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# Wage bargaining with non-stationary preferences under strike decision

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**Abstract.** In this paper, we present a non-cooperative wage bargaining model in which preferences of both parties, a union and a firm, are expressed by the sequences of discount rates varying in time. For such a wage bargaining with non-stationary preferences, we determine subgame perfect equilibria between the union and the firm for the case when the union is supposed to go on strike in each period in which there is a disagreement. A certain generalization of the original Rubinstein bargaining model is applied to determine these equilibria.

**JEL Classification:** J52, C78

**Keywords:** union - firm bargaining, alternating offers, varying discount rates, subgame perfection

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## 1 Introduction

As mentioned by many labor economists, collective wage bargaining is one of the most important problems in most of the markets. Collective bargaining between firms and unions (or workers' representatives, workers' groups, etc) can be either cooperative or non-cooperative. Labor economists have studied many versions of wage bargaining between firms and unions. They have determined different ways to solve wage bargaining problems and have drawn different conclusions from such a bargain. Nevertheless, a common assumption in the literature on wage bargaining is the stationarity of sides' preferences.

There are essentially two approaches to bargaining, i.e., a static (axiomatic) approach and a dynamic (strategic) approach. The first traditional model of collective bargaining is based on Nash static approach (Nash 1950, [16]), where the equilibrium of the bargaining is found by the maximization of the utility levels of both sides. The focus is on bargaining over a jointly owned surplus. The payoffs are found by the agreement point and they do not depend on the history of the game. The same axiomatic approach to bargaining is applied in Kalai and Smorodinsky (1975, [12]), where the authors define another solution to the bargaining problem.

By using Nash's approach, many of the labor economists have determined the levels of wage and employment between unions and firms; see e.g. McDonald and Solow (1981, [14]), Nickell and Andrews (1983, [17]). Also the equilibria that are obtained have been

used to measure the bargaining power of the both sides (Doiron 1992, [6]). Another approach to wage bargaining - the approach based on cooperative games - is presented in Levy and Shapley (1997, [13]), where wage negotiation is modeled as an oceanic game and the Shapley value (Shapley 1953, [29]) is used for the solution concept.

One of the disadvantages of the static approach to bargaining concerns a difficulty to model the non-cooperative wage bargaining. Since the static approach is based only on the parties preferences over the set of possible agreements, other effects cannot be explained by this kind of modeling. Although using a von Neumann-Morgenstern utility function (von Neumann and Morgenstern 1944, [31]) can lead to a solution of agents' preferences toward risk, the time preferences are not included. These difficulties may be overcome by using a dynamic approach to bargaining initiated by Rubinstein (1982, [21]); see also Fishburn and Rubinstein (1982, [8]), Osborne and Rubinstein (1990, 1994, [18, 19]), and Muthoo (1999, [15]). The works of Rubinstein and, e.g., Binmore et al. (1986, [1]) have made a new way to understand the bargaining process. They combine the strategic approach to bargaining with a repeated game mechanism and create the dynamic bargaining approach. The history of the game, where players make the alternating offers, influences the sides' payoffs. Players share a unique divisible good and they make offers in each period. The aim is to choose an accord from a set of possible agreements. The chosen accord depends on the parties preferences over the set of possibilities, their compartments toward risk and time, bargaining environment and the bargaining procedure. The players will reach either an immediate agreement or a late agreement (which involves some discount rates), or they will never reach an agreement. Usually the analysis is based on determining subgame perfect equilibria of the game (Selten 1975, [28]), which is a natural refinement of Nash equilibrium for a game with complete information.

Several authors have applied the dynamic approach to bargaining. Conlin and Furusawa (2000, [3]), for instance, consider a three-stage firm-union bargaining game and they investigate subgame perfect equilibria of the game. Cripps (1997, [5]) who analyzes the model of investment, considers e.g. the alternating-offer bargaining game over binding long-term wage contracts and describes a stationary subgame perfect equilibrium of the game. Also Haller and Holden (1990, [9]), and Fernandez and Glazer (1991, [7]) use Rubinstein's model to determine the wage level in bargaining between two important players of labor economics - union and firm; see also Bolt (1995, [2]). They investigate the model based on a monopolistic situation, where there exists only one firm who needs labor and a unique union who supplies the labor for the firm. It is assumed that if a wage contract offer proposed by one party is rejected by another one, then the union can either go for strike or not to go for strike. The authors consider bargaining games with different strike decisions of the union, and they determine subgame perfect equilibria of the games.

The wage bargaining models presented in Haller and Holden (1990, [9]), and in Fernandez and Glazer (1991, [7]) are not sufficient to be naturally applied to real life situations, because of the stationarity assumption. In real bargaining, due to the sides' time preferences, discount rates of the players may vary in time. Hence, modeling the wage bargaining by constant discount rates may lead to some serious simplifications and errors. To the best of our knowledge, not many works study the consequences of different discounting. By using collective bargaining contract and industry wage survey data, Kahn (1993, [11]) tests the effect of discounting on cooperative bargaining behavior by unions and firms. Using some experimental results, Rubinstein (2003, [22]) questions the use of

‘hyperbolic discounting utility function’ instead of the standard ‘constant discount utility function’. Cramton and Tracy (1994, [4]) emphasize that stationary bargaining models are very rare in real situations. They study wage bargaining with time-varying threats in which the union is uncertain about the firm’s willingness to pay. Rusinowska (2000, 2001, 2002, 2003, 2004, [23–27]) generalizes the original model of Rubinstein to a bargaining model with non-stationary preferences, e.g., to the models with preferences varying in time, like varying discount rates or bargaining costs. In her analysis, the author uses the same strategic approach (subgame perfection) as the one used by Rubinstein.

Strikes in bargaining between unions and firms have been studied in numerous works, both from a theoretical and an empirical point of view. In Hayes (1984, [10]) it is shown that although a strike seems to be a Pareto-inefficient outcome of bargaining, it can be the outcome of rational behavior of both agents. In a situation with asymmetric information, for instance, strikes can be used to gain more information. Sopher (1990, [30]) reports on an experiment on the frequency of disagreement (strikes) in a set of “shrinking pie” games in which parties bargain in consecutive periods over how to divide a quantity of money. Although bargaining theory predicts that no disagreement is involved in the outcome of a two-person pie-splitting game with complete information, in the experiment strikes occurred frequently in the games and they did not disappear over time. This can be supported by the joint-cost theory of strikes which attributes strikes to the costs of negotiation. Robinson (1999, [20]) uses the theory of repeated games to present a dynamic model of strikes as part of a constrained efficient enforcement mechanism of a labor contract. In particular, he shows that under imperfect observations strikes occur in equilibrium.

The *aim of this paper* is to contribute to the wage bargaining literature by emphasizing the importance of the non-stationarity of preferences in union-firm bargaining. In particular, we study the effect of the non-stationarity of parties’ discount rates on solutions (subgame perfect equilibria) of a wage bargaining model related to Rubinstein’s approach (Rubinstein 1982, [21]). To be more precise, we generalize the model of Fernandez and Glazer (1991, [7]) to the wage bargaining between firm and union in which both sides have preferences expressed by discount rates varying in time. Especially, we determine subgame perfect equilibria for a bargaining game in which the union decides to go on strike in each period until the agreement is reached. We apply the generalization of the original Rubinstein bargaining model investigated in Rusinowska (2000, 2001, [23, 24]) to determine these equilibria.

The remaining sections of the paper are organized as follows. In Section 2, we present the wage bargaining model of Fernandez and Glazer (1991, [7]). In Section 3, we investigate the generalized union-firm bargaining game with non-stationary preferences and describe subgame perfect equilibria of the game. Some numerical examples of this wage bargaining between the firm and the union with preferences expressed by varying discount rates are presented in Section 4. Our conclusions, including the future research agenda, are presented in Section 5. The proofs of the theorems formulated for the wage bargaining are sketched in the Appendix (Section 6).

## 2 Non-cooperative wage bargaining with stationary time preferences

In the paper, we deal with a non-cooperative bargaining game, where each player (both union and firm) has complete information. As a simplification, we suppose a monopolistic market share with one union on the labor supply and a monopole of a firm which is the only agent who hires the workers on the market. In the basic non-cooperative bargaining game of Rubinstein (1982, [21]), which produces a unique and Pareto efficient equilibrium, there exists a certain cost of bargaining. This cost can be either a fixed bargaining cost or a fixed discounting factor.

An applied version of this model on wage determination between union and firm has been studied by Fernandez and Glazer (1991, [7]), where the union has a strike possibility as a response to the firm's offer. In this section, we describe an efficient equilibrium on wages between the union and the firm by using a game theoretical approach of Rubinstein. It is supposed that a union and a firm bargain on workers wages, where there is perfect information between these two agents. There can be an infinite number of periods of bargaining and each party makes offers alternately. While in Rubinstein's model the players bargain on the division of one divisible good (of value 1), here it is supposed that in each period of a normal production the firm has an added value of one unit of a good, which the union and the firm can divide.

The bargaining procedure between the union and the firm, as presented in Fernandez and Glazer (1991, [7]) and Haller and Holden (1990, [9]) is the following. There is an existing wage contract between the union and the firm, but it has reached its end. So, the union tries to determine a new wage contract in favor of workers, but the firm can either accept or reject it. The share of the union under the previous contract is  $W_0$ , where  $W_0 \in (0, 1]$ . By the new contract, the union and the firm will divide the added value (normalized to 1) with new shares of the parties, where the union's share is  $W \in [0, 1]$  and the firm's share is  $1 - W$ . Figure 1 presents the first three periods of this wage bargaining.

ABOUT HERE FIGURE 1

The union moves first and makes an offer  $x_0$ , where  $x_0$  is the share of the added value proposed by the union for itself in the new contract. The firm can either accept the offer or refuse it. If it accepts the new wage contract, then the agreement is reached and the payoffs are  $(x_0, 1 - x_0)$ . If the firm rejects the new share, then the union can either go on strike and then both parties will get nothing in the current period, or go on with previous contract with payoffs  $(W_0, 1 - W_0)$ . If the union decides to go on strike, it is the firm's turn to make a new offer. This procedure goes on until the agreement is reached, i.e., in each even-numbered period  $t \in \mathbb{N}$  the union makes a wage contract offer  $x_t$ , where  $x_t \geq 0$  and the firm responds either by accepting this offer or by rejecting it. If the firm accepts the proposal, the bargaining ends, and if the firm rejects the new wage contract, then the decision turn passes to the union. If the union decides not to strike, the workers will be paid upon the previous contract. If the union decides to go for strike, then both parties will get nothing and the turn to make an offer in the next period passes to the firm. In each odd-numbered period  $t$ , the firm makes a wage contract offer  $y_t \geq 0$ . The

union responds either by accepting this offer or by rejecting it. If the union accepts it, the wages are set by the new contract. If the union rejects the offer, it will decide either to go on strike or not. The same rules as described previously govern the strike decision. The payoff function of the union is equal to:

$$U = \sum_{t=0}^{\infty} \delta_u^t \cdot u_t \quad (1)$$

where  $0 < \delta_u < 1$  is the discount rate of the union, and

$$\begin{aligned} u_t &= 0 && \text{if there is a strike in period } t \in \mathbb{N} \\ u_t &= W_0 && \text{if there is no strike in period } t \text{ and agreement has yet to be reached} \\ u_t &= W && \text{if agreement } W \text{ is reached in period } t. \end{aligned}$$

The payoff function of the firm is equal to:

$$V = \sum_{t=0}^{\infty} \delta_f^t \cdot v_t \quad (2)$$

where  $0 < \delta_f < 1$  is the discount rate of the firm, and

$$\begin{aligned} v_t &= 0 && \text{if there is a strike in period } t \\ v_t &= 1 - W_0 && \text{if there is no strike in period } t \text{ and agreement has yet to be reached} \\ v_t &= 1 - W && \text{if agreement } W \text{ is reached in period } t. \end{aligned}$$

Fernandez and Glazer (1991, [7]) consider three different situations of the equilibrium:

- (i) The first one is the Minimum Wage Contract which means that there is a subgame perfect equilibrium in which an agreement of  $W_0$  is reached in the first period. It is supposed that the union's strategy is never to strike, it offers  $x_t = W_0$  in every even period  $t \in \mathbb{N}$ , and in every odd period  $t$  it accepts an offer  $y_t$  of the firm if  $y_t \geq W_0$  and refuses it otherwise. The firm's strategy is to offer  $y_t = W_0$  in every odd period  $t$ , and in every even period  $t$  to accept an offer  $x_t$  if  $x_t \leq W_0$  and to reject it otherwise.
- (ii) The second equilibrium depends on the union's striking decision. Fernandez and Glazer show that if the union is committed to strike in every period in which the parties did not reach an agreement, then there is a unique subgame perfect equilibrium of the bargaining game between the union and the firm. This equilibrium leads to an agreement that is reached in the first period of the negotiations and results in a wage contract  $\bar{W}$  if bargaining starts in an even period (by the union), and it has a contract  $\bar{Z}$  if the bargaining starts in an odd period (by the firm), where

$$\bar{W} = \frac{1 - \delta_f}{1 - \delta_u \cdot \delta_f} \quad (3)$$

$$\bar{Z} = \frac{\delta_u \cdot (1 - \delta_f)}{1 - \delta_u \cdot \delta_f} \quad (4)$$

This result is the same as the one in the original Rubinstein bargaining model.

- (iii) The third situation is the Maximum Wage Contract. Fernandez and Glazer show that if  $W_0 \leq \delta_u \cdot Z'$ , then there is a subgame perfect equilibrium in which an agreement  $W'$  is obtained in the first period, where

$$W' = \bar{W} + \frac{\delta_f \cdot W_0 \cdot (1 - \delta_u)}{1 - \delta_u \cdot \delta_f} \quad (5)$$

and

$$Z' = \bar{Z} + \frac{W_0 \cdot (1 - \delta_u)}{1 - \delta_u \cdot \delta_f} \quad (6)$$

This is the maximum wage contract that the union can receive in any subgame perfect equilibrium. The strategies of the parties are the following. In even-numbered periods, the union offers the contract  $W'$  and strikes if this offer is rejected, and in odd-numbered periods the union accepts only offers that are greater than or equal to  $Z'$ , but never strikes.

As it can be seen from this short presentation of the non-cooperative wage bargaining equilibria, the offers in the subgame perfect equilibrium depend on the discount factors  $\delta_u$ ,  $\delta_f$ , and in some cases additionally on the previous wage contract  $W_0$ .

In the next section, we will use the same bargaining procedure with the union's strike decision as in case (ii), i.e., where the union strikes in every disagreement period, but we will assume that the discount factors do not have to be constant anymore. With the constant discount rates, when the union's decision is to strike in every disagreement period, the result of Rubinstein's original bargaining model is obtained, i.e., the offers by the union and the firm are  $\bar{W}$  and  $\bar{Z}$ , respectively. When the discount factors vary in time (non-stationary time preferences), the equilibria will change. For this reason, we will use the model introduced in Rusinowska (2000, 2001, [23, 24]) to show the effects of the non-stationarity on the equilibria.

### 3 Non-cooperative wage bargaining with non-stationary time preferences

#### 3.1 The model

In this section, instead of assuming stationary preferences expressed by constant discount rates, we present the wage bargaining between union and firm with non-stationary time preferences of both parties. Such a generalization of Rubinstein's bargaining to the model with discount rates varying in time has been introduced in Rusinowska (2000, 2001, [23, 24]). Following this generalization of Rubinstein's model, we generalize the model of Fernandez and Glazer (1991, [7]) and introduce the wage bargaining game with discount rates varying in time. We focus on the case when the union decides to go on strike in every period before the agreement is reached. The equilibrium result obtained by Fernandez and Glazer does not hold if the sides' preferences are expressed by non-stationary discount rates. Consequently, solutions to such a generalized wage bargaining have to be found.

We consider the wage bargaining with the bargaining procedure similar to the one introduced in Fernandez and Glazer (1991, [7]); see description of the procedure given



in Section 2. The key difference between their model and our wage bargaining lies in preferences of the union and the firm and, as a consequence, in the payoff functions of both sides. While Fernandez and Glazer assumed stationary preferences described by constant discount rates  $\delta_u$  and  $\delta_f$ , we consider a wage bargaining in which preferences of the union and the firm are described by the *sequences of discount rates varying in time*,  $(\delta_{u,t})_{t \in \mathbb{N}}$  and  $(\delta_{f,t})_{t \in \mathbb{N}}$ , respectively, where

$\delta_{u,t}$  = the discount rate of the union in period  $t \in \mathbb{N}$

$\delta_{f,t}$  = the discount rate of the firm in period  $t \in \mathbb{N}$

$\delta_{u,0} = \delta_{f,0} = 1$ ,  $0 < \delta_{i,t} < 1$  for  $t \geq 1$  and  $i = u, f$ .

The payoff function of the union in the non-stationary wage bargaining is equal to:

$$\tilde{U} = \sum_{t=0}^{\infty} \left( \prod_{k=0}^t \delta_{u,k} \right) \cdot u_t \quad (7)$$

where

$u_t = 0$  if there is a strike in period  $t \in \mathbb{N}$   
 $u_t = W_0$  if there is no strike in period  $t$  and agreement has yet to be reached  
 $u_t = W$  if agreement  $W$  is reached in period  $t$ .

The payoff function of the firm in the non-stationary wage bargaining is equal to:

$$\tilde{V} = \sum_{t=0}^{\infty} \left( \prod_{k=0}^t \delta_{f,k} \right) \cdot v_t \quad (8)$$

where

$v_t = 0$  if there is a strike in period  $t$   
 $v_t = 1 - W_0$  if there is no strike in period  $t$  and agreement has yet to be reached  
 $v_t = 1 - W$  if agreement  $W$  is reached in period  $t$ .

Consequently, if the agreement  $W \in [0, 1]$  is reached in period  $t \in \mathbb{N}$  and in all previous periods there was a strike, then the payoffs of the union and the firm denoted by  $\tilde{U}(W, t)$  and  $\tilde{V}(W, t)$ , respectively, are equal to:

$$\tilde{U}(W, t) = W \cdot \prod_{k=0}^t \delta_{u,k} \quad (9)$$

and

$$\tilde{V}(W, t) = (1 - W) \cdot \prod_{k=0}^t \delta_{f,k} \quad (10)$$



One of the possible results of the wage bargaining is the (permanent) *disagreement* denoted by  $(0, \infty)$ , i.e., a situation in which the union and the firm never reach the agreement. The utility of the disagreement is equal to

$$\tilde{U}(0, \infty) = \tilde{V}(0, \infty) = 0 \quad (11)$$

In the next section, we will present subgame perfect equilibria for the model with non-stationary time preferences of the sides under strike decision, which generalize the equilibrium obtained in Fernandez and Glazer (1991, [7]).

### 3.2 Subgame perfect equilibria

In this section, we will apply results obtained in Rusinowska (2001, [24]) to the wage bargaining between union and firm with non-stationary preferences. Proofs of all theorems presented in this section are sketched in the Appendix. First of all, we introduce the following definition of the sides' strategies.

**Definition 1** *Strategy A of the union and strategy B of the firm are defined as follows:*

- A - *The union in each period  $2t$  ( $t \in \mathbb{N}$ ) submits an offer  $\overline{W}_u^{2t}$  and in each period  $2t + 1$  accepts an offer  $y$  by the firm if and only if  $y_u \geq \overline{Z}_u^{2t+1}$ , and it goes on strike in every period in which there is a disagreement*
- B - *The firm in each period  $2t + 1$  submits an offer  $\overline{Z}_u^{2t+1}$  and in each period  $2t$  accepts an offer  $x$  by the union if and only if  $x_f \geq \overline{W}_f^{2t}$ .*

We adopt the convention that

$$\overline{W}^{2t} = \overline{W}_u^{2t} = 1 - \overline{W}_f^{2t}, \quad \overline{Z}^{2t+1} = \overline{Z}_u^{2t+1} = 1 - \overline{Z}_f^{2t+1}$$

$$x = x_u = 1 - x_f, \quad y = y_u = 1 - y_f$$

In particular, the strategies for the first and second periods (that is, when  $t = 0$ ) are as follows:

- The strategy of the union is to offer  $(1 - \overline{W}_u^0)$  to the firm and  $\overline{W}_u^0$  to itself in period 0, and to accept an offer  $y$  in period 1 if and only if the share offered by the firm to the union is greater than or equal to the share  $\overline{Z}_u^1$  that the firm proposes to the union in period 1. As we assume the union to start the game, there will be no proposition of the firm to the union in period 0.
- The strategy of the firm in period 1 is to make an offer which assigns the share  $\overline{Z}_u^1$  to the union and  $(1 - \overline{Z}_u^1)$  to the firm, and in period 0 to accept an offer  $x$  of the union if and only if the share offered by the union is greater than or equal to the share  $(1 - \overline{W}_u^0)$  proposed by the union to the firm in period 0.
- If the union and the firm use the strategies A and B, respectively, then they reach an agreement in period 0 which assigns  $\overline{W}_u^0$  to the union and  $1 - \overline{W}_u^0$  to the firm.

We can prove the following result:

**Theorem 1** *If in the wage bargaining game the preferences of the union and the firm are described by the sequences of discount rates  $(\delta_{i,t})_{t \in \mathbb{N}}$ , where  $\delta_{i,0} = 1$ ,  $0 < \delta_{i,t} < 1$  for  $t \geq 1$ ,  $i = u, f$ , and the union goes on strike in every period in which there is a disagreement, then the pair of strategies  $(A, B)$  described above is a subgame perfect equilibrium of this game if and only if the offers of the parties satisfy the following conditions:*

$$\bar{W}_f^{2t} = \bar{Z}_f^{2t+1} \cdot \delta_{f,2t+1} \quad (12)$$

and

$$\bar{Z}_u^{2t+1} = \bar{W}_u^{2t+2} \cdot \delta_{u,2t+2} \quad (13)$$

for each  $t \in \mathbb{N}$ .

The main consequence of moving from the constant discount rates to the sequences of discount rates varying in time is the determination of the subgame perfect equilibrium offers by the infinite system of equations instead of just two equations. The subgame perfect equilibrium described in Theorem 1 depends on the solutions of this infinite system of equations (12) and (13) for each  $t \in \mathbb{N}$ , which cannot be solved step by step, since the offers of the union and the firm are given in a recursive way (for instance, for  $t = 0$  we have  $\bar{W}_f^0 = \bar{Z}_f^1 \cdot \delta_{f,1}$  and  $\bar{Z}_u^1 = \bar{W}_u^2 \cdot \delta_{u,2}$ ). By using some techniques of mathematical analysis to solve such an infinite system of equations that determine the offers under every subgame perfect equilibrium, two theorems have been proved. Theorem 2 mentions all the possibilities for  $\bar{W}_u^0$  in a general case.

**Theorem 2** *If in the bargaining game the preferences of the union and the firm are described by the sequences of discount rates  $(\delta_{i,t})_{t \in \mathbb{N}}$ , where  $\delta_{i,0} = 1$ ,  $0 < \delta_{i,t} < 1$  for  $t \geq 1$ ,  $i = u, f$ , and the union goes on strike in every period in which there is a disagreement, then each pair of the strategies  $(A, B)$  such that:*

$$\bar{W}_u^0 \geq 1 - \delta_{f,1} + \sum_{n=1}^{+\infty} \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1}) \quad (14)$$

and

$$\bar{W}_u^0 \leq 1 - \delta_{f,1} + \sum_{n=1}^{+\infty} \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1}) + \prod_{j=1}^{+\infty} \delta_{u,2j} \cdot \delta_{f,2j-1} \quad (15)$$

$$\bar{W}_u^{2t+2} = \frac{\bar{W}_u^{2t} + \delta_{f,2t+1} - 1}{\delta_{u,2t+2} \cdot \delta_{f,2t+1}} \quad \text{and} \quad \bar{Z}_u^{2t+1} = \bar{W}_u^{2t+2} \cdot \delta_{u,2t+2} \quad \text{for each } t \in \mathbb{N} \quad (16)$$

is the subgame perfect equilibrium of this game.

In Theorem 2 we have formulated the subgame perfect equilibria for the general case. As a consequence of replacing the constant discount rates by the discount rates varying in time, there may exist infinitely many subgame perfect equilibria. As recapitulated in Section 2, in the stationary model studied by Fernandez and Glazer (1991, [7]), under the union's strike decision in each disagreement period, there exists the unique subgame perfect equilibrium and the equilibrium offers of the union and the firm depend on the constant discount rates  $\delta_u, \delta_f$ . In the non-stationary model with the discount rates varying

in time, the equilibria depend on the union's discount rates in even periods and the firm's discount rates in odd periods. In particular, the offer made by the union in period 0 depends on the sum of a certain convergent series and on the product of the discount rates of the firm and the union in consecutive periods. Note that it is consistent with the fact that the union makes the offer to the firm in every even period ( $2t$ ) and the firm makes its offer to the union in every odd period ( $2t + 1$ ). We can also remark that the parties' offers are determined in a recursive way. While in the stationary wage bargaining the offers of the party (the union or the firm) are the same in each period in which this party is supposed to make a proposal, in the non-stationary wage bargaining the offers of the party are obviously different in all periods of the party's turn to make a proposal.

A natural question is under which conditions there exists only one subgame perfect equilibrium in the non-stationary model. If we assume that  $\prod_{j=1}^{t+1} \delta_{u,2j} \cdot \delta_{f,2j-1} \rightarrow_{t \rightarrow +\infty} 0$ , then the equilibria given in Theorem 2 collapse to the unique subgame perfect equilibrium, as presented in Theorem 3.

**Theorem 3** *If in the wage bargaining game the preferences of the union and the firm are described by the sequences of discount rates  $(\delta_{i,t})_{t \in \mathbb{N}}$ , where  $\delta_{i,0} = 1$ ,  $0 < \delta_{i,t} < 1$  for  $t \geq 1$ ,  $i = u, f$ , and*

$$\prod_{j=1}^{t+1} \delta_{u,2j} \cdot \delta_{f,2j-1} \rightarrow_{t \rightarrow +\infty} 0 \quad (17)$$

*and the union goes on strike in every period in which there is a disagreement, then there is the only one subgame perfect equilibrium of the form  $(A, B)$ , where the offers of the players are as follows:*

$$\bar{W}_u^0 = 1 - \delta_{f,1} + \sum_{n=1}^{+\infty} \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1}) \quad (18)$$

$$\bar{W}_u^{2t+2} = \frac{\bar{W}_u^{2t} + \delta_{f,2t+1} - 1}{\delta_{u,2t+2} \cdot \delta_{f,2t+1}} \quad \text{and} \quad \bar{Z}_u^{2t+1} = \bar{W}_u^{2t+2} \cdot \delta_{u,2t+2} \quad \text{for each } t \in \mathbb{N}.$$

The first step when calculating this equilibrium is therefore to check if the assumption that the product of the discount rates of the firm and the union in consecutive periods goes to zero (condition (17)). When we apply our results to the wage bargaining studied by Fernandez and Glazer (case (ii) presented in Section 2), we get obviously their result. In order to see this, assume that the union goes on strike in every period in which there is a disagreement, and that  $\delta_{u,t} = \delta_u$ ,  $\delta_{f,t} = \delta_f$  for  $t \geq 1$ ,  $\delta_u, \delta_f \in (0, 1)$ . Then the payoff functions  $\tilde{U}$  and  $\tilde{V}$  defined in (7) and (8) become the payoff functions (1) and (2), respectively. Moreover,

$$\prod_{j=1}^{t+1} \delta_{u,2j} \cdot \delta_{f,2j-1} = (\delta_u \cdot \delta_f)^{t+1} \rightarrow_{t \rightarrow +\infty} 0$$

and hence we can apply Theorem 3 to this model. When calculating the share  $\overline{W}_u^0$  that the union proposes for itself at the beginning of the game, we get the sum of the convergent geometric series (since  $0 < \delta_u \cdot \delta_f < 1$ ), that is,

$$\overline{W}_u^0 = 1 - \delta_f + \sum_{n=1}^{+\infty} (\delta_u \cdot \delta_f)^n \cdot (1 - \delta_f) = 1 - \delta_f + \frac{\delta_u \cdot \delta_f \cdot (1 - \delta_f)}{1 - \delta_u \cdot \delta_f} = \frac{1 - \delta_f}{1 - \delta_u \cdot \delta_f} = \overline{W}$$

which is the result obtained by Fernandez and Glazer (1991, [7]).

## 4 Examples

In this section we apply Theorems 2 and 3 presented in the previous section to two examples of wage bargaining models with non-stationary discount rates. In one example there are infinitely many subgame perfect equilibria, in another example there exists the only one subgame perfect equilibrium.

### 4.1 Example 1 - Application of Theorem 2

Let us consider the wage bargaining model in which the discount rates are

$$\delta_{i,0} = 1 \quad \text{and} \quad \delta_{i,t} = 1 - \frac{1}{(t+2)^2} \quad \text{for} \quad i = u, f, \quad t \geq 1.$$

First of all, we verify condition (17):

$$\begin{aligned} \prod_{j=1}^{t+1} \delta_{u,2j} \cdot \delta_{f,2j-1} &= \left(1 - \frac{1}{3^2}\right) \cdot \left(1 - \frac{1}{4^2}\right) \cdot \dots \cdot \left(1 - \frac{1}{(2t+3)^2}\right) \cdot \left(1 - \frac{1}{(2t+4)^2}\right) = \\ &= \frac{2 \cdot (2t+5)}{3 \cdot (2t+4)} \xrightarrow{t \rightarrow +\infty} \frac{2}{3} > 0 \end{aligned}$$

Hence the assumption of Theorem 3 is not satisfied for the given discount rates. Consequently, we cannot use this theorem, but we can apply Theorem 2 to this example.

$$\begin{aligned} \sum_{n=1}^{+\infty} \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1}) &= \frac{2}{3} \cdot \sum_{n=1}^{+\infty} \frac{1}{(2n+2) \cdot (2n+3)} = \frac{2}{3} \cdot \sum_{n=1}^{+\infty} \left( \frac{1}{2n+2} - \frac{1}{2n+3} \right) \\ &= \frac{2}{3} \cdot \left( \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} + \dots \right) \end{aligned}$$

Since

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{(-1)^{n-1}}{n} \pm \dots = \ln 2$$

we have

$$\begin{aligned} 1 - \delta_{f,1} + \sum_{n=1}^{+\infty} \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1}) &= \frac{2}{3} \cdot (1 - \ln 2) \\ 1 - \delta_{f,1} + \sum_{n=1}^{+\infty} \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1}) + \prod_{j=1}^{+\infty} \delta_{u,2j} \cdot \delta_{f,2j-1} &= \frac{2}{3} \cdot (2 - \ln 2) \end{aligned}$$

Note that  $\frac{2}{3} \cdot (1 - \ln 2) > 0$  and  $\frac{2}{3} \cdot (2 - \ln 2) < 1$ . Since condition (17) is not satisfied in this example, there are infinitely many subgame perfect equilibria. According to Theorem 2, each pair of strategies  $(A, B)$  such that the offer of the union proposed in period 0 is

$$\frac{2}{3} \cdot (1 - \ln 2) \leq \overline{W}_u^0 \leq \frac{2}{3} \cdot (2 - \ln 2)$$

is the subgame perfect equilibrium in this model.

## 4.2 Example 2 - Application of Theorem 3

Let us analyze a model in which the union and the firm have the following sequences of discount rates varying in time:

$$\delta_{u,0} = \delta_{f,0} = 1 \quad \text{and for each } t \geq 1$$

$$\delta_{u,t} = \begin{cases} \frac{1}{t+2} & \text{if } t \text{ is even} \\ \frac{t}{t+1} & \text{if } t \text{ is odd} \end{cases} \quad \delta_{f,t} = \begin{cases} \frac{1}{2} & \text{if } t \text{ is even} \\ \frac{1}{t+2} & \text{if } t \text{ is odd} \end{cases}$$

First of all, we verify (17):

$$\prod_{j=1}^{t+1} \delta_{u,2j} \cdot \delta_{f,2j-1} = \frac{2}{(2t+4)!} \rightarrow_{t \rightarrow +\infty} 0$$

$$\sum_{n=1}^{+\infty} \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1}) = 2 \cdot \sum_{n=1}^{+\infty} \frac{2n+2}{(2n+3)!} = 2 \cdot \left( \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} + \dots \right)$$

Since

$$1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots + \frac{(-1)^n}{n!} \pm \dots = \frac{1}{e}$$

we get

$$\sum_{n=1}^{+\infty} \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1}) = 2 \cdot \left( \frac{1}{e} - \frac{1}{3} \right)$$

According to Theorem 3, there is the only one subgame perfect equilibrium in which the union in each even period  $2t$ , where  $t \in \mathbb{N}$ , submits its offer  $\overline{W}^{2t}$  and in each odd period  $2t+1$  it accepts an offer of the firm if and only if the share given to the union under this offer is at least  $\overline{Z}_u^{2t+1}$ . Moreover, the union goes to strike in every period in which a disagreement appears. Similarly, the firm in each odd period  $2t+1$  ( $t \in \mathbb{N}$ ) submits its offer  $\overline{Z}_u^{2t+1}$  and in each even period  $2t$  it accepts an offer of the union if and only if the share given to the firm under this offer is at least  $\overline{W}_f^{2t}$ , where:

$$\overline{W}_u^0 = 1 - \delta_{f,1} + \sum_{n=1}^{+\infty} \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1}) = \frac{2}{e} < 1.$$

## 5 Conclusions

In this paper, we have investigated a non-cooperative wage bargaining between union and firm with discount rates varying in time. Each party, starting with the union, makes its offer sequentially and tries to reach an agreement point which satisfies both sides. We have described solutions of such a bargaining for a particular case in which the union is supposed to go on strike always when the agreement is not reached. Based on certain generalization of Rubinstein's alternating offers model, we have determined subgame perfect equilibria for this case. While with the constant discount rates the equilibrium is the same as in the original Rubinstein bargaining model, the result obtained for the stationary case does not hold if the discount rates vary in time. Hence, the non-stationary model introduced in Rusinowska (2000, 2001, [23, 24]) has been applied to the wage bargaining in question. We have stressed the differences between the stationary and non-stationary wage bargaining models. In particular, if the union is committed to strike in every period in which the sides did not reach an agreement, then under each subgame perfect equilibrium of the model with discount rates varying in time, the offer of the union in period 0 depends e.g. on the sum of a certain convergent series, and the sides' offers are determined in a recursive way. If the product of the discount rates of both parties in consecutive periods goes to zero, then there is the only one subgame perfect equilibrium, otherwise there exist infinitely many equilibria. In the wage bargaining with constant discount rates, which is a particular case of our generalized model, this product of the discount rates always converges to 0. Consequently, according to our results, there is always the unique subgame perfect equilibrium, which confirms the result of Fernandez and Glazer. We have presented two different examples, i.e., an example with infinitely many subgame perfect equilibria, and an example with the unique equilibrium.

This paper is our first step to investigate wage bargaining models with non-stationary preferences of unions and firms. As mentioned before, we have focused on the generalization of the wage bargaining analyzed in Fernandez and Glazer (1991, [7]) with the assumption that the union's decision is to go on strike in every period in which there is a disagreement. Hence, we have generalized case (ii) of their model recapitulated in Section 2. The next step will be to relax this strike condition and to analyze, e.g., the situations of the Minimum Wage Contract (no strike decision; see case (i) presented in Section 2), of the Maximum Wage Contract (see case (iii) in Section 2), or the situation of free choice of the union's decision. Moreover, we could also analyze wage bargaining, where preferences of unions and firms are varying in time, but they are expressed not by discount rates, but by bargaining costs. In particular, if the union is committed to strike in every disagreement period, then for such a wage bargaining we could apply solutions of other generalizations of Rubinstein's bargaining model, in which the preferences are defined by bargaining costs, either constant or varying in time (Rusinowska, 2000, 2002, [23, 25]). Another possibility could be to consider a "mixed" wage bargaining with preferences expressed both by discount rates and bargaining costs. For such a generalization of Rubinstein's model with mixed preferences varying in time we refer to Rusinowska (2004, [27]). Also, some further extensions of this studies are possible, such as multiple firm and union schemes or strike periods determinations.

We believe that wage bargaining with non-stationary preferences is a very useful improvement of the wage bargaining procedure with stationary preferences. The wage

bargaining with discount factors varying in time is much more realistic than the model with constant bargaining rates. It can model real life situations in a more accurate way, and consequently, it can explain actual strike and wage bargaining problems much better than the traditional wage bargaining based on stationary preferences.

## 6 Appendix

### Proof of Theorem 1

If the union is supposed to go on strike in each period in which there is a disagreement and the preferences of the union and the firm are expressed by the discount rates varying in time, then we get the same payoff functions of the parties as the ones defined in the bargaining model investigated in Rusinowska (2000, 2001, [23, 24]) - see formulas from (7) till (11). The union and the firm are players 1 and 2, respectively. In order to find the subgame perfect equilibria of the wage bargaining, we can apply the results obtained for the model with discount rates varying in time to our updated wage bargaining situation. By virtue of Rusinowska (2000, [23]), the pair of strategies  $(A, B)$  is a subgame perfect equilibrium if and only if for each  $t \in \mathbb{N}$  offers of the union and of the firm satisfy the following conditions:

$$(\bar{W}^{2t}, 2t) \sim_f (\bar{Z}^{2t+1}, 2t+1) \quad \text{and} \quad (\bar{Z}^{2t+1}, 2t+1) \sim_u (\bar{W}^{2t+2}, 2t+2)$$

where  $\sim_i$  means the indifference preference relation of player  $i$ ,  $i = u, f$ . By using (9) and (10), these conditions are equivalent to (12) and (13).  $\square$

### Proof of Theorem 2 and Theorem 3

We sketch the proof given in Rusinowska (2001, [24]) updated to the wage bargaining between the union and the firm. When the sides use the strategies  $(A, B)$ , the solution of Theorem 1 gives us for each  $t \geq 1$ :

$$\bar{W}_u^0 = 1 - \delta_{f,1} + \sum_{n=1}^t \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1}) + \bar{W}_u^{2t+2} \cdot \left( \prod_{j=1}^{t+1} \delta_{u,2j} \cdot \delta_{f,2j-1} \right)$$

Since  $\bar{W}_1^{2t+2} \in [0, 1]$ , we have for each  $t \geq 1$ :

$$\bar{W}_u^0 \geq 1 - \delta_{f,1} + \sum_{n=1}^t \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1})$$

and

$$\bar{W}_u^0 \leq 1 - \delta_{f,1} + \sum_{n=1}^t \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1}) + \prod_{j=1}^{t+1} \delta_{u,2j} \cdot \delta_{f,2j-1}$$

Moving on to the limit, we obtain (14) and (15), and if condition (17) is satisfied, then by virtue of the three sequences theorem we get (18). Let

$$S_t = \sum_{n=1}^t \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1})$$



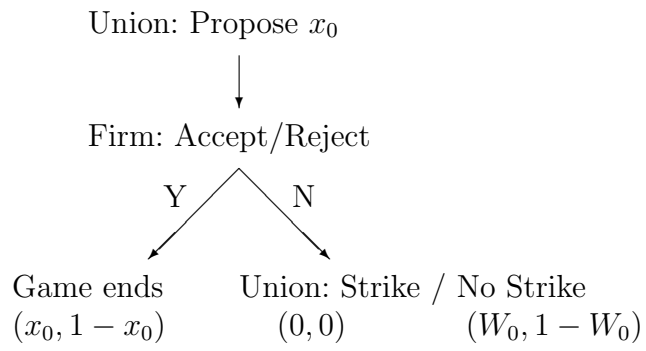
Note that the sequence of partial sums  $(S_t)$  is increasing (i.e., for each  $t \geq 1$ ,  $S_{t+1} > S_t$ ) and bounded from above (i.e., for each  $t \geq 1$ ,  $S_t < \delta_{f,1}$ ), and therefore  $(S_t)$  is convergent and  $\sum_{n=1}^{+\infty} \left( \prod_{k=1}^n \delta_{u,2k} \cdot \delta_{f,2k-1} \right) \cdot (1 - \delta_{f,2n+1})$  is a convergent series. For a more detailed proof, see Rusinowska (2000, [23]).  $\square$

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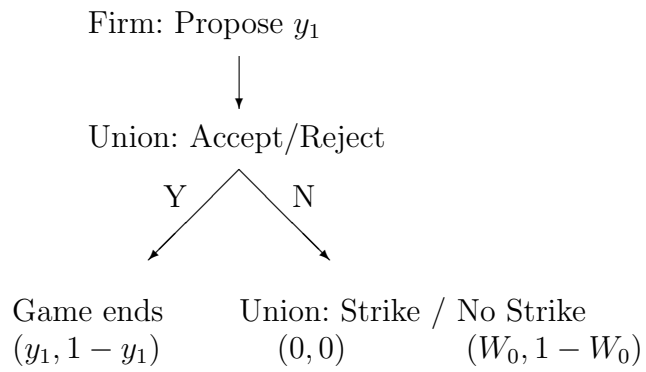
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Period

0



1



2

Union: Propose  $x_2$   
etc.

**Figure 1:** Non-cooperative bargaining game between the union and the firm