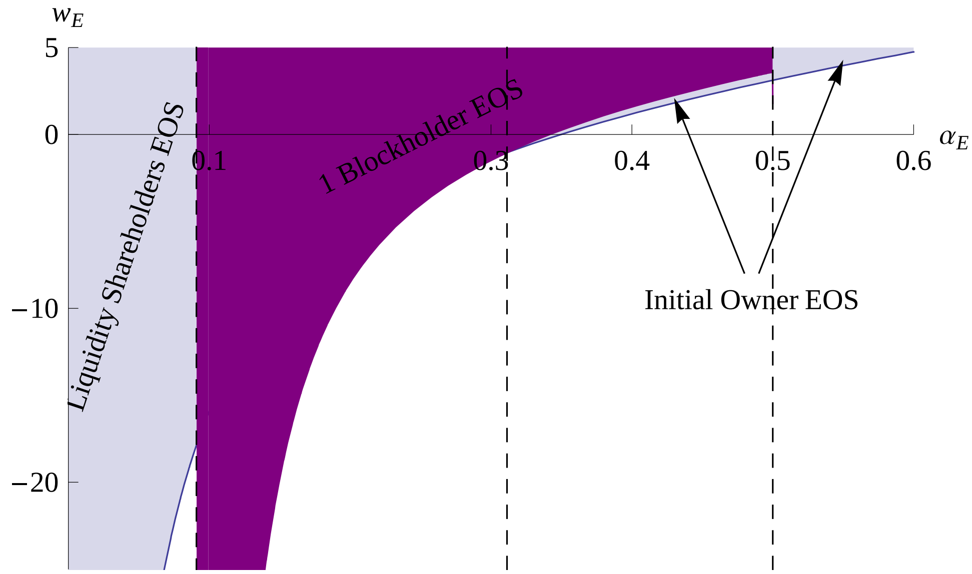

Figure 2: Equilibrium Ownership Structures ($\bar{R} = 1.3$, $\gamma = 10$, $\sigma = 0.3$, $\bar{\alpha} = 0.05$, $K = 10$, $\lambda = 0.1$, $\epsilon = 0.00001$).

Proposition 1 *Suppose the initial owner chooses not to monitor in period 3, in period 0 he sells the firm when the firm has a positive NPV, otherwise he does not raise the capital and invests in the risk free asset (in both cases, $\alpha_E = 0 < \bar{m}$). The corresponding EOS is a Liquidity Shareholder EOS. The initial owner's value function is given by:*

$$V_E^{NM} = \max(\bar{R}\bar{X} - K + 1, 1) \tag{30}$$

When the initial owner chooses not to monitor, he does not want to remain a shareholder in the firm. This is because he can choose between two possibilities: not raising the capital and investing in the riskfree asset, i.e. $w_E = \alpha_E = 0$, or selling the firm, i.e. $\alpha_E = 0$, $w_E < 0$.[20] If the firm is a positive NPV project even without monitoring, investors are willing to pay the full value of the firm. The initial owner extracts the full surplus without bearing any risk. In such a case given $\alpha_E = 0$ and $\lambda > 0$ the active investors ensure that the outcome is $\bar{X}$. So, there is no incentive to have blocks. If the firm has a negative present value, investors do not want to pay for the losses (they prefer to invest their wealth in the risk free asset or go short on the shares) and hence the initial owner would bear all the losses of the investment. He then also prefers not to raise money and to invest his wealth in the riskfree asset.

Proposition 1 shows that the choice of $\alpha_E > 0, m = 0$ by the initial owner is dominated by the choice of either selling the firm or not raising capital. Hence, in what follows it is sufficient to show that the participation constraint and the non selling constraints are satisfied, to ensure that the initial owner prefers $\alpha_E \geq \bar{m}$, $m = 1$ to any non-monitoring equilibrium.

[20]We may interpret $w_E < 0$ as rent for the initial owner for the entrepreneurial idea.

5.2 Monitoring Equilibria

Our aim in this paper is to show the conditions under which blockholder equilibria exist. The existence of blockholder equilibria depend on three necessary conditions (1) a conflict of interest between investors that is generated endogenously by the fact that if they have different shares in the firm, they have different preferences on the risk/return profile of the firm; (2) they are able to influence the voting decision if they strategically buy more shares than a liquidity shareholder would; (3) In equilibrium the initial owner's shareholding is high enough that active liquidity investors in the firm cannot jointly ensure that $X_{med} = \bar{X}$, even without holding more than the liquidity shares. If any of these three requirements is not met, then we do not have a blockholder equilibrium.

Section 5.2.1 analyzes the case (1) where monitoring costs are so low ($\bar{m} \leq \bar{\alpha}$) that the final choice is $X_{med} = X_E = \bar{X}$. Since this is the first best point for outside investors (see Lemma 7), this implies that there is no conflict of interest: hence no blockholders in the ownership structure. Section 5.2.2, on the other hand considers case (2) where the Blockholders cannot influence the voting decision or they have no incentive because in equilibrium the initial owner holds a very big fraction of shares when monitoring and hence an Initial Owner equilibrium arises. Section 5.2.3 discusses case (3) of Liquidity Shareholder equilibria where liquidity shareholder are in control because the initial owner chooses to hold so few shares that the active liquidity shareholders can shift control without holding a block.

Section 5.2.4 shows that when the monitoring costs are intermediate (so the conflict of interest is not too big and not too small) blockholders emerge endogenously and help to mitigate the conflict of interests between the initial owner and the liquidity shareholders.

Finally section 5.3 shows that the initial owner in equilibrium raises the minimum capital needed, so that the assumption that he needs to raise exactly $K - w_E$ is without loss of generality.

Before moving to the equilibria we show that investors are willing to receive less shares than what they proportionally contribute. Conditional on monitoring, investing in the firm increases the utility of the investors because it widens the possible portfolios they can choose among (the minimum expected return of the project is at least 1, otherwise it would not be chosen as it would be better to invest in the risk free asset). Since the initial owner is a monopolist he can push share prices up to the point where the the participation constraint of investors is satisfied with equality. Put another way, the initial owner contributes less than proportionally to what he receive as shares: $\alpha_E K < w_E$, i.e. $w_E < \underline{\underline{w}}_E(\alpha_E)$ where $m = 1$. We show this in the next Lemma:

Lemma 12 *Assume that $m = 1$. In any subgame perfect equilibrium, the price per share $\frac{K-w_E}{1-\alpha_E} > K$.*

The initial owner sets the price per share as high as possible to avoid dilution of his shareholdings. If the initial owner monitors for sure the expected firm value is above the return on the risk-free asset. Hence, the minimum possible price that guarantees the participation of the investors is above the price of the risk-free asset.

5.2.1 No Conflicts Equilibrium

Recall that $\bar{\alpha}_l$ denotes the demand for shares by liquidity investors when the anticipated $X_{med} = \bar{X}$ (see Lemma 3). Define:

$$\bar{m}_{NC}^{RC} \equiv \frac{\sqrt{K\left(K + 2\bar{X}^2\gamma\sigma^2\right)} - K}{\bar{X}^2\gamma\sigma^2} \tag{31}$$

and

$$\bar{m}_{NC}^{S} \equiv \frac{\sqrt{2\bar{R}\gamma\sigma^2\bar{X}^3 + K^2} - K}{\bar{X}^2\gamma\sigma^2} \tag{32}$$

Proposition 2 *Suppose that*

$$\bar{m} \leq \min\left[\bar{\alpha}, \bar{m}_{NC}^{RC}, \bar{m}_{NC}^{S}\right] \tag{33}$$

then there exists a No Conflicts (NC) equilibrium where the initial owner monitors, $m = 1$, $\alpha_E = \bar{m}$, $X_{med} = X_E = \bar{X}$ and $w_E = \underline{w}_E^{LS}$.

When the monitoring costs are low ($\bar{\alpha} \leq \bar{m}$) or the projects available are not very risky (as $\bar{X} = \frac{\bar{R}}{\gamma\sigma\bar{\alpha}}$), the initial owner's preferred risk/return is the same as for the outside investors, i.e. $\bar{X}$. Even when he holds a non-perfectly diversified portfolio because of the monitoring costs, i.e. $\bar{\alpha}_l < \alpha_E$, the initial owner would still vote for the maximum risk/return because the "friction" in the model is low. Therefore there are no conflicts of interest. The initial owner can extract all rents from liquidity shareholders using his position as a monopolist.

Liquidity shareholders are willing to buy the shares only if the returns are high enough to compensate for the risk. The monitoring sets a maximum threshold on the shares that can be distributed to the liquidity shareholders. Given this threshold there is a maximum fraction of wealth that liquidity shareholders are willing to invest. The rest of the capital (if needed) must be pledged by the initial owner ($w_E = \underline{w}_E^{LS}$). The higher the amount of capital the initial owner needs for the project, K, the higher the wealth he needs to pledge.

Note that given that there are no financial constraints, the initial owner could finance the project completely on his own. However, he prefers to rely on outside equity in order to limit the risk exposure and extract the maximum rent from the liquidity shareholders.[21] As the firm is a risky asset, he prefers to invest the minimum to guarantee the liquidity shareholder's participation and to invest the remaining of his wealth (if any is left) in the risk free asset.

If the value created is very high the initial owner does not need to invest *any* money; indeed, he can be compensated by the investors for the monitoring exerted and the entrepreneurial idea ($\underline{w}_E^{LS} < 0$). These characteristics of the capital invested by the initial owner are common to all equilibria we are going to find.

Note that the no-conflicts case equilibrium does not imply that the initial owner has the same shareholdings as the liquidity investors, $\bar{\alpha}_l$. He always wants to minimize his shareholdings to minimize his exposure to risk. Hence, his shareholdings are determined by the monitoring costs, $\bar{m}$ and

[21] Adding financial constraint would only make it easier to get our results.

depending on the monitoring costs, the initial owner can hold more or less than the liquidity shareholders. In the extreme case when the monitoring costs are 0, he would like to sell the firm. In this way he extracts all the rent from the investment without incurring any risk. The liquidity shareholders instead would always like to hold some shares for diversification reasons.

Figure 3 offers a graphic representation of the No Conflicts Equilibrium case. The equilibrium exists when the area in the graph that satisfies both constraints is positive. Since $\bar{m}_{NC}^{RC}, \bar{m}_{NC}^{S}, \bar{\alpha} > 0$, we can conclude that this is the case.

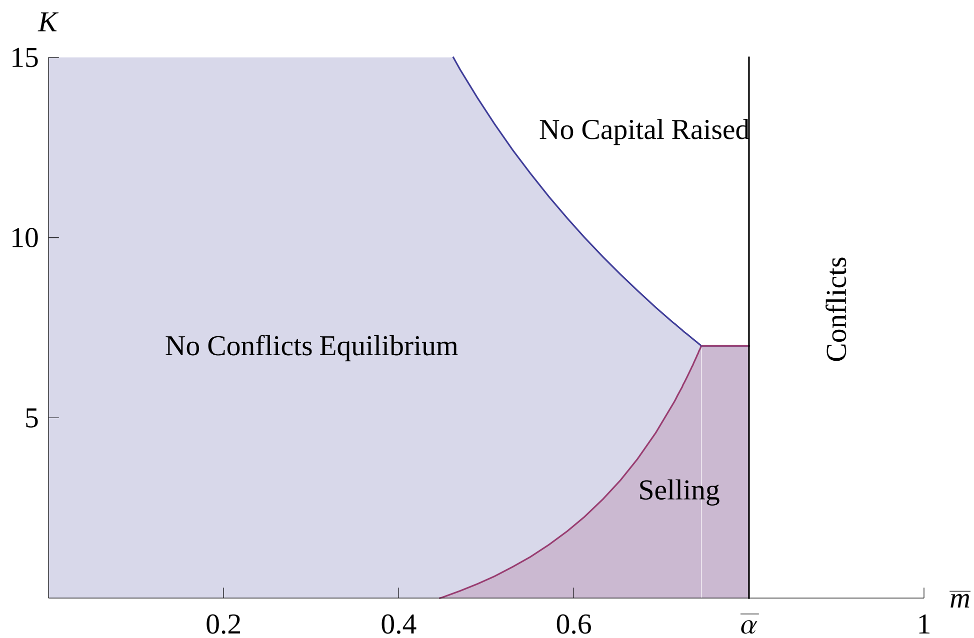

Figure 3: No Conflicts Equilibrium ($\bar{R} = 0.8$, $\gamma = 10$, $\sigma = 0.1$, $\bar{X} = 10$).

Alternatively the initial owner could choose not to raise the capital or to sell the firm. To be a viable project for the initial owner, the value remaining to the initial owner after compensating the investors, must be high enough to compensate him for the money invested and the risk (i.e. $\bar{m} \geq \bar{m}_{NC}^{RC}$).

At the same time to be willing to remain a shareholder of the firm, rather than sell it outright, the value created by monitoring ($f(m) = K$) has to be high enough. The extra value due to monitoring compensates the initial owner for the direct cost of monitoring as well as the indirect costs related to holding a sub-optimal portfolio. If the extra utility created by monitoring can be achieved by a dispersed ownership structure without monitoring then he would prefer to sell the firm ($\bar{m} \geq \bar{m}_{NC}^{S}$ and $\bar{R}\bar{X} - K \geq 0$).

5.2.2 Initial Owner Equilibrium

The second equilibrium we consider is the one where requirement (2) is partly relaxed, i.e. outside investors *cannot* or *do not want* to influence the voting decision and there will be no blockholders apart from the initial owner. We obtain the Initial Owner equilibrium in two cases. First, when the initial owner has more than 50% of the shares and hence no outside investor can influence the decision: in this case the Initial Owner equilibrium is unique. The second case occurs when the initial owner holds less than 50% but 1 Blockholder would not have a unilateral incentive to deviate from holding liquidity shares. Let

$$\bar{m}_E^{RC} \equiv \frac{1}{2}\left(1-\frac{\bar{R}\bar{X}}{K}\right)-\frac{\bar{R}^2}{4K\gamma\sigma^2}+\frac{\sqrt{16K\bar{R}^2\gamma\sigma^2+\left(\bar{R}^2+2\gamma\sigma^2\left(\bar{R}\bar{X}-K\right)\right)^2}}{4K\gamma\sigma^2} \quad (34)$$

$$\bar{m}_E^{S} \equiv -\frac{\bar{R}^2}{4K\gamma\sigma^2}+\frac{\bar{R}\sqrt{\bar{R}^2+16K\gamma\sigma^2}}{4K\gamma\sigma^2} \quad (35)$$

Proposition 3 *Suppose that*

$$\bar{m} \in \left(\max\left[\bar{\alpha},\min\left[\frac{1}{2},\max\left[\hat{\alpha}(1),\tilde{\alpha}(1)\right]\right]\right],\min\left[\bar{m}_E^{RC},\bar{m}_E^{S},1\right]\right] \quad (36)$$

then there exists an Initial Owner (IO) equilibrium where the initial owner is the only blockholder, $m=1$, $\alpha_E=\bar{m}$, $X_{med}=X_E=\frac{\bar{R}}{\gamma\sigma^2\alpha_E^E}$ *and* $w_E=\underline{w}_E^E$.

When $\bar{m}\in\left(\max\left[\bar{\alpha},\frac{1}{2}\right],\min\left[\bar{m}_E^{RC},\bar{m}_E^{S},1\right]\right]$ *the Initial Owner equilibrium is unique.*

If there is a wide range of projects such that there is a conflict of interests between investors and initial owner and the monitoring technology is sufficiently productive, $\bar{m}$ small, such an equilibrium always exists, i.e interval (36) is not empty and $\bar{m}_E^{RC}$ and $\bar{m}_E^{S}$ are large.

Consider first the case, when the monitoring costs are very high ($\bar{m}\geq\frac{1}{2}$), such that the initial owner is willing to monitor only if he holds more than 50% of the shares. This implies that he is highly exposed to firm risk and because he is in control of the vote outcome he chooses a low risk/return project. Hence, disagreement arises between the initial owner and outside investors on the project's choice ($X_E=X_{med}<\bar{X}$). Because the initial owner has the majority of the votes there exists a unique monitoring equilibrium where the initial owner is the only blockholder and he has control over the outcome. No other blockholders arise because under these conditions control is concentrated in the hands of the initial owner in all circumstances and there is no reason to hold a suboptimally diversified portfolio. So this equilibrium exists if $\bar{m}\geq\frac{1}{2}$ and participation constraints for liquidity shareholders are satisfied ($w_E\geq\underline{w}_E^E$) and as long as it is worthwhile to monitor rather than not (i.e. $\bar{m}\leq\min[\bar{m}_E^{RC},\bar{m}_E^{S}]$.

An Initial Owner equilibrium can also arise when the monitoring costs less than $\frac{1}{2}$. As we discussed above: this is because no single investor has a unilateral incentive to deviate.[22] The conditions that

[22]However it is possible to have blockholder equilibria if $n\geq 2$. This case is discussed in Section 5.2.4.

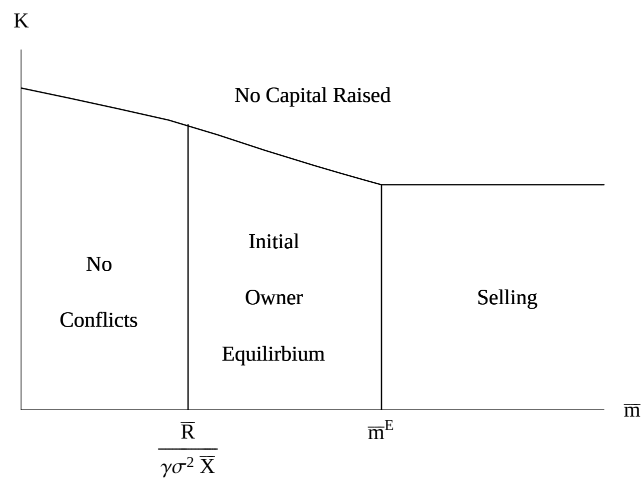

Figure 4: Initial Owner Equilibrium ($\bar{R} = 2$, $\gamma = 12$, $\sigma = 0.2$, $\bar{X} = 10$, $\lambda = 0.05$).

ensure this are $\alpha_E \geq \max[\hat{\alpha}(1), \tilde{\alpha}(1)]$. Finally $\alpha_E > \bar{\alpha}$, otherwise there will be no conflicts and $X_E = \bar{X}$.

Analogously to the no conflicts equilibrium, liquidity shareholders are willing to buy shares if the price is so low to be compensated for the risk they bear. This implies that the amount of capital that they will contribute is determined by their belief in the vote outcome and hence in the risk profile of the firm. This capital can be greater or smaller than the capital needed. If it is smaller the initial owner needs to pledge the rest ($\underline{w}_E^E > 0$), if it is greater the initial owner cashes in the rest as compensation for the entrepreneurial idea ($\underline{w}_E^E < 0$).

The monitoring not only implies a direct cost of $\bar{m}$, but it also has an indirect cost for the initial owner in terms of holding a sub-optimally diversified portfolio. Hence the initial owner is willing to monitor and hold a large fraction of shares if he is able to increase the firm value enough to compensate both the monitoring and the risk of holding a suboptimal portfolio. When the marginal returns on firm value from monitoring are not very high the initial owner prefers not to be involved in the project. In this case two are the possible outcomes. If the extra value created by monitoring is not very high and the firm is very valuable even without monitoring (high $\bar{X}$), i.e. $\bar{m} > \bar{m}_E^S$, the initial owner sells the firm to the investors. In this case the firm will show dispersed ownership (a liquidity shareholder equilibrium). Instead, when the value created by the project is not high enough (with or without monitoring) to compensate the initial owner for the capital invested $\bar{m} > \bar{m}_E^{RC}$, the initial owner does not raise the capital and invests all his wealth in the risk free asset. The possible outcomes are shown in Figure 4

5.2.3 Liquidity Shareholders equilibrium

We now move to consider the equilibrium where the liquidity shareholders are in control and no blockholders arise.

Proposition 4 *When the following conditions are satisfied:*

$$\bar{\alpha} < \bar{m} \leq \min\left[\frac{1}{2}, \tilde{\alpha}(n), \bar{m}_1^{NC}, \bar{m}_2^{NC}\right] \tag{37}$$

then there exists a Liquidity Shareholders equilibrium with $N_A + n$ active investors where $m = 1$, $\alpha_E = \bar{m}$, $X_{med} = \bar{X}$ and $w_E = \underline{w}_E^{LS}$.

A Liquidity Shareholder equilibrium arises when the monitoring costs are relatively low. In such a case the initial owner can distribute a large enough fraction of shares so that there are enough active investors so that the liquidity shareholders are pivotal. As in the other equilibria, the initial owner needs to invest an amount of wealth in order to guarantee that the participation constraint of liquidity shareholders is satisfied when $X = \bar{X}$, i.e. $w_E = \underline{w}_E^{LS}$. If monitoring costs are too high ($\bar{m} > \max[\tilde{\alpha}(n), \frac{1}{2}]$) then the initial owner holds a much higher fraction of shares so that the liquidity shareholders are in control and other ownership structures arise, either an n Blockholder or an Initial Owner one (recall that these were the conditions on α_E for the LS EOS to exist according to Lemma (11)). Finally, as in the case of the No Conflicts and the Initial Owner equilibrium, when the monitoring costs are too high the initial owner prefers to raise no capital or to sell the firm. This occurs when the monitoring costs are too high that the gains for the initial owner to monitor are so low that they do not cover the costs ($\bar{m} \leq \bar{m}_1^{NC}, \bar{m}_2^{NC}$).

Notice too that the initial owner could always choose to retain a strictly higher fraction of shares ($\alpha_E > \bar{m}$) to induce a blockholder EOS or even to hold α_E bigger than half, to induce an Initial Owner EOS. In either of these cases, the vote outcome is closer to his own preferred point. In equilibrium, he chooses not to do this as it is very costly– it implies both a lower price paid by the investors and hence more dilution and a less diversified portfolio for the initial owner.

5.2.4 Blockholder equilibria

This section demonstrates our main result: for intermediate monitoring costs, there exist blockholder equilibria. As we discussed before, n-Blockholder equilibria exist only if blockholders can change the decision. A necessary condition therefore is that the initial owner holds less than the majority of the shares and thus his most preferred decision is no longer guaranteed. However, if the initial owner already chooses $X_E = \bar{X}$ (see Proposition 2), or if there are sufficiently many (active) liquidity shareholders who are expected to vote and guarantee that $\bar{X}$ is chosen then again, no second blockholder will arise in equilibrium.

Define as $\bar{m}_{1,n}^{RC}(n)$ and $\bar{m}_{2,n}^{RC}(n)$ the first two biggest solutions of the equation $V_E^n = 1$ and $\bar{m}_{1,n}^{S}(n)$ and $\bar{m}_{2,n}^{S}(n)$ the two biggest solutions of the equation $V_E^n = \bar{R}\bar{X} + 1 - K$: these are the values of $\bar{m}$

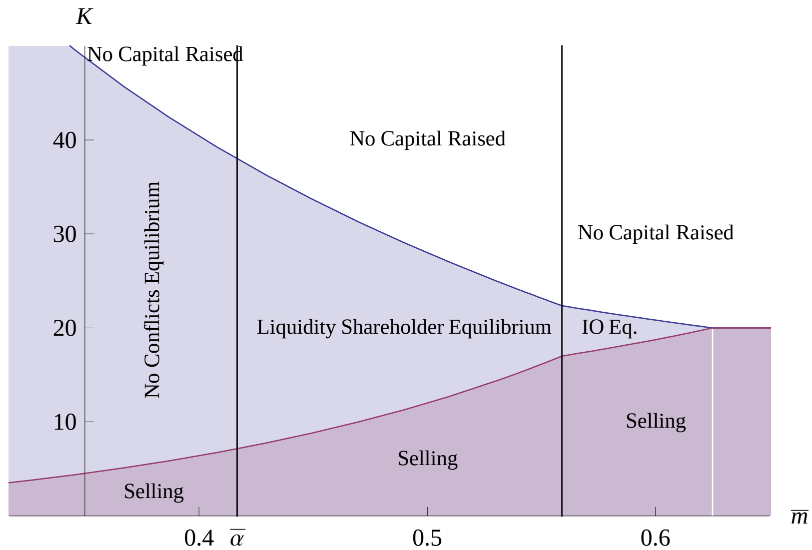

Figure 5: Initial Owner Equilibrium ($\bar{R} = 2$, $\gamma = 12$, $\sigma = 0.2$, $\bar{X} = 10$, $\lambda = 0.05$).

such that the n Blockholder equilibrium gives a higher payoff than the non -monitoring One. Let:

$$\bar{m}_1^E(n) \equiv \frac{\lambda(1+\lambda) + n(1-\lambda)\left(1 - \sqrt{8(3n-1)\lambda^2 + (4-24n)\lambda + 1}\right)}{11n^2(1-\lambda)^2 + (n(1-\lambda) - (1+\lambda))^2} \tag{38}$$

$$\bar{m}_2^E(n) \equiv \frac{\lambda(1+\lambda) + n(1-\lambda)\left(1 + \sqrt{8(3n-1)\lambda^2 + (4-24n)\lambda + 1}\right)}{11n^2(1-\lambda)^2 + (n(1-\lambda) - (1+\lambda))^2} \tag{39}$$

where $\bar{m}_1^E(n)$ and $\bar{m}_2^E(n)$ are the two monitoring costs for which the initial owner is indifferent between being in a n Blockholder equilibrium or in an Initial Owner equilibrium holding the majority of the shares. For monitoring costs values within this interval the initial owner prefers to be in an n Blockholder equilibrium.

Proposition 5 *When:*

$$\max\left[\bar{\alpha}, \tilde{\alpha}(n), \bar{m}_1^E(n), \bar{m}_{1,n}^{RC}(n), \bar{m}_{1,n}^S(n)\right] < \bar{m} \le \min\left[\frac{1}{2}, \hat{\alpha}(n), \bar{m}_2^E(n), \bar{m}_{2,n}^{RC}(n), \bar{m}_{2,n}^S(n)\right] \tag{40}$$

then there exists an n Blockholder equilibrium where the initial owner is monitoring, $m = 1$, $\alpha_E = \bar{m}$, $X_{med} = X_1$ and $w_E = \underline{w}_E^n$.

Proposition 5 demonstrates our main result: i.e there exists a blockholder equilibrium where at least n investors prefers to hold a large block of shares, and hence a sub-optimally diversified portfolio in order to shift the decision to a higher level of risk/return. This mitigates the conflicts of interests between the initial owner and the investors.

Let us try to explain some of the sufficient conditions for existence and where they come from. The conditions for the n Blockholder equilibrium obviously subsume those for the existence of the n

BH EOS (Lemma (10)), and this is where the conditions $\max\left[\bar{\alpha}, \tilde{\alpha}(n)\right] < \bar{m} \leq \min\left[\frac{1}{2}, \hat{\alpha}(n)\right]$ come from, noting that in the n Blockholder equilibrium α_E is always chosen to be equal to $\bar{m}$.

Secondly, in equilibrium the initial owner must get a higher return from monitoring than non monitoring: this yields the conditions $\max\left[\bar{m}^{RC}_{1,n}(n), \bar{m}^{S}_{1,n}(n)\right] \leq \bar{m} \leq \min\left[m^{RC}_{2,n}(n), \bar{m}^{S}_{2,n}(n)\right]$. Finally, since the initial owner could always choose $\alpha_E > \frac{1}{2} > \bar{m}$ leading to an Initial Owner equilibrium and ownership structure, the monitoring costs must be in the range $\bar{m}^E_1(n) \leq \bar{m} \leq \bar{m}^E_2(n)$, to ensure that he does not prefer the Initial Owner equilibrium.

In our model, after having chosen (α_E, w_E), the initial owner does not directly choose the ownership structure, nor can he choose the number of blockholders[23]. In addition the initial owner cannot influence the n BH outcome by setting a different price: Increasing the price, (reducing w_E) there is no participation (see Lemma 8) and so the initial owner would not be able to raise capital. Reducing the price would not change the preferences of the investors but would just distribute more rent to the investors. Only if the price is too low, i.e. $w_E \geq \underline{\underline{w}}_E$, then each investor demands all the shares tendered. We showed already (Lemma (12)) that this is the case. However this is not optimal for the initial owner.

Corollary 4 showed that in the blockholder equilibrium, liquidity shareholders free ride on blockholders as they hold the optimal portfolio. The price which satisfies the participation constraints of the blockholders is lower than the maximum price that liquidity shareholders are willing to pay. Therefore the existence of blockholders allows the liquidity shareholders to extract some of the rent, that in all other equilibria goes entirely to the initial owner. In all the other equilibria the initial owner sets the price low enough to satisfy the participation constraint of the liquidity shareholders with equality, and hence he extracts all the rent. As the initial owner is not able to extract all the rent, when switching from an Initial Owner to an n Blockholder equilibrium there is jump in the level of utility of the initial owner. This discontinuity can be seen in Fig. 6 at $\hat{\alpha}$ which represents the switching point between the Initial Owner to 1 Blockholder equilibrium as the monitoring costs decrease. Because the initial owner's utility decreases at $\hat{\alpha}$, the possibility to sell or not raise money becomes more attractive.

Corollary 6 *When either of these conditions are satisfied:*

$$\max\left[\bar{\alpha}, \tilde{\alpha}(n)\right] < \quad \bar{m} \quad < \max\left[\bar{m}^E_{1,n}(n), \bar{m}^{RC}_E, \bar{m}^S_E\right] \tag{41}$$

$$\bar{m}^E_{2,n}(n) < \quad \bar{m} \quad < \max\left[\bar{m}^{RC}_E, \bar{m}^S_E\right] \tag{42}$$

an Initial Owner Equilibrium arises where $m = 1$, $\alpha_E = \frac{1}{2} + \eta_E$, $X_{med} = X_E$ *and* $w_E = \underline{w}^E_E$.,

The only way for the initial owner to influence the ownership structure is to hold the majority of the shares and be in full control of the vote outcome. Given the high costs, the initial owner is

[23] There are mutiple EOS, involving different n and we get different equilibria parametrized by n depending on the beliefs in equilibrium.

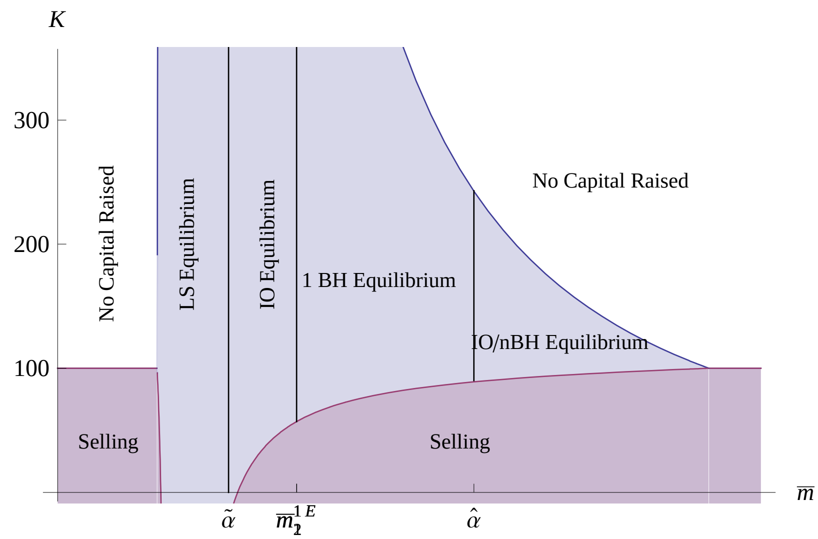

Figure 6: 1 Blockholder equilibrium ($\bar{R} = 1$, $\gamma = 12$, $\sigma = 0.2$, $\bar{X} = 50$, $\lambda = 0.1$). Note that in the graph there is not the boundary $\bar{\alpha}$ and $\bar{m}_2^{1E}$ as they are not binding and far out from the plot range considered.

willing to do so only when the monitoring costs are very high, $\bar{m} > \bar{m}^E_{2,n}$ or very low $\bar{m} < \bar{m}^E_{1,n}$. The intuition behind this result is that the initial owner faces a trade off between two types of costs: the cost of holding a very risky asset (this cost increases the bigger the gap between α_E and α_1) if he does not control the vote outcome, and the cost from being relatively undiversified (this cost increases with α_E). When the monitoring costs are very high, $\bar{m} > \bar{m}^E_{2,n}$, the cost of increasing α_E a little bit and reducing diversification is very small relative to the gain from controlling the vote outcome and choosing his preferred risk/return combination (Corollary 6). When the monitoring costs are very low, on the other hand, blockholders face a very low cost in terms of diversification to be in control as now the vote of the liquidity shareholders has relatively more weight. Hence, the initial owner faces a large cost in terms of diversification from increasing α_E to a point where he can control the vote outcome, $\alpha_E > \frac{1}{2}$, but the cost arising from the conflict of interest on the risk/return vote outcome is even larger if he does not (Corollary 6).

When the initial owner remains a shareholder of the firm he obtains the residual value after distributing the profits to the blockholders and to the liquidity shareholders depending on their ownership structure. If this residual value is not high enough to compensate for the investment, w_E, the monitoring costs, $\bar{m}$, and for the risk he bears, he prefers either to do nothing and not raise money or to sell the firm (See Fig. 6).

This occurs both when the monitoring costs are too high or too low. If the monitoring costs are too high the residual value for the initial owner after compensating the blockholder for the extra risk they bear due to the undiversified portfolio, is not high enough to compensate for holding an undiversified portfolio and to bear the monitoring costs. Hence the initial owner prefers not to raise money or to sell the firm. When the monitoring costs are too low, the active liquidity shareholders are relevant in determining the vote outcome and so the blockholders do not need to hold many shares to be pivotal. In such a case the risk/return outcome is very high compared to the initial owner's preferred point. In such a situation the risk the initial owner bears is so high and also the gain from monitoring is so low that he prefers to sell the firm or not to raise capital.

A possible objection to our main result is that there are multiple equilibria for this configuration of monitoring costs. In particular it can be that for the same monitoring costs an n Blockholder and an Initial Owner equilibrium can arise. Indeed this is true for n- blockholder equilibria where $n > 1$ – this result depends on the multiplicity of EOS for a given α_E, w_E combination. However, Corollary 7 shows that for intermediate values of the monitoring costs only n Blockholder equilibria are possible. The uniqueness follows from Corollary 3, which shows that when an 1 Blockholder EOS exists then the initial owner EOS does not exist. The intuition behind this result is that when monitoring costs are low enough then the cost of losing diversification is low enough for a single blockholder to deviate and demand a block. If monitoring costs are too low of course, there may be no conflict between the initial owner and the outside investors and hence no desire to hold blocks.

Corollary 7 *When the following conditions are satisfied:*

$$\max[\bar{\alpha}, \tilde{\alpha}(1), \bar{m}_1^E(1), \bar{m}_1^{RC}(1), \bar{m}_1^S(1)] < \bar{m} \leq \min\left[\frac{1}{2}, \hat{\alpha}(1), \bar{m}_2^E(1), \bar{m}_2^{RC}(1), \bar{m}_2^S(1)\right] \tag{43}$$

then there exist n *Blockholder equilibria with* $n \in [1, M]$ *where the initial owner is monitoring,* $m = 1$*,* $\alpha_E = \bar{m}$*,* $X_{med} = X_1 < \bar{X}$*,* $w_E = \underline{w}_E^1$*. No other equilibria with positive trade arises for these parameter values.*

Among the possible blockholder equilibria we might want to consider the one which is most preferred by the Initial Owner.

Proposition 6 *The optimal number of blockholders for the initial owner is:*

$$n^* = \left[\frac{(1+\bar{m})(\bar{m} - \lambda + \bar{m}\lambda)}{2\bar{m}^2(1-\lambda)}\right] > 0 \tag{44}$$

The optimal number of blockholders for the initial owner arises from the trade-off to the initial owner from having few blockholders and a vote outcome closer to his optimal outcome or having many blockholders and being able to sell the shares at a high price, i.e. low w_E. The preferred number of blockholders is a function of the monitoring costs and hence of the amount of shares he retains and the proportion of active liquidity investors.

When the monitoring costs are high a further increase of the monitoring costs would induce the initial owner to prefer less blockholders. In such a case blockholders are already holding a highly undiversified portfolio and the initial owner needs to set a very low price in order to induce them to buy the shares. Decreasing further the number of blockholders does not reduce the share price much further. At the same time, decreasing the number of blockholders would allow him to determine a vote outcome closer to his preferred point and hence decrease the costs of holding a suboptimal portfolio.

Instead, when the monitoring costs are low an increase in the monitoring costs induces the initial owner to prefer more blockholders. In such a case the discrepancy between the preferences of the blockholders and the initial owner is not that high. At the same time increasing the number of blockholders allows the initial owner to raise the price at which the shares are tendered. Hence the initial owner would prefer to have more blockholders.

Finally the effect of the proportion of active investors among the liquidity shareholders, λ has a negative effect on the optimal number of blockholders preferred by the initial owner. When more liquidity investors vote it becomes cheaper for blockholders to hold sufficiently large blocks so that they are jointly pivotal in the voting. This is very costly for the initial owner in terms of risk exposure. Hence when the λ is higher the initial owner would prefer to set a lower price, but to have the vote outcome closer to his preferred point. Hence the higher the participation of the liquidity shareholders to the vote the lower the number of blockholders the initial owner would like to have.

5.3 Optimal amount of capital raised

In this section we relax the assumption that the initial owner raises just enough capital needed to implement the project, K. We allow the initial owner to raise more capital $I \geq K$ and invest the difference, $I - K$, in the risk free asset. This would offer the possibility to the initial owner to achieve the preferred degree of diversification through the firm's investment in the risk free asset: hence the conflict of interest between the initial owner and outside investors may disappear. We show in the proposition below that our results are robust to relaxing this assumption.

Proposition 7 *The initial owner always strictly prefers to raise the minimum amount of capital, i.e.* $I = K$.

When raising $I > K$ the initial owner has to lower the price per share increasing w_E. The decrease in price decreases the initial owner's utility in such a way that it more than offsets the increase in utility due to a lower risk of the project. The compensation demanded by the investors is higher than the gain that the initial owner obtains. Hence the initial owner prefers to hold a suboptimally diversified portfolio rather than issue the shares at a lower price.

6 Comparative Statics and Empirical Implications

In this section we discuss some of the important predictions of our model and discuss how they relate to the empirical literature on ownership structure.

6.1 What does Ownership Structure depend on?

Our first prediction is that the *number* and *size* of the blockholders will depend on the degree of coordination required for voting decisions – the size of the first blockholder (hence e.g. on the monitoring technology or the degree of complementarity between the firms production and the input of the initial owner). We expect e.g. family held firms when rising capital generate blockholders only in cases where they hold less than 50% of the stock. No blockholders should be observed in firms that keep the voting control when they release the rest of the stock.[24]

Secondly, we find that the degree of concentration in the ownership structure is decreasing in the monitoring costs, $\bar{m}$, or in the size of the first block. For very small monitoring costs (or small size of the initial block) there is dispersed ownership, for intermediate levels of monitoring costs there are multiple large blocks and for high monitoring costs there is a single large block. High monitoring costs are common in firms dedicated to innovation or R&D where moral hazard issues are much more pervasive. In such firms we should see an initial owner who has control.

Third, we can capture the effect of riskier sectors through $\bar{X}$. High $\bar{X}$ corresponds to industries where risk can be potentially very high. Then our model predicts that blockholder ownership structures emerge in more risky industries.Thus, in more mature industries the projects among which the shareholders can choose is limited to the low risk ones, that is in our models, i.e. low $\bar{X}$. The projects available in more innovative sectors should present a choice of very high return/high return projects, that is there is a high $\bar{X}$ and eventually a lower boundary on $\bar{X}$. Ceteris Paribus, in more mature sectors having a choice of low risk/low return projects, blockholders are less likely. Anecdotal evidence suggests that in more mature sectors it is more common to see families in control of the firms. In very innovative industries, on the other hand, we should see blockholders which ease the conflicts of interests between investors and initial investor and induce more risky decisions. These blockholders are usually represented by institutional investors, e.g. venture capitalists, who professionally look for firms with a high risk/return profile.

There is also a positive relationship between ownership structure and the range of projects in term of size K. When K is small, the initial owner prefers not to raise the capital as the monitoring costs are too high to compensate for the risk incurred. On the other hand, when the project size K is very big the extra value added by the monitoring is such a small fraction of the total cost that it becomes more attractive to sell the idea and have a totally dispersed ownership structure. Hence, different ownership structures can occur only for intermediate values of K. If monitoring costs are low however, the range of project size for which ownership structure is less concentrated is bigger.

The absence of a financial constraint in our model implies that there is no direct relationship between firm size and ownership structure. Hence for the same size there can be any type of ownership

[24] Of course, our predictions apply only to firms where the initial owners choose the price of shares but cannot choose who buys them: if they could choose how much to sell to whom, blockholders would still emerge but would be very close in size to the size of the initial block.

structure.

Fourth, we analyze the effects of λ on the ownership structure. The effect of higher λ in our model is ambiguous because we do not impose any capital constraints on investors: on the one hand it helps to reduce the costs of participation for blockholders implying ceteris paribus, a more diversified structure. On the other hand, it becomes more costly for the initial owner to give up control since the vote outcome will be further from his first best. The balance between these two forces determines the equilibrium outcome (see Fig. 6). We may interpret higher λ as equivalent to increasing minority shareholder protection. Hence in financial markets with high minority shareholder protection and where the investors are not capital constrained we either witness a situation where there is an initial owner with a big block and many small investors or we see situations where there are few blocks – the initial owner and some others and they need to hold relatively few shares to be able to gain control over firm's decision. If investors were credit constrained, however, our results would unambiguously imply more diversified ownership structures.[25]

Finally notice that because the utility of the initial owner is decreasing in λ, the threshold of monitoring costs below which he is willing to invest in the project is lower. Hence minority shareholder protection has a positive and a negative effect on investments. On one side minority shareholder protection leads to higher value projects indirectly through its effect on the ownership structure. However it also reduces the willingness of the initial owner to invest money or to exert monitoring. Hence minority shareholder protection is detrimental to induce the investment, but on the other side if an investment is taken minority shareholder protection is a tool to undertake more value enhancing projects.

6.2 What does Ownership Structure determine?

Let us now look at the predictions we make on how ownership structure influences firm choices.

In our model, the risk/return decision depends not only on whether there is concentration of ownership but rather on the number and size of blockholders: firm value decreases with the size of the first block. This is because for low values of monitoring costs the initial owner prefers a high return as well so both agree on the choice of projects. Ceteris paribus, the smaller the size of the median block the higher is the predicted firm value. These results are consistent with the empirical literature which shows that the effect of the first blockholder on the value of the firm is usually negative, (Barclay and Holderness (1989) and Kirchmaier and Grant (2004) and Lehmann and Weigand (2000)). This is in keeping with our results on the Initial Owner equilibrium. Some of the papers distinguish between the role of the first and subsequent blockholders: The consensus seems to be (again in keeping with our results) that a second blockholder increases firm value. This phenomenon is present across

[25] The initial owner would be required to invest too much of his own capital in order to retain control (the boundary $\bar{m}_1^{1E}$ in Fig. 6 would shift right).

countries and across publicly listed or private firms (Volpin (2002), Maury and Pajuste (2005), Faccio, Lang, and Young (2001), Gutierrez and Tribo (2004) and Laeven and Levine (2008)). Roosenboom and Schramade (2006) studying French IPOs find that when the owner is powerful, the firm is less valued; when the initial owner is sharing control with other blockholders the value increases. Helwege, Pirinsky, and Stulz (2005) find that firms where insider ownership gets reduced over time or firms with widely held ownership are more highly valued.

Our paper also offers an extra prediction to differentiate our theoretical results from the private benefits story: in our theory the risk profile of a firm also depends on how widely dispersed is the ownership. There are few empirical papers looking at this relationship. An exception is Carlin and Mayer (2000; 2003) who find that multiple blockholders are present in high risk firms, while a single blockholder is common in low risk firms.

Our paper offers an explanation for the underpricing observed in IPOs. Empirically, it has been found that IPOs are usually associated with a first day positive return (i.e. the underpricing).[26] In literature there is no agreement on the reasons why this phenomenon arises (see Jenkinson and Ljungqvist (2001) and Welch and Ritter (2002) for reviews). Brennan and Franks (1997) among others (Boulton, Smart, and Zutter (2006) and Yeh and Shu (2004)) found that ownership structure contributes to the degree of underpricing. In particular they argue that underpricing can be more severe when the initial owner wants to avoid blocks. However, they note that this is not a stable outcome and over time blocks are formed anyway. The findings of Brennan and Franks (1997) are in line with our predictions. If the initial owner could retain control, he would be willing to do so even though this implies a lower price. However, if share trade is allowed this outcome cannot be stable. They contrast instead with the predictions of Stoughton and Zechner (1998) and DeMarzo and Urosevic (2006) where the higher the ownership concentration the higher the underpricing. However, in our model, underpricing arises when blockholders are present and it is higher the higher the degree of concentration. In fact our paper has underpricing only when there are blockholders. We show that when the initial owner has no discretion on share allocation and trade is allowed, blockholder equilibria cannot be avoided and underpricing occurs in the sense that liquidity shareholders would be willing to pay more than the equilibrium price to buy shares. Our theory predicts therefore that underpricing occurs when the size of the initial block (initial owner) in the shares is not too large (in particular less than the relevant voting threshold) and not too small.

Recall the discussion in Section (6.1): The range of values of K which would be financed by the initial owner increases when the ownership structure changes. This is because more value is created by the firm per unit of capital with these ownership structures.[27]

Empirically this implies that as the ownership structure is more dispersed we should see more

[26] The average underpricing is between 15% and 17% though it varies a lot over time

[27] This can be seen observing Fig. 6 and noting that m_j^{RC} and m_j^s where $j \in E, LS, n$ are respectively positively and negatively related to K

variance in the size of projects that are financed while firms that shows concentrated ownership should have little diversity in terms of size.

7 Conclusions

This paper attempts to reconcile two well documented empirical regularities in the corporate governance literature: the presence of multiple large shareholders, some of them without a controlling interest on the one hand and increased firm value when such large shareholders are present on the other hand (e.g. Carlin and Mayer (2000; 2003) and Laeven and Levine (2008)among others).

Our model relies on an endogenous conflict of interest over risk management decisions between the initial owner of the firm and outside investors. This problem is similar to a public good contribution game: if there are multiple blockholders which are intermediate in size between the entrepreneur and small shareholders, then the decision on the project is more favorable to all investors, but it comes at a cost to the large shareholders as they hold suboptimal portfolios. Hence blockholders provide a public good to other investors. We show that at very low levels and at very high levels of monitoring costs, we get equilibria where either the initial owner has full control when costs are high or liquidity shareholders having full control when costs are low. For intermediate values we get blockholder equilibria. The corresponding choice of projects goes from low risk/return (the entrepreneur's first best) when monitoring costs are low, to intermediate levels of risk/return in the blockholder equilibria (depending on how many blockholders there are), to very high risk/return projects when the monitoring costs are very high.

The main contribution of our paper is to generate a multiple blockholder equilibrium with a very simple and standard model of corporate control. Unlike many of the papers that explain this phenomenon, we show that blockholders arise without any coordination between investors and without conscious choice on the part of the initial owner – indeed the initial owner only chooses the ownership structure indirectly through the pricing of shares. This simple model is able to explain a variety of stylized facts about the links between ownership size and firm characteristics.

Although our paper offers assumes that all outside investors are identical, the paper could be easily extended to the case of heterogenous agents. Indeed this would help to reduce the problem of multiple equilibria. In this case less risk averse investors would be the natural blockholders and we would expect that if this occurs, the risk/return choice is even higher. This is in line with empirical papers which find that the presence of institutions as blockholders enhances firm performance (Ben Dor (2003), Hartzell, Kallberg, and Liu (2004), Barber (2006) and Chen, Harford, and Li (forthcoming)).

A possible objection to our model might be as some authors argue, that not all institutional investors are active (Rock, 1991) whereas our model *assumes* that blockholders emerge in order to influence voting decisions. In general, institutions like mutual funds are reticent in being active in a firm as the firm itself can be a future client Davis and Kim (forthcoming), or their managers do not receive the right incentives to be willing to intervene in the firm's decisions. Free rider issues are part of co-ordination failures when taking collective actions Del Guercio, Wallis, and Woidtke (2006). Finally, political issues or regulatory issue (such as a limit on the amount of blocks) may limit the action of the institution's managers. Our model seems most suited to active institutional investors such as hedge funds. It is well known that the value creation effect of hedge funds is highly significant both in the short and in the long run (Brav, Jiang, Partnoy, and Thomas, 2006). Brav, Jiang, Partnoy, and Thomas (2006) find that the events triggered by hedge funds are usually related to changes in business strategies, such as refocusing or spinning off, or providing new finances to new projects in the capital structure. This is in line with our ideas of how the blockholders are exerting control.

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A Appendixes

A.1 The Model

A.1.1 Lemma 1:

Proof. The proof is obvious. We just maximize the objective functions (over X) of the outside investors, equation (4), and of the initial owner, equation (5), given the fraction of shares held, α_j. Concavity ensures uniqueness of the solution. ■

A.2 The equilibria.

A.2.1 Lemma 2:

Proof. At date 3, the ownership structure, $\vec{\alpha}$, and thus X_{med} are already fixed. Given the initial owner's objective function (5), he monitors iff the utility from monitoring is geater than from not monitoring, that is when:

$$\alpha_E \left(X_{med}\bar{R} + K \right) - \frac{\gamma}{2}\alpha_E^2 X_{med}^2 \sigma^2 - \bar{m}K \geq \alpha_E X_{med}\bar{R} - \frac{\gamma}{2}\alpha_E^2 X_{med}^2 \sigma^2 \tag{45}$$

Rearranging, we get the condition $\alpha_E \geq \bar{m}$. ■

A.2.2 Lemma 3

Proof. Investor l chooses $\alpha_{l,j}$ to maximize equation (8) where $X_{med} = X_j$. The first order condition implies equation (9). The second order condition is satisfied as long as $X_j > 0$, so this is a maximum. ■

A.2.3 Lemma 4

Proof. As the liquidity shareholding $\alpha_{l,j}$ maximizes the utility of an investor given a vote outcome, any shareholding $\alpha_1 \neq \alpha_{l,j}$ gives less utility. ■

A.2.4 Lemma 5

Proof. Consider first the case where $\alpha_j \geq \bar{\alpha}$ and hence by Lemma 1 $X_j = \frac{\bar{R}}{\gamma\sigma^2\alpha_j}$. Using Lemma 3 $\alpha_{l,j} = \alpha_j + \frac{f(m) - \frac{K - w_E}{1-\alpha_E}}{\frac{\bar{R}^2}{\gamma\sigma^2\alpha_j^2}}$. By assumption, $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$ and thus $\alpha_{l,j} < \alpha_j$.

Now let $\alpha_j < \bar{\alpha}$. By definition, j is the median shareholder, hence $X_{med} = X_j = \bar{X}$. As $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$ a liquidity investor always chooses $\bar{\alpha}_l < \bar{\alpha}$ and hence it is sufficient to show that no active investors hold $\alpha_j < \bar{\alpha}_l$. Assume to the contrary investors j holds $\alpha_j < \bar{\alpha}_l$. (Active) Investor j can improve his utility by choosing $\bar{\alpha}_l$ and voting for $X = \bar{X}$ without changing X_{med}. Contradiction to the equilibrium definition where shareholders are maximizing their utility when X is fixed. This proves that in equilibrium $\alpha_j \geq \alpha_{l,j}$. ■

A.3 The Ownership Subgame

A.3.1 Lemma 6:

Proof. We first compute the utility of the median shareholder (a) when $X_{med} = X_1 < \bar{X}$ and (b) when $X_{med} = X_1 = \bar{X}$. We then prove the Lemma by contradiction.

(a) Since $X_{med} = X_1 < \bar{X}$, in Definition 2, shareholder 1 is the median shareholder and $\alpha_1 > \bar{\alpha}$. Using equation (8), the utility of the median shareholder is:

$$U_1^{nBH} = 1 + \alpha_1 \left(X_1 \bar{R} + f(m) - \frac{K - w_E}{1 - \alpha_E} \right) - \frac{\gamma}{2} \alpha_1^2 X_1^2 \sigma^2$$

By Lemma 1 and the fact that $X_{med} = X_1 < \bar{X}$, this is equivalent to

$$\alpha_1 (f(m) - \frac{K - w_E}{1 - \alpha_E}) + 1 + \frac{\bar{R}^2}{2\gamma\sigma^2} \tag{46}$$

By assumption, $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$, hence U_1^{nBH} is decreasing in α_1.

(b) Suppose instead that $X_{med} = X_1 = \bar{X}$, U_1^{nBH} is decreasing in α_1 as long as $\bar{\alpha}_l < \alpha_1$, since $\bar{\alpha}_l = \text{argmax}\, U_i(\alpha_i)|_{X=\bar{X}}$ by definition of a liquidity shareholder. By Definition 2, $\bar{\alpha}_l < \alpha_1$ in an n-BH EOS.

Now we are ready to prove the Lemma:

(1) Suppose to the contrary that $\alpha_{-1} = 0$ in an n-Blockholder EOS. In order to have $X_{med} = X_1$ we must have $\alpha_1 \geq \alpha_E$, since investor 1 is the only outside investor who votes. From part (a), we know that U_1^{nBH} above is decreasing in α_1 and hence he is better off setting $\alpha_1 = \alpha_E$, otherwise the initial owner becomes the median shareholder. But in this case, $X_{med} = X_1 = X_E$ and *de facto* investor 1 does not affect the vote outcome. Hence he prefers to be a liquidity shareholder and his optimal shareholding is given by $\alpha_{l,E}$. Contradiction to the fact that this is an EOS (since investor 1 wants to deviate unilaterally). Hence, $\alpha_{-1} > 0$ in any n-BH EOS.

(2) $\alpha_E > \alpha_1$ follows from the proof of part (1) above. We need to prove that $\alpha_1 + \alpha_{-1} \geq \alpha_E$. Suppose to the contrary that $\alpha_1 + \alpha_{-1} < \alpha_E$. Then $\alpha_E = \alpha_{med}$ and so $X_{med} = X_E$, so this is not an n BH EOS by Definition (2). Contradiction.

(3) $\alpha_E \geq N_A \alpha_{l,1}$. Given (2) above, this holds iff $\alpha_1 = 0$ which contradicts Definition (2). This implies that $N_A \alpha_{l,1} + n\alpha_1 > \alpha_E$ (since $\alpha_1 > 0$ and $\alpha_E \geq N_A \alpha_{l,1}$). ■

A.3.2 Corollary 1 :

Proof. Lemma 6 shows that U_1^{nBH} is decreasing in $\alpha_1 (\geq \bar{\alpha}_l)$. So, $n\alpha_1 = \alpha_E - N_A \alpha_{l,1}$. Recall that the total shares of the firm (assuming full subscription) must add up to 1 and that we consider equilibria where the proportion of active liquidity investors among the liquidity shareholders is given by λ. Hence N_A must satisfy $\lambda(1 - \alpha_E - n\alpha_1) = N_A \alpha_{l,1}$. Hence $n\alpha_1 + \lambda(1 - n\alpha_1 - \alpha_E) = \alpha_E$. Solving for α_1 we get equation 10. ■

A.3.3 Lemma 7:

Proof. By Lemma 1, given the shareholding α_i of investor i the first best X for the investor is given $X_i = \min\left[\frac{\bar{R}}{\gamma\sigma^2\alpha_i}, \bar{X}\right]$. By the proof of Lemma 6, we know that the investors' utility function is decreasing in α_i when $\alpha_i \geq \bar{\alpha}_l$. Hence, the first best choice of $\alpha_i = \bar{\alpha}_l$. ■

A.3.4 Lemma 8:

Proof. Suppose to the contrary that there exists a No trade equilibrium and $w_E \geq \underline{w}_E^{NT}$. Then a single shareholder can buy $1 - \alpha_E$ shares and ensure full subscription since $w_E \geq \underline{w}_E^{NT}$, contradiction. ■

A.3.5 Lemma 9:

Proof. We use Definition 1: An Initial Owner EOS exists for any combination of (α_E, w_E) iff (a) the participation constraint of investors is satisfied; (b) the incentive constraint of active investors is satisfied; (c) $X_{med} = X_E < \bar{X}$ in the EOS and (d) no shareholder is willing to sell his participation at a price lower than the price at which the excluded investors are willing to buy.

(a) The participation constraint for liquidity shareholders is satisfied iff $U_{l,E} \geq 1$. A sufficient condition for $U_{l,E} \geq 1$ is that $\alpha_{l,E} \geq 0$, i.e. iff $w_E \geq \underline{w}_E^E$. This is the first part of condition (21).

(b) We check that no liquidity investor has an incentive to switch to becoming a blockholder. Notice that if $X_1 < \bar{X}$, $\underline{w}_E^E < \underline{w}_1^E$ whenever $\alpha_E > \hat{\alpha}(1)$. This implies that, given the conditions of the lemma, if a liquidity investor switches to becoming a blockholder, his utility is negative while if he stays as a liquidity investor his utility is strictly positive since $w_E \geq \underline{w}_E^E$. Hence condition (21) implies that $U_{l,E} \geq U_1^{1BH}$ (assuming that $\frac{K - w_E}{1 - \alpha_E} > f(m)$ – this is the case since $w_E \leq \underline{\underline{w}}_E$).

(c)$\alpha_E > \bar{\alpha}$ ensures that $X_E < \bar{X}$.

(d) Recall that there are a fraction λ of active investors among the liquidity shareholders. For the EOS we also need to guarantee that none of the investors who hold shares want to sell them given the maximum price that excluded investors are willing to pay. Any active investor who is excluded can do better by buying liquidity shares from a *passive* investor (who does not vote) at a price slightly higher than the initial owner's price, if by voting he is able to become pivotal and change the outcome. This occurs when

$$\lambda(1 - \alpha_E - \bar{\alpha}_l) + \alpha_{l,E} > \alpha_E$$

which corresponds to $\alpha_E \leq \tilde{\alpha}_E$. In such a case the participation constraint of the investors is also satisfied since it was satisfied for the investors who originally held the shares.

To rule this out we set $\alpha_E > \max[\hat{\alpha}(1), \tilde{\alpha}]$. However these conditions become irrelevant when the initial owner has the majority of the shares since then, no active investors can change the vote outcome. This implies that $\alpha_E > \min\left[\frac{1}{2}, \max[\hat{\alpha}(1), \tilde{\alpha}]\right]$.

Putting together the constraints on α_E from point (c) and (d), the lower bound on α_E is given by (20).

Observe that the interval in condition (21) is non empty whenever $m = 1$ i.e. $f(m) = K$. This is because then $\underline{w}_E^E < \underline{\underline{w}}_E$, and we already showed that $\underline{w}_E^E < \underline{w}_1^E$ whenever $\alpha_E < \hat{\alpha}(1)$.

■

A.3.6 Lemma 10:

Proof. An n-Blockholder EOS with $X_1 < \bar{X}$ exists iff the conditions of Definition 1 are satisfied: (a) $U_1^{nBH} \geq 1$, $U_{l,1} \geq 1$; (b) No investor wants to unilaterally increase or decrease his shares; (c) No investor wants to sell his shares at a price lower than the maximum willingness to pay of excluded investors.

Observe that $X_1 < \bar{X}$ iff $\alpha_1 > \bar{\alpha}$ (Lemma 1). From Corollary 1, this condition is equivalent to $\alpha_E > \check{\alpha}(n)$, the lower boundary of condition (22) .

(a) The participation constraint of blockholders is satisfied iff

$$U_1^{nBH} = \left(\bar{R}X_1 + f(m)\right)\alpha_1 - \frac{K - w_E}{1 - \alpha_E}\alpha_1 - \frac{\gamma}{2}X_1^2\sigma^2\alpha_1^2 + 1 \geq 1 \tag{47}$$

Rearranging, this is equivalent to $w_E \geq \underline{w}_E^n(\alpha_E)$, the lower boundary of condition (23). By Lemma 4, liquidity shareholders participation constraint is always satisfied whenever the blockholders' is, hence $U_{l,1} \geq 1$.

(b) Claims 1 and 2 show that given conditions (22) and (23), no blockholder wants to change his shares unilaterally from α_1. Claim 3 shows that no liquidity investor wants to increase or decrease unilaterally his shareholding.

Claim 1: Suppose conditions (22) and (23) are satisfied. No blockholder wants to decrease his shares unilaterally from α_1.

Proof. Observe that $w_E \leq \underline{\underline{w}}_E$ by condition (23). By the proof of Lemma 6 and Corollary 1, a blockholder chooses α_1 as given in equation (10). If any blockholder reduces his shares, given α_{-i}, X_{med} shifts to $X_E < X_1 < \bar{X}$ as $\alpha_E > \alpha_1$. In such a case the highest possible utility a blockholder investor can achieve, is given by being a liquidity shareholder. Hence, it is sufficient to consider the deviation to liquidity shareholding only. Hence, the incentive compatibility constraint is $U_1^{nBH} \geq U_{l,E}$, where $U_{l,E}$ denotes the utility of a liquidity shareholder when $X_{med} = X_E$.

Note that, the utility of the blockholders increases monotonically in w_E, while the liquidity shareholders' utility is convex quadratic in w_E with the minimum equal to 1 at $\underline{w}_E^E$. However, because the firms' shares are issued only if the initial owner raises the capital, the liquidity shareholders' utility is convex quadratic in w_E for $w_E \geq \underline{w}_E^E$ and it is 1 for $w_E < \underline{w}_E^E$. Hence $U_1^{nBH} \geq U_{l,E}$ iff $\underline{w}_E^E > \underline{w}_E^n$ and $w_E \leq \bar{w}_E^*$ where $\bar{w}_E^*$ the biggest solutions of $U_1^{nBH} = U_{l,E}$. Note also that when $\underline{w}_E^E > \underline{w}_E^n$, $\bar{w}_E^*$ is real and is smaller than $\underline{\underline{w}}_E$ (this can be seen substituting $\underline{\underline{w}}_E$ in $U_1^{nBH} - U_{l,E}$ and verifying that this is always positive). Hence the only relevant condition to ensure IC is $\underline{w}_E^E > \underline{w}_E^n$. This condition is always satisfied when $2\alpha_1 \leq \alpha_E$ and substituting for α_1 from expression (10), this becomes:

$$\alpha_E \leq \hat{\alpha}_1(n) \tag{48}$$

This is the first upper bound in condition (22) .

Finally because an investor wants to hold a block only if he is pivotal in the vote outcome, we need to impose that $\alpha_E \leq \frac{1}{2}$. Suppose $\alpha_E > \frac{1}{2}$ then $X_{med} = X_E$ always, so there is no BH EOS. This gives the second upper bound in condition (22). ■

Claim 2: Suppose conditions (22) and (23) are satisfied. No blockholder wants to increase his shares from α_1.

Proof. Since $w_E \leq \underline{\underline{w}}_E$ by condition (23) his utility U_1^{nBH} is decreasing in α_1 so the blockholder holds just enough to be pivotal. ■

Claim 3: Liquidity Shareholders cannot gain from unilateral deviation.

Proof. No (active) liquidity shareholder has an incentive to hold shares bigger than α_1 in order to change $X_{med} < X_1$, as this would reduce their utility (which is increasing in X and decreasing in shareholdings when $\alpha_i > \alpha_{l,j}$).

If the investors choose any $\alpha_i < \alpha_1$ they do not change the outcome, hence $\alpha_{l,1}$ maximizes their utility.

(c) No shareholder, either liquidity or blockholder, is willing to sell his shares for a price lower than what he paid to the initial owner. Excluded investors are willing to buy the shares at a higher price from the liquidity shareholders, but not from the blockholders as this would shift the vote outcome to X_E. Hence the EOS does not change. ■

■

A.3.7 Corollary 2:

It follows directly from Lemma 10 and taking into account that when $\tilde{\alpha}(n) < \alpha_E < \check{\alpha}_E(n)$, $\bar{\alpha}_l < \alpha_1 < \bar{\alpha}$.

A.3.8 Corollary 3:

Proof. The proof of the first part follows directly from Lemma 10 and Corollary 2, setting $n = 1$ and observing that $\min\left[\hat{\alpha}(1), \frac{1}{2}\right] = \hat{\alpha}(1)$. For the second part, suppose to the contrary that there is an EOS with $X_{med} = X_E$ when α_E is in the interval (26). We know from the proof of Lemma 10, that any active liquidity shareholder has an incentive to switch to becoming a single blockholder when $\alpha_E \leq \hat{\alpha}_1$. Contradiction to the Definition 1 of EOS. ■

A.3.9 Lemma 11:

Proof. We divide the proof in two parts. We first consider the case $n \geq 1$ and then $n = 1$.

(A) $n \geq 1$

Let $U_{l,\bar{\alpha}}$ denote the value function of a liquidity shareholder when $X = \bar{X}$, and he holds the optimal shareholdings. A liquidity shareholder EOS exists iff the following conditions are satisfied: (1) $U_{l,\bar{\alpha}} \geq 1$; (2) No active investor wants to unilaterally increase or decrease his shares; (3) Passive investors maximize their utility conditional on $X_{med} = \bar{X}$. Moreover all active investors are liquidity shareholders. (4) No investor is willing to sell his shares at any price lower than the maximum that excluded investors are willing to pay.

1. By the proof of Lemma 3, $U_{l,\bar{\alpha}} \geq 1$ iff $w_E \geq \underline{w}_E^{LS}(\alpha_E)$, where $\underline{w}_E^{LS}(\alpha_E)$ is given by equation (18).

2. No active liquidity investor wants to increase or decrease his shareholdings since this is the most preferred point (see Lemma 7), as long as $\alpha_E \leq \tilde{\alpha}_E(n)$, since this condition guarantees that $X_{med} = \bar{X}$.

3. Passive investors hold $\bar{\alpha}_l$ which maximizes their utility.

4. No investor is willing to sell his shares to any excluded investors as the maximum price at which excluded investors are willing to buy the shares is the minimum price at which the liquidity shareholders are willing to sell.

(B) $n = 0$

In such a case the interval (28) becomes $\alpha_E \in \left(\bar{\alpha}, \frac{\lambda}{1+\lambda}\right]$. In such a case there are sufficiently many active liquidity shareholders that even with liquidity shares, $\bar{\alpha}_l$ they can get their most preferred point and $X_{med} = \bar{X}$. The participation constraint of the liquidity shareholders is satisfied when $w_E \geq \underline{w}_E^{LS}(\alpha_E)$. No active liquidity shareholders find it profitable to hold a suboptimal portfolio to switch the decision to $X < \bar{X}$, as this is their first best. Note that no (passive) investors are willing to sell their shares to the excluded investors. The excluded investors cannot improve the vote outcome and hence the maximum price they are willing to buy is equal to the minimum price the shareholders are willing to sell.

■

A.3.10 Corollary 4:

Proof. When $\alpha_E \in (0, \bar{\alpha}]$ there are no conflicts of interests between outside investors and the initial owner, $X_{med} = X_E = \bar{X}$. Active investors hold $\bar{\alpha}_l$, and are at their most preferred X. Hence they have no incentive to deviate. The participation constraint of the liquidity shareholders is satisfied when $w_E \geq \underline{w}_E^{LS}$. To prove necessity note that if $\alpha_E > \bar{\alpha}$, $X_E < \bar{X}$ hence there are conflicts between investors. If $w_E < \underline{w}_E^{LS}$, no investors would buy the shares hence it is not an EOS. Finally note no investor is willing to sell shares at a price lower than the maximum that an excluded investor will pay.

■

A.3.11 Corollary 5:

Proof. When $\alpha_E = 0$, and there is at least one active investor ($\lambda > 0$), and all active investors vote for $\bar{X}$, so $X_{med} = \bar{X}$. The participation constraint of outside investors is satisfied if $w_E \geq \underline{w}_E^{LS}$. This is the first best for outside investors hence there is no incentive to become a blockholder. Also it is trivial to see that investor is willing to sell his shares at a price lower than the maximum that an excluded investor is willing to pay. ■

A.3.12 Proposition 1:

Proof. We solve the problem using backward induction. The objective function of the initial owner is different depending on the anticipated ownership structure. The ownership subgame could have one of the following EOS: (A) An Initial Owner EOS, (B) n-Blockholder EOS (C) Liquidity shareholder EOS (D) No Conflicts EOS.

The initial owner maximizes the following general objective function:

$$\max_{\alpha_E, w_E} U\,(m=0) = \bar{R}X_{med}(\alpha_E)\alpha_E - \frac{\gamma}{2}X_{med}(\alpha_E)^2\sigma^2\alpha_E^2 + 1 - w_E \tag{49}$$

Given the objective function, regardless of the ownership structure, he chooses the lowest w_E, that is $w_E = w_E^j(\alpha_E)$ where $w_E^j(\alpha_E) = \{\underline{w}_E^E, \underline{w}_E^n, \underline{w}_E^{LS}\}$.

Substituting for w_E^j in the objective function, it can be checked that $U\,(m=0)$ is decreasing in α_E for all w_E^j, for $X_{med} \leq \bar{X}$. Therefore $\alpha_E = 0$ is the optimal choice of the initial owner for *any* ownership structure, by Corollary 5 the unique EOS is the Liquidity Shareholder EOS, $w_E = \underline{w}_E^{LS}$ and the initial owner's utility is given by:

$$\bar{R}\bar{X} - K + 1 \tag{50}$$

Finally we check if the participation constraint of the initial owner is satisfied: i.e if he invests in the riskfree asset his utility is 1. Hence when the project has a positive NPV he sells the firm, otherwise he does not raise capital. The initial owner's value function is given by equation (30). ■

A.4 Monitoring Equilibria

A.4.1 Lemma 12:

Proof. In order to show the Lemma first note that in any EOS the utility of the initial owner is decreasing in w_E. Further, in order to satisfy the participation constraint of the outside investors, blockholders or liquidity investors, their utility has to be greater than 1. Finally from the Section 5.1 we saw that all the possible EOS are characterized either by outside investors who are only liquidity shareholders or who are either liquidity shareholders or blockholders.

First consider the case where no blockholders exist. Hence only the participation constraint of the liquidity investors needs to be satisfied in equilibrium.

Now, suppose to the contrary, that there is an equilibrium with $\frac{K-w_E}{1-\alpha_E} < K$, $\alpha_{l,j} > 0$ and $U_{l,j} > 1$. By assumption there are sufficiently many investors in the market, so there always exist passive shareholders who have a strictly positive demand for shares for any $0 < X_j \leq \bar{X}$. Hence, the initial owner can increase his utility by decreasing w_E, for any α_E and still ensure that there is a (smaller) positive demand by passive investors, ensuring full subscription. As the demand of the liquidity shareholders is given by equation (9), the initial owner will do this until $\frac{K-w_E}{1-\alpha_E} > K$. Contradiction.

Now consider the case where blockholders and liquidity shareholders exist in equilibrium. In such a case either blockholders or liquidity shareholders will have the most binding constraint. We already showed above that when the liquidity investors participation is more binding then $\frac{K-w_E}{1-\alpha_E} > K$. So it is sufficient to show that this is true when the binding constraint is that of blockholders.

When instead the blockholders's participation constraint is more binding given $m = 1$ the value function for blockholders given $X_{med} = X_j$ is given by :

$$U_1^{nBH} = \bar{R}X_j\alpha_1 + \left(K - \frac{K-w_E}{1-\alpha_E}\right)\alpha_1 - \frac{\gamma}{2}X_j^2\sigma^2\alpha_1^2 + 1 \tag{51}$$

By the same logic as for the first part, suppose that there is an equilibrium with $\frac{K-w_E}{1-\alpha_E} < K$. In Section 5.1 we showed that $\alpha_1 \leq \max[\alpha_j, \bar{\alpha}]$. Hence when $\frac{K-w_E}{1-\alpha_E} < K$ then the participation constraint of the blockholders is satisfied with strict inequality, i.e. $U_1^{nBH} > 1$, The initial owner can decrease w_E and still satisfy the constraints and ensure full subscription (by assumption there are sufficiently many investors in the market, so there always exist passive shareholders who have a strictly positive demand for shares for any $0 < X_j \leq \bar{X}$). Contradiction to the fact that it is an equilibrium. ■

A.4.2 Proposition 2:

Proof. Before we prove the next proposition, we need a few lemmas which provide expressions for the value function of the initial owner under the alternative ownership structures that could be obtained in a subgame perfect equilibrium:

Lemma 13 *Suppose the conditions for the NC EOS (Corollary 4) are satisfied and the equilibrium of the game is the No Conflicts equilibrium, then the initial owner sets $\alpha_E = \bar{m}$, $X_{med} = X_E = \bar{X}$, $w_E = \underline{w}_E^{LS}$ and the value function of the Initial Owner is given by:*

$$V_E^{NC} = \bar{R}\bar{X} + 1 - \frac{\gamma}{2}\bar{X}^2\bar{m}^2\sigma^2 - \bar{m}K - \epsilon \tag{52}$$

Proof. By Lemma 2, in any monitoring equilibrium, $\alpha_E \geq \bar{m}$. By Corollary 4, the No Conflicts EOS with $X_{med} = X_E = \bar{X}$ exists if $\alpha_E \in (0, \bar{\alpha}]$ and $w_E \geq \underline{w}_E^{LS}(\alpha_E)$. Therefore the maximization problem of the Initial Owner in the NC equilibrium is:

$$\max_{\alpha_E, w_E} U_E = (\bar{R}\bar{X} + K)\alpha_E - \frac{\gamma}{2}\bar{X}^2\alpha_E^2\sigma^2 + 1 - w_E - \bar{m}K \tag{53}$$

$$\text{s.t} \quad w_E \geq \underline{w}_E^{LS}(\alpha_E) \tag{54}$$

$$\alpha_E \in [\bar{m}, \bar{\alpha}] \tag{55}$$

The initial owner's utility is decreasing in the wealth invested, w_E. Hence he chooses w_E such that it satisfies the participation constraint of the liquidity investors, (54), at equality. Hence $w_E = \underline{w}_E^{LS}$ where $\underline{w}_E^{LS}$ is given by equation (18). Inserting it in the initial owner's objective function we obtain:

$$\bar{R}\bar{X} + 1 - \frac{\gamma}{2}\bar{X}^2\alpha_E^2\sigma^2 - \bar{m}K - \epsilon \tag{56}$$

This expression is decreasing in α_E. Hence the initial owner will retain just enough shares to satisfy the monitoring constraint with equality: $\alpha_E = \bar{m}$. Inserting $\alpha_E = \bar{m}$ in the initial owners utility function, we have expression (52). ■

Let $\underline{b} = \left(\max\left[\bar{\alpha}, \min\left[\frac{1}{2}, \max\left[\hat{\alpha}(1), \tilde{\alpha}\right]\right]\right]\right) + \eta_E$. Remember that η_E is the fraction correspondent to one share when the vote outcome is X_E.

Lemma 14 *Suppose the conditions of the IO EOS are satisfied (Lemma 9), and the equilibrium of the game is an Initial Owner equilibrium, then* $X_{med} = X_E < \bar{X}$, $w_E = \underline{w}_E^E$, $\alpha_E = \max[\bar{m}, \bar{b}]$, *and the value function of the Initial Owner is given by:*

$$V_E^E = \bar{R}X_E + 1 - \frac{\gamma}{2}X_E^2 \max[\bar{m}, \bar{b}]^2\sigma^2 - \bar{m}K - \epsilon = \frac{\bar{R}^2}{\gamma\sigma^2}\left(\frac{1}{\max[\bar{m}, \bar{b}]} - \frac{1}{2}\right) + 1 - \bar{m}K - \epsilon \tag{57}$$

Proof. By Lemma 2, in any monitoring equilibrium, $\alpha_E \geq \bar{m}$. By definition of the Initial Owner EOS, $X_{med} = X_E < \bar{X}$. By Lemma 9 an Initial Owner equilibrium exists if conditions (20) and (21) are satisfied. Hence, the maximization problem of the initial owner in the IO equilibrium is:

$$\max_{\alpha_E, w_E} U = (\bar{R}X_E + K)\alpha_E - \frac{\gamma}{2}X_E^2\sigma^2\alpha_E^2 + 1 - w_E - \bar{m}K \tag{58}$$

$$\text{s.t} \quad \alpha_E \in (\underline{b}, 1] \tag{59}$$

$$\alpha_E \geq \bar{m} \tag{60}$$

$$w_E \in \left[\underline{w}_E^E, \min\left[\underline{w}_E^1, \underline{\underline{w}}_E\right]\right) \tag{61}$$

Given his objective function, the initial owner minimizes w_E, that is he sets $w_E = \underline{w}_E^E(\alpha_E)$. Substituting it into the utility of the initial owner, this is:

$$\bar{R}X_E - \frac{\gamma}{2}X_E^2\sigma^2\alpha_E^2 + 1 - \bar{m}K - \epsilon \tag{62}$$

Substituting X_E, it can be seen that the initial owner's utility becomes:

$$\frac{\bar{R}^2}{\gamma\sigma^2}\left(\frac{1}{\alpha_E} - \frac{1}{2}\right) + 1 - \bar{m}K - \epsilon$$

This is decreasing in α_E. Hence, $\alpha_E = \max[\bar{m}, \bar{b}]$ and substituting it in the initial owner's utility function expression (57) is obtained. ■

Let $\underline{c} = \max[\bar{\alpha}, \tilde{\alpha}_E(n)] + \bar{\eta}$ where $\bar{\eta}$ is the fraction correspondent to one share when the vote outcome is $\bar{X}$.

Lemma 15 *Suppose the conditions of the n BH EOS are satisfied (Lemma 10 and Corollary 2) and the equilibrium of the game is an n Blockholder equilibrium with $X_{med} = X_1$, the initial owner sets $\alpha_E = \max[\underline{c}, \bar{m}]$, $w_E = \underline{w}_E^n$ and the value function of the Initial Owner is given by:*

$$V_E^n = \bar{R}X_1 + 1 - \bar{m}K - \frac{\gamma}{2}X_1^2\sigma^2(\max[\underline{c}, \bar{m}]^2 + \alpha_1 - \alpha_1 \max[\underline{c}, \bar{m}]) \tag{63}$$

Proof. Again by Lemma 2 the initial owner monitors iff $\alpha_E \geq \bar{m}$. By Lemma 10, conditions (22) and (23) or (25) need to be satisfied in order to have an n Blockholder EOS. Hence the maximization problem of the Initial Owner in an n Blockholder equilibrium is given by:

$$\max_{\alpha_E, w_E} U = (\bar{R}X_1(n, \alpha_E) + K)\alpha_E - \frac{\gamma}{2}X_1(n, \alpha_E)^2\sigma^2\alpha_E^2 + 1 - \bar{m}K - w_E \tag{64}$$

$$\text{s.t} \quad \alpha_E \in \left(\underline{c}, \min\left[\hat{\alpha}(n), \frac{1}{2}\right]\right] \tag{65}$$

$$w_E(\alpha_E) \in [\underline{w}_E^n(\alpha_E), \underline{\underline{w}}_E(\alpha_E)) \tag{66}$$

$$\alpha_E \geq \bar{m} \tag{67}$$

Again, using the fact that his utility is decreasing in w_E, he chooses $w_E = \underline{\mathrm{w}}_E^n(\alpha_E)$. Substituting for this in his utility function we get equation (63). ■

Lemma 16 *Suppose the conditions for the LS EOS are satisfied and there exists a Liquidity Shareholder equilibrium with monitoring (Lemma 11). Then, $\alpha_E = \max(\bar{\alpha} + \bar{\eta}, \bar{m})$, there are at least $N_A + n$ active investors, $w_E = \underline{w}_E^{LS}$ and the value function of the Initial Owner is given by:*

$$V_E^{LS} \equiv \bar{R}\bar{X} + 1 - \bar{m}K - \frac{\gamma}{2}\bar{X}^2\sigma^2 \max[\bar{m}, \bar{\alpha} + \bar{\eta}]^2 - \epsilon \tag{68}$$

Proof. By Lemma 2 the initial owner monitors iff $\alpha_E \geq \bar{m}$. By Lemma 11, the LS EOS exists iff conditions (28) and (29) are satisfied. Hence the maximization problem of the initial owner assuming that the Liquidity Shareholder equilibrium exists is:

$$\max_{\alpha_E, w_E} U = (\bar{R}\bar{X} + K)\alpha_E - \frac{\gamma}{2}\bar{X}^2\sigma^2\alpha_E^2 + 1 - w_E - \bar{m}K \tag{69}$$

$$\text{s.t} \quad \alpha_E \in \left(\bar{\alpha}, \min\left[\tilde{\alpha}_E(n), \frac{1}{2}\right]\right] \tag{70}$$

$$w_E(\alpha_E) \geq \underline{w}_E^{LS}(\alpha_E) \tag{71}$$

$$\alpha_E \geq \bar{m} \tag{72}$$

Hence, the initial owner minimizes w_E, that is he sets $w_E = \underline{w}_E^{LS}(\alpha_E)$. Substituting it into the utility of the initial owner, this becomes:

$$\bar{R}\bar{X} - \frac{\gamma}{2}\bar{X}^2\sigma^2\alpha_E^2 + 1 - \bar{m}K - \epsilon \tag{73}$$

This is decreasing in α_E. Hence $\alpha_E = \max[\bar{m}, \bar{\alpha} + \bar{\eta}]$ and substituting it, the value function V_E^{LS} is given by equation (68). ■

We are now ready to prove the proposition. We solve the game by backward induction. The initial owner chooses α_E and w_E, anticipating the ownership structure. His problem can be broken into the following: (1) $\alpha_E \in [\bar{m}, \bar{\alpha}]$, (2) $\alpha_E \in (\bar{\alpha}, 1]$, (3) $\alpha_E \in [0, \bar{m}]$.

We first describe the beliefs on the EOS and the corresponding value functions in each interval.

Case (1). By Corollary 4 there exists a No Conflicts EOS. Hence in this interval the beliefs of all players on the EOS are the No Conflicts EOS when $w_E \geq \underline{w}_E^{LS}$. As $w_E < \underline{w}_E^{LS} \leq \underline{w}_E^{NT}$ for any possible X there exists a No Trade EOS by Lemma 8, and we assume that the belief is on the No Trade EOS. The initial owner's value function is given by equation (52) when $w_E \geq \underline{w}_E^{LS}$ and by the no trade value function, $V_E^{NT} = 1$ in case $w_E < \underline{w}_E^{LS}$.

Case (2). By Lemmas 9, 10 and 11 the possible EOS in this interval are the Initial Owner, the n Blockholder or the Liquidity Shareholder ones if $w_E \geq \underline{w}_E^j$ where $j = \{IO, n, LS\}$. Again if $w_E < \underline{w}_E^j$ the No Trade EOS exists and $V_E^{NT} = 1$. If an IO EOS exists the initial owner sets $\alpha_E = \max[\bar{m}, \underline{b}] = \underline{b}$ as $\bar{m} \leq \bar{\alpha}$ and his value function, V_E^E is given by equation (62). If an n BH EOS exists, Lemma 15 shows that the initial owner's utility is decreasing in α_E. Observe that for a BH equilibrium to exist $\tilde{\alpha}(n) > \bar{\alpha}$ otherwise the initial owner would choose $\alpha_E = \bar{\alpha}$. Hence $\underline{c} = \tilde{\alpha}(n)$ and the value function V_E^n is given by (63). If a LS EOS arises, Lemma 16 shows that $\alpha_E = \max[\bar{\alpha} + \bar{\eta}, \bar{m}] = \bar{\alpha} + \bar{\eta}$. Hence the value function V_E^{LS} is given by equation (68)

Case (3). In this interval the Liquidity Shareholder (LS) EOS exists by Corollary (5), hence we assume that the belief is that if $w_E \geq \underline{w}_E^{LS}$ it is the LS EOS. Lemma 2 shows that the initial owner choose not to monitor in this interval, and by Proposition 1 $\alpha_E = 0$ and the initial owner's value function is $V_E^{NM} = \max\left[1, \bar{R}\bar{X} - K + 1\right]$. If $w_E < \underline{w}_E^{LS}$ then the belief is on the no-trade EOS (Lemma 8) and the corresponding value function is $V_E^{NT} = 1$.

The initial owner will choose α_E to maximize his value function across the intervals (1)–(3) above.

First consider case 2, ignoring V_E^{NT} for the moment: It is easy to see from (62) that $V_E^E|_{\alpha_E = \underline{b}} < V_E^E|_{\alpha_E = \bar{m}} = V_E^{NC}$. Hence the initial owner is better off in a NC equilibrium. If an n BH EOS exists, it is easy to see from equation (63)that $V_E^n|_{\alpha_1 < \bar{m}} < V_E^n|_{\alpha_1 = \bar{m}}$. $V_E^n|_{\alpha_1 = \bar{X}} < V_E^{NC}$ i.e. iff

$$\frac{1}{\alpha_1}\left(1 + \alpha_E - \frac{\alpha_E^2}{\alpha_1}\right) < \frac{2}{\bar{\alpha}} - \frac{\bar{m}^2}{\bar{\alpha}^2}$$

which is always true as $\bar{m} \leq \bar{\alpha} \leq \alpha_E$. Hence the initial owner is better off in a NC equilibrium. If an Liquidity Shareholder EOS then by Lemma 16 $V_E^{LS}|_{\alpha_E > \bar{\alpha}} < V_E^E|_{\alpha_E = \bar{m}} = V_E^{NC}$. Hence the initial owner is better off in a NC equilibrium.

Now consider case 3: The initial owner's value function is $V_E^{NM} = \max\left(1, \bar{R}\bar{X} - K + 1\right)$. Hence he prefers to monitor iff $V_E^{NC} \geq V_E^{NM}$, i.e. iff $V_E^{NC} \geq \bar{R}\bar{X} - K + 1$ and $V_E^{NC} \geq 1$.

This condition is satisfied when $\bar{m} \in [c, \bar{m}_{NC}^S]$, where $c < 0$. Hence $V_E^{NC} \geq \bar{R}\bar{X} - K + 1$, whenever $\bar{m} < \bar{m}_{NC}^S$.[28] When the above condition is not satisfied the initial owner sells out the firm. The

[28]Note that as ϵ is a very small number we just consider a strict inequality.

second condition, $V_E^{NC} \geq 1$, is satisfied when $\bar{m} \in [d, \bar{m}_{NC}^{RC}]$, where $d < 0$. Hence, if $\bar{m} > \bar{m}_{NC}^{RC}$ then the initial owner does not raise capital.

Finally, consider the No Trade equilibrium. Clearly this gives the same value to the initial owner as not raising capital, so under the conditions of the proposition, the NC equilibrium is preferred by the initial owner. ■

A.4.3 Proposition 3:

Proof. The proof follows the same steps as the proof of Proposition 2. We break up the maximization problem of the initial owner into the following cases: (1) $\alpha_E \in [\max[\underline{b} + \eta_E, \bar{m}], 1]$ (2) $\alpha_E \in [0, \bar{m}]$.

We first describe the beliefs on the EOS and corresponding value functions in each interval.

Case (1). By Lemma 2 the initial owner monitors. By Lemma 9 an Initial Owner EOS exists in this interval of α_E as long as w_E satisfies condition (21) and we assume that the anticipated EOS is the Initial Owner EOS for $w_E \geq \underline{w}_E^E$ and otherwise the No Trade EOS (with value function V_E^{NT}) is anticipated. Lemma 14 implies then that: $\alpha_E = \bar{m}$, $w_E = \underline{w}_E^E$ and the initial owner's value function, V_E^E, is given by equation (57).

Case (2). This case is the same as in Proposition 2. The initial owner's value function is $V_E^{NM} = \max[\bar{R}\bar{X} - K + 1, 1]$.

Maximizing across intervals (1) and (2) the initial owner will choose $\alpha_E = \bar{m}$ as long as $V_E^E \geq \max(V_E^{NM}, V_E^{NT}) = V_E^{NM}$. This occurs when $\bar{m} \leq \min\left[\bar{m}_E^{RC}, \bar{m}_E^S, 1\right]$.

Note that when $\bar{m} > \left[\frac{1}{2}, \bar{\alpha}\right]$ this is the unique equilibrium (under the conditions of the proposition), induced by the uniqueness of the IO EOS. ■

A.4.4 Proposition 4:

Proof. Following the same steps as in the proof of Proposition 2, we break up the maximization problem into the following intervals of α_E : (1) $\alpha_E \in \left[\bar{m}, \min\left(\tilde{\alpha}_E(n), \frac{1}{2}\right)\right)$; 2) $\alpha_E \in \left[\tilde{\alpha}_E(n), \min\left[\hat{\alpha}(n), \frac{1}{2}\right], 1\right]$; (3) $\alpha_E > \min\left[\hat{\alpha}(n), \frac{1}{2}\right]$; (4) $\alpha_E \in [0, \bar{m})$. As before, we first describe the beliefs on the EOS in each interval and the corresponding value functions.

Case (1). By Lemma 2, $m = 1$. Then all investors anticipate monitoring in the last stage. We will assume the following beliefs about the EOS in date 1: if $w_E \geq \underline{w}_E^{LS}$ then the anticipated EOS is the Liquidity Shareholder EOS which exists by Lemma 11. If $w_E < \underline{w}_E^{LS}$ the EOS is the No Trade EOS with value function V_E^{NT} (Lemma 8). By Lemma 16, if a Liquidity Shareholder equilibrium exists the initial owner's value function, V_E^{LS}, is given by equation (68).

Case (2). If this interval is non-empty, and $w_E \geq \underline{w}_E^n$ there exists an n-Blockholder EOS by Lemma 15 and Corollary 2. The proof of Lemma 15 shows that the initial owner's minimizes α_E and the initial owner's utility function is continuous between these two intervals of α_E and is given by expression (64). Hence he prefers to minimizes α_E, i.e. $\alpha_E = \tilde{\alpha}(n)$. The value function is therefore given by V_E^n, expression (63) with $\underline{c} = \tilde{\alpha}(n) + \bar{\eta}$. If $w_E < \underline{w}_E^n < \underline{w}_E^{NT}$ then the belief on the EOS is the No Trade EOS, with value function V_E^{NT}.[29]

Case (3). In this case the unique EOS is the Initial Owner EOS for $w_E \geq \underline{w}_E^E$. Using the proof of Lemma 14 we know that the initial owner minimizes α_E, i.e. $\alpha_E = \underline{d} \equiv \min[\hat{\alpha}(n), \frac{1}{2}]$ and the value function is given by V_E^E. If $w_E < \underline{w}_E^E$ the belief on the EOS is the No Trade EOS, with value function V_E^{NT}.

Case (4). This interval is the same as in Proposition 2. By Proposition (1), the unique equilibrium is the no monitoring equilibrium and the value function is given by V_E^{NM} (expression (30)).

We now show that the initial owner chooses $\alpha_E = \bar{m}$, i.e. he chooses the LS equilibrium. This is true whenever $V_E^{LS} \geq \max(V_E^E, V_E^{NM}, V_E^n, V_E^{NT})$. Because the initial owner's value function is decreasing in α_E, $V_E^E|_{\alpha_E = \underline{d}} < V_E^E|_{\alpha_E = \bar{\alpha}} = V_E^{NC}$. Hence the liquidity shareholder ownership structure of case (1) is preferred over the initial owner one. Second, as in the proof of Proposition (2) $V_E^n|_{\alpha_E = \tilde{\alpha}(n)} < V_E^n|_{\alpha_E = \bar{m}} < V_E^{NC}$. Hence the liquidity shareholder ownership structure of case (1) is preferred over the n Blockholder ownership structure of case (2). Moreover $V_E^{NM} \geq V_E^{NT}$. Hence we only need to check that $V_E^{LS} \geq V_E^{NM}$ and this is true iff $\bar{m} \leq \min\left(\bar{m}_{NC}^{RC}, \bar{m}_{NC}^{S}\right)$. ■

A.4.5 Proposition 5:

Proof. Following the steps of the proof of Proposition 2 above, we break up the maximization problem of the initial owner into the following intervals of α_E : (1) $\alpha_E \in \left[\bar{m}, \frac{1}{2}\right]$; (2) $\alpha_E \in (\frac{1}{2}, 1]$ (3) $\alpha_E \in [0, \bar{m}]$. We first describe the beliefs on the EOS and the corresponding value functions in each of these intervals.

Case (1). By Lemma 2, $m = 1$. Then all investors anticipate monitoring in the last stage. We will assume the following beliefs about the EOS in date 1: if $w_E \geq \underline{w}_E^n$ then the anticipated EOS is the n blockholder equilibrium which exists by Lemma (10). If $w_E < \underline{w}_E^n < \underline{w}_E^{NT}$ then the EOS is the no-trade EOS with corresponding value function V_E^{NT}. By Lemma 15 we know that the initial owner's value function, V_E^n is given by equation (63) in an n-Blockholder equilibrium.

Case (2). By Lemma 2, $m = 1$. In this interval the Initial Owner EOS, whenever $w_E \geq \underline{w}_E^E$. By Lemma 14, he minimizes α_E, i.e. $\alpha_E = \frac{1}{2}$ and his value function is:

$$V_E^E = \frac{3}{2}\frac{\bar{R}^2}{\gamma\sigma^2} + 1 - \bar{m}K - \epsilon \tag{74}$$

Case (3). This is the same as Proposition 2 above and generates a value of V_E^{NM}.

Now we show that the conditions under which the initial owner chooses $\alpha_E = \bar{m}$ i.e. the first interval.

[29] In this interval there can be also an Initial Owner EOS if $n > 1$ and $w_E \geq w_E^E$. In such a case the proof that shows that the initial owner prefers the LS EOS follow the same steps as part (3).

We first check that $V_E^n \geq V_E^E$ that is:

$$\frac{1}{\alpha_1}\left(1-\frac{\bar{m}^2}{\alpha_1}+\bar{m}\right) \geq 3$$

This condition is satisfied iff:

$$\frac{1+\bar{m}-\sqrt{1+2\bar{m}-11\bar{m}^2}}{6} \leq \alpha_1 \leq \frac{1+\bar{m}+\sqrt{1+2\bar{m}-11\bar{m}^2}}{6} \tag{75}$$

Substituting α_1 we obtain that $V_n^E \geq V_E^E$ iff $\bar{m} \in [\bar{m}_{1,n}^E, \bar{m}_{2,n}^E]$. Note also that in order to guarantee real values of condition (75) $\bar{m}$ has to be below 40%. This means that in order to guarantee an n BH equilibrium $\bar{m} < 40\%$.

Second we check that $V_n^E \geq V_{NM}^E$:

(i) $V_n^E \geq 1$ iff:

$$n(1-\lambda)\lambda\bar{R}^2 + \bar{m}\left(\bar{m}n^2(1-\lambda)^2\bar{R}^2 - n(1-\lambda)(\lambda\bar{m}+\bar{m}+1)\bar{R}^2 + 2K\gamma(\lambda\bar{m}+\bar{m}-\lambda)^2\sigma^2\right) < 0 \tag{76}$$

The left hand side is a third degree inequality which goes from $-\infty$ to ∞, and it is positive at $\bar{m} = \frac{\lambda}{1+\lambda}$. Note also that when $\bar{m} = \frac{1}{2}$, the left hand side can be either positive or negative. Hence of the 3 potential roots for which the left hand side is equal to 0, we are interested for the two biggest ones which are defined as $\bar{m}_{1,n}^{RC}$ and $\bar{m}_{2,n}^{RC}$ and the negative values are between these two values, that is $\bar{m}_{1,n}^{RC} < \bar{m} < \bar{m}_{2,n}^{RC}$.

(ii) $V_n^E \geq \bar{R}\bar{X} - K + 1$: This condition is satisfied iff

$$\begin{aligned} &2\bar{\alpha}K\gamma(1+\lambda)^2\sigma^2\bar{m}^3 + \left(\bar{R}^2\left(2(1+\lambda)^2 - \bar{\alpha}n(1-\lambda)(\lambda+1)\right) - n(1-\lambda) - 2\bar{\alpha}K\gamma\left(3\lambda^2+4\lambda+1\right)\sigma^2\right)\bar{m}^2 \\ &+ \left(\bar{\alpha}\left(2K\gamma\lambda(3\lambda+2)\sigma^2 - n\bar{R}^2(1-\lambda)\right) - 4\bar{R}^2\lambda(1+\lambda)\right)\bar{m} + \lambda\left(2\lambda\bar{R}^2 + \bar{\alpha}\left(n\bar{R}^2(1-\lambda) - 2K\gamma\lambda\sigma^2\right)\right) < 0 \end{aligned} \tag{77}$$

The left hand side has the same features of the left hand side of condition (76). Hence also this condition is satisfied when $\bar{m}_{1,n}^S < \bar{m} < \bar{m}_{2,n}^S$ where $\bar{m}_1^S$ and $\bar{m}_2^S$ are the biggest solutions of the left hand side set equal to zero. ■

A.4.6 Corollary 6:

Proof. It follows directly from Propositions 3 and 5. ■

A.4.7 Corollary 7:

Proof. This is special case of Proposition (5). Note that in such a case no Initial Owner equilibrium can arise as one active investor is always willing to unilaterally deviate and hold a block and become pivotal. ■

A.4.8 Proposition 6:

The initial owner's value function has a maximum for $n = n^*$. As $\bar{m} > \frac{\lambda}{1+\lambda}$, $n > 0$.

A.4.9 Proposition 7

Proof. When the initial owner can raise an amount of capital $I \geq K$ and invest the remaining amount in the risk free asset his objective function becomes:

$$\alpha_E(X\bar{R} + K + I - K) - \frac{\gamma}{2}\alpha_E^2 X^2\sigma^2 - w_E - \bar{m}$$

The objective function of the investors becomes instead:

$$(1 - \alpha_i \frac{I - w_E}{\alpha_I}) + \alpha_i \left[X\bar{R} + K + I - K\right] - \frac{\gamma}{2}\alpha_i^2 X^2\sigma^2$$

For each levels of monitoring costs following the same steps of Propositions 2, 3, 4, 5, we obtain the optimal w_E. Inserting it in the initial owner objective function, we obtain that the initial owner's objective function is decreasing in I. ■