Social Conformity in Games with Many Players

Myrna Wooders

Edward Cartwright

And

Reinhard Selten

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Myrna Wooders†
Department of Economics
University of Warwick
Coventry CV4 7AL, UK
M.Wooders@warwick.ac.uk
http://www.warwick.ac.uk/fac/soc/Economics/wooders/

Edward Cartwright
Department of Economics
University of Warwick
Coventry CV4 7AL, UK
E.J.Cartwright@warwick.ac.uk

Reinhard Selten
Department of Economics
University of Bonn
Adenauerallee 24-26, 53113 Bonn
Germany

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Abstract: In the literature of psychology and economics it is frequently observed that individuals tend to imitate similar individuals. A fundamental question is whether the outcome of such imitation can be consistent with self-interested behavior. We propose that this consistency requires the existence of a Nash equilibrium that induces a partition of the player set into relatively few groups of similar individuals playing the same or similar strategies. In this paper we define and characterize a family of games admitting existence of approximate Nash equilibria in pure strategies that induce partitions of the player sets with the desired properties. We also introduce the Conley-Wooders concept of 'crowding types' into our description of players and distinguish between the crowding type of a player - those characteristics of a player that have direct effects on others– and his tastes, taken to directly affect only that player. With the assumptions of 'within crowding type anonymity' and a 'convexity of taste-types' assumption we show that the number of groups can be uniformly bounded.

1 Social learning

Individuals belonging to the same society typically have commonalities of language, social and behavioral norms, and customs. Social learning consists, at least in part, in learning the norms and behavior patterns of the society into which one is born and in those other groups that one may join – professional associations, faculty clubs, and communities, for example. Individuals may learn by observing and imitating individuals in the same groups. A fundamental question is whether the outcome of such imitation can be consistent with self-interested behavior. From the perspective of game theory, we propose that this consistency requires the existence of a Nash equilibrium or an approximate equilibrium where individuals within the same society play the same or similar strategies and where most or all societies are nontrivial in size; the Nash equilibrium captures a notion of self-interested behavior while the existence of societies that are nontrivial in size facilitates imitation within societies. In this paper we provide a family of games where an equilibrium with the desired properties exists.

In an economic context there are many reasons why an individual may be influenced by the actions of others, leading to imitation or conformity. For example, an individual who is boundedly rational or has imperfect information may choose to imitate someone he believes is better informed than himself (Gale and Rosenthal 1999, Schleifer 2000). Similarly, in a coordination game with multiple equilibria an individual may benefit from observing and conforming to the actions of others (Ellison and Fudenberg 1995, Young 2001). Finally, a player may be motivated by desires for prestige, popularity or acceptance and so conforms to the actions of others to 'fit in' (Bernheim 1994). An individual will typically, however, only be influenced by a certain groups of individuals with whom he identifies (Gross 1996). He may not, for example, imitate people he perceives as less informed than himself or people whose acceptance he does not desire.

To address the question of whether conformity is individually rational we introduce a structure generating games with the property that, in games with many players, for most players, there are many similar players. We take as given a metric space Ω of player attributes and a finite set S of pure strategies. An attribute $\omega \in \Omega$ is interpreted as specifying the characteristics of a player and the metric on attribute space Ω allows measurement of similarity of players. A universal payoff function h is also taken as given. These three elements, Ω, S, and h, are called a (noncooperative) pregame. Given a finite set of players and an attribute function, ascribing a point in attribute space to each player, a pregame induces a game, according to standard definitions, on the player set. The definition of h ensures that any induced game has a certain anonymity property – only the attributes, and not the names, of other players are relevant. A pregame allows us to model a family of games all induced from a common strategic structure.

The pregame framework accommodates diverse situations. As a simple example, consider an economy where each agent offers 0 or 1 units of labour towards the provision of a public good. The amount of the public good produced depends on the average offer. The payoff function of an individual is increasing in the amount of public good provided but decreasing in the amount of labour he offers. Suppose payoff functions are drawn from some compact set Ω. A universal payoff function h, with domain in terms of strategy vectors, can be constructed to reflects these preferences. The construction of h reflects the anonymity property implicit in the description of the economy; the payoff to an individual player depends only on his contribution and the average contribution of the other players in the economy. The set Ω, the function h, and the set of strategies $\{0, 1\}$ determine a pregame. Note that since a total player set has not been specified, a pregame is not a game. But given a finite population of players, described by the the preferences of the individuals in the population, a game satisfying the standard definition is induced on that population by the pregame. An infinite number of games can be induced by this one pregame – two player games, three player games, ... and the players in the games may have different preferences (albeit all drawn from Ω). Many such examples can be generated; three more appear in the body of the paper.

Given a pregame, an induced game and a strategy profile for the game, we define a collection of players A as a society if every player belonging to A has attributes in some convex subset of attribute space Ω_A and every player with attributes in in the interior of Ω_A plays the same strategy.[1] Further, since it is difficult to motivate the use of mixed strategies if players imitate or conform, we require that all players belonging to A to play the same pure strategy.

To obtain our results, we make two assumptions on a pregame. The first ensures that players whose attributes are close in the metric on attribute space are indeed similar as players in induced games and the second ensures that the strategy choices of individual players have near-negligible impacts on other players. Our main result can be summarized:

Conformity: Given any $\varepsilon > 0$ there are integers $\eta(\varepsilon)$ and $L(\varepsilon)$ such that any game with at least $\eta(\varepsilon)$ players has a Nash ε-equilibrium in pure strategies that induces a partition of the population into at most $L(\varepsilon)$ societies.

Note that the bound on the number of societies L is independent of the size of the population. Thus, if there are 'many' players then most societies must be large. Moreover, the smaller the number of societies, the greater the possible difference between players in the same society and the stronger the conformity. Generally, however, the bound L is not independent of ε. With an additional assumption, we also demonstrate:

Uniform boundedness of the number of societies: With 'convexity in taste types,' the bound on the number of societies L is independent of ε.

To the best of our knowledge, our conformity results have no analogues in the extant literature. Bernheim (1994) provides conditions for the existence of a Nash equilibrium consistent with conformity. As we discuss in Section 3.2, however, Bernheim assumes players have explicit desires to 'fit in' with social norms; we treat games where this may or may not be the case. Conformity within subgroups (clubs or jurisdictions) of the population has been an issues in the literature of local public goods and clubs; see, for example,

[1] Our definition of a society is, of course, highly stylized. In part, this is due to the complete information aspect of our model. Nevertheless, the definition captures the idea that individuals within a society are similar and conform to the same norms or standards of behavior. In the framework of a standard noncooperative game, conformity appears in the choice of strategy.

Wooders (1978), Greenberg and Weber (1986), Conley and Wooders (1996) and Demange (1994), where equilibrium/core jurisdictions consist of similar individuals. The approach of these papers however is based on cooperative and/or price-(or tax) taking behavior. A further related literature concerns dynamic models of social learning; see. for example, Kandori, Mailath and Rob (1993), Ellison and Fudenberg (1995), and Young (2001). The motivation for these dynamic models is analogous to ours in questioning the effectiveness of social learning. Typically, however, in these dynamic models there trivially exists a Nash equilibrium that is consistent with the assumed social learning dynamic; the motivating issues are whether play converges to a Nash equilibrium and, if so, which one.

We proceed as follows: Section 2 introduces notation and definitions. In Section 3 we treat conformity beginning with some simple examples before providing our two main results and a discussion on normative influence. In Section 4 we conclude and an Appendix contains remaining proofs.

2 Notation and definitions

A *game* Γ is given by a triple $(N, S, \{u_i\}_{i\in N})$ consisting of a finite player set N, a finite set of *pure strategies* S, and a set of payoff functions $\{u_i\}_{i\in N}$. A *pure strategy vector* for game Γ is given by $m = \left(m_1, ..., m_{|N|}\right)$ where $m_i \in S$ denotes the *pure strategy of player* i. The set of pure strategy vectors is given by S^N. We note that for each $i \in N$ the payoff function u_i maps S^N into the real line.

Let Ω be a metric space, called an *attribute space*, let S be a finite set of strategies, and let W be the set of all mappings from $\Omega \times S$ into $\mathbb{R}_+$ with finite support.[2] A member of W is called a *weight function.* A *non-cooperative pregame* is a triple $\mathcal{G} = (\Omega, S, h)$ consisting of an attribute space Ω, a set of pure strategies S and a function $h : \Omega \times S \times W \longrightarrow \mathbb{R}_+$. As we formalize below, the function h determines a payoff function for each player in any game induced by a pregame; the payoff to a player depends on the attributes of that player, his strategy choice, and the weight function induced by the strategy choices of the other players.

Take as given a pregame $\mathcal{G} = (\Omega, S, h)$. Let $N = \{1, ..., |N|\}$ be a finite set and let α be a mapping from N to Ω, called an *attribute function.* The pair (N, α) is a *population*, a set of players and their attributes. A pure strategy vector for the population (N, α) is given by a vector $m = \left(m_1, ..., m_{|N|}\right)$ where $m_i \in S$ ascribes a pure strategy to $i \in N$.

[2] Where $\mathbb{R}_+$ denotes the non-negative real numbers.

Given a population (N, α) and a pure strategy vector $m \in S^N$ we say that weight function $w_{\alpha,m} \in W$ is *relative to* m if,

$$w_{\alpha,m}(\omega, s_k) = |\{i \in N : \alpha(i) = \omega \text{ and } m_i = s_k\}|$$

for all $s_k \in S$ and all $\omega \in \Omega$. Thus, $w_{\alpha,m}(\omega, s_k)$ denotes the number of players with attribute ω who play strategy s_k. An *induced game* $\Gamma(N, \alpha)$ can now be defined:

$$\Gamma(N, \alpha) = \left(N, S, \{u_i^\alpha : S^N \longrightarrow \mathbb{R}_+\}_{i \in N}\right)$$

where

$$u_i^\alpha(m) \stackrel{\text{def}}{=} h(\omega, m_i, w_{\alpha,m})$$

for all $\omega \in \alpha(N)$ and $m \in \Sigma_\alpha$. We note that players who are ascribed the same attribute have the same payoff function.

Other than finiteness of the strategy set, a pregame need not imply any assumptions on the games induced. A pregame, however, provides a useful framework in which (a) to treat a family of games all induced from a common strategic situation as given by the attribute space Ω and pure strategy set S, and (b) to be able, relatively simply, to impose assumptions on that family of games through the function h. We demonstrate this later point in Section 3.

We will assume throughout that players play pure strategies. We invoke, however, the standard von Neumann Morgenstern assumptions with regard to expected utility of (mixed) strategies. The standard definition of a Nash equilibrium applies. Given $\varepsilon \geq 0$, a strategy vector m is a *Nash ε-equilibrium in pure strategies* or, informally, *an approximate Nash equilibrium in pure strategies*, only if,

$$u_i^\alpha(m_i, m_{-i}) \geq u_i^\alpha(s_k, m_{-i}) - \varepsilon \tag{1}$$

for all $i \in N$ and $s_k \in S$.

2.1 Societies

Throughout we assume, for convenience, a particular form of attribute space. Let $\mathcal{C} = \{1, 2, ..., C\}$ be a finite set of *crowding types*.[3] We assume that Ω is given by $\mathcal{C} \times [0,1]^F$ for some finite integer F.[4] We will typically denote an

[3]The term 'crowding type' is taken from Conley and Wooders (1996, 1997,2001). Crowding types are described further in the next section, where they play a larger role.

[4]This appears more than general enough to cover many potential applications. Results with a more general form of attribute space are obtained by Wooders, Cartwright and Selten (2001).

attribute by $\omega = (c, t)$ where $c \in \mathcal{C}$ and $t \in [0,1]^F$. We use the metric on Ω whereby the distance between two attributes $\omega = (c, t)$ and $\omega' = (c', t')$ is 2 if $c \neq c'$ and equals the $\max_f \left| t_f - t'_f \right|$ otherwise. In interpretation, therefore, two players i and j with the same crowding type are always seen as 'more similar' than two players with different crowding types. The attribute space will be treated in more detail in Section 3.3 and 3.4.

Given a set A we denote by $con(A)$ the convex hull of A and by $int(A)$ the interior of A.

A society: Given population (N, α) and strategy vector m a set of players $D \subset N$ is a *society* (relative to α and m) if

1. all players $i \in D$ play the same pure strategy,
2. all players $i \in D$ have the same crowding type, and
3. for any player $i \in N$, if $\alpha(i) \in int(con(\alpha(D))$ then $i \in D$.

We say that a pure strategy vector m *induces a partition of the population* (N, α) into a set of societies $\mathcal{S} = \{N_1, ..., N_Q\}$ if each player $i \in N$ belongs to a unique society $N_q \in \mathcal{S}$ and if each society $N_q \in \mathcal{S}$ is relative to α and m. Note that if m induces a partition of the population (N, α) into Q societies then there exists a partition of Ω into Q convex subsets $\{\Omega_q\}_{q=1}^{Q}$ such that, for any two players $i, j \in N$ and any Ω_q, if $\alpha(i), \alpha(j) \in int(\Omega_q)$ then $m_i = m_j$.

The definition of a society captures two key features. First, players in the same society play the same strategy; this is clearly motivated by the observation that conformity within a society may lead to common behavior. Second, players in the same society have a commonality of attribute; this is motivated by the observation that a player will only conform to those with whom he identifies. That players in a society have a commonality of attribute is demonstrated by the fact that players in a society have the same crowding type and that to each society we can associate a convex subset of attribute space.

We note that any strategy vector induces a partition of a population (N, α) into $|N|$ societies where each society consists of one player.[5] A crucial aspect of our main results will thus be to bound the number of societies

[5] That a society could have just one member is not unreasonable as this may represent a player who chooses not to conform (Bernheim 1994) to the actions of similar players.

independently of the size of the player set. In games with many players, this will ensure that most societies contain many players. We treat other implications of the definition of a society after stating our main result in Section 3.3.

3 Conformity

In this section we demonstrate conditions under which all sufficiently large games have approximate Nash equilibria in pure strategies that induce partitions of the player sets into a uniformly bounded number of societies. Before introducing our results we provide two simple examples to show why the desired equilibrium may not exist.

Example 1: Players have to choose between two locations A and B. The attribute space is given by $\{X, P\}$ where a player with crowding type X is a celebrity and a player with crowding type P an 'ordinary' member of the public. Members of the public like living in the same location as celebrities. Thus, the payoff of a player with attribute P is equal to the proportion of celebrities whose choice of location he matches. Celebrities, by contrast, prefer avoiding the public and thus the payoff of a player with attribute X is equal to the proportion of members of the public whose choice of location she mismatches. Arbitrarily large games induced from this pregame need not have an approximate Nash equilibrium in pure strategies consistent with conformity. This follows from the fact that there may not exist an approximate Nash equilibrium in pure strategies. This is easily seen by supposing that there is only one celebrity.⧫

Example 1 illustrates that some conditions will be required to guarantee the existence of an approximate Nash equilibrium in pure strategies. Our second example demonstrates that even if there exists an approximate Nash equilibrium in pure strategies there need not exist one that is consistent with conformity.

Example 2: Players choose between locations A and B. The attribute space is $[0, 1]$. A player's attribute determines whether he prefers location A or B. Whether a player prefers A or B can, however, be seen as essentially a random event. More formally, assume that if a player has attribute ω where ω is a rational number then he is assigned a payoff of 1 for choosing A and 0 for choosing B. If a player has attribute ω where ω is an irrational number then he is assigned a payoff of 1 for choosing B and 0 for choosing A.

Games induced from this pregame clearly have a Nash equilibrium in pure strategies. For arbitrarily large games, however, it is clear that no bound can be put on the number of societies that a Nash equilibrium would induce. For example, in a game where alternate players (in terms of the size of their attribute) have rational and irrational attributes the number of societies is as large as the player set.♦

Example 2 illustrates that some continuity assumption on attributes is necessary. In particular, we require that players with close attributes are similar. It would appear that a simple redefinition of an attribute would solve the observed problem with conformity in Example 2; for example, we could state that there are two attributes to represent those who like location A and those who like location B. Note, however, that the number ω may signify an observable characteristic of a player that is irrelevant in terms of his payoff but does influence whether or not other players will identify with him; for example, ω may represent age and a player conforms to those with a similar age to himself. This suggests that conformity on the basis of ω may be observed, implying that $[0, 1]$ is a relevant attribute space to consider.

3.1 Large games

To derive our main result we make two assumptions on pregames - *continuity in attributes* and *global interaction*. We introduce each in turn.

Continuity in Attributes: The pregame $\mathcal{G} = (\Omega, S, h)$ satisfies *continuity in attributes* if for any $\varepsilon > 0$ there exists a real number $\delta_c(\varepsilon) > 0$ such that, for any two games $\Gamma(N, \alpha)$ and $\Gamma(N, \overline{\alpha})$, if for all $i \in N$ it holds that

$$dist(\alpha(i), \overline{\alpha}(i)) < \delta_c(\varepsilon)$$

then for any $j \in N$ and for any pure strategy vector m,

$$\left| u_j^{\alpha}(m) - u_j^{\overline{\alpha}}(m) \right| < \varepsilon.$$

Continuity in attributes dictates that, given strategy choices, if the attribute function changes only slightly, then payoffs change only slightly. We note that the pregame of Example 2 does not satisfy continuity in attributes.

To define global interaction we introduce a metric ρ_α on pure strategy vectors for a given game $\Gamma(N, \alpha)$. Consider two arbitrary pure strategy vectors $m, s \in S^N$ and denote by w_m and g_s the respective induced weight functions. Define ρ_α by

$$\rho_\alpha(m,s) \stackrel{\text{def}}{=} \frac{1}{|N|} \sum_{s_k \in S} \sum_{\omega \in \alpha(N)} |w_m(\omega, s_k) - g_s(\omega, s_k)| \, .$$

Thus, pure strategy vectors m and s are seen as 'close' if the *proportion* of players with each attribute playing each strategy is approximately the same.

Global Interaction: The pregame $\mathcal{G} = (\Omega, S, h)$ satisfies *global interaction* when for any $\varepsilon > 0$ there exists a real number $\delta_g(\varepsilon) > 0$ such that, for any game $\Gamma(N, \alpha)$ and any two pure strategy vectors m and s, if

$$\rho_\alpha(m,s) < \delta_g(\varepsilon)$$

then for any $j \in N$ where $m_j = s_j$

$$\left| u_j^\alpha(m) - u_j^\alpha(s) \right| < \varepsilon. \tag{2}$$

The assumption of global interaction states that a player is nearly indifferent to small changes in the *proportion*, relative to the total population, of players of each attribute playing each strategy. Thus, the actions of any one player have little influence on the payoffs of others. We note that the pregame of Example 1 does not satisfy global interaction.

The pregame $\mathcal{G} = (\Omega, S, h)$ is said to satisfy the *large game property* if it satisfies both continuity in attributes and global interaction. The large game property implies a form of continuity of h with respect to changes in the weight function and attribute. Indeed, to summarize: continuity in attributes is a bound on the payoff difference when the *attributes* of players change but their *strategies* do not. By contrast, global interaction is a bound on the payoff difference when the *strategies* of players change but their *attributes* do not. The pregame of Example 1, for instance, satisfies continuity in attributes but not global interaction. The pregame of Example 2, by contrast, satisfies global interaction but not continuity in attributes.

3.2 Purification

It is difficult to motivate the use of mixed strategies if players imitate or conform. A preliminary to treating the individual rationality of conformity is thus to treat the individual rationality of using pure strategies. The following result demonstrates that in sufficiently large games induced from a pregame satisfying the large game property there exists an approximate Nash equilibrium in pure strategies. This result is most easily obtained using a purification theorem due to Kalai (2000). Wooders, Cartwright and Selten (2001) provide generalizations of Theorem 1.[6]

Theorem 1: Consider a pregame $\mathcal{G} = (\Omega, S, h)$ that satisfies the large game property. Given any real number $\varepsilon > 0$ there exists a real number $\eta(\varepsilon)$ such that for any population (N, α) where $|N| > \eta(\varepsilon)$ the induced game $\Gamma(N, \alpha)$ has a Nash ε-equilibrium in pure strategies.

Proof: Suppose not. Then there exists an $\varepsilon > 0$ and, for every integer ν, a game $\Gamma(N^\nu, \alpha^\nu)$ such that $|N^\nu| > \nu$ and game $\Gamma(N^\nu, \alpha^\nu)$ has no Nash ε-equilibrium. Let $\delta = \delta_c\left(\frac{\varepsilon}{3}\right)$ be the real number implied by continuity in attributes for a payoff bound of $\frac{\varepsilon}{3}$. Partition Ω into a finite number of subsets $\Omega_1, ..., \Omega_Q$ each of diameter less than δ. For each Ω_q pick a point $\omega_q \in \Omega_q$. For each ν consider a population $(N^\nu, \overline{\alpha}^\nu)$ satisfying, for all $i \in N^\nu$, the property that $\overline{\alpha}^\nu(i) = \omega_q$ if and only if $\alpha^\nu(i) \in \Omega_q$. We note that, by the well known Nash existence theorem, each game $\Gamma(N^\nu, \overline{\alpha}^\nu)$ has a Nash equilibrium. Consider the set of games $G = \{\Gamma(N^\nu, \overline{\alpha})\}_\nu$. Given that the set of attributes for G is finite it can trivially be seen that G is a subset of a family of semi-anonymous Bayesian games as defined by Kalai (2000). From this (global interaction and the existence of Nash equilibrium) it is equally trivial that Theorem 1 of Kalai (2000) implies there exists ν^* such that any game $\Gamma(N^\nu, \overline{\alpha}^\nu)$ where $\nu > \nu^*$ has a Nash $\frac{\varepsilon}{3}$-equilibrium in pure strategies m^ν. By continuity in attributes and the choice of δ

$$\left|u_i^\alpha(s_k, m_{-i}^\nu) - u_i^{\overline{\alpha}}(s_k, m_{-i}^\nu)\right| < \frac{\varepsilon}{3}$$

for all $s_k \in S$. Thus, m^ν is a Nash ε-equilibrium in pure strategies of game $\Gamma(N^\nu, \alpha^\nu)$ if $\nu > \nu^*$.$\blacksquare$

3.3 Main Result

Theorem 2 demonstrates that in sufficiently large games there exists an approximate Nash equilibrium in pure strategies that partitions the population

[6] We note that Kalai (2000) and Wooders, Cartwright and Selten (2001) are independent and concurrent research endeavours. The purification result of Kalai (2000) is sufficient for the purposes of the current paper (and the reader interested in this paper may also be interested in the other results of Kalai 2000). Seminal results on purification appear in Schmeidler (1973). Pascoa (1993) studies situations with a continuum of agents but nevertheless uses assumptions similar to ours. The literature on purification is surveyed in Khan and Sun (2002).

into a bounded number of societies. A fundamental aspect of Theorem 2 is that the bound is independent of population size. Note that the smaller is the bound the more dissimilar players in the same society may be. Theorem 2 is proved in an appendix.

Theorem 2: Let $\mathcal{G} = (\Omega, S, h)$ be a pregame satisfying the large game property. Given any real number $\varepsilon > 0$ there exists real number $\eta(\varepsilon)$ and integer $A(\varepsilon)$ such that for any population (N, α) where $|N| > \eta(\varepsilon)$ the induced game $\Gamma(N, \alpha)$ has a Nash ε-equilibrium in pure strategies that induces a partition of the population (N, α) into $Q \leq A(\varepsilon)K$ societies.

Theorem 2 suggests that conformity can be individually rational in sufficiently large games. We highlight that the result applies to games in which all players have different attributes.

Note how the definitions of a society and of a partition of the population into societies leaves open the possibility that two players $i, j \in N$ with the same attributes could belong to different societies and play different strategies. A result such as Theorem 2 cannot be obtained unless this is permitted.[7] To see this, consider a game $\Gamma(N, \alpha)$ where all players have the same attribute and any Nash equilibrium has the property that a positive fraction of the players choose one strategy and a positive fraction choose another strategy.

Another important feature of Theorem 2 is the convexity aspect of societies. For some attribute spaces, for example, $\Omega = \mathcal{C} \times [0, 1]$, Theorem 2 implies the existence of an approximate Nash equilibria in pure strategies with the property that most players are playing the same strategy as their nearest neighbors in attribute space. This, however, is a special case; see Wooders, Cartwright and Selten (2001) for further discussion.

3.4 Bounding the number of societies independently of ε

In some cases it is possible to bound the number of societies independently of ε. We provide one such example.

Recall that the attribute space is given by $\Omega = \mathcal{C} \times [0, 1]^F$. In this section we shall assume that if a player has attribute (c, t) the value c characterizes

[7] Wooders, Cartwright and Selten (2001) obtain a complementary result to Theorem 1 in which players with the same attribute do belong to the same society. This is possible by treating populations in which the number of players of any one attribute is bounded. Similarly the literature on non-atomic games demonstrates the existence of a symmetric Nash equilibrium in pure strategies in games with a continuum of players if the distribution of players over attributes is atomless (see Pascoa 1993).

his external influence on others - his *crowding type* - while t characterizes his payoff function - his *taste type*. We think of the crowding type of a player as such characteristics as gender, educational level, height, ability to salsa, and so on, that are observable to other players and may have direct effects on them. In contrast, we think of a player's taste type as of direct relevance only to himself, for example, whether he enjoys school or whether he likes to dance. Two assumptions on crowding and taste types are required.

The first assumption, within type anonymity, implies that two players of the same crowding type, playing the same strategy, have the same influence on the payoffs of others.

Within (crowding) type anonymity: Pregame $\mathcal{G} = (\Omega, S, h)$ satisfies within type anonymity when for any induced game $\Gamma(N, \alpha)$ and for any two pure strategy vectors m and s if

$$\sum_{\omega:\omega=(c,\cdot)} w_m(\omega, s_k) = \sum_{\omega:\omega=(c,\cdot)} w_s(\omega, s_k)$$

for all $c \in \mathcal{C}$ then for any $i \in N$ where $m_i = s_i$

$$u_i^\alpha(m_i, m_{-i}) = u_i^\alpha(s_i, s_{-i}).$$

That is, if any two strategy vectors have the property that, from the perspective of player i, the weight functions induced by these strategy vectors assign the same weight to each strategy chosen by players of each crowding type, then player i's is indifferent between two situations.

Our second assumption is on payoff functions. An explanation follows the definition.

Convexity in taste types: Pregame $\mathcal{G} = (\Omega, S, h)$ satisfies *convexity in taste types* if there exists a function $y : \mathcal{C} \times S \times W \to \mathbb{R}$ and a function $x : \mathcal{C} \times S \times W \to \mathbb{R}^F$ such that for any induced game $\Gamma(N, \alpha)$ and any pure strategy vector m the payoff of player $i \in N$ where $\alpha(i) = (c, t)$ is given by

$$u_i^\alpha(m_i, m_{-i}) = y(c, m_i, w_m) + t \cdot x(c, m_i, w_m).$$

We recall that a player's taste type is determined by a vector $t \in [0, 1]^F$. All else equal, if a player's taste type is a convex combination of the taste types of two other players (and all three players have the same crowding type), then his payoff is the same convex combination of the payoffs of the other two players. Intuitively, we could think of there being a 'representative player' for each crowding type with, say, taste type $(0.5, 0.5, ..., 0.5)$. The payoff of a player with taste type t can then be thought of as a linear function of how much his attribute differs from that of the representative for his crowding type.

In Section 3.5 we consider a pregame that satisfies within type anonymity and convexity in taste types. Our second main result, Theorem 3, places a bound that is independent of ε on the number of societies. For simplicity, we state and prove the Theorem for the case of only two pure strategies and one dimensional space of taste types. After the proof we discuss the general case.

Theorem 3: Let $\mathcal{G} = (\Omega, S, h)$ be a pregame satisfying the large game property, within type anonymity and convexity in taste types. Let $K = 2$ and $F = 1$. Given any real number $\varepsilon > 0$ there exists real number $\eta(\varepsilon)$ such that for any population (N, α) where $|N| > \eta(\varepsilon)$ the induced game $\Gamma(N, \alpha)$ has a Nash ε-equilibrium in pure strategies that induces a partition of the population (N, α) into $Q \leq CK$ societies.

Proof: Suppose not. By Theorem 1 for any sufficiently large population (N, α) the induced game $\Gamma(N, \alpha)$ has a Nash ε-equilibrium in pure strategies m^*. Let M denote the set of pure strategy vectors such that $\overline{m} \in M$ if and only if $\overline{m}$ is a Nash ε-equilibrium and

$$\sum_{\omega:\omega=(c,\cdot)} w_{m^*}(\omega, s_k) = \sum_{\omega:\omega=(c,\cdot)} w_{\overline{m}}(\omega, s_k)$$

for all c and s_k. We note that $m^* \in M$ and so M is non-empty.

Consider pure strategy vector $m \in M$. For each c and s_k let $T_{ck} \subset [0,1]^F$ be such that $t \in T_{ck}$ if and only if there exists a player $i \in N$ such that $\alpha(i) = (c, t)$ and $m_i = s_k$. Let $X_{ck} \subset N$ be such that $j \in X_{ck}$ if and only if $\alpha(j) = (c, t)$ where $t \in int(con(T_{ck}))$ and $m_j \neq s_k$. Let $X = \cup_{c,k} X_{ck}$. Suppose, without any loss, that m minimizes $|X|$.[8] If $X = 0$ then m induces a partition of the population into CK societies; this is a contradiction to our initial supposition. Therefore $|X| > 0$. We will construct from m a pure strategy vector $m^i \in M$ that diminishes $|X|$ by one, thus providing a contradiction and completing the proof.

Given that $|X| > 0$, we can select a nonempty set X_{ck} for some c and s_k and a player $j \in X_{ck}$. Thus, $\alpha(j) = (c, t)$ and $t \in int(con(T_{ck}))$ yet

[8] Of course $|X|$ depends on our inital choice of m.

$m_j \neq s_k$. Assume that $m_j = s_{\overline{k}}$. Let $A_{ck} \subset N$ be such that $i \in A_{ck}$ where $\alpha(i) = (c,t)$ if and only if $m_i = s_k$ and t belongs to the boundary of $con(T_{ck})$.[9] For each $i \in A_{ck}$ let m^i be the pure strategy vector with the properties that $m^i_i = s_{\overline{k}}$, $m^i_j = s_k$ and $m^i_l = m_l$ for all other $l \in N$; thus players i and j have 'exchanged' pure strategies. We conjecture (*) that for some $i^* \in A_{ck}$ the pure strategy vector m^{i^*} is a Nash ε-equilibrium. Provided this conjecture holds, given that i^* belongs to the boundary of $con(T_{ck})$ and $K = 2$ the value of $|X|$ is one less for m^{i^*} then for m giving the desired contradiction.

To prove the conjecture (*) observe that within type anonymity and convexity in taste types implies that for some $\beta_1, \beta_2, ..., \beta_{|A_{ck}|}$ [where $1 \geq \beta_i \geq 0$ and $\sum \beta_i = 1$]

$$u^\alpha_j(z, m_{-j}) = \sum_{i \in A_{ck}} \beta_i u^\alpha_i(z, m^i_{-i})$$

for all $z \in S$. Given that m is a Nash ε-equilibrium

$$u^\alpha_j(s_{\overline{k}}, m_{-j}) \geq u^\alpha_j(s_k, m_{-j}) - \varepsilon.$$

Thus, there exists some $i^* \in A_{ck}$ and corresponding m^{i^*} where

$$u^\alpha_{i^*}(m^{i^*}_{-i^*}, m^{i^*}_{-i^*}) \geq u^\alpha_{i^*}(s_k, m^{i^*}_{-i^*}) - \varepsilon.$$

It is clear, by within type anonymity, that

$$u^\alpha_l(m^{i^*}_l, m^{i^*}_{-l}) \geq u^\alpha_l(z, m^{i^*}_{-l}) - \varepsilon$$

for all $z \in S$ and $l \in N$, $l \neq i^*, j$. It thus remains to consider player j. Let $l \in A_{ck}$ and $l \neq i^*$. Within type anonymity and convexity of taste types implies that

$$u^\alpha_j(z, m^{i^*}_{-j}) = \sum_{i \in A_{ck} \setminus i^*} \beta_i u^\alpha_i(z, m^{i^*}_{-i}) + \beta_{i^*} u^\alpha_{i^*}(z, m^l_{-i^*})$$

for all $z \in S$. Within type anonymity and that m is a Nash ε-equilibrium implies that

$$u^\alpha_j(m^{i^*}_j, m^{i^*}_{-j}) \geq u^\alpha_j(s_{\overline{k}}, m^{i^*}_{-j}) - \varepsilon.$$

Thus m^{i^*} is a Nash ε-equilibrium.■

[9] There must be some such player since the convex hull $int(con(T_{ck}))$ is determined by players in A_{ck} and $int(con(T_{ck})) \neq \varnothing$.

Remark: Analogous results to that of Theorem 3 can be obtained for $K > 2$ and $F > 1$; a proof is available from the authors on request. Using the notation from the proof of Theorem 3, we briefly explain, however, why extending Theorem 3 is not trivial - and significantly complicates the analysis. Consider a player j where $\alpha(j) = (c, t)$ and $t \in int(con(T_{ck}))$ yet $m_j \neq s_k$. As we show in the proof of Theorem 3 there will exist a player i belonging to the boundary of $con(T_{ck})$ for whom $u_i^\alpha(s_{\overline{k}}, m_{-i}^i) > u_i^\alpha(s_k, m_{-i}^i) - \varepsilon$. If $K = 2$ this is sufficient to show that playing $s_{\overline{k}}$ is an 'ε-best response' for player i. As a result we can exchange the strategies of players i and j and retain a Nash ε-equilibrium. If, however, $K = 3$ we have not done enough to show that $s_{\overline{k}}$ is a 'ε-best response' for player i - there is the third pure strategy option that must be considered. As a result playing $s_{\overline{k}}$ may be only a '2ε-best response' for player i. Thus, for the $K = 3$ case, modifying the above proof would require commencing with a pure strategy vector m that is a Nash $\frac{\varepsilon}{2}$-equilibrium. Also note that simply exchanging the pure strategies of players i^* and j may not be enough to reduce $|X|$ by one if $K > 2$ or $F > 1$; thus, an additional exchange of pure strategies may be required.

3.5 Normative Influence and conformity

In a number of game theoretic models of social situations, there is some feature built into the model that ensures conformity. For example, Bernheim (1994) obtains such a result for a model in which individuals care about status and behavior (or strategy) serves as a signal of status. Since individuals gain from playing the most commonly chosen strategy this creates a normative influence. The following example illustrates that with such normative influence it is 'easier' to have conformity as defined in this paper. Indeed, we demonstrate that the existence of normative influence can imply the existence of exact equilibria satisfying conformity with a fewer number of societies.

The attribute space is given by $\Omega = \{P, R\} \times [0, 1]$ where a player with attribute $(P, \cdot)$ is referred to as *poor* and a player with attribute $(R, \cdot)$ as *rich.* The set of pure strategies is $S = \{A, B\}$ where A and B are interpreted as different locations. Given pure strategy vector m let $rb(m)$ denote the proportion of the population that is rich and chooses location B; formally

$$rb(m) = \frac{1}{|N|} \sum_{\omega:\omega=(R,\cdot)} w_m(\omega, B).$$

We define $ra(m), pb(m)$ and $pa(m)$ in an analogous way.

The payoff function of a poor player $i \in N$ with attribute $\alpha(i) = (P, t)$ is given by

$$\begin{aligned} u_i^\alpha(A, m_{-i}) &= 2ra(m_{-i}) + t \text{ and} \\ u_i^\alpha(B, m_{-i}) &= 2rb(m_{-i}) + 1 - t. \end{aligned}$$

Thus, a poor player receives a higher payoff the more rich players he matches in choice of location. The value t can be interpreted as player i's preference for living in location A as opposed to B.

The payoff function of a rich player can be seen as composed of two parts - one part that is free from normative influence and another part that captures normative influence. Let real number β denote the degree of normative influence. The payoff function of a rich player $i \in N$ with attribute $\alpha(i) = (R, t)$ is given by

$$\begin{aligned} u_i^\alpha(A, m_{-i}) &= 1 + t - pa(m_{-i}) + \beta ra(m_{-i}) \text{ and} \\ u_i^\alpha(B, m_{-i}) &= 2 - t - pb(m_{-i}) + \beta rb(m_{-i}). \end{aligned}$$

A rich player thus receives a higher payoff the fewer poor people he matches in choice of location. The value t can be interpreted as the player i's preference for living in location A. If $\beta = 0$ then we say that there is no normative influence. If $\beta > 0$ then we say that there is normative influence – a player's payoff is increasing in the number of rich players that choose the same location as himself.

It is clear that Theorem 3 can be applied to demonstrate the existence, for sufficiently large populations, of an approximate Nash equilibrium in pure strategies that partitions the population into 4 societies. For this example, it is, of course, fairly simple to explicitly derive the Nash equilibria of induced games. We find, however, that if $\beta = 0$ then we generally cannot improve upon Theorem 2. If $\beta > 0$ then we may be able to do so. To illustrate, consider the following two cases.

- Suppose that $\beta = 0$. Consider a population (N, α) in which there are n rich and n poor players where n is an odd number. Further, assume each player has attribute $(\cdot, 0.5)$. It is clear (using argument by contradiction) that game $\Gamma(N, \alpha)$ does not have a Nash equilibrium in pure strategies. It is equally simple to see that any approximate Nash equilibrium would induce four societies - 'rich societies' in locations A and B and 'poor societies' in locations A and B.

- Suppose that $\beta = 4$. Consider a population (N, α) in which there are n players of attribute $(R, 0)$, n players of attribute $(R, 1)$ and similarly n players of attributes $(P, 0)$ and $(P, 1)$. Thus, half of the population is poor and half rich. Also, half of the population have a strong preference for location A and half for location B. It is easily checked that there exists an exact Nash equilibria of game $\Gamma(N, \alpha)$ wherein every player chooses the same location, thus inducing only two societies.

This illustrates how the presence of normative influence can lead to existence of an exact equilibrium in pure strategies that induces a smaller number of societies. Another interesting aspect of this example is how the normative influence of the rich has a 'knock on' effect on the conformity amongst poor; the desire of the rich to conform in their choice of location leads to observed conformity by the poor.

4 Conclusions

If individuals are influenced by, imitate, and conform to the actions of others then this poses a challenge to the individual rationality assumption of game theory. This challenge leads us to question the possible existence of an approximate Nash equilibrium consistent with conformity and imitation. In this paper we demonstrate the existence of such an equilibrium in games with many players. This result is made more interesting by observing that in these games, where for most players there are many similar players, it seems intuitively most likely that players will base decisions on processes such as conformity and imitation.

Issues that still remain include: (i). We only demonstrate the *existence* of an approximate Nash equilibrium with conformity; we do not address directly, the question of whether players actually learn to play that equilibrium. (ii). Conformity in mixed strategy equilibrium is not treated. To treat conformity in mixed strategies may seem unmotivated given our insistence on pure strategy equilibria. Note, however, that while it may seem unnatural that a player would use a mixed strategy it need not be unnatural that a society would 'play a mixed strategy'. In Cartwright and Wooders (2003) we consider this possibility by formulating conformity in terms of mixed strategies and incomplete information. Turning to question (ii), there are many reasons why players who learn through imitation or conformity fail to learn to play an approximate Nash equilibrium even if an equilibrium consistent with conformity exists. This is discussed by Cartwright (2003) where sufficient conditions are provided under which play will indeed converge to the desired equilibrium.

5 Appendix

Lemma 1: Let $\mathcal{G} = (\Omega, S, h)$ be a pregame satisfying the large game property. For any induced game $\Gamma(N, \alpha)$, for any partition of Ω into a finite number of subsets $\Omega_1, ..., \Omega_A$, each of diameter less than $\delta_c\left(\frac{\varepsilon}{3}\right)$, and for any two pure strategy vectors m and $\overline{m}$ where

$$\sum_{\omega \in \Omega_a} w_{\alpha,m}(\omega, s_k) = \sum_{\omega \in \Omega_a} w_{\alpha,\overline{m}}(\omega, s_k), \tag{3}$$

if m is a Nash $\frac{\varepsilon}{3}$-equilibrium in pure strategies then $\overline{m}$ is a Nash ε-equilibrium in pure strategies.[10]

Proof: Given an induced game (N, α) and two pure strategy vectors m and $\overline{m}$ satisfying (3), it is immediate that there exists a one-to-one mapping $R(i) : N \to N$ such that,

$$\overline{m}_i = m_{R(i)} \tag{4}$$

for all $i \in N$ and,

$$dist(\alpha(i), \alpha(R(i))) < \delta_c\left(\frac{\varepsilon}{3}\right). \tag{5}$$

Informally, we can treat equivalently: (a) player i having attribute $\omega = \alpha(i)$ and playing strategy $m_{R(i)}$ and (b) player $R(i)$ playing strategy $m_{R(i)}$ and having attribute $\omega = \alpha(i)$. Thus, consider the population $(N, \overline{\alpha})$ where,

$$\overline{\alpha}(R(i)) = \alpha(i) \tag{6}$$

for all $i \in N$. Our method of proof is to (i) demonstrate that m is a Nash ε-equilibrium in pure strategies of game $\Gamma(N, \overline{\alpha})$ before (ii) demonstrating how this implies that $\overline{m}$ is a Nash ε-equilibrium in pure strategies of game $\Gamma(N, \alpha)$.

The assumption of continuity in attributes and (5) implies,

$$\left|u_i^{\alpha}(s_k, m_{-i}) - u_i^{\overline{\alpha}}(s_k, m_{-i})\right| < \frac{\varepsilon}{3} \tag{7}$$

[10] Where $\delta_c\left(\frac{\varepsilon}{3}\right)$ is the real number implied by continuity in attributes for a payoff bound of $\frac{\varepsilon}{3}$.

for all $s_k \in S$ and all $i \in N$. Given that m is a Nash $\frac{\varepsilon}{3}$-equilibrium in pure strategies for $\Gamma(N, \alpha)$ it follows that

$$u_i^{\alpha}(m_i, m_{-i}) \geq u_i^{\alpha}(s_k, m_{-i}) - \frac{\varepsilon}{3}.$$

The above two inequalities yield

$$\begin{aligned} u_i^{\overline{\alpha}}(m_i, m_{-i}) &\geq u_i^{\alpha}(m_i, m_{-i}) - \tfrac{\varepsilon}{3} \\ &\geq u_i^{\alpha}(s_k, m_{-i}) - \tfrac{2\varepsilon}{3} \\ &\geq u_i^{\overline{\alpha}}(s_k, m_{-i}) - \varepsilon \end{aligned} \qquad (8)$$

for all $i \in N$ and $s_k \in S$. Thus, m is a Nash ε-equilibrium in pure strategies of game $\Gamma(N, \overline{\alpha})$.

By (4) and (6)

$$\left(\overline{\alpha}(R(i)), m_{R(i)}\right) = \left(\alpha(i), m_{R(i)}\right) = (\alpha(i), \overline{m}_i) \qquad (9)$$

for all $i \in N$. It follows that

$$u_{R(i)}^{\overline{\alpha}}(s_k, m_{-R(i)}) = u_i^{\alpha}(s_k, \overline{m}_{-i}) \qquad (10)$$

for all $i \in N$ and all $s_k \in S$. It is immediate from (8) and (10) that $\overline{m}$ is a Nash ε-equilibrium in pure strategies of game $\Gamma(N, \alpha)$.■

We recall that $\Omega = \{1, 2, ..., C\} \times [0,1]^F$ for some finite integers C and F. We make use of a lexicographic ordering on elements of $[0,1]^F$. Formally, we define the binary relations $<_L$ and $=_L$ as follows: Take any two points $t = (t_1, ..., t_F), \tau = (\tau_1, ..., \tau_F) \in [0,1]^F$. We say that $t =_L \tau$ if and only if $t_f = \tau_f$ for all $f = 1, ..., F$. We say that $t <_L \tau$ if either:

1. $\sum_f t_f < \sum_f \tau_f$ or,
2. $\sum_f t_f = \sum_f \tau_f$ and for some f^* we have $t_{f^*} < \tau_{f^*}$ and $t_f = \tau_f$ for all $f < f^*$.

We say that $t \leq_L \tau$ if either $t <_L \tau$ or $t =_L \tau$.

Lemma 2: Given any two finite sets of points $\Omega_J = \{t^1, ..., t^J\}$ and $\Omega_Q = \{\tau^1, ..., \tau^Q\}$ (where $t^1, ..., t^J, \tau^1, ..., \tau^Q \in [0,1]^F$) if $t^j \leq_L \tau^q$ for all j and q then the interior of the convex hulls of Ω_J and Ω_Q are either distinct or both empty.

[Note that given $t^j \in \Omega_J$, the hypotheses of the Lemma require that $t^j \leq_L \tau^q$ for *all* $\tau^q \in \Omega_Q$. If $F = 1$, for example, this implies that the interiors of the convex hulls are both empty or that the two sets have in common at most one point. If the two sets have no points in common, then all the points in Ω_J are smaller, according to $\leq_L$, than all the points in Ω_Q.]

Proof: Suppose the claim is false. Then there exists a point $\omega \in \Omega$ such that $\omega \in I(Co(\Omega_J))$ and $\omega \in I(Co(\Omega_Q))$. Thus, for non-negative numbers $\gamma_1, ..., \gamma_J$ and $\beta_1, ..., \beta_Q$

$$\sum_q \beta_q = \sum_j \gamma_j = 1 \tag{11}$$

and, for each $f = 1, ..., F$,

$$\omega_f = \sum_j \gamma_j t_f^j = \sum_q \beta_q \tau_f^q. \tag{12}$$

This implies that

$$\sum_j \left[\gamma_j \sum_f t_f^j \right] = \sum_q \left[\beta_q \sum_f \tau_f^q \right]. \tag{13}$$

By assumption, for each $t^j \in \Omega_J$ and $t^q \in \Omega_Q$ it holds that $\sum_f t_f^j \leq \sum_f t_f^q$. Suppose, for some $t^{\overline{j}} \in \Omega_J$ and $\tau^{\overline{q}} \in \Omega_Q$, that $\sum_f t_f^{\overline{j}} < \sum_f \tau_f^{\overline{q}}$. Given (13) it must be that either $\gamma_{\overline{j}} = 0$ or $\beta_{\overline{q}} = 0$. Let Ω_J^+ denote the set of $t^j \in \Omega_J$ given positive weight $\gamma_j > 0$ and Ω_Q^+ the set of all $\tau^q \in \Omega_Q$ given positive weight $\beta_q > 0$. It is immediate that $\sum_f t_f^j = \sum_f \tau_f^q$ for each $t^j \in \Omega_J^+$ and $\tau^q \in \Omega_Q^+$.

If $\Omega_J^+ = \Omega_Q^+$ then we easily obtain the desired contradiction. When $\Omega_J^+ = \Omega_Q^+$ and for *each* element t^j in Ω_J^+ it holds that $t^j \leq_L t^q$ then the sets must each contain only one element and, in this case, the interiors of the convex hulls are both empty.

Let $\Omega_J^{++} = \Omega_J^+ \backslash \Omega_Q^+$ and $\Omega_Q^{++} = \Omega_Q^+ \backslash \Omega_J^+$. Either Ω_J^{++} or Ω_Q^{++} is non-empty. Suppose that Ω_J^{++} is non-empty. For every $t^j \in \Omega_J^{++}$ and $\tau^q \in \Omega_Q^+$ there is some $f^* \in \{1, ..., F-1\}$ for which $t_{f^*}^j < \tau_{f^*}^q$ and $t_f^j = \tau_f^q$ for all $f < f^*$. Take the minimum of these f^* over all points $t^j \in \Omega_J^{++}$ and $\tau^q \in \Omega_Q^+$. By choice of f^* it holds that $t_{f^*}^j \leq \tau_{f^*}^q$ for all $j \in \Omega_J^+$ and $q \in \Omega_Q^+$ and $t_{f^*}^{\overline{j}} < \tau_{f^*}^{\overline{q}}$ for some $\overline{j} \in \Omega_J^{++}$ and $\overline{q} \in \Omega_Q^+$. This must contradict either (11) or (12). The case where Ω_Q^{++} is non-empty can be treated in an analogous manner.■

Proof of Theorem 2: We proceed by contradiction. Suppose that the statement of the Theorem does not hold for some $\varepsilon_0 > 0$. We proceed by first determining a value for $A(\varepsilon_0)$ and then showing that this $A(\varepsilon_0)$ satisfies the claim of the Theorem for $\varepsilon = \varepsilon_0$. Given ε_0 set $\delta = \delta_c\left(\frac{\varepsilon_0}{3}\right) > 0$, where $\delta_c\left(\frac{\varepsilon_0}{3}\right)$ is the real number implied by continuity in attributes for a payoff bound of $\frac{\varepsilon_0}{3}$. Use compactness of Ω to write Ω as the disjoint union of a finite number $A \stackrel{\text{def}}{=} A(\varepsilon_0)$ of *convex* non-empty subsets $\Omega_1, ..., \Omega_A$, each of diameter less than δ. We now claim that A satisfies the conditions required by the Theorem. Suppose not. Then, for each integer ν there is a population (N^ν, α^ν) where $|N^\nu| > \nu$ and induced game $\Gamma(N^\nu, \alpha^\nu)$ does not have a Nash ε_0-equilibrium in pure strategies that induces a partition of the population (N^ν, α^ν) into $Q \leq AK$ societies. By Theorem 1, for sufficiently large ν, $\Gamma(N^\nu, \alpha^\nu)$ has a Nash $\frac{\varepsilon_0}{3}$-equilibrium in pure strategies, say m^ν.

Consider a change of pure strategy vector from m^ν to $\overline{m}^\nu$, for all ν, where $\overline{m}^\nu$ satisfies:

1. for all Ω_a and $s_k \in S$,
$$\sum_{\omega \in \Omega_a} w_{\alpha^\nu, m^\nu}(\omega, s_k) = \sum_{\omega \in \Omega_a} w_{\alpha^\nu, \overline{m}^\nu}(\omega, s_k),$$
2. for any $i, j \in N^\nu$ where $\alpha^\nu(i), \alpha^\nu(j) \in \Omega_a$ for some a, if $m_i^\nu = s_k$ and $m_j^\nu = s_{\overline{k}}$ where $k < \overline{k}$ then $\alpha^\nu(i) \leq \alpha^\nu(j)$.

Given that the finite set of points $\alpha(N^\nu)$ is well ordered for all ν, it is always possible to construct such a pure strategy vector $\overline{m}^\nu$ by a simple 'reassignment' of pure strategies. Given the choice of δ and that m^ν is a Nash $\frac{\varepsilon_0}{3}$-equilibrium in pure strategies it is immediate from Lemma 1 that, for sufficiently large ν, $\overline{m}^\nu$ is a Nash ε_0-equilibrium in pure strategies. By applying Lemma 2 and recalling that each Ω_a is convex it is clear that $\overline{m}^\nu$ is a Nash ε_0-equilibrium in pure strategies that induces a partition of the population into at most AK societies.■

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