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# Testing for seasonal unit roots in heterogeneous panels

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## ABSTRACT

This paper uses the approach of Im, Pesaran and Shin (2003) to propose seasonal unit root tests for dynamic heterogeneous panels based on the means of the individuals HEGY test statistics. The standardised t-bar and F-bar statistics are simply averages of the HEGY tests across groups. These statistics converge to standard normal variates.

Keywords: Heterogeneous dynamic panels; Monte Carlo; seasonal unit roots;

JEL classification: C12; C15; C22; C23

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## 1. Introduction

Im, Pesaran and Shin (2003) (IPS) proposed a test for the presence of unit roots in panels, that combines information from the time-series dimension with that from the cross-section dimension, such that fewer time observations are required for the test to have power.

Many economic time series contain important seasonal components and a variety of tests have been proposed to test for seasonal unit roots see Osborn and Ghysels (2001) for a review of these tests. Of these tests the one proposed by Hylleberg, Engle, Granger and Yoo (1990) (HEGY) has proved to be the most popular.

In this paper, we look at using the approach of IPS to investigate the performance of the HEGY test in dynamic heterogeneous panels. Based on Monte Carlo simulations we find that the standardised averaged test statistics from the HEGY auxiliary regression follow a standard normal distribution even for a relatively small number of data points.

The plan of the paper is as follows. Section 2 briefly reviews the IPS approach to unit root testing in panels and sets up the model used to develop the HEGY panel seasonal unit root tests. Section 3 presents the Monte Carlo results.

## 2. IPS unit root test and basic framework

IPS presented a method to test for the presence of unit roots in dynamic heterogeneous panels. They consider a sample of  $N$  cross section units observed over  $T$  time periods. The IPS test averages the (Augmented) Dickey-Fuller statistic obtained across the  $N$  cross-sectional units of the panel (denoted as  $\tilde{tbar}_{NT} = \frac{1}{N} \sum_{i=1}^N \tilde{t}_{iT}$ , where  $\tilde{t}_{iT}$  is the ADF test for the  $i^{\text{th}}$  cross-sectional unit). and show that a suitable

standardisation of the  $\tilde{tbar}_{NT}$  statistic, denoted as  $Z_{\tilde{tbar}}$ , follows a standard normal distribution.

Generalising the HEGY test for seasonal unit roots, to a panel in which there is sample of  $N$  cross sections (industries, countries) observed over  $T$  time periods:

$$\varphi_i(L)y_{4it} = \mu_{it} + \pi_{1i}y_{1it-1} + \pi_{2i}y_{2it-1} + \pi_{3i}y_{3it-2} + \pi_{4i}y_{3it-1} + \varepsilon_{it}, i = 1, \dots, N, t = 1, \dots, T \quad (1)$$

$$\text{where } \mu_{it} = \alpha_i + \beta_i t + \sum_{j=1}^{s-1} \gamma_{is} D_{st}, \quad D_{st} = \begin{cases} 1 & \text{in season } s \\ 0 & \text{otherwise} \end{cases}, \quad \varphi_i(L) \text{ is a } p_i^{\text{th}} \text{ ordered}$$

polynomial in the lag operator,  $L$ ,  $\varepsilon_{it} \sim N(0, \sigma_{\varepsilon_i}^2)$  and  $y_{1it} = y_{it} + y_{it-1} + y_{it-2} + y_{it-3}$ ,

$$y_{2it} = -y_{it} + y_{it-1} - y_{it-2} + y_{it-3}, \quad y_{3it} = -y_{it} + y_{it-2} \text{ and } y_{4it} = \Delta_4 y_{it} = y_{it} - y_{it-4}.$$

HEGY test for the existence of a unit root by testing  $H_0 : \pi_1 = 0$  against  $H_1 : \pi_1 < 0$ , and for the existence of a seasonal unit root by testing  $H_0 : \pi_2 = 0$  against  $H_1 : \pi_2 < 0$  and simultaneously testing  $H_0 : \pi_3 = \pi_4 = 0$  against  $H_1 : \pi_3 < 0, \pi_4 \neq 0$ . A null hypothesis of a seasonal unit root is only rejected when both the t-test for  $\pi_2$  and the joint F-test for  $\pi_3$  and  $\pi_4$  are rejected. Subsequently, Ghysels *et. al.* (1994) suggest using a test of  $H_0 : \pi_2 = \pi_3 = \pi_4 = 0$  against  $H_1 : \pi_2 < 0, \pi_3 < 0, \pi_4 \neq 0$ .

In a panel context, the null hypothesis to test the presence of a unit root, for example, becomes  $H_0 : \pi_{1i} = 0 \quad \forall i$  against  $H_0 : \pi_{1i} < 0$  for  $i = 1, 2, \dots, N_1$ ,  $\pi_{1i} = 0$ , for  $i = N_1 + 1, N_1 + 2, \dots, N$ . This allows some, but not all, of the individual series to have a unit root, but assumes that a non-zero fraction of the processes are stationary.

### 3. Monte Carlo simulation results

In this section we undertake Monte Carlo simulation to examine the finite sample properties of the HEGY-IPS test. Simulations are undertaken under the null hypothesis,  $\pi_{1i} = \pi_{2i} = \pi_{3i} = \pi_{4i} = 0$  in equation (1):

$$y_{it} - y_{it-4} = \mu_{it} + \sum_{j=1}^{p_i} \varphi_{ji} \Delta_4 y_{it} + \varepsilon_{it}, \quad i = 1, \dots, N, t = 1, \dots, T \quad (2)$$

where  $\varepsilon_{it} \sim N(0, \sigma_{\varepsilon_i}^2)$  and  $\sigma_{\varepsilon_i}^2 \sim U[0.5, 1.5]$ , and  $\sigma_{\varepsilon_i}^2$  are generated independently of  $\varepsilon_{it}$  and are fixed for all replications, where  $N = (5, 7, 10, 15, 25, 40)$  and  $T = (20, 32, 40, 60, 100)$ .

The HEGY statistics from estimating equation (1) for the  $i^{\text{th}}$  group are given by the t-ratios on  $\pi_{ji}$ ,  $j = 1, 2$  and the F-tests of the joint significance of  $\pi_{2i}, \pi_{3i}$  and  $\pi_{2i}, \pi_{3i}, \pi_{4i}$ . Denote the estimated t-ratio as  $\tilde{t}_{jiT}$ ,

$$\tilde{t}_{jiT} = \frac{\hat{\pi}_j - 0}{se(\hat{\pi}_j)} \quad j = 1, 2$$

and the F-test as  $\tilde{F}_{jiT}$ ,

$$F_{jiT} = (R_j \hat{\pi}_i)' \left[ R_j \hat{V}_{\hat{\pi}_i} R_j' \right]^{-1} (R_j \hat{\pi}_i) / j, \quad j = 2, 3$$

where  $R_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ ,  $R_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ ,  $\hat{\pi}_i' = (\hat{\pi}_{1i} \quad \hat{\pi}_{2i} \quad \hat{\pi}_{3i} \quad \hat{\pi}_{4i})'$  and the

estimated variance-covariance matrix from equation (1), is written in partitioned form

$$\text{as: } \hat{V}_i = \begin{bmatrix} \hat{V}_{\pi_i} & \hat{C}_{\pi_i \mu_i} & \hat{C}_{\pi_i \varphi_i} \\ \hat{C}_{\pi_i \mu_i} & \hat{V}_{\mu_i} & \hat{C}_{\mu_i \varphi_i} \\ \hat{C}_{\pi_i \varphi_i} & \hat{C}_{\mu_i \varphi_i} & \hat{V}_{\varphi_i} \end{bmatrix},$$

where, for example,  $\hat{V}_{\pi_i}$  is the estimated  $(4 \times 4)$  variance-covariance matrix for the coefficients on  $\pi_i$ , and  $\hat{C}_{\pi_i \mu_i}$  is the estimated variance-covariance matrix between the  $\pi_i$  and  $\mu_i$  terms.

For a fixed  $T$  define the average statistics:

$$\tilde{t}_j \text{bar}_{NT} = \frac{1}{N} \sum_{i=1}^N \tilde{t}_{jiT} \quad j=1,2$$

and

$$\tilde{F}_j \text{bar}_{NT} = \frac{1}{N} \sum_{i=1}^N \tilde{F}_{jiT}, \quad j=2,3.$$

Following IPS, consider the standardised statistics:

$$W_{\tilde{t}_j \text{bar}} = \frac{\sqrt{N} \left\{ \tilde{t}_j \text{bar}_{NT} - \frac{1}{N} \sum_{i=1}^N E[\tilde{t}_{jiT}(p_i, 0 | \pi_i = 0)] \right\}}{\sqrt{\frac{1}{N} \sum_{i=1}^N \text{Var}[\tilde{t}_{jiT}(p_i, 0 | \pi_i = 0)]}} \Rightarrow N(0,1), \quad j=1,2$$

and

$$W_{\tilde{F}_j \text{bar}} = \frac{\sqrt{N} \left\{ \tilde{F}_j \text{bar}_{NT} - \frac{1}{N} \sum_{i=1}^N E[\tilde{F}_{jiT}(p_i, 0 | \pi_i = 0)] \right\}}{\sqrt{\frac{1}{N} \sum_{i=1}^N \text{Var}[\tilde{F}_{jiT}(p_i, 0 | \pi_i = 0)]}} \Rightarrow N(0,1), \quad j=2,3$$

where  $E[\tilde{t}_{jiT}(p_i, 0 | \pi_i = 0)]$  ( $E[\tilde{F}_{jiT}(p_i, 0 | \pi_i = 0)]$ ) and  $\text{Var}[\tilde{t}_{jiT}(p_i, 0 | \pi_i = 0)]$  ( $\text{Var}[\tilde{F}_{jiT}(p_i, 0 | \pi_i = 0)]$ ) are the mean and variance of  $\tilde{t}_{jiT}$  ( $\tilde{F}_{jiT}$ ) in the HEGY model, when  $\pi_{1i} = \pi_{2i} = \pi_{3i} = \pi_{4i} = 0$ .

Table 1 reports the values of  $E[\tilde{t}_{jiT}(p_i, 0 | \pi_i = 0)]$  and  $\text{Var}[\tilde{t}_{jiT}(p_i, 0 | \pi_i = 0)]$ ,  $j=1,2$  and  $E[\tilde{F}_{jiT}(p_i, 0 | \pi_i = 0)]$  and  $\text{Var}[\tilde{F}_{jiT}(p_i, 0 | \pi_i = 0)]$ ,  $j=2,3$ , for different values of  $T$  and  $p$ , and for different combinations of deterministic components in the HEGY model. These results are based upon 20,000 replications.

Through simulations it appears that in the HEGY model (when  $p_i = 0$ ), the second moment of  $t_{jiT}$  exists only for  $T \geq 16$  (when there is a constant and trend) and for  $T \geq 20$  (when there are seasonal dummy variables). In addition, for the second

moment of  $F_{jIT}$  to exist requires at least  $T \geq 20$  for all combinations of the deterministic components.

We now consider three Monte Carlo experiments to examine the size and power (at the 5% significance level) of the HEGY-IPS test, using 5,000 replications. Table 2 reports the size of the tests when there is no serial correlation and the model includes a constant and a constant and trend as deterministic components. The tests for both  $W_{t_1,bar}$  and  $W_{t_2,bar}$  are approximately correctly sized. However, both the  $W_{F_2,bar}$  and the  $W_{F_3,bar}$  tests are slightly over-sized especially for smaller  $N$  and  $T$ . This table also reports the power of the HEGY-IPS test, when the data is generated as  $y_{it} = 0.9y_{it-4} + \varepsilon_{it}$ ,  $i = 1, \dots, N, t = 1, \dots, T$ .

In a second set of experiments, we allow for the presence of heterogeneous AR(1) serial correlation in  $\varepsilon_{it}$ , such that,

$$\varepsilon_{it} = \rho_i \varepsilon_{it-1} + \eta_{it}, \quad i = 1, \dots, N, t = 1, \dots, T$$

where  $\eta_{it} \sim N(0, \sigma_{\eta_i}^2)$ ,  $\rho_i \sim U[0.2, 0.4]$  and  $\rho_i$  is generated independently of  $\eta_{it}$ . Table 3 reports the size of the HEGY-IPS test for  $p=0,1,2,3,4$ , when there is only a constant in the HEGY model. The table demonstrates the importance of not underestimating the order of the lag length, with the empirical size for  $p=0$ , substantially different from the nominal 5%, with  $W_{t_1,bar}$  markedly under-sized, but all of the other tests becoming increasingly over-sized as  $T$  increases. There are little costs in terms of size to over-specifying the lag length, with the empirical size of  $W_{F_2,bar}$  and  $W_{F_3,bar}$  actually improving. However, the power of all of the tests falls with an over-specified lag length.

In a third set of experiments, we allow for the presence of heterogeneous MA(1) serial correlation in  $\varepsilon_{it}$ , such that,

$$\varepsilon_{it} = \theta_i \eta_{it-1} + \eta_{it}, \quad i = 1, \dots, N, t = 1, \dots, T$$

where  $\eta_{it} \sim N(0, \sigma_{\eta_i}^2)$ ,  $\theta_i \sim U[-0.4, -0.2]$  and  $\theta_i$  is generated independently of  $\eta_{it}$ .

Table 4 reports the size of the HEGY-IPS test for  $p=0,1,2,3,4$ . In this case there are severe size distortions for  $p=0$ , with  $W_{t,bar}$  massively over-sized. The other tests are also over-sized and this becomes increasingly so as both  $T$  and  $N$  increase. Increasing  $p$  improves the size of these tests, but even for  $p=3$  there is consistent evidence that  $W_{t,bar}$ ,  $W_{F_2,bar}$  and  $W_{F_3,bar}$  are all still marginally over-sized.

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Table 1: Mean and variance correction for  $t_1bar$  and  $t_2bar$

| $p$  | $t_1bar$ |        |        |        |         | $t_2bar$ |        |        |        |         |
|--|----------|--------|--------|--------|---------|----------|--------|--------|--------|---------|
|  | $T=20$   | $T=32$ | $T=40$ | $T=60$ | $T=100$ | $T=20$   | $T=32$ | $T=40$ | $T=60$ | $T=100$ |
| Constant, seasonal dummies, trend          |          |        |        |        |         |          |        |        |        |         |
| 0 mean                                     | -1.862   | -2.004 | -2.042 | -2.091 | -2.130  | -1.320   | -1.416 | -1.443 | -1.474 | -1.501  |
| 0 variance                                 | 0.725    | 0.622  | 0.604  | 0.575  | 0.567   | 0.720    | 0.667  | 0.676  | 0.673  | 0.684   |
| 1 mean                                     | -1.721   | -1.915 | -1.972 | -2.044 | -2.102  | -1.107   | -1.277 | -1.330 | -1.398 | -1.454  |
| 1 variance                                 | 0.751    | 0.629  | 0.611  | 0.578  | 0.567   | 0.734    | 0.683  | 0.691  | 0.687  | 0.693   |
| 2 mean                                     | -1.692   | -1.903 | -1.965 | -2.042 | -2.101  | -1.178   | -1.332 | -1.376 | -1.431 | -1.475  |
| 2 variance                                 | 0.848    | 0.666  | 0.641  | 0.595  | 0.575   | 0.793    | 0.710  | 0.715  | 0.700  | 0.706   |
| 3 mean                                     | -1.561   | -1.820 | -1.898 | -1.996 | -2.073  | -0.994   | -1.207 | -1.273 | -1.360 | -1.431  |
| 3 variance                                 | 0.900    | 0.685  | 0.656  | 0.605  | 0.580   | 0.825    | 0.738  | 0.735  | 0.713  | 0.719   |
| 4 mean                                     | -1.517   | -1.817 | -1.903 | -2.007 | -2.084  | -1.054   | -1.264 | -1.326 | -1.401 | -1.459  |
| 4 variance                                 | 1.002    | 0.732  | 0.684  | 0.623  | 0.591   | 0.890    | 0.766  | 0.756  | 0.724  | 0.724   |
| Constant, seasonal dummies, no trend       |          |        |        |        |         |          |        |        |        |         |
| 0 mean                                     | -1.314   | -1.406 | -1.432 | -1.470 | -1.493  | -1.317   | -1.407 | -1.435 | -1.468 | -1.496  |
| 0 variance                                 | 0.737    | 0.697  | 0.685  | 0.682  | 0.688   | 0.739    | 0.687  | 0.690  | 0.682  | 0.690   |
| 1 mean                                     | -1.210   | -1.339 | -1.378 | -1.434 | -1.471  | -1.210   | -1.340 | -1.380 | -1.431 | -1.475  |
| 1 variance                                 | 0.766    | 0.720  | 0.705  | 0.697  | 0.694   | 0.764    | 0.704  | 0.707  | 0.697  | 0.700   |
| 2 mean                                     | -1.184   | -1.325 | -1.369 | -1.429 | -1.468  | -1.183   | -1.327 | -1.371 | -1.426 | -1.472  |
| 2 variance                                 | 0.818    | 0.750  | 0.724  | 0.712  | 0.703   | 0.814    | 0.732  | 0.732  | 0.710  | 0.713   |
| 3 mean                                     | -1.093   | -1.266 | -1.320 | -1.395 | -1.447  | -1.093   | -1.267 | -1.322 | -1.392 | -1.451  |
| 3 variance                                 | 0.862    | 0.771  | 0.746  | 0.727  | 0.717   | 0.860    | 0.758  | 0.753  | 0.724  | 0.726   |
| 4 mean                                     | -1.070   | -1.262 | -1.321 | -1.400 | -1.452  | -1.071   | -1.263 | -1.323 | -1.397 | -1.456  |
| 4 variance                                 | 0.919    | 0.804  | 0.768  | 0.744  | 0.726   | 0.922    | 0.793  | 0.777  | 0.736  | 0.732   |
| Constant, no seasonal dummies, trend       |          |        |        |        |         |          |        |        |        |         |
| 0 mean                                     | -1.886   | -2.007 | -2.044 | -2.093 | -2.134  | -0.253   | -0.300 | -0.319 | -0.347 | -0.377  |
| 0 variance                                 | 0.707    | 0.625  | 0.613  | 0.587  | 0.577   | 0.897    | 0.907  | 0.928  | 0.945  | 0.962   |
| 1 mean                                     | -1.893   | -2.017 | -2.051 | -2.096 | -2.133  | -0.191   | -0.246 | -0.270 | -0.308 | -0.349  |
| 1 variance                                 | 0.746    | 0.657  | 0.640  | 0.597  | 0.582   | 1.036    | 0.997  | 0.995  | 0.986  | 0.985   |
| 2 mean                                     | -1.885   | -2.019 | -2.057 | -2.102 | -2.136  | -0.277   | -0.314 | -0.330 | -0.354 | -0.380  |
| 2 variance                                 | 0.826    | 0.694  | 0.672  | 0.617  | 0.591   | 0.839    | 0.886  | 0.913  | 0.939  | 0.964   |
| 3 mean                                     | -1.960   | -2.050 | -2.077 | -2.109 | -2.138  | -0.187   | -0.250 | -0.275 | -0.313 | -0.351  |
| 3 variance                                 | 0.977    | 0.747  | 0.706  | 0.638  | 0.600   | 0.969    | 0.966  | 0.975  | 0.978  | 0.987   |
| 4 mean                                     | -1.624   | -1.852 | -1.926 | -2.019 | -2.091  | -0.189   | -0.254 | -0.281 | -0.321 | -0.359  |
| 4 variance                                 | 0.979    | 0.758  | 0.711  | 0.642  | 0.603   | 0.941    | 0.949  | 0.958  | 0.966  | 0.977   |
| Constant, no seasonal dummies, no trend    |          |        |        |        |         |          |        |        |        |         |
| 0 mean                                     | -1.322   | -1.407 | -1.433 | -1.472 | -1.495  | -0.241   | -0.290 | -0.309 | -0.339 | -0.371  |
| 0 variance                                 | 0.761    | 0.737  | 0.719  | 0.711  | 0.706   | 0.985    | 0.962  | 0.970  | 0.972  | 0.978   |
| 1 mean                                     | -1.335   | -1.412 | -1.436 | -1.471 | -1.493  | -0.214   | -0.266 | -0.288 | -0.322 | -0.359  |
| 1 variance                                 | 0.811    | 0.765  | 0.744  | 0.726  | 0.714   | 1.044    | 1.003  | 1.000  | 0.990  | 0.988   |
| 2 mean                                     | -1.326   | -1.409 | -1.435 | -1.471 | -1.493  | -0.262   | -0.302 | -0.319 | -0.346 | -0.374  |
| 2 variance                                 | 0.860    | 0.797  | 0.766  | 0.743  | 0.723   | 0.924    | 0.940  | 0.955  | 0.966  | 0.980   |
| 3 mean                                     | -1.354   | -1.420 | -1.441 | -1.472 | -1.491  | -0.221   | -0.274 | -0.295 | -0.328 | -0.362  |
| 3 variance                                 | 0.934    | 0.829  | 0.791  | 0.758  | 0.736   | 0.969    | 0.970  | 0.980  | 0.983  | 0.991   |
| 4 mean                                     | -1.128   | -1.284 | -1.335 | -1.408 | -1.457  | -0.180   | -0.245 | -0.272 | -0.313 | -0.353  |
| 4 variance                                 | 0.965    | 0.863  | 0.818  | 0.778  | 0.747   | 1.030    | 1.007  | 1.004  | 0.994  | 0.993   |
| No constant, no seasonal dummies, no trend |          |        |        |        |         |          |        |        |        |         |
| 0 mean                                     | -0.223   | -0.281 | -0.299 | -0.332 | -0.367  | -0.226   | -0.278 | -0.299 | -0.331 | -0.367  |
| 0 variance                                 | 1.071    | 1.034  | 1.027  | 1.010  | 0.995   | 1.089    | 1.020  | 1.014  | 0.999  | 0.993   |
| 1 mean                                     | -0.240   | -0.290 | -0.306 | -0.336 | -0.368  | -0.242   | -0.287 | -0.306 | -0.334 | -0.367  |
| 1 variance                                 | 1.030    | 1.018  | 1.021  | 1.007  | 0.995   | 1.045    | 1.008  | 1.005  | 0.996  | 0.994   |
| 2 mean                                     | -0.242   | -0.293 | -0.309 | -0.339 | -0.370  | -0.244   | -0.290 | -0.309 | -0.338 | -0.370  |
| 2 variance                                 | 1.004    | 1.008  | 1.011  | 1.002  | 0.994   | 1.024    | 0.999  | 0.999  | 0.993  | 0.995   |
| 3 mean                                     | -0.254   | -0.300 | -0.315 | -0.342 | -0.371  | -0.256   | -0.297 | -0.315 | -0.341 | -0.370  |
| 3 variance                                 | 0.958    | 0.986  | 1.001  | 0.997  | 0.994   | 0.974    | 0.978  | 0.988  | 0.990  | 0.997   |
| 4 mean                                     | -0.169   | -0.239 | -0.265 | -0.308 | -0.350  | -0.171   | -0.237 | -0.264 | -0.307 | -0.349  |
| 4 variance                                 | 1.124    | 1.076  | 1.065  | 1.033  | 1.009   | 1.140    | 1.069  | 1.050  | 1.022  | 1.008   |

Table 1 (cont'd): Mean and variance correction for  $F_2\bar{bar}$  and  $F_3\bar{bar}$

| $p$  | $F_2\bar{bar}$ |        |        |        |         | $F_3\bar{bar}$ |        |        |        |         |
|--|----------------|--------|--------|--------|---------|----------------|--------|--------|--------|---------|
|  | $T=20$         | $T=32$ | $T=40$ | $T=60$ | $T=100$ | $T=20$         | $T=32$ | $T=40$ | $T=60$ | $T=100$ |
| Constant, seasonal dummies, trend          |                |        |        |        |         |                |        |        |        |         |
| 0 mean                                     | 2.583          | 2.728  | 2.784  | 2.861  | 2.924   | 2.745          | 2.839  | 2.876  | 2.926  | 2.970   |
| 0 variance                                 | 5.433          | 4.111  | 3.982  | 3.903  | 3.738   | 4.822          | 3.294  | 3.055  | 2.815  | 2.603   |
| 1 mean                                     | 2.481          | 2.653  | 2.723  | 2.821  | 2.899   | 2.521          | 2.674  | 2.740  | 2.832  | 2.912   |
| 1 variance                                 | 5.361          | 3.985  | 3.870  | 3.841  | 3.703   | 4.342          | 2.985  | 2.822  | 2.682  | 2.528   |
| 2 mean                                     | 2.173          | 2.412  | 2.518  | 2.676  | 2.808   | 2.529          | 2.668  | 2.734  | 2.827  | 2.909   |
| 2 variance                                 | 5.054          | 3.668  | 3.621  | 3.674  | 3.603   | 4.771          | 2.980  | 2.835  | 2.684  | 2.526   |
| 3 mean                                     | 2.085          | 2.337  | 2.457  | 2.632  | 2.783   | 2.247          | 2.476  | 2.578  | 2.723  | 2.848   |
| 3 variance                                 | 5.113          | 3.571  | 3.531  | 3.596  | 3.583   | 4.428          | 2.752  | 2.638  | 2.549  | 2.467   |
| 4 mean                                     | 2.186          | 2.418  | 2.533  | 2.695  | 2.828   | 2.329          | 2.530  | 2.629  | 2.765  | 2.878   |
| 4 variance                                 | 6.429          | 3.793  | 3.666  | 3.675  | 3.636   | 5.879          | 3.013  | 2.803  | 2.649  | 2.517   |
| Constant, seasonal dummies, no trend       |                |        |        |        |         |                |        |        |        |         |
| 0 mean                                     | 2.643          | 2.763  | 2.813  | 2.881  | 2.935   | 2.788          | 2.861  | 2.894  | 2.937  | 2.976   |
| 0 variance                                 | 5.287          | 4.178  | 4.062  | 3.983  | 3.787   | 4.605          | 3.309  | 3.078  | 2.850  | 2.624   |
| 1 mean                                     | 2.513          | 2.680  | 2.746  | 2.838  | 2.910   | 2.621          | 2.750  | 2.804  | 2.877  | 2.941   |
| 1 variance                                 | 5.179          | 4.066  | 3.942  | 3.907  | 3.748   | 4.368          | 3.136  | 2.935  | 2.768  | 2.579   |
| 2 mean                                     | 2.299          | 2.526  | 2.616  | 2.747  | 2.853   | 2.609          | 2.741  | 2.797  | 2.872  | 2.938   |
| 2 variance                                 | 4.967          | 3.890  | 3.797  | 3.807  | 3.689   | 4.515          | 3.095  | 2.914  | 2.745  | 2.565   |
| 3 mean                                     | 2.222          | 2.458  | 2.561  | 2.706  | 2.830   | 2.463          | 2.636  | 2.710  | 2.813  | 2.904   |
| 3 variance                                 | 4.983          | 3.785  | 3.702  | 3.738  | 3.674   | 4.502          | 2.981  | 2.813  | 2.674  | 2.541   |
| 4 mean                                     | 2.243          | 2.466  | 2.571  | 2.719  | 2.841   | 2.388          | 2.571  | 2.660  | 2.782  | 2.887   |
| 4 variance                                 | 5.663          | 3.872  | 3.753  | 3.759  | 3.690   | 5.092          | 3.053  | 2.844  | 2.690  | 2.542   |
| Constant, no seasonal dummies, trend       |                |        |        |        |         |                |        |        |        |         |
| 0 mean                                     | 0.965          | 0.957  | 0.971  | 0.991  | 1.017   | 0.980          | 0.990  | 1.006  | 1.029  | 1.055   |
| 0 variance                                 | 1.244          | 1.034  | 1.029  | 1.024  | 1.037   | 0.881          | 0.739  | 0.730  | 0.718  | 0.728   |
| 1 mean                                     | 0.926          | 0.945  | 0.964  | 0.989  | 1.016   | 0.993          | 1.001  | 1.014  | 1.033  | 1.055   |
| 1 variance                                 | 1.181          | 1.022  | 1.019  | 1.020  | 1.032   | 0.951          | 0.768  | 0.746  | 0.723  | 0.725   |
| 2 mean                                     | 1.019          | 1.013  | 1.022  | 1.029  | 1.039   | 1.029          | 1.034  | 1.046  | 1.056  | 1.070   |
| 2 variance                                 | 1.431          | 1.161  | 1.142  | 1.108  | 1.068   | 1.040          | 0.817  | 0.796  | 0.761  | 0.742   |
| 3 mean                                     | 1.109          | 1.049  | 1.045  | 1.039  | 1.041   | 1.129          | 1.077  | 1.073  | 1.068  | 1.072   |
| 3 variance                                 | 1.790          | 1.259  | 1.200  | 1.128  | 1.074   | 1.310          | 0.891  | 0.840  | 0.777  | 0.746   |
| 4 mean                                     | 1.071          | 1.012  | 1.008  | 1.011  | 1.022   | 1.070          | 1.035  | 1.038  | 1.045  | 1.059   |
| 4 variance                                 | 1.673          | 1.158  | 1.097  | 1.059  | 1.033   | 1.171          | 0.808  | 0.772  | 0.736  | 0.724   |
| Constant, no seasonal dummies, no trend    |                |        |        |        |         |                |        |        |        |         |
| 0 mean                                     | 1.006          | 0.999  | 1.008  | 1.019  | 1.034   | 1.035          | 1.034  | 1.044  | 1.055  | 1.070   |
| 0 variance                                 | 1.332          | 1.128  | 1.111  | 1.082  | 1.071   | 0.969          | 0.805  | 0.785  | 0.754  | 0.748   |
| 1 mean                                     | 1.024          | 1.008  | 1.015  | 1.023  | 1.035   | 1.065          | 1.050  | 1.054  | 1.060  | 1.071   |
| 1 variance                                 | 1.424          | 1.160  | 1.131  | 1.088  | 1.069   | 1.061          | 0.838  | 0.803  | 0.758  | 0.747   |
| 2 mean                                     | 1.019          | 1.013  | 1.022  | 1.030  | 1.040   | 1.056          | 1.051  | 1.058  | 1.064  | 1.075   |
| 2 variance                                 | 1.382          | 1.151  | 1.137  | 1.106  | 1.071   | 1.049          | 0.832  | 0.809  | 0.769  | 0.749   |
| 3 mean                                     | 1.054          | 1.029  | 1.032  | 1.035  | 1.041   | 1.093          | 1.068  | 1.070  | 1.070  | 1.076   |
| 3 variance                                 | 1.524          | 1.200  | 1.166  | 1.116  | 1.073   | 1.156          | 0.863  | 0.830  | 0.778  | 0.750   |
| 4 mean                                     | 1.100          | 1.052  | 1.045  | 1.038  | 1.039   | 1.118          | 1.080  | 1.076  | 1.071  | 1.074   |
| 4 variance                                 | 1.654          | 1.243  | 1.180  | 1.118  | 1.066   | 1.197          | 0.874  | 0.832  | 0.773  | 0.744   |
| No constant, no seasonal dummies, no trend |                |        |        |        |         |                |        |        |        |         |
| 0 mean                                     | 1.099          | 1.058  | 1.056  | 1.050  | 1.051   | 1.130          | 1.092  | 1.089  | 1.084  | 1.086   |
| 0 variance                                 | 1.572          | 1.260  | 1.214  | 1.148  | 1.104   | 1.149          | 0.896  | 0.851  | 0.794  | 0.768   |
| 1 mean                                     | 1.135          | 1.075  | 1.067  | 1.055  | 1.053   | 1.146          | 1.101  | 1.095  | 1.087  | 1.087   |
| 1 variance                                 | 1.718          | 1.312  | 1.246  | 1.158  | 1.104   | 1.199          | 0.914  | 0.863  | 0.797  | 0.768   |
| 2 mean                                     | 1.006          | 1.011  | 1.021  | 1.032  | 1.043   | 1.079          | 1.066  | 1.071  | 1.073  | 1.081   |
| 2 variance                                 | 1.297          | 1.132  | 1.129  | 1.109  | 1.078   | 1.053          | 0.848  | 0.822  | 0.780  | 0.756   |
| 3 mean                                     | 1.024          | 1.021  | 1.029  | 1.036  | 1.044   | 1.078          | 1.069  | 1.074  | 1.076  | 1.082   |
| 3 variance                                 | 1.364          | 1.163  | 1.150  | 1.117  | 1.079   | 1.065          | 0.850  | 0.830  | 0.784  | 0.758   |
| 4 mean                                     | 1.184          | 1.108  | 1.091  | 1.068  | 1.056   | 1.212          | 1.138  | 1.121  | 1.099  | 1.090   |
| 4 variance                                 | 1.844          | 1.369  | 1.287  | 1.185  | 1.099   | 1.349          | 0.964  | 0.903  | 0.815  | 0.764   |

Table 2: Size and power of the HEGY-IPS test: No serial correlation

| N  | T=20         |              |              |              | T=32         |              |              |              | T=40         |              |              |              | T=60         |              |              |              | T=100        |              |              |              |
|--|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|  | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ |
| SIZE                                     |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |
| Constant , no seasonal dummies. no trend |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |
| 5  | 4.52         | 4.56         | 6.88         | 6.70         | 4.68         | 5.02         | 7.20         | 6.62         | 5.00         | 4.84         | 6.44         | 6.54         | 5.28         | 4.94         | 6.64         | 6.42         | 4.72         | 4.96         | 6.14         | 5.74         |
| 7  | 4.70         | 4.34         | 6.98         | 7.02         | 5.18         | 4.98         | 6.50         | 6.44         | 5.38         | 5.22         | 6.76         | 6.48         | 4.90         | 4.94         | 6.50         | 6.08         | 4.60         | 4.72         | 6.86         | 6.04         |
| 10                                       | 5.04         | 4.76         | 6.58         | 6.70         | 5.12         | 4.74         | 6.88         | 6.54         | 4.90         | 4.90         | 6.20         | 5.90         | 4.68         | 4.60         | 6.36         | 5.80         | 4.52         | 4.78         | 6.18         | 6.26         |
| 15                                       | 5.16         | 4.54         | 6.58         | 6.58         | 5.40         | 4.72         | 6.90         | 6.32         | 5.24         | 5.12         | 6.34         | 5.82         | 4.68         | 4.82         | 5.88         | 5.92         | 4.56         | 4.44         | 6.72         | 6.30         |
| 25                                       | 5.04         | 4.68         | 6.32         | 5.98         | 4.90         | 4.66         | 6.36         | 6.00         | 5.72         | 4.46         | 6.06         | 6.10         | 4.94         | 4.40         | 6.30         | 6.14         | 4.66         | 4.54         | 6.10         | 5.88         |
| 40                                       | 5.08         | 4.90         | 5.88         | 5.80         | 5.12         | 4.38         | 6.02         | 5.60         | 5.44         | 4.74         | 5.72         | 5.66         | 4.86         | 5.06         | 6.30         | 6.02         | 4.82         | 4.82         | 5.82         | 5.82         |
| Constant, no seasonal dummies. trend     |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |
| 5  | 5.06         | 4.76         | 6.86         | 6.76         | 5.44         | 5.18         | 7.06         | 6.26         | 5.34         | 4.92         | 6.30         | 6.52         | 5.46         | 5.16         | 6.68         | 6.68         | 5.02         | 4.92         | 6.12         | 5.82         |
| 7  | 5.00         | 4.34         | 7.06         | 7.36         | 5.30         | 5.10         | 6.86         | 6.56         | 5.58         | 5.46         | 6.52         | 6.24         | 5.38         | 5.10         | 6.60         | 6.42         | 4.38         | 4.70         | 6.94         | 6.18         |
| 10                                       | 4.76         | 4.66         | 6.44         | 6.52         | 5.12         | 4.80         | 6.66         | 6.36         | 5.92         | 4.88         | 5.76         | 5.96         | 5.14         | 4.68         | 6.48         | 5.96         | 4.94         | 4.78         | 6.16         | 6.36         |
| 15                                       | 5.38         | 4.92         | 6.68         | 6.76         | 5.10         | 5.22         | 6.90         | 6.38         | 5.66         | 5.06         | 6.00         | 5.66         | 5.40         | 4.86         | 6.04         | 5.98         | 4.54         | 4.48         | 6.80         | 6.36         |
| 25                                       | 5.64         | 4.52         | 6.16         | 6.30         | 4.76         | 4.86         | 6.56         | 5.94         | 5.44         | 4.48         | 5.96         | 5.72         | 5.36         | 4.36         | 6.14         | 6.10         | 4.62         | 4.56         | 5.96         | 5.88         |
| 40                                       | 5.08         | 4.80         | 5.92         | 5.88         | 4.76         | 4.34         | 6.12         | 5.42         | 5.66         | 4.68         | 5.66         | 5.72         | 5.12         | 5.00         | 6.28         | 6.06         | 5.22         | 4.76         | 5.92         | 5.84         |
| POWER                                    |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |
| Constant , no seasonal dummies. no trend |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |
| 5  | 6.94         | 24.02        | 11.94        | 14.10        | 8.00         | 34.70        | 20.36        | 24.44        | 9.00         | 40.90        | 25.94        | 31.42        | 11.10        | 57.16        | 43.96        | 53.74        | 17.44        | 82.26        | 76.72        | 87.34        |
| 7  | 8.40         | 31.92        | 12.98        | 15.66        | 9.06         | 46.40        | 23.88        | 29.08        | 11.14        | 55.34        | 30.44        | 37.98        | 12.66        | 74.22        | 53.50        | 65.78        | 23.18        | 94.60        | 87.86        | 95.02        |
| 10                                       | 9.46         | 42.72        | 15.34        | 18.08        | 10.58        | 62.74        | 29.34        | 35.88        | 13.30        | 71.22        | 38.48        | 47.88        | 16.04        | 89.62        | 65.66        | 77.48        | 31.20        | 99.20        | 95.62        | 99.02        |
| 15                                       | 10.68        | 59.96        | 17.14        | 22.54        | 12.82        | 81.02        | 36.74        | 46.12        | 16.46        | 89.18        | 49.06        | 60.94        | 21.04        | 97.60        | 79.48        | 89.44        | 43.00        | 100.0        | 99.44        | 99.96        |
| 25                                       | 13.54        | 82.24        | 21.42        | 29.44        | 16.40        | 96.20        | 49.90        | 61.92        | 21.56        | 98.66        | 67.24        | 79.06        | 31.00        | 99.92        | 94.18        | 98.46        | 65.18        | 100.0        | 100.0        | 100.0        |
| 40                                       | 17.20        | 95.42        | 28.20        | 40.62        | 22.94        | 99.66        | 66.52        | 79.24        | 29.50        | 99.90        | 84.10        | 92.80        | 44.66        | 100.0        | 99.26        | 99.82        | 84.04        | 100.0        | 100.0        | 100.0        |
| Constant, no seasonal dummies. trend     |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |              |
| 5  | 7.08         | 24.72        | 12.24        | 15.92        | 7.28         | 35.44        | 22.20        | 26.62        | 7.22         | 41.72        | 27.70        | 33.72        | 7.58         | 57.50        | 45.90        | 56.10        | 8.86         | 82.38        | 77.84        | 88.18        |
| 7  | 7.14         | 33.40        | 13.72        | 17.34        | 7.26         | 47.98        | 25.48        | 31.76        | 8.34         | 56.26        | 32.72        | 40.70        | 8.04         | 74.90        | 56.18        | 67.80        | 10.22        | 94.58        | 88.80        | 95.68        |
| 10                                       | 8.02         | 44.40        | 15.68        | 20.36        | 8.18         | 63.82        | 31.38        | 39.98        | 8.44         | 72.06        | 40.64        | 51.14        | 9.14         | 89.72        | 68.10        | 80.28        | 12.28        | 99.18        | 96.28        | 99.18        |
| 15                                       | 9.14         | 61.26        | 18.34        | 25.74        | 9.18         | 82.10        | 40.06        | 51.12        | 9.12         | 89.34        | 52.16        | 65.30        | 10.44        | 97.54        | 81.58        | 91.22        | 14.60        | 100.0        | 99.48        | 99.98        |
| 25                                       | 10.44        | 83.50        | 22.10        | 33.32        | 10.16        | 96.50        | 54.10        | 67.38        | 10.90        | 98.70        | 70.80        | 83.16        | 12.08        | 99.92        | 95.24        | 98.88        | 19.62        | 100.0        | 100.0        | 100.0        |
| 40                                       | 12.30        | 95.82        | 29.82        | 45.78        | 11.64        | 99.72        | 70.76        | 83.82        | 13.12        | 99.92        | 86.68        | 95.10        | 14.82        | 100.0        | 99.52        | 99.92        | 27.08        | 100.0        | 100.0        | 100.0        |

NOTE: For power the DGP is written as  $y_{it} = \alpha_i + \phi_i y_{it-1} + \varepsilon_{it}$ , where  $\phi_i = 0.9$ ,  $\alpha_i = (1 - \phi_i)\delta_i$ ,  $\delta_i \sim N(0,1)$ ,  $\varepsilon_{it} \sim N(0, \sigma_i^2)$  and  $\sigma_i^2 \sim U[0.5, 1.5]$ .

Table 3: Size of the HEGY-IPS test: AR(1) errors  $\rho_i \sim U[0.2, 0.4]$ , constant, no seasonal dummies, no trend

| N  | p | T=20         |              |              |              | T=32         |              |              |              | T=40         |              |              |              | T=60         |              |              |              | T=100        |              |              |              |
|----|---|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|    |   | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ |
| 5  | 0 | 1.54         | 7.36         | 6.24         | 6.04         | 1.08         | 9.98         | 6.68         | 7.62         | 0.92         | 11.00        | 6.82         | 8.32         | 0.72         | 14.02        | 8.46         | 10.80        | 1.00         | 16.40        | 9.62         | 14.10        |
| 7  | 0 | 1.10         | 8.72         | 5.96         | 6.14         | 0.78         | 11.20        | 6.60         | 7.28         | 0.68         | 13.44        | 6.78         | 9.02         | 0.62         | 16.52        | 7.96         | 10.72        | 0.62         | 20.58        | 10.16        | 15.00        |
| 10 | 0 | 0.74         | 10.28        | 5.70         | 5.52         | 0.40         | 13.62        | 6.08         | 6.94         | 0.32         | 15.66        | 6.82         | 8.64         | 0.18         | 19.84        | 8.26         | 10.80        | 0.14         | 25.32        | 10.44        | 16.28        |
| 15 | 0 | 0.40         | 11.78        | 5.72         | 4.90         | 0.22         | 17.12        | 5.74         | 6.36         | 0.22         | 20.54        | 6.82         | 9.02         | 0.18         | 25.32        | 7.98         | 11.40        | 0.10         | 33.34        | 11.10        | 17.60        |
| 25 | 0 | 0.16         | 15.76        | 5.04         | 4.26         | 0.04         | 24.34        | 5.02         | 6.16         | 0.04         | 29.08        | 6.30         | 8.84         | 0.04         | 37.42        | 8.64         | 12.62        | 0.00         | 48.00        | 11.82        | 22.34        |
| 40 | 0 | 0.04         | 21.16        | 4.48         | 3.86         | 0.00         | 34.54        | 4.82         | 6.54         | 0.00         | 41.04        | 5.86         | 8.90         | 0.00         | 53.62        | 8.10         | 14.82        | 0.00         | 66.04        | 13.46        | 27.46        |
| 5  | 1 | 6.54         | 4.10         | 7.54         | 6.76         | 5.94         | 4.58         | 7.26         | 6.90         | 5.02         | 4.96         | 7.20         | 7.00         | 4.96         | 5.32         | 7.16         | 6.76         | 5.36         | 4.46         | 6.50         | 6.40         |
| 7  | 1 | 6.84         | 4.44         | 7.08         | 7.04         | 5.96         | 4.36         | 6.80         | 6.70         | 5.30         | 4.92         | 6.88         | 6.70         | 5.32         | 4.84         | 6.76         | 6.46         | 5.34         | 4.12         | 6.64         | 6.30         |
| 10 | 1 | 6.90         | 4.82         | 7.18         | 6.74         | 6.58         | 4.44         | 6.80         | 6.10         | 5.40         | 4.98         | 7.44         | 6.70         | 5.18         | 4.68         | 6.84         | 6.32         | 5.46         | 4.84         | 6.48         | 5.66         |
| 15 | 1 | 7.18         | 4.46         | 7.32         | 6.66         | 6.92         | 4.40         | 6.36         | 5.96         | 5.48         | 5.14         | 6.98         | 6.64         | 5.48         | 4.84         | 6.90         | 6.40         | 5.60         | 4.28         | 6.08         | 5.94         |
| 25 | 1 | 7.96         | 4.62         | 7.52         | 6.34         | 6.98         | 4.56         | 6.06         | 5.68         | 6.18         | 4.68         | 7.06         | 6.58         | 6.00         | 4.78         | 6.48         | 5.70         | 6.10         | 4.50         | 6.20         | 5.90         |
| 40 | 1 | 8.74         | 4.48         | 7.94         | 6.26         | 7.06         | 4.58         | 6.38         | 5.46         | 7.36         | 5.64         | 6.22         | 6.28         | 5.86         | 4.96         | 5.86         | 6.16         | 6.22         | 4.18         | 6.56         | 5.54         |
| 5  | 2 | 5.60         | 4.46         | 7.52         | 6.68         | 5.70         | 4.60         | 7.48         | 7.44         | 4.82         | 5.16         | 6.76         | 6.86         | 4.66         | 5.34         | 6.78         | 6.76         | 5.30         | 5.08         | 6.56         | 6.34         |
| 7  | 2 | 5.78         | 4.34         | 7.32         | 6.88         | 5.48         | 4.50         | 7.06         | 7.14         | 5.26         | 4.84         | 7.04         | 6.68         | 5.28         | 4.80         | 6.72         | 6.34         | 5.02         | 5.08         | 6.64         | 5.96         |
| 10 | 2 | 5.72         | 4.80         | 7.18         | 6.74         | 5.90         | 4.50         | 6.56         | 6.10         | 5.28         | 5.24         | 6.90         | 6.88         | 5.14         | 4.64         | 7.04         | 6.24         | 5.06         | 5.03         | 6.16         | 5.82         |
| 15 | 2 | 6.04         | 4.56         | 7.34         | 6.62         | 5.96         | 4.28         | 6.32         | 5.98         | 5.06         | 5.20         | 6.96         | 6.20         | 4.98         | 4.80         | 6.22         | 5.92         | 5.26         | 4.95         | 6.16         | 5.76         |
| 25 | 2 | 6.36         | 4.58         | 7.60         | 6.78         | 5.72         | 4.58         | 6.38         | 5.38         | 5.42         | 4.72         | 6.38         | 6.52         | 5.36         | 4.58         | 6.34         | 5.24         | 5.68         | 4.94         | 6.02         | 5.74         |
| 40 | 2 | 6.40         | 4.58         | 7.52         | 6.96         | 6.36         | 4.44         | 6.68         | 5.62         | 6.64         | 5.22         | 6.26         | 5.80         | 5.14         | 4.64         | 5.80         | 5.70         | 5.96         | 4.88         | 6.26         | 5.52         |
| 5  | 3 | 4.26         | 4.54         | 5.74         | 5.68         | 4.54         | 4.84         | 6.38         | 6.46         | 4.12         | 5.26         | 6.02         | 6.36         | 4.16         | 5.22         | 6.44         | 6.38         | 5.08         | 5.09         | 6.46         | 5.90         |
| 7  | 3 | 4.58         | 4.72         | 5.62         | 5.48         | 4.44         | 4.52         | 5.94         | 5.90         | 4.52         | 5.24         | 5.98         | 6.30         | 4.88         | 4.84         | 6.26         | 5.90         | 4.72         | 5.17         | 6.50         | 5.76         |
| 10 | 3 | 4.08         | 5.20         | 4.82         | 5.36         | 4.80         | 4.66         | 5.30         | 5.20         | 4.32         | 5.60         | 5.76         | 5.96         | 4.48         | 4.94         | 6.48         | 5.72         | 4.98         | 5.09         | 6.08         | 5.82         |
| 15 | 3 | 3.76         | 5.08         | 5.20         | 4.98         | 4.62         | 5.00         | 4.54         | 4.72         | 3.72         | 5.40         | 5.54         | 5.22         | 4.14         | 5.06         | 5.60         | 5.42         | 5.46         | 5.06         | 5.84         | 5.44         |
| 25 | 3 | 3.72         | 5.54         | 4.50         | 4.34         | 3.64         | 5.56         | 4.58         | 4.12         | 4.22         | 5.18         | 4.92         | 5.22         | 4.32         | 5.14         | 5.44         | 4.68         | 5.10         | 5.06         | 5.56         | 5.16         |
| 40 | 3 | 3.26         | 5.68         | 3.48         | 4.02         | 3.70         | 5.56         | 4.18         | 3.94         | 4.40         | 5.84         | 4.68         | 4.74         | 4.14         | 5.46         | 4.62         | 4.58         | 5.12         | 5.07         | 5.86         | 5.04         |
| 5  | 4 | 4.44         | 4.54         | 6.08         | 5.96         | 4.36         | 4.76         | 6.64         | 5.88         | 4.00         | 5.14         | 6.22         | 6.70         | 4.14         | 5.24         | 6.56         | 6.44         | 5.00         | 4.68         | 6.72         | 6.32         |
| 7  | 4 | 4.50         | 4.90         | 5.56         | 5.74         | 4.58         | 4.32         | 5.80         | 5.76         | 4.34         | 4.98         | 5.50         | 6.24         | 5.04         | 4.92         | 6.10         | 6.00         | 4.56         | 4.34         | 6.38         | 5.62         |
| 10 | 4 | 4.32         | 5.44         | 5.34         | 5.52         | 4.76         | 5.00         | 5.74         | 5.30         | 4.44         | 5.70         | 5.84         | 5.74         | 4.20         | 4.92         | 6.28         | 5.96         | 5.02         | 4.80         | 6.14         | 5.88         |
| 15 | 4 | 4.40         | 4.80         | 5.42         | 5.42         | 4.64         | 5.06         | 5.00         | 4.68         | 4.20         | 5.36         | 5.78         | 5.48         | 4.32         | 5.18         | 5.66         | 5.44         | 5.42         | 4.02         | 5.74         | 5.72         |
| 25 | 4 | 4.46         | 5.56         | 5.00         | 4.80         | 4.20         | 5.22         | 4.56         | 4.22         | 4.64         | 5.20         | 5.16         | 5.14         | 4.30         | 5.14         | 5.54         | 4.98         | 5.38         | 4.78         | 5.66         | 5.74         |
| 40 | 4 | 4.06         | 5.64         | 4.24         | 4.38         | 4.12         | 5.64         | 4.26         | 4.28         | 5.18         | 5.92         | 4.12         | 4.50         | 4.62         | 5.58         | 4.92         | 4.80         | 5.36         | 4.54         | 5.74         | 5.00         |

Table 4: Size of the HEGY-IPS test: MA(1) errors  $\theta_i \sim U[-0.4, -0.2]$ , constant, no seasonal dummies, no trend

| N  | p | T=20         |              |              |              | T=32         |              |              |              | T=40         |              |              |              | T=60         |              |              |              | T=100        |              |              |              |
|----|---|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|    |   | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ | $W_{t_1bar}$ | $W_{t_2bar}$ | $W_{F_2bar}$ | $W_{F_3bar}$ |
| 5  | 0 | 35.02        | 4.88         | 6.76         | 9.32         | 44.04        | 4.22         | 7.22         | 9.82         | 46.68        | 4.02         | 6.46         | 9.08         | 52.62        | 3.62         | 8.72         | 10.70        | 57.04        | 2.34         | 9.64         | 10.68        |
| 7  | 0 | 44.70        | 4.90         | 6.08         | 9.70         | 54.38        | 3.98         | 6.82         | 10.00        | 58.82        | 3.72         | 6.52         | 8.96         | 63.56        | 3.26         | 8.24         | 10.78        | 68.34        | 1.84         | 9.80         | 11.02        |
| 10 | 0 | 56.06        | 3.98         | 6.08         | 9.64         | 67.48        | 3.34         | 6.88         | 10.50        | 71.08        | 3.38         | 6.32         | 9.66         | 76.84        | 2.68         | 8.88         | 11.60        | 81.56        | 2.04         | 10.28        | 12.14        |
| 15 | 0 | 71.76        | 3.94         | 6.20         | 10.34        | 81.92        | 2.90         | 6.26         | 11.04        | 85.50        | 2.76         | 6.44         | 10.34        | 89.22        | 2.32         | 9.20         | 12.96        | 92.04        | 1.34         | 11.06        | 12.56        |
| 25 | 0 | 88.76        | 3.04         | 5.88         | 10.84        | 94.74        | 2.38         | 6.48         | 12.34        | 96.56        | 2.00         | 6.48         | 12.08        | 97.66        | 1.38         | 9.68         | 14.50        | 98.92        | 1.10         | 12.72        | 15.16        |
| 40 | 0 | 97.10        | 2.84         | 5.08         | 11.72        | 99.46        | 1.86         | 6.04         | 13.58        | 99.62        | 1.44         | 6.62         | 12.84        | 99.78        | 0.94         | 10.26        | 17.22        | 99.96        | 0.42         | 14.36        | 17.82        |
| 5  | 1 | 7.16         | 5.34         | 5.88         | 7.70         | 9.08         | 5.82         | 5.86         | 9.50         | 9.74         | 6.00         | 6.74         | 8.72         | 10.54        | 5.88         | 6.80         | 9.80         | 12.44        | 6.12         | 7.14         | 9.86         |
| 7  | 1 | 8.06         | 5.58         | 5.82         | 6.94         | 10.58        | 5.94         | 6.48         | 9.04         | 11.56        | 6.12         | 6.64         | 9.06         | 12.94        | 6.68         | 7.14         | 10.36        | 14.46        | 6.26         | 7.42         | 10.28        |
| 10 | 1 | 9.02         | 5.52         | 6.14         | 7.52         | 11.64        | 6.28         | 6.46         | 9.44         | 13.34        | 6.78         | 6.46         | 9.42         | 14.34        | 6.84         | 7.04         | 11.16        | 17.52        | 7.14         | 7.32         | 10.74        |
| 15 | 1 | 10.90        | 6.10         | 5.22         | 7.80         | 14.40        | 6.26         | 6.84         | 9.70         | 15.86        | 7.30         | 6.36         | 9.62         | 17.26        | 7.70         | 7.50         | 11.44        | 21.74        | 7.20         | 7.64         | 11.98        |
| 25 | 1 | 13.48        | 5.92         | 6.18         | 8.54         | 18.60        | 7.08         | 7.16         | 11.26        | 20.82        | 7.60         | 7.44         | 10.72        | 24.54        | 9.14         | 7.34         | 14.00        | 29.64        | 8.36         | 7.82         | 13.52        |
| 40 | 1 | 16.80        | 7.08         | 6.26         | 8.34         | 25.74        | 7.82         | 7.28         | 11.72        | 28.26        | 8.82         | 7.64         | 12.80        | 33.70        | 9.22         | 8.30         | 15.46        | 39.76        | 9.50         | 8.34         | 15.52        |
| 5  | 2 | 4.16         | 5.06         | 5.76         | 6.98         | 5.32         | 5.26         | 7.24         | 7.08         | 5.14         | 5.46         | 6.26         | 6.50         | 5.32         | 4.96         | 6.62         | 7.12         | 6.62         | 4.95         | 7.18         | 6.84         |
| 7  | 2 | 4.46         | 4.96         | 5.62         | 6.50         | 5.38         | 5.06         | 6.74         | 6.60         | 5.74         | 5.12         | 5.96         | 6.50         | 6.20         | 4.88         | 6.72         | 6.96         | 6.88         | 4.97         | 6.58         | 6.20         |
| 10 | 2 | 4.06         | 5.18         | 5.86         | 6.68         | 5.04         | 5.08         | 6.58         | 7.18         | 5.24         | 5.30         | 5.90         | 6.40         | 6.08         | 4.82         | 7.24         | 7.14         | 6.92         | 4.84         | 7.02         | 6.32         |
| 15 | 2 | 4.28         | 5.34         | 5.76         | 6.82         | 5.56         | 4.70         | 6.06         | 6.70         | 6.30         | 5.24         | 5.72         | 5.90         | 6.28         | 5.38         | 6.72         | 7.12         | 7.74         | 4.74         | 6.60         | 6.14         |
| 25 | 2 | 4.70         | 4.66         | 5.30         | 6.70         | 5.94         | 5.16         | 6.38         | 6.62         | 6.04         | 4.90         | 5.48         | 6.16         | 7.56         | 5.18         | 6.82         | 7.36         | 9.96         | 4.58         | 6.90         | 6.38         |
| 40 | 2 | 4.44         | 4.96         | 5.20         | 6.06         | 6.86         | 5.40         | 5.34         | 6.22         | 6.68         | 5.04         | 5.76         | 5.92         | 8.46         | 4.44         | 6.66         | 6.94         | 10.74        | 4.56         | 6.56         | 6.20         |
| 5  | 3 | 4.70         | 5.22         | 6.60         | 7.50         | 5.32         | 4.82         | 7.94         | 7.70         | 5.08         | 5.26         | 6.90         | 6.42         | 4.60         | 5.14         | 6.64         | 6.80         | 5.34         | 5.03         | 6.70         | 6.92         |
| 7  | 3 | 5.28         | 5.04         | 6.62         | 7.28         | 5.28         | 4.96         | 7.64         | 7.56         | 5.74         | 5.10         | 6.84         | 6.74         | 5.22         | 5.12         | 6.84         | 7.00         | 5.80         | 5.17         | 5.76         | 6.44         |
| 10 | 3 | 5.30         | 5.18         | 7.50         | 7.70         | 5.56         | 4.74         | 7.22         | 7.72         | 5.62         | 5.48         | 6.46         | 6.94         | 5.38         | 4.74         | 7.04         | 6.98         | 5.66         | 5.05         | 6.24         | 6.56         |
| 15 | 3 | 5.64         | 5.02         | 7.66         | 8.18         | 5.92         | 4.52         | 6.96         | 7.14         | 6.02         | 5.18         | 6.58         | 6.38         | 5.50         | 5.28         | 6.56         | 6.52         | 5.98         | 4.99         | 6.14         | 6.28         |
| 25 | 3 | 5.66         | 4.40         | 7.76         | 8.18         | 6.22         | 5.02         | 7.02         | 7.22         | 6.14         | 4.78         | 6.34         | 6.58         | 6.28         | 5.58         | 7.08         | 6.88         | 7.12         | 4.94         | 6.00         | 6.26         |
| 40 | 3 | 6.86         | 4.44         | 8.34         | 8.62         | 7.24         | 4.94         | 7.22         | 7.50         | 7.06         | 5.32         | 7.02         | 6.56         | 6.60         | 5.18         | 6.74         | 6.60         | 7.14         | 5.02         | 5.62         | 6.30         |
| 5  | 4 | 5.06         | 5.18         | 6.42         | 7.30         | 5.02         | 5.22         | 8.02         | 7.54         | 4.80         | 5.44         | 6.96         | 6.84         | 4.66         | 5.58         | 6.76         | 7.34         | 5.04         | 4.46         | 7.10         | 6.88         |
| 7  | 4 | 5.26         | 5.24         | 6.50         | 7.00         | 4.98         | 4.92         | 7.16         | 7.54         | 5.32         | 5.26         | 6.96         | 7.10         | 5.20         | 5.12         | 6.94         | 7.00         | 5.48         | 4.52         | 6.46         | 6.94         |
| 10 | 4 | 5.44         | 5.06         | 6.98         | 7.48         | 5.42         | 4.82         | 7.42         | 7.38         | 5.12         | 5.38         | 6.94         | 7.18         | 4.88         | 4.94         | 7.30         | 7.12         | 5.42         | 4.96         | 6.50         | 6.76         |
| 15 | 4 | 5.56         | 5.22         | 7.66         | 7.90         | 6.50         | 4.82         | 7.32         | 7.24         | 6.28         | 5.22         | 6.74         | 7.08         | 4.64         | 5.38         | 6.66         | 7.16         | 5.50         | 4.76         | 6.02         | 6.90         |
| 25 | 4 | 6.06         | 4.68         | 7.64         | 8.12         | 6.12         | 4.88         | 7.14         | 7.24         | 6.04         | 5.00         | 6.96         | 6.82         | 5.74         | 5.52         | 7.50         | 7.58         | 6.64         | 4.38         | 6.48         | 7.04         |
| 40 | 4 | 7.18         | 4.86         | 8.28         | 8.78         | 7.10         | 5.22         | 6.98         | 7.76         | 6.22         | 5.00         | 7.16         | 7.00         | 5.92         | 4.84         | 7.34         | 7.00         | 6.52         | 4.44         | 6.16         | 6.90         |