Market Selection and Payout Policy Under Majority Rule

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Abstract

The purpose of this paper is to explain how the choice between distributing cash through dividends or shares repurchases affects the firm's ability to raise capital in the financial market. I assume investors have quadratic preferences over wealth but different prior beliefs about the likelihood a distribution takes place. At date zero agents purchase shares given their expectation about the firm's payout method. At date 1 the firm announces whether the payout takes place that period. As in Brennan and Thakor [3], investors with different shareholdings have different incentives to gather information and, therefore, heterogeneous preferences about payout methods at date 1. I assume the firm adopts the payout method preferred by the majority of shareholders at date 1 under the one share/one vote rule. At date 2 the firm is liquidated and the remaining output is distributed among its shareholders. If at date zero agents disagree but not too much on the probability a distribution takes place, I show that a firm expected to pay dividends raises strictly more financial capital than an otherwise identical firm which is expected to repurchase shares. Therefore, a larger fraction of cash is distributed as dividend than through repurchases. One concludes that even in the presence of a small tax disadvantage financial markets favor dividend paying firms.

Key words: Market Selection Hypothesis - Payout Policy - Production under Incomplete Markets

J.E.L. Classification Numbers: G35, D52, D84

1. INTRODUCTION

A corporation that wants to distribute cash to its shareholders may do so by way of a dividend payment or a share repurchase. In their seminal paper, Miller and Modigliani [6] proved that in a world without taxes or transaction costs, where all information is symmetrically distributed and there are complete markets and contracts, shareholders would be indifferent between share repurchases and dividends. However, this prediction is clearly counterfactual since firms rely heavily on dividends even when they offer a tax disadvantage over repurchases. The choice of dividends as a method to distribute cash by corporations has been rationalized by a number of authors (see [1] and references therein) as a costly signalling device that the manager of the firm uses to convey her private information about the prospects of the firm to investors. These models, however, are highly sensitive to the precise specification of the managerial objective function, and the difficulties to justify an appropriate objective function, even when shareholders are symmetrically informed, are well known.[1]

In an illuminating paper, Brennan and Thakor [3] offer a theory of why shareholders have different preferences for dividends and repurchases that does not rely upon an assumed asymmetry of information between managers and investors. They assume that the share price is not a perfect aggregator of the private information investors have about the prospects of the firm, and that the collection of information by shareholders is costly. Under those circumstances, share repurchases are no longer a costless alternative to dividends if some of the shareholders are better informed than others about the prospects of the firm. Hence, share repurchases generate a redistribution of wealth from the uninformed to the informed. When money is paid out in the form of dividends, instead, there is no adverse selection because the informed and uninformed investors receive a pro rata amount. As a result, they argue, uninformed shareholders prefer dividends to repurchases. According to this theory, the manner in which the cash distribution is made creates different incentives to collect information by the shareholders. If there is a fixed cost of acquiring information, large shareholders will have a greater incentive to become informed than will small shareholders. Therefore, repurchases are associated with redistribution of wealth from small to large shareholders.

To explain the choice of payout method by the firm, Brennan and Thakor assume that the manager takes the decision preferred by the owners of the majority of shares. Under this hypothesis, they show that dividends are likely to be observed for small cash distributions while for large cash distributions share repurchases are more likely. The precise outcome depends on the size distribution of shares, which they take as exogenous.

Brennan and Thakor's theory rationalizes why firms use different payout methods. To argue that it explains why a large fraction of cash disbursements takes the form of dividends, however, one must be ready to accept that most distributions tend to be small. This paper explores an alternative explanation for this phenomena. It might well be the case that dividend paying firms simply raise more capital than otherwise identical firms that

[1] A common feature of these models is the dependence of the utility of managers on both the current stock price as well as the end of period distribution of cash ows from the firm.

repurchase shares. If this were the case, then the fraction of cash distributed through dividends would exceed, *ceteris paribus*, the fraction of cash distributed by means of share repurchases. That is, capital markets favor those firms that pay dividends.

The purpose of this paper is to explain how the choice of the payout method affects the firm's ability to raise funds in the capital market. The theory proposed by Brennan and Thakor begs the question on how markets selects among firms which adopt different payout methods, a question that could not be addressed in the Miller and Modigliani framework where the payout method is irrelevant. Brennan and Thakor, however, did not focus on this issue because they take both the distribution of shares as well as the firm's financial capital as given. To address this issue, the size distribution of shares must be endogenous because it is the equilibrium price of shares what determines the funds an equity financed firm can raise in the stock market. In my model, as in Brennan and Thakor's model, agents with different portfolios have different incentives to gather information. Investors who anticipate they will remain uninformed in the event a payout takes place, demand a higher risk premium to hold shares if the company is expected to repurchase its stock rather than paying dividends; the opposite is true for those who anticipate they will obtain informational rents in an open market repurchase. The main question I ask is whether the choice of one payout method over another allows an equity financed firm to raise more funds in the capital market.

Even though for a given size distribution of shares and cash payment there is a unique payout method preferred by the majority of shareholders, I argue that ex-ante identical firms can actually have different size distribution of shares and, therefore, choose different methods to distribute cash. This is because the size distribution of shares depends on the expectation that investors hold at the moment they buy shares about what the preferred form of cash distribution will be when the distribution takes place. If investors conjecture that the firm will repurchase shares then those who anticipate they will obtain information buy more shares and those who expect to remain uninformed buy less shares than if the firm were to pay dividends, changing the identity and preferences of the median voter in the shareholders' assembly.

I develop my results in a general equilibrium model of a stock market with two firms that at date zero issue equity shares to finance a project. Investors, who have quadratic preferences and maximize expected utility of wealth, disagree only about the probability the project will yield some output at date 1 and this creates the heterogeneity in their shareholdings. At date 1, after learning whether the technology yields some output in that period, each shareholder can pay a fixed cost in order to learn what the output will be at date 2. At date 2, the firm is liquidated and each shareholder receives a fraction of the proceedings proportional to her shares. If the project generates some output at date 1, investors agree that the firm should distribute the proceedings among its shareholders because they all have access to a better investment opportunity than the firm has. The form to distribute cash is chosen by the manager who follows the method preferred by the owners of the majority of shares under the one share/one vote rule.

If the firm decides to repurchase shares, it has to announce it publicly and has to place an order with a dealer.[2] The open market repurchase takes place between dates 1 and 2. My model of the open market repurchase is an adaptation to a finite number of rounds of Brennan and Thakor's model.[3] The market maker is risk neutral; she does not take a net position and simply crosses orders at the announced price. Once the repurchase program is announced, investors follow their optimal information gathering strategy. The initial price depends only on the available public information. After all feasible trades have been made at the current market price, the market closes and the market maker quotes a new price which reflects the expected value of the shares given the information in the previous order flow. I show that, informed investors tender their shares if and only if the quoted price exceeds the true value of the firm while uniformed never tender. Therefore, after two rounds the behavior of the informed shareholders fully reveals the true value of the assets to the market maker.

I show that there exist equilibria where ex-ante identical firms choose different payout methods. The main result of this paper is that if agents disagree but not too much about the probability a distribution takes place, a dividend paying firm raises strictly more funds in the capital market (and produce more) than an otherwise identical firm which conducts open market repurchases.

The intuition behind this result is as follows. Each would-be uninformed shareholder of a firm that repurchases shares anticipates that with positive probability she will lose wealth at the hand of the informed shareholders. Therefore, the stock of that firm has a lower expected value for the would-be uninformed shareholders than the stock of an otherwise identical firm which pays dividends. However, the opposite is true for the would-be informed shareholders. Hence, the total effect on the demand for shares of the firm and the shares price is not obvious. Since shares of the dividend paying firm do not suffer from adverse selection, I show that the aggregate demand for shares of both firms coincides with the demand of a representative agent. This agent holds all the stock of the dividend paying and the repurchasing firms but she believes the marginal variance of the firm which repurchase shares is greater than that of the dividend paying firm while the expected rate of return of the former is smaller than the mean of the latter. The risk premium of the firm that repurchases shares, therefore, must be higher. Thus, *ceteris paribus*, the shares of a firm expected to conduct an open market repurchase must sell at a lower price than those of a firm expected to pay dividends, which implies the dividend paying firm raises more capital. An additional insight of this paper is that a manager who wants to maximize the firm's market share should pay dividends instead of repurchasing shares.

1.1 Related Literature

This paper is based on the theory proposed by Brennan and Thakor [3] to explain why shareholders are not

[2] In the US this is regulated by Rule 10b-18 which requires that firms repurchasing shares on the open market should publicly announce the repurchase program, only use one dealer on any single day, avoid trading on an up tick or during the last half-hour before the closing of the market, and limit the daily volume of purchases to a specified amount.

[3] The adaptation is needed because in their model the value of the assets has a uniform distribution while I assume that the return on shares can only take two values.

indifferent between dividends and open market repurchases. Barclay and Smith [2] also argue those firms that repurchase shares face a higher cost of capital. They suggest that open market repurchases in which managers participate give them an opportunity to expropriate uninformed shareholders. When an open market repurchase is announced, they argue, the specialist recognizes that more informed traders (the corporation's managers) enter the market. Therefore, he widens the bid-ask spread until he earns again a competitive rate of return which causes the price at which he buys shares to fall. Anticipating this, the investors raise the required rate of return for the shares of firms that are expected to repurchase shares. This implies that the price of the shares of a firm which conduct open market repurchases is lower than the price of an otherwise identical firm which pays dividends. In this respect, their conclusions about the effect of the payout method on the price of shares are the same I obtain. However, their result is based on the assumption there is an agency problem while in this paper I do not make that assumption. In addition, since Barclay and Smith do not model the firm's decisions, they cannot address how the investor's expectation about the payout method affects the ability of the firm to raise funds in the capital market neither they show that *ex-ante* identical firms can actually choose different payout methods as I do.

In its focus on the preferences of the majority as a way to solve the con ict of interests associated to the decision problem of the firm in incomplete markets, this paper is close to those by De Marzo [5] and Crès [4]. However, those papers focus only on the effect of the institution of majority voting on the choice of the firm's production plan while I make assumptions so that such con ict does not exists and study the consequences of the choice of different payout methods.

1.2 Overview

The structure of the paper is as follows. In section 2, I present the main features of the model and explain how information unfolds over time. In section 3, I analyze the equilibrium of an economy where firms pay dividends and provide conditions such that a unique equilibrium exists. Section 4 describes a version of Brennan and Thakor's model of an open market repurchase. I take both the cash distribution as well as the size distribution of shares as given and solve for the equilibrium strategies of informed and uninformed investors. I also discuss the incentives to gather information of shareholders and derive the preferences over dividends and open market repurchases of investors with different shareholdings. In section 5, I introduce the concept of a Majority Equilibrium in which managers are restricted to choose the payout policy preferred by the majority of shareholders. This section contains the main result of the paper: If agents disagree but not too much on the probability a distribution takes place, a dividend paying firm raises more capital and has a larger market share than an otherwise identical firm which repurchases shares. Section 6 concludes the paper. All other proofs are gathered in the Appendix.

2. THE MODEL

In this section I describe the main features of the model. First I discuss the firm's investment opportunities and how information about the return of assets and the yield of the technology unfolds over time. Then, I describe investors' preferences and beliefs. Finally, I characterize the individual's demand function for shares.

2.1 Firms

Consider an economy where there is one consumption good produced by two *ex-ante* identical firms.[4] Time is indexed by $t \in \{0, 1, 2\}$. The state space of the economy is $\mathcal{S} \equiv \{1, 2, ..., S\}$ and $\mathcal{F}$ is the partition which consists of all the subsets of $\mathcal{S}$. The function $P : \mathcal{F} \mapsto [0, 1]$ is a probability measure and the triple $(\mathcal{S}, \mathcal{F}, P)$ is the probability space of this economy.

The technology firm $i \in \{1, 2\}$ uses to produce its output can be described by two independent random variables, $d : \mathcal{S} \mapsto \{1, 2\}$ and $\theta_i : \mathcal{S} \mapsto \{\theta_L, \theta_H\}$

$$d(s) = \begin{cases} 1 & \text{with prob. } \widehat{\varepsilon} \\ 2 & \text{with prob. } 1 - \widehat{\varepsilon} \end{cases} \qquad \theta_i(s) = \begin{cases} \theta_L & \text{with prob. } \pi \\ \theta_H & \text{with prob. } 1 - \pi \end{cases}$$

where $0 \leq \theta_L < \theta_H$. It follows that θ_i has mean $\overline{\theta} \equiv (1 - \pi) \cdot (\theta_H - \theta_L) + \theta_L > 0$ and variance $\sigma^2 \equiv \pi \cdot (1 - \pi) \cdot (\theta_H - \theta_L)^2$. In addition, I assume θ_1 and θ_2 are independent. Let $y_{i,t}$ denote the output of firm i at date t. Then, $y_i : \mathcal{S} \times \Re_+ \mapsto \Re_+^2$, with typical element $(y_{i,1}, y_{i,2})$, is the output stream of firm i. If firm i invests $y_{i,0} \geq 0$ at date zero, the technology yields

$$y_i(s) = \begin{cases} (c, \theta_i) \cdot y_{i,0}^{\alpha} & \text{if } d(s) = 1 \\ \\ (0, \theta_i + c) \cdot y_{i,0}^{\alpha} & \text{if } d(s) = 2 \end{cases}$$

where $\alpha \in (0, 1]$. Therefore, $c \cdot y_{i,0}^{\alpha}$ can be interpreted as a cash ow that can arrive either at date 1 or 2 and $d(s)$ is the cash ow random arrival date which is common to all firms.

At date zero, each firm issues equity to finance production. The realization of d is revealed to every agent at date 1. Therefore, the knowledge of the agents at date 1 is represented by $\mathcal{F}^U$, the partition generated by d. The realization of θ_1 and θ_2, instead, becomes public information only at the beginning of period 2. Nonetheless, it can be privately learned by any investor at date 1 at a physical cost of $f > 0$. The knowledge of an investor who acquires information at date 1 can be represented by the partition $\mathcal{F}^I \equiv \mathcal{F}$. Finally, at date 2, after the value of θ_i becomes public information, the firm is liquidated and output is distributed proportionally to the shares owned by each shareholder at that moment. The only function the firm manager is to decide how to distribute the firm's earnings at date 1 in the event the output is positive on that period.

[4] Actually, I like to think that there are two groups of firms and these two firms represent the average behavior of all other firms in the group. Most of the result in this paper apply to both interpretations. The interpretation of firms as representing the average behavior of a group is only needed in section 5.2.

2.2 Financial Markets

There are markets for shares and bonds which open at date zero and achieve a perfectly competitive equilibrium.[5] To simplify the analysis, I do not allow agents to sell shares short.[6] Let $p_i \in \Re_+$ and $r_i : \mathcal{S} \mapsto \Re_+$ denote the price and the stochastic gross rate of return of firm i's shares. Let q be the price of a bond which yields 1 unit of the consumption good at date 2.

2.3 Investors

Investors are born at date zero endowed with wealth w_0. They know π but they disagree on the value of $\widehat{\varepsilon}$. I assume that there exists two types of investors, a fraction $g(\underline{\varepsilon}) = \lambda \in (0,1)$ of them believe $\widehat{\varepsilon}$ equals $\underline{\varepsilon} \in (0,1)$ while the fraction $g(\overline{\varepsilon}) = 1 - \lambda$ believe $\widehat{\varepsilon}$ equals $\overline{\varepsilon} \in (\underline{\varepsilon}, 1)$. Let $A \equiv \{\underline{\varepsilon}, \overline{\varepsilon}\}$ denote the set of investors in this economy. The knowledge of the investors at date zero is represented by $\mathcal{F}_0$, the trivial partition. Let P_ε be investor ε's date zero beliefs about the states of nature.

Investors consume only at date 2, do not discount future consumption and have preferences over wealth that can be represented by a quadratic utility function

$$u(z) = z - \frac{\delta}{2} \cdot z^2 \quad ; z \geq 0$$

If investor ε's wealth is represented by the random variable $w : \mathcal{S} \mapsto \Re_+$, the expected utility of wealth of investor ε with information partition $\mathcal{F}'$ is[7]

$$E_\varepsilon \left[u(w) \left\| \mathcal{F}' \right. \right] = E_\varepsilon \left[w \left\| \mathcal{F}' \right. \right] - \tfrac{\delta}{2} \cdot E_\varepsilon \left[w^2 \left\| \mathcal{F}' \right. \right]$$

Then, the marginal utility of wealth is positive in every state s if and only if

$$w(s) < \frac{1}{\delta} \quad \forall s \in \mathcal{S} \tag{1}$$

Let $w_0, w(s)$ denote the aggregate wealth at date 0 and in state s at date 2, respectively. Since only w_0 is available for investment at date zero, then the most that can be consumed at date 2 is $2 \cdot (\theta_H + c) \cdot \left(\frac{w_0}{2}\right)^\alpha$. In addition, the wealth of each individual at date 2 cannot exceed $\frac{2}{\min\{\lambda, 1-\lambda\}} \cdot (\theta_H + c) \cdot \left(\frac{w_0}{2}\right)^\alpha$ because individual wealth is non-negative. The following assumption, therefore, ensures that for each individual the marginal utility of wealth is positive in every state of nature.

Assumption NS: $\frac{2}{\min\{\lambda, 1-\lambda\}} \cdot (\theta_H + c) \cdot \left(\frac{w_0}{2}\right)^\alpha < \dfrac{1}{\delta}$.

Let $x_i \in \Re_+$ be the shares of firm i the investor purchases at date zero and let $x \equiv (x_1, x_2)$, $p \equiv (p_1, p_2)$,

[5] At date zero, there are no information asymmetries so it seems reasonable to assume that trading would yield a competitive equilibrium outcome.
[6] The assumption that investors cannot sell shares short simplifies the analysis because it keeps the relationship one share/one vote.
[7] For any random variable $\xi : \mathcal{S} \mapsto \Re_+$, $E_\varepsilon(\xi \left| \mathcal{F}' \right.)$ and $var_\varepsilon(\xi \left| \mathcal{F}' \right.)$ denotes investor ε's perception of its mean and variance, respectively, given information $\mathcal{F}'$. $E_\varepsilon(\xi \left| \mathcal{F}_0 \right.)$ and $var_\varepsilon(\xi \left| \mathcal{F}_0 \right.)$ are denoted $E_\varepsilon(\xi)$ and $var_\varepsilon(\xi)$, respectively.

and $r \equiv (r_1, r_2)$ be her portfolio, the prices of shares and the rates of return of the shares, respectively. Then, the investor's wealth in state s is

$$w(r_1, r_2, p, q, x)(s) \equiv \left(r_1(s) - \tfrac{p_1}{q}\right) \cdot x_1 + \left(r_2(s) - \tfrac{p_2}{q}\right) \cdot x_2 + \tfrac{w_0}{q}$$

The preferences of investors over random wealth naturally induce preferences over portfolios. Investor ε's utility of holding portfolio (x_1, x_2) at date zero is:

$$E_\varepsilon\left[u(w(r_1, r_2, p, q, x)) \| \mathcal{F}_0\right] = E_\varepsilon\left[w(\cdot, x) \| \mathcal{F}_0\right]\left(1 - \tfrac{\delta}{2} E_\varepsilon\left[w(\cdot, x) \| \mathcal{F}_0\right]\right) - \tfrac{\delta}{2} \cdot var_\varepsilon\left[w(\cdot, x) \| \mathcal{F}_0\right]$$

which depends only on the mean and the variance of the investor's wealth.

In order to explain the portfolio choice of the investor, I define the set of no arbitrage prices. In this paper, the following conditions are always met:

- $\exists$ an agent $\varepsilon \in A$ such that $P_\varepsilon\left[(\theta_L + c) \cdot p_i^\alpha \leq r_i(s) \leq (\theta_H + c) \cdot p_i^\alpha, \forall i = 1, 2\right] = 1$ (2)
- For every agent $\varepsilon \in A$, $P_\varepsilon\left[r_i(s) = (\theta_L + c) \cdot p_i^\alpha, \forall i = 1, 2\right] > 0$ (3)

If the agent who meets condition (2) is not satiated, the absence of arbitrage opportunities implies that

$$\theta_L + c < \frac{p_i^{1-\alpha}}{q} < \theta_H + c \quad i = 1, 2 \tag{4}$$

Define the set

$$\Psi = \left\{(p_1, p_2, q) \in \Re_+^3 : p_1 + p_2 = w_0 \text{ and } (4) \text{ holds}\right\}.$$

Let $(p, q) \in \Psi$. Investor ε's decision problem is

$$\max E_\varepsilon\left[u(w(r_1, r_2, p, q, x)) \| \mathcal{F}_0\right]$$
$$s.t.\ w(r_1, r_2, p.q, x)(s) \geq 0, \forall s \in \mathcal{S}$$

Let $x(r, p, q, \varepsilon) \in \Re_+^2$ be the solution to this problem.

3. Dividend Equilibrium

In this section I assume that if a firm needs to distribute cash at date 1, then it does it in the form of a dividend. Therefore, shareholders have no in uence on the choice of the payout method. First, I describe the rate of return of the firm's shares and find the aggregate demand for shares. I define a dividend equilibrium and provide a necessary and sufficient condition for the existence of a unique equilibrium.

Total output of firm i depends on the capital it raises at date zero. Since the firm is equity financed, then its financial capital at date zero is p_i and its output stream is

$$y_{i,t} = \begin{cases} (c, \theta_L) \cdot p_i^{\alpha} & \text{with prob. } \widehat{\varepsilon} \cdot \pi \\ (c, \theta_H) \cdot p_i^{\alpha} & \text{with prob. } \widehat{\varepsilon} \cdot (1-\pi) \\ (0, \theta_L + c) \cdot p_i^{\alpha} & \text{with prob. } (1-\widehat{\varepsilon}) \cdot \pi \\ (0, \theta_H + c) \cdot p_i^{\alpha} & \text{with prob. } (1-\widehat{\varepsilon}) \cdot (1-\pi) \end{cases}$$

At date 1, investors have access to an investment opportunity which is not available to the firms. For simplicity, I suppose that investors have a storage technology that transfers one unit of the consumption good from one period to the next.[8] If the project yields positive output at date 1, therefore, all shareholders agree on the need to distribute it. Hence, firm i's rate of return per share is $r_i^D \equiv (\theta_i + c) \cdot p_i^{\alpha}$ and $E_{\varepsilon}\left(r_i^D\right) = \left(\overline{\theta} + c\right) \cdot p_i^{\alpha}$ is the same for every investor $\varepsilon \in A$.

The aggregate demand for shares and bonds is

$$\begin{aligned} X_i^D\left(r^D, p, q\right) &\equiv \lambda \cdot x_i\left(r^D, p, q, \underline{\varepsilon}\right) + (1-\lambda) \cdot x_i\left(r^D, p, q, \overline{\varepsilon}\right) = x_i\left(r^D, p, q, \underline{\varepsilon}\right) \\ B(p,q) &\equiv \frac{w_0 - p_1 \cdot X_1^D(r^D, p, q) - p_2 \cdot X_2^D(r^D, p, q)}{q} \end{aligned}$$

Definition 3.1 *A Dividend Equilibrium (DE) is a $(p_1, p_2, q) \in \Re^3_+$ such that $X_i^D\left(r^D, p, q\right) = 1$ for $i = 1, 2$ and $B(p,q) = 0$.*

In a DE investors take asset prices as given, no short-selling is allowed, firms distribute cash by means of dividends and markets clear. Since agents consume only at date 2, they fully invest their wealth at date 0. Because bonds are in zero net supply, the market value of the firms equals the initial wealth. Moreover, since firms are ex-ante identical and follow the same payout policy, their shares sell at the same price.

Proposition 3.1 *If a DE exists, $p_1 = p_2 = \frac{w_0}{2}$.*

After some algebra, one can show that the market for bonds clears if and only if q equals

$$q^* \equiv \left(\frac{w_0}{2}\right)^{1-\alpha} \cdot \frac{\frac{1}{\delta w_0}\left(\frac{w_0}{2}\right)^{1-\alpha} - \left(\overline{\theta}+c\right)}{\left(\overline{\theta}+c\right)\left[\frac{1}{\delta w_0}\left(\frac{w_0}{2}\right)^{1-\alpha} - \left(\overline{\theta}+c\right)\right] - \sigma^2/2} \tag{5}$$

which implies that there can be at most one DE. It follows that a necessary and sufficient condition for the existence of a DE is that the left hand side of (5) is positive, which is always true under assumption NS.

Proposition 3.2 *If assumption NS holds, $\left(\frac{w_0}{2}, \frac{w_0}{2}, q^*\right)$ is the unique DE. Moreover, $\left(\frac{w_0}{2}, \frac{w_0}{2}, q^*\right) \in \Psi$.*

In the unique DE the investors' marginal rates of substitution between mean and variance of wealth are equalized. Moreover, since every investor buys the same portfolio, has the same preferences and is endowed with the same initial wealth, one concludes that in a DE markets are effectively complete.

[8] What is really needed is some reason so that investors unanimously prefer the firm to distribute its earnings at date 1.

4. A MODEL OF ENDOGENOUS PAYOUT METHODS

In this section I take the cash payment and the size distribution of shares as given and consider the problem of a firm that must choose to distribute cash either by means of an open market repurchase (OMR) or dividends. First I describe the process of an open market repurchase. Then, I show that under some weak assumptions, uninformed shareholders never tender and the informed ones tender all shares if and only if the price is above the true value of the firm. I analyze the incentives of shareholders to acquire information and derive their preferences between OMR and dividend payments before they collect information. Finally I obtain the date zero demand for shares.

4.1 The model of an open market repurchase

Let $(p_1, p_2, q) \in \Psi$ be the price of assets at date zero. Let $\mathcal{S}_1 \equiv \{s : d(s) = 1\}$ denote the event in which a cash distribution takes place at date 1. Let $\mathcal{J} \neq \varnothing$ be the set of firms which choose to repurchase shares. On $\mathcal{S}_1$, company $i \in \mathcal{J}$ has some cash, $c \cdot p_i^{\alpha}$, it will distribute by means of an open market repurchase; furthermore those agents who decided to become informed about (θ_1, θ_2) have already gathered information and sunk the cost f. Let $\omega_i \in [0, 1]$ denote the fraction of shares in hand of firm i's uninformed investors and let $\omega \equiv (\omega_1, \omega_2)$.

Since open market repurchase programs often extend over several months, it may be difficult for investors to determine whether repurchases are actually taking place on any given day. Following Brennan and Thakor, in order to re ect in a simple manner the assumption that the share price is not fully revealing of the private information held by the informed investors, I assume that between dates 1 and 2 the market maker opens twice. At each round of transactions, $n \in \{1, 2\}$, the market maker announces a price ρ_i^n. Therefore, in round n the fraction β_i^n of its outstanding shares that the firm wants to repurchase satisfies $\beta_i^n \cdot \rho_i^n = c \cdot p_i^{\alpha}$. Investors place orders with the market maker to trade at the announced price.[9] The repurchase program ends as soon as the offer is fully subscribed. The market maker is risk neutral; she does not take a net position and simply crosses orders at the announced price. If there is an order imbalance, the market maker randomly rations the long side of the market. The initial price depends only on the public information available. Therefore, the first price announced by the market maker is $\rho_i^1 = E[r_i \| \mathcal{S}_1] = \left(\overline{\theta} + c\right) \cdot p_i^{\alpha}$. After all feasible trades have been made at the current market price, the market closes and the market maker then quotes a new price which re ects the expected value of the shares given the information in the previous order ow.

The payoff each investor obtains when an open market repurchase is conducted depends not only on her decision about tendering or not her shares in a given round but also on the decision taken by every other investor in that round. This is because the extent to which her market order will be fulfilled depends on whether the sum

[9] As in Brennan and Thakor's model, if the cash distribution is not too large with respect to the liquidation value of the firm, then the informed shareholders have enough shares to fully subscribe the offer.

of all orders exceeds or not the offer β_i^n. For example, if every investor tenders her shares in a given round, then the market maker rations the supply side and purchases only a fraction β_i^n of the shares tendered by each investor. Likewise, if $1-\omega_i > \beta_i^n$ and every informed shareholders tender their shares, the market maker only buys a fraction $\frac{\beta_i^n}{1-\omega_i}$ of each order at the quoted price. Therefore, there is strategic interaction and the optimal decision for each investor depends on her conjectures about the decision of the remaining shareholders.

4.2 Timeline of events

In this model, a cash distribution is needed at date 1 with probability $\widehat{\varepsilon}$, independently of the mode of payout the firm chooses. In figure 1, I describe the time of events for a firm that distributes cash through an OMR.

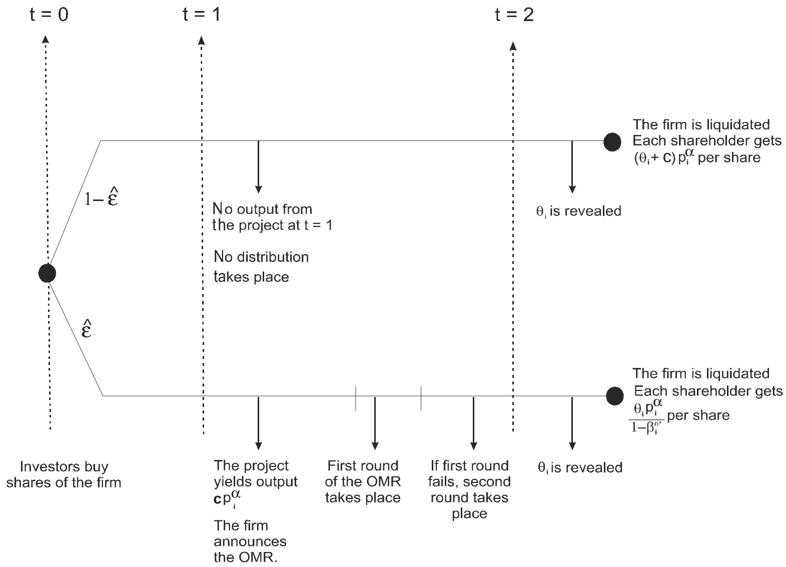

Figure 1. Timeline for a firm that repurchases shares.

At date zero, investors buy shares of the firm without knowing whether a distribution will take place at date 1 or not. On the event that no distribution takes place at date 1, at date 2, after θ_i is publicly revealed, the firm is liquidated and each shareholder is paid $(\theta_i + c) \cdot p_i^\alpha$ per share. These events are represented in the upper branch of the tree of events described in Figure 1. In the lower branch of that tree, which occurs with probability $\widehat{\varepsilon}$, the project yields output $c \cdot p_i^\alpha$ at date 1. The firm announces the OMR and submits an order to the market maker to buy $\frac{c \cdot p_i^\alpha}{\rho_i^1}$ shares at the current price ρ_i^1. The first round takes place. If the offer is fully subscribed, then no second round takes place. Otherwise, the second round does take place. At date 2, after θ_i is revealed, the firm is liquidated and each shareholder who did not tender her shares receives $\frac{\theta_i \cdot p_i^\alpha}{1-\beta_i^{n^*}}$ per share, where n^* denotes the round in which the offer was subscribed.

The timeline for a dividend paying firm only differs from that in Figure 1 in the lower branch, when the firm needs to distribute cash, because each shareholder receives $c \cdot p_i^\alpha$ per share at date 1.

4.3 Rates of Return

In this section, I obtain the return of the shares belonging to firms that employ different payout methods. Since agents have different information, they perceived the rate of return of a given firm's shares differently and so I consider the rate of return for both the informed as well as the uninformed ones.

Suppose the distribution of shares is exogenously given and assume that $1 - \omega_i > \frac{c}{\overline{\theta}}$.[10] In each round of the OMR, the uninformed investor can choose the fraction of her shares she wants to tender. Since her actions cannot be contingent on the informed investors' actions, the uninformed investor strategy depends only on the round of the open market repurchase and the price quoted by the market maker in that round. Consider the following strategies for the informed and the uninformed shareholders:

$\widehat{\gamma}^I$: Tender all shares of firm $i \in \mathcal{J}$ if and only if the quoted price is greater or equal to the true value of the firm. Do not tender any share otherwise.

$\widehat{\gamma}^U$: Do not tender any share of firm $i \in \mathcal{J}$.

If $\widehat{\gamma}^U$ and $\widehat{\gamma}^I$ are the strategies of the uninformed and the informed, then

1. On $\{s : \theta_i(s) = \theta_L\} \cap \mathcal{S}_1$, the repurchase ends in the first round and the repurchase price is $\left(\overline{\theta} + c\right) \cdot p_i^\alpha$.

2. On $\{s : \theta_i(s) = \theta_H\} \cap \mathcal{S}_1$, the repurchase ends in the second round and the repurchase price is $(\theta_H + c) \cdot p_i^\alpha$.

3. The rate of return per share of firm i for an informed shareholder who follows strategy $\widehat{\gamma}^I$ is

$$r_i^I(\omega_i)(s) \equiv \left[\theta_i(s) + c_i^I(\omega_i)(s)\right] \cdot p_i^\alpha \quad \text{where } c_i^I(\omega_i)(s) \equiv \begin{cases} c \cdot \left(1 + \frac{\omega_i}{1-\omega_i} \cdot \tau\right) & \text{if } (\theta_i(s), d(s)) = (\theta_L, 1) \\ c & \text{otherwise} \end{cases}$$

where $\tau \equiv \left(1 - \frac{\theta_L}{\overline{\theta}}\right)$ is the implicit tax rate per share the uninformed shareholder faces in an OMR if she does not tender her shares. It follows that $E_\varepsilon\left(r_i^I\right) = \left(\overline{\theta} + c\right) + \pi \cdot \varepsilon \cdot \tau \cdot c \cdot \frac{\omega_i}{1-\omega_i}$ and $var_\varepsilon\left(r_i^I\right)$ decreases with ω_i.

4. The rate of return per share of firm i for an uninformed shareholder who follows strategy $\widehat{\gamma}^U$ is

$$r_i^U(s) \equiv \left[\theta_i(s) + c_i^U(s)\right] \cdot p_i^\alpha \quad \text{where } c_i^U(s) \equiv \begin{cases} c \cdot (1 - \tau) & \text{if } (\theta_i(s), d(s)) = (\theta_L, 1) \\ c & \text{otherwise} \end{cases}$$

and, therefore, her expected rate of return conditional on $\mathcal{S}_1$ is $E_\varepsilon\left(r_i^U \| \mathcal{S}_1\right)(s) = \left[\overline{\theta} + c - \pi \cdot \tau \cdot c\right] \cdot p_i^\alpha$.

Lemma 4.1 *Let $(p, q) \in \Psi$. If every uninformed investor follows $\widehat{\gamma}^U$ and every informed investor follows $\widehat{\gamma}^I$, there is $\overline{\delta}(p, q)$ such that for all $0 < \delta < \overline{\delta}(p, q)$ no investor wants to deviate unilaterally from $\left(\widehat{\gamma}^I, \widehat{\gamma}^U\right)$.*

In what follows, therefore, for each $k \in \{I, U\}$ the vector of rates of return is

$$r_{\mathcal{J}}^k(\omega) \equiv \begin{cases} \left(r_1^k, r_2^k\right) & \text{if } \mathcal{J} = \{1, 2\} \\ \left(r_i^k, r_{-i}^D\right) & \text{if } \mathcal{J} = \{i\} \\ \left(r_1^D, r_2^D\right) & \text{if } \mathcal{J} = \emptyset \end{cases}$$

[10] Below I provide sufficient conditions so that this inequality is implied by market clearing at date zero.

4.4 Endogenous Information Acquisition

To this point, I have taken as exogenous the relative proportions of informed and uninformed investors in the shareholders assembly. Now, I consider which shareholders choose to pay the fixed cost $f > 0$ of observing (θ_1, θ_2) after the firms announce a repurchase. I characterize the information decision of each investor as a function of her shareholdings.

Consider an investor who holds portfolio $x \in \Re_+^2$ and believes $\omega \in [0,1] \times [0,1]$ is the fraction of shares in hands of the uninformed shareholders when the announced payout policy is $\mathcal{J} \in \mathcal{P} \equiv \{\{1\}, \{2\}, \{1,2\}\}$. The value of information for that investor, conditional on the event a repurchase is announced by firms in $\mathcal{J}$, is

$$\Delta_{\mathcal{J}}(\omega, p, q, x) \equiv E_\pi\left(u\left[w\left(r_{\mathcal{J}}^I(\omega), p, q, x\right)\right] | \mathcal{S}_1\right) - E_\pi\left(u\left[w\left(r_{\mathcal{J}}^U, p, q, x\right)\right] | \mathcal{S}_1\right)$$

and he chooses to acquire information if and only if $\Delta_{\mathcal{J}}(\omega, p, q, x) > f$.

As one could expect, the value of information increases with the fraction of shares in hand of the uninformed and with the number of firms which announce a share repurchase.

Lemma 4.2 *Suppose $\delta < \overline{\delta}(p,q)$ and $\mathcal{J}' \in \mathcal{P}$. The function $\Delta_{\mathcal{J}'}(\omega, p, q, x)$ has the following properties:*
i. It is an increasing function of ω_i for every $i \in \mathcal{J}'$.
ii. $\mathcal{J} \subset \mathcal{J}' \Rightarrow \Delta_{\mathcal{J}}(\omega, p, q, x) \leq \Delta_{\mathcal{J}'}(\omega, p, q, x)$ for all $\omega \in [0,1)$.

For fixed distribution of shares $[x(\underline{\varepsilon}), x(\overline{\varepsilon})]$, the fraction owned by investors who remain uninformed when firms in $\mathcal{J} \in \mathcal{P}$ repurchase shares, $\omega_{\mathcal{J}}$, is implicitly defined by the following system of equations

$$\omega_{\mathcal{J}} = \sum_{\varepsilon : \Delta_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, x(\varepsilon)\right) \leq f} g(\varepsilon) \cdot x(\varepsilon)$$

Let $\Omega \equiv \{\omega_{\mathcal{J}}\}_{\mathcal{J} \in \mathcal{P}}$ be the collection of expectations about the fraction of shares owned by the uninformed.

4.5 Preferences for Dividends and OMR

Shareholders express their preferences over payout methods before the information about (θ_1, θ_2) is collected. To determine each shareholder's preferred mode of cash distribution, therefore, it is necessary to calculate, for each investor, the unconditional expected utility under dividends and open market repurchases.

For $\mathcal{J} \in \mathcal{P} \cup \varnothing$, define,

$$V_{\mathcal{J}}(\omega, p, q, x) \equiv \max\left\{E_\pi\left(u\left[w\left(r_{\mathcal{J}}^I(\omega), p, q, x\right)\right] \| \mathcal{S}_1\right) - f, E_\pi\left(u\left[w\left(r_{\mathcal{J}}^U, p, q, x\right)\right] \| \mathcal{S}_1\right)\right\}$$

Clearly, $V_{\mathcal{J}}(\omega, p, q, x)$ is the expected utility of an investor who holds portfolio x and optimally decides whether to acquire information or not, under the expectation that ω will be the fraction of shares in hands of the uninformed shareholders. Let

$$\Delta V_{\mathcal{J}'\mathcal{J}}(\Omega, p, q, x) \equiv V_{\mathcal{J}'}[\omega_{\mathcal{J}'}, p, q, x] - V_{\mathcal{J}}[\omega_{\mathcal{J}}, p, q, x]$$

be investor ε's expected utility gain if the firms' payout policy changes from $\mathcal{J}$ to $\mathcal{J}' \in \mathcal{P} \cup \varnothing$. If investor ε holds portfolio x, then she prefers $\mathcal{J}$ over $\mathcal{J}'$, conditional on $\mathcal{S}_1$, if and only if $\Delta V_{\mathcal{J}'\mathcal{J}}(\Omega, p, q, x) < 0$.

If the underlying size distribution of shares is $[x(\underline{\varepsilon}), x(\overline{\varepsilon})]$, one says that the majority of shareholders of firm i prefers $\mathcal{J}$ over $\mathcal{J}'$ if and only if

$$\sum_{\varepsilon:\Delta V_{\mathcal{J}'\mathcal{J}}(\Omega,p,q,x(\varepsilon))<0} g(\varepsilon) \cdot x_i(\varepsilon) > \frac{1}{2}$$

Suppose the majority of shareholders of firm i prefers $\mathcal{J}$ over $\mathcal{J}'$. If $i \in \mathcal{J}$ and $\mathcal{J}' = \mathcal{J} \setminus \{i\}$, then the majority prefers firm i to repurchase shares. Likewise, if $i \notin \mathcal{J}$ and $\mathcal{J}' = \mathcal{J} \cup \{i\}$, the majority prefers firm i to pay dividends. Therefore, given prices $(p, q) \in \Psi$, $\delta < \overline{\delta}(p, q)$ and a fixed set $\mathcal{J} \in \mathcal{P} \cup \varnothing$, there exists a unique payout method which is preferred by the majority of shareholders of each firm.

4.6 The Distribution of Shares

At date zero, investors decide their shareholdings under the expectation that firm $i \in \mathcal{J}$ would repurchase shares in the event a repurchase takes place and firm $j \notin \mathcal{J}$ pays dividends.

Suppose investor ε expects at date zero that firms in $\mathcal{J} \in \mathcal{P}$ will repurchase shares and that ω will be the fraction of shares of each firm owned by the would-be uninformed investors. Define

$$\begin{aligned} x^I_{\mathcal{J}}(\omega, p, q, \varepsilon) &\equiv \underset{x\in\Re^2_+}{\arg\max} E_\varepsilon\left(u\left[w\left(r^I_{\mathcal{J}}, p, q, x\right)\right]\right) \;\; s.t.\; w\left(r^I_{\mathcal{J}}, p, q, x\right)(s) \geq 0 \text{ for all } s \in \mathcal{S} \\ x^U_{\mathcal{J}}(p, q, \varepsilon) &\equiv \underset{x\in\Re^2_+}{\arg\max} E_\varepsilon\left(u\left[w\left(r^U_{\mathcal{J}}, p, q, x\right)\right]\right) \;\; s.t.\; w\left(r^U_{\mathcal{J}}, p, q, x\right)(s) \geq 0 \text{ for all } s \in \mathcal{S} \end{aligned}$$

$x^I_{\mathcal{J}}(\omega, p, q, \varepsilon)$ and $x^U_{\mathcal{J}}(p, q, \varepsilon)$ are the portfolios chosen by an investor who at date zero anticipates that in the event a repurchase takes place at date 1 she will acquire information and remain uninformed, respectively.[11] Since investor ε maximizes expected utility, then her optimal portfolio at date zero is

$$x_{\mathcal{J}}(\omega, p, q, \varepsilon) \equiv \begin{cases} x^I_{\mathcal{J}}(\omega, p, q, \varepsilon) & \text{if } E_\varepsilon\left(u\left[w\left(r^I_{\mathcal{J}}, p, q, x^I_{\mathcal{J}}\right)\right]\right) - \varepsilon \cdot f > E_\varepsilon\left(u\left[w\left(r^U_{\mathcal{J}}, p, q, x^U_{\mathcal{J}}\right)\right]\right) \\ x^U_{\mathcal{J}}(p, q, \varepsilon) & \text{otherwise} \end{cases}$$

The reader may conjecture that whenever $\underline{\varepsilon}$ anticipates that she will become informed in the event a repurchase takes place, $\overline{\varepsilon}$ also does it. This is not always true, however, because the date zero expected cost of information increases with ε.[12]

[11] As in Brennan and Thakor, I implicitly assume that agents do not alter their date 0 shareholdings unless a stock repurchase causes them to.

[12] This conjecture would be correct, for example, if $\frac{E\left(u\left[w\left(r^I_{\mathcal{J}}, p, q, x^I_{\mathcal{J}}\right)\right] \| \mathcal{S}_1\right) - E\left(u\left[w\left(r^U_{\mathcal{J}}, p, q, x^U_{\mathcal{J}}\right)\right] \| \mathcal{S}_1\right)}{\varepsilon}$ were increasing in ε.

5. MAJORITY EQUILIBRIUM

In this section I address how the investors' date zero expectation about the firm's payout method affects the ability of the company to raise capital in the stock market. This point is not addressed by Brennan and Thakor because they assume the size distribution of shares is exogenous and, therefore, they did not solve for the equilibrium in the capital market.

Since firms are equity financed, the capital each firm is able to raise depends only on the market value of the firm, i.e. the price of its shares at date zero. The market value of the firm, however, depends on the investors' expectation about the payout method that would be preferred by the majority of shareholders. In equilibrium the asset market clears, firms use the method of cash disbursement most preferred by the majority of shareholders, investors demand shares in order to maximize their expected utility given their expectation about the choice of payout method and expectations are correct.

Fix a distribution of shares $[x(\underline{\varepsilon}), x(\overline{\varepsilon})]$. The demand for firm i's shares and bonds are

$$\begin{aligned} X_i &\equiv \lambda \cdot x_i(\underline{\varepsilon}) + (1-\lambda) \cdot x_i(\overline{\varepsilon}) \quad \text{for } i = 1,2 \\ B &\equiv \tfrac{w_0 - p_1 \cdot X_1 - p_2 \cdot X_2}{q} \end{aligned}$$

Definition 5.1 *A Majority Equilibrium* (ME) *is a collection* $\{\mathcal{J}, \Omega, x, (p_1, p_2, q)\}$ *consisting of a set* $\mathcal{J} \in \mathcal{P} \cup \varnothing$ *of distribution policies, a distribution of shares* $[x(\underline{\varepsilon}), x(\overline{\varepsilon})] \in \Re^4_+$*, expectations* Ω *and prices* $(p_1, p_2, q) \in \Psi$ *such that*

■ INFORMED AND UNINFORMED FOLLOW $\left(\widehat{\gamma}^U, \widehat{\gamma}^I\right)$ IN AN OMR

E.1. $\overline{\delta}(p,q) > \delta$ *and* $1 - \omega_{h,\mathcal{J}'\mathcal{J}} > \frac{c}{\theta}$, $\forall h \in \{1,2\}$ *and* $\mathcal{J}' \in \mathcal{P}$.

■ PORTFOLIO IS OPTIMAL GIVEN EXPECTATIONS

E.2 $x(\varepsilon) = x_{\mathcal{J}}(\omega_{\mathcal{J}}, p, q, \varepsilon)$ *for every* $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$

■ MARKETS CLEAR

E.3. $X_i = 1, \forall i = 1,2.$

E.4. $B = 0.$

■ EXPECTATIONS ARE CORRECT

E.5. $\omega_{\mathcal{J}'} = \sum_{\varepsilon : \Delta_{\mathcal{J}'}(\omega_{\mathcal{J}'}, p, q, x(\varepsilon)) \leq f} g(\varepsilon) \cdot x(\varepsilon) \qquad \forall \mathcal{J}' \in \mathcal{P}.$

■ THE PAYOUT POLICY IS PREFERRED BY THE SHAREHOLDER'S MAJORITY

E.6. $\forall\, i \in \{1,2\}$, $\sum_{\varepsilon : \Delta V_{\mathcal{J}'\mathcal{J}}(\Omega, p, q, x(\varepsilon)) < 0} g(\varepsilon) \cdot x_i(\varepsilon) > \frac{1}{2}$ *for* $\mathcal{J}' = \begin{cases} \mathcal{J} \setminus \{i\} & \text{if } i \in \mathcal{J} \\ \mathcal{J} \cup \{i\} & \text{if } i \notin \mathcal{J} \end{cases}$.

Condition $E.1$ says that in a Majority Equilibrium uninformed and informed shareholders would optimally follow strategies $\widehat{\gamma}^U$ and $\widehat{\gamma}^I$, respectively, in the event some firm repurchases shares. $E.2$ states that the portfolio of each agent maximizes her expected utility given her expectation about the distribution policies and the fraction of shares in hand of the uninformed shareholders of each firm. $E.3$ and $E.4$ are the market clearing conditions, while $E.5$ says that investors at date zero have correct expectations about the fraction of shares that would be in hand of the uninformed in the event a firm decides to repurchase at date 1. Finally, $E.6$ requires that each firm's payout policy is prefered by the majority of its shareholders at date 1.

The assumption that conjectures are correct does not rule out the possibility that *ex-ante* identical firms choose different payout methods in equilibrium.[13] Therefore, this model is appropriate to address how, *ceteris paribus*, the choice of payout method affects the firm's ability to raise capital in the financial market.

In a Majority Equilibrium firms are partitioned in two sets. Firms in set $\mathcal{J}$ choose to conduct an open market repurchase in the event they need to distribute cash. Firms which are not in $\mathcal{J}$, choose to pay dividends in the event a payout is needed. If the set $\mathcal{J}$ is empty, then all firms use dividends while if $\mathcal{J} = \{1, 2\}$ every firm distributes cash by mean of shares repurchases. If $\mathcal{J} \notin \{\emptyset, \{1, 2\}\}$, then *ex-ante* identical firms choose different payout methods. Since firms are *ex-ante* identical, it seems reasonable to define a symmetric ME as one in which every firm chooses the same payout policy.

Definition 5.2 *A ME $\{\mathcal{J}, \Omega, x, (p_1, p_2, q)\}$ is symmetric if $\mathcal{J} = \emptyset$ or $\mathcal{J} = \{1, 2\}$.*

Now, I begin the analysis of the ME. The first proposition shows that under assumption NS no agent can be satiated in a ME because market clearing implies that individual wealth must be bounded above by $\frac{2}{\min\{\lambda, 1-\lambda\}} \cdot (\theta_H + c) \cdot \left(\frac{w_0}{2}\right)^\alpha$.

Proposition 5.1 *Suppose $\{\mathcal{J}, \Omega, x, (p_1, p_2, q)\}$ is a ME. If NS holds and $\omega_{i,\mathcal{J}} < 1$ for some firm $i = 1, 2$, condition (1) holds in every state $s \in \mathcal{S}$ and for every agent $\varepsilon \in A$.*

The following Proposition argues that share prices must add up to total initial wealth and that in any symmetric equilibria the value of the firms must be identical. It is important to notice, however, that it leaves open the possibility that firms that choose different payout policies have different market value.

Proposition 5.2 *If a ME exists, $p_1 + p_2 = w_0$. If the ME is symmetric, then $p_1 = p_2 = \frac{w_0}{2}$.*

5.1 Symmetric Equilibrium

In a symmetric equilibrium every firm raises the same amount of financial capital at date zero. This is because investors do not consume at date zero and, therefore, they fully invest their wealth independently of the firms' payout policy. However, the equilibrium price of bonds does depend on the payout policy of the firms. From the definition of a ME, it is straightforward to show that if $\{\mathcal{J}, \Omega, x, (p_1, p_2, q)\}$ is a ME in which every firm pays dividends, then (p_1, p_2, q) is also a DE and, therefore $q = q^*$. Since every symmetric ME in which

[13] I like to think about the case in which *ex-ante* identical firms choose different payout methods, as a way of modelling that managers may have different objectives. One may think that all managers find it worthwhile to do what the majority of shareholders prefer (perhaps to avoid being fired) but given that, some may prefer to pay dividends and others may prefer to conduct open market repurchases. Announcing their preferred payout method, managers can coordinate investors' expectations.

firms pay dividends induces a DE, a natural question is when a DE fails to be associated with a ME. To answer this question first notice that in a ME in which both firms pay dividends, every investor buys one share of each firm. Then, in the event an open market repurchase takes place, they all want to be on the same side of the market: either all of them want to acquire information or to remain uninformed. Since informational rents become infinitely large as everybody remains uninformed, it cannot be the case that everybody stays uninformed. It follows that the necessary and sufficient conditions for the existence of a ME in which every firm pays dividends are $E.1$ and

$$\Delta_{\{i\}}\left(\mathbf{0}, \tfrac{w_0}{2}, \tfrac{w_0}{2}, q^*, 1\right) > f \qquad \text{for some } i = 1, 2 \tag{6}$$

where $\mathbf{0} \equiv (0, 0)$. Condition (6) requires that every shareholder would acquire information in the case an open market repurchase takes place.

Proposition 5.3 *Suppose NS holds. A ME with $\mathcal{J} = \emptyset$ exists iff $\overline{\delta}\left(\frac{w_0}{2}, \frac{w_0}{2}, q^*\right) > \delta$ and (6) holds.*

Proposition 5.3, completely characterizes a ME where every firm pays dividends. Consider now the case in which every firm repurchases shares. If $\omega_{\mathcal{J}} = g\left(\varepsilon^U\right) \cdot x^U_{\mathcal{J}}\left(\frac{w_0}{2}, \frac{w_0}{2}, q, \varepsilon^U\right)$ for some $\varepsilon^U \in A$, market clearing holds iff q solves

$$g\left(\varepsilon^U\right) \cdot x^U_{i,\mathcal{J}}\left(\tfrac{w_0}{2}, \tfrac{w_0}{2}, q, \varepsilon^U\right) + \left(1 - g\left(\varepsilon^U\right)\right) \cdot x^I_{i,\mathcal{J}}\left(\omega_{\mathcal{J}}, \tfrac{w_0}{2}, \tfrac{w_0}{2}, q, \varepsilon^I\right) = 1 \tag{7}$$

Let $q\left(\varepsilon^U, \varepsilon^I\right)$ be the solution to equation (7).[14]

Proposition 5.4 *A ME with $\mathcal{J} = \{1, 2\}$ exists if and only if*

$R.1$ $\overline{\delta}\left(\frac{w_0}{2}, \frac{w_0}{2}, q\left(\varepsilon^U, \varepsilon^I\right)\right) > \delta.$

$R.2$ $\mathbf{0} < \omega_{\mathcal{J}'} = g\left(\varepsilon^U\right) \cdot x^U_{\mathcal{J}}\left(\frac{w_0}{2}, \frac{w_0}{2}, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right) < \left(\frac{1}{2}, \frac{1}{2}\right)$ *for every $\mathcal{J}' \in \mathcal{P}$.*[15]

$R.3$ $\Delta_{\mathcal{J}}\left(\omega_{\mathcal{J}}, \frac{w_0}{2}, \frac{w_0}{2}, q\left(\varepsilon^U, \varepsilon^I\right), x_{\mathcal{J}}\left(\cdot, \varepsilon^U\right)\right) \leq f < \Delta_{\{i\}}\left(\omega_{\{i\}}, \frac{w_0}{2}, \frac{w_0}{2}, q\left(\varepsilon^U, \varepsilon^I\right), x_{\mathcal{J}}\left(\cdot, \varepsilon^I\right)\right)$ *for some i.*

Condition $R.2$ says that in a ME in which both firms repurchase shares, some investors remain uninformed in the event a repurchase takes place. Those investors, however, do not hold a majority of shares in any firm. Therefore, as expressed by condition $R.3$, the cost of acquiring information must be high enough to discourage some shareholders to purchase information but not too high so that the rest still purchase information even if one firm deviates and pays dividends.

In a ME with $\mathcal{J} = \{1, 2\}$, investors differ in their decision about purchasing information in the event a payout takes place. As a consequence, there is a wedge between the rate of return per share of the optimistic

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[14] Since the left hand side equals zero at $q = \frac{\left(\frac{W_0}{2}\right)^{1-\alpha}}{\overline{\theta}+c}$ and is strictly increasing in q, a solution to the equation above exists iff

$$\lim_{q \to \frac{\left(\frac{w_o}{2}\right)^{1-\alpha}}{\theta_L + c}} g\left(\varepsilon'\right) \cdot x^U_{i,\mathcal{J}}\left(\tfrac{w_0}{2}, \tfrac{w_0}{2}, q, \varepsilon'\right) + \left(1 - g\left(\varepsilon'\right)\right) \cdot x^I_{i,\mathcal{J}}\left(\omega_{\mathcal{J}\mathcal{J}}, \tfrac{w_0}{2}, \tfrac{w_0}{2}, q, \varepsilon''\right) \geq 1.$$

[15] For any vectors $x, y \in \Re^2$, $x > y$ means that $x_i > y_i$ for every $i = 1, 2$.

and the pessimistic investors, which causes that their marginal utility of wealth must differ in some state s at date 2. Therefore, markets are effectively incomplete. The following Lemma shows a stronger result: agents disagree on the price of risk. For $k \in \{I, U\}$, let $w^k_{\mathcal{J}} \equiv w\left(r^k_{\mathcal{J}}, p, q, x^k_{\mathcal{J}}\right)$ be the wealth of an agent who holds portfolio $x^k_{\mathcal{J}}$ and faces rates of return $r^k_{\mathcal{J}}$.

Lemma 5.1 *If $\{\mathcal{J}, \Omega, x, (p_1, p_2, q)\}$ is a ME with $\mathcal{J} = \{1, 2\}$, then $MRS\left[E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right), var_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right)\right] < MRS\left[E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right), var_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right)\right]$.*

In the equilibria analyzed so far, the shares of both firms trade at the same price. However, this need not be the case if firms choose different payout methods. In the next section, I characterize the equilibrium price of shares of firms which are expected to choose different payout policies.

5.2 Asymmetric Equilibrium

If $\{(p_1, p_2, q), \mathcal{J}, \omega_{\mathcal{J}}\}$ is an asymmetric ME, then Proposition 5.2 implies that $p_1 + p_2 = w_0$. The question I analyze in this section is which payout method raises more capital in the financial market. I show that in any asymmetric equilibrium, the firm expected to pay dividends raises more capital than an otherwise identical firm expected to repurchase shares.

To fix ideas, suppose firm j pays dividends and firm i repurchases shares. For any $k \in \{I, U\}$, the wealth of an investor who purchases (x_1, x_2) shares is $w\left(r^k_{\mathcal{J}}, p, q, x\right) = x_i \cdot \left(r^k_i - \frac{p_i}{q}\right) + x_j \cdot \left(r^D_j - \frac{p_j}{q}\right) + \frac{w}{q}$. Let $\eta \equiv \frac{p_i \cdot x_i}{p_i \cdot x_i + p_j \cdot x_j}$. Therefore, the mean and variance of wealth associated with portfolio (x_1, x_2) is

$$\begin{aligned} E_\varepsilon\left(w^k_{\mathcal{J}}\right) &= \left[\eta \cdot \left(E_\varepsilon\left(\frac{r^k_i}{p_i}\right) - \frac{1}{q}\right) + (1-\eta) \cdot \left(E_\varepsilon\left(\frac{r^D_j}{p_j}\right) - \frac{1}{q}\right)\right] \cdot (p_i \cdot x_i + p_j \cdot x_j) + \frac{w}{q} \\ var\left(w^k_{\mathcal{J}}\right) &= \left[\eta^2 \cdot var_\varepsilon\left(\frac{r^k_i}{p_i}\right) + (1-\eta)^2 \cdot var_\varepsilon\left(\frac{r^D_j}{p_j}\right)\right] \cdot (p_i \cdot x_i + p_j \cdot x_j)^2 \end{aligned}$$

If the investor is not satiated, as it is the case in any ME under assumption NS, the expression above implies that she allocates a positive fraction of wealth to the shares of both firms only if

$$E_\varepsilon\left(\frac{r^k_i}{p_i}\right) < E_\varepsilon\left(\frac{r^D_j}{p_j}\right) \Rightarrow var_\varepsilon\left(\frac{r^k_i}{p_i}\right) < var_\varepsilon\left(\frac{r^D_j}{p_j}\right) \quad \text{for } k \in \{U, I\} \tag{8}$$

$$E_\varepsilon\left(\frac{r^D_j}{p_j}\right) < E_\varepsilon\left(\frac{r^k_i}{p_i}\right) \Rightarrow var_\varepsilon\left(\frac{r^D_j}{p_j}\right) < var_\varepsilon\left(\frac{r^k_i}{p_i}\right) \quad \text{for } k \in \{U, I\} \tag{9}$$

When $\alpha = 1$, $\frac{r^k_i}{p_i}$ and $\frac{r^D_j}{p_j}$ are independent of (p_i, p_j). For the would-be uninformed investor, the rate of return of firm i has lower mean and higher variance than the rate of return of firm j, i.e. $E_\varepsilon\left(\frac{r^U_i}{p_i}\right) < E_\varepsilon\left(\frac{r^D_j}{p_j}\right)$ and $var_\varepsilon\left(\frac{r^D_j}{p_j}\right) < var_\varepsilon\left(\frac{r^k_i}{p_i}\right)$ for any $\varepsilon \in A$. Hence, condition (8) is violated. Because of this, in this section, I consider only the cases in which $\alpha \in (0, 1)$. It is important to observe that when $\alpha \in (0, 1)$, in principle, one can always find prices (p_1, p_2) such that conditions (8) and (9) hold. Therefore, to address the question posed above, one has to use some equilibrium conditions. I turn to that problem now.

First, I argue that in any asymmetric equilibrium some shareholders remain uninformed while some others acquire information in the event a repurchase takes place. The explanation is as follows. If only firm i repurchased shares, then an open market repurchase in which everybody obtains information would be equivalent to a dividend payment. It follows that every shareholder would prefer firm i to pay dividends to avoid paying the fixed cost of acquiring information. Therefore, if the majority of shareholders of firm i prefers an open market repurchase, it must be the case they anticipate that some shareholders will remain uninformed. That is, $\omega_{i,\mathcal{J}\mathcal{J}} > 0$. Because information rents are infinitely large when everybody else remains uninformed and the cost of acquiring information is finite, it must be the case that some investors do obtain information when firm i announces a repurchase. That is $\omega_{\mathcal{J}} = g\left(\varepsilon^U\right) \cdot x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^U\right)$ for some $\varepsilon^U \in \{\underline{\varepsilon}, \overline{\varepsilon}\}$ as stated in $A.1$ below. Since the returns of firms are independent and agents are nonsatiated, it follows that both investors hold a positive fraction of each firm. This is condition $A.2$. Condition $A.3$ follows because, on the one hand, the majority in firm i prefers repurchases and, on the other hand, the informed investors' ownership of the dividend paying firm is bounded away from zero. Finally, since the incentives to gather information in the event the two firms conduct an OMR are larger than when only one does it, agents conjecture that at most $g\left(\varepsilon^U\right) \cdot x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^U\right)$ shares would be in hands of the uninformed if both firms were to repurchase shares. This is the content of condition $A.1$.

Proposition 5.5 *If $\{\mathcal{J}, \Omega, x, (p_1, p_2, q)\}$ is a ME with $\mathcal{J} = \{i\}$, then*
$A.1$ $\omega_{\mathcal{J}'} \leq \omega_{\mathcal{J}} = g\left(\varepsilon^U\right) \cdot x^U_{\mathcal{J}}\left(p, q, \varepsilon^U\right)$ *for* $\mathcal{J}' = \{1, 2\}$.
$A.2$ $\omega_{h,\mathcal{J}} \in (0, 1)$ *for* $h = 1, 2$.
$A.3$ $\omega_{i,\mathcal{J}} < \frac{1}{2}$.

Since the optimal portfolio of investor ε is interior, it equalizes the price of risk of each asset to her marginal rate of substitution between mean and variance of wealth. That is,

$$\begin{aligned}
\frac{E_{\varepsilon^U}\left(r_i^U - \frac{p_i}{q}\right)}{x^U_{i,\mathcal{J}}(p,q,\varepsilon^U)\cdot var_{\varepsilon^U}\left(r_i^U\right)} &= \frac{E\left(r_j^D - \frac{p_j}{q}\right)}{x^U_{j,\mathcal{J}}(p,q,\varepsilon^U)\cdot var\left(r_j^D\right)} = \frac{\delta}{1-\delta E_{\varepsilon^U}\left[x^U_{i,\mathcal{J}}(\cdot,\varepsilon^U)\cdot\left(r_i^U - \frac{p_i}{q}\right) + x^U_{j,\mathcal{J}}(\cdot,\varepsilon^U)\cdot\left(r_j^D - \frac{p_j}{q}\right) + \frac{w}{q}\right]} \\
\frac{E_{\varepsilon^I}\left(r_i^I - \frac{p_i}{q}\right)}{x^I_{i,\mathcal{J}}(\omega_{\mathcal{J}},p,q,\varepsilon^I)\cdot var_{\varepsilon^I}\left(r_i^I\right)} &= \frac{E\left(r_j^D - \frac{p_j}{q}\right)}{x^I_{j,\mathcal{J}}(\omega_{\mathcal{J}},p,q,\varepsilon^I)\cdot var\left(r_j^D\right)} = \frac{\delta}{1-\delta\cdot E_{\varepsilon^I}\left[x^I_{i,\mathcal{J}}(\cdot,\varepsilon^I)\cdot\left(r_i^I - \frac{p_i}{q}\right) + x^I_{j,\mathcal{J}}(\cdot,\varepsilon^I)\cdot\left(r_j^D - \frac{p_j}{q}\right) + \frac{w}{q}\right]}
\end{aligned}$$

The following Lemma shows that in any asymmetric ME agents disagree on the price of risk. Thus, markets are effectively incomplete.

Lemma 5.2 *If $\{\mathcal{J}, \Omega, x, (p_1, p_2, q)\}$ is a ME with $\mathcal{J} = \{i\}$, then $MRS\left[E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right), var_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right)\right] < MRS\left[E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right), var_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right)\right]$ and $\omega_{i,\mathcal{J}} < \omega_{j,\mathcal{J}}$.*

The main question I want to address in this section is whether a firm that repurchases shares can raise more financial capital than an otherwise identical firm that pays dividends. That is, can it exist a ME such that $p_i \geq p_j$? I argue that this can never happen provided agents do not disagree much about the likelihood a distribution takes place at date 1. Since the shares of the dividend paying firm do not suffer from adverse selection, investors agree on the Sharpe ratio of firm j. That is,

$$
\begin{aligned}
\frac{E_{\varepsilon^U}\left(r_i^U-\frac{p_i}{q}\right)}{var_{\varepsilon^U}\left(r_i^U\right)}\cdot\frac{x_{j,\mathcal{J}}^U\left(\omega_{\mathcal{J}},p,q,\varepsilon^U\right)}{x_{i,\mathcal{J}}^U\left(\omega_{\mathcal{J}},p,q,\varepsilon^U\right)} &= \frac{E\left(r_j^D-\frac{p_j}{q}\right)}{var\left(r_j^D\right)}\\
\frac{E_{\varepsilon^I}\left(r_i^I-\frac{p_i}{q}\right)}{var_{\varepsilon^I}\left(r_i^I\right)}\cdot\frac{x_{j,\mathcal{J}}^I\left(\omega_{\mathcal{J}},p,q,\varepsilon^I\right)}{x_{i,\mathcal{J}}^I\left(\omega_{\mathcal{J}},p,q,\varepsilon^I\right)} &= \frac{E\left(r_j^D-\frac{p_j}{q}\right)}{var\left(r_j^D\right)}
\end{aligned}
$$

Manipulating these conditions one obtains that

$$
\frac{\omega_{j,\mathcal{J}}\cdot E_{\varepsilon^U}\left(r_i^U-\frac{p_i}{q}\right)+\left(1-\omega_{j,\mathcal{J}}\right)\cdot E_{\varepsilon^I}\left(r_i^I-\frac{p_i}{q}\right)}{\omega_{i,\mathcal{J}}\cdot var_{\varepsilon^U}\left(r_i^U\right)+\left(1-\omega_{i,\mathcal{J}}\right)\cdot var_{\varepsilon^I}\left(r_i^I\right)}=\frac{E\left(r_j^D-\frac{p_j}{q}\right)}{var\left(r_j^D\right)} \tag{10}
$$

That is, (p_1, p_2, q) is a DE in a representative agent economy where agent i holds all the stock of both companies, she has correct beliefs about firm j's rate of return but she believes firm i offers an expected return per share of $\omega_{j,\mathcal{J}}\cdot E_{\varepsilon^U}\left(r_i^U\right)+\left(1-\omega_{j,\mathcal{J}}\right)\cdot E_{\varepsilon^I}\left(r_i^I\right)$ and variance per share of $\omega_{i,\mathcal{J}}\cdot var_{\varepsilon^U}\left(r_i^U\right)+\left(1-\omega_{i,\mathcal{J}}\right)\cdot var_{\varepsilon^I}\left(r_i^I\right)$.

To see the intuition behind the result that $p_i<p_j$, it is useful to rewrite (10) as

$$
\frac{\left(\overline{\theta}+c\right)+e\left(\omega_{\mathcal{J}}\right)-\frac{p_i^{1-\alpha}}{q}}{\omega_{i,\mathcal{J}}\cdot var_{\varepsilon^U}\left(\frac{r_i^U}{p_i^\alpha}\right)+\left(1-\omega_{i,\mathcal{J}}\right)\cdot var_{\varepsilon^I}\left(\frac{r_i^I}{p_i^\alpha}\right)}\cdot\frac{1}{p_i^\alpha}=\frac{\left(\overline{\theta}+c\right)-\frac{p_j^{1-\alpha}}{q}}{\sigma^2}\cdot\frac{1}{p_j^\alpha}
$$

where

$$
e\left(\omega_{\mathcal{J}}\right)\equiv\left(-\omega_{j,\mathcal{J}}\cdot\varepsilon^U+\left(1-\omega_{j,\mathcal{J}}\right)\cdot\frac{\omega_{i,\mathcal{J}}}{1-\omega_{i,\mathcal{J}}}\cdot\varepsilon^I\right)\cdot\pi\cdot\tau\cdot c \tag{11}
$$

is the weighted difference between the subjective expected gain and loss per capita of the informed and the uninformed when an OMR takes place.[16] Hence, to argue that it cannot be the case that $p_i\geq p_j$, it suffices to show that the following two conditions hold

$$
\omega_{i,\mathcal{J}}\cdot var_{\varepsilon^U}\left(\frac{r_i^U}{p_i^\alpha}\right)+\left(1-\omega_{i,\mathcal{J}}\right)\cdot var_{\varepsilon^I}\left(\frac{r_i^I}{p_i^\alpha}\right)>\sigma^2 \tag{12}
$$

$$
e\left(\omega_{\mathcal{J}}\right)<0 \tag{13}
$$

Consider first the case in which $\varepsilon^U=\varepsilon^I=\varepsilon$. Since $\omega_{i,\mathcal{J}}\cdot\frac{r_i^U}{p_i^\alpha}+\left(1-\omega_{i,\mathcal{J}}\right)\cdot\frac{r_i^I}{p_i^\alpha}=\theta_i+c$ for any $\omega_{i,\mathcal{J}}\in(0,1)$

[16] The agent who anticipates she will remain uninformed in the event an OMR takes place, expects a loss per share of $\varepsilon'\cdot\pi\cdot\tau\cdot c$, compared to the case in which firm i distributes cash by means of dividends. However, the shareholder who anticipates she will become informed expects a gain per share of $\varepsilon''\cdot\pi\cdot\tau\cdot c\cdot\frac{\omega_{i,\mathcal{J}}}{1-\omega_{i,\mathcal{J}}}$. The expected net transfer under the true probability distribution, therefore, is zero. Expression (11) differs from the true expected net transfer in two aspects. On the one hand, it underestimates the true expected net transfer because $\omega_{j,\mathcal{J}\mathcal{J}}>\omega_{i,\mathcal{J}\mathcal{J}}$. On the other hand, it uses the agents's beliefs about the occurrence of an OMR.

and variance is a strictly convex function,

$$\begin{aligned}\omega_{i,\mathcal{J}} \cdot var_{\varepsilon}\left(\frac{r_i^U}{p_i^\alpha}\right) + (1-\omega_{i,\mathcal{J}}) \cdot var_{\varepsilon}\left(\frac{r_i^I}{p_i^\alpha}\right) &> var_{\varepsilon}\left(\omega_{i,\mathcal{J}} \cdot \frac{r_i^U}{p_i^\alpha} + (1-\omega_{i,\mathcal{J}}) \cdot \frac{r_i^I}{p_i^\alpha}\right) \\ &= var\left(\theta_i + c\right) = \sigma^2\end{aligned}$$

Rearranging terms in expression (11), it is straightforward to see that

$$e\left(\omega_{\mathcal{J}}\right) \equiv \left(\frac{1-\omega_{j,\mathcal{J}}}{1-\omega_{i,\mathcal{J}}} \cdot \frac{\omega_{i,\mathcal{J}}}{\omega_{j,\mathcal{J}}} - 1\right) \cdot \omega_{j,\mathcal{J}} \cdot \varepsilon^I \cdot \pi \cdot \tau \cdot c < 0$$

since Lemma 5.2 implies that $\frac{1-\omega_{j,\mathcal{J}}}{1-\omega_{i,\mathcal{J}}} \cdot \frac{\omega_{i,\mathcal{J}}}{\omega_{j,\mathcal{J}}} < 1$. Hence, both (12) and (13) holds for $\varepsilon^U = \varepsilon^I = \varepsilon$ and, therefore, $p_i < p_j$. A continuity argument shows that these conditions also hold for ε^U close to ε^I.

Proposition 5.6 *Suppose $\{\mathcal{J}, \Omega, x, (p_1, p_2, q)\}$ is a ME with $\mathcal{J} = \{i\}$. There exists $\kappa \in (0,1)$ sucht that $\varepsilon^U/\varepsilon^I \geq \kappa$ implies that $p_i < \frac{w_0}{2} < p_j$. In particular, it holds whenever $\varepsilon^U = \overline{\varepsilon}$ and $\varepsilon^I = \underline{\varepsilon}$.*

Proposition 5.6 shows that if investors do not disagree much about the probability that a distribution takes place, a firm that pays dividends raises more capital in the financial market than an otherwise identical firm which repurchases shares. Therefore, the market share of the dividend paying firm exceeds, *ceteris paribus*, that of a firm which conducts open market repurchases. One concludes that financial markets favor firms which choose to payout through dividends.

6. CONCLUSIONS

The choice of method to use to distribute cash to the firm's shareholders is one of the fundamental decisions that managers take. As Brennan and Thakor show, that choice is not a matter of indifference for the shareholders. Therefore, it is important to asses the effect that the expectation about the payout method has on asset prices as well as the output decisions of the firm. I show that a firm expected to payout dividends faces a lower cost of capital and raises more funds than an otherwise identical firm expected to repurchases shares. In addition, *ex-ante* identical firms can choose different payout methods. Therefore, this model allows for the managers to differ in their objectives without entering into con ict with the preferences of the majority of shareholders. A manager whose objectives dictate that she should maximize the market share of the firm chooses to pay dividends, while a manager who wants to favor the would-be informed shareholders may choose to repurchase shares. Therefore, this model gives some intuition on how the market selects among managers that have objectives which do not enter into con ict with the interests of the shareholders' majority.

APPENDIX

The following Proposition shows that the absence of arbitrage opportunities together with the requirement that the value of assets add up to initial wealth imply that asset prices are uniformly bounded.

Proposition 6.1 *If $(p_1, p_2, q) \in \Psi$, there exists $\underline{l}, l$ and u such that $0 < l < p_i, q < u < +\infty$ and $p_i - (\theta_L + c) \cdot p_i^{\alpha} \cdot q \geq \underline{l}$.*

Proof of Proposition 6.1: Suppose $(p_1, p_2, q) \in \Re_+^3$ satisfies (4) and $p_1 + p_2 = w_0$. Since $\frac{p_i^{1-\alpha}}{q} > \theta_L + c$, there exists $\underline{l} > 0$ such that $p_i - (\theta_L + c) \cdot p_i^{\alpha} \cdot q \geq \underline{l}$. Clearly $p_i \leq w_0$ for $i = 1, 2$. Since $\theta_L + c \leq \frac{p_i^{1-\alpha}}{q}$, it follows that $q \leq \frac{w_0^{1-\alpha}}{\theta_L + c}$. If one defines $u = \max\left\{w_0, \frac{w_0^{1-\alpha}}{\theta_L + c}\right\}$, then $p_i, q \leq u < +\infty$. Suppose there is no lower bound for q. Then, there exists $\{p^m, q^m\} \in \Psi$ such that $q^m \rightarrow 0$. Since $\frac{(p_1^m)^{1-\alpha}}{q^m} \leq \theta_H + c$, it follows that $p_1^m \rightarrow 0$ and $p_2^m \rightarrow w_0$. But then, $\frac{(p_1^m)^{1-\alpha}}{q^m} \rightarrow \infty$, a contradiction. It follows that there exists $l' > 0$ such that $l' \leq q$. Since, $\theta_L + c \leq \frac{p_i^{1-\alpha}}{q}$, it follows that $p_i \geq [(\theta_L + c) \cdot l']^{\frac{1}{1-\alpha}} > 0$. If one defines $l = \min\left\{l', [(\theta_L + c) \cdot l']^{\frac{1}{1-\alpha}}\right\}$, then $p_i, q > l > 0$, as desired. ■

It should be easy to see that whenever $(p, q) \in \Psi$, to require wealth to be nonnegative in every state, i.e. $w(r_1, r_2, p, q, x)(s) \geq 0$ for all $s \in \mathcal{S}$, implies that $x \in \mathcal{B}$ where

$$\mathcal{B} \equiv \left\{x \in \Re_+^2 : x_i \leq \frac{w}{\underline{l}}, \forall i = 1, 2\right\}$$

is a compact set. Thus, for each price vector in Ψ the set of portfolios such that the agent's wealth is non-negative in every state is a compact set of $\Re_+^2$.

Lemma 6.1 *Suppose $(p, q) \in \Psi$, $r_i : \mathcal{S} \times \Re_+ \mapsto \Re_+$ is homogeneous of degree α in p_i, $x_i(r, p, q, \varepsilon) > 0$ for some asset $i \in \{1, 2\}$ and $1 - \delta \cdot E_\varepsilon[w(r_1, r_2, p, q, x(\cdot))] > 0$. If $\frac{E_\varepsilon(r_i)}{p_i^\alpha} = \frac{E_\varepsilon(r_j)}{p_j^\alpha}$ and $var_\varepsilon\left(\frac{r_i}{p_i^\alpha}\right) = var_\varepsilon\left(\frac{r_j}{p_j^\alpha}\right)$ then $x_i(r, p, q, \varepsilon) > x_j(r, p, q, \varepsilon)$ if and only if $p_i < p_j$.*

Proof of Lemma 6.1: Suppose $(p, q) \in \Psi$, $r_i : \mathcal{S} \times \Re_+ \mapsto \Re_+$ is homogeneous of degree α in p_i, $x_i(r, p, q, \varepsilon) > 0$ for some asset $i \in \{1, 2\}$ and $1 - \delta \cdot E_\varepsilon[w(r_1, r_2, p, q, x(\cdot))] > 0$. Assume $\frac{E_\varepsilon(r_i)}{p_i^\alpha} = \frac{E_\varepsilon(r_j)}{p_j^\alpha}$ and $var_\varepsilon\left(\frac{r_i}{p_i^\alpha}\right) = var_\varepsilon\left(\frac{r_j}{p_j^\alpha}\right)$.

First I show that for $x_j(r, p, q, \varepsilon) > 0$, it is true that $x_i(r, p, q, \varepsilon) > x_j(r, p, q, \varepsilon) \Leftrightarrow p_i < p_j$. Since $x_i(r, p, q, \varepsilon) > 0$, the necessary conditions for an interior global maximum are that

$$\frac{E_\varepsilon\left(r_i - \frac{p_i}{q}\right)}{var_\varepsilon\left(\frac{r_i}{p_i^\alpha}\right) \cdot (p_i^\alpha)^2 \cdot x_i + cov_\varepsilon\left(\frac{r_i}{p_i^\alpha}, \frac{r_j}{p_j^\alpha}\right) \cdot p_j^\alpha \cdot p_i^\alpha \cdot x_j} = \frac{E_\varepsilon\left(r_j - \frac{p_j}{q}\right)}{var_\varepsilon\left(\frac{r_j}{p_j^\alpha}\right) \cdot \left(p_j^\alpha\right)^2 \cdot x_j + cov_\varepsilon\left(\frac{r_i}{p_i^\alpha}, \frac{r_j}{p_j^\alpha}\right) \cdot p_i^\alpha \cdot p_j^\alpha \cdot x_i}$$

dividing numerator and denominator by p_i^α on the left hand side and by p_j^α on the right hand side one obtains

$$\frac{E_\varepsilon\left(\frac{r_i}{p_i^\alpha}-\frac{p_i^{1-\alpha}}{q}\right)}{var_\varepsilon\left(\frac{r_i}{p_i^\alpha}\right)\cdot p_i^\alpha\cdot x_i+cov_\varepsilon\left(\frac{r_i}{p_i^\alpha},\frac{r_j}{p_j^\alpha}\right)\cdot p_j^\alpha\cdot x_j}=\frac{E_\varepsilon\left(\frac{r_j}{p_j^\alpha}-\frac{p_j^{1-\alpha}}{q}\right)}{var_\varepsilon\left(\frac{r_j}{p_j^\alpha}\right)\cdot p_j^\alpha\cdot x_j+cov_\varepsilon\left(\frac{r_i}{p_i^\alpha},\frac{r_j}{p_j^\alpha}\right)\cdot p_i^\alpha\cdot x_i}$$

and since $\frac{E_\varepsilon(r_i)}{p_i^\alpha}=\frac{E_\varepsilon(r_j)}{p_j^\alpha}$, it follows that

$$\begin{aligned}
p_i<p_j \quad&\Leftrightarrow\quad var_\varepsilon\left(\tfrac{r_i}{p_i^\alpha}\right)\cdot p_i^\alpha\cdot x_i+cov_\varepsilon\left(\frac{r_i}{p_i^\alpha},\tfrac{r_j}{p_j^\alpha}\right)\cdot p_j^\alpha\cdot x_j>var_\varepsilon\left(\tfrac{r_j}{p_j^\alpha}\right)\cdot p_j^\alpha\cdot x_j+cov_\varepsilon\left(\tfrac{r_i}{p_i^\alpha},\tfrac{r_j}{p_j^\alpha}\right)\cdot p_i^\alpha\cdot x_i\\
&\Leftrightarrow\quad \left[var_\varepsilon\left(\frac{r_i}{p_i^\alpha}\right)-cov_\varepsilon\left(\frac{r_i}{p_i^\alpha},\frac{r_j}{p_j^\alpha}\right)\right]\cdot p_i^\alpha\cdot x_i>\left[var_\varepsilon\left(\frac{r_j}{p_j^\alpha}\right)-cov_\varepsilon\left(\frac{r_i}{p_i^\alpha},\frac{r_j}{p_j^\alpha}\right)\right]\cdot p_j^\alpha\cdot x_j\\
&\Leftrightarrow\quad p_i^\alpha\cdot x_i>p_j^\alpha\cdot x_j
\end{aligned}$$

where the last line follows from the assumption that $var_\varepsilon\left(\frac{r_i}{p_i^\alpha}\right)=var_\varepsilon\left(\frac{r_j}{p_j^\alpha}\right)$ and the Cauchy Schwarz inequality which together imply that $var_\varepsilon\left(\frac{r_i}{p_i^\alpha}\right)-cov_\varepsilon\left(\frac{r_i}{p_i^\alpha},\frac{r_j}{p_j^\alpha}\right)>0$. It follows that $x_j(r,p,q,\varepsilon)>0$ implies

$$x_i(r,p,q,\varepsilon)>x_j(r,p,q,\varepsilon)\Leftrightarrow p_i<p_j$$

Now suppose $x_j(r,p,q,\varepsilon)=0$. Clearly, the equivalence above can only fail if $p_j\geq p_i$. From the first order conditions

$$\begin{aligned}
(1-\delta\cdot E_\varepsilon[w(\cdot)])\cdot E_\varepsilon\left(\frac{r_j}{p_j^\alpha}-\frac{p_j^{1-\alpha}}{q}\right)&\leq\delta\cdot x_i\cdot cov\left(\frac{r_i}{p_i^\alpha},\frac{r_j}{p_j^\alpha}\right)\cdot p_i^\alpha\\
(1-\delta\cdot E_\varepsilon[w(\cdot)])\cdot E_\varepsilon\left(\frac{r_i}{p_i^\alpha}-\frac{p_i^{1-\alpha}}{q}\right)&=\delta\cdot x_i\cdot var\left(\frac{r_i}{p_i^\alpha}\right)\cdot p_i^\alpha
\end{aligned}$$

and since $(1-\delta\cdot E_\varepsilon[w(\cdot)])>0$ and $\frac{E_\varepsilon(r_i)}{p_i^\alpha}=\frac{E_\varepsilon(r_j)}{p_j^\alpha}$, it follows that $\delta\cdot x_i\cdot cov\left(\frac{r_i}{p_i^\alpha},\frac{r_j}{p_j^\alpha}\right)\cdot p_i^\alpha\geq\delta\cdot x_i\cdot var\left(\frac{r_i}{p_i^\alpha}\right)\cdot p_i^\alpha$. Since $x_i(r,p,q,\varepsilon)>0$, $\delta>0$ and $p_i>0$, this implies $cov\left(\frac{r_i}{p_i^\alpha},\frac{r_j}{p_j^\alpha}\right)\geq var\left(\frac{r_i}{p_i^\alpha}\right)$, a contradiction because the assumption $var_\varepsilon\left(\frac{r_i}{p_i^\alpha}\right)=var_\varepsilon\left(\frac{r_j}{p_j^\alpha}\right)$ together with the Cauchy-Schwarz inequality imply that $var_\varepsilon\left(\frac{r_i}{p_i^\alpha}\right)-cov_\varepsilon\left(\frac{r_i}{p_i^\alpha},\frac{r_j}{p_j^\alpha}\right)>0$. ■

Proof of Proposition 3.1: Let (p_1,p_2,q) be a DE. By definition, $B(p_1,p_2,q)=w_0-p_1\cdot X_1^D(p,q)-p_2\cdot X_2^D(p,q)$. Since in equilibrium $B(p_1,p_2,q)=0$ and $X_i^D\left(r^D,p,q\right)=1$ for all $i=1,2$, it follows that $p_1+p_2=w_0$. To get a contradiction, suppose $p_1\neq p_2$. Without loss of generality assume $p_1<p_2$. Since $X_i^D\left(r^D,p,q\right)=x_i\left(r^D,p,q,\underline{\varepsilon}\right)$, it follows by Lemma 6.1 that $X_1^D\left(r^D,p,q\right)>X_2^D\left(r^D,p,q\right)=1$, a contradiction. One concludes that $p_1=p_2=\frac{w_0}{2}$, as desired. ■

Proof of Proposition 3.2: Suppose assumption NS holds. By proposition 3.1**,** $p_1=p_2=\frac{w_0}{2}$. In addition, q

solves $X_i^D\left(\frac{w_0}{2},\frac{w_0}{2},q\right)=1$ if and only if

$$\frac{1-\frac{\delta w_0}{q}}{\delta\frac{w_0}{2}}\frac{\overline{\theta}+c-\frac{\left(\frac{w_0}{2}\right)^{1-\alpha}}{q}}{\sigma^2+2\cdot\left[\overline{\theta}+c-\frac{\left(\frac{w_0}{2}\right)^{1-\alpha}}{q}\right]^2}=1\Leftrightarrow q=\left(\frac{w_0}{2}\right)^{1-\alpha}\frac{\frac{1}{\delta w_0}\cdot\left(\frac{w_0}{2}\right)^{1-\alpha}-\left(\overline{\theta}+c\right)}{\left(\overline{\theta}+c\right)\cdot\left(\frac{1}{\delta w_0}\left(\frac{w_0}{2}\right)^{1-\alpha}-\left(\overline{\theta}+c\right)\right)-\frac{\sigma^2}{2}}\equiv q^*$$

It follows that if a DE exists then it is unique. I shall show that $\left(\frac{w_0}{2},\frac{w_0}{2},q^*\right)$ is a DE. It suffices to show that $q^*>0$, that is

$$\left(\overline{\theta}+c\right)\cdot\left(\frac{1}{\delta\cdot w_0}\cdot\left(\frac{w_0}{2}\right)^{1-\alpha}-\left(\overline{\theta}+c\right)\right)-\frac{\sigma^2}{2}>0$$

Rearranging terms, one needs to show that

$$\frac{1}{\delta}>\left(\frac{w_0}{2}\right)^{\alpha}\cdot\frac{2\cdot\left(\overline{\theta}+c\right)^2+\pi\cdot(1-\pi)\cdot(\theta_H-\theta_L)^2}{\overline{\theta}+c}$$

and since assumption NS holds, it suffices to show that $\frac{2\cdot\left(\overline{\theta}+c\right)+\pi\cdot(1-\pi)\cdot(\theta_H-\theta_L)^2}{\overline{\theta}+c}<2\cdot(\theta_H+c)$. Notice that

$$\begin{aligned}
\frac{2\cdot\left(\overline{\theta}+c\right)^2+\pi\cdot(1-\pi)\cdot(\theta_H-\theta_L)^2}{\overline{\theta}+c}<2\cdot(\theta_H+c)\quad&\Leftrightarrow\quad 2\cdot\left(\overline{\theta}+c\right)^2+\pi\cdot(1-\pi)\cdot(\theta_H-\theta_L)^2<2\cdot\left(\overline{\theta}+c\right)\cdot(\theta_H+c)\\
&\Leftrightarrow\quad \pi\cdot(1-\pi)\cdot(\theta_H-\theta_L)^2<2\cdot\left(\overline{\theta}+c\right)\cdot\left(\theta_H-\overline{\theta}\right)\\
&\Leftrightarrow\quad (1-\pi)\cdot(\theta_H-\theta_L)^2<2\cdot\left(\overline{\theta}+c\right)\cdot(\theta_H-\theta_L)\\
&\Leftrightarrow\quad (1-\pi)\cdot(\theta_H-\theta_L)<2\cdot\left(\overline{\theta}+c\right)\\
&\Leftrightarrow\quad (1-\pi)\cdot(\theta_H-\theta_L)<2\cdot((1-\pi)\cdot(\theta_H-\theta_L)+\theta_L+c)\\
&\Leftrightarrow\quad 0<(1-\pi)\cdot(\theta_H-\theta_L)+2\cdot(\theta_L+c)
\end{aligned}$$

which is always true since $\theta_H-\theta_L>0$ and $(\theta_L+c)>0$. Hence, $\left(\frac{w_0}{2},\frac{w_0}{2},q^*\right)$ is the unique DE. Next, I show that $\left(\frac{w_0}{2},\frac{w_0}{2},q^*\right)\in\Psi$. First notice that

$$\frac{\left(\frac{w_0}{2}\right)^{1-\alpha}}{q^*}=\frac{\left(\overline{\theta}+c\right)\cdot\left(\frac{1}{\delta w_0}\left(\frac{w_0}{2}\right)^{1-\alpha}-\left(\overline{\theta}+c\right)\right)-\frac{\sigma^2}{2}}{\frac{1}{\delta w_0}\cdot\left(\frac{w_0}{2}\right)^{1-\alpha}-\left(\overline{\theta}+c\right)}=\left(\overline{\theta}+c\right)-\frac{\frac{\sigma^2}{2}}{\frac{1}{\delta w_0}\cdot\left(\frac{w_0}{2}\right)^{1-\alpha}-\left(\overline{\theta}+c\right)}<\theta_H+c$$

where the last inequality uses the fact that $\frac{1}{\delta w_0}\cdot\left(\frac{w_0}{2}\right)^{1-\alpha}-\left(\overline{\theta}+c\right)>\frac{1}{\delta\cdot w_0}\cdot\left(\frac{w_0}{2}\right)^{1-\alpha}-\left(\overline{\theta}+c\right)-\frac{\sigma^2}{2\cdot\left(\overline{\theta}+c\right)}>0$. So it suffices to show that

$$\begin{aligned}
\frac{\left(\frac{w_0}{2}\right)^{1-\alpha}}{q^*}>\theta_L+c\quad&\Leftrightarrow\quad\frac{\frac{1}{\delta w_0}\cdot\left(\frac{w_0}{2}\right)^{1-\alpha}-\left(\overline{\theta}+c\right)}{\left(\overline{\theta}+c\right)\cdot\left(\frac{1}{\delta w_0}\left(\frac{w_0}{2}\right)^{1-\alpha}-\left(\overline{\theta}+c\right)\right)-\frac{\sigma^2}{2}}<\frac{1}{\theta_L+c}\\
&\Leftrightarrow\quad\left(\frac{w_0}{2}\right)^{\alpha}\cdot\frac{2\cdot\left(\overline{\theta}+c\right)^2+\pi\cdot(1-\pi)\cdot(\theta_H-\theta_L)^2-2\cdot\left(\overline{\theta}+c\right)\cdot(\theta_L+c)}{\overline{\theta}-\theta_L}<\frac{1}{\delta}
\end{aligned}$$

and since assumption NS holds, it suffices to show that $\frac{2\cdot\left(\overline{\theta}+c\right)^2+\pi\cdot(1-\pi)\cdot(\theta_H-\theta_L)^2-2\cdot\left(\overline{\theta}+c\right)\cdot(\theta_L+c)}{\overline{\theta}-\theta_L}<2\cdot(\theta_H+c)$.

Notice that

$$
\begin{aligned}
& \tfrac{2\cdot\left(\overline{\theta}+c\right)^2+\pi\cdot(1-\pi)\cdot(\theta_H-\theta_L)^2-2\cdot\left(\overline{\theta}+c\right)\cdot(\theta_L+c)}{\overline{\theta}-\theta_L} < 2\cdot(\theta_H+c) \\
\Leftrightarrow\quad & \pi\cdot(1-\pi)\cdot(\theta_H-\theta_L)^2+2\cdot\left(\overline{\theta}+c\right)\cdot\left(\overline{\theta}-\theta_L\right) < 2\cdot(\theta_H+c)\cdot\left(\overline{\theta}-\theta_L\right) \\
\Leftrightarrow\quad & \pi\cdot(\theta_H-\theta_L)\cdot\left(\overline{\theta}-\theta_L\right)+2\cdot\left(\overline{\theta}+c\right)\cdot\left(\overline{\theta}-\theta_L\right) < 2\cdot(\theta_H+c)\cdot\left(\overline{\theta}-\theta_L\right) \\
\Leftrightarrow\quad & \pi\cdot(\theta_H-\theta_L) < 2\cdot\left(\theta_H-\overline{\theta}\right) \\
\Leftrightarrow\quad & \pi\cdot(\theta_H-\theta_L) < 2\cdot\pi\cdot(\theta_H-\theta_L)
\end{aligned}
$$

which is always true since $\pi\cdot(\theta_H-\theta_L)>0$. It follows that $\left(\frac{w_0}{2},\frac{w_0}{2},q^*\right)\in\Psi$, as desired. ■

Lemma 6.2 *Let $(p,q)\in\Psi$. If every uninformed investor follows $\widehat{\gamma}^U$ and every informed investor follows $\widehat{\gamma}^I$, there is $\delta^I(p,q)$ such that for all $0<\delta<\delta^I(p,q)$ no informed investor wants to deviate unilaterally from $\widehat{\gamma}^I$.*

Proof of Lemma 6.2: Since the action of a single informed investor neither affects the price quoted by the market maker nor the round in which the OMR ends, the set of rates of return associated with the strategies of an informed agent is

$$\mathcal{R}_i^I \equiv \left\{r_i=\phi_i^I\cdot r_i^I+\left(1-\phi_i^I\right)\cdot r_i' \text{ where } \phi_i^I:\mathcal{S}\mapsto[0,1] \text{ is } \mathcal{F}\text{-measurable}\right\}$$

where

$$r_i'(s)\equiv\begin{cases}\left(\overline{\theta}+c\right)\cdot p_i^\alpha & \text{if } \theta_i(s)=\theta_H \ \&\ d(s)=1\\ (\theta_L+c\cdot(1-\tau))\cdot p_i^\alpha & \text{if } \theta_i(s)=\theta_L \ \&\ d(s)=1\\ (\theta_i(s)+c)\cdot p_i^\alpha & \text{otherwise}\end{cases}$$

Clearly, $r_i^I(\omega_i)$ dominates r_i', that is $r_i^I(\omega_i)(s)\geq r_i'(s)$ for all $s\in S$ and there is some s in which the inequality is strict.

I would like to argue that if every other informed shareholder follows strategy $\widehat{\gamma}^I$ and every uninformed shareholder follows $\widehat{\gamma}^U$, each informed shareholder finds it optimal to follow $\widehat{\gamma}^I$ regardless of her portfolio. There are two cases to consider, depending on whether only one or both firms repurchase shares. Formally, $\widehat{\gamma}^I$ is optimal for the informed shareholder regardless of her portfolio and the identity of the firms that announce an OMR if

$$u\left[w\left(r_1^I,r_2^I,p,q,x\right)(s)\right] \geq u\left[w\left(r_1,r_2,p,q,x\right)(s)\right] \quad \forall x\in\mathcal{B}, \forall(r_1,r_2)\in\mathcal{R}_1^I\times\mathcal{R}_2^I, \forall s\in\mathcal{S}_1 \tag{14}$$

$$u\left[w\left(r_i^I,r_{-i}^D,p,q,x\right)(s)\right] \geq u\left[w\left(r_i,r_{-i}^D,p,q,x\right)(s)\right] \quad \forall x\in\mathcal{B}, \forall r_i\in\mathcal{R}_i^I, \forall s\in\mathcal{S}_1 \tag{15}$$

where the first inequality corresponds to the case in which both firms repurchase shares and the second one to the case in which only one firm performs a repurchase.

Since an agent who follows $\widehat{\gamma}^I$ maximizes her wealth in each state, she may only find it optimal to deviate if she is satiated in some state. Therefore, to show that (14) - (15) holds is equivalent to argue that for every portfolio in the compact set $\mathcal{B}$ and every state s the agent is not satiated. Since portfolios are in a compact set and the assets return is bounded then the investors wealth is also bounded and, therefore, condition (1) holds provided δ is not too large as the following argument shows.

First notice that $w\left(r_1, r_2, p, q, x\right)(s) \leq w\left(r_1^I, r_2^I, p, q, x\right)(s)$ for all $x \in \mathcal{B}$, $(r_1, r_2) \in \mathcal{R}_1^I \times \mathcal{R}_2^I$ and $s \in \mathcal{S}_1$. Therefore, to show that (14) and (15) holds, it suffices to argue that when $\phi(s) = 1$ for all $s \in \mathcal{S}_1$, the marginal utility of wealth is positive in every state $s \in \mathcal{S}_1$, i.e. (1) holds at every $s \in \mathcal{S}_1$. Since $r_i^I(s) \leq (\theta_H + c) \cdot p_i^\alpha$, it follows that

$$w\left(r_1^I, r_2^I, p, q, x\right)(s) \leq \left[(\theta_H + c) \cdot p_1^\alpha - \tfrac{p_1}{q}\right] \cdot x_1 + \left[(\theta_H + c) \cdot p_2^\alpha - \tfrac{p_2}{q}\right] \cdot x_2 + \tfrac{w_0}{q}$$

Now, let

$$\delta^I(p, q) \equiv \min_{x \in \mathcal{B}} \frac{1}{\left[(\theta_H + c) \cdot p_1^\alpha - \frac{p_1}{q}\right] \cdot x_1 + \left[(\theta_H + c) \cdot p_2^\alpha - \frac{p_2}{q}\right] \cdot x_2 + \frac{w_0}{q}}$$

Since $\mathcal{B}$ is a compact set and the objective function is continuous in x, it follows that $\delta^I(p, q)$ is well defined. Then,

$$w\left(r_1^I, r_2^I, p, q, x\right)(s) < \frac{1}{\delta^I(p, q)} < \frac{1}{\delta}$$

for any $0 < \delta < \delta^I(p, q)$, as desired. ■

Let $(p, q) \in \Psi$. If every uninformed investor follows $\widehat{\gamma}^U$ and every informed investor follows $\widehat{\gamma}^I$, there is $\delta^U(p, q)$ such that for all $0 < \delta < \delta^U(p, q)$ no uninformed investor wants to deviate unilaterally from $\widehat{\gamma}^U$.

Proof of Lemma 15**:** The rate of return obtained by an uninformed shareholder who tenders in the first round is

$$r_i^{T_1}(\omega_i)(s) \equiv \begin{cases} \left(\overline{\theta} + c\right) \cdot p_i^\alpha & \text{if } \theta_i(s) = \theta_H \\ \left(\theta_L + c + \frac{\omega_i}{1-\omega_i} \cdot \tau \cdot c\right) \cdot p_i^\alpha & \text{if } \theta_i(s) = \theta_L \end{cases}$$

and $E\left(r_i^{T_1} \| \mathcal{S}_1\right)(s) = \left[\overline{\theta} + c - \pi \cdot \tau \cdot \left(c + \overline{\theta} - \frac{c}{1-\omega_i}\right)\right] \cdot p_i^\alpha$. Therefore,

$$E\left(r_i^U \| \mathcal{S}_1\right)(s) > E\left(r_i^{T_1} \| \mathcal{S}_1\right)(s) \Leftrightarrow 1 - \omega_i > \frac{c}{\overline{\theta}} \tag{16}$$

Notice also that to tender in the second round is never better than not tendering at all for the uninformed shareholder. Indeed, if she follows the strategy of tendering in the second round, then her rate of return per share is

$$r_i^{T_2}(s) = \begin{cases} (\theta_i + c) \cdot p_i^\alpha & \text{if } \theta_i = \theta_H \\ (\theta_L + c - \tau \cdot c) \cdot p_i^\alpha & \text{if } \theta_i = \theta_L \text{ and } d(s) = 1 \end{cases}$$

and, therefore, $r_i^U(s) \geq r_i^{T_2}(s)$ for all $s \in \mathcal{S}_1$. So we can assume the uninformed either tender in the first round or does not tender at all.

Since the action of a single uninformed investor neither affects the price quoted by the market maker nor the round in which the OMR ends, the set of rates of return associated with the strategies of the uninformed is

$$\mathcal{R}_i^U \equiv \left\{ r_i = \phi_i^U \cdot r_i^U + \left(1 - \phi_i^U\right) \cdot r_i^{T_1} \text{ where } \phi_i^U \in [0,1] \right\}$$

Formally, $\widehat{\gamma}^U$ is optimal for the uninformed shareholder regardless of her portfolio and the identity of the firms that announce an OMR if

$$\begin{aligned}
E\left(u\left[w\left(r_1^U, r_2^U, p, q, x\right)\right] - u\left[w\left(r_1, r_2, p, q, x\right)\right] \|\mathcal{S}_1\right)(s) &\geq 0 \ \forall x \in \mathcal{B}, \forall\left(r_1, r_2\right) \in \mathcal{R}_1^I \times \mathcal{R}_2^I \\
E\left(u\left[w\left(r_i^U, r_{-i}^D, p, q, x\right)\right] - u\left[w\left(r_i, r_{-i}^D, p, q, x\right)\right] \|\mathcal{S}_1\right)(s) &\geq 0 \ \forall x \in \mathcal{B}, \forall r_i \in \mathcal{R}_i^U, \forall i = 1,2
\end{aligned}$$

where the first inequality corresponds to the case in which the two firms repurchase shares and the second one to the case in which only one of them performs a repurchase. Since the investor is uninformed and her objective function is concave in ϕ_i^U, the expressions above are equivalent to

$$D_{\phi_i^U} E\left(u\left[w\left(\phi_i^U \cdot r_i^U + \left(1 - \phi_i^U\right) \cdot r_i', r_{-i}^U, p, q, x\right)\right] \|\mathcal{S}_1\right)(s)\Big|_{\phi_i^U = 1} \geq 0 \ \forall x \in \mathcal{B}, \forall i \in \mathcal{J} \tag{17}$$

$$D_{\phi_i^U} E\left(u\left[w\left(\phi_i^U \cdot r_i^U + \left(1 - \phi_i^U\right) \cdot r_i', r_{-i}^D, p, q, x\right)\right] \|\mathcal{S}_1\right)(s)\Big|_{\phi_i^U = 1} \geq 0 \ \forall x \in \mathcal{B}, \forall i \in \mathcal{J} \tag{18}$$

where D_x denotes the partial derivative with respect to x.

Let $(p,q) \in \Psi$, $x \in \mathcal{B}$ and $i \in \mathcal{J}$. Notice that for $r_{-i} \in \left\{r_{-i}^U, r_{-i}^D\right\}$

$$D_{\phi_i^U} E\left(u\left[w\left(\phi_i^U \cdot r_i^U + \left(1 - \phi_i^U\right) \cdot r_i', r_{-i}, p, q, x\right)\right] \|\mathcal{S}_1\right)(s)\Big|_{\phi_i^U = 1} > 0 \Leftrightarrow \frac{E\left(r_i^U - r_i^{T_1} \|\mathcal{S}_1\right)}{E\left[w(r_i^U, r_{-i}, p, q, x) \cdot \left(r_i^U - r_i^{T_1}\right) \|\mathcal{S}_1\right]} > \delta$$

However, since $E\left(r_{-i}^U \|\mathcal{S}_1\right) < E\left(r_{-i}^D \|\mathcal{S}_1\right)$ and r_i^U is conditionally independent of r_{-i}^D,

$$\frac{E\left(r_i^U - r_i^{T_1} \|\mathcal{S}_1\right)}{E\left[w(r_1^U, r_2^U, p, q, x) \cdot \left(r_i^U - r_i^{T_1}\right) \|\mathcal{S}_1\right]} \geq \frac{E\left(r_i^U - r_i^{T_1} \|\mathcal{S}_1\right)}{E\left[w\left(r_i^U, r_{-i}^D, p, q, x\right) \cdot \left(r_i^U - r_i^{T_1}\right) \|\mathcal{S}_1\right]}$$

then it suffices to show that the right hand side exceeds δ. Define

$$\delta^U(p,q) \equiv \min_{x \in \mathcal{B}} \frac{E_\varepsilon\left(r_i^U - r_i^{T_1} \|\mathcal{S}_1\right)}{E_\varepsilon\left[w\left(r_i^U, r_{-i}^D, p, q, x\right) \cdot \left(r_i^U - r_i^{T_1}\right) \|\mathcal{S}_1\right]}$$

Then, for all $0 < \delta < \delta^U(p,q)$ and $r_{-i} \in \left\{r_{-i}^U, r_{-i}^D\right\}$,

$$\delta < \delta^U(p,q) \leq \frac{E\left(r_i^U - r_i^{T_1} \|\mathcal{S}_1\right)}{E\left[w(r_i^U, r_{-i}, p, q, x) \cdot \left(r_i^U - r_i^{T_1}\right) \|\mathcal{S}_1\right]}$$

Therefore, (17) and (18) holds, as desired.■

Proof of Lemma 4.1: Set $\overline{\delta}(p,q) = \min\left\{\delta^U(p,q), \delta^I(p,q)\right\}$ and the desired results follows by Lemmas 6.2 and 15. ■

Proof of Lemma 4.2: Suppose $\delta < \delta^I$ and $\mathcal{J}' \in \mathcal{P}$. First I show property (i). By definition

$$\Delta_{\mathcal{J}'}(\omega,p,q,x) \equiv E\left(u\left[w\left(r^I_{\mathcal{J}'},p,q,x\right)\right]|\mathcal{S}_1\right) - E\left(u\left[w\left(r^U_{\mathcal{J}'},p,q,x\right)\right]|\mathcal{S}_1\right)$$

Since $r^U_{\mathcal{J}'}$ is constant with respect to ω, $\Delta_{\mathcal{J}'}(\omega,p,q,x)$ increases with ω if and only if $E\left(u\left[w\left(r^I_{\mathcal{J}'},p,q,x\right)\right]|\mathcal{S}_1\right)$ does. Let $w\left(r^I_{\mathcal{J}'},\cdot\right) = w\left(r^I_{\mathcal{J}'},p,q,x\right)$. Notice that

$$E\left(u\left[w\left(r^I_{\mathcal{J}'},\cdot\right)\right]|\mathcal{S}_1\right) = E\left(w\left(r^I_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right)\cdot\left(1-\tfrac{\delta}{2}\cdot E\left(w\left(r^I_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right)\right) - \tfrac{\delta}{2}\cdot var\left(w\left(r^I_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right)$$

where

$$\begin{aligned} E\left(w\left(r^I_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right) &= E\left(r^I_{1,\mathcal{J}'} - \tfrac{p_1}{q}|\mathcal{S}_1\right)\cdot x_1 + E\left(r^I_{2,\mathcal{J}'} - \tfrac{p_2}{q}|\mathcal{S}_1\right)\cdot x_2 + \tfrac{w}{q} \qquad (19)\\ var\left(w\left(r^I_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right) &= var\left(r^I_{1,\mathcal{J}'}|\mathcal{S}_1\right)\cdot x_1 + var\left(r^I_{2,\mathcal{J}'}|\mathcal{S}_1\right)\cdot x_2 \qquad (20)\end{aligned}$$

where the last line uses the property that $cov\left(r^I_{1,\mathcal{J}'}, r^I_{2,\mathcal{J}'}|\mathcal{S}_1\right) = 0$ for every $\mathcal{J}' \in \mathcal{P}$. Since $\delta < \delta^I$, it follows that $\Delta_{\mathcal{J}'}(\omega,p,q,x)$ increases with $E\left(w\left(r^I_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right)$ and decreases with $var\left(w\left(r^I_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right)$. So it suffices to show that $E\left(w\left(r^I_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right)$ and $var\left(w\left(r^I_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right)$ increases and decreases with ω, respectively. On the one hand, for every $i \notin \mathcal{J}'$, $E\left(r^I_{i,\mathcal{J}'}|\mathcal{S}_1\right)$ and $var\left(r^I_{i,\mathcal{J}'}|\mathcal{S}_1\right)$ are constant with respect to ω. On the other hand, for every $i \in \mathcal{J}'$, $E\left(r^I_{i,\mathcal{J}'}|\mathcal{S}_1\right) = E\left(r^I_i|\mathcal{S}_1\right)$ increases with ω and $var\left(r^I_{i,\mathcal{J}'}|\mathcal{S}_1\right) = var\left(r^I_i|\mathcal{S}_1\right)$ decreases with ω. It follows from (19) and (20) that $\Delta_{\mathcal{J}'}(\omega,p,q,x)$ increases with ω, as desired.

Now I show property (ii) holds. Suppose $\mathcal{J} \subset \mathcal{J}'$. Let $\omega \in [0,1)$. Since $E\left(r^I_{i,\mathcal{J}}|\mathcal{S}_1\right) \leq E\left(r^I_{i,\mathcal{J}'}|\mathcal{S}_1\right)$ and $var\left(r^I_{i,\mathcal{J}}|\mathcal{S}_1\right) \geq var\left(r^I_{i,\mathcal{J}'}|\mathcal{S}_1\right)$ $\forall i = 1,2$, then $E\left(w\left(r^I_{\mathcal{J}},\cdot\right)|\mathcal{S}_1\right) \leq E\left(w\left(r^I_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right)$ and $var\left(w\left(r^I_{\mathcal{J}},\cdot\right)|\mathcal{S}_1\right) \geq var\left(w\left(r^I_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right)$. Therefore, $E\left(u\left[w\left(r^I_{\mathcal{J}},p,q,x\right)\right]|\mathcal{S}_1\right) \leq E\left(u\left[w\left(r^I_{\mathcal{J}'},p,q,x\right)\right]|\mathcal{S}_1\right)$. Likewise, since $E\left(r^U_{i,\mathcal{J}}|\mathcal{S}_1\right) \geq E\left(r^U_{i,\mathcal{J}'}|\mathcal{S}_1\right)$ and $var\left(r^U_{i,\mathcal{J}}|\mathcal{S}_1\right) \leq var\left(r^U_{i,\mathcal{J}'}|\mathcal{S}_1\right)$ $\forall i = 1,2$, $E\left(w\left(r^U_{\mathcal{J}},\cdot\right)|\mathcal{S}_1\right) \geq E\left(w\left(r^U_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right)$ and $var\left(w\left(r^U_{\mathcal{J}},\cdot\right)|\mathcal{S}_1\right) \leq var\left(w\left(r^U_{\mathcal{J}'},\cdot\right)|\mathcal{S}_1\right)$. Therefore, $E\left(u\left[w\left(r^U_{\mathcal{J}},p,q,x\right)\right]|\mathcal{S}_1\right) \geq E\left(u\left[w\left(r^U_{\mathcal{J}'},p,q,x\right)\right]|\mathcal{S}_1\right)$. Thus,

$$\begin{aligned} \Delta_{\mathcal{J}}(\omega,p,q,x) &= E\left(u\left[w\left(r^I_{\mathcal{J}},p,q,x\right)\right]|\mathcal{S}_1\right) - E\left(u\left[w\left(r^U_{\mathcal{J}},p,q,x\right)\right]|\mathcal{S}_1\right)\\ &\leq E\left(u\left[w\left(r^I_{\mathcal{J}'},p,q,x\right)\right]|\mathcal{S}_1\right) - E\left(u\left[w\left(r^U_{\mathcal{J}'},p,q,x\right)\right]|\mathcal{S}_1\right) = \Delta_{\mathcal{J}'}(\omega,p,q,x)\end{aligned}$$

as desired. ■

Proof of Proposition 5.1: Suppose $\{(p_1,p_2,q), \mathcal{J}, \omega_{\mathcal{J}}\}$ is a ME, NS holds and $\omega_{i,\mathcal{J}} < 1$ for some firm

$i = 1, 2$. By $E.2$ there exists some agent ε^I for whom $x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^I\right) = x^I_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^I\right)$. Suppose there exists some agent ε^U for whom $x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^U\right) = x^U_{\mathcal{J}}\left(p, q, \varepsilon^U\right)$. Then,

$$\begin{aligned} w\left(r^U_{\mathcal{J}}, p, q, \varepsilon^U\right) &= \left(r^U_{1,\mathcal{J}} - \tfrac{p_1}{q}\right) \cdot x^U_{1,\mathcal{J}}\left(p, q, \varepsilon^U\right) + \left(r^U_{2,\mathcal{J}} - \tfrac{p_2}{q}\right) \cdot x^U_{2,\mathcal{J}}\left(p, q, \varepsilon^U\right) + \tfrac{w}{q} \\ w\left(r^I_{\mathcal{J}}, p, q, \varepsilon^I\right) &= \left(r^I_{1,\mathcal{J}} - \tfrac{p_1}{q}\right) \cdot x^I_{1,\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^I\right) + \left(r^I_{2,\mathcal{J}} - \tfrac{p_2}{q}\right) \cdot x^I_{2,\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^I\right) + \tfrac{w}{q} \end{aligned}$$

It follows that

$$\begin{aligned} g\left(\varepsilon^U\right) \cdot w\left(r^U_{\mathcal{J}}, p, q, \varepsilon^U\right) + \left(1 - g\left(\varepsilon^U\right)\right) \cdot w\left(r^I_{\mathcal{J}}, p, q, \varepsilon^I\right) &= \sum_{i=1,2} \left[r^U_{i,\mathcal{J}} \cdot \omega_{i,\mathcal{J}} + r^I_{i,\mathcal{J}} \cdot \left(1 - \omega_{i,\mathcal{J}\mathcal{J}}\right)\right] \\ &= \sum_{i=1,2} \left(\theta_i + c\right) \cdot p_i^\alpha \\ &\leq \sum_{i=1,2} \left(\theta^H + c\right) \cdot p_i^\alpha \\ &= \left(\theta^H + c\right) \cdot \left[\left(\tfrac{p_1}{w}\right)^\alpha + \left(\tfrac{p_2}{w}\right)^\alpha\right] \cdot w^\alpha \\ &\leq 2 \cdot \left(\theta^H + c\right) \cdot \left(\tfrac{w}{2}\right)^\alpha \end{aligned}$$

and since $w\left(r^U_{\mathcal{J}}, p, q, \varepsilon^U\right)(s), w\left(r^I_{\mathcal{J}}, p, q, \varepsilon^I\right)(s) \geq 0$ for all $s \in \mathcal{S}$, then

$$\max\left\{w\left(r^U_{\mathcal{J}}, p, q, \varepsilon^U\right)(s), w\left(r^I_{\mathcal{J}}, p, q, \varepsilon^I\right)(s)\right\} < \frac{2 \cdot \left(\theta^H + c\right) \cdot \left(\frac{w}{2}\right)^\alpha}{\min\{\lambda, 1 - \lambda\}} < \frac{1}{\delta} \text{ for all } s \in \mathcal{S}$$

as desired. Now, suppose everybody is informed. Then, $\omega_{\mathcal{J}} = \mathbf{0} \equiv (0, 0)$ and $r^I_{i,\mathcal{J}} = \left(\theta_i + c\right) \cdot p_i^\alpha$ for all $i = 1, 2$. Then,

$$g\left(\varepsilon^U\right) \cdot w\left(r^I_{\mathcal{J}}, p, q, \varepsilon^U\right) + \left(1 - g\left(\varepsilon^U\right)\right) \cdot w\left(r^I_{\mathcal{J}}, p, q, \varepsilon^I\right) = \sum_{i=1,2} \left(\theta_i + c\right) \cdot p_i^\alpha \leq 2 \cdot \left(\theta^H + c\right) \cdot \left(\frac{w}{2}\right)^\alpha$$

and since $w\left(r^I_{\mathcal{J}}, p, q, \varepsilon^U\right)(s), w\left(r^I_{\mathcal{J}}, p, q, \varepsilon^I\right)(s) \geq 0$ for all $s \in \mathcal{S}$, then

$$\max\left\{w\left(r^I_{\mathcal{J}}, p, q, \varepsilon^U\right)(s), w\left(r^I_{\mathcal{J}}, p, q, \varepsilon^I\right)(s)\right\} < \frac{2 \cdot \left(\theta^H + c\right) \cdot \left(\frac{w}{2}\right)^\alpha}{\min\{\lambda, 1 - \lambda\}} < \frac{1}{\delta} \text{ for all } s \in \mathcal{S}$$

as desired. ■

Proof of Proposition 5.2: Let $\left\{\left(p_1, p_2, q\right), \mathcal{J}, \omega_{\mathcal{J}}\right\}$ be a ME. By definition,

$$B_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q\right) = w_0 - p_1 \cdot X_{1,\mathcal{J}}\left(\omega, p, q\right) - p_2 \cdot X_{2,\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q\right)$$

Since in equilibrium $B_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q\right) = 0$ and $X_{i,\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q\right) = 1$ for all $i = 1, 2$, it follows that $p_1 + p_2 = w_0$.

Suppose the ME is symmetric, i.e. $\mathcal{J} \in \{\varnothing, \{1, 2\}\}$. The case in which $\mathcal{J} = \varnothing$ follows from proposition 3.1

since (p_1, p_2, q) is a DE. So I consider here only the case in which $\mathcal{J} = \{1,2\}$.To get a contradiction, suppose $p_1 \neq p_2$. Without loss of generality assume $p_1 < p_2$. By Proposition 5.1 $1 - \delta \cdot E_\varepsilon (w(\cdot)) > 0$ for every $\varepsilon \in A$.
$\cdot$ Assume first that everybody is informed. Then $\omega_{i,\mathcal{J}} = 0$ and $r^I_{i,\mathcal{J}} = r^D_i$ for all $i = 1,2$. In addition, $X_{i,\mathcal{J}}(\mathbf{0}, p, q) = \lambda \cdot x^I_{i,\mathcal{J}}(\mathbf{0}, p, q, \underline{\varepsilon}) + (1-\lambda) \cdot x^I_{i,\mathcal{J}}(\mathbf{0}, p, q, \overline{\varepsilon}) = x_i\left(r^D, p, q, \underline{\varepsilon}\right)$ for every firm i. Since $E_\varepsilon\left(\frac{r^D_i}{p^\alpha_i}\right) = E_\varepsilon\left(\frac{r^D_{-i}}{p^\alpha_{-i}}\right)$ and $var_\varepsilon\left(\frac{r^D_i}{p^\alpha_i}\right) = var_\varepsilon\left(\frac{r^D_{-i}}{p^\alpha_{-i}}\right)$ for every $\varepsilon \in A$ and $x_{i,\mathcal{J}}\left(r^D, p, q, \underline{\varepsilon}\right) > 0$ for all $i = 1,2$ by $E.2$, it follows by Lemma 6.1 that $X_{1,\mathcal{J}}(\mathbf{0}, p, q) > X_{2,\mathcal{J}}(\mathbf{0}, p, q) = 1$, a contradiction.
$\cdot$ Assume now there exists some investor $\varepsilon^U \in A$ who remains uninformed. The case in which $\omega_{i,\mathcal{J}} = 0$ for all $i = 1,2$ is similar to the one in which everybody is informed. So, assume that $\omega_{i,\mathcal{J}} > 0$ for some $i \in \{1,2\}$. Then, $x^U_{i,\mathcal{J}}\left(p, q, \varepsilon^U\right) > 0$. Since $E_{\varepsilon^U}\left(\frac{r^U_i}{p^\alpha_i}\right) = E_{\varepsilon^U}\left(\frac{r^U_{-i}}{p^\alpha_{-i}}\right)$ and $var_{\varepsilon^U}\left(\frac{r^U_i}{p^\alpha_i}\right) = var_{\varepsilon^U}\left(\frac{r^U_{-i}}{p^\alpha_{-i}}\right)$, then Lemma 6.1 implies that $x^U_{1,\mathcal{J}}\left(p, q, \varepsilon^U\right) > x^U_{2,\mathcal{J}}\left(p, q, \varepsilon^U\right) \geq 0$. So it cannot be the case that everybody is uninformed. That is, $\omega_{i,\mathcal{J}} < 1$ for some $i = 1,2$. Hence, there exists $\varepsilon^I \in A$ that is informed. Then $\omega_{1,\mathcal{J}} > \omega_{2,\mathcal{J}} \geq 0$ implies that $E_{\varepsilon^I}\left(\frac{r^I_i}{p^\alpha_i}\right) > E_{\varepsilon^I}\left(\frac{r^I_{-i}}{p^\alpha_{-i}}\right)$ and $var_{\varepsilon^I}\left(\frac{r^I_i}{p^\alpha_i}\right) < var_{\varepsilon^I}\left(\frac{r^I_{-i}}{p^\alpha_{-i}}\right)$. Since $\omega_{i,\mathcal{J}} < 1$, Proposition 5.1 implies that no agent is satiated. Therefore, it must be the case that $x^I_{2,\mathcal{J}}\left(\omega_\mathcal{J}, p, q, \varepsilon^I\right) = 0$. Hence, $g\left(\varepsilon^U\right) \cdot x^U_{2,\mathcal{J}}\left(p, q, \varepsilon^U\right) = 1$. It follows that $X_{1,\mathcal{J}}(\omega_\mathcal{J}, p, q) = g\left(\varepsilon^U\right) \cdot x^U_{1,\mathcal{J}}\left(p, q, \varepsilon^U\right) + g\left(\varepsilon^I\right) \cdot x^I_{1,\mathcal{J}}\left(\omega_\mathcal{J}, p, q, \varepsilon^I\right) > g\left(\varepsilon^U\right) \cdot x^U_{2,\mathcal{J}}\left(p, q, \varepsilon^U\right) = 1$, a contradiction. Hence, $p_1 = p_2 = \frac{w_0}{2}$, as desired. ■

Proof of Proposition 5.3: Suppose $\overline{\delta}\left(\frac{w_0}{2}, \frac{w_0}{2}, q^*\right) > \delta$ and (6) holds. Let $p^* \equiv \left(\frac{w_0}{2}, \frac{w_0}{2}\right)$ and $\mathbf{0} \equiv (0,0)$. I show that (p^*, q^*) together with $\mathcal{J} = \emptyset$ and $\omega_{\mathcal{J}'} \equiv \mathbf{0}$ for all $\mathcal{J}' \in \mathcal{P}$ is a ME. Clearly, $E.1$ holds. Since (p^*, q^*) is a DE, conditions $(E.2)$ and $(E.3)$ hold. Since $x_\mathcal{J}(\omega_\mathcal{J}, p^*, q^*, \varepsilon) = x\left(r^D, p^*, q^*, \varepsilon\right) = 1$,

$$\left\{\varepsilon : \Delta_{\{i\}}\left(\omega_{\{i\}}, p^*, q^*, x_\mathcal{J}(\omega_\mathcal{J}, p^*, q^*, \varepsilon)\right) \leq f\right\} = \left\{\varepsilon : \Delta_{\{i\}}(\mathbf{0}, p^*, q^*, 1) \leq f\right\} = \emptyset$$

Since $x_{1,\mathcal{J}}(\mathbf{0}, p^*, q^*, \varepsilon) = x_{2,\mathcal{J}}(\mathbf{0}, p^*, q^*, \varepsilon) = 1$, it follows that

$$\left\{\varepsilon : \Delta_{\{-i\}}(\mathbf{0}, p^*, q^*, x_\mathcal{J}(\mathbf{0}, p^*, q^*, \varepsilon)) \leq f\right\} = \left\{\varepsilon : \Delta_{\{i\}}(\mathbf{0}, p^*, q^*, x_\mathcal{J}(\mathbf{0}, p^*, q^*, \varepsilon)) \leq f\right\} = \emptyset$$

Finally, since $\{i\} \subset \{1,2\}$ then property (ii) in Lemma 4.2 implies that

$$\left\{\varepsilon : \Delta_{\{1,2\}}(\mathbf{0}, p^*, q^*, x_\mathcal{J}(\mathbf{0}, p^*, q^*, \varepsilon)) \leq f\right\} = \left\{\varepsilon : \Delta_{\{i\}}(\mathbf{0}, p^*, q^*, x_\mathcal{J}(\mathbf{0}, p^*, q^*, \varepsilon)) \leq f\right\} = \emptyset$$

Therefore, $\omega_{\mathcal{J}'} = \mathbf{0}$ satisfies $(E.4)$ for all $\mathcal{J}' \in \mathcal{P}$. Finally, for $\mathcal{J}' = \{i\}$

$$\begin{aligned}\Delta V_{\mathcal{J}'\mathcal{J}}(\omega_\mathcal{J}, p^*, q^*, \varepsilon) &= V_{\mathcal{J}'}\left[\omega_{\mathcal{J}'}, p^*, q^*, x_\mathcal{J}(\omega_\mathcal{J}, p^*, q^*, \varepsilon)\right] - V_\mathcal{J}\left[\omega_\mathcal{J}, p^*, q^*, x_\mathcal{J}(\omega_\mathcal{J}, p^*, q^*, \varepsilon)\right] \\ &= V_{\mathcal{J}'}\left[\mathbf{0}, p^*, q^*, x_\mathcal{J}(\mathbf{0}, p^*, q^*, \varepsilon)\right] - V_\mathcal{J}\left[\mathbf{0}, p^*, q^*, x_\mathcal{J}(\mathbf{0}, p^*, q^*, \varepsilon)\right] < 0\end{aligned}$$

Hence, $\sum_{\varepsilon : \Delta V_{\mathcal{J}'\mathcal{J}}(\omega_\mathcal{J}, p^*, q^*, \varepsilon) < 0} g(\varepsilon) \cdot x_{i,\mathcal{J}}(\mathbf{0}, p^*, q^*, \varepsilon) = 1$ implies that $(E.5)$ holds. Therefore, $\left\{\left(\frac{w_0}{2}, \frac{w_0}{2}, q^*\right), \mathcal{J}, \omega_\mathcal{J}\right\}$ is a ME.

Now suppose $\{(p_1, p_2, q), \mathcal{J}, \omega_{\mathcal{J}}\}$ is a ME in which $\mathcal{J} = \emptyset$. Since a ME with $\mathcal{J} = \emptyset$ is a DE, it follows that $(p_1, p_2, q) = (p^*, q^*)$. Then, $\overline{\delta}\left(\frac{w_0}{2}, \frac{w_0}{2}, q^*\right) > \delta$. Suppose (6) does not hold. Then, $\Delta_{\{i\}}(\mathbf{0}, p^*, q^*, 1) \leq f$ for all i. Since $x_{\mathcal{J}}(p^*, q^*, \varepsilon) = (1,1)$ for all $\varepsilon \in A$ and $(E.4)$ holds, it follows that $\omega_{\{i\}} \neq (0,0)$ for all i. Let $\overline{\omega}$ be such that $\Delta_{\{i\}}(\omega, p^*, q^*, 1) = f$. On the one hand, since $\omega_{\{i\}}$ satisfies $(E.4)$ and $x_{\mathcal{J}}(p^*, q^*, \varepsilon) = (1,1)$ for all ε, it must be the case that $\Delta_{\{i\}}\left(\omega_{\{i\}}, p^*, q^*, 1\right) \leq f$ for all $i = 1, 2$. It follows by property (i) in Lemma 4.2 that $\omega_{\{i\}} \leq \overline{\omega} < (1,1)$ for all i. But on the other hand, $\omega_{\{i\}} < (1,1)$ together with $(E.2)$ and $(E.4)$ implies that some investor purchases information. Hence, $\Delta_{\{i\}}\left(\omega_{\{i\}}, p^*, q^*, 1\right) > f$ for some i, a contradiction. It follows that (6) does hold, as desired. ■

Proof of Proposition 5.4: Suppose $(R.1)$, $(R.2)$ and $(R.3)$ holds. I show that $\left\{\left(\frac{w_0}{2}, \frac{w_0}{2}, q\left(\varepsilon^U, \varepsilon^I\right)\right), \mathcal{J}, \omega_{\mathcal{J}}\right\}$ is a ME. Clearly $(E.1)$, $(E.2)$ and $(E.3)$ hold. By symmetry, $x^U_{1,\mathcal{J}}\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right) = x^U_{2,\mathcal{J}}\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right)$. Then, $\omega_{1,\mathcal{J}} = \omega_{2,\mathcal{J}} > 0$. By $R.3$ it follows that $x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right) = x^U_{\mathcal{J}}\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right)$. By $R.3$ and since property (ii) in Lemma 4.2 implies that $\Delta_{\{i\}}(\omega, p, q, x) \leq \Delta_{\mathcal{J}}(\omega, p, q, x)$, one obtains that

$$\begin{aligned}
\Delta_{\{i\}}\left(\omega_{\{i\}}, p^*, q(\cdot), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q(\cdot), \varepsilon^U\right)\right) &\leq \Delta_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q(\cdot), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q(\cdot), \varepsilon^U\right)\right) \leq f \\
\Delta_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q(\cdot), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q(\cdot), \varepsilon^I\right)\right) &\geq \Delta_{\{i\}}\left(\omega_{\{i\}}, p^*, q(\cdot), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q(\cdot), \varepsilon^I\right)\right) > f
\end{aligned}$$

This implies that

$$\left\{\varepsilon : \Delta_{\mathcal{J}'}\left(\omega_{\mathcal{J}'}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right)\right) \leq f\right\} = \left\{\varepsilon^U\right\} \quad \forall \mathcal{J}' \in \mathcal{P}$$

Then,

$$\omega_{\mathcal{J}'} = g\left(\varepsilon^U\right) \cdot x^U_{\mathcal{J}}\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right) = \sum_{\{\varepsilon : \Delta_{\mathcal{J}'}(\omega_{\mathcal{J}'}, p^*, q(\varepsilon^U, \varepsilon^I), x_{\mathcal{J}}(\omega_{\mathcal{J}}, p^*, q(\varepsilon^U, \varepsilon^I), \varepsilon)) \leq f\}} g(\varepsilon) \cdot x^U_{\mathcal{J}}\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon\right)$$

and, therefore, $E.4$ holds. Notice that

$$\begin{aligned}
\Delta V_{\{i\}\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right) &= V_{\{i\}}\left(\omega_{\{i\}}, \cdot, x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, \cdot, \varepsilon^U\right)\right) - V_{\mathcal{J}}\left(\omega_{\mathcal{J}}, \cdot, x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, \cdot, \varepsilon^U\right)\right) \geq 0 \\
\Delta V_{\{i\}\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^I\right) &= V_{\{i\}}\left(\omega_{\{i\}}, \cdot, x_{\mathcal{J}}\left(\omega_{\mathcal{J}\mathcal{J}}, \cdot, \varepsilon^I\right)\right) - V_{\mathcal{J}}\left(\omega_{\mathcal{J}}, \cdot, x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, \cdot, \varepsilon^I\right)\right) < 0
\end{aligned}$$

where the strict inequality holds because $\omega_{i,\{i\}} = \omega_{i,\mathcal{J}} > 0$ for every $i \in \{1, 2\}$. Hence, $\left\{\varepsilon : \Delta V_{\mathcal{J}'\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon\right) < 0\right\} = \left\{\varepsilon^I\right\}$ and

$$\sum_{\varepsilon : \Delta V_{\{i\}\mathcal{J}}(\omega_{\mathcal{J}}, p^*, q(\cdot), x_{\mathcal{J}}(\omega_{\mathcal{J}}, p^*, q(\cdot), \varepsilon)) < 0} g(\varepsilon) \cdot x^U_{i,\mathcal{J}}(p^*, q(\cdot), \varepsilon) = g\left(\varepsilon^I\right) \cdot x_{i,\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q(\cdot), \varepsilon^I\right) = 1 - \omega_{\mathcal{J}} > \tfrac{1}{2}$$

Therefore, $E.5$ holds. It follows that $\left\{\left(\frac{w_0}{2}, \frac{w_0}{2}, q\left(\varepsilon^U, \varepsilon^I\right)\right), \mathcal{J}, \omega_{\mathcal{J}}\right\}$ is a ME.

Assume there exists a ME with $\mathcal{J} = \{1,2\}$. Then, there is $\omega_{\mathcal{J}}$ such that $\left\{\left(\frac{w_0}{2}, \frac{w_0}{2}, q\right), \mathcal{J}, \omega_{\mathcal{J}}\right\}$ satisfies $E.1 - E.5$. Suppose $\omega_{\mathcal{J}} = (0,0)$. Clearly, $E.4$ implies that everybody is informed. Hence, for every $\varepsilon \in A$

$$\begin{aligned}
V_{\mathcal{J}\mathcal{J}}\left(\mathbf{0}, p, q, x_{\mathcal{J}}\left(\mathbf{0}, p, q, \varepsilon\right)\right) &= E\left(u\left[w\left(r^I, p, q, x_{\mathcal{J}}\left(\mathbf{0}, p, q, \varepsilon\right)\right)\right]\right) - f \\
&= E\left(u\left[w\left(r^D, p, q, x_{\mathcal{J}}\left(\mathbf{0}, p, q, \varepsilon\right)\right)\right]\right) - f \\
&\leq E\left(u\left[w\left(r_i^I, r_{-i}^D, p, q, x_{\mathcal{J}}\left(\mathbf{0}, p, q, \varepsilon\right)\right) | \mathcal{S}_1\right]\right) - f \\
&\leq V_{\mathcal{J}'\mathcal{J}}\left(\omega_{\mathcal{J}'}, p, q, x_{\mathcal{J}}\left(\mathbf{0}, p, q, \varepsilon\right)\right)
\end{aligned}$$

where the first inequality follows because $\omega_{\mathcal{J}'} \geq \omega_{\mathcal{J}}$. Hence, $\Delta V_{\{i\}\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q^*, \varepsilon\right) \geq 0$ for all ε, contradicting condition $E.5$. It follows that $\omega_{\mathcal{J}} \neq (0,0)$. Hence, there exists some firm i such that $\omega_{i,\mathcal{J}} > 0$. Then, $E.4$ implies there exists ε^U such that $\Delta_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right)\right) \leq f$. Let $\overline{\omega} \equiv \left(\overline{\omega}_1, \overline{\omega}_2\right) < (1,1)$ be the unique solution to $\Delta_{\mathcal{J}}\left(\omega, p^*, q\left(\varepsilon^U, \varepsilon^I\right), x_{\mathcal{J}}\left(\omega, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right)\right) = f$ such that $\overline{\omega}_1 = \overline{\omega}_2$. Since $x_{1,\mathcal{J}}^U\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right) = x_{2,\mathcal{J}}^U\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right)$, then $\omega_{1,\mathcal{J}} = \omega_{2,\mathcal{J}}$ and $\omega_{i,\mathcal{J}} \leq \overline{\omega} < 1$. By $E.2$, there exists ε^I such that $\Delta_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^I\right)\right) > f$. Hence, $x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^I\right) = x_{\mathcal{J}}^I\left(\omega_{\mathcal{J}}, p, q, \varepsilon^I\right)$. Since $\omega_{\mathcal{J}} \leq \overline{\omega} < (1,1)$, then $E.4$ implies that $\omega_{\mathcal{J}} = g\left(\varepsilon^U\right) \cdot x_{\mathcal{J}}^U\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right)$.

Suppose $\omega_{\{i\}} = (0,0)$. By properties (ii) and (i) in Lemma 4.2,

$$\begin{aligned}
\Delta_{\{i\}}\left(\mathbf{0}, p^*, q\left(\cdot\right), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\cdot\right), \varepsilon^U\right)\right) &\leq \Delta_{\mathcal{J}}\left(\mathbf{0}, p^*, q\left(\cdot\right), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\cdot\right), \varepsilon^U\right)\right) \\
&\leq \Delta_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\cdot\right), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\cdot\right), \varepsilon^U\right)\right) \leq f
\end{aligned}$$

But then, $E.4$ implies that $x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right) = x_{\mathcal{J}}^U\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right) = (0,0)$, a contradiction since $\omega_{i,\mathcal{J}} > 0$ for some i. It follows that $\omega_{\{i\}} \neq (0,0)$ and there exists some firm i such that $\omega_{i,\{i\}} > 0$. A reasoning analogous to the one used to prove that $\omega_{i,\mathcal{J}} > 0 \Rightarrow \omega_{\mathcal{J}} = g\left(\varepsilon^U\right) \cdot x_{\mathcal{J}}^U\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right)$ can be used here to show that $\omega_{i,\{i\}} > 0 \Rightarrow \omega_{\{i\}} = g\left(\varepsilon^U\right) \cdot x_{\mathcal{J}}^U\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right)$. By symmetry, $g\left(\varepsilon^U\right) \cdot x_{1,\mathcal{J}}^U\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right) = g\left(\varepsilon^U\right) \cdot x_{2,\mathcal{J}}^U\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right)$. Hence, $\omega_{j,\{i\}} > 0$ for all j. Since $\omega_{\mathcal{J}} = \omega_{\{i\}} = g\left(\varepsilon^U\right) \cdot x_{\mathcal{J}}^U\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right)$, then it must be the case that

$$\Delta_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\cdot\right), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\cdot\right), \varepsilon^U\right)\right) \leq f < \Delta_{\{i\}}\left(\omega_{\{i\}}, p^*, q\left(\cdot\right), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\cdot\right), \varepsilon^I\right)\right)$$

and, therefore, $R.3$ holds. Therefore, $\left\{\varepsilon : \Delta V_{\{i\}\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon\right) < 0\right\} = \left\{\varepsilon^I\right\}$ and by $E.5$, it follows that

$$\omega_{\mathcal{J}} = g\left(\varepsilon^U\right) \cdot x_{\mathcal{J}}^U\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right) = 1 - \sum_{\varepsilon : \Delta V_{\{i\}\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon\right) < 0} g\left(\varepsilon\right) \cdot x_{i,\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon\right) < \frac{1}{2}$$

for all i. Thus, condition $R.2$ holds, as desired. ■

Proof of Lemma 5.1: Suppose $\{(p_1, p_2, q), \mathcal{J}, \omega_{\mathcal{J}}\}$ is a ME with $\mathcal{J} = \{1, 2\}$. By Proposition 5.2, $p_1 = p_2 = \frac{w_0}{2}$. From the no arbitrage conditions,

$$\frac{p_i}{q} = E_{\varepsilon^U}\left(\frac{1-\delta \cdot w^U_{\mathcal{J}}}{1-\delta \cdot E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right)} \cdot r^U_i\right) = E_{\varepsilon^I}\left(\frac{1-\delta \cdot w^I_{\mathcal{J}}}{1-\delta \cdot E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right)} \cdot r^I_i\right)$$

and, after some algebra, one obtains that

$$E_{\varepsilon^U}\left(r^U_i\right) - \frac{\delta \cdot x^U_{i,\mathcal{J}}}{1-\delta \cdot E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right)} \cdot \left[var_{\varepsilon^U}\left(r^U_i\right) + cov_{\varepsilon^U}\left(r^U_i, r^U_j\right)\right] = E_{\varepsilon^I}\left(r^I_i\right) - \frac{\delta \cdot x^I_{i,\mathcal{J}}}{1-\delta \cdot E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right)} \cdot \left[var\left(r^I_i\right) + cov_{\varepsilon^I}\left(r^I_i, r^I_j\right)\right]$$

where I use the property that $x^k_{1,\mathcal{J}} = x^k_{2,\mathcal{J}}$ for any $k \in \{I, U\}$. By Proposition 5.1, $1 - \delta \cdot E_{\varepsilon^U}\left(w^k_{\mathcal{J}}\right) > 0$ for all $k \in \{I, U\}$. Since $E_{\varepsilon^U}\left(r^U_i\right) < \left(\overline{\theta} + c\right) \cdot p^{\alpha}_i \leq E_{\varepsilon^I}\left(r^I_i\right)$ and $\underline{var_{\varepsilon^U}\left(r^U_i\right) + cov_{\varepsilon^U}\left(r^U_i, r^U_j\right) > \sigma^2 \cdot p^{2 \cdot \alpha}_i \geq var_{\varepsilon^I}\left(r^I_i\right) + cov_{\varepsilon^I}\left(r^I_i, r^I_j\right)}$, then

$$\frac{\delta \cdot x^U_{i,\mathcal{J}}\left(\frac{w_0}{2}, \frac{w_0}{2}, q, \varepsilon^U\right)}{1-\delta \cdot E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right)} < \frac{\delta \cdot x^I_{i,\mathcal{J}}\left(\omega_{\mathcal{J}}, \frac{w_0}{2}, \frac{w_0}{2}, q, \varepsilon^I\right)}{1-\delta \cdot E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right)} \tag{21}$$

Since $MRS\left[E_{\varepsilon}(w), var_{\varepsilon}(w)\right] = \frac{\delta}{1-\delta \cdot E_{\varepsilon}(w)}$, I shall show that $\frac{\delta}{1-\delta \cdot E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right)} < \frac{\delta}{1-\delta \cdot E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right)}$. Suppose not. Then,

$$\begin{aligned} E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right) \geq E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right) \quad &\Leftrightarrow \quad 2 \cdot E_{\varepsilon^U}\left(r^U_1 - \frac{w_0/2}{q}\right) \cdot x^U_{1,\mathcal{J}}\left(\cdot, \varepsilon^U\right) \geq 2 \cdot E_{\varepsilon^I}\left(r^I_1 - \frac{w_0/2}{q}\right) \cdot x^I_{1,\mathcal{J}}\left(\cdot, \varepsilon^I\right) \\ &\Rightarrow \quad x^U_{1,\mathcal{J}}\left(\frac{w_0}{2}, \frac{w_0}{2}, q, \varepsilon^U\right) > x^I_{1,\mathcal{J}}\left(\omega_{\mathcal{J}}, \frac{w_0}{2}, \frac{w_0}{2}, q, \varepsilon^I\right) \end{aligned}$$

But this implies

$$\frac{\delta \cdot x^U_{1,\mathcal{J}}\left(\frac{w_0}{2}, \frac{w_0}{2}, q, \varepsilon^U\right)}{1-\delta \cdot E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right)} \geq \frac{\delta \cdot x^I_{1,\mathcal{J}}\left(\omega_{\mathcal{J}}, \frac{w_0}{2}, \frac{w_0}{2}, q, \varepsilon^I\right)}{1-\delta \cdot E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right)}$$

which contradicts (21). Therefore, $MRS\left[E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right), var_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right)\right] < MRS\left[E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right), var_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right)\right]$. ■

Proof of Proposition 5.5: Suppose $\{(p_1, p_2, q), \mathcal{J}, \omega_{\mathcal{J}}\}$ is a ME with $\mathcal{J} = \{i\}$. Suppose $\omega_{i,\mathcal{J}} = 0$. Then,

$$\begin{aligned} \Delta V_{\emptyset\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon\right) &= V_{\emptyset}\left(\omega_{\emptyset}, p, q, x_{\mathcal{J}}\left(\omega_{\mathcal{J}\mathcal{J}}, p, q, \varepsilon\right)\right) - V_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon\right)\right) \\ &= E\left(u\left[w\left(r^D, p, q, x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon\right)\right)\right] | \mathcal{S}_1\right) - V_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon\right)\right) > 0 \end{aligned}$$

where the last inequality uses the fact that for any $x \in \Re^2_+$,

$$\max\left\{E\left(u\left[w\left(r^I_{\mathcal{J}}, p, q, x\right)\right] | \mathcal{S}_1\right) - f, E\left(u\left[w\left(r^U_{\mathcal{J}}, p, q, x\right)\right] | \mathcal{S}_1\right)\right\}\Big|_{\omega_{i,\mathcal{J}}=\mathbf{0}} < E\left(u\left[w\left(r^D, p, q, x\right)\right] | \mathcal{S}_1\right)$$

But $\Delta V_{\emptyset\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon\right) > 0$ for all ε, contradicts condition $E.5$. Hence, $\omega_{i,\mathcal{J}} \neq 0$ and there exists

$\varepsilon^U \in A$ such that $\Delta_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^U\right)\right) \leq f$. Then, $x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^U\right) = x^U_{\mathcal{J}}\left(p, q, \varepsilon^U\right)$ and $x^U_{i,\mathcal{J}}\left(p, q, \varepsilon^U\right) > 0$. Let $\overline{\omega}$ be a solution to $\Delta_{\mathcal{J}}\left(\omega, p, q, x_{\mathcal{J}}\left(\omega, p, q, \varepsilon^U\right)\right) = f$. Clearly, $0 < \omega_{i,\mathcal{J}} \leq \overline{\omega}_i < 1$. Then, there also exists $\varepsilon^I \in A$ such that $\Delta_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^I\right)\right) > f$. By $(E.2)$ and $(E.4)$, $\omega_{i,\mathcal{J}} = g\left(\varepsilon^U\right) \cdot x^U_{i,\mathcal{J}}\left(p, q, \varepsilon^U\right) \in (0,1)$ and $\omega_{j,\mathcal{J}} = g\left(\varepsilon^U\right) \cdot x^U_{j,\mathcal{J}}\left(p, q, \varepsilon^U\right)$.

Suppose $\omega_{\mathcal{J}'} > g\left(\varepsilon^U\right) \cdot x^U_{j,\mathcal{J}}\left(p, q, \varepsilon^U\right)$ for $\mathcal{J}' = \{1,2\}$. Then, ε^I will not acquire information. But

$$\Delta_{\mathcal{J}'}\left(\omega_{\mathcal{J}'}, p^*, q\left(\cdot\right), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\cdot\right), \varepsilon^I\right)\right) > \Delta_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\cdot\right), x_{\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\cdot\right), \varepsilon^I\right)\right) > f$$

a contradiction. It follows that $\omega_{\mathcal{J}'} \leq g\left(\varepsilon^U\right) \cdot x^U_{j,\mathcal{J}}\left(p, q, \varepsilon^U\right)$. Hence, $A.1$ holds.

To show that $\omega_{j,\mathcal{J}} \in (0,1)$, suppose there exists some ε such that $x_{j,\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon\right) = 0$. Since $x_{i,\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon\right) > 0$ for all $\varepsilon \in A$, it follows that for some $k \in \{I, U\}$

$$\begin{aligned}
& E_{\varepsilon}\left(r^k_j - \frac{p_j}{q}\right) - \delta \cdot E_{\varepsilon}\left[\left(r^k_i - \frac{p_i}{q}\right) \cdot \left(r^k_j - \frac{p_j}{q}\right) \cdot x_i + \frac{w}{q}\left(r^k_j - \frac{p_j}{q}\right)\right] \leq 0 \\
\Leftrightarrow \quad & E_{\varepsilon}\left(r^k_j - \frac{p_j}{q}\right) - \delta \cdot E_{\varepsilon}\left(r^k_j - \frac{p_j}{q}\right) E_{\varepsilon}\left[\left(r^k_i - \frac{p_i}{q}\right) \cdot \cdot x_i + \frac{w}{q}\right] \leq 0 \\
\Leftrightarrow \quad & \delta \cdot E_{\varepsilon}\left[\left(r^k_i - \frac{p_i}{q}\right) \cdot x_i + \frac{w}{q}\right] \geq 1
\end{aligned}$$

The last inequality implies that there exists some state $s \in S$ such that $w\left(r^k_{\mathcal{J}}, p, q, \varepsilon\right)(s) \geq \frac{1}{\delta}$ which contradicts Proposition 5.1 since I argued that $\omega_{i,\mathcal{J}} < 1$. It follows that $x_{j,\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon\right) > 0$ for all $\varepsilon \in A$. Therefore, $\omega_{j,\mathcal{J}} \in (0,1)$, as desired. Hence, $A.2$ holds.

Clearly, $\left\{\varepsilon : \Delta V_{\varnothing\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon\right) < 0\right\} = \left\{\varepsilon^I\right\}$ and by $E.5$ it follows that $A.3$ holds because

$$\omega_{i,\mathcal{J}} = g\left(\varepsilon^U\right) \cdot x^U_{i,\mathcal{J}}\left(p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon^U\right) = 1 - \sum_{\varepsilon : \Delta V_{\varnothing\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon\right) < 0} g\left(\varepsilon\right) \cdot x_{i,\mathcal{J}}\left(\omega_{\mathcal{J}}, p^*, q\left(\varepsilon^U, \varepsilon^I\right), \varepsilon\right) < \frac{1}{2} \quad \blacksquare$$

Proof of Lemma 5.2: Suppose $\left\{\left(p_1, p_2, q\right), \mathcal{J}, \omega_{\mathcal{J}}\right\}$ is a ME with $\mathcal{J} = \{i\}$. By Proposition 5.1, $1 - \delta \cdot E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right) > 0$ and $1 - \delta \cdot E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right) > 0$. In order to get a contradiction, assume $MRS\left[E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right), var_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right)\right] \geq MRS\left[E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right), var_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right)\right]$. Then $E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right) \geq E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right)$. It follows that

$$\begin{aligned}
\frac{E\left(r^D_j - \frac{p_j}{q}\right)}{x^U_{j,\mathcal{J}}\left(p,q,\varepsilon^U\right) \cdot var\left(r^D_j\right)} \quad & \geq \quad \frac{E\left(r^D_j - \frac{p_j}{q}\right)}{x^I_{j,\mathcal{J}}\left(\omega_{\mathcal{J}},p,q,\varepsilon^I\right) \cdot var\left(r^D_j\right)} \Leftrightarrow x^U_{j,\mathcal{J}}\left(p, q, \varepsilon^U\right) \leq x^I_{j,\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^I\right) \\
\frac{E_{\varepsilon^U}\left(r^U_i - \frac{p_i}{q}\right)}{x^U_{i,\mathcal{J}}\left(p,q,\varepsilon^U\right) \cdot var_{\varepsilon^U}\left(r^U_i\right)} \quad & \geq \quad \frac{E_{\varepsilon^I}\left(r^I_i - \frac{p_i}{q}\right)}{x^I_{i,\mathcal{J}}\left(\omega_{\mathcal{J}},p,q,\varepsilon^I\right) \cdot var_{\varepsilon^I}\left(r^I_i\right)} \Rightarrow x^U_{i,\mathcal{J}}\left(p, q, \varepsilon^U\right) \leq x^I_{i,\mathcal{J}}\left(\omega_{\mathcal{J}}, p, q, \varepsilon^I\right)
\end{aligned}$$

But then,

$$\begin{aligned} E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right) &< E_{\varepsilon^U}\left[x^U_{i,\mathcal{J}}\left(\cdot,\varepsilon^U\right)\cdot\left(\left(\overline{\theta}+c\right)\cdot p^\alpha_i-\frac{p_i}{q}\right)+x^U_{j,\mathcal{J}}\left(\cdot,\varepsilon^U\right)\cdot\left(r^D_j-\frac{p_j}{q}\right)\right] \\ &= E_{\varepsilon^I}\left[x^U_{i,\mathcal{J}}\left(\cdot,\varepsilon^U\right)\cdot\left(\left(\overline{\theta}+c\right)\cdot p^\alpha_i-\frac{p_i}{q}\right)+x^U_{j,\mathcal{J}}\left(\cdot,\varepsilon^U\right)\cdot\left(r^D_j-\frac{p_j}{q}\right)\right] \\ &\le E_{\varepsilon^I}\left[x^I_{i,\mathcal{J}}\left(\cdot,\varepsilon^I\right)\cdot\left(r^I_i-\frac{p_i}{q}\right)+x^I_{j,\mathcal{J}}\left(\cdot,\varepsilon^I\right)\cdot\left(r^D_j-\frac{p_j}{q}\right)\right]=E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right) \end{aligned}$$

a contradiction. It follows that $MRS\left[E_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right),var_{\varepsilon^U}\left(w^U_{\mathcal{J}}\right)\right] < MRS\left[E_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right),var_{\varepsilon^I}\left(w^I_{\mathcal{J}}\right)\right]$. Suppose $p_i \ge p_j$. Then by the no arbitrage conditions

$$\frac{1-\omega_{j,\mathcal{J}}}{1-\omega_{i,\mathcal{J}}}\cdot\frac{\omega_{i,\mathcal{J}}}{\omega_{j,\mathcal{J}}}=\frac{E_{\varepsilon^U}\left(\frac{r^U_i}{p^\alpha_i}-\frac{p^{1-\alpha}_i}{q}\right)}{E_{\varepsilon^I}\left(\frac{r^I_i}{p^\alpha_i}-\frac{p^{1-\alpha}_i}{q}\right)}\cdot\frac{var_{\varepsilon^I}\left(\frac{r^I_i}{p^\alpha_i}\right)}{var_{\varepsilon^U}\left(\frac{r^U_i}{p^\alpha_i}\right)}<\frac{\sigma^2}{var_{\varepsilon^U}\left(\frac{r^U_i}{p^\alpha_i}\right)}$$

Since $E_{\varepsilon^U}\left(\frac{r^U_i}{p^\alpha_i}\right) < E\left(\frac{r^D_j}{p^\alpha_j}\right)$ and $var_{\varepsilon^U}\left(\frac{r^U_i}{p^\alpha_i}\right) > \sigma^2$, it follows that $\omega_{i,\mathcal{J}} < \omega_{j,\mathcal{J}}$, as desired. ■

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