CER-ETH - Center of Economic Research at ETH Zurich

Economics Working Paper Series

ETH

Elections, Contracts and Markets

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This version: May 2006

First version: February 2005

Abstract

As the performance of long-term projects is not observable in the short run politicians may pander to public opinion. To solve this problem, we propose a triple mechanism involving political information markets, reelection threshold contracts, and democratic elections. An information market is used to predict the long-term performance of a policy, while threshold contracts stipulate a price level on the political information market that a politician must reach to have the right to stand for reelection. Reelection thresholds are offered by politicians during campaigns. We show that, on balance, the triple mechanism increases social welfare.

Keywords: elections, threshold contracts, democracy, information markets, triple
mechanism

JEL Classification: D72, D82

1   Introduction

In democracies elections are the primary mechanisms for making politicians accountable. Holding reelections may induce incumbents to act in the public interest and allow the electorate to replace an incumbent with a more promising candidate. However, at a particular election date citizens may sometimes lack the information required to decide wisely about whether an incumbent deserves to be reelected. Lack of information may arise for several reasons. Voters may be rationally ignorant, since in a large electorate the likelihood of a single citizen affecting the outcome of an election is negligible. Alternatively, voters may have no access to information, e.g. in cases where policies have mainly long-term effects, and precise information about the consequences of a project is not available at the election date.

A typical example of a long-term policy is the pressing issue of unemployment. Reforming the labor market is generally considered inevitable for remedying unemployment. However, introducing labor market reforms may initially cause disruptions and even higher unemployment, because some layoffs will occur immediately, while the creation of new jobs may take time. Thus in the short term it may be impossible for voters to judge the politician's performance in the field of labor market policy. A policy problem with a longer time horizon is global warming, where due to the complex structure of the global warming problem it is difficult to assess the effect that reducing greenhouse gases will have on the climate and the well-being of people in the future.[1]

In this paper we propose a triple mechanism involving political information markets, threshold incentive contracts, and democratic elections to solve this fundamental information problem. At the end of the first term, a political information market takes place, where investors can bet on whether the incumbent will be reelected at the end of the second term and hence whether he has undertaken socially beneficial long-term policies. As it is uncertain whether the politician will be reelected for the first time at the end of period 1 this is a conditional information market. It aggregates the information on whether the

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[1]Most predictions suggest that the temperature associated with thermal equilibrium on earth will increase as a result of rapidly rising emissions of greenhouse gases (IPPC (2001)). Such temperature changes may have a sizable impact on the well-being of future generations (see e.g. Nordhaus (1991), Cline (1992), and Fankhauser (1995)).

incumbent has undertaken socially desirable long-term projects or whether the incumbent has merely pandered to current public opinion. A high price in the political information market indicates high probability that the incumbent will be elected a second time.

The second mechanism on which our proposal is built involves reelection threshold contracts that competing politicians can offer before they start on their first term. The reelection threshold contract stipulates a critical price threshold the information market must reach or exceed for the incumbent to have the right to stand for first reelection. The critical price thresholds are offered competitively by politicians campaigning for their first term in office.

Political information markets, price thresholds on these markets, and democratic elections increase the motivation of politicians to undertake long-term beneficial policies that may be unpopular at the time at which they are introduced. This is the main idea of this paper, and we develop it in the framework of a simple political agency model. We show that a carefully designed combination of political information markets and threshold contracts can – on balance – improve welfare.

Our model is most closely related to the proposal for combining contracts and democratic elections introduced by Gersbach (2003) and extended by Gersbach and Liessem (2005). A comprehensive presentation of the ideas, chances, and problems of incentive contracts for politicians can be found in Gersbach (2005). These papers show how the dual mechanism – contracts offered competitively during campaigns and elections – can improve political outcomes. All these papers rely on verifiable data by which contracts can be conditioned. By contrast, we analyze in this paper the case where the results from current policy can only be observed in a future period and may never be verifiable. We propose a novel triple mechanism where a political information market produces verifiable information in the form of prices at a time when policy results are not observable.

Political information markets have attracted a lot of attention recently. Information markets have been proposed to improve public policy decisions (see e.g. the recent surveys and discussions by Hanson (2003), or Wolfers and Zitzewitz (2004)). A comprehensive summary on this relatively new topic can be found, for example, in Hahn and Tetlock (2004). The basic idea behind information markets is the accumulation of scattered in-

formation in order to predict uncertain future events. Political information markets have turned out to be very successful in predicting election results (see e.g. Berg, Forsythe and Rietz (1996) or Berlemann and Schmidt (2001)), and are already established in practice. We propose a new type of information market. While standard markets predict the result of the next election, we use a market that predicts the result of the next but one election in order to obtain an approximation of the long-term effects of the current policies. The idea is that the incumbent will only be reelected in the next but one election if the voters are satisfied with the long-term project results they learn about over time.

Our paper is broadly related to political agency and accountability theory. While this literature developed by Barro (1973), Ferejohn (1986), and Persson, Roland and Tabellini (1997) has established the advantages and drawbacks of democratic elections in making office-holders accountable, we propose new institutional frameworks to improve the potential of democratic decision-making.

The paper is organized as follows. In the next section we introduce the model. The results for elections only are analyzed in section 3. In section 4 we examine the triple mechanism involving political information markets, threshold incentive contracts, and democratic elections. In section 5 we look at various extensions to our basic model. Section 6 concludes. The appendix contains the proofs.

2 The Model

Our model draws on Maskin and Tirole (2004), Gersbach (2005) and Gersbach and Liessem (2005). It contains democratic election rules, information acquisition, an information market, and reelection threshold contracts. There are three periods, denoted by $t = 1, 2, 3$.

2.1 The Election Framework

We assume that there are two politicians, denoted by $i = 1, 2$. They compete for office before the first period starts. The elected politician has to take some kind of action during the first period. He can choose between action $a_1 = 1$ and action $a_1 = 0$. All voters have the same preference ranking for the two possible actions, but they do not know their

preferences when they decide about the office-holder for the first term. There are two possible states of the world $s_1 = 1$ and $s_1 = 0$, which are drawn randomly. State $s_1 = 1$ will occur with probability z, and state $s_1 = 0$ will occur with probability $1 - z$. We assume that $\frac{1}{2} < z < 1$. The state of the world determines which action is optimal for the voters. If state $s_1 = 1$ is drawn, then the optimal action for the voters will be $a_1 = 1$. The optimal action for the voters will be $a_1 = 0$ in state $s_1 = 0$. As $z > \frac{1}{2}$, we will refer to $a_1 = 1$ as the popular action and to $a_1 = 0$ as the unpopular action. If $a_1 = s_1$ voters get a payoff of 1. Otherwise they get a payoff of 0. Voters are risk-neutral and want to maximize their expected utility.

There are two types of politicians, either congruent or dissonant. Both politicians know their own type and the type of their opponent.[2] However, voters cannot observe the politicians' types. A politician is congruent with probability π ($0 < \pi < 1$). In this case he has the same preferences as the voters. A politician is dissonant with probability $1 - \pi$, i.e. if $a_1 = 1$ is optimal for the voters, then $a_1 = 0$ is optimal for the dissonant politician and vice versa. The two political candidates may differ as to congruence or dissonance. In all other respects they are identical.

2.2 The Information Structure

At the beginning of the whole game, both voters and politicians have a priori probabilities of z that state $s_1 = 1$ will occur and of $1 - z$ that state $s_1 = 0$ will occur. In the first period, the elected politician can learn precisely which state of the world will occur, thus knowing with certainty which action is best for the voters and which action is best for himself.

We assume that while it is impossible to verify which state of the world has occurred the voters will be able to observe the realized state. However, it is not clear when the voters will make this observation. We assume that before their first reelection decision voters will observe with probability μ which state of the world has occurred, while the probability that they will observe the realized state in period 2 (i.e. after their first reelection decision) is $1 - \mu$. Furthermore, we assume that $0 \leq \mu \leq \frac{1}{2}$, which means that early observability

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[2]The assumption that politicians have knowledge about each other's type appears to be plausible because of their daily interaction. However, a candidate cannot use his knowledge about the type of his opponent in his election campaign, since he is not able to credibly communicate his information.

is unlikely. We use this assumption to analyze a situation where the possibility that the performance of a project is not observable in the short run is a really serious problem.[3] Note that regardless of whether there is early observability or not the project result will never be verifiable. Thus the problem of non-verifiability is given in all cases.

The value of μ does not depend on the realized state of the world. This means that early observability is as likely in state $s_1 = 1$ as in state $s_1 = 0$. The incumbent has to undertake the action in the first period before he knows whether the voters will be able to observe the realized state in period 1.

Some remarks about our informational assumptions are in order here. We model a situation where politicians obtain information earlier than voters. At the time the policy is undertaken, the incumbent can precisely identify the correct state of the world, while voters are still completely ignorant. Voters will observe the state of the world at a later point in time. If voters only observe the realized state in period 2, they do not know whether the incumbent has undertaken the socially optimal action at the time of their first reelection decision.

2.3 The Information Market

We allow for a political information market organized during the first period and after politicians have chosen their actions. There are N potential investors.[4] Investors are a subgroup of voters, but they are assumed to be a small group of the electorate so that the influence on the voting outcome is negligible. Investors have log utility with

$$U_j(Y_j + W_j) = \ln(Y_j + W_j) \tag{1}$$

where W_j is the investor's wealth and Y_j is gain or loss in the information market.[5] Investors can obtain a signal regarding the state of the world and can judge about whether the politician has undertaken the optimal action.[6] We use q_j to denote the subjective prob-

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[3]The assumption that $\mu \leq \frac{1}{2}$ is not crucial for our qualitative results. It is only of importance for our quantitative welfare analysis in section 4.

[4]It is sensible, that only individuals should be allowed to trade in such information markets and that trading volume per person is limited to avoid large-scale manipulation attempts.

[5]Note that we neglect the utility from the action of the politician in the utility function of the investors as policy outcomes have no influence on the trading behavior of the investors.

[6]One might assume that investors spend time for collecting information concerning the state of the world and thus have additional knowledge in comparison to ordinary voters.

ability that investor j believes that the politician has chosen the optimal action. We use q_j^g and q_j^b to distinguish the subjective probabilities when the politician acts congruently or dissonantly, i.e. q_j^g denotes the probability that investor j believes that the politician has chosen the optimal action if $a_1 = s_1$, while q_j^b denotes the probability in the case $a_1 \neq s_1$. Hence, $q_j \in \{q_j^g, q_j^b\}$.[7] Of course, we have $q_j^g > \frac{1}{2} > q_j^b \ \forall j$, i.e. the signals are at least partially informative regarding the quality of the incumbent's policy.

With probability μ there is already complete certainty in period 1 regarding the state of the world. In this case, all investors have the same level of information, and nobody will make gains or losses in the market. As it is already possible to observe the state of the world, it is not necessary to run the information market in order to predict the state. Thus, in the case of early observability we can either assume that the market will take place or that it will be canceled. As the market has no effect in this case, it will not affect our analysis.

There are two assets D and E. If the politician gets reelected after the second period the owners of asset D receive one monetary unit for a single unit of D. If the politician stands for reelection but is not reelected after the second period the owners of asset E receive one monetary unit for a single unit of E. This means that the settlement of the information market will occur at the beginning of period 3, when the result of the second reelection decision is known. If the politician is not able to run for his second reelection (for example due to the fact that he was already deselected at the first reelection) or if he does not want to stand for reelection, then all trades that have occurred will be neutralized.[8]

The information market works as follows: A bank or an issuer offers an equal amount of assets D and E. On the secondary market traders can buy assets D or E.[9] Trading in the secondary market results in price p for one unit of asset D. As buying one unit of D and one unit of E pays one monetary unit with certainty, the price of asset E must be $1 - p$. Otherwise either traders or the issuer could make riskless profits. An equilibrium in the information market is a price p such that traders demand an equal amount of assets D and

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[7]In principle q_j^g and q_j^b might differ across states, but we ignore this in our analysis.

[8]Alternatively, it would be possible in this case to make the payoffs of the assets on the information market dependent on the performance of the politician's party in the election at the end of period 2.

[9]We could allow for short-selling, but this is immaterial to our analysis.

$E.$[10]

It is useful to look more closely at the event tree associated with the assets. If, for example, an investor buys one unit of asset D at price p, then the event tree and the payoffs for the information market are given as:
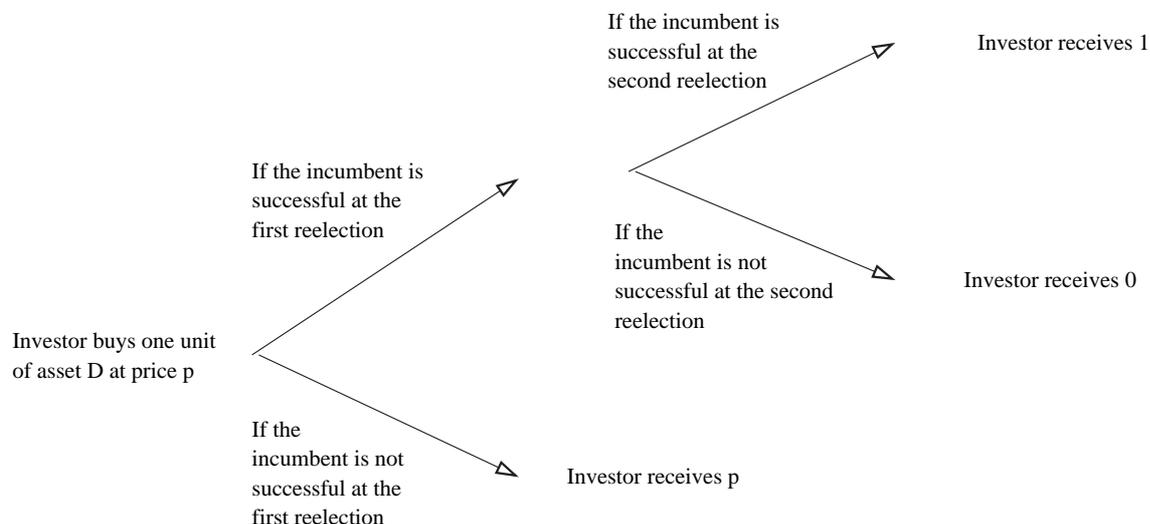

Figure 1

In this paper we specifically design information markets to allow for the design of reelection threshold contracts introduced in the next subsection.

2.4 Reelection Thresholds

Before the first period starts, politician i can offer a threshold contract $C_i(\hat{p}_i)$, which means that he will only be allowed to stand for reelection after the first period if price p on the political information market fulfills the condition $p \geq \hat{p}_i$. Candidates compete by offering $C_i(\hat{p}_i)$. We will see how competition for threshold contracts impacts on the policy decisions of the incumbent. If threshold contracts are offered, then the event tree and the payoffs for the information market have to be modified in the following way:

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[10]This is equivalent to an information market with only asset D, where traders can buy or sell D, and an equilibrium is obtained when supply equals demand.
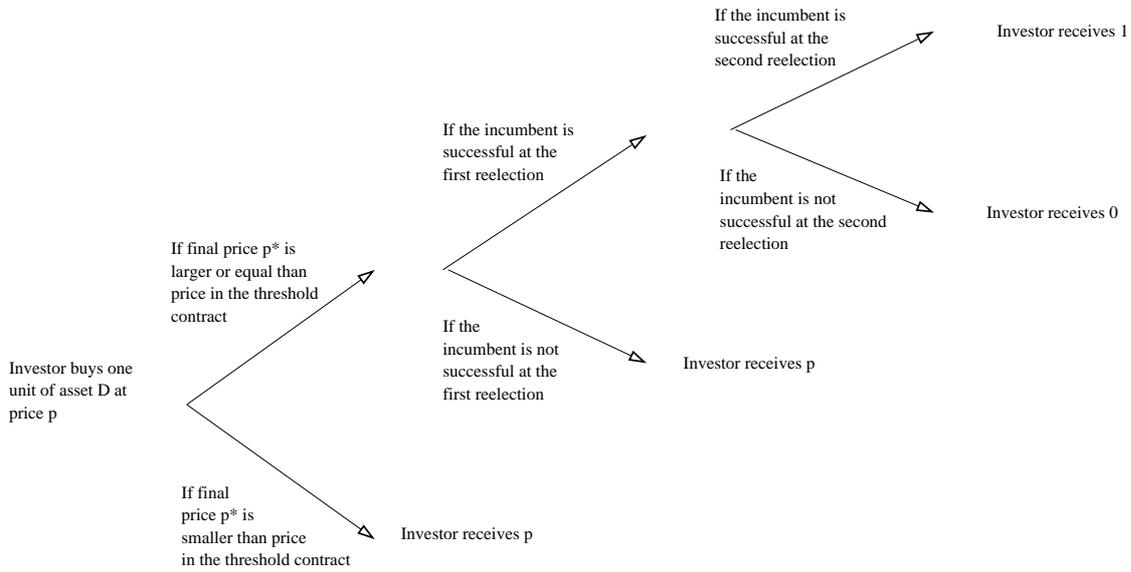
Figure 2

2.5 Summary

The timing of the whole game in its basic version is summarized in the following figure:
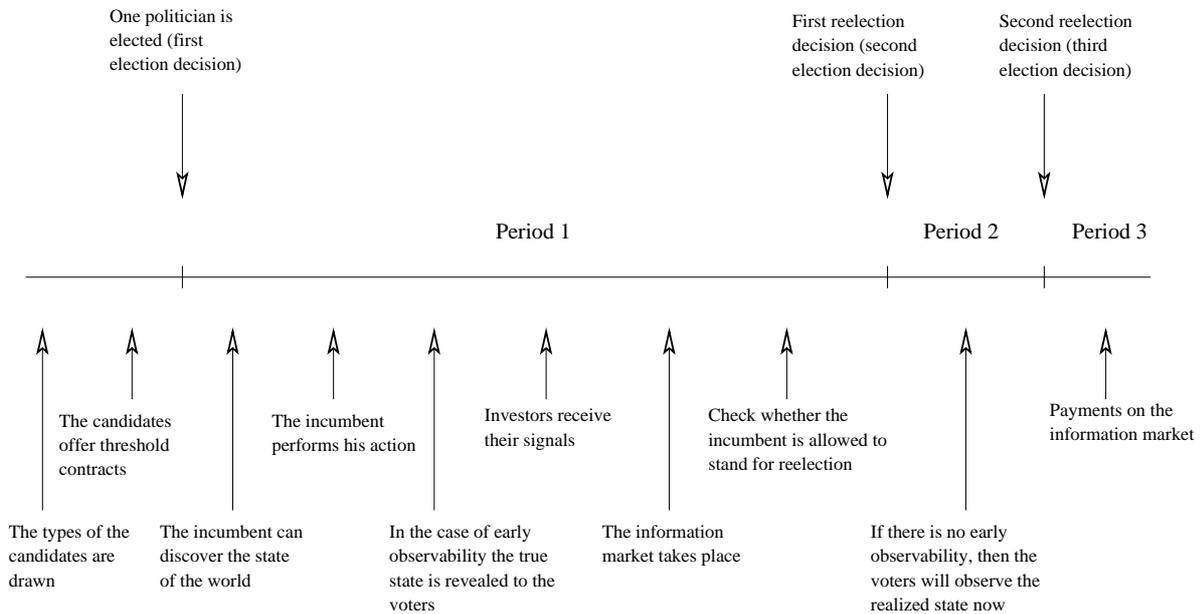

Figure 3

3 Elections Only

We first consider the behavior of both types of politicians in the scenario without threshold contracts and information markets. Here, elections are the only instrument used to discipline the incumbent.

3.1 Reelection Schemes

We use $r_1(a_1, s_1)$ to denote reelection probability for the incumbent after his first period in office.[11] Voters are able to observe the realized state in period 1 with probability μ. In this case they know whether the politician has undertaken the socially optimal action and will reelect the incumbent if $a_1 = s_1$, while they will deselect him if $a_1 \neq s_1$. If voters are not able to observe the state of the world in period 1, which happens with probability $1 - \mu$, they do not know whether the incumbent has acted congruently. In this case voters will reelect the politician if $a_1 = 1$, while they will deselect him if $a_1 = 0$. The idea behind the voting behavior in this case is the following: Voters do not know the state of the world. However, they know that $s_1 = 1$ is more likely than $s_1 = 0$, and thus they will reelect a politician who has undertaken the popular action $a_1 = 1$. Hence, when politicians undertake their actions, their beliefs regarding reelection are given as

$$
\begin{align}
r_1(a_1 = 1, s_1 = 1) &= \mu + (1 - \mu) = 1, \tag{2}\\
r_1(a_1 = 0, s_1 = 1) &= 0, \tag{3}\\
r_1(a_1 = 1, s_1 = 0) &= 1 - \mu, \tag{4}\\
r_1(a_1 = 0, s_1 = 0) &= \mu. \tag{5}
\end{align}
$$

We assume that reelection probability at the end of period 2 depends only on the outcomes realized in period 2 from the policy action undertaken in period 1. Further policy actions during the second term are assumed to be irrelevant for reelection chances at the end of period 2. This assumption greatly simplifies our analysis and can be justified

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[11]Note that voters are indifferent between reelection schemes, as the politician will undertake no further action during his second or third term in office. The retrospective voting scheme used in this paper is an optimal response of voters in our simple model and hence an equilibrium outcome. Retrospective voting is a particular resolution of the indifference of voters creating the highest possible disciplining device. The voting behavior can be further justified as a unique equilibrium outcome when we allow for an arbitrarily small amount of reciprocity. This justification has been developed by Hahn (2004). Of course, retrospective voting is a polar case, and thus highlights the tradeoffs the politician faces.

in several ways. First, if the politician undertakes only long-term policies in the second period, then no new information may be available at the end of the second period when the second reelection decision takes place. Second, the policy actions during his second term in office may be much less relevant than the first-period choices, so the performance of his policy depends only on his first-period action. Later we will extend our model to cover the case where the incumbent has to undertake further actions and discuss how this influences our result.

We use $r_2(a_1, s_1 | a_1 = s_1)$ to denote reelection probability for the incumbent at the end of period 2 if he has undertaken the action that is optimal for the voters and $r_2(a_1, s_1 | a_1 \neq s_1)$ to denote reelection probability if he has undertaken the wrong action from the voters' point of view. We assume that voters will reelect the incumbent if and only if he has acted congruently. Thus, the beliefs of the politicians regarding reelection at the end of period 2 are given as:

$$r_2(a_1, s_1 | a_1 = s_1) = 1 \tag{6}$$

and

$$r_2(a_1, s_1 | a_1 \neq s_1) = 0. \tag{7}$$

3.2 Preferences of Politicians

The elected politician has personal benefits R from being in office. Furthermore, he obtains a private benefit or personal satisfaction G if he undertakes the action that is optimal for himself. This benefit G accrues to the politician in the period in which he performs the action. We assume that the candidate receives no utility from the realization of his preferred action if another politician undertakes the action.[12] We use δ with $0 < \delta \leq 1$ to denote the discount factor for the politician. We will use the following tie-breaking rule: If the elected politician is indifferent as to the two actions, he will undertake the action that is optimal for the voters.

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[12]We might also assume that the politician receives the same utility as an ordinary voter if his opponent performs the action. However, this assumption may be less plausible in the case of a dissonant politician. At all events, the results of our analysis are not affected, as long as the value of G is sufficiently large in comparison to the utility of ordinary voters.

There are eight different cases depending on which state has occurred, which type of politician is involved, and which action he is undertaking. We use the following notation: The superscript in the utility function denotes the type of the politician, while the subscript in the utility function denotes his behavior. An elected politician who is congruent has utility

$$U_c^{cP} = R + G + r_1(a_1, s_1 | a_1 = s_1) \left[\delta R + r_2(a_1, s_1 | a_1 = s_1) \delta^2 R \right] \tag{8}$$

if he undertakes the optimal action in the first period. Note that reelection probability $r_1(a_1, s_1 | a_1 = s_1)$ for a politician who behaves congruently depends on the state of the world. If $s_1 = 1$ has occurred he will be reelected with certainty, while his reelection chance will be equal to μ if $s_1 = 0$ has occurred. The notation $r_1(a_1, s_1 | a_1 = s_1)$ covers both cases. An elected politician of the congruent type has utility

$$U_d^{cP} = R + r_1(a_1, s_1 | a_1 \neq s_1) \left[\delta R + r_2(a_1, s_1 | a_1 \neq s_1) \delta^2 R \right] \tag{9}$$

if he behaves dissonantly in the first period. A dissonant politician has utility

$$U_c^{dP} = R + r_1(a_1, s_1 | a_1 = s_1) \left[\delta R + r_2(a_1, s_1 | a_1 = s_1) \delta^2 R \right] \tag{10}$$

if he undertakes the socially optimal action in the first period, while his utility is

$$U_d^{dP} = R + G + r_1(a_1, s_1 | a_1 \neq s_1) \left[\delta R + r_2(a_1, s_1 | a_1 \neq s_1) \delta^2 R \right] \tag{11}$$

if he behaves dissonantly in the first period. Note that because of $r_2(a_1, s_1 | a_1 \neq s_1) = 0$ both types of politicians are deselected with certainty after the second period at the latest if they behaved dissonantly in the first period. By contrast, as $r_2(a_1, s_1 | a_1 = s_1) = 1$ both types of politicians are reelected with certainty at the end of the second period[13] if they behaved congruently in the first period. We now need to examine the circumstances under which the elected politician will act congruently. Obviously, it is always optimal for the voters if the incumbent behaves congruently.[14]

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[13] Note that it is possible that a politician who behaved congruently in his first term may be dropped from office by the voters at their first reelection decision.

[14] Note that, in contrast to Maskin and Tirole (2004), there is no "selection effect" in our model, as the politician only acts during his first term in office. Thus there is no welfare-enhancing effect when the voters discover that the incumbent is of the dissonant type and accordingly select a new one.

3.3 Behavior of Dissonant Politicians

We first look at the case $s_1 = 1$, where the popular action is optimal from the voters' point of view but the politician would prefer the unpopular action. The dissonant politician will only undertake the correct action if

$$R + \delta R + \delta^2 R \geq R + G$$
$$\Leftrightarrow \quad \delta R(1 + \delta) \geq G. \tag{12}$$

Condition (12) will be violated if personal gain from choosing the optimal action for himself is sufficiently larger than the gains from holding office.

We next examine $s_1 = 0$, where voters prefer the unpopular action while the politician prefers the popular action. The dissonant politician will only undertake the optimal action if

$$R + \mu(\delta R + \delta^2 R) \geq R + G + (1 - \mu)\delta R$$
$$\Leftrightarrow \quad \delta R(2\mu + \delta\mu - 1) \geq G. \tag{13}$$

This condition can only be fulfilled for certain values of δ and μ, as (13) cannot be satisfied if $(2\mu + \delta\mu - 1)$ is not positive. Note that $(2\mu + \delta\mu - 1)$ is monotonically increasing in δ. For $\delta = 1$ the condition $(2\mu + \delta\mu - 1) > 0$ is equivalent to $\mu > \frac{1}{3}$. This means that even in the case of $\delta = 1$ (the best value to fulfill the condition) it is only possible to fulfill equation (13) for $\frac{1}{3} < \mu < \frac{1}{2}$. Hence there are large parameter ranges where it is not possible to motivate a dissonant politician to perform the optimal action if the unpopular state has occurred. In particular, this will not be possible if the probability of early observation by the voters is small, as reflected in a low value for μ. Furthermore, it is obvious that condition (12) is easier to fulfill than condition (13).

Finally, we obtain the following intuitive results. If the parameters are such that condition (12) is fulfilled while condition (13) is not fulfilled, then there will be a distortion in favor of the popular action $a_1 = 1$. If neither condition (12) nor condition (13) are fulfilled, then there will be a distortion in favor of the unpopular action $a_1 = 0$.[15] It is useful to summarize the key observations in the following proposition.

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[15]Note that $z > \frac{1}{2}$, so – under the assumption that neither (12) nor (13) are fulfilled – the probability that the incumbent will undertake $a_1 = 0$ in a situation where he should perform $a_1 = 1$ is higher than the probability for undertaking $a_1 = 1$ instead of the optimal action $a_1 = 0$.

Proposition 1

Dissonant politicians will not choose the optimal action

(i) if $s_1 = 1$ and $\delta R(1+\delta) < G$ or

(ii) if $s_1 = 0$ and $\delta R(2\mu + \delta\mu - 1) < G$.

Four particularly interesting special cases of Proposition 1 are summarized in the following corollary:

Corollary 1

Suppose $\delta = 1$. A dissonant politician will not choose the optimal action,

α) if $s_1 = 1$ has occurred and $G > 2R$ or

β) if $s_1 = 0$ has occurred and $G > \frac{1}{2}R$ or

γ) if $s_1 = 0$ has occurred and $\mu < \frac{1}{3}$ or

δ) if $s_1 = 0$ has occurred and $\delta < \frac{1-2\mu}{\mu}$.

Note that $\delta = 1$ is most favorable for the public. If it is not possible to motivate a dissonant incumbent to act congruently in the case $\delta = 1$, then it will never be possible.

3.4   Behavior of Congruent Politicians

The congruent politician will undertake the optimal action in state $s_1 = 1$ if

$$R + G + \delta R + \delta^2 R \geq R. \tag{14}$$

This condition is always fulfilled, which means that in this state of the world congruent politicians always undertake the optimal action as both voters and the politician prefer the popular action.

We now look at the case $s_1 = 0$, meaning that both the voters and the politician prefer the unpopular action. The congruent politician will only undertake the optimal action if

$$R + G + \mu(\delta R + \delta^2 R) \geq R + (1-\mu)\delta R$$
$$\Leftrightarrow \quad G + \delta R(2\mu + \delta\mu - 1) \geq 0. \tag{15}$$

In contrast to the case of $s_1 = 1$ it may now be the case that even a congruent politician will not undertake the optimal policy although he too would prefer this optimal action, since the optimal action is unpopular but the politician would like to be reelected. This condition resembles equation (13) from above, but now G is on the left side because a congruent politician receives personal benefits G by acting congruently, while a dissonant politician receives G by acting dissonantly. Hence, if condition (13) is fulfilled, then condition (15) will also hold. Obviously, if it is possible to motivate a dissonant politician to undertake the optimal action, then it is always possible to motivate a congruent politician to undertake the optimal action. Clearly, the reverse is not true. Furthermore, we have a distortion in favor of the popular action given that it is possible for $a_1 = 1$ to be chosen too often, while the incumbent might not always carry out the unpopular action $a_1 = 0$ when he should. We summarize the results in the following proposition:

Proposition 2

A politician of the congruent type will not undertake the socially optimal action if $s_1 = 0$ and $G + \delta R(2\mu + \delta\mu - 1) < 0$.

4   The Triple Mechanism

We now analyze the effects of reelection threshold contracts on the behavior of the politicians. A reelection threshold contract $C_i(\hat{p}_i)$ (with $\hat{p}_i > 0$) consists of an announcement by politician i about the price p that will at least be reached on the information market. The threshold contract has the following consequence: If politician i offers $C_i(\hat{p}_i)$, then he will only be allowed to stand for reelection after the first period if the price p on the information market fulfills the condition $p \geq \hat{p}_i$. There will be competition concerning the threshold contracts.

The following questions will be of interest: Does price p on the information market correctly predict the quality of the politician's action? Will politicians offer reelection threshold contracts if this is voluntary? Is it possible to motivate politicians to behave optimally by introducing threshold contracts?

4.1 Reelection Schemes

If information markets are allowed and actually used, they might be taken into account by voters when making reelection decisions. Such feedback effects will be discussed in our extensions. In this section we abstract from such feedback effects in order to identify the pure effect of reelection contracts.

When there is no feedback from information markets to voting, reelection decisions by voters are the same as when no such markets and threshold contracts are present. Thus we still have the reelection probabilities from the section above. Note that the scheme for the first reelection is such that a politician will always be deselected if he acts dissonantly in the state $s_1 = 1$. Thus threshold contracts will have no effect in the state $s_1 = 1$, as the reelection scheme with $r_1(a_1 = 1, s_1 = 1) = 1$ and $r_1(a_1 = 0, s_1 = 1) = 0$ is already the maximal possible spread for deterring the politician from acting dissonantly. Adding threshold contracts forbidding a politician who has behaved dissonantly to run for reelection will not change the results, as the politician would not get reelected anyway. Nevertheless, threshold contracts will have a positive effect in the state $s_1 = 0$, where the reelection chances of a politician who has chosen the dissonant action will decrease from $1 - \mu$ to 0.

4.2 Pricing on the Information Market

In the next stage we determine the equilibrium price in the information market. We assume that all investors acquire a signal.

Suppose that the incumbent, say politician i, has offered a threshold contract $C_i(\hat{p}_i)$. Hence, for a price $p < \hat{p}_i$ no investor will have a strict incentive to buy assets as he will be paid back p. Suppose $p \geq \hat{p}_i$. An investor j who has obtained signal q_j has to weigh up the state of his information and the information the market price will reveal. A standard way to model the information aggregation process is as follows:

$$Prob(RE|p) = b_j \, Prob(RE) + (1 - b_j) \, p \tag{16}$$

where b_j (with $0 \leq b_j \leq 1$) is a weighting term describing self-assessed confidence, i.e. the subjective confidence of an investor in his own signal q_j relative to the market signal

expressed by price p.[16]

Given price p and signal q_j, an investor j maximizes

$$\max_{d_j} EU_j = Prob(RE|p) \, ln(W_j + d_j(1-p)) + (1 - Prob(RE|p)) \, ln(W_j - d_j p) \quad (17)$$

where d_j is the demand. If d_j is positive, investor j wants to buy d_j units of asset D. If d_j is negative, investor j wants to buy d_j units of asset E. The solution of the investor's problem yields

$$
\begin{aligned}
d_j^* &= W_j \, \frac{b_j \, q_j + (1 - b_j)p - p}{p(1-p)} \\
\Leftrightarrow \quad d_j^* &= W_j \, \frac{b_j \, q_j - p \, b_j}{p(1-p)}.
\end{aligned} \quad (18)
$$

We thus obtain

Proposition 3

There is a unique equilibrium in the information market given by

$$p^* = \sum_{j=1}^{N} q_j \, \frac{W_j \, b_j}{\sum\limits_{k=1}^{N} W_k \, b_k} \, . \quad (19)$$

Proof of Proposition 3

Equilibrium in the information market requires that the condition $\sum\limits_{j=1}^{N} d_j^* = 0$ be fulfilled, which implies $\sum\limits_{j=1}^{N} W_j \, b_j \, q_j - p \sum\limits_{j=1}^{N} W_j \, b_j = 0$ and the assertion follows from that.

■

The market price is a wealth- and confidence-weighted average belief on the part of investors. We note that the market price is equal to the simple average belief of investors if traders are homogeneous with respect to wealth and confidence in their own signal. If confidence levels are homogeneous, the market price is a wealth-weighted average belief on the part of traders. We summarize both cases in the following corollary:

Corollary 2

(i) Suppose $W_j = W \ \forall j$ and $b_j = b \ \forall j$. Then, $p^* = \frac{1}{N} \sum\limits_{j=1}^{N} q_j$.

---

[16]For a statistical foundation see Morris (1983) and Rosenblueth and Ordaz (1992).

(ii) Suppose $b_j = b \ \forall j$. Then $p^* = \sum\limits_{j=1}^{N} q_j \dfrac{W_j}{\sum\limits_{k=1}^{N} W_k}$.

There are two equilibrium price realizations, depending on whether or not the politician has undertaken the socially desirable policy. We use p_u^* to denote the upper equilibrium price in the case $a_1 = s_1$ and p_l^* to denote the lower equilibrium price in the case $a_1 \neq s_1$, i.e.

$$p_u^* = \sum_{j=1}^{N} q_j^g \frac{W_j b_j}{\sum\limits_{k=1}^{N} W_k b_k} \text{ and } p_l^* = \sum_{j=1}^{N} q_j^b \frac{W_j b_j}{\sum\limits_{k=1}^{N} W_k b_k}.$$

Since $q_j^g > q_j^b \ \forall j$, we have $p_u^* > p_l^*$.[17]

4.3 Election Decision

Politicians are free to offer threshold contracts. We assume that both politicians have to decide simultaneously about offering a threshold contract. As voters can only observe the threshold contracts of candidates and not their type, they are in expected terms equally or better off when they elect the candidate who offers a tighter constraint on his reelection threshold, as long as politicians can secure reelection by undertaking the socially optimal policy. Hence we obtain the following result, where $e_1(\hat{p}_1, \hat{p}_2)$ denotes the probability that candidate 1 will be elected at the first election decision:

Proposition 4

The sophisticated election scheme (SES)

$$e_1(\hat{p}_1, \hat{p}_2) = \begin{cases} 1 & \text{if } p_u^* \geq \hat{p}_1 > \hat{p}_2 \ \text{ or if } \ \hat{p}_2 > p_u^* \ \text{ and } \ \hat{p}_1 \leq p_u^*, \\ \frac{1}{2} & \text{if } \hat{p}_1 = \hat{p}_2, \text{ or if } \ \hat{p}_1 > p_u^* \ \text{ and } \ \hat{p}_2 > p_u^*, \\ 0 & \text{if } \hat{p}_1 < \hat{p}_2 \leq p_u^* \ \text{ or if } \ \hat{p}_1 > p_u^* \ \text{ and } \ \hat{p}_2 \leq p_u^*. \end{cases}$$

is optimal for voters.[18]

The proof of Proposition 4 is given in the appendix. While we will work for the moment with the sophisticated election scheme from Proposition 4, we will comment in section 5 on other optimal and simpler election schemes. All these schemes will produce the same welfare results.

---

[17]Note that the offer of a threshold contract with $\hat{p}_i < p_l^*$ is equivalent to offering no contract at all, since it is not possible for the price on the information market to be smaller than p_l^*.

[18]One could refine *SES* by adding $e_1(\hat{p}_1, \hat{p}_2) = \frac{1}{2}$ if $\hat{p}_1 < p_l^*$ and $\hat{p}_2 < p_l^*$. This is immaterial to our analysis.

It is important to note that the point in time when threshold contracts can be offered is at the beginning of the game, that is, even before the politicians have learned which state of the world has occurred. The politicians only know the probabilities of the two states. We thus obtain

Proposition 5

Both politicians will offer incentive contracts $C_i(\hat{p}_i = p_u^*)$ irrespective of their own type and irrespective of the type of their opponent.

The proof is given in the appendix. The next proposition is our main result.

Proposition 6

The conditions under which politicians in state $s_1 = 0$ behave congruently with threshold contracts are less strict, and dissonant behavior is less attractive, than without thresholds. This holds for both types of politicians. In the scenario with the triple mechanism we obtain:

(i) A dissonant politician behaves congruently in $s_1 = 1$ if $\delta R(1 + \delta) \geq G$.

(ii) A dissonant politician behaves congruently in $s_1 = 0$ if $\delta R\mu(1 + \delta) \geq G$.

(iii) A congruent politician always behaves congruently in both states.

The proof is given in the appendix. The intuition is as follows: Given equilibrium threshold contracts $C_i(p_u^*)$, politicians who behave dissonantly in the state $s_1 = 0$ have no chance of being reelected. If they behave congruently, their reelection chances are given by the probability μ. If no threshold contracts are written, a politician who behaves dissonantly still has a chance to get reelected, while congruent behavior does not yield higher reelection probabilities than μ. Hence threshold contracts contingent on prices in the political information market make dissonant behavior in the state $s_1 = 0$ less attractive relative to congruent behavior.

4.4 Welfare Gains

In this section we provide a brief example of the welfare gains that can be achieved with the triple mechanism. Suppose that, at a time when this institution is introduced, it is only

known that δ is equal to 1 and that μ is uniformly distributed in $[0,\frac{1}{2}]$. Since only the proportion of R and G is important for our analysis, we write $G = \alpha R$ with $0 \leq \alpha < \infty$. In the following we calculate the values of μ that enable congruent behavior by the incumbent. We use *eo* to denote the case with elections only and *tm* to denote the scenario with the triple mechanism. From condition (14) we obtain the conclusion that in the case of elections only a congruent politician will only behave congruently in state $s_1 = 1$ if

$$\alpha R + 3R \geq R.$$

This condition is always fulfilled. In the same way, we obtain the other conditions that are summarized in the following table:

	Congruent Politician		Dissonant Politician	
	$s_1 = 1$	$s_1 = 0$	$s_1 = 1$	$s_1 = 0$
Elections Only	$\alpha \geq -2$	$\mu \geq \dfrac{1-\alpha}{3}$	$\alpha \leq 2$	$\mu \geq \dfrac{1+\alpha}{3}$
Triple Mechanism	$\alpha \geq -2$	$\mu \geq -\dfrac{\alpha}{2}$	$\alpha \leq 2$	$\mu \geq \dfrac{\alpha}{2}$

Table 1

Note that congruent politicians always behave congruently in the scenario with the triple mechanism, as the conditions $\alpha \geq -2$ and $\mu \geq -\frac{\alpha}{2}$ are always fulfilled. Furthermore, congruent politicians always behave congruently in the scenario with elections only if $\alpha \geq 1$. Finally, it can be seen that a dissonant politician will never act congruently for $\alpha \geq 2$, which clearly derives from Corollary 1 and Proposition 6. In the next stage we calculate the expected utilities

$$EU^{tm} = \pi + (1-\pi)z \begin{cases} \int\limits_{0}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq 2 \\ \int\limits_{\frac{1}{2}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha > 2 \end{cases} + (1-\pi)(1-z) \begin{cases} \int\limits_{\frac{\alpha}{2}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \leq 1 \\ \int\limits_{\frac{1}{2}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha > 1 \end{cases}$$

20

and

$$
EU^{eo} \;=\; \pi z + \pi(1-z)
\begin{cases}
\int_{\frac{1-\alpha}{3}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \le 1 \\[2ex]
\int_{0}^{\frac{1}{2}} 2d\mu & \text{if } \alpha > 1
\end{cases}
$$

$$
+ (1-\pi)z
\begin{cases}
\int_{0}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \le 2 \\[2ex]
\int_{\frac{1}{2}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha > 2
\end{cases}
+ (1-\pi)(1-z)
\begin{cases}
\int_{\frac{1+\alpha}{3}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha \le \frac{1}{2} \\[2ex]
\int_{\frac{1}{2}}^{\frac{1}{2}} 2d\mu & \text{if } \alpha > \frac{1}{2}
\end{cases}
$$

These expressions can be simplified to

$$
EU^{tm} =
\begin{cases}
\pi + (1-\pi)[1 - \alpha(1-z)] & \text{if } \alpha \le 1 \\
\pi + (1-\pi)z & \text{if } 1 < \alpha \le 2 \\
\pi & \text{if } \alpha > 2
\end{cases}
\tag{20}
$$

and

$$
EU^{eo} =
\begin{cases}
z + (1-z)\dfrac{1 - 2\alpha + 4\alpha\pi}{3} & \text{if } \alpha \le \frac{1}{2} \\[2ex]
z + (1-z)\dfrac{(1+2\alpha)\pi}{3} & \text{if } \frac{1}{2} < \alpha \le 1 \\[2ex]
\pi + (1-\pi)z & \text{if } 1 < \alpha \le 2 \\[1ex]
\pi & \text{if } \alpha > 2
\end{cases}
\tag{21}
$$

We illustrate the relationships by calculating the utilities for four different values of α. We choose one value of α that is smaller than 1, one value larger than 1, and α equal to 1. These values correspond to the cases where for the politician the utility G is lower/higher than or equal to the utility R. Furthermore, we add the special case $\alpha = 0$, where the politician has no private benefits G. The expected utilities in these four cases are summarized in the following table:

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	$\alpha = 3$	$\alpha = 1$	$\alpha = 0.1$	$\alpha = 0$
EU^{eo}	π	$\pi + (1-\pi)z$	$\dfrac{8 + 4\pi + 22z - 4\pi z}{30}$	$\dfrac{1 + 2z}{3}$
EU^{tm}	π	$\pi + (1-\pi)z$	$\dfrac{27 + 3\pi + 3z - 3\pi z}{30}$	1
$EU^{tm} - EU^{eo}$	0	0	$\dfrac{(19-\pi)(1-z)}{30}$	$\dfrac{2(1-z)}{3}$
$\Delta_{EU} = \dfrac{EU^{tm} - EU^{eo}}{EU^{eo}}$	0	0	$\dfrac{(19-\pi)(1-z)}{8 + 4\pi + 22z - 4\pi z}$	$\dfrac{2(1-z)}{1 + 2z}$

Table 2

Note that in all cases we have $EU^{tm} \geq EU^{eo}$. Furthermore, we see that EU^{tm} is strictly larger than EU^{eo} if $z < 1$ and $\alpha < 1$. The difference between EU^{tm} and EU^{eo} depends on π and z for $0 < \alpha < 1$. The last row of the table shows the relative welfare gains (Δ_{EU}). Δ_{EU} is maximum for $\alpha = 0$. The example illustrates the following insights:

(i) Threshold contracts have the highest effect in the case $\alpha = 0$, i.e. if the politicians are only motivated by benefits R acquired from holding office.

(ii) If α is at least equal to 1, i.e. if politicians are at least as motivated by G as by R, then there is no effect from threshold contracts. This is due to the fact that congruent politicians always behave congruently, while dissonant candidates always behave dissonantly in state $s_1 = 0$. The conditions for congruent behavior in state $s_1 = 1$ are the same in the scenarios with or without threshold contracts.

If α is at least equal to 2, then congruent politicians always behave congruently, while dissonant candidates always behave dissonantly. Thus the expected utility is equal to π.

(iii) Finally, for a given value of α we discover that Δ_{EU} is (weakly) increasing when π decreases. Thus the more politicians are dissonant, the greater is the effect of threshold contracts.

5 Extensions

In the following we extend our basic scenario in various directions. First, we consider other election schemes and explore their robustness. Second, we examine feedback effects when voters take the price on the information market into consideration for their first reelection decision. Finally, we extend our analysis to the case of more than two periods, where the incumbent has to undertake an action during each period. The three extensions are discussed independently of each other, which means that we start with our basic model and then introduce one particular modification.

5.1 Other Election Schemes and Overpromising

The sophisticated election scheme used in our basic version of the model requires common knowledge concerning the values q_j, W_j and b_j for voters to be able to calculate the value p_u^* before the first election takes place. One may ask whether other election schemes might produce the same equilibrium threshold contracts. We first consider a simple scheme called monotonic election scheme (*MES*):

$$e_1(\hat{p}_1, \hat{p}_2) = \begin{cases} 1 & \text{if } \hat{p}_1 > \hat{p}_2, \\ \frac{1}{2} & \text{if } \hat{p}_1 = \hat{p}_2, \\ 0 & \text{if } \hat{p}_1 < \hat{p}_2. \end{cases}$$

Such a scheme, however, might invite extreme short-termism in the following sense: Given for example $\hat{p}_1 = p_u^*$, candidate 2 could select a threshold $\hat{p}_2 > p_u^*$ in order to be elected with certainty. Although the second politician will never be reelected, this may be profitable compared to an election chance of $\frac{1}{2}$. The question whether *MES* invites extreme short-termism is covered by the following proposition:

Proposition 7

Case (i): Suppose that voters use MES and that both politicians are of the congruent type.

(α) *The scheme MES is optimal (and leads to $C_i(\hat{p}_i = p_u^*)$) if*

$$R\{[z + \mu(1-z)](\delta + \delta^2) - 1\} \geq G \tag{22}$$

is fulfilled. Then both politicians will offer $C_i(\hat{p}_i = p_u^*)$ and have election probabilities of $\frac{1}{2}$. The elected politician will act congruently in both states of the world.

(β) *If condition (22) is not fulfilled, then both politicians will offer $C_i(\hat{p}_i = 1)$, they have election probabilities of $\frac{1}{2}$, and the elected politician will act congruently in both states.*

Case (ii): Suppose that voters use MES and that both politicians are of the dissonant type.

(α) *The scheme MES is optimal (and leads to $C_i(\hat{p}_i = p_u^*)$) if either the condition*

$$\frac{1}{2}R\{[z + \mu(1-z)](\delta + \delta^2) - 1\} \geq G. \tag{23}$$

or the condition

$$\frac{R}{1+z}[z(\delta + \delta^2) - 1] \geq G. \tag{24}$$

is fulfilled. Then both politicians will offer $C_i(\hat{p}_i = p_u^*)$ and have election probabilities of $\frac{1}{2}$. The elected politician will act congruently in the state $s_1 = 1$. In the state $s_1 = 0$ he will only act congruently if $\delta R \mu(1+\delta) \geq G$.

(β) *If neither condition (23) nor condition (24) is fulfilled, then both politicians will offer $C_i(\hat{p}_i = 1)$, they have election probabilities of $\frac{1}{2}$, and the elected politician will act dissonantly in both states.*

Case (iii): Suppose that voters use MES and that one politician (without loss of generality candidate 1) is of the congruent type, while his opponent is of the dissonant type.

(α) *The scheme MES is optimal (and leads to $C_i(\hat{p}_i = p_u^*)$) if either (23) or (24) is fulfilled. Then both politicians will offer $C_i(\hat{p}_i = p_u^*)$ and have election probabilities of $\frac{1}{2}$. The elected politician will act congruently in the state $s_1 = 1$. If candidate 1 is elected, he will also behave congruently in the state $s_1 = 0$. If candidate 2 is elected, he will only act congruently in the state $s_1 = 0$ if $\delta R \mu(1+\delta) \geq G$.*

(β) *If neither condition (23) nor condition (24) is fulfilled, then both politicians will offer $C_i(\hat{p}_i = 1)$ and have election probabilities of $\frac{1}{2}$. If candidate 1 is elected, he will behave congruently. If candidate 2 is elected, he will behave dissonantly.*

The proof is given in the appendix. Note that if condition (23) is not fulfilled, then dissonant politicians will promise a higher reelection probability than they can actually

achieve. This behavior can be interpreted as overpromising. Overpromising invites extreme short-termism, where dissonant politicians behave dissonantly in all cases. While it is obvious that overpromising is detrimental in the case of dissonant politicians, the negative effect of overpromising is more difficult to detect when the incumbent is of the congruent type. A congruent politician will behave congruently even if he overpromises. Thus his first-period behavior is not influenced in a negative way by overpromising. However, a politician who has practiced overpromising is never allowed to run for reelection. Hence a congruent incumbent has to be replaced by a new politician who can be either congruent or dissonant. Since a congruent politician creates higher expected welfare than a dissonant candidate, overpromising may have a negative dynamic feedback effect in the case of a congruent incumbent. Note that this argument is only relevant under the assumption that there might be another action of the politicians in period 2 or 3, while under the assumptions of our basic model the type of the incumbent in period 2 and 3 does not matter at all, as he will undertake no further action.

Now we examine another voting scheme called robust election scheme (*RES*):

$$
e_1(\hat{p}_1, \hat{p}_2) = \begin{cases} 1 & \text{if } \hat{p}_1 \geq \frac{1}{2} \text{ and } \hat{p}_2 < \frac{1}{2}, \\ \frac{1}{2} & \text{if } \hat{p}_1 \geq \frac{1}{2} \text{ and } \hat{p}_2 \geq \frac{1}{2}, \\ \frac{1}{2} & \text{if } \hat{p}_1 < \frac{1}{2} \text{ and } \hat{p}_2 < \frac{1}{2}, \\ 0 & \text{if } \hat{p}_1 < \frac{1}{2} \text{ and } \hat{p}_2 \geq \frac{1}{2}. \end{cases}
$$

The idea of this voting scheme is the following: Voters will elect a politician if he announces that he will undertake the optimal policy, which is reflected in a value $\hat{p} \geq \frac{1}{2}$, whereas the absolute value of $\hat{p}$ does not matter. As $p_u^* \geq \frac{1}{2}$ there is no overpromising problem in this case. We obtain the following proposition:

Proposition 8

The robust election scheme is optimal for voters. Both politicians will offer threshold contracts $C_i(\hat{p}_i = \frac{1}{2})$.

The proof of Proposition 8 is given in the appendix. The *RES* greatly enhances the applicability of our triple mechanism. Under this scheme voters do not need to have specific information regarding the parameters of projects or the wealth and the signals of investors in the information market. They simply judge whether politicians are willing to

compete against a fair coin when they hold office.

5.2 Forward-Looking Voters and Political Information Markets

In our basic model we have assumed that price p on the information market has no influence on reelection probability $r_1(a_1)$. In this subsection we assume that the voters pay attention to the price on the information market at the stage when they have to decide about the first reelection of the incumbent. Imagine an extreme case where voters only use the price on the information market as a basis for their reelection decision. As price p will be $p = p_u^*$ for $a_1 = s_1$ and $p = p_l^*$ for $a_1 \neq s_1$, the following scheme is optimal:

$$r_1(p(a_1, s_1)) = \begin{cases} 1 & \text{if } p \geq p_u^*, \\ 0 & \text{if } p < p_u^*. \end{cases}$$

Note that reelection probability no longer depends directly on the action undertaken but only on price p which measures the quality of the politician's action. We start by looking at the scenario without threshold contracts. In this case, a dissonant politician will undertake the optimal action if

$$R + (\delta R + \delta^2 R) \geq R + G$$

$$\Leftrightarrow \delta R(1 + \delta) \geq G \tag{25}$$

In the next stage we look at the scenario where politicians are allowed to offer threshold contracts and obtain exactly the same condition as in equation (25). Hence in this case threshold contracts are without effect (either positive or negative). The existence of a political information market that predicts the reelection chances after the next term is sufficient to generate all efficiency gains when voters use this forward-looking reelection scheme.

This result is due to the fact that the reelection scheme of the voters reproduces the consequences of equilibrium reelection contracts. If the incumbent undertakes an action that would result in his deselection after the second term, then he is already rejected after his first term in office. This reelection scheme is indeed optimal for the voters. The incumbent has no opportunity to behave in a populistic manner. Obviously, the case where voters fully adopt the assessments from the information market is less plausible, and only

the combination of reelection threshold contracts, information markets, and elections actually promises to produce all possible welfare gains.

5.3 Repeated Actions by the Politician

So far we have restricted the game to three periods. In the following we analyze the case where the incumbent is allowed to stay in office as long as he gets reelected. The incumbent has to undertake an action a_t in each period t ($t = 1, ..., T-2$) in office, with the exception of the last two periods.[19] T denotes the last term in office of the incumbent and hence his maximum lifetime as a potentially active politician. The candidates are allowed to offer threshold contracts before each election. The functioning of these contracts is the same as in the basic model, where threshold contracts were offered only once. All the assumptions of our basic model concerning the actions, the reelection probability after the first period etc. are still valid, as the policy problem in $t = 1$ is repeated $T - 2$ times. In particular, we have $r_1(a_1 = 1, s_1 = 1) = 1$, $r_1(a_1 = 0, s_1 = 1) = 0$, $r_1(a_1 = 1, s_1 = 0) = 1 - \mu$, and $r_1(a_1 = 0, s_1 = 0) = \mu$. A new component in our model is the reelection probability at the end of each period t with $t > 1$. There are two possible extreme cases for the reelection scheme. First suppose that voters take only the last action of the politician into consideration. In this case, reelection probabilities, denoted by r_t^P, are given as

$$r_t^P(a_t, s_t | a_t = 1, s_t = 1) = 1 \ , \tag{26}$$

$$r_t^P(a_t, s_t | a_t = 0, s_t = 1) = 0 \ , \tag{27}$$

$$r_t^P(a_t, s_t | a_t = 1, s_t = 0) = 1 - \mu \ , \tag{28}$$

$$r_t^P(a_t, s_t | a_t = 0, s_t = 0) = \mu \ . \tag{29}$$

This reelection scheme captures popularity voting, i.e. cases where voting is determined by the current attraction of a politician's action. Second, imagine a reelection scheme that only takes into consideration the action of the politician in period $t - 1$ (i.e. voters behave retrospectively), denoted by r_t^R, and given by

$$r_t^R(a_{t-1}, s_{t-1} | a_{t-1} = s_{t-1}) = 1 \tag{30}$$

---

[19]The assumption that the politician undertakes no action during the last two periods is equivalent to our assumption in the basic model that the politician chooses no action in the second and third period.

and

$$r_t^R(a_{t-1}, s_{t-1} | a_{t-1} \neq s_{t-1}) = 0. \tag{31}$$

Note that this retrospective reelection scheme is equivalent to the one used in our basic scenario to calculate the reelection probability at the end of the second period, while the popularity reelection scheme is equivalent to the one used to calculate reelection probability at the end of the first period.

Of course, a general voting scheme would be a weighted combination of the two schemes. In the following, we analyze the behavior of the politicians under both reelection schemes.

Proposition 9

(i) *Suppose voters behave retrospectively in all periods t with $t > 1$. Then the conditions under which politicians behave congruently with threshold contracts are less strict, and dissonant behavior is less attractive, than without thresholds. This holds for both types of politicians and for both states.*

(ii) *Suppose voters behave according to the popularity reelection scheme in all periods t with $t > 1$. Then the conditions under which politicians behave congruently with threshold contracts are less strict, and dissonant behavior is less attractive, than without thresholds. This holds for both types of politicians and for both states.*

The proof is given in the appendix. Under both analyzed reelection schemes, a congruent politician will always behave optimally if threshold contracts and information markets are introduced, while it is possible that he will undertake the socially undesirable action in the scenario without threshold contracts. Note that under retrospective voting a politician will be deselected if he acts dissonantly, while a politician who always acts congruently will definitely not be deselected for all periods $t > 1$. This contrasts with the popularity voting scheme, where it is possible that even a politician who always acts congruently will be deselected. If the politician is of the dissonant type, then it is possible that he will act dissonantly even in the scenario with threshold contracts. Nevertheless, the conditions under which dissonant politicians behave congruently with threshold contracts are less strict than without thresholds.

The extension of our model to T periods shows that the results of the two-period case are still valid. The scenario with the combination of elections and threshold incentive contracts is always socially advantageous compared to the case with elections alone, since the probability of a politician behaving congruently is higher when threshold contracts exist.[20]

6 Conclusion

In this paper we have proposed a triple mechanism to improve the functioning of democracies when information is not observable or not verifiable. The results seem to be quite robust under various extensions. Moreover, the idea of the triple mechanism might be extended to multi-task settings where the politician decides on many issues in his first term. As the threshold contract depends on the average long-term performance of the politician, the standard problem may aggravate distortions in favor of tasks with better observability.

Political information markets are an instrument for solving the problem of short-term unobservability coupled with long-term non-verifiability. Hence threshold contracts combined with information markets can be used successfully when projects have long-term effects and information on project results is not available in the short term. Of course, any proposal for a new institution such as the one we have made in this paper, has to be subjected to further scrutiny.[21] Such scrutiny will be undertaken in our future research work.

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[20]Note that, in contrast to our basic version of the model, there might now exist a welfare-enhancing "selection effect", as there are repeated actions. Nevertheless, the triple mechanism is still advantageous. Suppose that there is a dissonant incumbent and the parameters are such that he would act congruently and be reelected under the triple mechanism, while he would act dissonantly and be deselected in the case with elections only. Then he will either always act congruently under the triple mechanism, or he will act dissonantly and be deselected in a later period. In both cases, the benefits are higher under the triple mechanism, as there is either no dissonant behavior at all, or the dissonant behavior takes place in the more distant future. In the other case, where the incumbent is of the congruent type, the lower probability of being deselected under the triple mechanism is advantageous at all events.

[21]One might, for example, wonder how the triple mechanism can be introduced. We think that this might be triggered in election campaigns. If one party proposes the idea then competing parties might be forced to offer the same in order to avoid a decline of supporting votes.

7 Appendix

Proof of Proposition 4

Note that a politician offering a contract with a price higher than p_u^* will definitely not be allowed to stand for reelection and thus will behave in full accordance with his first-period preferences. Hence such a politician can never achieve a higher utility than a politician offering a price equal to or smaller than p_u^*. The same argument holds for a politician, say $i = 2$, who offers $\hat{p}_2$ with $\hat{p}_2 < \hat{p}_1 \leq p_u^*$. In this case electing politician 1 can never be worse than electing politician 2.

■

Proof of Proposition 5

Note that both candidates decide simultaneously about their threshold contracts. Hence they do not know the proposal of their opponent when they have to offer their contracts. We want to show that the offer $C_i(\hat{p}_i = p_u^*)$ for $i = 1, 2$ is a unique Nash equilibrium. Given that $\hat{p}_2 = p_u^*$, politician 1 will definitely not be elected if he does not offer $\hat{p}_1 = p_u^*$. Thus offering $C_i(\hat{p}_i = p_u^*)$ is a Nash equilibrium. In the next stage we show that it is unique.

We start by considering a politician (without loss of generalization candidate 1) who is of the congruent type and show that $C_i(\hat{p}_i = p_l^*)$ for $i = 1, 2$ is not a Nash equilibrium. Given that $\hat{p}_2 = p_l^*$, politician 1 has the following choices: He can offer a threshold contract with $p_u^* \geq \hat{p}_1 > p_l^*$, then he will definitely be elected. If he offers the contract $C_1(\hat{p}_1 = p_l^*)$ instead, then his election probability is $\frac{1}{2}$. Since the politicians are not yet conversant with the state of the world when they have to decide about offering a threshold contract, they have to base their utility comparison on the expected utility EU. State $s_1 = 1$ will occur with probability z, while $s_1 = 0$ will occur with probability $1 - z$. Hence the necessary condition for a congruent politician 1 to offer a threshold contract with $p_u^* \geq \hat{p}_1 > p_l^*$ in the case of $\hat{p}_2 = p_l^*$ is given by

$$EU^{cP}\left(\hat{p}_1 | p_u^* \geq \hat{p}_1 > p_l^*\right) \geq EU^{cP}\left(\hat{p}_1 | \hat{p}_1 = p_l^*\right). \tag{32}$$

If a congruent politician offers a threshold incentive contract with $p_u^* \geq \hat{p}_1 > p_l^*$ and gets elected, then he will always behave congruently.[22] If a congruent politician offers a contract $\hat{p}_1 = p_l^*$ and gets elected, then his behavior in state $s_1 = 0$ will depend on whether $R + G + \mu[\delta R + \delta^2 R]$ is larger or smaller than $(R + (1 - \mu)\delta R)$. Thus condition (32) takes the following form:

$$z[R + G + \delta R + \delta^2 R] + (1 - z)\{R + G + \mu[\delta R + \delta^2 R]\}$$
$$\geq \tag{33}$$
$$\frac{1}{2}z[R + G + \delta R + \delta^2 R] + \frac{1}{2}(1 - z)\max\{R + G + \mu[\delta R + \delta^2 R]; R + (1 - \mu)\delta R\}.$$

---

[22]This is obvious in state $s_1 = 1$. In state $s_1 = 0$, the politician has utility $R + G + \mu[\delta R + \delta^2 R]$ when he behaves congruently and utility R when be behaves dissonantly. Hence the politician will always behave congruently.

To analyze this inequality, we consider the two possible cases starting with $R + G + \mu[\delta R + \delta^2 R] \geq (R + (1 - \mu)\delta R)$. In this case, inequality (33) can be simplified to $1 \geq \frac{1}{2}$, which is always fulfilled. Next we look at $R + G + \mu[\delta R + \delta^2 R] < (R + (1 - \mu)\delta R)$. This time inequality (33) can be simplified to

$$
\begin{aligned}
\frac{1}{2}z[R + G + \delta R + \delta^2 R] \quad &+ \quad (1 - z)\{R + G + \mu[\delta R + \delta^2 R]\} \\
&\geq \\
\frac{1}{2}(1 - z) \quad &\cdot \quad [R + (1 - \mu)\delta R].
\end{aligned}
$$

This condition is always fulfilled because $\frac{1}{2}z[R + \delta R] > \frac{1}{2}(1 - z)[R + (1 - \mu)\delta R]$ and the other terms on the left hand side of the condition are positive. Thus we have $EU^{cP}\left(\hat{p}_1 | p_u^* \geq \hat{p}_1 > p_l^*\right) \geq EU^{cP}\left(\hat{p}_1 | \hat{p}_1 = p_l^*\right)$. According to this consideration of expected utilities, politician 1 will offer a contract with $p_u^* \geq \hat{p}_1 > p_l^*$. Thus $C_i(\hat{p}_i = p_l^*)$ is not a Nash equilibrium. The same argument holds for all contracts with $\hat{p}_i < p_u^*$, as the politicians will always overbid themselves up to $\hat{p}_i = p_u^*$. There remains the question about offers $\hat{p}_i > p_u^*$. Given that politician 2 offers $\hat{p}_2 > p_u^*$, it is optimal for candidate 1 to deviate to $\hat{p}_1 = p_u^*$. We can summarize the above considerations as follows: If a politician is of the congruent type, he will always offer an incentive contract $C_i(\hat{p}_i = p_u^*)$, irrespective of his opponent's behavior.

In the next step we analyze the behavior of a politician (without loss of generalization candidate 1) who is of the dissonant type and look at the case where his opponent offers $\hat{p}_2 = p_l^*$. In contrast to our considerations above for congruent politicians, it is no longer clear this time whether politician 1 will behave congruently or dissonantly. Nevertheless, we can still predict that he will offer a contract $C_1(\hat{p}_1 = p_u^*)$. This can be done in the following way: If the value of G is large enough, then a dissonant politician will behave in a dissonant manner regardless of the threshold contract he has offered. Given that $\hat{p}_2 = p_l^*$, we obtain

$$
EU_d^{dP}\left(\hat{p}_1 | p_u^* \geq \hat{p}_1 > p_l^*\right) = z(R + G) + (1 - z)(R + G) = R + G \tag{34}
$$

and

$$
\begin{aligned}
EU_d^{dP}\left(\hat{p}_1 | \hat{p}_1 = p_l^*\right) &= \frac{1}{2}\{z[R + G] + (1 - z)[R + G + (1 - \mu)\delta R]\} \\
&= \frac{1}{2}[R + G + (1 - z)(1 - \mu)\delta R] < R + \frac{1}{2}G. \tag{35}
\end{aligned}
$$

In this case where the politician always behaves dissonantly, it is obvious that the expected utility is larger if the politician offers $\hat{p}_1 > p_l^*$. Since a threshold contract with $\hat{p}_1 = p_u^*$ is as easy to fulfill as a contract with $p_u^* > \hat{p}_1 > p_l^*$, it will be optimal for candidate 1 to offer $\hat{p}_1 = p_u^*$.

For certain parameter ranges the politician acts congruently regardless of the thresh-

old contract he has offered. Given that $\hat{p}_2 = p_l^*$, we obtain

$$
\begin{aligned}
EU_c^{dP} & \left(\hat{p}_1 | p_u^* \geq \hat{p}_1 > p_l^* \right) \\
& = z[R + \delta R + \delta^2 R] + (1 - z)[R + \mu(\delta R + \delta^2 R)] \\
& = R + [z + (1 - z)\mu](1 + \delta)\delta R
\end{aligned}
\tag{36}
$$

and

$$
\begin{aligned}
EU_c^{dP} & \left(\hat{p}_1 | \hat{p}_1 = p_l^* \right) \\
& = \frac{1}{2}\{z[R + \delta R + \delta^2 R] + (1 - z)[R + \mu(\delta R + \delta^2 R)]\} \\
& = \frac{1}{2}\{R + [z + (1 - z)\mu](1 + \delta)\delta R\}.
\end{aligned}
\tag{37}
$$

In the case of a politician who always acts congruently, it is obvious that the expected utility is larger if the politician offers $\hat{p}_1 > p_l^*$. In line with the argumentation set out above, it will be optimal for candidate 1 to offer $\hat{p}_1 = p_u^*$.

There remains the scenario where the politician behaves congruently in one case and dissonantly in the other. It will never be the case that the politician acts congruently after offering $\hat{p}_1 < p_u^*$ while acting dissonantly after offering $\hat{p}_1 = p_u^*$. Hence the only case left to check is the scenario where the politician behaves dissonantly with contract $C_1(\hat{p}_1 < p_u^*)$ and congruently with contract $C_1(\hat{p}_1 = p_u^*)$. We know the following: If the politician has offered the contract $C_1(\hat{p}_1 = p_u^*)$, he will only act congruently if his utility is larger than it would be by acting dissonantly. Furthermore, the utility of acting dissonantly with contract $C_1(\hat{p}_1 < p_u^*)$ is smaller than the utility of acting dissonantly with contract $C_1(\hat{p}_1 = p_u^*)$. Thus it is clear that the utility of acting dissonantly with $C_1(\hat{p}_1 < p_u^*)$ has to be smaller than the utility of behaving congruently with contract $C_1(\hat{p}_1 = p_u^*)$.

Thus $C_i(\hat{p}_i = p_l^*)$ is not a Nash equilibrium. The same argument holds for all contracts with $\hat{p}_i < p_u^*$, as the politicians will always overbid themselves up to $\hat{p}_i = p_u^*$. Furthermore, contracts with $\hat{p}_i > p_u^*$ are not Nash equilibria because if politician 2 offers $\hat{p}_2 > p_u^*$, it is optimal for candidate 1 to deviate to $\hat{p}_1 = p_u^*$. Therefore we obtain the following result: If a politician is of the dissonant type, he will always offer an incentive contract $C_i(\hat{p}_i = p_u^*)$, irrespective of his opponent's behavior.

Thus the offer $C_i(\hat{p}_i = p_u^*)$ for $i = 1, 2$ is a unique Nash equilibrium.

■

Proof of Proposition 6

We start with dissonant politicians. We look first at the case $s_1 = 1$. Here the state of the world is such that the popular action is optimal from the voters' point of view, but the

politician would prefer the unpopular action. In the scenario with threshold contracts, the dissonant politician will undertake the right action if

$$
\begin{aligned}
R + \delta R + \delta^2 R &\geq R + G \\
\Leftrightarrow \quad \delta R(1 + \delta) &\geq G. \quad (38)
\end{aligned}
$$

Comparison with the condition in the scenario without threshold contracts shows that condition (38) is identical to condition (12). This is due to the fact that in state $s_1 = 1$ threshold contracts are without effect.

We next consider the case $s_1 = 0$. In this state, voters prefer the unpopular action, while the politician prefers the popular action. The dissonant politician will only undertake the optimal action if

$$
\begin{aligned}
R + \mu(\delta R + \delta^2 R) &\geq R + G \\
\Leftrightarrow \quad \delta R \mu(1 + \delta) &\geq G. \quad (39)
\end{aligned}
$$

Comparison with the condition in the scenario without threshold incentive contracts shows that in the case of $s_1 = 0$ it is easier to fulfill condition (39) than to fulfill condition (13). In particular, the expression $\mu(1 + \delta)$ is positive. Thus it is always possible to fulfill equation (39) by choosing a high enough value of R. This contrasts with the scenario without contracts, where it is impossible for many values of δ and μ to motivate the politician to undertake the socially optimal action.

We continue our analysis with congruent politicians. In the case $s_1 = 1$, we have the following condition for a congruent politician to undertake the optimal action:

$$
R + G + \delta R + \delta^2 R \geq R. \quad (40)
$$

This condition is always fulfilled. In the case of $s_1 = 0$, a congruent politician will undertake the optimal action if

$$
R + G + \mu(\delta R + \delta^2 R) \geq R. \quad (41)
$$

Again, this condition is always fulfilled. Hence in both states of the world, the politician will always pursue the policy optimal for the voters if he has offered a threshold contract. As we showed above in equation (15), this is not necessarily true for congruent politicians in the scenario without threshold contracts.

∎

Proof of Proposition 7

Case (i): We start our considerations with two congruent politicians and want to establish whether $\hat{p}_i = p_u^*$ is a unique Nash equilibrium. We look at the utility calculations of politician 1 and analyze whether he wants to deviate from $\hat{p}_1 = p_u^*$, given that $\hat{p}_2 = p_u^*$. If he offers $\hat{p}_1 < p_u^*$, then he will definitely not be elected. Thus he will offer $\hat{p}_1 \geq p_u^*$. If he offers $\hat{p}_1 > p_u^*$, then he will be elected with certainty, but he will never get reelected as

it is not possible to fulfill this contract. If he offers $\hat{p}_1 = p_u^*$, then he will be elected with a probability of $\frac{1}{2}$. Candidate 1 has to compare his expected utility under both proposals in order to offer the threshold contract that maximizes his expected utility. The necessary condition for not deviating from $\hat{p}_1 = p_u^*$ to $\hat{p}_1 > p_u^*$ is given by

$$EU^{cP}\left(\hat{p}_1 | \hat{p}_1 = p_u^*\right) \geq EU^{cP}\left(\hat{p}_1 | \hat{p}_1 > p_u^*\right). \tag{42}$$

Note that a congruent politician will always behave congruently in both states of the world if he has offered $\hat{p}_1 = p_u^*$ or $\hat{p}_1 > p_u^*$. Thus condition (42) gives us the following equation:

$$\frac{1}{2}\{z[R + G + \delta R + \delta^2 R] + (1 - z)[R + G + \mu(\delta R + \delta^2 R)]\} \geq (R + G)$$
$$\Leftrightarrow R\{[z + \mu(1 - z)](\delta + \delta^2) - 1\} \geq G. \tag{43}$$

If $\hat{p}_2 = p_u^*$ and (43) is fulfilled, then candidate 1 will offer $\hat{p}_1 = p_u^*$. If $\hat{p}_2 = p_u^*$ and (43) is not fulfilled, then candidate 1 will offer $\hat{p}_1 > p_u^*$. However, politician 1 would always behave congruently.

No politician has an incentive to deviate to $\hat{p}_i < p_u^*$, as he would be outbidded by his opponent. Furthermore, if (43) is not fulfilled, then both candidates will offer $\hat{p}_i = 1$. No politician would have an incentive to deviate from $\hat{p}_i = 1$, since his election chances would be 0 in the case of deviation. This overpromising case $\hat{p}_i = 1$ can only be avoided if (43) is fulfilled.[23] Hence we have a unique subgame perfect Nash equilibrium, where both politicians offer $\hat{p}_i = p_u^*$ when (43) is fulfilled and $\hat{p}_i = 1$ otherwise.

Case (ii): Now we assume that both politicians are dissonant and want to establish when $\hat{p}_i = p_u^*$ is a unique Nash equilibrium. We analyze whether candidate 1 deviates from $\hat{p}_1 = p_u^*$ to $\hat{p}_1 > p_u^*$, given that $\hat{p}_2 = p_u^*$. The necessary condition for not deviating is $EU^{dP}\left(\hat{p}_1 | \hat{p}_1 = p_u^*\right) \geq EU^{dP}\left(\hat{p}_1 | \hat{p}_1 > p_u^*\right)$. Note that a dissonant politician will never behave congruently when he offers $\hat{p}_i > p_u^*$. When he offers $\hat{p}_i = p_u^*$ and behaves dissonantly, he has a utility of $\frac{1}{2}(R + G)$, while he has a utility of $R + G$ when he offers $\hat{p}_i > p_u^*$ and behaves dissonantly. Therefore it is not optimal to offer $\hat{p}_i = p_u^*$ and to behave dissonantly in both states of the world. Thus politician 1 has to compare the expected utilities of three possible strategies, which we will denote in the following by c, d, and γ:

- Strategy c: Offering $\hat{p}_1 = p_u^*$ and behaving congruently in $s_1 = 1$ and $s_1 = 0$.

- Strategy d: Offering $\hat{p}_1 > p_u^*$ and behaving dissonantly in $s_1 = 1$ and $s_1 = 0$.

- Strategy γ: Offering $\hat{p}_1 = p_u^*$ and behaving congruently in $s_1 = 1$ and dissonantly in $s_1 = 0$.

Candidate 1 will offer $\hat{p}_1 = p_u^*$ if either $EU_c \geq EU_d$ or if $EU_\gamma \geq EU_d$. The condition $EU_c \geq EU_d$ gives us the following inequality:

$$\frac{1}{2}R\{[z + \mu(1 - z)](\delta + \delta^2) - 1\} \geq G \tag{44}$$

---

[23]The politicians do not know the offer made by their opponent when they make their own proposal. Nevertheless, they know whether condition (43) is fulfilled, so they also know what their opponent's optimal offer would be.

while the condition $EU_\gamma \geq EU_d$ can be transformed in the following way:

$$\frac{1}{2}[z(R+\delta R+\delta^2 R)+(1-z)(G+R)] \geq G+R$$

$$\Leftrightarrow \frac{R}{1+z}[z(\delta+\delta^2)-1] \geq G. \qquad (45)$$

If $\hat{p}_2 = p_u^*$ and (44) or (45) is fulfilled, then candidate 1 will offer $\hat{p}_1 = p_u^*$. In the following we analyze which action will actually be chosen by the incumbent. As he knows the state of the world at the date when he has to choose his action, his behavior may deviate from the behavior he used to calculate the expected utilities of the three strategies. Actual behavior depends on the state of the world. According to equation (38) he will behave congruently in state $s_1 = 1$ if $\delta R(1+\delta) \geq G$. As

$$\delta R(1+\delta) \geq \max\left\{\frac{1}{2}R\{[z+\mu(1-z)](\delta+\delta^2)-1\}; \frac{R}{1+z}[z(\delta+\delta^2)-1]\right\},$$

condition (38) is always fulfilled in the case where the politician offers $\hat{p}_1 = p_u^*$. In state $s_1 = 0$ the incumbent will only behave congruently if condition (39) is fulfilled, that is if $\delta R \mu(1+\delta) \geq G$. As it is not clear whether $\delta R \mu(1+\delta)$ is larger or smaller than $\max\left\{\frac{1}{2}R\{[z+\mu(1-z)^2](\delta+\delta^2)-1\}; \frac{R}{1+z}[z(\delta+\delta^2)-1]\right\}$, the behavior of the politician can be either congruent or dissonant, depending on whether condition (39) is fulfilled.

If neither condition (44) nor condition (45) is fulfilled, then both candidates will offer $\hat{p}_i = 1$ and will behave dissonantly. No politician would have an incentive to deviate from $\hat{p}_i = 1$, since his election chances would then be 0. Thus we have a unique Nash equilibrium, where both dissonant politicians offer $\hat{p}_i = p_u^*$ when (44) or (45) is fulfilled and $\hat{p}_i = 1$ otherwise.

Case (iii): We know that a dissonant politician will deviate to $\hat{p}_i = 1$ if neither condition (44) nor condition (45) is fulfilled. If one politician offers $\hat{p}_i = 1$, then his opponent will also offer $\hat{p}_i = 1$, irrespective of his type. Since the candidates know the type of their opponent, both politicians will offer $\hat{p}_i = 1$ if both (44) and (44) are violated, no matter whether condition (43) for the congruent politician is fulfilled or not. The candidates will only offer $\hat{p}_i = p_u^*$ if either inequality (44) or inequality (45) is fulfilled.

∎

Proof of Proposition 8

First it is important to note that $p_u^* > \frac{1}{2}$ and that $p_l^* < \frac{1}{2}$ since $q_j^g > \frac{1}{2} > q_j^b \forall j$. As the market price is a wealth- and confidence-weighted average belief on the part of the investors, it is not possible for the market price to exceed $\frac{1}{2}$ if each single investor has a belief that is smaller than $\frac{1}{2}$ and vice versa. By using *RES*, voters are able to avoid giving the politicians an incentive for overpromising. As there is no overpromising problem in this case, the absolute values of $\hat{p}_i$ do not matter. It is only important if $\hat{p}_i$ is $\geq \frac{1}{2}$ or $< \frac{1}{2}$. Furthermore, we note that under *RES* a politician (say $i = 2$) who offers a contract with a price smaller

than $\frac{1}{2}$ will never generate a higher utility than a politician who offers a price equal to or larger than $\frac{1}{2}$. Thus electing politician 1 can never be worse than electing politician 2 in this case.

Both politicians will offer $C_i(\hat{p}_i = \frac{1}{2})$. Given that candidate 2 offers $C_2(\hat{p}_i = \frac{1}{2})$, politician 1 will not deviate to $\hat{p}_i < \frac{1}{2}$, since then he has no chance of winning the election. Furthermore, he will not deviate to $\hat{p}_i > \frac{1}{2}$, since this does not increase his chances of winning the election while his threshold contract gets more demanding. Thus he will offer $C_1(\hat{p}_1) = \frac{1}{2}$.

∎

Proof of Proposition 9

Case (i) *Retrospective Reelection Scheme*

We start with a congruent politician. It is obvious that he will act congruently in state $s_1 = 1$, because if he behaved dissonantly then he would have a lower utility in the first period, his reelection chances after the first period would be lower, and he would be deselected with certainty after his second term in office.

The analysis is more difficult in the case of $s_1 = 0$. First we look at the scenario without threshold contracts. It is obvious that a congruent politician will always behave congruently for $t \geq 2$ irrespective of the state of the world, as a politician who acts congruently has additional utility G and will be reelected with certainty in the next period according to $r_t^R(a_{t-1}, s_{t-1} | a_{t-1} = s_{t-1}) = 1$, while he will be deselected with certainty if he undertakes the wrong action. Even a politician who behaved dissonantly in the first period will act congruently in the second period and will be deselected afterwards. The only remaining question is the behavior of the politician in the first period. A congruent politician will act congruently in the first period (and in all following periods) if

$$R + G + \mu[\delta(R+G) + ... + \delta^{T-3}(R+G) + \delta^{T-2}R + \delta^{T-1}R] \geq R + (1-\mu)\delta(R+G),$$

which can be simplified to

$$G + \mu R \sum_{k=1}^{T-1} \delta^k + \mu G \sum_{k=1}^{T-3} \delta^k > (1-\mu)\delta(R+G). \tag{46}$$

In the scenario with threshold contracts and $s_1 = 0$, a congruent politician will act congruently in the first period if

$$R + G + \mu[\delta(R+G) + ... + \delta^{T-3}(R+G) + \delta^{T-2}R + \delta^{T-1}R] \geq R. \tag{47}$$

This condition is always fulfilled. Hence under the triple mechanism the politician acts congruently in the first period and, as we showed above, in all subsequent periods as well.

We can summarize our results as follows:

(α) If $s_1 = 1$, then a congruent politician will behave congruently in each period and will always be reelected. This holds both with and without threshold contracts.

(β) If $s_1 = 0$ and there are no threshold contracts, then a congruent politician will only behave congruently in each period if condition (46) is fulfilled. Otherwise the politician will behave dissonantly in the first period, congruently in the second period, and be deselected afterwards.

(γ) If $s_1 = 0$ and there are threshold contracts, then a congruent politician will behave congruently in each period and will always be reelected.

We continue our analysis with a dissonant politician. The procedure is similar to the case of a congruent politician and is therefore omitted here. However, note that it is no longer clear that the incumbent will behave congruently for $t \geq 2$. Thus we obtain the following results:

(α1) If the triple mechanism is not introduced, then a dissonant politician will act congruently in the first period if

$$R \sum_{k=1}^{T-1} \delta^k \geq G$$

in state $s_1 = 1$ and if

$$\mu R \sum_{k=1}^{T-1} \delta^k \geq G + (1 - \mu)\delta(R + G)$$

in state $s_1 = 0$ is fulfilled. Otherwise the politician will behave dissonantly in the first and second period and be rejected at the end of his second term in office.

(α2) If the triple mechanism is not introduced, then a dissonant politician will act congruently in period t with $t \geq 2$ if

$$R \sum_{k=1}^{T-t} \delta^k \geq G + \delta(R + G).$$

Otherwise the politician will behave dissonantly in period t and $t+1$ and be rejected afterwards.

(β1) If there are threshold incentive contracts, then a dissonant politician will behave congruently in period 1 if

$$R \sum_{k=1}^{T-1} \delta^k \geq G$$

in the state $s_1 = 1$ and if

$$\mu R \sum_{k=1}^{T-1} \delta^k \geq G$$

in the state $s_1 = 0$ is fulfilled. Otherwise the politician will behave dissonantly in the first period and will not be allowed to stand for reelection in the second period.

(β2) If there are threshold incentive contracts, then a dissonant politician will behave congruently in period t with $t \geq 2$ if

$$R \sum_{k=1}^{T-t} \delta^k \geq G$$

Thus we have shown that under the retrospective voting scheme the conditions under which politicians behave congruently with threshold contracts are less strict than they would be without thresholds. This holds for both types of politicians and for both states.

Case(ii) *Popularity Reelection Scheme*

We start with a congruent politician in the scenario without threshold contracts. If $s_t = 0$ and the condition

$$R + G + \mu \left\{ \sum_{k=1}^{T-t} \delta^k R[z + \mu(1-z)]^{k-1} + \sum_{k=1}^{T-t-2} \delta^k G[z + \mu(1-z)]^{k-1} \right\}$$

$$\geq R + (1-\mu) \left\{ \sum_{k=1}^{T-t} \delta^k R[z + \mu(1-z)]^{k-1} + \sum_{k=1}^{T-t-2} \delta^k G[z + \mu(1-z)]^{k-1} \right\} \quad (48)$$

is fulfilled, then it will be better for a congruent politician to behave congruently in period t and in each subsequent period than to behave dissonantly in period t and always behave congruently afterwards. Note that the term $[z + \mu(1-z)]$ denotes the expected reelection probability in the case of congruent behavior. If $s_t = 0$ and the condition

$$R + G + \mu \left\{ \sum_{k=1}^{T-t} \delta^k R[z + \mu(1-z)]^{k-1} + \sum_{k=1}^{T-t-2} \delta^k G[z + \mu(1-z)]^{k-1} \right\}$$

$$\geq R + (1-\mu) \sum_{k=1}^{T-t} \delta^k R[(1-\mu)(1-z)]^{k-1} \quad (49)$$

is fulfilled, then it will be better for a congruent politician to behave congruently than to behave dissonantly in period t and in all subsequent periods. As the right-hand side of condition (48) is larger than the right-hand side of condition (49), the politician uses condition (48) to calculate his optimal behavior. The politician has to undertake this calculation in each period, and if condition (48) is fulfilled, then the politician will behave congruently.

If $s_t = 1$, then the politician will behave congruently, since the condition

$$R + G + \left\{ \sum_{k=1}^{T-t} \delta^k R[z + \mu(1-z)]^{k-1} + \sum_{k=1}^{T-t-2} \delta^k G[z + \mu(1-z)]^{k-1} \right\} \geq R \quad (50)$$

is always fulfilled. When state $s_t = 0$ occurs for the first time, the politician will only act congruently if condition (48) is satisfied.

In the scenario with elections and threshold contracts, the politician will always behave congruently, since the condition

$$R+G+\left\{\sum_{k=1}^{T-t}\delta^k R[z+\mu(1-z)]^{k-1}+\sum_{k=1}^{T-t-2}\delta^k G[z+\mu(1-z)]^{k-1}\right\}\geq R \quad (51)$$

in the case of $s_t=1$ or the condition

$$R+G+\mu\left\{\sum_{k=1}^{T-t}\delta^k R[z+\mu(1-z)]^{k-1}+\sum_{k=1}^{T-t-2}\delta^k G[z+\mu(1-z)]^{k-1}\right\}\geq R \quad (52)$$

in the case of $s_t=0$ is always fulfilled.

We can summarize our results as follows:

(α) In the scenario without threshold contracts, a congruent politician will behave congruently in period t in both states of the world if

$$G\geq(1-2\mu)\left(\sum_{k=1}^{T-t}\delta^k R[z+\mu(1-z)]^{k-1}+\sum_{k=1}^{T-t-2}\delta^k G[z+\mu(1-z)]^{k-1}\right). \quad (53)$$

If condition (53) is violated, then he will behave congruently in state $s_t=1$ and dissonantly in state $s_t=0$. The politician will be deselected if he acts congruently in state $s_t=0$ and the voters are not able to observe this state in period t, or if he acts dissonantly in state $s_t=0$ and the voters are able to observe this state in period t.

(β) In the scenario with threshold contracts, a congruent politician will always behave congruently in both states of the world.

We continue our analysis with a dissonant politician. The procedure is similar to the case of a congruent politician and is therefore omitted here. We obtain the following results:

(α) If there are no threshold contracts, then a dissonant politician will act dissonantly in state $s_t=0$ because of our assumption that $\mu<\frac{1}{2}$. In state $s_t=1$ he will act congruently in period t if

(a) $R(z+\mu(1-z))\geq(R+G)(1-\mu)(1-z)$ and if

$$[\sum_{k=1}^{T-t}\delta^k R(z+\mu(1-z))^{k-1}]\geq G \quad (54)$$

or if

(b) $R(z+\mu(1-z))<(R+G)(1-\mu)(1-z)$ and if

$$[\sum_{k=1}^{T-t}\delta^k R((1-\mu)(1-z))^{k-1}+\sum_{k=1}^{T-t-2}\delta^k G((1-\mu)(1-z))^{k-1}]\geq G \quad (55)$$

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The politician will be deselected if he acts congruently in state $s_t = 0$ and the voters are not able to observe this state in period t or if he acts dissonantly in state $s_t = 0$ and the voters are able to observe this state in period t.

(β) If there are threshold contracts, then a dissonant politician will act congruently in period t in state $s_t = 1$ if

$$[\sum_{k=1}^{T-t} \delta^k R(z^2 + (1-z)^2)^{k-1}] \geq G \tag{56}$$

and he will act congruently in period t in state $s_t = 0$ if

$$\mu[\sum_{k=1}^{T-t} \delta^k R(z^2 + (1-z)^2)^{k-1}] \geq G. \tag{57}$$

If condition (56) or condition (57), respectively, is violated, then the politician will act dissonantly in period t and will not be allowed to stand for reelection afterwards because of the threshold contract.

Thus we have shown that under popularity voting the conditions under which politicians behave congruently with threshold contracts are less strict than they would be without thresholds in state $s_t = 0$, while threshold contracts have no effect in state $s_t = 1$.

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References

Barro, R. (1973), "The Control of Politicians: An Economic Model", *Public Choice*, 14, 19-42.

Berg, J., R. Forsythe, and T. Rietz (1996), "What Makes Markets Predict Well? Evidence from the Iowa Electronic Markets", in *Understanding Strategic Interaction: Essays in Honor of Reinhard Selten*, Springer-Verlag, Berlin.

Berlemann, M., and C. Schmidt (2001), "Predictive Accuracy of Political Stock Markets. Empirical Evidence from an European Perspective", *Discussion Paper, Humboldt-University of Berlin*.

Cline, W.R. (1992), *The Economics of Global Warming*, Institute for International Economics, Washington, DC.

Fankhauser, S. (1995), *Valuing Climate Change: The Economics of the Greenhouse*, Earthscan Publications, London.

Ferejohn, J. (1986), "Incumbent Performance and Electoral Control", *Public Choice*, 50, 5-25.

Gersbach, H. (2003), "Incentive Contracts and Elections for Politicians and the Down-Up Problem", in *Advances in Economic Design*, ed. by M. Sertel, and S. Koray. Springer-Verlag, Berlin.

Gersbach, H. (2005), *Designing Democracy*, Springer-Verlag, Berlin.

Gersbach, H., and V. Liessem (2005), "Reelection Threshold Contracts in Politics", *CEPR Discussion Paper*, 5175.

Hahn, R., and P. Tetlock (2004), "Using Information Markets to Improve Policy", *AEI-Brookings Joint Center Working Paper*, 04-18.

Hahn, V. (2004), "Reciprocity and Voting", mimeo, University of Heidelberg.

Hanson, R. (2003), "Shall We Vote on Values, But Bet on Beliefs", mimeo, George Mason University.

IPCC (2001), *Climate Change: The IPCC Scientific Assessment*, Working Group 1, J.T. Houghton, G.J. Jenkins, J.J. Ephraums, Cambridge University Press, Cambridge.

Maskin, E., and J. Tirole (2004), "The Politician and the Judge: Accountability in Government", *American Economic Review*, 94 (4), 1034-1054.

Morris, P. (1983), "An Axiomatic Approach to Expert Resolution", *Management Science*, 29 (1), 24-32.

Nordhaus, W.D. (1991), "To Slow or not to Slow: The Economics of the Greenhouse Effect", *Economic Journal*, 101 (6), 920-937.

Persson, T., G. Roland, and G. Tabellini (1997), "Separation of Powers and Political Accountability", *Quarterly Journal of Economics*, 112, 1136-1202.

Romer, D. (2003), "Misconceptions and Political Outcomes", *The Economic Journal*, 113, 1-20.

Rosenblueth, E., and M. Ordaz (1992), "Combination of Expert Opinions", *Journal of Scientific and Industrial Research*, 51, 572-580.

Wolfers, J., and E. Zitzewitz (2004), "Prediction Markets", *Journal of Economic Perspectives*, 18(2), 107-126.

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