Voting Power and Voting Blocs

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No 716

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

THE UNIVERSITY OF WARWICK

"Voting Power and Voting Blocs"

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July 2004

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Abstract
We investigate the applicability of voting power indices, in particular the Penrose index (aka absolute Banzhaf index), in the analysis of voting blocs by means of a hypothetical voting body. We use the power of individual bloc members to study the implications of the formation of blocs and how voting power varies as bloc size varies. This technique of analysis has many real world applications to legislatures and international bodies. It can be generalised in many ways: the analysis is *a priori* (assuming formal voting and ignoring actual voting behaviour) but can be made empirical with voting data; it examines the consequences of two blocs but can easily be extended to more.

It has long been argued that power indices, as objective measures of voting power, can be used as tools for a precise political theory leading to rigorous analysis. This suggestion was first made by Simon (1957). One of the first to take up the challenge and to investigate the possibility was Riker (1959) who sought to use power indices to make statements about Political Man whose rational behaviour would lead him to maximise power as Economic Man maximises profit or wealth. Many studies subsequently have used power indices empirically, but almost all of them have focused on the rather more limited objective of analysing the relative voting powers of members, or power distribution, within a given decision-making system, and very few have followed Riker's lead. One result of this has been that voting power analysis is often dismissed, by those who might be its users, such as government ministers and public officials, as being only capable of giving results for static situations, and of being useless for generating insights into the consequences of institutional changes, requiring an essentially dynamic mode of analysis.[1]

An important exception is the work of Coleman (1970, 1971, 1973) whose approach to the measurement of power is fundamentally dynamic, in which voting is conceived as being about decisions leading to an action taken by a 'collectivity'. In Coleman's framework a voting body may or may not decide to take an action and the main questions are, first, how likely it is that the collectivity might take such action, and second, how much control social actors could exert over it, given the rules of decision making. In his famous 1971 paper, he proposed power measures within this framework, and subsequently applied them in his 1973 paper.

[1] Another common criticism is that the analysis of relative voting power does not reflect the importance of the decisions to be taken by the given voting body of interest. A voting body is taken as a given and the results are not dependent on whether for example it is a major international organisation or a minor organ of local government.

Our paper follows Coleman (1973) and Riker (1959) in exploring the possibility of using voting power indices for dynamic analysis when the voting body changes by the formation of voting blocs. Our approach is different from that of Riker however in two major respects: we avoid the use of the Shapley-Shubik index (SSI), for which there are compelling grounds[2], and we make no attempt at empirical testing here. We follow the general approach of Coleman (1973), an important and neglected paper, with the difference that our measure of power is the Penrose (1946) index, which, in this particular context, differs only in name from that used by Coleman ('the power to prevent action') but its use by name makes for greater clarity[3].

We begin with a short discussion of Coleman's approach and his critique of the use of game theoretic power indices. This is followed by a discussion of voting blocs, a description of the Penrose index, and then the results of applying this to a hypothetical legislature. Our conclusion is that this framework is applicable and capable of generating useful results in real contexts.

Coleman's Contribution to Voting Power Theory

Coleman's 1971 paper argued against the use of cooperative game theory in general and the SSI in particular. In fact his paper contains a fundamental theoretical critique of that index based, first, on its use of orderings of members to give different weight to coalitions of different sizes and, second, its characterisation of voting as a group of rivals bargaining among

[2] See Leech (2002). Riker did a lot of other work on voting power measurement but his contribution was limited by his reliance on the Shapley-Shubik index.

[3] Actually the more commonly used name for this index in the literature is the absolute Banzhaf index. We prefer to use the term Penrose index (after its original inventor) and reserve the name Banzhaf index for its normalised version as a measure of relative voting power. We make this distinction to emphasise the importance we attach to the non-normalised index as an analytical tool for answering a different set of questions than computing power shares.

themselves over a fixed payoff in a game. There does not seem ever to have been a proper reply to Coleman's arguments but the SSI continues to be taken seriously and used by some scholars.

Coleman's approach was based on the dynamic idea of collective decisions being taken which would lead to action and not the static idea of decisions being taken about how to divide up a given fixed quantity such as the spoils of office. This allowed the relaxation of some of the analytical constraints that came from game theory, such as the requirement that the power indices of the different players should add up to a constant (an idea often referred to as the efficiency axiom) and the restriction that the quota has to be at least half the total number of votes (the restriction to 'proper games'). This meant that voting power, in Coleman's sense, was conceived in absolute not relative terms. It shifted the focus of the analysis from the powers of the members in relation to each other to the relationship between the powers of individual members and that of the collectivity, which relationship is where much of the real concern lies in discussing institutions. Coleman made a distinction between the negative power to (in his terminology) prevent action and the positive power to initiate action, which again is a key distinction of much practical value in many cases where decisions are taken by supermajorities. It disappears when the voting rule is a simple majority, and the power to prevent and to initiate action are identical to each other, and also to the Penrose index and the absolute Banzhaf index. Mathematically, within this framework, a power index is a probability and it is inappropriate to normalise it. In this sense there is a fundamental difference between what we refer to as the Banzhaf index (that is, the normalised Banzhaf index) and the Penrose index. Coleman's perspective is useful for considering how power changes as a result of members participating in coalitions, for which game theory is ill suited.

We do not wish to argue here against the use of cooperative game theory in general as a model of voting. Only that the results it leads to are of limited empirical interest. We do argue strongly against the SSI, which is by no means the only game-theoretic power index, on grounds of the lack of realism of its assumptions and its failure to produce results that are acceptable from an empirical perspective. (Coleman, 1971, Leech, 2002)

Voting Power and Voting Blocs

When a social actor, whether an individual or a group, relinquishes independent political power by joining a group, or a larger group, and agrees to be bound by its decisions, his, or its, power will either increase or decrease. For example, a country which, as a member of a global organisation, gives up its independence in certain matters within the organisation, in order to join a powerful bloc within it, may gain or lose power. The bloc will be more powerful than the country could be by itself because of its greater size, but the country has only limited power over decisions taken by the bloc's members about how it should vote in the global organisation. The country's power, as a member of the bloc, is a compound of these two factors. Another example is a parliament containing one or more party groups whose representatives agree to a strict whipping discipline combined with majority voting within the group.

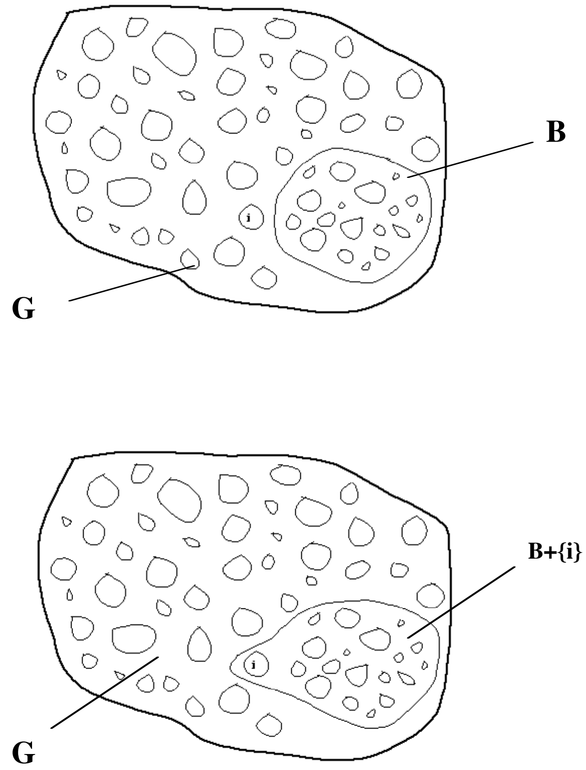
This paper is a theoretical investigation that uses power indices to find the trade-offs involved when blocs are formed in a legislature. We assume a simple model of a legislature and use the Penrose power index to measure formal voting power when there are blocs of members who act as one in accordance with a prior agreement such as a party whip.

Power Indices and Voting Blocs within a Global Voting Body: Formal Definitions and Notation

We begin by stating our assumptions and defining our notation. We assume a legislature with a large number of members; where notation is needed this number is denoted by n. Decisions are made by simple majority; all members vote, there are no abstentions. Every member has the independent right to vote 'aye' or 'no' in any ballot or roll-call. We model the formal power of an actor, whether an individual or a bloc, who is a member of this body, as a probability.

The global legislature, denoted by *G, H,* etc, is assumed to consist of one or more blocs, denoted (for example) *B, C, W, W1, W2*, etc, and individuals, *i, j,* etc. An individual is treated formally as a bloc consisting of a single member. It is sometimes convenient to denote the global body, using set notation, in terms of its membership, as for example, $G = \{B, C, D, \ldots, \{i\},\{j\}, \ldots\}$. The scenario is shown schematically in Figure 1.

Figure 1: Schematic Representation of a Voting Bloc



The power of an actor (whether an individual or bloc) internally within the body in which it votes is the probability that it swings the vote, a power index. The power of actor *a* in voting body *V* can be written formally as P_a^V,

$$P_a^V = \Pr[\text{Actor } a \text{ swings the vote in } V] \qquad (1)$$

Expression (1) is the probability that the combined votes of the other members of V are just short of a majority such that adding the vote(s) of a to them will produce a majority. This obviously depends on the particular data for the voting body consisting of the sizes of all the blocs, their number, the number of votes cast by actor a, the decision rule in terms of what sort of majority is required (in this paper it is always a simple majority) and the model of probabilistic voting. Thus, the power of an individual i within a bloc B is then written P_i^B while the power of the bloc within the global body G is denoted by P_B^G.

The power indices for all the actors are found using the general definition in equation (1) applied to the voting model assumed and the data. We can think of the model of probabilistic voting as either a description of actual behaviour, taking into account relationships between members and party blocs, or a stylised model in which all actors vote for or against an action with equal probability and independently. The power indices from the former approach would measure power empirically, while the latter would be an *a priori* power index measuring power deriving from the bloc structure and the voting rules in a purely constitutional sense. The former requires data on actual voting behaviour; Coleman showed how an estimate of the variance of the size of the 'aye' vote could be used for this purpose. The latter, which is followed here, requires only a stylised model of probabilistic voting to compute the *a priori* power indices.[4]

In this study the power indices are found in two general ways.

(i) If a voting body consists only of individuals, and does not contain any blocs, the power index for a member is a binomial probability. Thus, the power of an actor who is an individual

[4] A recent application where *a priori* power indices are appropriate for the study of the fairness of voting rules, is Leech and Leech (2004b)

member within bloc *B*, which has *m* members is simply the binomial probability that the number of other members who vote ‘aye’ is exactly one vote less than the number required for a decision. That is *m*/*2*, or *(m-1)*/*2*, depending on whether m is even or odd.

(ii) To find the power of an actor which is a particular bloc within a legislature which also contains other blocs, that are in general of different sizes, is more difficult computationally, and requires the use of a computer program that implements an appropriate power indices algorithm. In this study we use the algorithm known as the method of generating functions to compute the power indices for bodies that have blocs. (Brams and Affuso, 1976; Leech and Leech, 2004a).

Each of these calculations gives us the absolute voting power of a certain actor within a given voting body. Our main interest however is in the power of individuals in relation to voting blocs, for which we need further notation. It is unnecessary for this purpose now to label the individual so we can drop the actor subscript from the power index. It is however necessary to label the bloc structure. Thus we denote the power of an individual acting as a member of bloc *B* in global body *G* as *P(B,G),* and the power of an individual acting independently in the same body as *P({i},G)*. In this notation, when we consider variation in the first argument of *P(B,G), B*, with *G* held constant, it is understood that the other blocs do not change. Changes in the size of the bloc *B* occur by way of changes in the number of individuals who do not belong to the other blocs, all of which are assumed constant.

Thus we can write the voting power of an individual member of bloc *B*:[5]

$$P(B,G) = P_i^B P_B^G. \qquad (2)$$

[5] It is sometimes appropriate to refer to this as the indirect voting power to emphasise that the member is working through the group.

The power of a member of bloc *B* is the product of his or her power over decisions of the bloc times the power of the bloc over the decisions of the global legislature. This can be compared with $P(\{i\},G) = P_{\{i\}}^{G}$, the power of an independent member, in order to determine if there is a net power gain or loss when *i* joins *B*[6].

Riker's Study of the French Assembly

Riker (1959) was an attempt to test the SSI as a measure of absolute voting power by looking at migrations between party blocs in an actual legislature, the French Assembly, over two years, 1953 and 1954. He computed the indices for all party blocs before and after every migration and sought evidence that these could have been motivated by the deputies concerned seeking to increase their a priori voting power in the Assembly. His findings were negative.

However, the study was deficient in several respects and its findings should not be taken as serious evidence against power indices, but rather as inconclusive. First, it was methodologically flawed in its use of the SSI which measures only the relative voting powers of parties. The measure he used for the voting power of an individual member was the SSI per head, computed as the index for the bloc divided by the number of bloc members. That is, Riker assumed the index could be composed in the same way as the Penrose indices in (2), and multiplied together the SSI of the bloc in the assembly and the internal SSI of the member within the bloc (which is just $1/m$). But this is quite unjustified, as Owen (1995) shows. Owen discusses the derivation of power indices for a composed game at length, including a proof of the validity of equation (2). He also describes an approximation method of computation for the

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[6] This comparison assumes that when individual i joins the bloc the characteristics of the global voting body do not change. This is strictly false but has been ignored for ease of exposition. Write $G = \{B, C, D, E, \dots\}$. Then the relevant comparison should be between $P(B,G)$ and $P(\{i\},H)$ where $H = \{B\text{-}\{i\}, \{i\}, C,D,E, \dots\}$ because a member of B is better off after leaving the bloc if his/her power increases as a result. But the size of bloc B falls and it is necessary to allow for that, as well as the greater number of individuals.

properly defined SSI for the composed game and applies it to the US presidential election game. The indices he obtains in this way are very different from the SSI's per head derived simply from the results for the states game with the same data, and illustrate how inaccurate Riker's approach is. [7]

A second major criticism is that his data set was not good for empirically testing the adequacy of the power index since very few of the migrations he observed involved members of the large and powerful party blocs, and the period he took was very short. This suggests a need for empirical testing of power indices using better data.

Power Index Calculations for a Hypothetical Legislature

We report the calculations for the power indices for a legislature assuming one and then two blocs. The one-bloc case is described first in order to demonstrate the power of blocs and to show the trade-off faced by individuals, described above, and also the optimum bloc size. Then we generalise it and show that the two-bloc situation gives rise to a rich variety of cases including monopolar and bipolar power structures. We then discuss the incentives that individual members have to migrate that the differences in voting power create.

Power with One Bloc. We assume there is one bloc, labelled W, whose number of members is w. Then we can write, for the global legislature, $G=\{W, \{i\}, \{j\}, \dots\}$, the indirect power of a bloc member:

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[7] Owen (1997), chapter XII. The fact that the SSI does not compose in the simple way assumed by Riker is not in itself a sufficient argument against using it to measure power in voting blocs, since, following Owen, an appropriate version of it can be defined, and can also be calculated with the right algorithm, either the approximate method based on multilinear extensions and probabilistic voting described in his book or a more exact method. However we consider the theoretical arguments and empirical evidence against its use described above as decisive.

$$P(W,G) = P_i^W . P_W^G. \tag{3}$$

The two components of (3) are evaluated separately. The value of P_i^W is found analytically as a binomial probability. This depends on the parity of *w*, and we must use different formulae for odd and even bloc sizes:

$$P_i^W = \binom{w-1}{\frac{w-1}{2}} 0.5^{w-1}, \text{ if w is odd; } \binom{w-1}{\frac{w}{2}} 0.5^{w-1} \text{ if w is even.} \tag{4}$$

The value of P_W^G can also be found analytically in this case, but it is better, as a general strategy for these calculations, where we wish to allow for a general bloc structure, to evaluate it numerically.

If *w* is large enough, then (4) can be replaced by the approximation[8],

$$P_i^W = \sqrt{\frac{2}{\pi w}} = \frac{0.79788}{\sqrt{w}} \tag{5}$$

Expression (5) is Penrose's square root rule which states that the power of a member of a large voting body is approximately inversely proportional to the square root of size of the body. (Penrose 1946, 1952). In this paper, since our interest is in voting blocs of all sizes, including very small ones, we will use (4) only. However (5) is useful when the voting blocs contain many members, for example, where they are constituencies with thousands of electors or countries with millions.

Power with Two Blocs. When there are two blocs, labelled *W1* and *W2*, with w_1 and w_2 members, the global legislature can be written, *G={W1, W2, {i}, {j}, . . . }*. The power indices we are interested in are written:

[8] This approximation is based on Stirling's formula. See Feller (1950, p180). See also Penrose (1946), Coleman (1973).

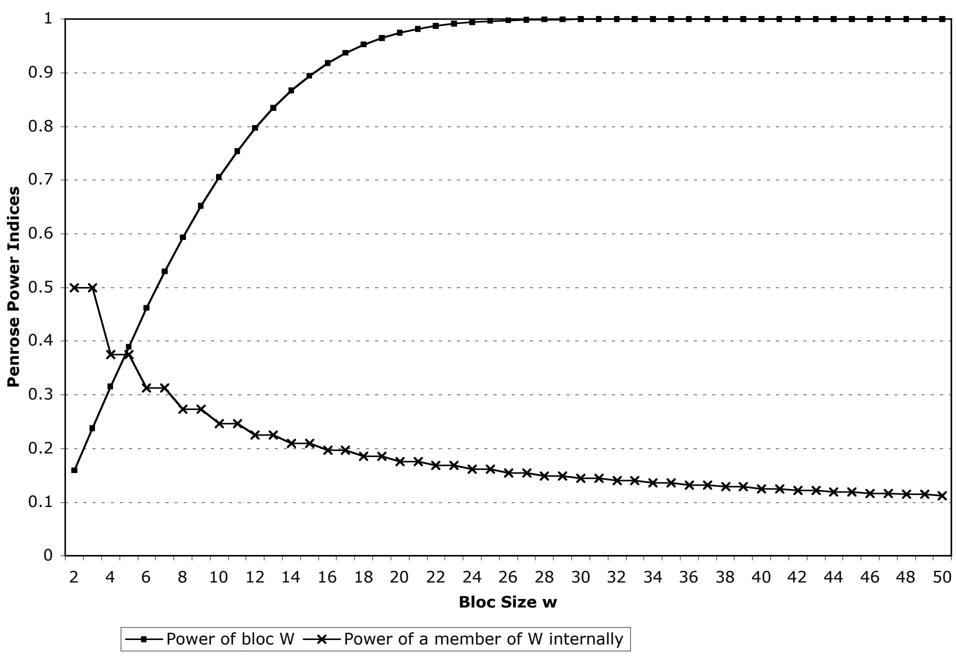
$$P(W1, G) = P_i^{W1}.P_{W1}^{G} \ , \quad P(W2, G) = P_i^{W2}.P_{W2}^{G} \ , \quad P(\{i\}, G)= P_{\{i\}}^{G}.$$

We find P_i^{W1} and P_i^{W} as binomial probabilities, and P_{W1}^{G} , P_{W2}^{G} and $P_{\{i\}}^{G}$ numerically as before.

Voting Power and Voting Blocks: An Example

Here we report the results for a legislature with 100 members. The assumptions throughout are that the legislature uses a simple majority of 51 votes and that each bloc uses a simple majority rule internally.

Figure 2: Bloc Power and Internal Power of a Bloc Member



We first consider the one-bloc case. Figure 2 illustrates the tradeoff between the power indices for the bloc as a whole, P_W^{G} , and of a bloc member within it, P_i^{W}, as the bloc size, w, increases, for all values of w from 2 to 50. As the size of the bloc increases its power increases, eventually approaching 1 when it has an absolute majority, $w=51$. Its power index gets very close to 1 long before it has an absolute majority, however, illustrating how very powerful even minority blocs can be. On the other hand, the power of one of its members to control the bloc in an internal vote falls continuously, in the limit when $w=100$, to about 0.08. Figure 3 shows the trade-off between these two power indices, indicating that there is an optimum bloc size in terms of indirect voting power $P(W,G)$.

Figure 3: Tradeoff between Bloc Power and Bloc Member's Internal Power

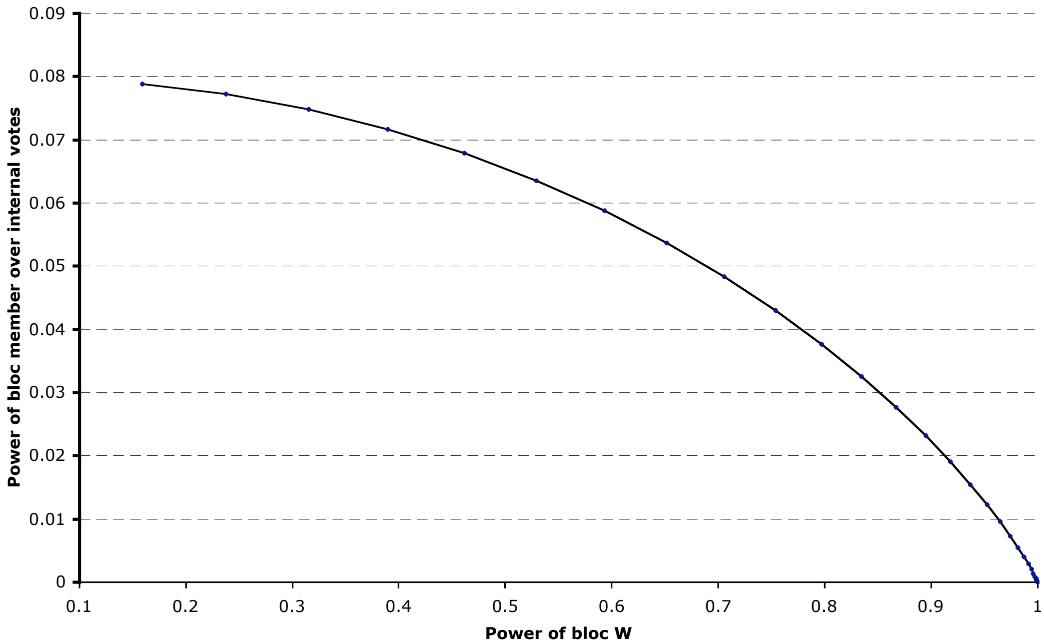


Figure 4 shows this relationship between the indirect power of a bloc member, $P(W,G)$, defined in equation (3), and bloc size. A bloc with more than 50 members has an absolute majority, and therefore the essential trade-off disappears: when $w>50$, $P_W^G=1$, and the power index for a bloc member, P_i^W , is just a declining function of w, and his or her voting power is diluted as the bloc membership grows. The saw-tooth appearance of the diagram shows the sensitivity of the power analysis to the parity of the bloc size, especially in small blocs. This comes about because,, for example, a member of a bloc with 10 members has the same internal voting power within the bloc as he or she would have if the bloc had 11 members, and both have the same value of P_i^W.[9] However the bloc with 11 members has more power in the legislature and a greater value of P_W^G.

Figure 4: Power of a Bloc Member and a Non-Member

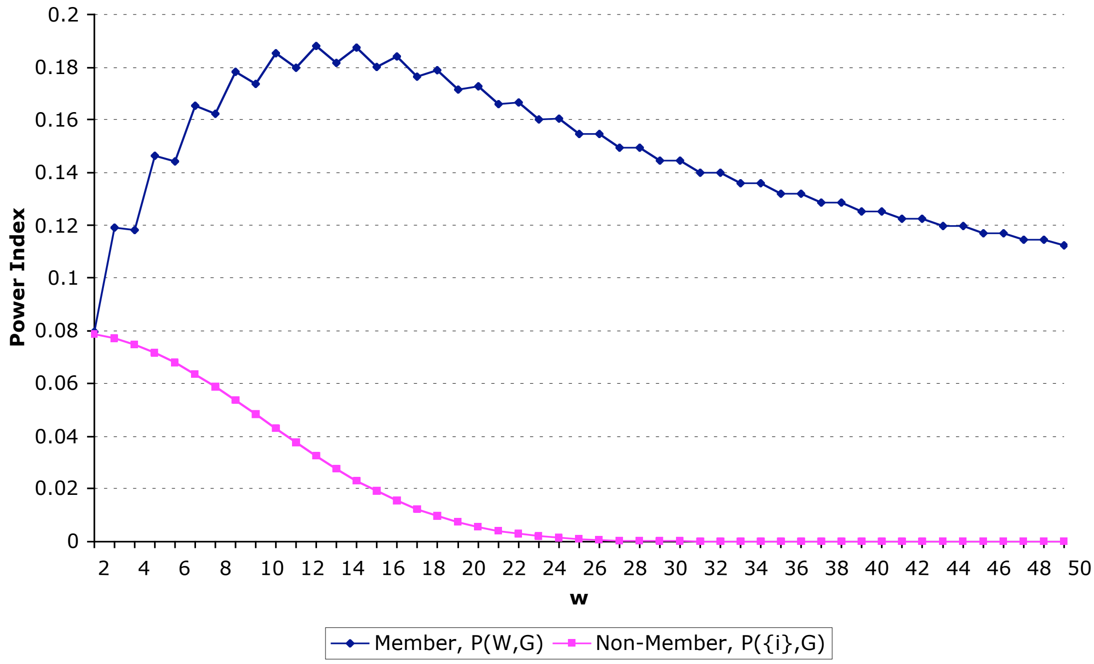


Figure 4 also shows how the bloc size affects the power of an independent member, *P({i},G)*. As the bloc grows in size and the number of independent members declines, it becomes rapidly more powerful. At the same time the power of each independent member falls rapidly and continuously, becoming virtually zero once the bloc has more than about 20 members, $w>20$. However the power of a member of the bloc does not grow continuously. It grows to a point and then declines. The size of voting bloc that makes the influence of one of its members a maximum is where $w=13$. Up to this point the bloc is powerful in the legislature but because the number of members is small, each individual member is influential internally;

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[9] If w is an even number, then the internal powers of a member of bloc W and of another bloc bigger by one member, say $W+\{j\}$, can easily be shown to be equal, that

$$P_i^W = \binom{w-1}{\frac{w}{2}} 0.5^{w-1} = P_i^{W+\{j\}} = \binom{w}{\frac{w}{2}} 0.5^{w} .$$

beyond that point the bloc power increases less rapidly while the addition of new members dilutes the internal power of individual members. On the other hand, however much this dilution proceeds, the power of a bloc member still far exceeds that of a non-member.

Figure 5: Power of a Bloc Member, Two Blocs

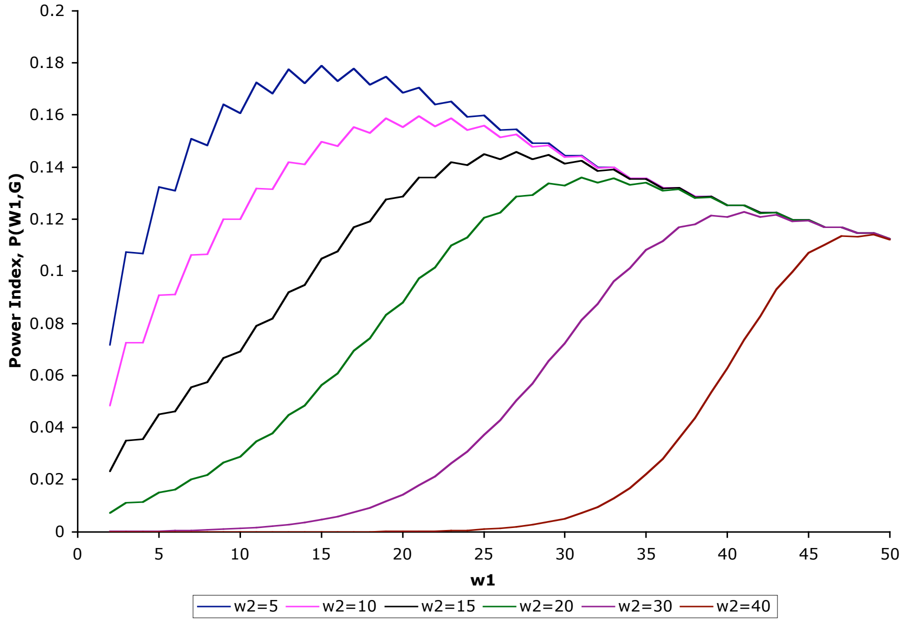


Figure 5 extends this analysis to the case where there are two blocs, *W1* and *W2,* and *W2* is of a fixed size. The chart shows the power of a member of the bloc *W1* as w_1 varies, for different values of w_2. The power of a member of bloc *W1* is lower the greater the size of *W2*. Table 1 shows the relation between the optimum value of w_1 and w_2.

Table 1: Optimum w_1		
w_2	Optimum w_1	Power of member of *W1*
0	13	0.1883
5	17	0.1789
10	21	0.1596
15	27	0.1459
20	31	0.1359
30	41	0.1227
40	49	0.1142

Figure 6 shows the powers of members of *W1, W2* and non-members, *i*, in terms of the size of the bloc *W1* for the four cases: $w_2 =$ 10, 20, 30, 40. It is noticeable how in all four diagrams a major effect is that the two large blocs reduce each other's power substantially when they are of comparable size while one of them is very dominant when their sizes differ. In some cases this is to the advantage of individuals who are not bloc members who become more powerful than bloc members.

Figure 6(a) is the case where w_2=10. When w_1 is small *P({i}, G)* is equal to *P{W1,G)* and the bloc is too small to matter. As *W1* increases in size and becomes more powerful, *W2* loses power, as does, also, after a while, the independent member i. The optimum size of *W1* is 21 when its members' power is at its maximum.

Figure 6(a): Two Blocs, w2=10, n=100

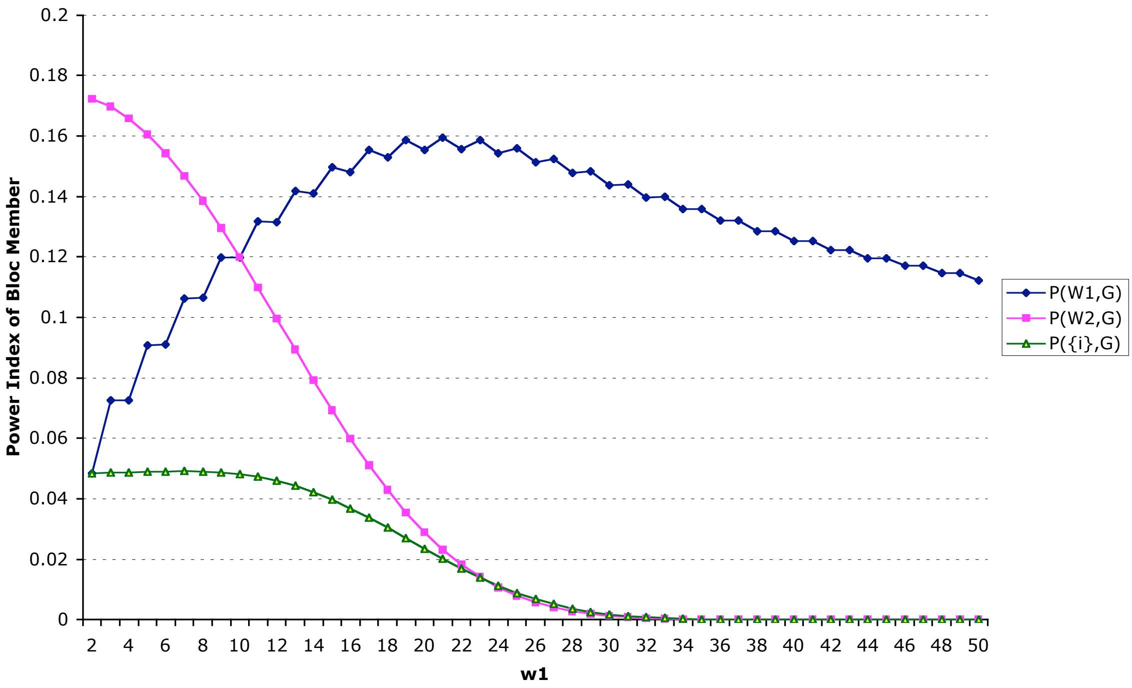


Figure 6(b): Two Blocs, w2=20, n=100

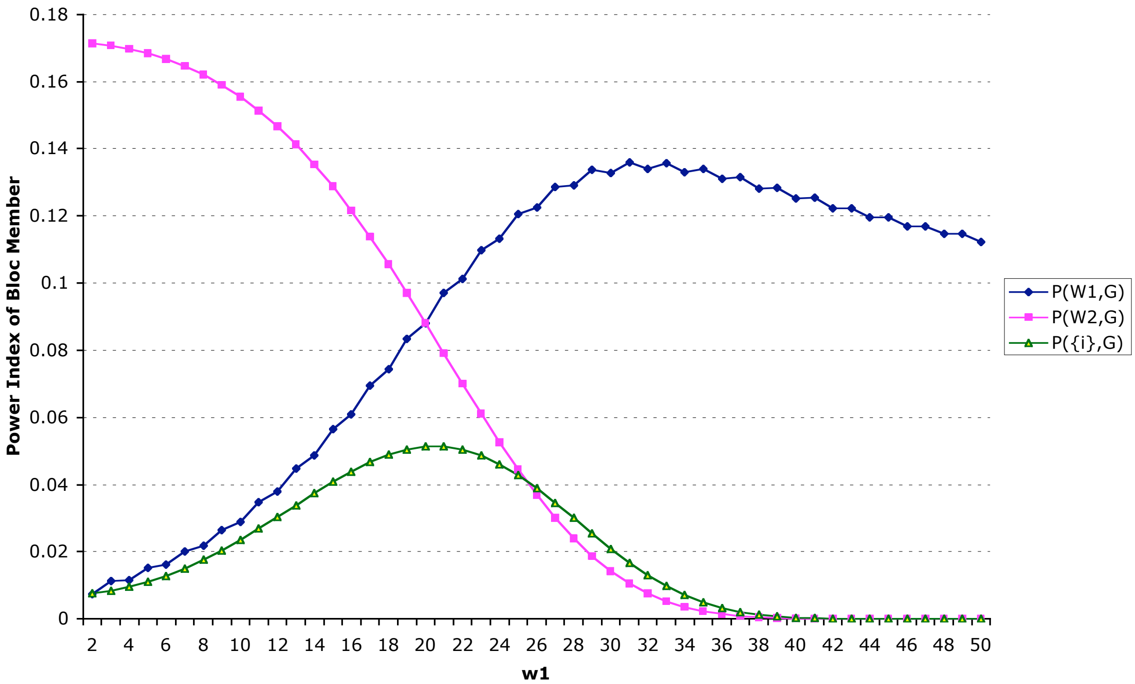


Figure 6(c): Two Blocs, w2=30, n=100

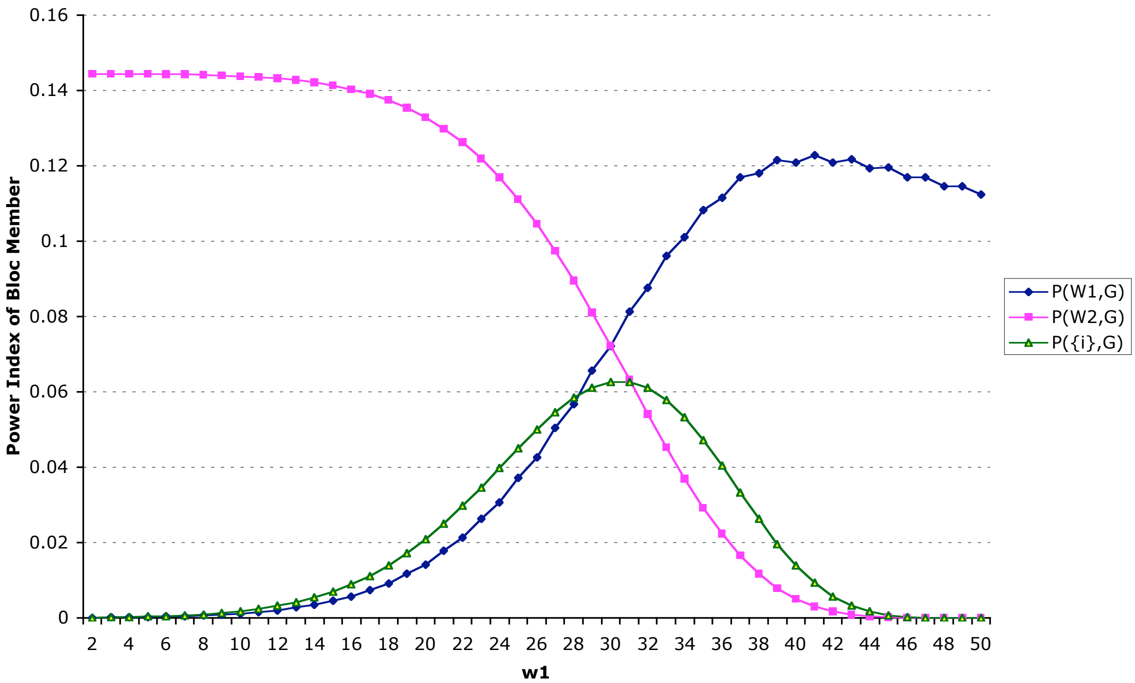


Figure 6(d): Two Blocs, w2=40, n=100

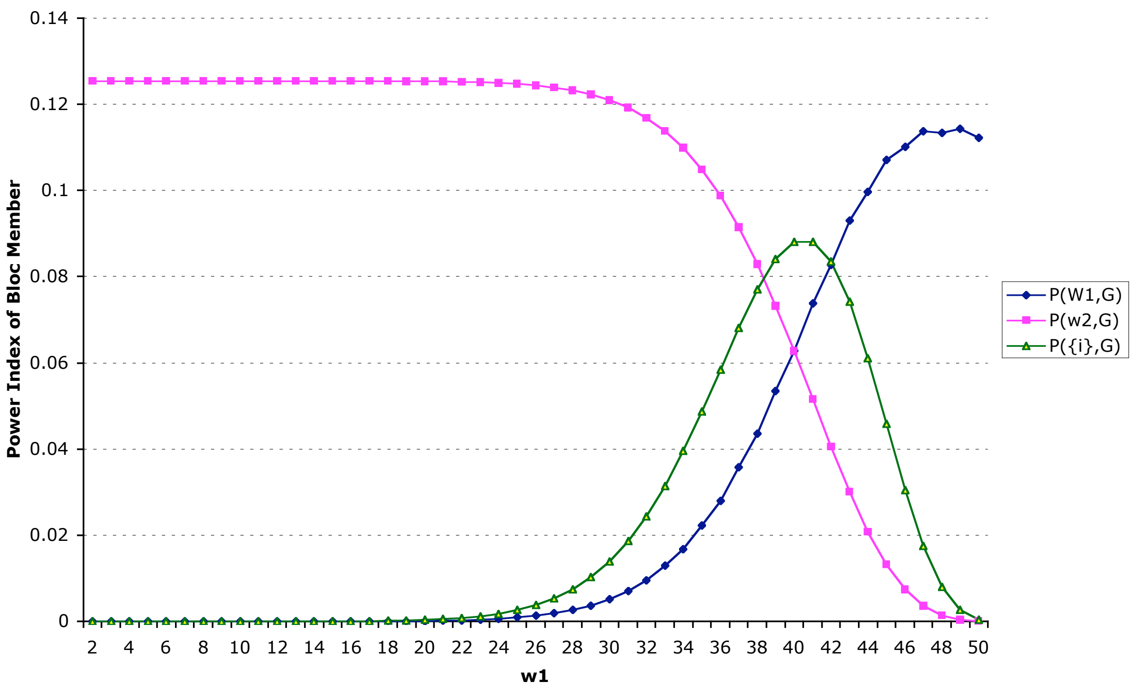


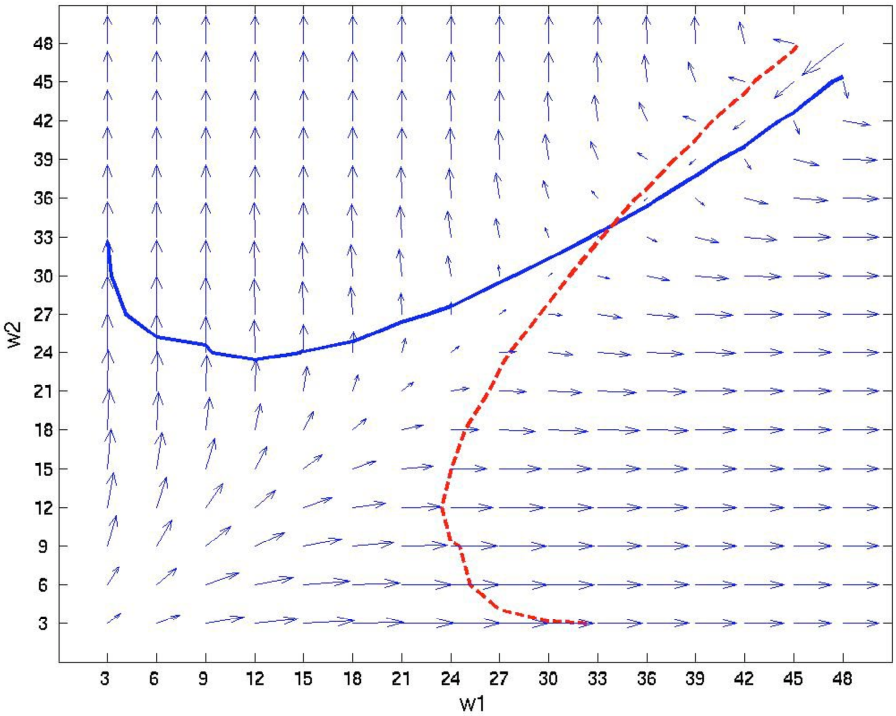
Figure 6(b) shows the case where *W2*=20. Now it is advantageous to belong to either *W2* or *W1* until w_1=26 when members of bloc *W2* have less power than independent members. For values of w_1>26 there is an incentive to join *W1* and to leave *W2*.

Figure 6(c) shows the situation when w_2=30. Now *W2* is very powerful when w_1 is small, and members of *W1* have less power than non-members of any bloc until w_1=28. In this range, there are strong incentives to join *W2* and weak incentives for members of *W1* to leave and become independent. Between w_1=28 and w_1=32 there is an intermediate range where the power of the independent member is at its peak but, because both blocs are powerful, his or her power is still below that of a bloc member. Above w_1=32 a non-member has greater power than a member of bloc *W2* (even though that bloc controls 30 percent of the votes), such is the power of *W1*. In this bipolar situation, the power of *W1*, even though it is the dominant bloc, is much less than that of *W2* was when *W1* was small.

Figure 6(d) shows the case where *W2* is just short of an overall majority, w_2=40. Now, when *W1* becomes big enough to rival *W2*, the powers of these 2 blocs are low enough for an independent member to be more powerful than any member of either bloc. This is a truly bipolar situation in which two blocs oppose one another and limit one another's power while at the same time they are each so large that their members' power over internal decisions is dissipated.

Figure 7 shows the incentives facing individual members to migrate between blocs when there are two blocs. The diagram shows the range of values of w_1 and w_2, where neither bloc has an absolute majority. The incentives to migrate are measured by the differences in power indices for an individual who is a bloc member and a non-member. The diagram is constructed from the vector $[P(W1,G) - P(\{i\},G), P(W2,G) - P(\{i\},G)]$ for every pair of values of (w_1, w_2). The arrows indicate the direction and strength of the resultant as an indication of the strength of the incentive to migrate and the consequent direction of change of the bloc sizes. The lines are the zero contours where there is no incentive that would lead one of the blocs to change: the power of a non-member of a bloc is equal to that of a bloc member.

Figure 7: Incentives to Migrate between Two Blocs



In Figure 7, along the 45-degree line, when both blocs are equal, there is an incentive for them both to change unless $w_1(=w_2)=33$. Below this value, the incentive is for both blocs to grow, above it to shrink. The set of points where $w_1= w_2$ has a knife-edge property, since when $w_1 \neq w_2$, the incentive is for the larger bloc to grow and the smaller one to decline. The point $w_1=w_2=33$ has a saddle point property where it is stable in one dimension and unstable in another.

Conclusion

This paper has considered the use of Penrose power indices to study the power of actors in a voting body with blocs. We have looked at the simple case of a legislature with 100 members where there are one or two blocs, or party groups, in which the whip is applied on the basis of simple majority voting among its members.

We have shown that the power of an individual bloc member can be modelled in terms of two contrasting components: the power of the bloc within the legislature deriving from the internal discipline that creates the power of combined forces, that increases with bloc size; and the power of the individual member within the bloc, which declines with bloc size. This tradeoff leads to useful insights for voting situations with more than one voting body or multiple layers of decision making.

The model and the general approach described here can be generalised in many ways. First, the analysis here is entirely *a priori* in the sense that no account is taken of preferences or actual voting behaviour. This analysis is especially useful for an understanding of the power implications of voting rules when considered formally. However, the approach is more general since the Penrose index can be adapted to allow for actual or empirically observed voting behaviour if the appropriate data on voting patterns is available. Second, we have considered a stylised legislature with only two blocs. This can be generalised trivially to take account of many voting blocs, as for example parties in a real legislature or where weighted voting is used, such as intergovernmental international organisations.

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