The Double Majority Voting Rule of the EU Reform Treaty as a Democratic Ideal for an Enlarging Union: an Appraisal Using Voting Power Analysis

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#### Abstract

The Double Majority rule in the Treaty is claimed to be simpler, more transparent and more democratic than the existing rule. We examine these questions against the democratic ideal that the votes of all citizens in whatever member country should be of equal value using voting power analysis considering possible future enlargements involving candidate countries and then to a number of hypothetical future enlargements. We find the Double Majority rule to fails to measure up to the democratic ideal in all cases. We find the Jagiellonian compromise to be very close to this ideal.


Keywords: European Union; Reform Treaty; Nice Treaty; Qualified Majority Voting; Power Indices.

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THE DOUBLE MAJORITY VOTING RULE OF THE EU REFORM TREATY AS A DEMOCRATIC IDEAL FOR AN ENLARGING UNION: AN APPRAISAL USING VOTING POWER ANALYSIS

The Reform Treaty agreed in Rome in September 2004 contains fundamental reforms to the voting system used by the Council of Ministers. The current triple-majority system would be replaced with a double-majority decision rule that is said to be simpler to understand, more democratic and more flexible. In this paper we investigate these claims using voting power analysis in a number of enlargement scenarios.

## 1. QUALIFIED MAJORITY VOTING AND FUTURE EU ENLARGEMENT

The council of ministers, the senior legislature of the EU, is an intergovernmental body in which some matters are decided by unanimity but the most important voting rule is qualified majority voting (QMV), that is being used for an increasing number of decisions. Under QMV each country has a different number of votes that it can cast that is related in some way to its size. Under the Reform Treaty proposals they will be strictly proportional to population sizes but under the system determined by the Nice Treaty and under the previous system the voting weights were not directly based on populations in a transparently mathematical way.

The problem of the determination of the voting weights is an important one because under the rules of the council each country must cast its votes as a bloc; a country is not permitted to divide its votes for any reason, as it might, for example in order to reflect a division of public opinion at home in the country. Alternatively if, instead of a single representative with many votes, the country's representation were by numbers of elected individuals who would vote individually as representatives or delegates rather than as a national group acting en bloc, as for example members of the European Parliament are able to do, the problem addressed in this paper would not exist.

In that case the voting power of the citizen of each country would be approximately the same. However in a body that uses weighted voting, there is not a simple relation between weight and voting power and each case must be considered on its merits by considering the possible outcomes of the voting process, making a voting power analysis.

The proposed new Double Majority rule is that a decision taken by QMV should require the support of 55 percent of the member countries whose combined populations are at least 65 percent of the EU total. This contrasts with the system currently in use (the Nice system) under which each country has a given number of weighted votes, all of which were laid down in the Nice Treaty. Specifically the Nice system is a triple-majority rule that works as follows. For a vote to lead to a decision, three requirements must be met: (i) the countries voting in favour must constitute a majority of members; (ii) they must contain at least 62 percent of the population of the union; and (iii) their combined weighted votes must exceed the specified threshold. The Nice Treaty specified a threshold that depended on the size of the membership: for the union of 15 countries it was about 71 percent of the total of the weighted votes, increasing gradually with enlargement to its present level, with 27 members, to almost 74 percent.

Studies using voting power analysis have concluded that the Nice system is broadly equitable in the sense that the resulting powers of individual countries are fair in relative terms (in an appropriately defined sense), with one or two exceptions ${ }^{\mathrm{i}}$, but that the threshold was set much too high for the Council to be able to deal with a greater range of decisions by qualified majority voting in an efficient manner. (Leech (2002), Felsenthal and Machover (2001))

Advocates of changing to the Double Majority rule argue, first, that it would be much simpler to understand than the Nice system (which has been described as
'fiendishly complex') which is lacking in transparency because of its use of artificially constructed voting weights. The Nice weights are criticised because they are not, even approximately, directly proportional to populations; the countries with larger populations are assigned larger weights than the smaller ones but the difference does not fully reflect relative populations. Superficially it appears that larger states are underrepresented, although it can be argued that such weights may well, in actual fact, be consistent with a reasonable degree of fairness in the distribution of voting power. But this argument by itself would not be decisive in favour of change given that the Nice system is already in place.

A second criticism of the Nice system is that the threshold is set too high, and moreover, increasing it as the membership increases, makes decisions harder by requiring a larger qualified majority, or making it easier for a blocking minority to form. Studies of the formal a priori decisiveness of the system have shown that the probability of a qualified majority emerging could be extremely small. (Felsenthal and Machover, 2001.) However, despite these fears, recent studies have found little evidence in practice of the sclerosis that was feared, and qualified majority voting appears to be working quite well. (Wallace and Hayes-Renshaw, 2006)

The third argument for change is that the Nice system was designed for certain specified anticipated enlargements of the union, which have now all occurred. It provided for a union of up to 27 members - the fifteen members at the time of the treaty, plus the ten countries that acceded in May 2004 followed by Bulgaria and Romania that joined in January 2007 - and further enlargement beyond that is therefore outside its scope. The formal position is that the accession of a new country would require a new treaty that included amendments to the system of qualified majority voting. But there are further candidates including Turkey and the former republics of Yugoslavia; and there is also the
remote possibility of further FSU countries, and perhaps also other European countries joining. It would clearly be impossibly inefficient to have to hold an Intergovernmental Conference every time further enlargement took place. So there is need for a system that embodies a principle that can be applied in a more or less routine manner each time a new member accedes. An example of such a voting system is the double-majority rule in the Reform Treaty.

It is this administrative simplicity that makes the double-majority voting rule most attractive. It enables us to know immediately how many votes a new member will have and in what ways the operation of qualified majority voting will be affected. All that it is necessary to know is the country's population.

## 2. APPRAISAL OF VOTING RULES BY POWER ANALYSIS

It does not follow that we understand all the consequences of enlargement for the fairness and efficiency of the voting system. It has been claimed that a weighted voting rule based directly on populations will implement a desirable democratic principle of equality: that each country will have a voting power proportional to its population. That is undoubtedly a major factor in the thinking behind the proposal. However it is a serious mistake because in a weighted voting body like the council, where members cast all their votes as a single bloc, power in the sense of the ability to influence decisions is not related straightforwardly to weight. It is possible for example for a country to have voting weight that is not translated into actual voting power ${ }^{\text {ii }}$. It is therefore necessary to make a voting power analysis to establish the properties of this system, and in particular the powers of the members.

In this paper we do this by considering voting in the Council of Ministers as a formal two-stage democratic decision process that allows us to compare voting power of
citizens of different countries. It is a fundamental principle of the EU that all citizens should have equal rights in whatever country they happen to live. This provides a natural criterion on which to judge the adequacy of the voting system, a benchmark against which to compare the fairness of the distribution of voting power. We use voting power analysis to do this, following the approach of Penrose (1946, 1952), treating the council of ministers as a delegate body on which individual citizens are represented by government ministers elected by them.

The voting power of a citizen is derived from two components: (i) power of his or her country in the council (a property of the system of weighted QMV in the council), and (ii) the power of the citizen in a popular election within the country. A citizen's voting power, as a structural property of the voting system, is measured by his or her Penrose power index, which is the product of these two voting power indices.

We make two analyses. First we compare the double-majority rule with the Nice system for the current EU of 27 countries. Secondly we investigate various scenarios for further enlargement. These begin with the expected accession of the known candidate countries and then become more and more speculative as further new members are presupposed. Our primary purpose is to test the claim that the Reform Treaty proposals are simple and transparent in the face of further enlargement ${ }^{\text {iii }}$. We also investigate the alternative voting rule that has recently been proposed, known as the Jagiellonian Compromise, and find it remarkably equitable. (Slomczynski and Zyczkowski, 2007)

We report analyses of the following Scenarios for possible future enlargement of the $\mathrm{EU}^{\text {iv }}$ :

O EU27: the current union. Member countries: Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, United Kingdom.

I EU29: as above plus Croatia, Macedonia.
II EU30: with Turkey.
III EU34: with Albania, Bosnia, Montenegro, Serbia.
IV EU37: with Norway, Iceland, Switzerland.
V EU40: with Belarus, Moldova, Ukraine.
VI EU41: with Russia.
In the next section we describe the mathematics of the voting power approach that we employ to analyse these scenarios.

## 3. THE PENROSE POWER INDEX APPROACH

The EU council of ministers at any time is assumed to consist of $n$ member countries, represented by a set $N=\{1,2, \ldots, n\}$, where each country is labelled by an integer $i=1$ to $n$. Each country has a population (which we take, as a first approximation, to be the same as its electorate ${ }^{\mathrm{v}}$ ), represented for country $i$ by $m_{i}$. The total population of the EU is $m=\sum_{j \in N} m_{j}$.

Under the Reform Treaty, any normal decision requires a double majority in favour of the proposal: at least $55 \%$ of member countries whose combined populations are at least $65 \%$ of the total population.

Suppose that in any vote concerning such a decision there are $s$ countries in favour, represented by a set $S$, a subset of $N$. Then the decision is taken if:
(i) $s \geq 0.55 n ; \quad$ and
(ii) $\sum_{j \in S} m_{j} \geq 0.65 m$

Let us denote the number of such subsets that satisfy conditions (1) by $\omega$; that is, $\omega$ is the number of possible outcomes of votes among the countries in the council of ministers that lead to a decision. The number $\omega$ reflects the general capacity of the body to act (a measure of its decisiveness).

The appropriate measure of a priori voting power for each country can now be defined in terms of its ability to influence decisions taken by this rule, strictly by its being able to swing a decision by adding its vote to those of the other members. A swing is defined for member $i$ as a subset $S$ which does not contain $i$ such that: it is just losing and becomes winning with the addition of member $i$. Formally, the two conditions for a swing are,
(i) $0.55 n-1 \leq s<0.55 n$, and
(ii) $0.65 m-m_{i} \leq \sum_{j \in S} m_{j}<0.65 m$, for any $S \subseteq N, i \notin S$.

The number of possible swings, $S$, taking account of all possible voting outcomes among the $n-1$ members other than $i$, that is the number of subsets of $N \backslash\{i\}$, is $2^{n-1}$. If the number of swings for country $i$ is denoted by $h_{\mathrm{i}}$, then the (modified ${ }^{\text {vi }}$ ) Penrose index for the council of ministers is defined as $P_{i}^{C}$

$$
\begin{equation*}
P_{i}^{C}=\frac{\eta_{i}}{2^{n-1}}, \quad \text { for } i=1,2, \ldots, n . \tag{3.}
\end{equation*}
$$

The basic interpretation of expression (3) is the proportion of all possible voting outcomes that are swings for country $i$. It is usually given a probabilistic interpretation by assuming that all possible voting outcomes are equally likely.

Computation of the indices $P_{i}^{C}$ is a substantial problem especially when the number of countries is large because the direct evaluation of expression (3.) requires a
search over all $2^{n}$ possible voting outcomes. Such a computation becomes prohibitively slow even when $n$ is relatively small and therefore alternative approaches have to be found. The method used in this study is a modification of the method of generating functions first defined by Brams and Affuso (1976) and extended to multiple decision rules by Algaba et al (2003). The method of generating functions is very fast even for very large voting bodies and therefore is suitable to the present analysis of a union with an expanding membership. This great speed comes at a cost however in terms of space complexity which limits the sizes of the weights that the method of generating functions can cope with. The method of generating functions is described and full details of computation are given in Aziz et al. (2007).

The Penrose index (3) is a measure of absolute voting power in the sense of meaning the country's likelihood of being decisive when all voting outcomes are considered on an equal basis. A measure of the decisiveness of the Council, its Power to Act, $A^{\mathrm{C}}$, is a measure of its ability to make decisions defined on the same basis. The power to act is the number of subsets of all $n$ countries that would lead to a decision (Coleman, 1971):

$$
\begin{equation*}
A^{C}=\frac{\omega}{2^{n}} . \tag{4.}
\end{equation*}
$$

The power of an individual citizen is defined formally by idealising the council as a representative body in which the determination of how a country will cast its weighted votes follows a simple majority vote among its citizens. This requires finding a measure of the power of a single citizen within the country.

The Penrose power index of a citizen assuming a one-person-one-vote electoral system ${ }^{\text {vii }}$ is the likelihood of his or her being able to swing the national election. This is
the binomial probability of his or her being the swing voter, considering all possible voting outcomes among the other voters to be equally likely. For country $i$, with $m_{i}$ voters, it is the probability that the number of votes cast by the $m_{i}-1$ voters other than the single citizen under consideration are precisely one vote short of a majority ${ }^{\text {viii }}$. Denote this power index for the single vote of any citizen in country i by $P_{i}^{S}$.

Then, if $m_{i}$ is even,

$$
\begin{align*}
& P_{i}^{S}=\operatorname{Pr}\left[\text { combined votes of } m_{i}-1 \text { voters }=\frac{m_{i}}{2}\right]=\binom{m_{i}-1}{\frac{m_{i}}{2}}(0.5)^{m_{i}-1}  \tag{5.}\\
& \text { or, if } m_{i} \text { is odd, } \\
& P_{i}^{S}=\operatorname{Pr}\left[\text { combined votes of } m_{i}-1 \text { voters }=\frac{m_{i}-1}{2}\right]=\binom{m_{i}-1}{\frac{m_{i}-1}{2}}(0.5)^{m_{i}-1}
\end{align*}
$$

When $m_{i}$ is large, as is well known, (5) and (6) can be approximated very accurately by Stirling's formula ${ }^{\mathrm{ix}}$, which gives $P_{i}^{S}$ a very simple form,

$$
\begin{equation*}
P_{i}^{S}=\sqrt{\frac{2}{\pi m_{i}}}=\frac{0.79788}{\sqrt{m_{i}}} \tag{7.}
\end{equation*}
$$

We can evaluate the indirect power index for a citizen of country $i$ as the product of (7.) and (3.). Let us denote this composed power index $P_{i}$. Therefore we can write, for the indirect power of a citizen of country $i$,

$$
\begin{equation*}
P_{i}=P_{i}^{C} P_{i}^{S} \tag{8.}
\end{equation*}
$$

The numerical value of $P_{i}$ is of course rather small because $P_{i}^{S}$ is small. However its value can vary enormously between countries, over changes in the membership of the union and changes in the voting system. Expression (8) provides a yardstick or benchmark to use in the evaluation of the weighted voting system on a consistent basis of
democratic legitimacy. Comparisons can be made using relative voting power indices to compare countries and therefore to test the extent to which the voting system is egalitarian.

Expression (8.) can be used as the basis of the Penrose Square Root rule for equalising voting power in all countries. The rule is that weighted voting be adopted in the council with a decision rule and weights chosen such that (8.) is constant for all i. The power indices $P_{i}^{C}$ therefore should be proportional to the square roots of the populations. This can be a decision rule with a single majority. An approximation to this that will be sufficient in many cases is to choose the weights themselves in proportion to population square roots. This has been applied recently in the so-called Jagiellonian compromise in which also the decision threshold is also adjusted to improve the approximation (Slomczynski and Zyczkowski, 2007). We have investigated the performance of this proposed voting rule in equality of voting power and find it works very well indeed.

## 4. ANALYSIS: VOTING POWER IN THE EU27

We compared the three voting systems for the present-day union, E27. The results are given in Table $1^{\mathrm{x}}$ in which the countries are in size order. The Penrose indices agree very closely with those of Felsenthal and Machover (2007), the only slight differences being due to the use of different weights.

The method of generating functions requires that the weights be integers that are not too large. This necessitates, in practice, replacing the population figures, which are mostly millions and too large numbers to be used as weights, by much smaller integers that are manageable computationally ${ }^{\mathrm{xi}}$. The table also gives the quota in terms of the weights.

Besides the Penrose index for each country, $P_{i}^{C}$, Table 1 also shows the indirect citizen power indices, $P_{i}$, defined in (8.). These are presented in two ways, as absolute values and relative to the power of a citizen of Germany. (In all our results the relative voting power of a German citizen is taken as equal to 1 .) The absolute citizen voting power indices are of course very small in all countries. Under the Reform treaty, that for Germany is equal to $1.76 \times 10^{-5}$, that is 0.0000176 , while that for the smallest member, Malta, is $3.41 \times 10^{-5}$.

The relative voting power indices show the inequity inherent in the voting rule, with all but those for the smallest group of countries being less than 1 . The effect is biggest for the medium sized countries, especially Belgium, with 0.673 , and Greece, 0.671. The conclusion is that the Double Majority system would create large disparities in voting power in different countries.

Inequality is measured by the Gini coefficient of citizen voting power for the whole union ${ }^{\text {xii }}$. Table 1 shows how much more unequal the proposed voting system would be compared with the existing Nice system, under which citizens of every country (with the slight exception of Latvia) have slightly greater voting power than those of Germany (a result due to the fact that Germany has no greater weight than France, Italy and the UK despite its much larger population): the Gini coefficient for Nice being 0.059 , that for the Reform Treaty, 0.080.

We have also reported the power to act of the council of ministers, $\mathrm{A}^{\mathrm{C}}$, which show that the Reform Treaty voting rule is a very much more decisive voting rule than Nice ${ }^{\text {xiii }}$, with a power to act of 0.129 compared to a very low value of 0.02 .

The results for the Jagiellonian Compromise are quite impressive in showing that this method would lead to the equalisation of voting power throughout the union of 27
countries ${ }^{\text {xiv }}$. There is almost no variation in the relative citizen voting power indices across countries, which indicates how good an approximation to the Penrose square root rule is obtained by using population square roots as weights.

| Table 1: Voting Power Analysis of the EU27 |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Nice |  |  | Ref | Treaty |  |  |  | onian |  |
| Country | Population | Penrose <br> Index | Czn <br> Power | Rel Czn <br> Power | Wei | Penrose <br> Index | Czn <br> Power | Rel Czn <br> Power | Weight | Penrose <br> Index | Czn <br> Power | Rel Czn Power |
| Germany | 82,437,995 | 0.03269 | $2.87 \mathrm{E}-06$ | 1.000 | 824 | 0.20011 | $1.76 \mathrm{E}-05$ | 1.000 | 9080 | 0.20080 | $1.76 \mathrm{E}-05$ | 1.00000 |
| France | 62,998,773 | 0.03269 | $3.29 \mathrm{E}-06$ | 1.144 | 630 | 0.15517 | $1.56 \mathrm{E}-05$ | 0.887 | 7937 | 0.17597 | $1.77 \mathrm{E}-05$ | 1.00246 |
| UK | 60,393,100 | 0.03269 | $3.36 \mathrm{E}-06$ | 1.168 | 604 | 0.14932 | $1.53 \mathrm{E}-05$ | 0.872 | 7771 | 0.17233 | $1.77 \mathrm{E}-05$ | 1.00267 |
| Italy | 58,751,711 | 0.03269 | $3.40 \mathrm{E}-06$ | 1.185 | 588 | 0.14587 | $1.52 \mathrm{E}-05$ | 0.863 | 7665 | 0.16999 | $1.77 \mathrm{E}-05$ | 1.00276 |
| Spain | 43,758,250 | 0.03116 | $3.76 \mathrm{E}-06$ | 1.308 | 438 | 0.11252 | $1.36 \mathrm{E}-05$ | 0.772 | 6615 | 0.14675 | $1.77 \mathrm{E}-05$ | 1.00310 |
| Poland | 38,157,055 | 0.03116 | $4.02 \mathrm{E}-06$ | 1.401 | 382 | 0.09816 | $1.27 \mathrm{E}-05$ | 0.721 | 6177 | 0.13702 | $1.77 \mathrm{E}-05$ | 1.00300 |
| Romania | 21,610,213 | 0.01789 | $3.07 \mathrm{E}-06$ | 1.069 | 216 | 0.07139 | $1.23 \mathrm{E}-05$ | 0.697 | 4649 | 0.10307 | $1.77 \mathrm{E}-05$ | 1.00256 |
| Netherlands | 16,334,210 | 0.01669 | $3.29 \mathrm{E}-06$ | 1.147 | 163 | 0.06006 | $1.19 \mathrm{E}-05$ | 0.674 | 4042 | 0.08959 | $1.77 \mathrm{E}-05$ | 1.00230 |
| Greece | 11,125,179 | 0.01547 | $3.70 \mathrm{E}-06$ | 1.288 | 111 | 0.04933 | $1.18 \mathrm{E}-05$ | 0.671 | 3335 | 0.07390 | $1.77 \mathrm{E}-05$ | 1.00182 |
| Portugal | 10,569,592 | 0.01547 | $3.80 \mathrm{E}-06$ | 1.322 | 106 | 0.04830 | $1.19 \mathrm{E}-05$ | 0.674 | 3251 | 0.07203 | $1.77 \mathrm{E}-05$ | 1.00184 |
| Belgium | 10,511,382 | 0.01547 | $3.81 \mathrm{E}-06$ | 1.325 | 105 | 0.04810 | $1.18 \mathrm{E}-05$ | 0.673 | 3242 | 0.07184 | $1.77 \mathrm{E}-05$ | 1.00184 |
| Czech | 10,251,079 | 0.01547 | $3.86 \mathrm{E}-06$ | 1.342 | 103 | 0.04768 | $1.19 \mathrm{E}-05$ | 0.676 | 3202 | 0.07095 | $1.77 \mathrm{E}-05$ | 1.00191 |
| Hungary | 10,076,581 | 0.01547 | $3.89 \mathrm{E}-06$ | 1.354 | 101 | 0.04727 | $1.19 \mathrm{E}-05$ | 0.676 | 3174 | 0.07031 | $1.77 \mathrm{E}-05$ | 1.00156 |
| Sweden | 9,047,752 | 0.01299 | $3.45 \mathrm{E}-06$ | 1.199 | 90 | 0.04500 | $1.19 \mathrm{E}-05$ | 0.679 | 3008 | 0.06664 | $1.77 \mathrm{E}-05$ | 1.00180 |
| Austria | 8,265,925 | 0.01299 | $3.60 \mathrm{E}-06$ | 1.255 | 83 | 0.04356 | $1.21 \mathrm{E}-05$ | 0.687 | 2875 | 0.06369 | $1.77 \mathrm{E}-05$ | 1.00168 |
| Bulgaria | 7,718,750 | 0.01299 | $3.73 \mathrm{E}-06$ | 1.299 | 77 | 0.04233 | $1.22 \mathrm{E}-05$ | 0.691 | 2778 | 0.06154 | $1.77 \mathrm{E}-05$ | 1.00149 |
| Denmark | 5,427,459 | 0.00916 | $3.14 \mathrm{E}-06$ | 1.092 | 54 | 0.03758 | $1.29 \mathrm{E}-05$ | 0.732 | 2330 | 0.05161 | $1.77 \mathrm{E}-05$ | 1.00164 |
| Slovakia | 5,389,180 | 0.00916 | $3.15 \mathrm{E}-06$ | 1.096 | 54 | 0.03758 | $1.29 \mathrm{E}-05$ | 0.735 | 2321 | 0.05141 | $1.77 \mathrm{E}-05$ | 1.00131 |
| Finland | 5,255,580 | 0.00916 | $3.19 \mathrm{E}-06$ | 1.110 | 53 | 0.03738 | $1.30 \mathrm{E}-05$ | 0.740 | 2293 | 0.05078 | $1.77 \mathrm{E}-05$ | 1.00161 |
| Ireland | 4,209,019 | 0.00916 | $3.56 \mathrm{E}-06$ | 1.240 | 42 | 0.03510 | $1.37 \mathrm{E}-05$ | 0.776 | 2052 | 0.04544 | $1.77 \mathrm{E}-05$ | 1.00145 |
| Lithuania | 3,403,284 | 0.00916 | $3.96 \mathrm{E}-06$ | 1.379 | 34 | 0.03344 | $1.45 \mathrm{E}-05$ | 0.822 | 1845 | 0.04086 | $1.77 \mathrm{E}-05$ | 1.00143 |
| Latvia | 2,294,590 | 0.00525 | $2.77 \mathrm{E}-06$ | 0.963 | 23 | 0.03116 | $1.64 \mathrm{E}-05$ | 0.933 | 1515 | 0.03355 | $1.77 \mathrm{E}-05$ | 1.00142 |
| Slovenia | 2,003,358 | 0.00525 | $2.96 \mathrm{E}-06$ | 1.030 | 20 | 0.03053 | $1.72 \mathrm{E}-05$ | 0.979 | 1415 | 0.03133 | $1.77 \mathrm{E}-05$ | 1.00070 |
| Estonia | 1,344,684 | 0.00525 | $3.61 \mathrm{E}-06$ | 1.257 | 13 | 0.02908 | $2.00 \mathrm{E}-05$ | 1.138 | 1160 | 0.02568 | $1.77 \mathrm{E}-05$ | 1.00148 |
| Cyprus | 766,414 | 0.00525 | $4.78 \mathrm{E}-06$ | 1.666 | 8 | 0.02803 | $2.56 \mathrm{E}-05$ | 1.453 | 875 | 0.01938 | $1.77 \mathrm{E}-05$ | 1.00069 |
| Luxembourg | 459,500 | 0.00525 | $6.18 \mathrm{E}-06$ | 2.151 | 5 | 0.02741 | $3.23 \mathrm{E}-05$ | 1.835 | 678 | 0.01501 | $1.77 \mathrm{E}-05$ | 1.00149 |
| Malta | 404,346 | 0.00396 | $4.97 \mathrm{E}-06$ | 1.730 | 4 | 0.02720 | $3.41 \mathrm{E}-05$ | 1.941 | 636 | 0.01408 | $1.77 \mathrm{E}-05$ | 1.00126 |
| Total | 492,964,961 | 0.41999 |  |  | 4930 | 1.71869 |  |  | 95921 | 2.12556 |  |  |
| Quota |  |  |  |  | 3205 |  |  |  | 59062 |  |  |  |
| Power to Act |  |  |  | 0.020 |  |  |  | 0.129 |  |  |  | 0.163 |
| Gini Coeff |  |  |  | 0.059 |  |  |  | 0.080 |  |  |  | $9.08 \mathrm{E}-05$ |

## 5. ANALYSIS: ENLARGEMENT SCENARIOS

Table 2 presents the results for the enlargement scenarios I to VI. They are presented diagrammatically for existing members in Figures 1 to 3 . We also present a parallel analysis for the Jagiellonian Compromise in Table 3. In all scenarios the same population figures have been used, the 2006 estimates taken from Eurostat ${ }^{\mathrm{xv}}$.

They show that the inequality in citizen voting power under the Double Majority rule persists although there are sharp changes in relative voting power following changes
in the membership. On the other hand, the Jagiellonian system turns out to be remarkably successful in creating a very equal distribution of citizen power in all scenarios, and to be quite robust to membership changes. The use of square root weights and adjustment of the quota gives an extremely good approximation to the Penrose square root rule.

The results for the Reform Treaty voting rule in Table 2 show that citizen voting power is relatively unequal under all scenarios. The Gini coefficient for Scenario VI (41 countries including Russia) is the same as in Scenario O (EU27) although it falls below this in some scenarios. Citizen voting power is most equal following the accession of Turkey, Gini $=0.059$, Scenario II, that may be largely due to the similarity in population of the two largest members, Germany and Turkey. Whereas having one country that is much larger than all the others creates an unequal power distribution, where there are two members with very large weight, a bipolar voting structure, there is a tendency for them to counteract one another. Thus the presence of Turkey would reduce Germany's power and increase the power of other members, making the distribution more equal. The accession of Turkey would substantially increase the voting power of citizens of Poland and Spain, from 0.718 and 0.772 to 0.822 and 0.815 . There is a similar effect for medium sized countries, but their relative voting power remains much lower: for example, the index for Belgium goes from 0.661 to 0.760 . The effects for small countries, whose citizen voting powers are already much larger than Germany's are quite large: for example, Malta's goes from 1.859 to 2.442 .

The power of the council to act, $\mathrm{A}^{\mathrm{C}}$, declines more or less steadily as the union enlarges, from $12.9 \%$ for O (EU27) to $9.3 \%$ in VI, although it is always much greater than under the current system.

| Country | Table 2: Citizen Indirect Power Indices Under All Scenarios: Reform Treaty |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Enlargement Scenarios |  |  |  |  |  |  |
|  | O | I | II | III | IV | V | VI |
| Albania |  |  |  | 0.933 | 1.030 | 0.818 | 1.231 |
| Austria | 0.687 | 0.673 | 0.788 | 0.743 | 0.800 | 0.679 | 0.917 |
| Belarus |  |  |  |  |  | 0.670 | 0.886 |
| Belgium | 0.673 | 0.661 | 0.760 | 0.721 | 0.770 | 0.667 | 0.871 |
| Bosnia \& H |  |  |  | 0.872 | 0.960 | 0.771 | 1.139 |
| Bulgaria | 0.691 | 0.676 | 0.796 | 0.749 | 0.808 | 0.682 | 0.931 |
| Croatia | 0.000 | 0.742 | 0.906 | 0.841 | 0.921 | 0.748 | 1.087 |
| Cyprus | 1.453 | 1.394 | 1.815 | 1.648 | 1.854 | 1.404 | 2.276 |
| Czech | 0.676 | 0.663 | 0.764 | 0.725 | 0.774 | 0.669 | 0.877 |
| Denmark | 0.732 | 0.712 | 0.861 | 0.802 | 0.875 | 0.718 | 1.025 |
| Estonia | 1.138 | 1.094 | 1.409 | 1.284 | 1.439 | 1.102 | 1.756 |
| Finland | 0.740 | 0.719 | 0.871 | 0.811 | 0.885 | 0.726 | 1.038 |
| France | 0.887 | 0.889 | 0.904 | 0.900 | 0.905 | 0.909 | 0.921 |
| Germany | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Greece | 0.671 | 0.659 | 0.755 | 0.718 | 0.765 | 0.665 | 0.862 |
| Hungary | 0.676 | 0.663 | 0.765 | 0.725 | 0.775 | 0.669 | 0.879 |
| Iceland |  |  |  |  | 2.876 | 2.152 | 3.554 |
| Ireland | 0.776 | 0.753 | 0.925 | 0.857 | 0.941 | 0.759 | 1.114 |
| Italy | 0.863 | 0.865 | 0.884 | 0.880 | 0.886 | 0.888 | 0.906 |
| Latvia | 0.933 | 0.900 | 1.139 | 1.044 | 1.161 | 0.907 | 1.401 |
| Lithuania | 0.822 | 0.796 | 0.989 | 0.913 | 1.007 | 0.803 | 1.201 |
| Luxembourg | 1.835 | 1.758 | 2.304 | 2.087 | 2.355 | 1.770 | 2.902 |
| Macedonia |  | 0.935 | 1.189 | 1.089 | 1.213 | 0.942 | 1.468 |
| Malta | 1.941 | 1.859 | 2.442 | 2.210 | 2.496 | $1.872$ | 3.080 |
| Moldova |  |  |  |  |  | 0.780 | 1.156 |
| Montenegro |  |  |  | 1.842 | 2.076 | 1.564 | 2.555 |
| Netherlands | 0.674 | 0.666 | 0.739 | 0.711 | 0.746 | 0.671 | 0.822 |
| Norway |  |  |  |  | 0.911 | 0.741 | 1.074 |
| Poland | 0.721 | 0.718 | 0.822 | 0.804 | 0.817 | 0.788 | 0.846 |
| Portugal | 0.674 | 0.662 | 0.761 | 0.722 | 0.771 | 0.668 | 0.872 |
| Romania | 0.697 | 0.692 | 0.746 | 0.724 | 0.752 | 0.692 | $0.812$ |
| Russia |  |  |  |  |  |  | 1.225 |
| Serbia |  |  |  | 0.727 | 0.778 | 0.670 | 0.883 |
| Slovakia | 0.735 | 0.715 | 0.864 | 0.805 | 0.878 | 0.721 | 1.029 |
| Slovenia | 0.979 | 0.943 | 1.200 | 1.098 | 1.224 | 0.951 | 1.481 |
| Spain | 0.772 | 0.772 | 0.815 | 0.802 | 0.814 | 0.815 | 0.854 |
| Sweden | 0.679 | 0.665 | 0.774 | 0.732 | 0.785 | 0.671 | 0.897 |
| Switzerland |  |  |  |  | 0.814 | 0.686 | 0.939 |
| Turkey |  |  | $0.959$ | 0.958 | 0.959 | 0.968 | 0.969 |
| UK | 0.872 | 0.874 | 0.892 | 0.888 | 0.893 | 0.896 | 0.911 |
| Ukraine |  |  |  |  |  | 0.825 | 0.861 |
| Power to Act | 0.129 | 0.126 | 0.110 | 0.106 | 0.092 | 0.096 | 0.093 |
| Gini Coefficient | 0.080 | 0.082 | 0.059 | 0.067 | 0.061 | 0.079 | 0.080 |

Weights used are given in Table 1. Gini coefficients computed at the citizen level for the whole union.

Our overall conclusion is that the Reform Treaty's Double Majority rule falls a long way short of the democratic ideal of ensuring that the votes of all members of the community are of equal value whatever country they are cast in. It is an endemic feature that citizens of middle sized countries have considerably less voting power than those in either large or small countries. This pattern persists under all the enlargement scenarios we have looked at.

| Table 3: Citizen Indirect Power Indices Under All Scenarios: Jagiellonian Compromise |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Enla | ement Scenar |  |  |  |
|  |  | O | I | II | III | IV | V | VI |
| Country | Weight | EU27 | EU29 | EU30 | EU34 | EU37 | EU40 | EU41 |
| Albania | 1786 |  |  |  | 1.0004 | 1.0005 | 1.0005 | 0.9987 |
| Austria | 2875 | 1.0017 | 1.0017 | 1.0006 | 1.0007 | 1.0008 | 1.0007 | 0.9989 |
| Belarus | 3113 |  |  |  |  |  | 1.0009 | 0.9991 |
| Belgium | 3242 | 1.0018 | 1.0019 | 1.0007 | 1.0008 | 1.0009 | 1.0008 | 0.9990 |
| Bosnia \& H | 1984 |  |  |  | 1.0007 | 1.0007 | 1.0007 | 0.9990 |
| Bulgaria | 2778 | 1.0015 | 1.0016 | 1.0004 | 1.0006 | 1.0007 | 1.0006 | 0.9988 |
| Croatia | 2134 |  | 1.0014 | 1.0003 | 1.0004 | 1.0005 | 1.0005 | 0.9987 |
| Cyprus | 875 | 1.0007 | 1.0007 | 0.9996 | 0.9998 | 0.9999 | 0.9999 | 0.9981 |
| Czech | 3202 | 1.0019 | 1.0020 | 1.0008 | 1.0009 | 1.0010 | 1.0009 | 0.9991 |
| Denmark | 2330 | 1.0016 | 1.0017 | 1.0006 | 1.0007 | 1.0008 | 1.0008 | 0.9990 |
| Estonia | 1160 | 1.0015 | 1.0017 | 1.0004 | 1.0007 | 1.0008 | 1.0008 | 0.9990 |
| Finland | 2293 | 1.0016 | 1.0018 | 1.0007 | 1.0008 | 1.0009 | 1.0008 | 0.9991 |
| France | 7937 | 1.0025 | 1.0025 | 1.0015 | 1.0014 | 1.0014 | 1.0011 | 1.0001 |
| Germany | 9080 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| Greece | 3335 | 1.0018 | 1.0018 | 1.0006 | 1.0007 | 1.0008 | 1.0007 | 0.9989 |
| Hungary | 3174 | 1.0016 | 1.0018 | 1.0006 | 1.0007 | 1.0008 | 1.0007 | 0.9989 |
| Iceland | 549 |  |  |  |  | 1.0011 | 1.0011 | 0.9993 |
| Ireland | 2052 | 1.0015 | 1.0017 | 1.0006 | 1.0007 | 1.0008 | 1.0008 | 0.9990 |
| Italy | 7665 | 1.0028 | 1.0028 | 1.0016 | 1.0016 | 1.0015 | 1.0013 | 1.0002 |
| Latvia | 1515 | 1.0014 | 1.0015 | 1.0004 | 1.0006 | 1.0006 | 1.0006 | 0.9989 |
| Lithaunia | 1845 | 1.0014 | 1.0016 | 1.0004 | 1.0006 | 1.0006 | 1.0006 | 0.9989 |
| Luxembourg | 678 | 1.0015 | 1.0013 | 1.0011 | 1.0005 | 1.0006 | 1.0006 | 0.9988 |
| Macedonia | 1428 |  | 1.0014 | 1.0006 | 1.0007 | 1.0008 | 1.0008 | 0.9990 |
| Malta | 636 | 1.0013 | 1.0013 | 1.0011 | 1.0005 | 1.0005 | 1.0006 | 0.9989 |
| Moldova | 1948 |  |  |  |  |  | 1.0006 | 0.9989 |
| Montenegro | 773 |  |  |  | 0.9999 | 1.0000 | 1.0000 | 0.9982 |
| Netherlands | 4042 | 1.0023 | 1.0024 | 1.0012 | 1.0012 | 1.0012 | 1.0011 | 0.9994 |
| Norway | 2167 |  |  |  |  | 1.0004 | 1.0004 | 0.9986 |
| Poland | 6177 | 1.0030 | 1.0030 | 1.0017 | 1.0017 | 1.0017 | 1.0014 | 0.9999 |
| Portugal | 3251 | 1.0018 | 1.0019 | 1.0007 | 1.0008 | 1.0009 | 1.0008 | 0.9990 |
| Romania | 4649 | 1.0026 | 1.0027 | 1.0013 | 1.0014 | 1.0014 | 1.0012 | 0.9995 |
| Russia | 11937 |  |  |  |  |  |  | 0.9943 |
| Serbia | 3140 |  |  |  | 1.0009 | 1.0010 | 1.0009 | 0.9991 |
| Slovakia | 2321 | 1.0013 | 1.0014 | 1.0003 | 1.0004 | 1.0005 | 1.0004 | 0.9986 |
| Slovenia | 1415 | 1.0007 | 1.0011 | 1.0000 | 1.0001 | 1.0002 | 1.0002 | 0.9984 |
| Spain | 6615 | 1.0031 | 1.0031 | 1.0018 | 1.0018 | 1.0018 | 1.0015 | 1.0000 |
| Sweden | 3008 | 1.0018 | 1.0018 | 1.0007 | 1.0008 | 1.0009 | 1.0008 | 0.9990 |
| Switzerland | 2736 |  |  |  |  | 1.0009 | 1.0008 | 0.9990 |
| Turkey | 8653 |  |  | 1.0007 | 1.0007 | 1.0007 | 1.0005 | 1.0001 |
| UK | 7771 | 1.0027 | 1.0026 | 1.0015 | 1.0015 | 1.0014 | 1.0012 | 1.0001 |
| Ukraine | 6797 |  |  |  |  |  | 1.0014 | 1.0000 |
| Total |  | 95921 | 99483 | 108136 | 115,819 | 121271 | 133129 | 145066 |
|  | Quota: | 59062 | 60917 | 66052 | 70076 | 72929 | 79451 | 86735 |

Table 3 shows the results for the Jagiellonian compromise under the same scenarios. For each scenario the weights, which are the population square roots, $\sqrt{ } m_{i}$, are shown in the first column, and the quota is equal to:

$$
q=\frac{1}{2}\left[\sqrt{m}+\sum_{i=1}^{n} \sqrt{m_{i}}\right]
$$

There is almost no variation in the relative citizen voting powers either between countries or over scenarios. We conclude that the method is therefore found to be extremely successful in equalising voting power in a wide range of circumstances.

Figure 1 Relative Citizen Power: Large Countries


Figure 2: Relative Citizen Voting Power: Middle-sized Countries


Figure 3 Relative Citizen Voting Power: Small Countries


## 6. CONCLUSIONS

We have tested the suitability of the proposed Double Majority rule in the EU Reform Treaty by looking at its implications for voting power under various enlargement scenarios, some of which are realistic prospects, while some are no more than speculations. Our scenarios include the possibility of virtually all European countries up to and even including Russia, acceding to membership. We have also tested the performance of the Jagiellonian compromise based on the Penrose Square Root rule whereby voting weights are determined by a simple formula as proportional to population square roots. In judging the voting rule we looked at two criteria: (i) equality of voting power as measured by the Penrose power index at the level of the citizen, assuming one-person-one-vote in national elections, and (ii) decisiveness of the Council of Ministers, as measured by the Coleman power to act.

We found that for the present union of 27, the Reform Treaty voting rule gives a much more unequal distribution of citizen voting power than the existing voting rule, although it leads to the Council of Ministers being more decisive. The Jagiellonian compromise leads to the equalisation of citizen voting power in all countries.

In considering enlargement scenarios, the inequality of citizen voting power persists with each enlargement. The common pattern is for citizens of the smallest countries to have the greatest voting power, sometimes by a factor of as much as 2 or 3 times those of other countries, such as in the cases of Malta and Luxembourg. The medium sized countries have the smallest citizen voting power. That for Netherlands, for example, varies from about two-thirds that of Germany in EU27 to about four fifths of it following the accession of Russia.

Our conclusion is that the Reform Treaty voting system is a flawed proposal that fails to reach the democratic ideal of equality of voting power of all citizens in the European Union. This ideal is reached by the Jagiellonian Compromise.

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[^0]:    ${ }^{i}$ Germany was deliberately underrepresented relative to its population and it turned out that Spain and Poland were overrepresented.
    ${ }^{\text {ii }}$ Luxembourg in the original EEC of six members is a case in point. It had one vote while France, West Germany and Italy each had 4, Netherlands and Belgium 2 each. Luxembourg did not have one quarter the voting power of France, as these figures suggest, but precisely zero because the decision threshold was set at 12 , and therefore in order that its vote could make a difference, the combined votes of the others would have to come to 11, which was impossible. See Leech (2003).
    iii Obviously, since the analysis is theoretical, we could have done this using hypothetical countries with assumed or randomly generated populations; but it is of considerable intrinsic interest to use real countries.
    ${ }^{\text {iv }}$ We do not include any microstates with populations under 100,000 such as Lichtenstein, Andorra, Monaco or San Marino in any future enlargements.
    ${ }^{v}$ We also assume this to be the number of electors who participate in elections. We make no allowance for possible variations in voter turnout.
    ${ }^{\text {vi }}$ The index is modified because we are considering a double-majority rule rather than the conventional single rule. The difference is conceptually trivial although important computationally.
    vii What Penrose called the power of a single vote. For simplicity this is modelled as a stylised plebiscite. More complex arrangements corresponding more closely to real world electoral systems await further research.

[^1]:    ${ }^{\text {viii }}$ We disregard the possibility of a tie to save space. It does not affect the analysis when $\mathrm{m}_{\mathrm{i}}$ is large and (7.) still holds. ${ }^{\text {ix }}$ Penrose (1946) p.53, (1952) p.11. See also for example Feller (1968) pp 50-3.
    ${ }^{\mathrm{x}} \mathrm{A}$ spreadsheet containing the detailed calculations is available from the authors. The power indices for the Nice system have previously been published in Leech (2002) and Felsenthal and Machover (2001). Indices for the Double-Majority rule and Jagiellonian compromise have been published by Felsenthal and Machover (2007).
    ${ }^{\text {xi }}$ For the case of EU27 we have also done these computations using direct enumeration, that is by searching over all voting outcomes and using expression (3.) directly, which does not require us to use this fudge, and the results are the same to a very close approximation. This exact method was not available to use for the larger enlargement scenarios of course and therefore we have used generating functions throughout.
    ${ }^{\text {xii }}$ The Gini coefficients have been calculated by treating all citizens from all countries as a single group. ${ }^{\text {xiii }}$ The powers to act have been reported in Felsenthal and Machover (2007).
    ${ }^{\text {xiv }}$ We found that we could compute these power indices to full accuracy using weights that are the square roots of the populations without adjustment.
    ${ }^{\mathrm{xv}}$ It would be an interesting analysis to allow for future population growth. That would suggest important new scenarios which did not necessarily involve new members.

