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Information Aggregation, Costly Voting and Common Values

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Abstract

In a model of majority voting with common values and costly but voluntary participation, we show that in the vicinity of equilibrium, it is *always* Pareto-improving for more agents, on the average, to vote. This demonstrates that the negative voting externality identified by Borgers(2001) in the context of private values is always dominated by a positive informational externality. In addition, we show that multiple Pareto-ranked voting equilibria may exist and moreover, majority voting with compulsory participation can Pareto dominate majority voting with voluntary participation. Finally, we show that the inefficiency result is robust to limited preference heterogeneity.

Keywords: Voting, information, pivot, externality, inefficiency.
JEL Classification Numbers: D72, D82.

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1. Introduction

Many decisions are made by majority voting. In most cases, participation in the voting process is both voluntary and costly. The question then arises whether the level of participation is efficient i.e. is there too much or too little voting?

In a model with costly voting and private values, Borgers (2001) identifies a negative externality from voting: the decision of one voter to vote lowers the probability that any other voter is pivotal, and thus reduces the benefit to voting of all other agents. A consequence of the negative externality is that compulsory voting is *never* desirable: all voters are strictly better off at the (unique) voluntary voting equilibrium. An implication of this global result is a local one: in the vicinity of an equilibrium it is *always* Pareto-improving for fewer agents, on the average, to vote.

In this paper, we re-examine the nature of inefficiency of majority voting in a model with costly participation and common values. Our motivation is two-fold. First, we believe there is good reason to suppose that many decisions made by majority voting have a significant common-values component. By a common-values component, we mean that voters basically agree on the correct course of action, given full information about the environment, but in practice, information is very imperfect due to the complexity of the issue. An example would be state referenda in the US over issues[1] such as gun control and legalization of marijuana[2].

What links these two issues is major disagreement over the evidence, combined with some consensus over what the right course of action would be, were there to be agreement on the evidence. For example, there is disagreement in the US over whether more relaxed gun control actually leads to greater crime, but general agreement that crimes such as murder, robbery and so on are unacceptable. Again, there is disagreement over whether consumption of marijuana is more harmful than consumption of alcohol, and whether the former leads on to harder drugs, but increasing consensus that if neither of these things is true, then it is not really consistent for consumption of alcohol to be legal, but

[1] Note that we do not include fiscal issues such as referenda on property taxes, etc. on this list, as almost by definition, there are gainers and losers from such measures, and so preferences will be heterogenous.

[2] For example, in the Cogressional elections of November 2000, 42 states had 204 ballot initiatives which offered voters the choice on this kind of issue. "Marijuana, a hardy perennial on the ballot, was approved for medical use in Colorado and Nevada, but a proposal for full-scale legalisation of pot in Alaska (where medical marijuana is already legal) fell well short of passing. In Oregon and Utah voters approved initiatives to reform asset-forfeiture laws, which often apply in drug cases...Californians voted, by a large margin, to mandate treatment rather than prison for non-violent drug offenders, but voters in Massachusetts narrowly rejected a similar initiative." (The Economist, 11 November 2000).

consumption of marijuana to be illegal.

Our second motivation is that in a common values environment, in addition to the negative "pivot" externality identified by Borgers (2001), there is[3] a positive informational externality from voting: an individual voter, by basing his voting decision on his informative signal, improves the quality of the collective decision for all voters. On the face of it, it is not at all obvious, in general, which of these two externalities dominates. To put it another way: will potential voters participate too little or too much in decision-making?

We show the following. First, we demonstrate that the payoff to voting for any one individual can be decomposed into two parts: a term equal to the probability that the voter is pivotal, times the probability that she makes the correct decision in this case, plus a term giving her expected payoff if she is not pivotal. The first term is decreasing in the probability that any other voter votes (this is the "pivot" externality), but with common values, the second term is increasing in the probability that any other voter votes (the information externality). Second - and this is our main result - we show that the second externality *always* dominates the first: overall any voter's payoff from voting (or indeed, not voting) is always increasing in the probability that any other voter votes[4].

We also demonstrate a simple intuition for the main result, which we call a *weak swing voter's curse*, following Fedderson and Pesendorfer(1996). In our setting, when any particular voter faces an odd number of other voters, conditional on being pivotal, she is indifferent about voting or abstaining, because this situation can only arise when in total there are equal numbers of signals in favor and against the two alternatives. This effect lowers the overall ex ante benefit to any voter from participation below what is socially optimal.

In moving to common values, we show additionally that voting equilibrium is no longer unique: typically, there will be several equilibria. Nevertheless, we show that starting at any voting equilibrium, an increase in the probability that agents vote *always* leads to a Pareto-improvement[5]. So, compared to the private values case, the inefficiency of voting equilibrium is reversed: with costly participation, majority voting fails to aggregate

[3] As Borgers remarks, *"In a common value model of voting...there will be positive externalities to voting which can mitigate or outweigh the negative externality which we identify. In such a model one cannot expect as clear-cut results as we obtain here".*

[4] It may seem surprising that this dominance holds even when the informativeness of the signals is low, as then the positive informational externality is weak. However, in this event the pivot externality must be weak also, as the value to any voter of being pivotal is small, as his voting decision is based on a signal that is not very informative.

[5] In section 4, we demonstrate the robustness of this key result to the introduction of limited preference heterogeneity.

information efficiently because not enough voters, on the average, participate in the voting process. One policy implication of our result is that by providing voting subsidies and recovering the cost through lump-sum taxes, all voters can be made better-off.

Additional results that follow are that under some conditions, voting equilibria can be Pareto-ranked, with an equilibrium with more voters Pareto-dominating the equilibrium with fewer voters, on the average. We also show that there are conditions under which compulsory voting Pareto-dominates voluntary majority voting. By contrast, in Borgers (2001), the unique voluntary voting equilibrium always Pareto dominates compulsory majority voting.

Finally, show that our main result on the nature of inefficiency of voting equilibria is robust to the introduction of preference heterogeneity. Specifically, we consider a utility function that is a convex combination of the common value specification of this paper, and the private values specification of Borgers. We characterize the maximum weight on the private values component such that the model has a voting equilibrium where all agents vote with their signals, rather than according to their idiosyncratic preference. We are then able to show that as long as the private values component is weighted by less than this maximum, the main results of the paper all carry over to this considerably more general specification.

Apart from Borgers' paper, this paper is related to a few other recent papers on voting with incomplete information[6]. In a model of voting with common values with partisan voters and uninformed agents, Feddersen and Pesendorfer (1996)[7] show that voting does not aggregate information efficiently in a finite electorate. The presence of partisan voters and uninformed voters adds noise to the voting process. In the absence of partisan voters, uninformed voters would abstain, implying that majority voting would aggregate information efficiently: as participation is costless, the remaining, symmetrically informed, non-partisan voters would all vote. In their set-up, voting fails to aggregate information efficiently as *too many* voters participate. In contrast, here we show that even with symmetrically informed, non-partisan voters, majority voting may fail to aggregate information efficiently if participation is costly because there is *too little* voting on the average.

[6] We should also mention Osborne, Rosenthal and Turner (2000) who study a model of costly participation. However, the focus of our paper and the formal model differs from their paper. They do not explicitly model voting and agents have complete information. Moreover, they do not consider the efficiency of participation equilibria.

[7] Other papers on information aggregation include Feddersen and Pessendorfer (1997) and Dekel and Piccione (2000).

Our model is also closely related to that of Persico(2001), who considers the design of voting rules and committee size when committee members have to pay for informative signals[8]. Our model is more general in what it assumes about the cost of observing signals (costs may be heterogenous, and are privately observed, whereas in Persico's paper, there is a homogenous cost which is common knowledge), but otherwise, more special (it does not allow for asymmetry in priors or over the cost of different types of mistaken decisions). However, we are addressing a rather different issue; Persico studies the optimal design of a committee (numbers of members, voting rule) subject to the constraint that members are given the correct incentives to acquire information: we are looking at how information acquisition is sub-optimal, given a voting particular rule (majority voting)[9] and fixed size of the electorate.

In the next section, we set out the model. Section 3 characterizes participation equilibria. Section 4 contains the main results on the sign of the externalities and efficiency of equilibrium. Section 5 discusses the extension to the case of heterogenous preferences, and the last section concludes.

2. The Model

There is a set $N = \{1, ..n\}$ of agents, who can collectively choose between two alternatives, A and B. Voters have identical payoffs over alternatives, but their payoffs are state-dependent. Specifically, there are two states of nature s_A, s_B. The the payoff for all voters is a map $u : \{A, B\} \times \{s_A, s_B\} \rightarrow \Re$ such that $u(A, s_A) = u(B, s_B) = 0$ and -1 otherwise: so, there is a cost of making the wrong decision, normalized at 1.

Agents have identical priors over the two states: all believe that each state is equally likely. However, prior to the decision to vote, voters receive private signals about the state of nature. Specifically, each $i \in N$ privately observes signal $\sigma^i \in \{\sigma_A, \sigma_B\}$, where σ^i is uncorrelated with σ^j for all $i, j \in N$. We assume that signals are informative i.e. the probability of signal σ_k, conditional on state s_k is $q > 0.5,\ k = A, B$.

We also assume that participation in the election is costly: i.e. it is costly to attend a meeting, or go to a polling station. Specifically, each voter $i \in N$ incurs a privately observed cost of participation, c^i: if he wishes to vote, he must pay this cost[10]. We

[8] This paper by Persico extends the Condorcet jury literature (Austen-Smith and Banks (1996), Feddersen and Pessendorfer (1998)) to allow for endogenous acquisition of information.

[9] However, we strongly conjecture that majority voting is optimal in our set-up, in a well-defined sense: study of the *choice* of voting rule in our set-up is a topic for future work.

[10] Below, we argue that under very weak assumptions, this can also be interpreted as the cost of

assume that the c^i are independently and identically distributed across individuals: c^i is distributed on support $[\underline{c}, \bar{c}] \subset \Re_{++}$ with the probability distribution $F(c)$. Moreover, we also assume that c^i is independently distributed from σ^i for each individual i. The sequence of events is as follows.

Step 0. Each $i \in N$ privately observes his cost of voting, and decides whether to participate or not.

Step 1. The state of the world is realized, and each $i \in N$ privately observes her signal σ^i of the state.

Step 2. All i who have decided to participate[11], vote either for A or for B.

Step 3. The alternative with the most votes is selected. If both A, B get equal numbers of votes, each is selected with probability 0.5.

Note that Step 3 embodies the assumption that there is no distinguished status quo. In particular, if no-one votes, each alternative is selected with equal probability. We focus on the subgame-perfect Bayesian equilibria of the above game. Also, we impose three relatively weak assumptions[12] on strategies. First, we suppose all agents behave alike in equilibrium (anonymity). Second, we rule out randomization. Second, we assume that player's equilibrium strategy at the voting stage is weakly undominated. Call any subgame-perfect Bayesian equilibrium satisfying these three conditions a *participation equilibrium.*

3. Participation Equilibrium

In the above environment, the n voters play a two-stage game of incomplete information. We solve the game backwards in the usual way, so we begin with the voting subgame when potential voters have made their participation decisions.

purchasing, or observing, the signal, σ^i.

[11] We have not specified whether voters observe the total number of participants, say l, at Step 2. One possibility is that they do i.e. at Step 2, $i's$ information set is (c^i, σ^i, l). This is the natural assumption to make if voting takes place at a meeting of some kind. An alternative is that they do not i.e. at Step 2, $i's$ information set is (c^i, σ^i). This is the natural assumption to make if voting is in a general election or referendum, where members of the public attend polling stations. Under the assumptions made below on strategies in the voting subgame, it makes no difference which of these assumptions hold: there is always a unique equilibrium at the voting stage.

[12] We are following Borgers(2001) in making these three assumptions.

3.1. Voting

At stage 2, a strategy for i is of the form $\gamma : \{\sigma_A, \sigma_B\} \rightarrow \{A, B\}$.In this case, a voter has four possible strategies: (i) ignore her information i.e. choose either A or B independently of her signal: (ii) vote with her signal (i.e. vote for A if $\sigma^i = \sigma_A$ and for B otherwise): (iii) voter against her signal i.e. vote for B if $\sigma^i = \sigma_A$ and for B otherwise. It is easy to see that only (ii) is weakly undominated, as the others are all sub-optimal in the event that i is pivotal, and i is always pivotal for some γ of the other voters[13]. Following Persico(2001), we will call strategy (ii) *sincere voting.* So, the only possible voting equilibrium is where all participants vote sincerely.

3.2. Participation

Let p denote the ex-ante probability, before learning (σ^i, c^i), that any individual i participates. So, the probability that exactly l voters other than i have chosen to participate is given by

$$v(l : p) = \begin{pmatrix} n-1 \\ l \end{pmatrix} p^l (1-p)^{n-1-l}. \tag{3.1}$$

We now calculate the gain to participating relative to not for some i : when combined with the cost of participating, this will allow us to characterize $i's$ equilibrium voting strategy. *Case 1: l even.* Here, i is pivotal only when there is a tie i.e. exactly $\frac{l}{2}$ voters vote for A while the other $\frac{l}{2}$ voters vote for B. In this event, what is $i's$ gain to participating? If he does not vote, both alternatives will be selected with probability 0.5, and this yields him a payoff of 0.5. If he does vote, how will he vote? As he is pivotal, he knows that the l agents who vote have received $l/2$ signals in favor of A, and $l/2$ in favor of B. In this case, the voting behavior of other players conveys no additional information to voter i about the state of the world, and so he votes according to his signal i.e. sincerely, so he will select the correct alternative with probability q. So, his gain to voting is $q - 0.5$.

The probability that i is pivotal, given exactly l other voters and his private information, c^i, is calculated as follows. Obviously, c^i is uninformative about the signals received by other voters. Given that other voters vote according to their signals, the unconditional

[13]This fact depends crucially on the fact that the majority voting rule is statistically optimal in the model - Persico(2001). Were it not, a voter who is pivotal would generally not wish to vote sincerely.

probability that $l/2$ voters receive a signal in favor of each alternative is simply[14]

$$\pi(l:q) = \begin{pmatrix} l \\ \frac{l}{2} \end{pmatrix} q^{\frac{l}{2}}(1-q)^{\frac{l}{2}} \tag{3.2}$$

Case 2: l odd. In this case, the only situation where $i's$ is pivotal is when $\frac{l+1}{2}$ voters have voted for one alternative, and $\frac{l-1}{2}$ for the other. In this case, the voting behavior of other players *does* convey information to i. In particular, suppose that i has observed a signal in favor of A. The first possibility is where $\frac{l+1}{2}$ others are voting for A. Then, the event that i is pivotal implies that i has $\frac{l+1}{2}+1$ signals favoring A and $\frac{l-1}{2}$ signals favoring B. Therefore, voter i prefers A, but knows that he does not need to vote for alternative A to be selected; there is *already* a majority for alternative A. In the second case, player $i's$ updated information set has $\frac{l-1}{2}+1$ signals favoring A and $\frac{l+1}{2}$ signals favoring B. Therefore, voter i is indifferent between A and B. So, we conclude that voter i has *zero gain from voting when l is odd, even when he is pivotal. This is the weak swing voter's curse referred to in the introduction.*

The preceding discussion implies that the unconditional expected gain to voting is:

$$B(p) = (q - \frac{1}{2}) \sum_{l=0}^{n-1} v(l:p)\beta(l:q) \tag{3.3}$$

where

$$\beta(l:q) = \begin{cases} \pi(l:q) & l \text{ even} \\ 0 & l \text{ odd} \end{cases} \tag{3.4}$$

It is now clear that if all other voters play a voting strategy γ with voting probability p, then $i's$ (strict) best response is to vote if $c^i < B(p)$ and not if $c^i > B(p)$. Following Borgers, we call this a *cutoff strategy,* and we denote the cutoff generally by $\hat{c}$. Generally, c^* is an *equilibrium cutoff strategy* if $c \leq B(F(c^*))$, all $c \leq c^*$, and $c \geq B(F(c^*))$, all $c \geq c^*$. A symmetric Bayesian equilibrium in cutoff strategies is a γ^* where every voter votes according to his signal if $c \leq c^*$ and abstains otherwise. We can now show that there is at least one symmetric Bayesian equilibrium in threshold strategies.

[14] If $l = 0$, i is pivotal with probability 1, so we set $\pi(0:\rho) = 1$.

Proposition 1. *There is at least one symmetric Bayesian equilibrium in cutoff strategies. If c^* solves $B(F(c^*)) = c^*$, then c^* is an equilibrium cutoff. If $B(1) > \overline{c}$, then $c^* = \overline{c}$ is an equilibrium cutoff. If $q - \frac{1}{2} < \underline{c}$, then $c^* = \underline{c}$ is the unique equilibrium cutoff. Finally,*

$$B(1) = \begin{cases} (q - \frac{1}{2}) \begin{pmatrix} n-1 \\ (n-1)/2 \end{pmatrix} q^{\frac{n-1}{2}} (1-q)^{\frac{n-1}{2}} & \text{if } n-1 \text{ even} \\ 0 & \text{if } n-1 \text{ odd} \end{cases}$$

Proof. Existence of some equilibrium follows from the continuity of $B(F(.))$ on $[\underline{c}, \overline{c}]$. The remaining parts follow directly from the definition of equilibrium, except the last part. This follows from the fact that $\sum_{l=0}^{n-1} v(l:p)\beta(l:q) < 1$, so $B(p) < B(0)$, all $p > 0$, so if $B(0) < \underline{c}$, neither of the other types of equilibria are possible. Finally, the formula for $B(1)$ follows from (1.1)-(1.5). □

This result leaves open the possibility that multiple equilibria exist, and the following example confirms this.

Example 1 (**Multiple Equilibria**).
Assume $n = 3$, and that c is uniform on $[0, \overline{c}]$. In this case, from (3.3),(3.4), we have:

$$B(p) = (q - 0.5)[2p^2 q(1-q) + (1-p)^2] \tag{3.5}$$

Note that $p^* = F(c^*) = c^*/\overline{c}$, so assuming an interior equilibrium, the equilibrium condition $B(F(c)) = c$ can be rewritten in terms of p as $B(p) = p\overline{c}$, or explicitly as

$$(q - 0.5)[2p^2 q(1-q) + (1-p)^2] = p\overline{c} \tag{3.6}$$

This is a quadratic in p, with two roots:

$$p = \frac{(2+\alpha) \overset{+}{\underset{-}{}} \sqrt{(2+\alpha)^2 - 8q(1-q) - 4}}{2[2q(1-q)+1]} \tag{3.7}$$

where $\alpha = \overline{c}/(q - 0.5)$. If we take $q = 0.75$, and $\overline{c} = 0.09$, then it is easy to check that the two roots are

$$p^* = \frac{1.3119}{1.375}, \; p^{**} = \frac{1.0481}{1.375}$$

i.e. the voting game has two interior equilibria.Note also for these numbers that $B(1) = 0.09375 > \overline{c}$, so there is also a corner equilibrium where $p^{***} = 1$. All these equilibria are illustrated in Figure 1 below.‖

Figure 1 in here

In the preceding example, the multiple equilibria are due to the non-monotonicity of the benefit function $B(p)$. This is in contrast to the case of private values, where the benefit from voting is strictly decreasing in p, and hence there is a unique equilibrium (Borgers(2001), Proposition 2).

3.3. Comparing Common and Private Values

The equilibria of our model can in fact be compared to the voting equilibrium with private values in Borgers(2001). The first step is to note that the gain from voting with private values is, in our notation:

$$B_{PV}(p) = \frac{1}{2}\sum_{l=0}^{n-1} v(l:p)\pi(l:0.5) \tag{3.8}$$

Note three differences between (3.3) and (3.8). First, in the private values case, there is a benefit to voting even when the number of voters is odd. Second q is replaced by 0.5 in $\pi(l:.)$ as any voter cannot predict how any other will vote, given that he decides to vote at all. As $q(1-q)$ is maximized at $q = 0.5$, we can assert that $\pi(l:0.5) > \pi(l:q)$, all $q \neq 0.5$. Finally, the benefit from one's most preferred alternative relative to random selection rises from $q - 0.5$ to 0.5 as in the private values case, voters are *sure* which alternative is best. It is clear that all these three differences raise the benefit to voting in the private values case, so that it is always true that

$$B_{PV}(p) > B(p),\ 0 \leq p \leq 1 \tag{3.9}$$

Moreover, as shown by Borgers, $B_{PV}(p)$ is decreasing in p, and so there is always a unique equilibrium in the private values case. Let c_{PV} be the unique equilibrium cost cutoff in the private values case, and let $c_{\max}$ be the *highest* equilibrium cutoff in the common values case (this is well-defined by Proposition 1). Then we have:

Proposition 2. $c_{\max} \leq c_{PV}$, and $c_{\max} < c_{PV}$ *if* $c_{PV} < \bar{c}$.

Proof. Case 1. $B_{PV}(1) > \bar{c}$. Then $c_{PV} = \bar{c}$. Also, $B_{PV}(1) > B(1)$ by (3.9), so by Proposition 1, with common values, $c_{\max} \leq c_{PV}$.

Case 2. $B_{PV}(c_{PV}) = c_{PV}$, some $\tilde{c} \in [\underline{c}, \bar{c})$. In this case, as $B(p)$ lies everywhere below $B_{PV}(p)$, and $p = F(c)$ is increasing in c, $B(F(c)) < B_{PV}(F(c))$, $c \in [\underline{c}, \bar{c}]$. Therefore, if c^* *solves* $B(F(c^*)) = c^*$ then $c^* < c_{PV}$, and in particular $c_{\max} < c_{PV}$. □

So, in a well-defined sense, a switch from private to common values lowers the probability of voting in equilibrium, and thus the fraction of the electorate who vote in equilibrium, when n is large.

4. The Inefficiency of Participation Equilibria

The central and striking result with private values and costly voting is that the negative externalities from voting decisions imply that compulsory voting is never desirable (Borgers (2001)). An implication of this "global" result is a "local" one: starting at the Bayes-Nash equilibrium cutoff c^*, it is *always* Pareto-improving to lower the cutoff slightly, so that fewer agents vote on average. This could be implemented (for example) by taxing voting and returning the revenue as a lump-sum. Here, we investigate the robustness of this result.

4.1. Participation Externalities

We begin by defining the ex ante payoff to any citizen (i.e. prior to observing σ^i, c^i), ignoring participation costs, but conditional on participating or not participating. Consider the ex ante payoff to any citizen if exactly m citizens vote sincerely i.e. according to their signals. As all agents have identical prior beliefs that each state is equally likely and losses from type-I errors (choosing B when the state is s_A) and type-II errors ((choosing A when the state is s_B) are the same, the ex ante payoff is simply minus the cost of making the wrong decision (a type-I or type-II error), given majority voting:

$$u(m) = \begin{cases} -\sum_{k=(m+1)/2}^{m} \begin{pmatrix} m \\ k \end{pmatrix} (1-q)^k q^{m-k} & \text{if } m \text{ is odd} \\ -\sum_{k=\frac{m}{2}+1}^{m} \begin{pmatrix} m \\ k \end{pmatrix} (1-q)^k q^{m-k} + 0.5 \begin{pmatrix} m \\ k \end{pmatrix} (1-q)^{m/2} q^{m/2} & \text{if } m \text{ is even} \end{cases}$$

It is well-known that $u(m+1) > u(m)$ i.e. more signals there are, the lower the probability of error. Now, let $u_1(p), u_0(p)$ be the expected payoffs from participation and non-participation for a given citizen i respectively, *given that all $j \neq i$ participate with probability p*. These are:

$$u_0(p) = \sum_{m=0}^{n-1} f_m(p) u(m), \;\; u_1(p) = \sum_{m=0}^{n-1} f_m(p) u(m+1)$$

where $\{f_m(p)\}_{m=0}^{n-1}$ is the Binomial distribution, with parameters $n-1, p$.

If $u_1(p), u_0(p)$ are increasing (resp. decreasing) in p, we will say that there is a *positive* (resp. *negative*) participation externality. In the private-values setting of Borgers, the participation externality is unambiguously negative, as an increase in p reduces the probability that any voter is pivotal, and with private values, voters always prefer to be

pivotal. In the common values setting, the first part of this statement is still true. The ambiguity arises because the increase in p can increase the expected utility of a voter when he is *not* pivotal, as more information is brought to the decision.

This can be shown most clearly in the case of three voters. Then, by direct calculation, we have:

$$u_1(p) = (1-p)^2 q + 2p(1-p)(q^2 + 2q(1-q)0.5) + p^2[q^3 + 3q^2(1-q)] \tag{4.1}$$

The explanation is as follows.

- With probability $(1-p)^2$, only 1 votes, so the correct decision will be taken with probability q.
- With probability $2p(1-p)$, only 1 and either 2 or 3 vote, so the correct decision will be taken with probability 1 when both receive the correct signal (which occurs with probability q^2) or with probability half when both receive different signals
- With probability p^2, all voters vote, so the correct decision will be taken with probability 1 when all receive the correct signal (which occurs with probability q^3) or when two out of three receive the correct signal (which occurs with probability $3q^2(1-q)$)

Now (4.1) can be decomposed in the following way:

$$u_1(p) = q\Lambda(p) + p^2q^2, \ \Lambda(p) = \left\{1 - p^2 + p^2 2q(1-q)\right\} \tag{4.2}$$

where $\Lambda(p)$ is the probability that 1 is pivotal. To see this, note that 1 is *always* pivotal if no, or one other citizens participate, which occurs with probability $1-p^2$, and is pivotal when two other citizens participate *only* if they receive different signals, which occurs with probability $2q(1-q)$. This decomposition is also possible in the general case, although the formula is complex and unenlightening[15].

Clearly, $\Lambda(p)$ is decreasing in p. This is as in private values case. But, *unlike* in private values case, 1 receives a positive benefit from an increase in p when he is *not* pivotal, as term p^2q^2 is increasing in p. By computation, it follows that when $n = 3$, this second effect dominates the first, and the overall participation externality is positive. We can prove that it is also true in the general case.

[15] Generally, $u_1(p) = q\Lambda(p) + g(p)$, where $\Lambda(p)$ is the probability of being pivotal, and $g(p)$ is the expected payoff when not pivotal.

Proposition 3. *Both $u_1(p), u_0(p)$ are increasing in p for all $p \in [0,1)$. That is, with common values, the overall participation externality is positive.*

Proof. First, let $f_m(p)$ denote the probability that $0 \leq m \leq n-1$ other voters than i vote when the probability of participation is p. Clearly, these are the probabilities of a Binomial distribution with parameters $n-1, p$. So

$$u_1(p) = \sum_{m=0}^{n-1} f_m(p) u(m+1)$$

So,

$$u_1(p') - u_1(p) = \sum_{m=0}^{n-1} (f_m(p') - f_m(p)) u(m+1).$$

Now, for $p' > p$, $\{f_m(p')\}_{m=0}^{n-1}$ first-order stochastically dominates $\{f_m(p)\}_{m=0}^{n-1}$. As $u(m+1)$ is monotonically increasing in m, we know that[16]

$$u_1(p') - u_1(p) = \sum_{m=0}^{n-1} (f_m(p') - f_m(p)) u(m+1) \geq 0$$

The proof is the same for $u_0(p)$. □

4.2. Inefficiency of Equilibrium

In view of Proposition 3, one might expect any of the participation equilibria characterized in Proposition 1 to be inefficient, with too little participation, and here we prove that this is the case. The ex ante expected utility of any voter (i.e. prior to observation of (σ^i, c^i)) in an equilibrium with participation probability p is:

$$U(p) = (1-p)u_0(p) + pu_1(p) - \int_{\underline{c}}^{F^{-1}(p)} cf(c)dc \tag{4.3}$$

So, differentiating:

$$\begin{aligned} U'(\hat{c}) &= (1-p)u_0'(p) + pu_1'(p) + \left(u_1(p) - u_0(p) - F^{-1}(p)\right) \\ &= (1-p)u_0'(p) + pu_1'(p) + \left(B(p) - F^{-1}(p)\right) \end{aligned} \tag{4.4}$$

where the second line follows from the definition of $B(p)$ in Section 2 above. Evaluated at an interior Bayes-Nash equilibrium i.e. $\hat{c} = c^*$, $\underline{c} < c^* < \overline{c}$, the last term vanishes, as $p = F(c^*)$, so $B(p) - F^{-1}(p) = B(F(c^*)) - c^* = 0$. So, from (4.4), we have:

$$U'(F(c^*)) = (1 - F(c^*))u_0'(F(c^*)) + F(c^*)u_1'(F(c^*)) \tag{4.5}$$

[16] See, for instance, Hadar and Russell(1969), Theorem 1, or Rothschild and Stiglitz (1970).

But by Proposition 3, as u_0, u_1 are increasing in p, $U'(F(c^*)) > 0$. So, we have proved:

Proposition 4. *For all $1 \geq q > 0.5$, starting any interior symmetric Bayesian equilibrium $c^* \in (\underline{c}, \overline{c})$, a small increase in the cutoff $\hat{c}$ from c^* is always ex ante Pareto-improving.*

Proposition 4 contrasts sharply with Borgers' results. His global result with private values establishes that it is never optimal to force agents to vote i.e. to raise $\hat{c}$ to $\overline{c}$. However, the proof of this result also establishes the local result that a small *decrease* in the cutoff $\hat{c}$ from c^* is always ex ante Pareto-improving. In this sense, Proposition 4 shows how a move from private values to common values reverses the nature of the inefficiency of voting equilibria.

Now consider two symmetric voting rules with cutoffs c^* and c^{**} such that $c^* < c^{**}$. Then, the difference between the expected payoffs at the two equilibria can be written as

$$U(c^{**}) - U(c^*) = \int_{c^*}^{c^{**}} \left(u_0'(F(c)) + F(c)B'(F(c))\right) f(c)dc + \int_{c^*}^{c^{**}} \left(B(F(c)) - c\right) f(c)dc \tag{4.6}$$

where $U(c) \equiv U(F(C))$. By Proposition 3, we know that the first integral is positive. However, the sign of the second integral is ambiguous as $B(p)$ is, in general, non-monotonic. This makes it impossible to obtain a general Pareto-ranking of equilibria. In particular, we cannot show that, in general, a Bayesian equilibrium with a higher cutoff value Pareto dominates a Bayesian equilibrium with a lower cutoff value. In general, it is also not possible to show that compulsory majority voting Pareto dominates Bayesian equilibrium outcomes with voluntary majority voting. However, the following results can be stated.

Proposition 5. *Suppose that there are multiple voting equilibria as represented by cutoffs: $c_1 < .. < c_k < ... < c_m$. If either (a) $m \geq 2$, and $B(1) < \overline{c}$ or (b) $m \geq 3$, there is some k, $1 \leq k \leq m-1$, such that the voting equilibrium c_{k+1} Pareto dominates the voting equilibrium c_k. If $B(1) \geq \overline{c}$, then $c_m = \overline{c}$, and this equilibrium Pareto-dominates equilibrium c_{m-1} i.e. starting at c_{m-1}, imposing compulsory voting is Pareto-improving.*

Proof. As $B(1) < \overline{c}$ remark that at $p = F(c_m)$, $B'(p) < 0$. As $m \geq 2$, it follows that there is at least one Bayesian equilibrium with cutoff c_k, for some k, $1 \leq k \leq m-1$ so that $B'(p) > 0$, $p = F(c_k)$ for some $k < m$. As $B'(p) > 0$, $p = F(c_k)$, for some $k < m$, $B(F(c)) > c$, $c \in (c_k, c_{k+1})$. Alternatively, suppose there exist at least three voting equilibria. Then, there is at least one voting equilibrium with cutoff c_k so that $B'(p) \geq 0$, $p = F(c_k)$ for some $k < m$. As $B'(p) \geq 0$, $p = F(c_k)$, for some $k < m$, $B(F(c)) \geq c$, $c \in (c_k, c_{k+1})$.So, in both cases, from (4.6), $U(c_{k+1}) > U(c_k)$ i.e. the voting equilibrium with the cutoff c_{k+1} Pareto dominates the voting equilibrium with cutoff c_k. Next, given that $B(1) \geq \overline{c}$, $c_m = \overline{c}$ follows directly from Proposition 1. By definition

of c_m, c_{m-1}, $B(F(c)) \geq c$, $c \in (c_{m-1}, c_m)$. So, from (4.6), $U(\overline{c}) = U(c_m) > U(c_{m-1})$ i.e. compulsory voting Pareto-dominates voluntary voting equilibrium c_m.

Can compulsory voting lead to a Pareto-improvement when $\bar{c}$ is not a voting equilibrium threshold? The following example shows that this a robust possibility. In this example, there is a unique equilibrium with $\hat{c} < \overline{c}$, and starting at this equilibrium, imposing compulsory voting leads to a strict Pareto-improvement.

Example 2 (Compulsory Voting May be Desirable).
The Example is the same as Example 1 i.e. $n = 3$ and uniform distribution of costs. Ex ante payoffs in this example can be computed from formula (4.3), which in this case simplifies to

$$U(p) = u_0(p) + pB(p) - \frac{1}{\overline{c}} \int_0^{p\overline{c}} cdc = u_0(p) + pB(p) - \overline{c}p^2/2$$

for any voting probability p. We already have computed a formula for $B(p)$ i.e. (3.5) in Example 1. Also, note that

$$u_0(p) = 0.5(1-p)^2 + 2p(1-p)q + p^2 q$$

So, using (3.5), in the above formula, we conclude that

$$U(p) = 0.5(1-p)^2 + 2p(1-p)q + p^2 q + p(q-0.5)[2p^2 q(1-q) + (1-p)^2] - \overline{c}p^2/2 \quad (4.7)$$

Now let $q = 0.75$, and ψ be the value of c for which the larger root of (3.6) is equal to 1. This will be the value for which $B(1) = \psi$, and $B(1) = (q-0.5)2q(1-q) = 0.09375$. Then from Figure 1, it is clear that for $\overline{c} > \psi$, there will be a unique equilibrium given by the smaller root to (3.6): the larger root is greater than 1 and so cannot be an equilibrium probability. So, take $\overline{c} = 0.0938$. Then $\alpha = \overline{c}/(q-0.5) = 0.3752$. In this case, there is a unique interior equilibrium with voting probability given by the smaller root to (3.7) i.e.

$$p^* = \frac{0.99947}{1.375} = 0.72689 \quad (4.8)$$

Now substituting $\bar{c} = 0.0938$ and $q = 0.75$ in (4.7), after some simplification, we get:

$$U(p) = 0.5 + 0.75p - 0.7969p^2 + 0.34375p^3 \quad (4.9)$$

So, $U(1) = 0.79685 > 0.75613 = U(p^*)$ i.e. compulsory voting leads to a strict Pareto-improvement. Indeed, from (4.9), it can be shown that $U(p)$ is everywhere increasing in $p \in [0, 1]$. ||

Finally, we conclude this section by examining what happens when signals become uninformative. The following proposition shows that as signals become uninformative both the negative pivot externality and the positive information externality, and thus their overall effect, become negligible.

Proposition 6. *As signals become uninformative, i.e.* $q \to 0.5$, $U'(F(c^*))$ *tends to zero i.e. the welfare gain from raising participation from its equilibrium level becomes negligible .*

Proof. As $q \to 0.5$, $u(m) \to -0.5$, all m. So, $u_1(p), u_0(p) \to -0.5$, all $p \in [0,1]$. So, $u_1'(p), u_0'(p) \to 0$ and the result then follows from (4.5). □

On the face of it, this result is somewhat counter-intuitive as the informativeness of the signal becomes small, one would think that the positive information externality becomes small, but not necessarily the pivot externality. However, the pivot externality becomes small because the *value* of being pivotal (to the pivotal voter) goes to zero with $q - 0.5$: if one's signal is almost uninformative, the gain to being pivotal becomes negligible.

5. Preference Heterogeneity

It may be argued that while many collective decision problems have a common values component, voters almost always have personal or idiosyncratic preferences on issues. So, it is interesting to ask how robust our results are to the introduction of preference heterogeneity across voters[17]. In this section, we study this issue and obtain some results that go beyond the usual claim that results are robust to "small enough" perturbations in preferences. In particular, we can explicitly characterize *the size of the deviation away from common values (in a well-defined sense) such that voters are willing to ignore their personal preferences on the alternatives and vote with their signal.* In this event, our main results, Propositions 4 and 5, still hold.

We model preference heterogeneity as follows. Let a^i be a random draw from $\{A, B\}$ with $\Pr(a^i = A) = 0.5$, and define $u_0 : \{A, B\} \times \{A, B\} \to \Re$ with $u_0(L, a^i) = 0$, when $a^i \neq L$, and $u_0(L, a^i) = 1$ if $a^i = L$. We also assume that the $(a^1..a^n)$ are independently distributed. The interpretation of a^i is that it is an individual preference param-

[17]Note also that all agents have identical prior beliefs that each state is equally likely and losses from type-I errors (choosing B when the state is s_A) and type-II errors ((choosing A when the state is s_B) are the same. Relaxing these assumptions would effectively generate the model studied in the Condorcet Jury literature ((Austen-Smith and Banks (1996), Feddersen and Pessendorfer (1998)). In an earlier version of this paper, it was shown that our main results are robust to small perturbations away from equal priors and equal losses.

eter. In Borgers' (2001) model of pure private values, agents all have utility functions $u_0(a^i, K)$. Now define we define a new utility function:

$$u_i(K, s) = (1-\varepsilon)u(K, s) + \varepsilon u_0(a^i, K),\ K = A, B,\ 0 \leq \varepsilon \leq 1$$

where $u(K, s)$ is the common values utility defined above. We now define the *mixed preference* (common values, private values) model as following. At step 1, nature now generates a pair (a^i, σ^i) for each $i \in N$ which is transmitted privately to each i. In subsequent play, agents are assumed to have utility functions $(u_1 ... u_n)$ over actions and states. In all other respects, the mixed model is the same as the common values model defined in Section 2. Note that if $\varepsilon = 0$, the mixed model reduces to the common values model. Our robustness result is the following:

Proposition 7. *In the mixed preference model, there is an equilibrium where those participating vote according to their signal if the weight on private values is sufficiently small, i.e.* $\varepsilon < \min\{\hat{\varepsilon}, \tilde{\varepsilon}\}$, $\tilde{\varepsilon} = (q-0.5)/(1+q-0.5)$, $\hat{\varepsilon} = (\chi - 0.5)/(2+(\chi-0.5))$, *where* $\chi = q^2/(q^2+(1-q)^2)$. *Moreover, expected payoffs at this equilibrium to non-participants and participants are* $(1-\varepsilon)u_0(p) - 0.5\varepsilon$, $(1-\varepsilon)u_1(p) - 0.5\varepsilon$ *respectively, where* p *is the equilibrium participation probability. Consequently, for* $\varepsilon < \min\{\hat{\varepsilon}, \tilde{\varepsilon}\}$, *Propositions 4 and 5 apply in the mixed preference model.*

Proof. (i) *The bound on* ε. We proceed by calculating the gain to a voter i from voting according to her signal, rather than voting according to her private preference, conditional on fixed l. We make this calculation under the assumption that all $j \neq i$ vote according to their signals.

Case 1: l even. Here, i is pivotal only when there is a tie i.e. exactly $\frac{l}{2}$ voters vote for A while the other $\frac{l}{2}$ voters vote for B. The gain to voting according to the signal is lowest when the signal and the personal preference parameter are opposed i.e. $\sigma^i = \sigma_K$, $a^i = L$, $L \neq K$. Then, the payoff to voting according to the signal is $(1-\varepsilon)(q-1) - \varepsilon$. The payoff to voting according to personal preference is $-(1-\varepsilon)0.5$. So, it is preferable to vote according to personal preference if $\varepsilon < \tilde{\varepsilon} = (q-0.5)/(1+q-0.5)$.

Case 2: l odd. In this case, there are two subcases where i is pivotal. Assume w.l.o.g. that $i's$ signal is in favor of alternative A. The first subcase is where $\frac{l+1}{2}$ voters have voted for A, and $\frac{l-1}{2}$ for B. In this case, i infers that the probability of A is $\chi = q^2/(q^2+(1-q)^2)$, as he has observed two signals more signals in favor of q than against[18]. Then, the payoff

[18] Formally, χ is equal to the posterior probability that the state is (say) A, given that there are $l+1$ (resp. $l-1$) signals in favour of A (resp. B). Using Bayes' rule, after some simplification, we get the formula in the text.

to voting according to signal is $(1-\varepsilon)(\chi-1)-\varepsilon$. The payoff to voting according to personal preference is $(1-\varepsilon)0.5$.

The second subcase is where $\frac{l+1}{2}$ voters have voted for B, and $\frac{l-1}{2}$ for A. In this case, i infers that the probability of A is 0.5, as he has effectively observed equal numbers of signals in favor of A and B. Then, the payoff to voting according to signal is $(1-\varepsilon)(0.5-1)-\varepsilon$, and the payoff to voting according to personal preference is $-(1-\varepsilon)0.5$.

Conditional on l odd, i does not know which of these two subcases has occurred when he decides whether to vote with his signal or with his personal preference. But, as the signals are i.i.d., knowing σ^i does not help predict the signals of others. So, these two events will be equally likely. So, the overall expected gain to i from voting according to his signal, rather than for his personal preference, is always at least

$$\Delta = 0.5(1-\varepsilon)(0.5-1)+0.5(1-\varepsilon)(\chi-1)-\varepsilon+(1-\varepsilon)0.5$$

So, $\Delta > 0$ if $\varepsilon < \hat{\varepsilon} = (\chi-0.5)/(2+(\chi-0.5))$. So, if $\varepsilon < \min\{\hat{\varepsilon},\tilde{\varepsilon}\}$, the best response to all $j \neq i$ voting with their signals is for i to vote with her signal, *whatever* l.

(ii) *Remainder of proof.* In an equilibrium where all participants vote according to their signals, the expected value of the common value component of u_i is $u_0(p)$ if i does not participate, and $u_1(p)$ if she does. The expected value of the private value component of u_i is simply -0.5, as the collective decision is taken on information that is uncorrelated with a^i. The formulae for equilibrium payoffs follow immediately. These formulae say that equilibrium payoffs are just affine transformations of equilibrium payoffs in the original common game, and it is clear from the inspection of proofs of Propositions 4 and 5 that these proofs are unaffected by affine transformations of payoffs. □

Several points on the bound $\min\{\hat{\varepsilon},\tilde{\varepsilon}\}$ are worth noting. First, it only depends on the informativeness of the signal, q. As the signal is informative $q > 0.5$, $\chi > 0.5$, and so $\hat{\varepsilon},\tilde{\varepsilon} > 0$ always: moreover, upper bounds on $\hat{\varepsilon},\tilde{\varepsilon}$ is when $q = 1$, in which case $\hat{\varepsilon} = 1/5$, $\tilde{\varepsilon} = 1/3$.

6. Conclusion

In this paper, we have shown that in a model of costly voting with common values, the nature of the inefficiency of voting equilibrium identified in Borgers (2001) is reversed: in the vicinity of a Bayesian equilibrium, it is *always* Pareto-improving for more agents, on the average, to vote. In addition, we have also shown that there Pareto ranked multiple Bayesian equilibria can exists and moreover, compulsory majority voting can Pareto dominate voluntary majority voting. The key behind all the results in this paper lies in the

finding that there are two different externalities at work: the negative "pivot" externality identified by Borgers (2001) and the positive information externality. In the vicinity of a Bayesian equilibrium, the positive informational externality always outweighs the negative "pivot" externality implying that too few voters, on the average, participate in the voting process.

References

[1] Austen-Smith, D. and J.S. Banks (1996), *"Information Aggregation, Rationality and the Condorcet jury theorem"*, *American Political Science Review, 90, pp.34-45.*

[2] Borgers, T. (2001), *"Costly Voting"*, mimeo, University College London.

[3] Dekel, E. and M. Piccione (2000), *"Sequential Voting Procedures in Symmetric Binary Elections"*, *Journal of Political Economy, 108(1), pp.34-55.*

[4] Feddersen, T and W. Pessendorfer (1996), *"The Swing Voter's Curse"*, *American Economic Review, 86(3), pp.3408-424.*

[5] Feddersen, T and W. Pessendorfer (1997), *"Voting Behavior and Information Aggregation in Elections with Private Information"*, *Econometrica, 65(5), pp.1029-1058.*

[6] Feddersen, T and W. Pessendorfer (1998) *"Convicting the Innocent: the Inferiority of Unanimuous Jury Verdicts under Strategic Voting"*, *American Political Science 92(1), pp.25-35.*

[7] Hadar,J. and W.R.Russell (1969), "Rules for Ordering Uncertain Prospects", *American Economic Review, 59(1), pp. 25-34.*

[8] Lohmann, S. (1994), *"Information Aggregation with Costly Political Action"*, *American Economic Review, 84(1), pp.518-530.*

[9] Osborne, M. and J. Rosenthal and M. Turner (2000), *"Meetings with Costly Participation"*, *American Economic Review, 90(4), pp.927-943.*

[10] Persico, N., (2001), "Committee Design with Endogeous Information", *Review of Economic Studies* , forthcoming

[11] Rothschild, M. and Stiglitz, J. (1970), *"Increasing Risk I: A definition"*, *Journal of Economic Theory, 2, pp.225-243.*
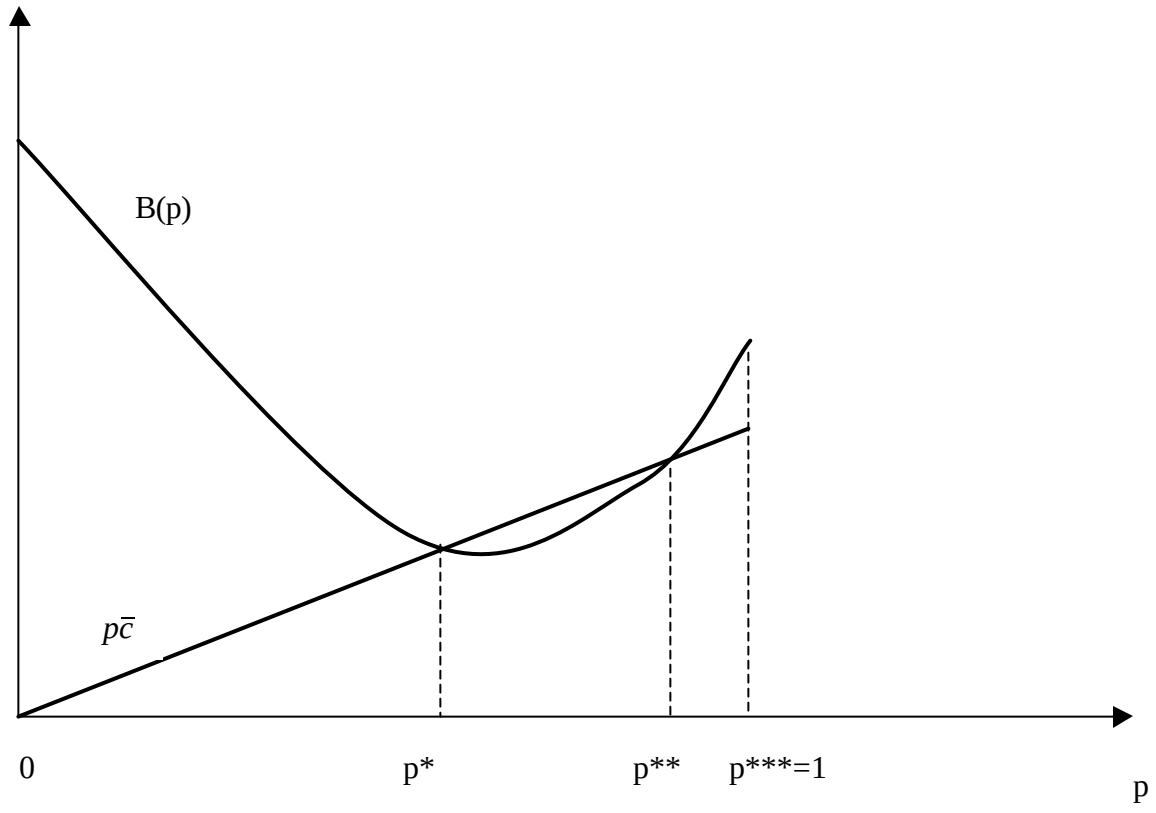
Figure 1 : Multiple Symmetric Bayesian Equilibria