Profitable Cost Increases and the Shifting of Taxation: Equilibrium Responses of Markets in Oligopoly

J Seade

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by

J. Seade

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1. Introduction

This paper considers the conjectural variations model of oligopoly and introduces a shift in its equilibrium solution : a cost-side shift, such as a change in technology or input prices, or the introduction of excise tax. The equilibrium effects of this cost-displacement are then found, deriving and examining explicit expressions for the resulting movements in individual outputs and hence in price, profits and market structure.

The main motivation we offer for the exercise is methodological : to derive, for the model adopted, certain industrial-organization results of general interest and applicability, which we then put to work mostly in a more specific public finance context. The results we are referring to are, very simply, the comparative statics (of our model) of oligopoly, in response to changes in costs. It is indeed surprising that the problem is not one which has been treated systematically in the literature except for particular cases, such as special functional forms and/or symmetric industry.[1/]

On the other hand we have the application of these results and ideas to questions of public finance. At the immediate level, we offer some results on the shifting - i.e. the effect on price - of taxation, and on its implications for profits, which are surprising for their simplicity and generality. These provide us with simple and empirically very plausible conditions under which the old questions of whether taxation can raise producers' (net) prices or even their profits admit affirmative answers. Price and profit *over-shifting* turn out to be, in oligopoly, a distinct possibility.

From another perspective, quite apart from whether overshifting does or does not occur in any particular case, we aim in this paper to contribute towards the restoration to tax theory of its original preoccupation with market structure, which has been conspicuously absent from the subject during the last two decades or so.[2/] The first thing one needs to know when evaluating the effects of excise tax, be this in a positive or normative framework, are its implications for prices, quantities and incomes, which are the variables whose movement with taxation we examine below.

The structure of the rest of the paper is as follows. Section 2 provides some background to our results on price- and profit-overshifting, discussing briefly some of the relevant literature and the intuition of why taxation behaves very differently under oligopoly from what conventional wisdom indicates. Section 3 presents the model - the standard homogenous-good conjectural-variations oligopoly - and the various formulae we shall use: first order, second order, and stability conditions. Section 4 analyzes, for the symmetric case, the effects of tax or cost increases on firms' outputs and on the other variables we look at, while section 5 extends the analysis to the asymmetric case. Section 6 illustrates some of the results for a concrete example and attempts a quick interpretation of some developments in the aftermath of the oil crisis of 1973 in the light of our results. Section 7 extends the results to a particular form of product differentiation, and finally section 8 contains some concluding remarks.

2. The shifting of taxation : some background

Consider first the effect of tax on price. The conventional-wisdom picture is, as usual, derived from the competitive case. Namely, an excise tax will raise the price consumers face, but not by as much as the size of the tax itself (or, at the margin, its increase), the reason being that producers' price falls along the supply curve, as the market in question contracts. Essentially this outcome, in a related form (the tax applying to a component of cost), was questioned by Krzyzaniak and Musgrave (1963), whose very empiricist analysis suggested the tax was more than fully shifted forward. But no satisfactory theorical explanation was offered by these authors or, largely, by the numerous authors who have commented on their work, as to why or when this price overshifting might arise. It is only by recourse to the general-equilibrium repercussions of taxation, working through other prices and incomes, that textbooks in the area typically challenge the conventional-wisdom belief that the extent of tax passed on to consumers is not more, and probably less, than the full amount of the tax. If the sector is sufficiently small (i.e. the tax sufficiently specific), it follows, conventional wisdom should reign.

That the assumption of perfect competition plays a crucial role in the story told above seems to have escaped our attention almost entirely. This I find extraordinary, given how obvious it is that in the presence of market power the tax may indeed raise the equilibrium net price received by producers. Consider for example the well-known formula for pure monopoly:

$$p = c'/(1-1/\varepsilon) \tag{1}$$

in obvious notation, where for a profit-maximum we need elasticity ε to exceed 1. Suppose, to keep things at their simplest, that ε is constant along the demand curve, which is a natural and widely-used particular case. We immediately see that cost increases $(c' \to c' + t)$ will result in amplified rather than only partially shifted price increases for the consumer: $dp/dt = 1/(1-1/\varepsilon) > 1$. The same conclusion follows from (1) for symmetric Cournot or conjectural-variations oligopoly too, now interpreting ε as $\equiv n\bar{\varepsilon}/\lambda$, where $\bar{\varepsilon}$ is true market elasticity (still a constant for the example), n is the number of firms, λ the conjectural-variation behavioural parameter, and ε the typical firm's perceived elasticity, which must still exceed 1 for equilibrium. Nevertheless, conventional wisdom can trick us sometimes. Thus, the very eminent tax theorist that was the late L.Johansen (1971, p.280), presumably failing to note the possibility of using (1) for a simple particular case, wrote, in his discussion precisely of the effects of excise tax on price under monopoly, that "it is theoretically possible (though this would hardly occur with any frequency in practice) that the price p will increase by *more* than the increase in the excise duty rate ..." (His italics and in-brackets remark.)

Similarly, or perhaps more particularly, the possibility of profits increasing, as a result of a tax or rise in costs, seems very alien to intuition, and yet not only can this happen but is in fact not at all exceptional in oligopoly, as we shall see. Indeed, it is interesting to note that, this time, both extreme forms of market structure accord with intuition. Apart from general equilibrium repercussions, which provide us with an almost infallible but for the same reason not very striking explanation for nearly anything we may wish to prove to be possible, it is impossible for the price-taking firms in a competitive industry to see their profits rise as a result of taxation. Their equilibrium position will move downwards along their collective supply curve with the tax, so that profits (the area under the curve, net of any fixed costs) must fall - unless their marginal cost schedule is downward sloping (and heavily so, 'inelastic', but less inelastic than demand, of course) which is not a possibility particularly amenable to intuition. But, at the other extreme, neither can a monopolist's profits rise with the tax. Clearly he could have imposed the tax upon himself, from the beginning (before there is tax), so to say, and reap the allegedly higher new profits plus the value of his tax bill. Strict optimization by him, and partial equilibrium, rule this out. Nevertheless, casual-observation real-world examples seem to be easy to find, in taxation contexts or otherwise, of apparently profitable cost increases. A notable case in point is the world oil industry in the years 1973-4, when the operating profits (exclusive of stock-revaluation) of the large multinational oil companies underwent marked increases, in the wake of the steep rise in the price of their prime input, crude. This tended to be explained through an "in adversity unite" effect the increases in costs had on the oligopolists' behaviour, which somewhow awoke the sense of comradeship or the conspiratory powers of the oil giants and made them more collusive. There may be something in that, but the story is not convincing, or still needs to be told. A good explanation in theory, for profit-overshifting of cost or tax rises, seems to be lacking.

But such a mechanism is provided, very simply, by oligopoly. The producers in such a market face what essentially amounts to a public-good problem: restraint by any one firm in the industry in question raises the price(s) they <u>all</u> face, for their outputs, hence constituting a common benefit. But the cost of this restraint, in the form of profitable revenue forlorn, is borne by the firm alone.[3/] Hence too little of that "good" is produced: too little restraint. A cost increase will necessarily induce output-reductions, as we shall see, thus raising the supply of the public-good, 'restraint', which will in itself be a good thing for the firms. The increase in costs can be seen as imposing upon the producers some of the collusion they themselves had been unable to achieve. One of our main purposes here will be to find out exactly when will this effect dominate the directly detrimental effect of the cost-rise, and to show that the supposedly 'perverse' qualitative outcome is not only possible in a formal sense, but likely indeed in practice. Clearly, the elasticity of demand will play a central role here, although the central variable turns out rather to be a second-order elasticity concept, the elasticity of the slope of demand, defined and found also to be central in Seade (1980a). This feature of demand naturally relates directly to ordinary demand elasticity (see footnote 7 below). Roughly speaking, it is clear that the more inelastic demand is, the greater the public-good benefit to the firms of having outside events induce them to cut output and raise price. And surely, the short-run demand for the oil industry's final products, such as gasoline, was low indeed in the early or mid 70's.

The above idea, on the special role oligopoly may play vis-à-vis exogenous shifts in equilibrium, has by and large gone unnoticed in the literature. Salop (1981, p.38 : fn 64) draws attention to the ambiguity one might have in establishing the net effects of cost rises on profits in oligopoly for the reasons we have described, without exploring the point formally. He and Scheffman (1983) then study the related problem of manipulation by a firm of other firms' cost curves in the industry, a question first raised by Williamson (1968). Closer to the spirit of this paper, analyzing the effects of environment (e.g. tax) changes on oligopoly, are de Meza (1982), Stern (1982) and Katz and Rosen (1983). David de Meza' interesting article is centered on the effects of entry on (i.e. the SR vs. LR comparison of) derived demand elasticities, but along the way he notes the result we referred to above in connection with equation (1) under oligopoly, on price-overshifting for the constant-elasticity identical-firms case, as well as its profit-overshifting analog. Stern analyzes a variety of oligopoly models to look at policy questions largely separate from our interests here, but also notes the price-overshifting result for the isoelastic-demand, symmetric-industry case. Lastly, Katz and Rosen concentrate on the same questions that motivate us (and some welfare-measurement counterparts) and derive interesting results by numerical simulation. Their purpose is primarily to illustrate possibilities rather than to characterize outcomes, however, hence they again rely on symmetry and specific examples of demand and cost functions.

"Paradoxical" results are pervasive in oligopoly; exceptions to "normal" behaviour are commonplace, but their interest is only commensurate to their robustness, which needs to be studied. Surprises can be important, aberrations much less so: it all depends on whether the former are seen to occur for a large and central set of circumstances or not. Unfortunately, a complete characterization of outcomes is usually hard to ascertain in oligopoly, on account of the algebraic barrier these problems can present. But then relying on special examples can be misleading: the generality of their behaviour remains open to question. We shall give in what follows a fairly full analytic characterization of the effects industry-wide cost-rises have on profit margins (price) and on profits, for the general one-good conjectural-variations oligopolistic equilibrium. It permeates that conventional wisdom is most unreliable in this problem. Price and profit "overshifting" are likely indeed.

3. Framework

The model I shall be concerned with is the conjectural variations model of oligopolistic equilibrium,[4/] initially under conditions of industry-wide symmetry as studied in Seade (1980a) although this will presently be relaxed to look at the general homogeneous-output case as in Seade (1980b). A further relaxation to introduce a form of product differentiation is presented in §7 below.

Faced with an inverse demand function for aggregate output $p(Y)$, a cost function for own output $c^f(y_f,\xi)$ where ξ is a shift parameter, and immersed in an industry consisting of n firms described by their cost functions $c^j(y_j,\xi)$, firm f chooses output to maximize profits:

$$\max_{y_f} \quad \Pi^f = y_f p(Y) - c^f(y_f,\xi) \tag{2}$$

given, in general, a conjectured functional dependence of responses of aggregate output $Y \equiv \Sigma_j y_j$ to changes in own production, say $Y^c = \Lambda_f(y_f, Y)$ (c = conjectured). More generally this function could also depend on the entire position of the industry as described by the vector (y_j). Given only the existence (and, for simplicity, differentiability) of such a function for each f, one could postulate and study the existence and properties of equilibrium. Little is lost, however, if, for local analysis, the derivatives $dY^c/dy_f \equiv \lambda_f$ are treated parametrically. More is lost, of course, if symmetry across the λ's is assumed: even firms of similar size (symmetry of structure as such) can have different styles of management, whose outlook on their industrial environment, as captured primarily by λ, will also differ. Since my purpose in this paper is to derive general conditions under which certain results obtain, rather than merely to establish them as possibilities, it seems important to allow for behavioural (and other) asymmetries.

The first and second order conditions for a maximum of (2) are, respectively,

$$p(\Sigma y_i) + \lambda_f y_f p'(\Sigma y_i) - c^f_y(y_f, \xi) = 0 \tag{3}$$

and
$$\lambda_f^2 y_f p'' + 2\lambda_f p' - c^f_{yy} < 0 \tag{4}$$

where suffixes of $c^f(.)$ denote partial derivatives and all arguments in the functions in (4) have been omitted. I shall also rely heavily on the following conditions for stability:

$$(n + \lambda_f)p' + n\lambda_f y_f p'' - c^f_{yy} < 0 \tag{5}$$

$$\lambda_f p' - c^f_{yy} < 0 \tag{5'}$$

for all f, whose common, stronger version $p' + \lambda_f y_f p'' < 0$ (plus (5')) would be weak enough for our purposes below.

These conditions are derived formally in Seade (1980b), under conditions of asymmetry. Taking (5') for granted, which can easily be shown to be a necessary condition for stability in its own right using arguments analogous to those in the paper referred to, (5) is a sufficient condition for stability of the process $\dot{y}_f = \kappa_f(\bar{y}_f - y_f)$, where $\bar{y}_f$ solves (3) and $\kappa_f > 0$. It moreover is also a necessary condition, relative to the class of cases where (5) either holds or it fails to hold for all firms together.

A heuristic argument to show the necessity of (5) and (5') can easily be given. Consider an initial equilibrium situation, which we disturb by changing each firm's output y_j by a specified amount δ_j. The total change in marginal profits

$$\Pi_f^f \equiv \partial\Pi^f/\partial y_f = p + \lambda_f y_f p' - c_y^f$$

can be found by noting that

$$\Pi_{ff}^f = (p' + \lambda_f y_f p'') + (\lambda_f p' - c_{yy}^f) \equiv u^f + v^f,$$

$$\Pi_{fj}^f = (p' + \lambda_f y_f p'') \equiv u^f,$$

so that

$$\Delta\Pi_f^f \approx u^f \Sigma\delta_j + v^f \delta_f \tag{6}$$

Now stability precisely means that whatever the choice of $\delta = (\delta_1, \ldots, \delta_n)$, equilibrium will be restored. Consider first a uniform disturbance: $\delta_j = 1\ \forall j$. In that case (6) becomes precisely the left hand side of (5), which thus requires that the total effect of the expansion on marginal profits be negative, so that re-contraction is sought by producers.[5/]

Similarly, to obtain (5'), set $\delta_1 = 1$, $\delta_2 = -1$ and $\delta_j = 0$ for $j > 2$ in (6). What we then get, from (6), is

$$\Delta\Pi_1^1 \approx v^1, \quad \Delta\Pi_2^2 \approx -v^2, \tag{6'}$$

with $\Delta\Pi_j^j = 0$ for other j's. Then, again, we intuitively want firm 1, which was 'forced' to expland, to wish to contract, hence $v^1 < 0$, and firm 2 to expand, again $v^2 < 0$. We thus require (5').[6/]

4. Displacing symmetric equilibrium

4.1 Output

Let us first consider, to simplify the exposition, the symmetric case, with all f's deleted. Equation (3) reduces to

$$\lambda y p'(ny) + p(ny) - c_y(y,\xi) = 0, \tag{7}$$

noting that $Y = ny$ under symmetry. Let us now introduce a shift in the parameter ξ of the cost function: this could for instance reflect an input-price increase, a specific tax on output, or some technological shift in the production function of all producers; any industry-wide change, but not economy-wide, given our partial equilibrium framework. To be specific, I assume that, for all y, $c_{y\xi} > 0$. Notice also that, if ξ is excise tax, $c(y,\xi) = b(y) + \xi y$ where $b(y)$ is before-tax cost, so that $c_{y\xi} = 1$ then.

Totally differentiating (7) and solving for $dy/d\xi$, we get:

$$\frac{dy}{d\xi} = \frac{c_{y\xi}}{(n+\lambda)p' + n\lambda y p'' - c_{yy}} \tag{8}$$

which, from (5), is unambiguously positive. Output always falls as marginal costs increase at the margin, at any rate under symmetry. Conventional wisdom, derived from the simple competitive case when c_y itself is the supply function, is proved correct in this regard, for a wide class of cases given the flexibility of interpretation the conjectural variations model lends itself to. The two usual limit members of the 'oligopoly' family can easily be obtained as special cases: monopoly setting $n = \lambda = 1$, and price-taking behaviour setting $\lambda = 0$.

4.2 Price

With output falling, price will clearly rise: some shifting will occur. Differentiating $p = p(ny)$

$$\frac{dp}{d\xi} = p'n \frac{dy}{d\xi} = \frac{p'nc_{y\xi}}{(n+\lambda)p' + n\lambda yp'' - c_{yy}} \tag{9}$$

which is, as expected, unambiguously positive. Now define the shifting coefficient $S \equiv (dp/d\xi)/c_{y\xi}$ $(\simeq \Delta p/\Delta c_y)$ whose being $\gtrless 1$ is equivalent to shifting being $\gtrless 100\%$. In the uniform excise-tax interpretation of ξ, S reduces to dp/dt. From (9),

$$S = \frac{p'n}{(n+\lambda)p' + n\lambda yp'' - c_{yy}} \tag{10}$$

so that $S > 1$ can easily arise: it will if $(\lambda/n)p' + \lambda yp'' - c_{yy}/n$ is a positive number, which can very well be the case, not under competition $(\lambda = 0)$, but in other structures. This expression can be simplified considerably and put in a more interpretable form as follows. We compute, for simplicity's sake, $S^{-1} - 1$ $(\simeq [\Delta c_y - \Delta p]/\Delta p)$, which conventional wisdom expects to be positive:

$$\begin{aligned} S^{-1} - 1 &= (\lambda p' + n\lambda yp'' - c_{yy})/np' \\ &= (\lambda/n)\left[1 + (nyp''/p') - (c_{yy}/\lambda p'\right], \end{aligned}$$

so that

$$S^{-1} - 1 = (\lambda/n)(k - E), \tag{11}$$

where

$$E \equiv -Yp''/p' \; ; \; k \equiv 1 - (c_{yy}/\lambda p') \tag{12}$$

The first of these parameters is the elasticity of the slope of inverse demand, whose value turned out to be the central element to determine the qualitative effects of entry in Seade (1980a), and which again will be very useful in what follows.[7/] Notice that E as defined in (12) is the negative of the the E of Seade (1982a). This simplifies notation and interpretation somewhat. The other parameter, k, can also be given an interpretation: the term $c_{yy}/\lambda p'$ measures the effect on own marginal cost in production per unit perceived change in market price, both brought about by a change in own output. One clearly expects $k \simeq 1$ in most interesting cases, which is exactly so under linear costs. The result in (11) says that, quite generally, over-shifting will occur if and only if the elasticity of the slope of demand is greater than this k.

The acceptability of this range of values of E needs, of course, to be checked against the second order and the stability conditions for the problem. In terms of E, these conditions [(4), and (5)-(5'), resp.] can be expressed as:

Second order $\quad E < (1+k)n/\lambda \qquad (13)$

Stability $\quad \begin{cases} E < (n/\lambda) + k & (14) \\ k > 0 & (14') \end{cases}$

But in most cases of interest n/λ will be a number considerably larger than 1, unless tacit collusion is very high. Under linear costs and for the iso-elastic case, with demand given by $p = AY^{-1/\varepsilon}$, the value of E is $1 + (1/\varepsilon)$, so that any iso-elastic demand function consistent with stable (symmetric) market equilibrium will result in overshifting,[8/] whatever the market structure and behavioural parameters, n and λ! Other demand functions can of course render this result less likely (or even rule it out, as linear demand or more generally any concave demand curve would do, but E and hence the sign of $E - 1$ (under $c_{yy} = 0$) are all that matters, which is useful.

4.3 Profits

Let us now turn our attention to the effects of ξ on profits. Producers' (net) price may well rise with taxation, but since output falls, their profits need not rise - they will probably fall, one would guess, as they will under the two extremes of price-taking behaviour and monopoly. The question of whether profits are in effect an increasing function of marginal cost is doubtless very important for allocative reasons, and not only distributionally.

Differentiating $\Pi \equiv yp(ny) - c(y,\xi)$, we get

$$\frac{d\Pi}{d\xi} = p\frac{dy}{d\xi} + yp'n\frac{dy}{d\xi} - c_y\frac{dy}{d\xi} - c_\xi$$

$$= \frac{(p + nyp' - c_y)c_{y\xi}}{(n+\lambda)p' + n\lambda yp'' - c_{yy}} - c_\xi$$

$$= \frac{(n-\lambda)yp'c_{y\xi}}{(n+\lambda)p' + n\lambda yp'' - c_{yy}} - c_\xi \tag{15}$$

using the first-order condition (3). Since n is a natural upper bound for λ in these models, and using the stability condition (5), the signs of the two terms in (15) are the signs of $c_{y\xi}$ and of $-c_\xi$ respectively. It seems reasonable to assume that a shift in input-price or production conditions that increases marginal cost, increases total cost too, so that these two effects will pull in opposite directions. This is what one would expect: the term c_ξ is the direct profit-loss suffered by producers in the absence of equilibrium effects, whilst the first term in (15) measures the beneficial effect of reduced equilibrium output which we were discussing earlier on in §2, the cost-induced collusion the producers themselves had been unable to achieve.

This expression can easily be shown to take one or other sign under a variety of cost and demand conditions. However, no neat classificatory results seem to emerge for the general cost-shift case. Let us then specialize, concentrating on the interpretation of ξ as an excise tax, i.e. $c(y,\xi) = b(y) + \xi y$, so that $c_\xi = y$ and $c_{y\xi} = 1$. Substituting these in (15),

$$\frac{d\Pi}{d\xi} = \frac{(n-\lambda)yp'}{(n+\lambda)p' + n\lambda yp'' - c_{yy}} - y$$

$$= \frac{y\left[(n-\lambda)p' - (n+\lambda)p' - n\lambda yp'' + c_{yy}\right]}{(n+\lambda)p' + n\lambda yp'' - c_{yy}} \tag{16a}$$

$$= \frac{\lambda y\left[-2\, p' - nyp'' + c_{yy}/\lambda\right]}{(n+\lambda)p' + n\lambda yp'' - c_{yy}}$$

$$= \frac{\lambda\, y\, (E - 1 - k)}{\left[(n+\lambda)p' + n\lambda yp'' - c_{yy}\right]/p'} \tag{16}$$

where E is again as defined in (12). The denominator of this expression is positive for stable equilibria.

We thus have a rather intriguing result, of general applicability to symmetric conjectural variations equilibria, which is easiest to state under linear costs: cost or excise-tax increases are profitable if and only if the elasticity of the slope of inverse demand exceeds the magic number 2, without reference to other aspects of demand or cost conditions, or indeed to the structural and behavioural parameters n and λ. Since constant elasticity ε makes $E = 1 + (1/\varepsilon)$, the result, for iso-elastic cases, says that <u>whenever ordinary demand elasticity is less than unity, profits increase with costs</u>.

This seems rather a perverse result, if by that we mean counter-intuitive, but is not at all unlikely. Under monopoly, of course, elasticity will not be below 1 in the optimum. Also at the other extreme, under price-taking behaviour: $\lambda = 0$ in (16a) yields $d\Pi/d\xi = yc_{yy}/(np'-c_{yy}) \leq 0$ ($= 0$ in linear case), so that again the paradox is excluded. The only requirement we can impose is (14), namely $E < (n/\lambda) + k$, which for the isoelastic case reduces to $\varepsilon > (\lambda/n) + k - 1$, a number perhaps closer to 0 than to 1 usually, at any rate if concentration or collusion are not too high.

Table 1 gives the ranges of values of the elasticity of the slope E for which profits increase with taxation, for different numbers of Cournot players and under constant cost. We also list, for easy reference, the E's that yield price overshifting, and those which are required by the second-order and the stability necessary conditions (only (5), for (5') is always met under $c^f_{yy} = 0$, $\lambda_f > 0$). In each case, the corresponding form of the requirement in the isoelastic case is also given, in terms of the ordinary demand elasticity ε. Casual inspection of the table suggests that, far from being a pathology which one can safely ignore, profit and price overshifting may be about as likely to occur as not to. And at any rate, it is important to realize how strongly the comparative statics of oligopoly depend on the <u>curvature</u> of demand more than on anything else, around the prevailing equilibrium point.

TABLE 1 : Restrictions on the values of the elasticity of the slope of inverse demand (and, in brackets, on ordinary demand elasticity for isoelastic case) to meet the requirements or obtain the results indicated in rows 2-5. Relations shown are for linear cost and Cournot behaviour (for other λ's, replace n by n/λ throughout).

n (number of firms, symmetric oligopoly)	2	3	5	10
Second-order condition: $E<2n$	$E<4$ $(\varepsilon > \frac{1}{3})$	$E<6$ $(\varepsilon > \frac{1}{5})$	$E<10$ $(\varepsilon > \frac{1}{9})$	$E<20$ $(\varepsilon > \frac{1}{19})$
Stability condition, necessary and sufficient under symmetry: $E<n+1$	$E<3$ $(\varepsilon > \frac{1}{2}$	$E<4$ $(\varepsilon > \frac{1}{3})$	$E<6$ $(\varepsilon > \frac{1}{5})$	$E<11$ $(\varepsilon > \frac{1}{10})$
Price overshifting, i.e. producers' net price rises with costs. $E>1$	$E>1$ (any ε)	$E>1$ (any ε)	$E>1$ (any ε)	$E>1$ (any ε)
Profit overshifting, i.e. profits rise with marginal cost: $E>2$	$E>2$ $(\varepsilon<1)$	$E>2$ $(\varepsilon<1)$	$E>2$ $(\varepsilon<1)$	$E>2$ $(\varepsilon<1)$

5. Displaying non-symmetric equilibria

The assumption of symmetry can be unduly strong or very acceptable, depending on the context in which it is imposed and the questions being asked. In the present case, it seems reasonable to conjecture that, whether a firm will gain or not from an industry-wide change in costs, may well depend on the position of that firm relative to the rest of the industry. That is, under asymmetry (and given a behavioural pattern for firms), it might be that some firms end up cutting their output much more than others, thus handing the latter a greater beneficial 'externality'. The robustness of our results to the introduction of asymmetry, and indeed the effects of taxation on this new dimension (say market-share distribution), need to be examined.

Let us initially proceed without restricting the way the parameter ξ shifts the various cost and marginal cost functions, deriving general expressions for the comparative statics of (y^f). We proceed as before and differentiate (3). Writing $x_f \equiv dy_f/d\xi$,

$$p'\sum_i x_i + \lambda_f y_f p''\sum_i x_i + \lambda_f p' x_f - c^f_{yy} x_f - c^f_{y\xi} = 0 \qquad (17)$$

Now, using (5'), write

$$\alpha_f \equiv (p' + \lambda_f y_f p'')/(\lambda_f p' - c^f_{yy}), \qquad (18)$$

$$\beta_f \equiv c^f_{y\xi}/(\lambda_f p' - c^f_{yy}). \qquad (18')$$

We can then express (17) as

$$\alpha_f \sum_i x_i + x_f = \beta_f \tag{19}$$

which upon addition, yields

$$\sum_i x_i = \sum_i \beta_i / (1 + \sum_i \alpha_i) \tag{20}$$

where we have used $1 + \sum_i \alpha_i > 0$, which can readily be established from (5) and (5'). Hence the change in total output $\sum_i x_i$ falls if and only if $\sum_i \beta_i < 0$, i.e. for instance, if $c^f_{y\xi} \geq 0 \ \forall f$ and > 0 for some f'. Using (20) in (19),

$$x_f = \beta_f - (\alpha_f \sum_i \beta_i)/(1 + \sum_i \alpha_i), \tag{21}$$

which can of course no longer be signed in general. If $\alpha_f > 0$, which is a stronger but common form of (5) (used by Hahn (1962)), the second term in (21) (sign included) is positive, whilst the first has the sign of $-c^f_{y\xi}$. Thus firms not harmed directly by ξ (whilst others are, say, and no-one is directly benefitted), will expand output when their competitors suffer a rise in costs, and since total output falls (in (20)) the output of at least some firms (ranked by β_f/α_f) will fall.

Let us revert to the interpretation of ξ as an excise tax or a similar across-the-board change, so that β_f in (18') reduces to $\tau/(\lambda_{fp'} - c^f_{yy})$. It turns out that (21) does not easily yield further results one can interpret or sign, in general. One central special case does give us strong explicit implications: Cournot oligopoly under linear technology, which retains its asymmetry through the different values of the marginal cost different firms have. The initial Cournot equilibrium can easily be seen to allocate market share in inverse relation to marginal cost: $y_f = (p-c_y^f)/(-p')$. Efficiency is rewarded. This, however, is not necessarily true of the tax, i.e. at the margin. If $E > 0$ (e.g. isoelastic), the tax penalizes efficiency, at least in terms of market share, whose change following the tax varies directly with the cost. To see this, we use $\lambda_f = 1$, $c_{y\xi}^f = \tau$ (> 0), and $c_{yy}^f = 0$ in (18), (18'), which transform α_f and β_f into $\alpha_f = 1 - \omega_f E$ where $\omega_f \equiv y_f/Y$, market share; and $\beta_f = \tau/p'$. This in turn allows us to put (21) as:

$$x_f = (E - \omega_f E-1)\tau/(-p')(1+n-E) \tag{22}$$

whose denominator is positive by (14). Hence, if $E > 0$, (2) will be 'most' negative (remember $\Sigma x_i < 0$) for firms with a large market share, which as we have just seen are those with lower costs. This is not surprising: the uniform cost increase due to the tax, say, reduces the relative cost differentials among firms, by adding a constant across the board.

We now turn to examine the counterparts of the results in the previous section: the effects on price and on profits. Let us assume, from the outset, that technology is linear and, again, $c_{y\xi}^f \equiv \tau$. Then

$$S \equiv (dp/d\xi)/c_{y\xi} = p'(dY/d\xi)/\tau = p'(\Sigma_i x_i)/\tau \ .$$

Hence,

$$S = p'\Sigma\beta_i/(1+\Sigma\alpha_i)\tau$$

$$= \Sigma \frac{1}{\lambda} /(1+\Sigma\frac{1}{\lambda} - E)$$

so that

$$S^{-1} - 1 = (1-E)/\Sigma(\frac{1}{\lambda_i}) \tag{23}$$

The result thus extends exactly its counterpart under symmetry, (setting $k = 1$ in) equation (11). Without linear costs, however, the expression remains rather involved and only has (23) as an approximation.

To compute profit effects we differentiate $\Pi^f = y_f p - c^f$, which yields $\frac{d\Pi}{dt} = (p - c_y^f)x_f + y_f \frac{dp}{dt}$. Using the first order conditions and earlier results on x_f and dp/dt, we obtain, after some manipulations,

$$d\Pi^f/dt = y_f\left\{(E-2) + E(1 - \frac{\lambda_f y_f}{\bar{\lambda}\,\bar{y}}\right\}/(1 - \Sigma\,\frac{1}{\lambda_i} - E), \tag{24}$$

where $\bar{\lambda}$ is the harmonic mean of λ_f and $\bar{y}$ the arithmetic mean of y_f. The sign of this expression for profit changes is that of the expression in curly brackets. We thus obtain an imperfect generalization of the result under symmetry, where profits increased iff $E > 2$. We now have two terms. One is $(E - 2)$, which is again the same for all firms and the same as before. This is the central value (sign) of the effect, or its 'trend', exactly true for a 'typical' firm, defined as one with $\lambda_f y_f = \bar{\lambda}\,\bar{y}$, of course. This term is then to be adjusted by the firm-specific term $E(1 - \lambda_f y_f/\bar{\lambda}\bar{y})$. The value of the latter is probably relatively small to the extent that asymmetries are not too pronounced, and is in any case lower for firms with high y_f (more efficient) and/or with high λ_f (more 'collusive', i.e. more cooperative with the others in the industry).

6. An example - or two

Let a given homogeneous industry consist of n Cournot firms, each with a constant marginal cost of m^f, $f = 1,\ldots,n$. Demand is isoelastic: $p = AY^{-\frac{1}{\varepsilon}}$. The first order condition (3) can be written as:

$$p(1 - y_f/\varepsilon Y) = m^f, \tag{25}$$

which yields equilibrium price of

$$p = \frac{\Sigma m^i}{n-(1/\varepsilon)} \tag{26}$$

Hence, if $m^i \to m^i + \xi$ for all i,

$$\frac{dp}{d\xi} = \frac{n\varepsilon}{n\varepsilon-1} > 1 \tag{27}$$

by the stability condition, which reduces to $n\varepsilon > 1$ here (perceived elasticity > 1).

Market shares, $\omega_f \equiv y_f/Y$, from (24), are

$$\omega_f = (\frac{p-m^f}{p})\varepsilon, \tag{28}$$

whose derivative w.r.t. a uniform shift in all m_f's is

$$\frac{d\omega_f}{d\xi} = -\frac{\varepsilon}{p} + \frac{\varepsilon m^f}{p^2}\frac{dp}{d\xi},$$

which upon some simplifications yields a neat strong variant of (22):

$$\text{sign}\ \frac{d\omega_f}{d\xi} = \text{sign}\ (m^f - \bar{m}) \tag{29}$$

where $\bar{m}$ is the (arithmetic) mean of m_f.

Lastly, the effects on profits turn out to be given by rather contorted expressions. We shall therefore not reproduce the derivations but simply refer the reader to the results on Table 1 in p.17 above, which are precisely applicable to the present example but under symmetry.

Our second example of the results we have derived is not an exact one worked out analytically, but one from the 'real world'. This is, of course, the experience of the 'Seven Sisters' (the seven dominant oil concerns) in the aftermath of the oil crisis of 1973-4. Readers will remember the surprise that was widely expressed by public opinion when it was realized that the operational profits of these companies (excluding capital gains on stocks held) had increased sharply following the equally sharp rise they had to pay for their crude. Popular explanations tended to imply that their oligopolistic <u>behaviour</u> (tacit collusion, λ) had changed, which is a difficult explanation to substantiate or accept (they all had the same incentives and opportunity to collude in earlier years, or again in more recent years). But our result, simple as it is in not taking into account distinctive features of the oil industry, notably inventories, can explain quite well this outcome. The short-run elasticity of demand for final oil products was low indeed in the years in question, 72-75. And in the medium run, with a much larger (in fact, as it was, surprisingly large) relevant elasticity, profits have again fallen from their abnormally high levels of the early expensive-oil years.

The general case with all n firms in the market producing differentiated goods is not one that can usefully be studied. To see this let $y \equiv (y_1, \ldots, y_n)$ be the vector of outputs and $p^f = p^f(y)$ the inverse demand faced by the f^{th} producer. Suppose, for notational simplicity, that producers in the game are Nash-Cournot players; the conjectural variations extension can easily be worked out along similar lines.

The first-order condition is now

$$p^f(y) + y_f^f \; p_f^f(y) - c_y^f(y_f) = 0 \tag{30}$$

where again subindices of functions denote derivatives with respect to the arguments indicated. Displacing (30) with respect to an excise tax (i.e. $c_{y\xi}^f = 1$), yields

$$\sum_j (p_j^f + y^f p_{fj}^f) x_j \quad + \quad (p_f^f - c_{yy}^f) x_f = 1 \tag{31}$$

or

$$\sum_j \alpha_{fj} x_j + x_f = \beta_f \, , \tag{32}$$

where

$$\alpha_{fj} \equiv (p_j^f + y^f p_{fj}^f)/(p_f^f - c_{yy}^f); \; \beta_f = 1/(p_f^f - c_{yy}^f) \tag{33}$$

Hence, in principle, the exercise is simple, requiring us to find the inverse of the matrix (α_{fj}) (with the identity matrix added to it), from which the x's can be found in (32). But there is no special structure to this matrix and therefore, unless more assumptions are put into it (which might be to limit its dimensionality), it cannot be inverted explicitly, nor can we proceed to look for qualitative results on the effects of the shift.

What we need in order to avoid general matrix inversions altogether and derive single-equation solutions as in earlier sections, is to be able to bring all the 'f' elements out of the summation in (31). That is, for that, we need p_j^f to be of the form $\phi^f.\psi^j$,[9/] or $p_j^f/p_i^f = \psi^j/\psi^i$. Similarly for p_{fj}^f. This in turn tells us what the requirement is: the f-demand function should be separable for the vector $y^{\tilde{}f}$ (defined as the vector y with y^f deleted from it), through a sub-aggregate function $\Theta(\cdot)$ that is not f-specific. That is, we need $P^f(y)$ to be of the form $P^f(y^f,\Theta(y^{\tilde{}f}))$, where we allow ourselves the liberty to retain 'P' as the name of the function. We can think of Θ as an indicator of the 'environment' that surrounds firm f. One can in fact be more general than that and allow for commodities to fall into two or even more groups through their respective group-specific Θ's - this would simply result in our having to do a dimension-two (or more) matrix inversion instead of our one-equation solution that follows. But the formulae became terribly complicated without adding any insights through the results.

In fact, to keep matters simple, we will do the opposite, and give $\Theta(\cdot)$ above its simplest possible form, namely $\Theta(y^{\tilde{}f}) \equiv \Sigma_j y_j - y_f$. Our demand function is therefore of the form

$$P^f = P^f(y_f, \Sigma_j y_j - y_f) \tag{34}$$

Hence 'other goods' appear as perfect substitutes. One can think of the goods as being distributed, in a space of characteristics, on the corners of a hypertetrahedron, with consumers preferring one or other particular corner to any other point, and being then indifferent among equidistantly-placed commodities. But the example is meant to be no more than that, and hence calls for no further justification.

Under (34), (31) becomes

$$(p_2^f + y_f p_{12}^f)(\Sigma_j x_j - x_f) + (p_1^f + y_f p_{11}^f)x_f + (p_1^f - c_{yy}^f)x_f = 1,$$

or

$$(p_2^f + y_f p_{12}^f)\Sigma_j x_j + \left[(p_1^f + y_f p_{11}^f) - (p_2^f + y_f p_{12}^f) + (p_1^f - c_{yy}^f)\right]x_f = 1 \qquad (35)$$

The analysis can clearly continue very much along the lines of that in section 5: we divide through by the expression in square brackets and take summation across f, which delivers a single equation in the collective-output effect $\Sigma_j x_j$ which is then used in (35) to find the x_f's. But the final result on price overshifting is worth noting.

Writing $s_f \equiv dp^f/d\xi$, we find, after some algebra,

$$s_f^{-1} - 1 = (1 - E_1^f - E_2^f)/D^f, \qquad (36)$$

where D^f is a positive expression that can be signed, again, from heuristic stability considerations similar to those used earlier (which, in all rigour, cover the symmetric case only: differentiation is not a problem for that; but symmetry means that all firms have the same $P(;)$ and cost functions. See footnote 5).

The expressions in the numerator of (36) are our main interest. They are the counterparts to the single elasticity of the slope we had before - namely, now, the elasticities of the own slope (p_1^f) with respect to own and to others' output respectively:

$$E_1^f \equiv -y_f p_{11}^f / p_1^f ; \quad E_2^f \equiv -Q_f p_{12}^f / p_1^f , \tag{37}$$

where $Q_f \equiv Y - y_f$. In the homogeneous-product limit case for this problem, where $p_{11}^f = p_{12}^f$, the sum $E_1^f + E_2^f$ reduces to the single E of previous sections.

The implications of (36) for the actual likelihood of overshifting, relative to the situation under our assumed homogeneity, can of course not be assessed too easily for the general case: it depends on the precise nature of the demand functions and the way they interact through cross derivatives. One can even think of (36) as referring to rather different kinds of circumstances, such as introducing differentiation into a market previously modelled as homogeneous which is the obvious interpretation, vis à vis recognizing interactions with goods which our partial-equilibrium approach initially kept out of the picture altogether. But it is reassuring to note that our very simple earlier result on when price overshifting does or does not occur admits of an equally simple extension to the wider present framework.

8. Concluding remarks

In this paper we have studied the comparative-statics effects of changes in cost conditions, such as excise tax or a wage or technology-shift, in an oligopolistic industry selling (mostly) homogeneous output to a market described by an arbitrary demand function, whose non-homogeneous firms behave in a parametric conjectural-variations fashion.

The main results that have obtained appear to us to be surprising, both for their generality or plausibility, being in some cases highly counter-intuitive results, and for their very simplicity. These are that, following a rise in excise taxation or some similar industry-wide flat cost rise, and assuming for simplicity linear costs,

(i) Output of all firms will unambiguously fall in all stable equilibria;

(ii) consumer's price will accordingly rise, but will do so to a greater extent than the shift in marginal cost, representing a more than 100% shift of excise tax (say) to consumers, if and only if the <u>elasticity of the slope of inverse demand</u> E is greater than 1 (its value for stable equilibria need only be less than $n + 1$, for n Cournot firms in the symmetric case), which for isoelastic demands means <u>always</u>; and

(iii) the increase in price will be sufficient to more than offset the fall in volume of sales and the rise in costs, thereby raising the profits of each firm in the industry, if and only if E is greater than a firm-specific number that clusters around 2 and takes that value exactly under symmetry.

Some instances of applications or situations where the above results may be of interest have already been mentioned, such as the analysis of tax-shifting both in the price and profit senses which are old topics of constant interest, or the interpretation of developments such as the example given in the text on the oil industry. In a foreign-trade context, the result on profits would call for an output tax (or even better, to better avoid retaliation, an input tax) on an oligopolistic export sector facing inelastic world demand: their profits would rise, apart from yielding revenue. This is reminiscent of optimum-tariff arguments, but refers to intervention considerably upstream in the production process, not taxing exports but total output, or even its inputs. Indeed, such a policy would not necessarily result in a net benefit, less so in a Paretian gain to government and producers alike, if conventional wisdom were necessarily right in placing tax revenue and private profits (or more generally surplus) on the two sides of the scale in choosing tax.

The motivation I offer for the paper, however, is also, and to a large extent, theoretical. Not that the actual results obtained are of much interest from that point of view, but more generally for the fact that it was found to be quite feasible to manipulate and study the
conjectural
conventional-variations model analytically, and in so doing to raise questions of fiscal policy under the richer industrial structures that are commonplace in industrial, but not in public economics. Indeed, much too often in the latter subject, or even as a rule, studies of the effects of taxes on pricing and output decisions restrict attention to the polar forms of monopoly and perfect competition, and immediately

shift attention from standard neoclassical tools to mark-up models if oligopoly is at all to be considered.

Of course the main questions to be asked in this connection lie ahead or elsewhere, and not in this paper, notably the welfare effects (and design) of taxation. Some interesting results have successfully been derived for special cases, by Stern (1982) and Katz and Rosen (1983). But imperfect competition is by and large still to make its full entry into public economics. One general difficulty is that, in the latter field, one tends to shy away these days from partial-equilibrium analysis, which is pretty much necessary to study many problems in industrial economics. One should perhaps be more open-minded about such matters.

This takes me to the question of limitations or extensions of the analysis. One that I think is <u>not</u> a limitation, firstly, at least in connection with the specific results obtained as opposed to the modelling done, is the assumption of partial equilibrium. In a general equilibrium context it is clear that one can have effects such as a firms' profits rising if the wage rate goes up, for example on account of the increased income of customers-workers. It is in a partial-equilibrium context that the possibility of profit-raising increases in costs is relatively striking.

Similarly, the no-entry structure we have adopted is not restrictive as far as our main results, on profits, are concerned. In a long-run, free-entry equilibrium context, the conditions under which short-run profits increase or decrease with costs, translate directly into entry, or exit from the industry until equilibrium is restored. Fortunately it <u>will</u>

then be restored, for as was shown in Seade (1980a) individual profits will unabiguously fall/rise with entry/exit. The result on price-over-shifting, in contrast, will be affected in the long-run. The reason is that this result holds in a larger class of cases than the profit-rises result: if profits rise with costs and entry is induced, aggregate output will unambiguously rise (again see the paper cited), and hence price will fall from its short-run increased level. I have not explored carefully conditions under which this effect may or may not overturn the initial overshifting, but I guess both possibilities will be there. On the other hand, in cases with profits falling but overshifting still occurring ($1 < E < 2$, or $\varepsilon > 1$ for isoelastic cases), exit will further contract output and hence result in a further degree of price overshifting in the long run.

Footnotes:

1/ The main exception here is an excellent recent paper by A.Dixit (1984), whose purpose is similar but complementary to ours, deriving results, some analogous to some below, for a wider range of models of oligopoly.

2/ It is perhaps Harberger's (1962) celebrated article that marks the point of departure in this regard, initiating the systematic application of general equilibrium ideas (which do not mix well too easily with market power) in the analysis of taxation, at the expense of a rigid enforcement of the perfect-markets neoclassical paradigm.

3/ For an interesting approach to oligopoly as a public-good problem see Kurz (1982).

4/ In its common form in which the variational parameters are treated as constants, which, to draw the distinction, I called quasi-Cournot oligopoly in my 1980a,b papers.

5/ Strictly speaking stability does not require that all firms move monotonically towards equilibrium: some might have their (5) reversed and still the path be convergent (cyclical). The requirement that can be put, as a general necessary condition, is only that the Euclidean norm of the vector of excess outputs above equilibrium levels be monotonically decreasing. But if all (5)'s hold, stability does follow (sufficiency), and if it failed to hold for all f then instability would obtain ('necessity' of (5) if it is to hold or fail for all firms).

6/ As in the previous footnote, we can really only demand the norm to decrease, not each component of the vector of excess outputs. But the norm, at the point (1, -1, 0, 0,...0), is simply (an increasing transform of)
$D \equiv \left\{(y_1 - \bar{\bar{y}}_1)^2 + (y_2 - \bar{\bar{y}}_2)^2\right\}/2$, ($\bar{\bar{y}}$ being equilibrium), so that $\dot{D} < 0$ under any reasonable adjustment mechanism such as the $\dot{y} = K_f(\bar{y}_f - y_f)$ suggested in the text or its near-equivalent $\dot{y}_f = k_f \Pi^f_f$, iff (5') holds at that point.

7/ The relation between E as defined in (12) and the ordinary demand elasticity ε is given by

$$E = 1 + \frac{1}{\varepsilon} + \eta_{\varepsilon Y}, \qquad (12')$$

where $\eta_{\varepsilon Y} \equiv Y(d\varepsilon/dY)/\varepsilon$, so that in the isoelastic case $E = 1 + 1/\varepsilon$.

8/ This result has also been noted by de Meza (1982) and Stern (1982).

9/ Or in fact, more generally, to be bilinear in the f- and j-components, i.e. of the form, $\mu^f + \nu^j + \theta^f.\psi^j$. But such extension does not appear to add anything of interest, and leads to the kind of algebraic messiness we are trying to avoid.

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