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"A Risk Management Approach for Portfolio Insurance Strategies"*

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Abstract

Controlling and managing potential losses is one of the main objectives of the Risk Management. Following Ben Ameur and Prigent (2007) and Chen *et al.* (2008), and extending the first results by Hamidi *et al.* (2009) when adopting a risk management approach for defining insurance portfolio strategies, we analyze and illustrate a specific dynamic portfolio insurance strategy depending on the Value-at-Risk level of the covered portfolio on the French stock market. This dynamic approach is derived from the traditional and popular portfolio insurance strategy (*Cf.* Black and Jones, 1987; Black and Perold, 1992): the so-called "Constant Proportion Portfolio Insurance" (CPPI). However, financial results produced by this strategy crucially depend upon the leverage – called the multiple – likely guaranteeing a predetermined floor value whatever the plausible market evolutions. In other words, the unconditional multiple is defined once and for all in the traditional setting.
The aim of this article is to further examine an alternative to the standard CPPI method, based on the determination of a conditional multiple. In this time-varying framework, the multiple is conditionally determined in order to remain the risk exposure constant, even if it also depends upon market conditions. Furthermore, we propose to define the multiple as a function of an extended Dynamic AutoRegressive Quantile model of the Value-at-Risk (DARQ-VaR). Using a French daily stock database (CAC40 and individual stocks in the period 1998-2008), we present the main performance and risk results of the proposed Dynamic Proportion Portfolio Insurance strategy, first on real market data and secondly on artificial bootstrapped and surrogate data. Our main conclusion strengthens the previous ones: the conditional Dynamic Strategy with Constant-risk exposure dominates most of the time the traditional Constant-asset exposure unconditional strategies.

Keywords: CPPI, Portfolio Insurance, VaR, CAViaR, Quantile Regression, Dynamic Quantile Model.

JEL Classification: G11, C13, C14, C22, C32.

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"A Risk Management Approach for Portfolio Insurance Strategies"

1 Introduction

Following Ben Ameur and Prigent (2007), Chen *et al.* (2008) and Hamidi *et al.* (2009), we apply the usual Risk Management approach to a particular type of portfolio insurance: the Constant Proportion Portfolio Insurance (CPPI - *Cf.* Black and Jones, 1987; Black and Perold, 1992). In other words, the risk of the new insured strategy is the true target and not the global asset weight exposure to risky assets as in the traditional approach.
The standard general method crucially depends upon the leverage – called the multiple – guaranteeing a predetermined floor whatever the plausible market evolutions. However, the unconditional multiple is defined once and for all in the traditional CPPI setting. We propose in this article an alternative to the standard CPPI method, based on the determination of a conditional multiple. In a time-varying framework, the multiple is conditionally determined in order the risk exposure to remain constant, but to depend on market conditions. In other words, while the traditional strategy is indeed a Constant-exposure Proportion Portfolio Insurance strategy, the strategy we examine has the main characteristic of being a Constant-risk Proportion Portfolio Insurance one.
Moreover, we propose to define the conditional multiple as a function of the Value-at-Risk (VaR) of the protected portfolio, which itself is modelled in a time-series framework following Engle and Manganelli (2004) and Gouriéroux and Jasiak (2008) through a Dynamic AutoRegressive Quantile modelling (DARQ-VaR model). Thus, when the forecasted risk increases, the leverage of the CPPI should decrease and *vice-versa*.
The paper is organized as follows. After having recalled some basics about Proportion Portfolio Insurance (PPI), we describe in section 3 the way we model the conditional multiple. In section 4, we present and estimate a particular specification of the general model presented in section 3 and compare it to the traditional unconditional strategy using real and several realistic artificial series based on the CAC40 Index and its components. Section 5 concludes. Appendix 1 justifies the proposed time-varying approach in an Insurance Portfolio context, whilst we briefly present in Appendix 2 the performance measures we use for evaluating the interest of a risk management approach for the Insurance of Portfolios.

2 Basics about Proportion Portfolio Insurance

The general Portfolio Insurance principle aims to allow investors to recover, at maturity, a given proportion of their initial capital. One of the standard PPI methods is the Constant Proportion Portfolio Insurance (CPPI). This strategy is based on a specific simple dynamic allocation on a risky asset and on a riskless one, aiming to guarantee a predetermined value at the end of the investment period.

The management of a cushioned portfolio follows a dynamic portfolio allocation and it is based on the definition of three virtual quantities: the floor, the cushion and the multiple. The floor is the minimum value of the portfolio that is acceptable for an investor at maturity. The value of the insured portfolio is invested in a risky asset and in a non-risky asset, in a proportion that varies in order to insure at any time the guaranteed floor value. Hence, the investment self-financed strategy aims that the portfolio has, at a certain maturity, a value equal, at minimum, to the floor, (*i.e.*, a predetermined percentage of the capital deposit at the beginning of the management period). The so-called cushion is defined as the difference between the portfolio value and the guaranteed floor. It represents a certain amount of the value of the portfolio that is dedicated to absorb some potential market shocks. Its size should be large enough for representing, each day, the *maximum* theoretical amount that can be lost without compromising the guaranteed capital.

The ratio between the risk-exposed asset value and the cushion corresponds, at any time, to the so-called multiple (defined once for all in the standard strategy). The multiple thus reflects the exposure of the portfolio. In its traditional version, the cushioned management strategy continuously targets a constant proportion of (unconditional) risk exposure. It means that the amount invested in the risky asset is determined by multiplying the cushion by the multiple. However, the crucial point of this simple strategy is to choose the targeted multiple. For instance, if the risky asset price drops, the value of the cushion must remain (by definition) superior or equal to zero. Therefore, the portfolio based on the cushion method will have (theoretically) a value superior or equal to the floor. Nevertheless, if the (fixed) multiple is too high (and/or the cushion is too low), a large fall in price of the risky asset may damage the value of the portfolio, which may fall below the guaranteed value. The cushion should thus allow the portfolio manager to absorb a market shock inferior or equal to the inverse of the multiple.

In a PPI framework, the multiple has to be at any time below the *maximum* of the (negative) realizations of the underlying risky asset return. The guarantee is thus perfect in the only case where the unconditional multiple is equal to one. In all other cases (for conditional or unconditional multiples), the guarantee is only provided according to plausible market conditions, that have to be defined by a set of assumptions regarding the potential loss on the risky asset one may face.

The probabilistic approach offers a *pseudo*-guarantee, mainly consisting in the respect, at any time, of the guarantee condition at a predefined significance level of probability. Using the quantile hedging approach, the guarantee constraint is associated to a significance level and the multiple must be lower than the inverse of the conditional quantile of the asset return distribution.

Thus, the target multiple can be re-interpreted as the inverse of the *maximum* loss that can bear the cushioned portfolio before the re-balancing of its risky component, at a given confidence level. Hamidi *et al.* (2009) propose a first conditional multiple model based on Value-at-Risk (VaR). This risk measure is based on a quantile function (*i.e.*, an inverse of the cumulative distribution function), and measures the potential loss of a portfolio over a defined period at a given confidence level. We complement hereafter their first results.

3 From the Extended DARQ-VaR Model to the Conditional Multiple in a CPPI Framework

Since it reflects the maximal exposure of the portfolio, the multiple is the crucial parameter of CPPI strategies. For a perfect capital guarantee, the multiple must be lower or equal to the inverse of the *maximum* loss of the risky asset return, until the portfolio manager can rebalance his position. For instance, if the risky asset drops drastically, the cushion must remain positive otherwise the predetermined floor is passed and the guarantee violated, (*i.e.*, the spread – varying across time – between the portfolio value and the guaranteed floor must be positive). Nevertheless, before the manager can re-adjust his position, the cushion allows the portfolio manager, by construction, for the absorption a shock smaller or equal to the inverse of the (superior limit of) the multiple.
Several unconditional multiple determination methods have been developed in the literature, but they all reduce the risk dimension of the strategy to the risky asset exposure (see Black and Perold, 1992). Thus, these traditional unconditional methods do not fully take into consideration the risk of the underlying asset that changes according, for instance, to market conditions. In other words, the risk of the risky asset proportion is considered as a constant through the whole life of the structured product. Looking at the time-variation of the amplitude and intensity of risk (see for instance Longin and Solnik, 1995), we propose to model the conditional multiple as a function of the VaR. The target multiple is then:

$$m_t = \left|VaR_t\left(r_{t-1};\beta\right) + d_t\right|^{-1} \tag{1}$$

where $VaR_t\left(r_{t-1};\beta\right)$ is the first percentile of the conditional distribution of daily returns of the underlying asset, r_t corresponds to the periodic return of the risky part of the portfolio covered, β is the vector of unknown parameters of the conditional percentile function, and d_t represents the exceeding *maximum* return during the estimation period.

When modelling the conditional multiple, we hereafter adopt a probabilistic quantile hedging approach, based on an extended Dynamic AutoRegressive Value-at-Risk model (DARQ-VaR), which is written in a particular extended Asymmetric Slope CAViaR specification - chosen for illustration purposes (see Engle and Manganelli, 2004), such as:

$$\begin{aligned} VaR_t\left(r_t;\beta\right) &= \beta_1 + \beta_2 \times VaR_{t-1}\left(r_{t-1};\beta\right) + \beta_3 \times \max\left(0;r_{t-1}\right) \\ &\quad +\beta_4 \times \left[-\min\left(0;r_{t-1}\right)\right] \end{aligned} \tag{2}$$

where the β_i, $i = [1,\ldots,4]$, are several parameters to estimate and r_t is the

risky asset return at time t.
The probability of 1% associated to the DARQ-VaR was chosen not only for focusing on true extremes but also for having enough data points for recovering good estimations. Without introducing the parameter d_t, the probability of violating the floor would have been equal to 1%. Working here at a daily frequency, this probability would thus have been too high for describing a realistic investor's demand (a multiple often equal to 30 or so). However, for a lower rebalancing time frequency (weekly or monthly), values of conditional multiples become more realistic. Moreover, if we assume that the portfolio manager can totally rebalance his position in one day, this particular estimation of the conditional multiple allows the portfolio manager for guaranteeing the predetermined floor defined by the investor. More generally, if the centile is well modelled (hit ratio not significantly different from 1%, no cluster of exceeding times, and limited exceeding *maximum* return from the centile) then the guarantee is (almost) insured (see Appendix 1). Finally, since the multiple is here modelled as a function of DARQ-VaR, it can also be interpreted in terms of Expected Shortfall. The parameter d_t allows for taking into account the risky asset dispersion of return in the (fat-)tail of the distribution of the risky asset returns. This parameter represents the highest failure of the model, and corresponds to one of the highest negative returns in the sample. The combination of both VaR and d_t is then closely linked to a measure of the Expected Shortfall. The VaR is here monitored (the risky asset allocation depending upon it), and extreme returns are taken into consideration through the parameter d_t. The proposed strategy can then be viewed as an application of Risk Management principles into a Portfolio Insurance context: the conditional multiple depends upon the forecasted Value-at-Risk, which depends on its turn to the lagged Value-at-Risk (and returns) and the highest failure of the model over the past. We propose in the next section to observe what type of results this kind of conditional approach can provide.

4 Data, Implementation Methods and Empirical Evidence of the Dynamic Strategy on the French Stock Market

We compare hereafter the performances of cushioned portfolios using a previously presented DARQ-VaR specification, and some of the traditional unconditional leveraged CPPI strategies associated to several levels of risk defined by an unconditional multiple fixed once and for all to values ranging from 3 to 13. We use CAC40 daily returns and single returns of its fifty main components since inception (stocks changing during the history of the series). The sample period consists of 21 years of daily data, from the 9th of July 1987 to 30th of April 2008. This total period consists of 5,242 returns which we split in two periods: we use a rolling window of 2,785 returns for dynamically in-sample estimating the parameters and a post-sample period consisting of 2,457 returns

for out-of-sample testing the various strategies. The following application on the CAC40 provides only a statistical illustration of the comparison between unconditional and conditional multiple-based portfolios built with the same series of returns. However, the proposed self-financed Dynamic PPI strategy can be easily applied using, for instance, an Exchange Trading Fund on the French Index, with some transaction costs; moreover, it is worth noticing that a fair buy-and-hold benchmark should also include the dividends.
After having estimated the DARQ-VaR model, we use it for defining daily conditional multiples and the related time-varying strategy. We then compare it with traditional CPPI strategies based on an unconditional multiple used in practice (between 3 and 13). Comparisons between the conditional multiple strategy and unconditional methods are presented in Tables 1 to 5. The first comparisons are based on observed prices: the CAC40 Index (see Table 1 and 2). For limiting the potential impact of the Index construction method, we complement the results of the former table by those of Table 3, that concern an equally weighted portfolio based on the fifty main components of the CAC40 Index since inception. Table 4 and 5 are related to comparisons based on realistic artificial series rebuilt from the CAC40 series, following first a simple stationary bootstrap (Politis and Romano, 1994) and secondly a surrogate data simulation procedure (Schreiber and Schmidzt, 2000).
All results, however, converge in the same way: the conditional Dynamic Strategy with Constant-risk exposure dominates most of the time the traditional Constant-asset exposure unconditional strategies in terms of return *per* unit of risk, combining a return close to the one of the best unconditional strategy, with a volatility amongst the lowest. While the risk of the conditional strategy is defined *ex ante* (with an almost Constant-risk exposure), it, however, appears - *ex post* - among the best portfolio strategies.

5 Concluding Remarks

The model and estimation methods proposed in this article provide a rigorous framework for fixing, at each date, a conditional multiple, preserving a constant exposition to risk defined by a shortfall constraint within an actual Risk Management approach. The dynamic setting starts with the conditioning of the time-varying multiple, through an extended DARQ-VaR for monitoring the true risk exposure of the structured product.
Hamidi *et al.* (2009) show that this strategy proves efficiency in the American stock market, whilst we complement here their results by both using CAC40 and a basket of French stocks, and artificial series built using bootstrap and surrogate techniques (thus limiting the dependency of the results to starting dates and asset price paths). This work will be improved in the near future, explicitly replacing the function of the conditional centile by a coherent measure of risk - namely the Expected Shortfall, expressed in a quantile regression conditional setting, for having a more robust and flexible estimation of the conditional multiple.

Table 1: Cushioned Portfolio Strategy Characteristics on the CAC40 Index from 1998 to 2008

	Return	**Volatility**	**VaR99%**	**Skewness**	***Kurtosis***	**Sharpe**	**Sortino**	**Omega**	**Kappa**	**Calmar**
Risky Asset	3.07%	23.03%	-4.14%	-.02 (.00%)	5.90 (.00%)	.00	.02	1.04	.02	.05
Cond. Multiple	3.03%	13.18%	-2.65%	-.36 (.00%)	10.03 (.00%)	.00	.03	1.06	.02	.10
Multiple 3	2.13%	6.80%	-1.36%	-.76 (.00%)	12.18 (.00%)	-.13	.03	1.07	.02	.11
Multiple 4	1.79%	9.00%	-1.98%	-.78 (.00%)	11.18 (.00%)	-.13	.02	1.05	.01	.07
Multiple 5	1.43%	11.70%	-2.58%	-.83 (.00%)	10.41 (.00%)	-.13	.01	1.04	.01	.04
Multiple 6	1.02%	13.64%	-3.11%	-.82 (.00%)	9.85 (.00%)	-.15	.01	1.03	.01	.03
Multiple 7	.67%	17.07%	-3.93%	-.84 (.00%)	10.59 (.00%)	-.14	.01	1.03	.01	.02
Multiple 8	.43%	19.13%	-4.45%	-.88 (.00%)	11.25 (.00%)	-.13	.01	1.03	.01	.01
Multiple 13	.13%	20.67%	-4.56%	-2.02 (.00%)	29.94 (.00%)	-.14	.01	1.03	.00	.00

Source: Bloomberg, daily data, CAC40 last prices from 07/09/1987 to 04/30/2008; computation by the authors. Returns and Volatilities are annualized. The VaR of each column is an historic daily VaR associated to a 99% confidence level. The skewness and *kurtosis* P-statistics (between parentheses) are related to Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991) and Kaplan and Knowles (2004). See Appendix 2, Aftalion and Poncet (2003) and related literature for other performance measures.

Table 2: Conditional Multiple Strategy Ranking vs Unconditional Strategies according to Performance Measures

	Sharpe	**Sortino**	**Omega**	**Kappa**	**Calmar**	**Information**	**Fama**	**Jensen**
Conditional Multiple Ranking	1	1	2	1	2	1	1	3

Source: Bloomberg, daily data, CAC40 last prices from 07/09/1987 to 04/30/2008; computation by the authors. Insured strategies presented in table 1 are ranked according to several performance measures (for definitions, see references in Table 1, Appendix 2 and Aftalion and Poncet, 2003).

Table 3: Cushioned Portfolio Strategy Characteristics on an Equally Weighted Components *pseudo*-CAC40 Portfolio from 1998 to 2008

	Return	**Volatility**	**VaR99%**	**Skewness**	***Kurtosis***	**Sharpe**	**Sortino**	**Omega**	**Kappa**	**Calmar**
Risky Asset	9.96%	20.17%	-3.75%	-.03 (.00%)	7.16 (.00%)	.33	.05	1.11	.03	.20
Cond. Multiple	4.59%	12.93%	-2.51%	-.68 (.00%)	9.1 (.00%)	.10	.03	1.08	.02	.13
Multiple 3	3.58%	8.29%	-1.86%	-.86 (.00%)	10.86 (.00%)	.04	.04	1.10	.02	.16
Multiple 4	3.36%	11.31%	-2.56%	-.91 (.00%)	11.84 (.00%)	.01	.03	1.07	.02	.11
Multiple 5	2.81%	14.77%	-3.34%	-1.01 (.00%)	12.00 (.00%)	-.03	.02	1.06	.01	.07
Multiple 6	2.85%	19.09%	-4.19%	-.97 (.00%)	12.15 (.00%)	-.02	.02	1.05	.01	.06
Multiple 7	2.65%	23.74%	-5.31%	-.94 (.00%)	11.89 (.00%)	-.03	.02	1.05	.01	.05
Multiple 8	2.24%	29.41%	-6.20%	-1.05 (.00%)	12.84 (.00%)	-.04	.02	1.05	.01	.03
Multiple 13	.31%	45.46%	-9.93%	-1.42 (.00%)	17.4 (.00%)	-.07	.02	1.06	.01	.00

Source: Bloomberg, daily data, CAC40 fifty main component last prices from 12/31/1987 to 01/16/2008; computation by the authors. The equally weighted portfolio is based on the fifty main CAC40 components since its inception and is rebalanced each day. Returns and volatilities are annualized. The VaR in each column is an historic daily VaR associated to a 99% confidence level. The skewness and *kurtosis* P-statistics (between parentheses) are related to Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991), Kaplan and Knowles (2004), and Young (1991). See Appendix 2, Aftalion and Poncet (2003) and related literature for other performance measures.

Table 4: Cushioned Portfolio Strategy Characteristics based on 500 Bootstrapped Simulated Series of the CAC40 Index Returns from 1987 to 2008

	Return	**Volatility**	**VaR99%**	**Skewness**	***Kurtosis***	**Sharpe**	**Sortino**	**Omega**	**Kappa**	**Calmar**
Risky Asset	6.07%	21.41%	-3.63%	-.15 (.00%)	7.21 (.00%)	.13	.03	1.07	.02	.07
Cond. Multiple	5.73%	14.08%	-2.65%	-.34 (.00%)	12.37 (.00%)	.18	.04	1.10	.02	.11
Multiple 3	5.26%	13.90%	-2.69%	-.38 (.00%)	27.71 (.00%)	.14	.04	1.11	.02	.06
Multiple 4	5.32%	19.09%	-3.75%	-.46 (.00%)	32.95 (.00%)	.11	.03	1.09	.02	.06
Multiple 5	5.08%	23.62%	-4.69%	-.51 (.00%)	38.11 (.00%)	.08	.03	1.09	.01	.05
Multiple 6	4.59%	27.19%	-5.45%	-.61 (.00%)	44.53 (.00%)	.05	.03	1.09	.01	.05
Multiple 7	4.23%	30.59%	-6.15%	-.68 (.00%)	49.67 (.00%)	.03	.03	1.09	.01	.04
Multiple 8	3.76%	33.09%	-6.62%	-.74 (.00%)	56.64 (.00%)	.01	.02	1.09	.01	.04
Multiple 13	1.87%	37.78%	-7.14%	-1.34 (.00%)	99.53 (.00%)	-.04	.02	1.09	.01	.02

Source: Bloomberg, daily data, CAC40 last prices from 07/09/1987 to 04/30/2008; computation by the authors. The strategies characteristics are calculated using 500 simulations of 5,242 daily returns based on stationary bootstrap (*Cf.* Politis and Romano, 1994): artificial series are composed with CAC40 random blocks of daily returns determined using a geometric probability law defined by a parameter equal to .9. Statistics presented here are the averages of the statistics computed for each strategy over every simulation. The VaR in each column is an historic daily VaR associated to a 99% confidence level. Returns and volatilities are annualized. The skewness and *kurtosis* P-statistics (between parentheses) are related to Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991), Kaplan and Knowles (2004), and Young (1991). See Appendix 2, Aftalion and Poncet (2003), and related literature for other performance measures.

Table 5: Cushioned Portfolio Strategy Characteristics based on 500 Surrogated Simulated Series of the CAC40 Index Returns from 1987 to 2008

	Return	**Volatility**	**VaR99%**	**Skewness**	***Kurtosis***	**Sharpe**	**Sortino**	**Omega**	**Kappa**	**Calmar**
Risky Asset	6.39%	21.38%	-3.62%	-.15 (.00%)	7.32 (.00%)	.15	.03	1.07	.02	.09
Cond. Multiple	4.92%	14.64%	-2.76%	-.21 (.00%)	14.28 (.00%)	.11	.04	1.09	.02	.09
Multiple 3	4.17%	14.26%	-2.75%	-.46 (.00%)	29.87 (.00%)	.06	.03	1.09	.02	.07
Multiple 4	3.65%	19.25%	-3.78%	-.53 (.00%)	38.12 (.00%)	.02	.02	1.07	.01	.05
Multiple 5	2.97%	23.37%	-4.61%	-.56 (.00%)	46.39 (.00%)	-.01	.02	1.07	.01	.04
Multiple 6	2.31%	26.42%	-5.24%	-.56 (.00%)	55.37 (.00%)	-.04	.02	1.06	.01	.03
Multiple 7	1.76%	28.54%	-5.62%	-.62 (.00%)	67.46 (.00%)	-.05	.02	1.06	.01	.02
Multiple 8	1.38%	29.82%	-5.78%	-.69 (.00%)	82.24 (.00%)	-.06	.02	1.06	.01	.01
Multiple 13	.47%	32.16%	-5.36%	-.52 (.00%)	185.63 (.00%)	-.09	.02	1.08	.01	.00

Source: Bloomberg, daily data, CAC40 last prices from 07/09/1987 to 04/30/2008; computation by the authors. The strategy characteristics are calculated using 500 simulations of 5,242 daily returns based on a surrogate data technique (*Cf.* Schreiber and Schmidzt, 2000): original series of daily returns are first randomly totally re-ordered and then second pair-wise permuted until the new series share some similarities with the original one (±10% of first correlation coefficients and of the long memory parameter of volatility). The statistics presented here are the averages of the statistics computed for each strategy over every simulation. The VaR in each column is an historic daily VaR associated to a 99% confidence level. The skewness and *kurtosis* P-statistics (between parentheses) are related to Pearson parametric tests. Performance measures are computed according to Sortino and van der Meer (1991), Kaplan and Knowles (2004), and Young (1991). See Appendix 2, Aftalion and Poncet (2003) and related literature for other performance measures.

6 References

Aftalion F. and P. Poncet, (2003), *Les techniques de mesure de performances*, Economica, 139 pages.

Ben Ameur H. and J.-L. Prigent, (2007), "Portfolio Insurance: Determination of a Dynamic CPPI Multiple as Function of State Variables", *THEMA Working Paper*, University of Cergy, 22 pages.

Bertrand Ph. and J.-L. Prigent, (2002), "Portfolio Insurance: the Extreme Value Approach to the CPPI Method", *Finance* 23(2), 69-86.

Black F. and R. Jones, (1987), "Simplifying Portfolio Insurance", *Journal of Portfolio Management* 14(1), 48-51.

Black F. and A. Perold, (1992), "Theory of Constant Proportion Portfolio Insurance", *Journal of Economic Dynamics and Control* 16(3), 403-426.

Bontemps Ch. and N. Meddahi, (2005), "Testing Normality: A GMM Approach", *Journal of Econometrics* 124(1), 149-186.

Chen J., C. Chang, J. Hou and Y. Lin, (2008), "Dynamic Proportion Portfolio Insurance using Genetic Programming with Principal Component Analysis", *Expert Systems with Applications: An International Journal* 35(1), 273-278.

Engle R. and S. Manganelli, (2004), "CAViaR: Conditional AutoRegressive Value-at-Risk by Regression Quantiles", *Journal of Business and Economic Statistics* 22(4), 367-381.

Gouriéroux Ch. and J. Jasiak, (2008), "Dynamic Quantile Models", *Journal of Econometrics* 147(1), 198-205.

Hamidi B., E. Jurczenko and B. Maillet, (2009), "A CAViaR Modelling for a Simple Time-Varying Proportion Portfolio Insurance Strategy", *Bankers, Markets & Investors*, forthcoming 2009, 21 pages.

Kaplan P. and J. Knowles, (2004), "Kappa: A Generalized Downside Risk-adjusted Performance Measure", *Journal of Performance Measurement* 8(3), 42-54.

Longin F. and B. Solnik, (1995), "Is the Correlation in International Equity Returns Constant: 1960-1990?", *Journal of International Money and Finance* 14(1), 3-26.

Politis D. and J. Romano, (1994), "The Stationary Bootstrap", *Journal of the American Statistical Association* 89(428), 1303-1313.

Schreiber T. and A. Schmidtz, (2000), "Surrogate Time-series", *Physica D* 142, 346-382.

Sortino F. and R. van der Meer, (1991), "Downside Risk", *Journal of Portfolio Management* 17(4), 27-31.

Young T., (1991), "Calmar Ratio: A Smoother Tool", *Futures* 20(1), 40.

7 Appendix

Appendix 1: Proportion Portfolio Insurance based on a Quantile Criterion in a Marked Point Process Framework

As we argue in the text, despite the fact that the multiple is conditional and thus time-varying, the portfolio is still guaranteed under some conditions. Indeed, a guaranteed portfolio is defined so that the portfolio value will always be above a predefined floor at a given high probability level. Assume that the risky price follows a marked point process, which is characterized by the sequence of marks $(S_l)_{l\in\mathbb{N}_+^*}$ and the increasing sequence of times $(T_l)_{l\in\mathbb{N}_+^*}$ at which the risky asset varies.
In the CPPI framework, the first following "global" quantile hedging condition can be considered (see Bertrand and Prigent, 2002):

$$\text{Prob}\left[\forall t \leq T, C_t \geq 0\right] \geq 1-\delta \tag{3}$$

where C_t is the cushion defined as the spread between the portfolio value and the guaranteed floor, Prob[.] stands for the unconditional probability and $(1-\delta)$ for a probability confidence level. Splitting the complete period, denoted $[0,...,T]$, into various L successive subperiods $[T_l, T_{l+1}[$, the previous equation is equivalent to define the multiple m as such (see Bertrand and Prigent, 2002):

$$m \leq \left[f_T^{-1}\left(1-\delta\right)\right]^{-1} \tag{4}$$

where $f_T^{-1}\left(.\right)$ is the quantile function, evaluated at a risky asset return for which the inverse function - denoted $f_T\left(.\right)$, is equal to $(1-\delta)$ - a specified unconditional quantile, as such:

$$f_T\left(r\right) = \sum_{l=1}^{+\infty}\left\{\text{Prob}\left[M_l \leq r \mid T_l \leq T < T_{l+1}\right] \times \text{Prob}\left[T_l \leq T < T_{l+1}\right]\right\} \tag{5}$$

with Prob$[. \mid T_l \leq T < T_{l+1}]$ denoting the conditional probability given the event $T_l \leq T < T_{l+1}$ and:

$$M_l = \underset{k=[1,...,L]}{Max}\left\{-r_1, ..., -r_k\right\} \tag{6}$$

where $r_t = \left(S_t - S_{t-1}\right)/S_{t-1}$ is the risky asset return at time t.

Following the same principle in a time-varying framework now, another "local" quantile condition can also be introduced, based this time on a conditional quantile corresponding to a conditional probability confidence level denoted $(1-\alpha)$, such as, for any time $t \in [T_l, T_{l+1}[$ with $t \leq T$:

$$\text{Prob}\left[C_{T_l} > 0 \mid \Omega_{T_{l-1}}\right] \geq 1-\alpha \tag{7}$$

where $\Omega_{T_{l-1}}$ is the σ-*algebra* generated by the set of all intersections of $\left\{C_{T_{l-1}} > 0\right\}$ with any subset $\Omega_{T_{l-1}}$ of the σ-*algebra* generated by the observation of the marked point process until time T_{l-1}.

From previous condition (7), an upper bound on the multiple can be deduced according to specific assumptions (see Ben Ameur and Prigent, 2007, for the special case of GARCH-type models with a deterministic transaction-time).

Appendix 2: About some Performance Measures

Sharpe Ratio

The Sharpe ratio is one of the most popular performance measures. It is defined as the ratio between the excess return about the risk free rate over the volatility of the analysed portfolio. However, use of the Sharpe ratio in performance measurement is subject to some criticisms since returns do not display a normal distribution. For example, the use of dynamic strategies results in an asymmetric return distribution, as well as fat tails, leading to the danger that the use of standard risk and performance measures will underestimate risk and overestimate performance *per* unit of risk.

Sortino, Omega and Kappa Measures

Lower partial moments measure risk by negative deviations of the realized returns, to a minimum acceptable return. The lower partial moment of order n is calculated using power n. Because lower partial moments consider only negative deviations to a minimal acceptable return (which could be zero), they are a more appropriate measure of risk than the standard deviation, which considers negative and positive deviations from expected return (see Sortino and van der Meer, 1991). The choice of the order n determines the extent to which the deviations are weighted. The lower partial moment of order 0 can be interpreted as the shortfall probability, the lower partial moment of order 1 as the expected shortfall, and the lower partial moment of order 2 as the semi-variance. The order of the lower partial moment to be chosen is linked to the downside-risk aversion of the investor. The more he is averse, the higher the order (since it gives extra weights to extreme pay-offs). The Omega (see Shadwick and Keating, 2002), the Sortino ratio (see Sortino and van der Meer, 1991), and Kappa 3 (see Kaplan and Knowles, 2004) make use respectively of the lower partial moments of order 1, 2 and 3.

Calmar Ratio

As the Sharpe ratio, the Calmar ratio is defined as the ratio between the excess return about the risk free rate over a risk measure of the analysed portfolio. The Calmar ratio (see Young, 1991), uses the *maximum* drawdown over a three-year period as the risk measure at the denumerator instead of the standard deviation of returns. The drawdown being the loss incurred over a certain investment period (peak-to-valley price difference), drawdown-based performance measures are particularly popular in practice, since they are better connected to the overall loss that can face an investor (without any reference to a specific observation frequency).

Jensen Measure

The Jensen measure considers the average return above what is explained by the capital asset pricing model. The beta factor is generally calculated using the correlation between the returns of a market index and the returns of the investment fund. The Jensen measure is, however, often criticized because it can be manipulated by leveraging the fund return, and because it is based on the assumption that *alpha* and *beta* can be clearly split.