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# Comparison of Negotiated Uniform versus Differentiated Abatement Standards for a Transboundary Pollution Problem

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## RESUME

Ce papier analyse un problème de pollution transfrontalière entre deux pays. Il étudie principalement la comparaison en termes d'efficacité des normes uniformes et différenciées d'abattement en présence de transferts monétaires imparfaits entre les pays. Pour le faire, on utilise un jeu de négociation et la solution de Nash bargaining comme équilibre. En ce qui concerne les pays symétriques, on remarque que l'utilisation des normes uniformes n'est pas approprié quand il s'agit de coûts fixes élevés dans la technologie d'abattement. Concernant les pays asymétriques, nos résultats théoriques montrent d'un côté que les normes différenciées avec transferts sont généralement mieux que les normes uniformes avec transferts en termes de bien-être total. De l'autre côté, les normes différenciées sans transferts dominent toujours les normes uniformes sans transferts. Enfin, les résultats des applications numériques mettent en évidence que l'hétérogénéité entre les pays sur les bénéfices d'abattement amène à la supériorité d'un accord de normes uniformes avec des transferts imparfaits sur un autre accord de normes différenciées sans la possibilité de transferts. Au contraire, une hétérogénéité entre les pays sur les coûts d'abattement donne le résultat opposé.

*Mots-clés:* pollution transfrontalière, jeux coopératifs, négociation, normes, transferts.

## ABSTRACT

This paper analyses a transboundary pollution problem between two countries, and studies the efficiency comparison of uniform versus differentiated abatement standards when there are imperfect transfers between countries. To achieve this goal, we use a negotiation game and the Nash bargaining solution as equilibrium. On the one hand, we remark that the argument of similarity of countries to defend the use of uniform standards is not appropriate, when there exists high level of fixed costs in abatement technology for symmetric countries. On the other hand, for asymmetric countries, according to the total welfare criteria, we notice first that differentiated standards with transfers are generally better than uniform standards with transfers. Secondly, differentiated standards without transfers always outperform uniform standards without transfers. Last, the numerical results show that the asymmetry on abatement benefits between the countries makes the uniform regime with imperfect transfers better than the differentiated regime without transfers, while an asymmetry on abatement costs gives the opposite result.

*Keywords:* transboundary pollution, cooperative games, bargaining, standards, transfers.

*JEL:* Q50, C71.

## 1 Introduction

This paper deals with the efficiency comparison of negotiated uniform and differentiated abatement standards for a transboundary pollution problem.

It is often admitted in the economic theory that uniform standards are inefficient instruments in the presence of asymmetric countries. We observe, however, a frequent use of these rules in international environmental agreements. Potential signatory countries in these agreements bargain over a uniform emission reduction rate. This means that countries have to reduce their individual emissions of an exogenously given year by the same percentage. The efficiency criteria would recommend, however, the use of differentiated emission reduction quotas specified for each country. These differentiated rules would take into account the different characteristics of countries (abatement and damage functions, preferences).

There exists an important number of international environmental agreements (IEA's) including the use of uniform emission reduction quotas. "For instance, the *Montreal Protocol* on Substances that Deplete the Ozone Layer specified an emission reduction of CFCs and halons by 20 percent based on 1986 emission levels to be accomplished by 1998. Another example is the *Helsinki Protocol*, which suggested a reduction of sulfur dioxide from 1980 levels by 30 percent by 1993. Moreover, the *Sofia Protocol* Concerning the Control of Emissions of Nitrogen Oxides or Their Transboundary Fluxes signed in 1988 called on countries to uniformly freeze their emissions at 1987 levels by 1995 and the *Geneva Protocol* concerning the Control of Emissions of Volatile Organic Compounds or Their Fluxes signed in 1991 required parties to reduce 1988 emissions by 30 percent by 1999" (Finus, (2001)).

We can find several reasons explaining the use of uniform standards. First, uniform solutions are more rapidly accepted by the signatory countries because of the fairness argument. Secondly, the negotiation process of differentiated standards specific to each country could be costly because of the informational problems between countries. Moreover, simple rules such as uniform quotas may form a "focal point" during negotiations (Schelling, (1960))<sup>1</sup> simplifying the coordination of expectations between countries (Schmidt, (2002)). Another argument in favor of the use of uniform standards is the presence of agency problems (Boyer and Laffont, (1999)). A constitution which mandates the use of uniform standards rather than differentiated ones might solve agency problems. It could prevent for example the imposition of lax environmental standards by politicians according to the political pressure from industrial groups.

In the context of a transboundary pollution problem between two countries, in this paper we ask the following question. Can the two countries benefit by constraining themselves to agreements with uniform standards?

We especially concentrate on a *transboundary* pollution problem between two countries. This means that there exist negative spillovers of pollution between these countries, namely the emissions of one country negatively affect the other

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<sup>1</sup>A detailed description of the "focal point" can be found in Schelling (1971).

country. In areas such as global warming, ozone layer depletion, acid rain or international water pollution, these negative effects are not taken into account by the polluting countries. The transboundary pollution problem studied in this paper is similar to international pollution issues in rivers and seas. The example of Colorado river <sup>2</sup> illustrates a transboundary issue between two countries. This Mexican-American river was subject to a salinity problem caused by U.S. Following the complaint of Mexico, U.S. accepted to undertake a costly measure, which is to construct a desalinization plant in Arizona. This issue led to the signature of the 1973 Agreement (Minute 242 of the International Water and Boundary Commission).

In the presence of transboundary pollution problems, the terms of an agreement can include situations where the countries that gain compensate the losers. In this case, *side payments* <sup>3</sup> will ensure the acceptability of the outcome to all countries. For this reason, side payments are often offered in order to increase participation in IEA's. Since most frequently cash payments are paid in IEA's including provisions for transfer schemes, it could be convenient to assume monetary side payments (Barrett (2003)).

Our purpose in this paper is to compare the efficiency of uniform versus differentiated emission reduction quotas when there is a possibility of a transfer scheme between countries. In this paper, we also ask the following question. What is the impact of the introduction of imperfect side payments on the comparison of uniform and differentiated standards? It could exist some arguments justifying the existence of imperfect transfers in IEA's. The first argument could be that it is costly to donor countries to collect transfers and deliver them to recipient countries. The second argument can be found in the paper of Finus and Rundshagen (1998). They argue that actual transfers being realized within some IEA's are less than (expected) abatement and damage costs.

To treat this problem, we use a negotiation game between two countries. The international negotiation is a one-stage game in which the governments holding regulatory powers *cooperatively* bargain over abatement levels<sup>4</sup>. We assume a cooperative spirit in the agreement in the sense that countries may wish to improve their payoffs through a bargaining process. So we use a cooperative game theoretic concept which is the Nash bargaining solution. This solution satisfies some axioms, especially *Pareto efficiency*. Defining all bargaining-efficient outcomes and the payoff level at the *threat point* which is the situation without bargaining, the bargaining can be defined axiomatically; the *Nash* (1950) *bargaining solution* is such an axiomatic solution<sup>5</sup>. More generally, the bargaining outcome will depend on the relative bargaining powers of countries (Jéhiel., (1997)).

<sup>2</sup>The discussion on Colorado river is taken from Maler (1990).

<sup>3</sup>"To introduce money side payments requires an assumption: each player's utility for money must be linear or equivalently, the marginal utility for money must be constant" (taken from Barrett, (2003)).

<sup>4</sup>For simplicity, we only take into account national government decisions in the negotiation process and exclude from the analysis national interest groups pressures on these decisions.

<sup>5</sup>A good introduction to (n) person simple Nash bargaining games can be found in Harsanyi (1977), chapter 10.

There are an important number of papers in the economic literature which study environmental problems by cooperative bargaining framework, like in the Nash bargaining solution: see among others Amacher and Malik (1996), Eckert (2003), Kampas and White (2003). In order to take into account the dynamic process of negotiations and the non-cooperative behavior of the agents, there is an alternative way to model negotiations by noncooperative bargaining games (Rubinstein (1982)). Some of the papers treating environmental problems by the noncooperative bargaining framework are among others Rotillon et al. (1996), Compte and Jéhiel (1997), Chen (1997) and Manzini and Mariotti (2003).

There exists one strand of the literature studying the efficiency comparison of uniform standards with other instruments. One of these papers is the model of Finus and Rundshagen (1998) in which a uniform emission reduction quota and an effluent charge are considered. They show that for global pollutants, where the number of the countries suffering from an externality is large, governments agree on a quota regime rather than an effluent tax. Their result widely depends on a special rule determining the uniform standard which will be imposed on all countries. This rule is the lowest common denominator meaning that the least proposal from one country will be imposed to all countries. Heyes and Simons (2003) also examine the comparative efficiency of uniform emission standards and firm-specific emission standards in the presence of a domestic pollution problem. They assume the existence of asymmetric information between firms and a regulator, firms having private information about their access to low-cost abatement technology. They show that if the regulator has the possibility to differentiate standards and doesn't use this possibility, this alternative to differentiate standards should be taken away as it is always harmful. Even if the regulator opts to use this possibility, it may be welfare-improving to remove it. Their welfare comparisons are made however, on numerical examples.

It is possible to count three possible instruments in IEA's. These instruments give incentives to the countries which would not be a part of the agreement without these "rewards". The first instrument which is studied in this paper is side payment scheme. Barrett (2001) indicates that the strong asymmetry between countries could allow to increase participation with the use of side payments in IEA's. Carraro and Siniscalco (1993) show that side payments can help to increase participation unless the signatories can commit being signatory. Chen (1997) shows in a two-country noncooperative bargaining model that some of the side payments will emerge as a result of the asymmetry in bargaining power between countries. Chang (1997) indicates in a signalling game that the only side payment regime without the use of punishments, would encourage greater environmental harm under conditions of asymmetric information between countries. The second instrument is issue linkage<sup>6</sup> which means a negotiation process on multiple issues, i.e., on the protection of the environment and trade or technological transfers etc. The last instrument is technological transfers for abatement process between countries<sup>7</sup>.

<sup>6</sup>See among others Folmer et al. (1993), Cesar and de Zeeuw (1996), Carraro and Siniscalco (1997), Conconi and Perroni (2002).

<sup>7</sup>See among others Lee (2001).

What concerns us in this paper is to compare the efficiency of uniform versus differentiated abatement standards in the presence of a transboundary pollution problem. The international aspect of the pollution problem conducts us to consider a negotiation game between two sovereign nations. To our knowledge, this is the first paper comparing both types of standards in the context of a bargaining game with perfect information. Moreover, we introduce the possibility of imperfect transfers between two countries in order to understand the robustness of uniform solutions to the introduction of these imperfect transfers. The simple two-country model allows us to take into account both symmetric and asymmetric country frameworks. Using very general functional forms, we do welfare comparisons in both analytical and numerical ways.

The paper is organized as follows. Section 2 presents a simple framework with *symmetric* countries. One of the arguments in favor of the use of uniform solutions is that countries are similar. We show that this argument is fragile when there are high fixed costs in abatement technology. In this symmetric country case, we study two different situations, *reciprocal* and *unilateral* action cases respectively. In the first situation, both countries undertake abatement efforts. In the second situation, we allow the country which does not provide any abatement effort to compensate the country providing a positive abatement effort by side payments. In Section 3, we consider the reciprocal action case of the *asymmetric* countries which differ by their abatement benefit and cost functions. Now, we investigate the efficiency comparison of the two standards in the presence of this asymmetric character between countries. The standards under consideration are: uniform standards with or without the possibility of transfers and differentiated standards with or without the possibility of transfers. To do all the welfare comparisons, we use analytical tools as well as numerical simulations. We summarize the results in section 4 and offer concluding remarks in section 5.

We start our comparative study with the case of symmetric countries.

## 2 Symmetric Countries

We assume that there exists one representative polluting firm in each country. The aggregate benefit function is  $B_i(a_1 + a_2)$  or  $B_i(A)$  for  $i = 1, 2$ , where  $a_1 = \bar{E}_1 \times \beta_1$  and  $a_2 = \bar{E}_2 \times \beta_2$  are respectively the abatement levels of the countries 1 and 2. Here,  $\bar{E}_1, \bar{E}_2$  represent respectively the emission levels of the countries 1 and 2 in a given year. These emission levels are assumed to be exogenous, since they are determined by scientific evidence in IEA's. The variables  $\beta_1, \beta_2$  are respectively differentiated percentage emission reduction rates for the countries 1 and 2, with  $\beta_1$  and  $\beta_2$  smaller than 1. The benefit from abatement for the country 1 also depends on the abatement level of the country 2 because of positive spillovers.  $B_i(A)$  is assumed to be an increasing and concave function of the variable  $A = (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)$ , i.e. the total abatement level. So we have  $B'_i(A) \geq 0$  and  $B''_i(A) \leq 0$ . The countries have the

same benefit function since they are symmetric.

The abatement cost function is written as  $C_i = co + C(a_i)$  for  $i = 1, 2$ , where  $a_1 = \bar{E}_1 \times \beta_1$  and  $a_2 = \bar{E}_2 \times \beta_2$ . So the total cost function of a country is the sum of a fixed cost  $co$  with a variable cost  $C(a_i)$  for  $i = 1, 2$ . The variable cost function is individual, since each country is lonely in its abatement effort. We introduce fixed costs in abatement technology in order to differentiate the incentives of completely symmetric countries to undertake abatement efforts. It is possible to justify the existence of fixed costs in reality, for the case of water pollution by industrial or agricultural activities. For instance, a polluting firm in a country must install a special plant for the cleaning of its discharge. The installation of such a plant could represent a fixed cost for the firm. The variable cost function is assumed to be increasing and convex, i.e.  $C'(a_i) \geq 0$  and  $C''(a_i) \geq 0$  for  $i = 1, 2$ . We assume that the cost is zero when there is no abatement effort, i.e.  $C_i = 0$  when  $a_i = 0$  for  $i = 1, 2$ .

Then, the net benefit function can be expressed as  $NB_1 = B(A) - co - C(a_1)$  for the country 1 and  $NB_2 = B(A) - co - C(a_2)$  for the country 2, which are assumed to be symmetric in this section.<sup>8</sup>

In the next section, we will look at the threat point for the bargaining parties, namely the non-cooperative Nash equilibrium of the game.

## 2.1 Threat Point: Nash Equilibrium

The country 1's program is to maximize the following function with respect to  $a_1$ <sup>9</sup>:

$$Max_{a_1} [B(a_1 + a_2) - co - C(a_1)]$$

We obtain the following first-order condition:

$$B'(A) = C'(a_1)$$

This condition means that individual marginal benefits from abatement are equal to individual marginal abatement costs, for the country 1.

Similarly, the country 2's program is to maximize the following function with respect to  $a_2$ :

$$Max_{a_2} [B(a_1 + a_2) - co - C(a_2)]$$

And we obtain the following first-order condition:

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<sup>8</sup>A quadratic payoff function could arise from linear demand and quadratic abatement technology combined with constant emissions. Any flow damages must be quadratic. At the aggregate level, the resource constraint is a labor one. The labor is split two sectors : pollution abatement sector and production of consumption good sector (see appendix for an example of introduction of resource constraint, section 6.1).

<sup>9</sup>Here, we consider a symmetric Nash equilibrium where both countries pay for fixed costs in order to abate. This can be true if the level of fixed costs is low. So, we implicitly assume that the level of fixed costs is sufficiently low. See the appendix (section 6.2) for the description of this specific value of fixed costs and a numerical example to illustrate it.

$$B'(A) = C'(a_2)$$

Since the two countries are symmetric, the first-order condition becomes:

$$B'(A) = C'\left(\frac{A}{2}\right)$$

where  $a_1 = a_2 = \frac{A}{2}$ .

The resolution of this equation will give us the value of the total abatement at the Nash equilibrium,  $\hat{A}$ . So we can calculate the welfare level for each country:

$$\hat{NB} = B(\hat{A}) - co - C(\hat{A}/2)$$

In the next section, we will look at the case where both countries undertake abatement efforts. Since the countries are symmetric, we assume an identical negotiation power. So we can only concentrate on the simple Nash bargaining solution (without introducing the negotiation powers of the countries).

## 2.2 A Case with Uniform Standards (Reciprocal Action Case)

The Nash bargaining solution is written in the following way when both of the countries realize abatement effort:

$$Max_{\beta_1, \beta_2} \left[ \begin{array}{l} (B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - co - C(\bar{E}_1 \times \beta_1) - \hat{NB}) \times \\ (B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - co - C(\bar{E}_2 \times \beta_2) - \hat{NB}) \end{array} \right]$$

Here, we remark that the bargaining variables are the percentage emission reduction rates, as this is the case in IEA's.

If we note as  $V$  the above value function, the first-order condition with respect to  $\beta_1$  is:

$$\begin{aligned} \frac{\partial V}{\partial \beta_1} &= 0 \iff \left[ \frac{\partial B}{\partial \beta_1}(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \frac{\partial C}{\partial \beta_1}(\bar{E}_1 \times \beta_1) \right] \times \\ &\quad \left[ B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - co - C(\bar{E}_2 \times \beta_2) - \hat{NB} \right] \\ &= - \left[ B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - co - C(\bar{E}_1 \times \beta_1) - \hat{NB} \right] \times \\ &\quad \left[ \frac{\partial B}{\partial \beta_1}(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \right] \end{aligned}$$

Similarly, the first-order condition with respect to  $\beta_2$  is:

$$\begin{aligned}
 \frac{\partial V}{\partial \beta_2} &= 0 \iff \left[ \frac{\partial B}{\partial \beta_2} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \right] \times \\
 &\quad \left[ B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - co - C(\bar{E}_2 \times \beta_2) - \hat{N}B \right] \\
 &= - \left[ B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - co - C(\bar{E}_1 \times \beta_1) - \hat{N}B \right] \times \\
 &\quad \left[ \frac{\partial B}{\partial \beta_2} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \frac{\partial C}{\partial \beta_2} (\bar{E}_2 \times \beta_2) \right]
 \end{aligned}$$

The symmetry assumption allows us to consider the same percentage emission reduction rate and naturally the same abatement level for both countries, i.e.  $a_1 = \beta_1 \bar{E}_1 = a_2 = \beta_2 \bar{E}_2 = a = \beta \bar{E}$ . This is why we call this situation the *uniform standard* case. The first condition becomes:

$$\begin{aligned}
 &\left[ B'(2a) - C'(a) \right] \times \left[ B(2a) - co - C(a) - \hat{N}B \right] = \\
 &\quad - \left[ B(2a) - co - C(a) - \hat{N}B \right] \times \left[ B'(2a) \right] \\
 &\iff B'(A) = \frac{1}{2} \times C'\left(\frac{A}{2}\right)
 \end{aligned}$$

where  $A = 2a$ .

The resolution of this equation will give us the value of the total abatement under the uniform standard case,  $A_U$ . Then we can calculate the welfare level for each country:

$$NB_U = B(A_U) - co - C(A_U/2)$$

Now, we can look at the case where only one of the countries undertakes an abatement effort. The reason behind this behavior could be rational. It could be economically efficient to avoid the payment of the fixed cost by both countries. Thus, we consider a case where only one country undertakes abatement and pays for fixed costs  $co$  and the other country does nothing in terms of abatement but compensates the country 1 by monetary side payments.

### 2.3 A Case with Differentiated Standards (Unilateral Action Case)

Here, we assume that only the country 1 abates and the country 2 pays a *transfer*  $t$  to compensate the country 1 ( $\beta_1 > 0, \beta_2 = 0$ ). The Colorado river salinity problem discussed in the introduction partially illustrates the unilateral action case studied here. The construction of a desalinization plant by U.S.,

which becomes the country 1 in our model, represents a fixed cost. Maler (1990) argues that the reason behind the unilateral effort of U.S. is to obtain non-cash benefits from Mexico in the future (for example possible future benefits related to the discovery of Mexican oil). Then, Mexico becomes the country 2 in our model, which pays side payments to the country 1. The major difference is that these countries are completely asymmetric in reality, and not symmetric as assumed in this part of the paper.

Since the country 1 will have a positive percentage emission reduction rate and the country 2 will have a zero rate, this situation could be similar to a *differentiated standard* case. Furthermore, we assume that these transfers are *imperfect* in the sense that the country 2 pays more than  $t$ , i.e.  $(1 + \lambda)t$ , and the country 1 receives only  $t$ , with  $\lambda > 0$ . So the parameter  $\lambda$  measures the imperfection of transfers between the countries.

The Nash bargaining solution is written in the following way:

$$\text{Max}_{\beta_1, t} \left[ \begin{array}{l} (B(\bar{E}_1 \times \beta_1) - co - C(\bar{E}_1 \times \beta_1) + t - \hat{N}B) \times \\ (B(\bar{E}_1 \times \beta_1) - (1 + \lambda)t - \hat{N}B) \end{array} \right]$$

We maximize this program with respect to two variables, namely the percentage abatement rate  $\beta_1$  and the transfer level  $t$ .

If we note as  $V$  the above value function, the first-order condition with respect to  $\beta_1$  is:

$$\begin{aligned} \frac{\partial V}{\partial \beta_1} &= 0 \iff \left[ \frac{\partial B}{\partial \beta_1}(\bar{E}_1 \times \beta_1) - \frac{\partial C}{\partial \beta_1}(\bar{E}_1 \times \beta_1) \right] \times \\ &\quad \left[ B(\bar{E}_1 \times \beta_1) - (1 + \lambda)t - \hat{N}B \right] \\ &= - \left[ B(\bar{E}_1 \times \beta_1) - co - C(\bar{E}_1 \times \beta_1) + t - \hat{N}B \right] \times \\ &\quad \left[ \frac{\partial B}{\partial \beta_1}(\bar{E}_1 \times \beta_1) \right] \end{aligned}$$

Similarly, the first-order condition with respect to  $t$  is:

$$\begin{aligned} \frac{\partial V}{\partial t} &= 0 \iff \left[ B(\bar{E}_1 \times \beta_1) - (1 + \lambda)t - \hat{N}B \right] = \\ &\quad - \left[ B(\bar{E}_1 \times \beta_1) - co - C(\bar{E}_1 \times \beta_1) + t - \hat{N}B \right] \times [-(1 + \lambda)] \end{aligned}$$

The ratio of these two first-order conditions is:

$$\frac{(\partial V / \partial \beta_1)}{(\partial V / \partial t)} \iff [B'(a_1) - C'(a_1)] = \frac{B'(a_1)}{-(1 + \lambda)}$$

$$\Leftrightarrow B'(a_1) = \frac{1 + \lambda}{2 + \lambda} C'(a_1)$$

The resolution of this equation will give us the value of the total abatement under the differentiated standard case,  $a_1 = A_D$ . So we can calculate the welfare level for each country:

$$NB_{1D} = B(A_D) - co - C(A_D) + t$$

$$NB_{2D} = B(A_D) - (1 + \lambda)t$$

In order to calculate the value of the transfer  $t$ , we have to use the second first-order conditions, i.e.  $\frac{\partial V}{\partial t} = 0$ . This gives us:

$$t = B(A_D) \times \left(\frac{-\lambda}{2(1 + \lambda)}\right) + \frac{co}{2} + \frac{C(A_D)}{2} + N\hat{B} \times \left(\frac{\lambda}{2(1 + \lambda)}\right)$$

In the next section, we look at the comparison of the abatement levels under different regimes.

## 2.4 Abatement Level Comparisons

To sum up, we can write the first-order conditions under the Nash equilibrium, the uniform standard and the differentiated standard cases:

- Nash equilibrium:  $B'(\hat{A}) = C'(\frac{\hat{A}}{2})$ .
- Uniform standards:  $B'(A_U) = \frac{1}{2} \times C'(\frac{A_U}{2})$ .
- Differentiated standards, imperfect transfers, i.e.  $\lambda > 0$ :  $B'(A_{ID}) = \frac{1+\lambda}{2+\lambda} \times C'(A_{ID})$ .
- Differentiated standards, perfect transfers, i.e.  $\lambda = 0$ :  $B'(A_{PD}) = \frac{1}{2} \times C'(A_{PD})$ .

See the figure 1 for the comparison of the abatement levels (note that the points A, B, C, D in the figure correspond respectively to the abatement levels  $A_{ID}$ ,  $A_{PD}$ ,  $\hat{A}$ ,  $A_U$ ).

**Proposition 1** *Given the assumptions on the respective curves of the marginal benefit and marginal cost functions, we observe the following order of the emission reductions:*

$$A_{ID} < A_{PD} < \hat{A} < A_U$$

The proposition 1 implies that when the two countries realize abatement efforts (i.e. uniform standard case), they abate more together than in the case where only the country 1 abates and the country 2 compensates it for this effort (i.e. differentiated standard case). This finding could be explained by the fact that the returns to scale in the abatement technology are decreasing despite the

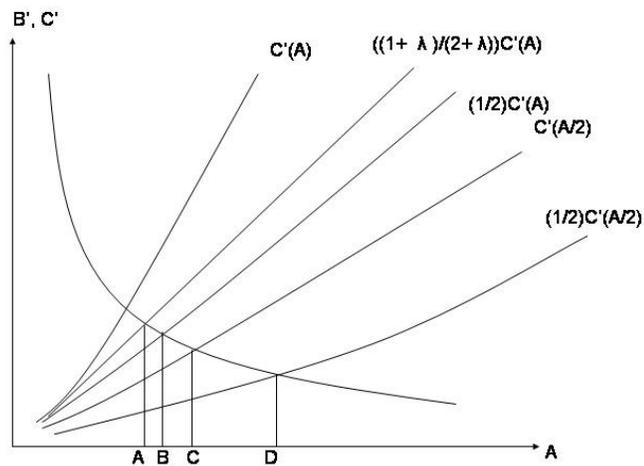


Figure 1:

existence of fixed costs. We could show for a specific functional form of the cost function in the abatement technology that it exists a level of abatement  $a^*$  (or fixed cost) such that the returns to scale in the abatement technology would be decreasing for  $a > a^*$ . In that case, the abatement of the pollution by two countries is advised. Furthermore, we remark that the total abatement level is higher in the uniform case than the one at the Nash equilibrium. Finally, the total emission reduction is lower in the differentiated case than the one at the Nash equilibrium. The fact that only one country abates in the differentiated case leads to a very low effort in terms of abatement.

In the next section, we will look at the comparison of the welfare levels under different regimes.

## 2.5 Welfare Level Comparisons

The total net benefit functions (for both countries) under the Nash equilibrium, the uniform standard and the differentiated standard cases are:

- Nash equilibrium:  $T\hat{N}B = 2 \times \hat{N}B = 2 \times [B(\hat{A}) - co - C(\hat{A}/2)]$ .
- Uniform standards:  $TNB_U = 2 \times NB_U = 2 \times [B(A_U) - co - C(A_U/2)]$ .
- Differentiated standards:  $TNB_D = NB_{1D} + NB_{2D} = 2B(A_D) - co - C(A_D) - \lambda t$ .

See the appendix (section 6.5) for a detailed discussion on the comparison of the welfare levels between different regimes. The proposition 2 sums up the main results.

**Proposition 2** *i)  $TNB_U > \hat{T}NB$  when the slope of the benefit function is large*  
*ii)  $\hat{T}NB > TNB_D$  when the level of fixed costs is sufficiently low*  
*iii)  $TNB_U > TNB_D$  when the level of fixed costs is sufficiently low*

First, we observe that the uniform standard case dominates the Nash equilibrium when the slope of the benefit function is sufficiently high, namely when the absolute value of the slope of the benefit function between  $A_U$  and  $\hat{A}$  is higher than the half of the absolute value of the slope of the cost function between  $(A_U/2)$  and  $(\hat{A}/2)$ . More interesting is the comparison of differentiated standards with the Nash equilibrium. We remark that the Nash equilibrium can outperform the differentiated standards if the level of fixed costs  $co$  is not so high, and this even when transfers are perfect ( $\lambda = 0$ ). The high level of fixed costs can, however, reverse this situation. Finally, we look at the comparison of the welfare levels between differentiated and uniform cases. The previous condition holds. The uniform standards can outperform the differentiated ones when the level of fixed costs is not so high, and this even when transfers are perfect. Again, the high level of fixed costs can reverse this situation. We know from the precedent section that the realization of the abatement investment in two countries, as in the uniform standard case, could be explained by the fact that the returns to scale in the abatement technology are decreasing. The welfare comparison shows on the one hand that the low level of fixed costs could be accompanied with decreasing returns to scale in the abatement. On the other hand, the argument of similarity of countries to defend the use of uniform standards is not appropriate when there exists high level of fixed costs in the abatement technology.

It is important to underline that we consider a symmetric Nash equilibrium where both countries pay for fixed costs. This implicitly requires a low level of fixed costs, which we define for a specific functional form in the appendix 6.2. Preliminary numerical results arising from a quadratic example show on the one hand, that this specific value of  $co$  exists in a large majority of cases. On the other hand, we notice that this value of  $co$  implies always the superiority of symmetric Nash equilibrium on differentiated standards, as well as the superiority of uniform standards on differentiated ones.

As was underlined by Barrett (2003), the countries doing transfer payments in IEA's were different from the countries accepting these payments in both the Fur Seal Treaty and the Montreal Protocol. In the Fur Seal Treaty, donor countries had an option to kill the seals on land because they possess their own breeding populations. This option was not available to the other countries in the agreement. Similarly, in the Montreal Protocol the donor countries were rich and benefitted more from the global protection of the ozone layer whereas, the

countries accepting the payments were poor and benefitted less. So it is more realistic to take into account asymmetric countries.

### 3 Asymmetric Countries

The structure of the benefit and cost functions is very similar to the previous symmetric country case. The total benefit function of a country is written as  $B_i(a_1 + a_2)$  or  $B_i(A)$ , for  $i = 1, 2$ , where  $a_1 = \bar{E}_1 \times \beta_1$  and  $a_2 = \bar{E}_2 \times \beta_2$ . Again,  $\bar{E}_1, \bar{E}_2$  represent respectively exogenous emission levels of the countries 1 and 2 in a given year. The parameters  $\beta_1, \beta_2$  (being smaller than 1) are respectively differentiated percentage emission reduction rates for the countries 1 and 2.  $B_i$  is assumed to be an increasing and concave function of the variable  $A = (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)$ , i.e. the total abatement level. So we have  $B'_i(A) \geq 0$  and  $B''_i(A) \leq 0$ .

The abatement cost function is written as  $C_1 = C(a_1)$  for the country 1 and,  $C_2 = C(a_2)$  for the country 2. The cost function is assumed to be increasing and convex, i.e.  $C'(a) \geq 0$  and  $C''(a) \geq 0$ . We do not introduce fixed costs in the cost function for this case. In the previous section, we introduced fixed costs to differentiate the incentives of completely symmetric countries to undertake abatement efforts. In the asymmetric country case, it exists another heterogeneity on marginal abatement benefit and cost functions. Furthermore, since we only study cases in which both countries undertake abatement effort, the welfare level comparisons will not depend on fixed costs.

To explain the existing asymmetry between the countries, we assume a simple structure for payoff functions:

$$\text{Country 1: } NB_1 = B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - C(\bar{E}_1 \times \beta_1)$$

$$\text{Country 2: } NB_2 = \alpha B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta C(\bar{E}_2 \times \beta_2)$$

Then the parameters  $\alpha$  and  $\delta$  explain respectively, the differences on abatement benefit and abatement cost functions between the countries.

We now turn to the resolution of the non-cooperative Nash equilibrium, which represents the threat point of the Nash bargaining solution.

#### 3.1 Threat Point: Nash Equilibrium

The country 1's program is to maximize the following payoff function with respect to  $a_1$ :

$$Max_{a_1} [B(a_1 + a_2) - C(a_1)]$$

We obtain the following first-order condition:

$$B'(A) = C'(a_1)$$

where  $A = \bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2$  and  $a_1 = \bar{E}_1 \times \beta_1$ .

Similarly, the country 2's program is to maximize the following payoff function:

$$Max_{a_2} [\alpha B(a_1 + a_2) - \delta C(a_2)]$$

The first-order condition with respect to  $a_2$  is:

$$\iff \alpha B'(A) = \delta C'(a_2)$$

where  $a_2 = \bar{E}_2 \times \beta_2$ .

The resolution of these equations gives us respectively, the abatement for the country 1, for the country 2 and the total abatement at the Nash equilibrium,  $\hat{a}_1, \hat{a}_2, \hat{A}$ . So we can calculate the welfare levels for each country:

$$NB_1 = B(\hat{A}) - C(\hat{a}_1)$$

$$NB_2 = \alpha B(\hat{A}) - \delta C(\hat{a}_2)$$

In the following, we study four different cases where uniform solutions can be applied by either the possibility of transfers or without this possibility and similarly differentiated solutions can be used by transfers or without transfers.

First, we study the uniform abatement standard case, which represents the most frequent recommendation in IEA's. We will first consider the situation where there is no possibility of transfers between countries.

### 3.2 A Case with Uniform Standards without the Possibility of Transfers

The Nash bargaining solution is written in the following way:

$$Max_{\beta} \left[ \begin{array}{l} (B(\beta(\bar{E}_1 + \bar{E}_2)) - C(\beta\bar{E}_1) - \hat{NB}_1)^\gamma \\ \times (\alpha B(\beta(\bar{E}_1 + \bar{E}_2)) - \delta C(\beta \times \bar{E}_2) - \hat{NB}_2)^{1-\gamma} \end{array} \right]$$

Here, we relax the assumption of equal bargaining power between the countries. We allow countries to have different bargaining powers in the negotiation, namely  $\gamma$  for the country 1 and  $(1 - \gamma)$  for the country 2. As  $\gamma$  increases, the weight of the utility of the country 1 increases and vice versa. This asymmetric Nash bargaining solution satisfies all other axioms required by the axiomatic theory but it does not result in a unique solution since the solution depends upon the pair of bargaining powers.

We also remark the following constraint,  $\beta_1 = \beta_2 = \beta$ . The countries must negotiate a uniform percentage emission reduction rate. Because it is a one-stage game, we do not consider the enforcement problems related to the respect of this uniform standard by the countries once the negotiation is achieved. It

could be possible to consider this very important aspect of IEA's in a repeated game framework by the use of punishment strategies from countries.

The resolution of this program will give us respectively, the abatement for the country 1, for the country 2 and the total abatement at the uniform standard case without transfers,  $a_{1U}, a_{2U}, A_U$ <sup>10</sup>. So we can calculate the welfare level for each country:

$$NB_{1U} = B(A_U) - C(a_{1U})$$

$$NB_{2U} = \alpha B(A_U) - \delta C(a_{2U})$$

Now we study the uniform abatement standard case with the possibility of transfers between countries. These transfers are assumed to be imperfect, however.

### 3.3 A case with Uniform Standards with the Possibility of Transfers

The Nash bargaining solution is written in the following way:

$$Max_{\beta, t} \left[ \begin{array}{l} (B(\beta(\bar{E}_1 + \bar{E}_2)) - C(\beta\bar{E}_1) - \hat{N}B_1 - (1 + \lambda)t)^\gamma \\ \times (\alpha B(\beta(\bar{E}_1 + \bar{E}_2)) - \delta C(\beta \times \bar{E}_2) + t - \hat{N}B_2)^{1-\gamma} \end{array} \right]$$

We remark that we dispose two choice variables, which are the levels of the uniform percentage emission reduction rate  $\beta$  and the transfer  $t$ .

The resolution of this program will give us respectively, the abatement for the country 1, for the country 2 and the total abatement at the uniform standard case with transfers,  $a_{1UT}, a_{2UT}, A_{UT}$ <sup>11</sup>. The welfare levels for each country are as follows:

$$NB_{1UT} = B(A_{UT}) - C(a_{1UT}) - (1 + \lambda)t$$

$$NB_{2UT} = \alpha B(A_{UT}) - \delta C(a_{2UT}) + t$$

We study in the next section the differentiated abatement standard case without the possibility of transfers between countries.

<sup>10</sup>See the appendix (section 6.3.1) for the entire resolution of the Nash bargaining solution in the uniform standard case without transfers.

<sup>11</sup>See the appendix (section 6.3.2) for the resolution of the Nash bargaining solution in the uniform standard case with transfers. Note also that the inefficiency of transfers from the country 2 to the country 1, when  $\delta > 1$  is proved in the appendix (section 6.4).

### 3.4 A Case with Differentiated Standards without the Possibility of Transfers

The Nash bargaining solution is written in the following way:

$$Max_{\beta_1, \beta_2} \left[ \begin{array}{l} (B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - C(\beta_1 \bar{E}_1) - \hat{N}B_1)^\gamma \\ \times (\alpha B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta C(\beta_2 \times \bar{E}_2) - \hat{N}B_2)^{1-\gamma} \end{array} \right]$$

Now we dispose two specific percentage abatement rates for each country  $\beta_1$  and  $\beta_2$ .

The resolution of this program will give us respectively, the abatement for the country 1, for the country 2 and the total abatement at the differentiated standard case without transfers,  $a_{1D}, a_{2D}, A_D$ <sup>12</sup>. The welfare levels for each country are:

$$NB_{1D} = B(A_D) - C(a_{1D})$$

$$NB_{2D} = \alpha B(A_D) - \delta C(a_{2D})$$

Now we study the differentiated abatement standard case with imperfect transfers between countries.

### 3.5 A Case with Differentiated Standards with the Possibility of Transfers

The Nash bargaining solution is written in the following way:

$$Max_{\beta_1, \beta_2, t} \left[ \begin{array}{l} (B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - C(\beta_1 \bar{E}_1) - (1 + \lambda)t - \hat{N}B_1)^\gamma \\ \times (\alpha B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta C(\beta_2 \times \bar{E}_2) + t - \hat{N}B_2)^{1-\gamma} \end{array} \right]$$

We remark that this case represents the largest set of bargaining variables because we maximize with respect to  $\beta_1, \beta_2$  and  $t$ .

The resolution of this program will give us respectively, the abatement for the country 1, for the country 2 and the total abatement at the differentiated standard case with transfers,  $a_{1DT}, a_{2DT}, A_{DT}$ <sup>13</sup>. The welfare levels for each country are as follows:

$$NB_{1DT} = B(A_{DT}) - C(a_{1DT}) - (1 + \lambda)t$$

$$NB_{2DT} = \alpha B(A_{DT}) - \delta C(a_{2DT}) + t$$

<sup>12</sup>See the appendix (section 6.3.3) for the resolution of the Nash bargaining solution in the differentiated standard case without transfers.

<sup>13</sup>See the appendix (section 6.3.4) for the resolution of the Nash bargaining solution in the differentiated standard case with transfers.

In the next section, we turn to the comparison of the abatement levels under different regimes.

### 3.6 Abatement Level Comparisons

#### 3.6.1 Abatement Levels at the Nash Equilibrium

We first compare the Nash equilibrium abatement levels of the countries. The first-order conditions at the Nash equilibrium are:

$$\text{Country 1: } B'(A) = C'(a_1) \quad (1)$$

$$\text{Country 2: } B'(A) = \frac{\delta}{\alpha} C'(a_2) \quad (2)$$

where  $A = \bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2$  and  $a_1 = \bar{E}_1 \times \beta_1$ ,  $a_2 = \bar{E}_2 \times \beta_2$ .

We assume that the values of the asymmetry parameters are as follows:

i)  $0 < \alpha < 1$  : it means that the country 2 is less affected by the global pollution compared to the country 1. So the country 2 benefits less from the global abatement,  $A$ .

ii)  $\delta > 1$  : it means that the country 2 has higher abatement costs than the country 1.

iii)  $\bar{E}_1 = \bar{E}_2 = \bar{E}$  : it is a simplifying assumption. It means that both countries emitted the same amount of pollution in the past in a given date.

Given the assumptions i) and ii), the ratio  $\frac{\delta}{\alpha}$  is superior to 1. Then, if we compare the right-hand side (RHS) of the equations (1) and (2),  $C'(a_1)$  must be superior to  $C'(a_2)$  to keep constant the left-hand side (LHS) of the equations.  $C'(a_1)$  higher than  $C'(a_2)$  implies that the abatement level for the country 1,  $a_1$  is higher than the country 2's,  $a_2$  since the cost function is convex. By the assumption iii), we can note that the percentage emission reduction rate of the country 1,  $\beta_1$  is higher than that of the country 2,  $\beta_2$ .

#### 3.6.2 Abatement Levels at the Differentiated Standard Case with Transfers

Here we compare the abatement levels of the countries under differentiated standards with transfers.

The first-order conditions are:

$$\text{Country 1} \iff B'(A) = \frac{C'(a_1)}{1 + (1 + \lambda)\alpha}$$

$$\text{Country 2} \iff \frac{1 + \lambda}{1 + (1 + \lambda)\alpha} \delta C'(a_2)$$

where  $A = \bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2$  and  $a_1 = \bar{E}_1 \times \beta_1$ ,  $a_2 = \bar{E}_2 \times \beta_2$ .

We know that  $\frac{1+\lambda}{1+(1+\lambda)\alpha}\delta$  is higher than  $\frac{1}{1+(1+\lambda)\alpha}$ , because  $\lambda > 0$  and  $\delta > 1$ .  $C'(a_1)$  higher than  $C'(a_2)$  implies that the abatement level for the country 1,  $a_1$  is higher than the country 2's,  $a_2$  since the cost function is convex. By the assumption iii), we can conclude that the percentage emission reduction rate of the country 1,  $\beta_1$  is higher than that of the country 2,  $\beta_2$ .

### 3.6.3 Abatement Levels at the Nash Equilibrium and the Differentiated Standards with Transfers

Here we compare the abatement levels of the countries at the Nash equilibrium and the differentiated standard case with transfers.

The respective first-order conditions (FOC's) for the countries 1 and 2 at the Nash equilibrium are:

$$\text{Country 1 } B'(A) = C'(a_1)$$

$$\text{Country 2 } B'(A) = \frac{\delta}{\alpha} C'(a_2)$$

The FOC's under the differentiated standard case with transfers are:

$$\text{Country 1 } B'(A) = \frac{C'(a_1)}{1 + (1 + \lambda)\alpha}$$

$$\text{Country 2 } B'(A) = \frac{1 + \lambda}{1 + (1 + \lambda)\alpha} \delta C'(a_2)$$

where  $A = \bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2$  and  $a_1 = \bar{E}_1 \times \beta_1, a_2 = \bar{E}_2 \times \beta_2$ .

We observe that  $C'(\hat{a}_1) < C'(a_{1DT})$  because  $\frac{1}{1+(1+\lambda)\alpha} < 1$ . So we can conclude that  $\hat{\beta}_1 < \beta_1^{DT}$ .

Similarly, we note that  $C'(\hat{a}_2) < C'(a_{2DT})$ , because  $\frac{1+\lambda}{1+(1+\lambda)\alpha} < \frac{1}{\alpha}$ . So we can conclude that  $\hat{\beta}_2 < \beta_2^{DT}$ .

Hence, we observe that the percentage abatement rates at the Nash equilibrium are lower than these under the differentiated standard case with transfers for both countries.

### 3.6.4 Abatement Levels under the Uniform versus Differentiated Standards with Transfers

Here we compare the abatement levels under the uniform and differentiated standard cases with transfers<sup>14</sup>.

The first-order condition in the uniform standard case is:

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<sup>14</sup>See the appendix, section 6.5 for this comparison of the abatement levels.

$$B'(A) = \frac{\bar{E}_1}{(\bar{E}_1 + \bar{E}_2)} \frac{C'(a_1)}{(1 + (1 + \lambda)\alpha)} + \frac{\bar{E}_2}{(\bar{E}_1 + \bar{E}_2)} \frac{(1 + \lambda)\delta C'(a_2)}{(1 + (1 + \lambda)\alpha)} \quad (1)$$

where  $A = \beta(\bar{E}_1 + \bar{E}_2)$ ,  $a_1 = \beta\bar{E}_1$ ,  $a_2 = \beta\bar{E}_2$ .

The respective first-order conditions for the countries 1 and 2 in the differentiated standard case are:

$$B'(A) = \frac{C'(a_1)}{1 + (1 + \lambda)\alpha} \quad (2)$$

$$B'(A) = \frac{1 + \lambda}{1 + (1 + \lambda)\alpha} \delta C'(a_2) \quad (3)$$

where  $A = \bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2$ ,  $a_1 = \beta_1\bar{E}_1$ ,  $a_2 = \beta_2\bar{E}_2$ .

Using the properties on the concavity of the benefit function and on the convexity of the cost function ( $B'' \leq 0$  and  $C'' \geq 0$ ) and the assumption iii)  $\bar{E}_1 = \bar{E}_2 = \bar{E}$ , we observe that  $\beta_2 < \beta < \beta_1$ , namely that the percentage emission reduction rate in the uniform case with transfers, i.e.  $\beta$ , is between  $\beta_1$  and  $\beta_2$ , the ones in the differentiated case with transfers.

In the next section, we compare the welfare levels of the two countries.

### 3.7 Welfare Level Comparisons

We first compare the Nash equilibrium welfare levels of the countries.

#### 3.7.1 Welfare Levels at the Nash Equilibrium

We assume for the sake of simplicity that  $\alpha = 1$ . Then the damage functions of the countries are assumed to be the same whereas, the only asymmetry existing between the countries is assumed to be on the abatement cost functions, measured by the parameter  $\delta$ . So the net benefit functions of the countries are:

$$NB_1 = B(A) - C(a_1)$$

$$NB_2 = B(A) - \delta C(a_2)$$

We can write the following equality by the envelope theorem (if  $\frac{da_2^*}{d\delta} \neq 0$ ):

$$\frac{\partial NB_2}{\partial \delta} = -C(a_2)$$

which is negative, because the abatement cost level of the country 2,  $C(a_2)$  is positive.

If there is no asymmetry between the countries, i.e.  $\delta = 1$  (symmetry case), then the welfare levels of the two countries are naturally equal. When the asymmetry between the countries increases ( $\delta > 1$ ), the welfare level of the country 2 decreases by the above lemma whereas, the welfare level of the country 1 is unaffected. So the welfare of the country 1 is superior to the country 2's at the Nash equilibrium, when the countries are asymmetric.

Now, we can study the welfare comparisons between uniform and differentiated standard cases.

### 3.7.2 Welfare Levels at the Uniform versus Differentiated Cases

Let suppose a situation where the countries 1 and 2 are in the uniform standard case with transfers. We would like to know if the total welfare level (for the two countries) can be increased if the country 1 moves into the differentiated standard case<sup>15</sup>. So we maintain constant the welfare level of the country 2 (leaving its surplus constant in the uniform case) and the transfer level in the uniform standard case. We investigate the conditions under which the welfare of the country 1 can be elevated. Remember that the net benefit functions of the countries are:

$$NB_1 = B(\beta_1 \bar{E}_1 + \beta_2 \bar{E}_2) - C(\beta_1 \bar{E}_1) - (1 + \lambda)t$$

$$NB_2 = \alpha B(\beta_1 \bar{E}_1 + \beta_2 \bar{E}_2) - \delta C(\beta_2 \bar{E}_2) + t$$

What we would like to investigate is to know if  $dNB_1 > 0$  when  $dNB_2 = 0$ , with:

$$dNB_1 = \frac{\partial NB_1}{\partial \beta_1} d\beta_1 + \frac{\partial NB_1}{\partial \beta_2} d\beta_2$$

$$dNB_2 = \frac{\partial NB_2}{\partial \beta_1} d\beta_1 + \frac{\partial NB_2}{\partial \beta_2} d\beta_2$$

**Proposition 3** *We remark that there is no improvement upon the welfare level of the country 1 ( $dNB_1 = 0$ ) by a movement from the uniform standard case with transfers to the differentiated standard case with transfers when  $\bar{E}_1 = \bar{E}_2 = \bar{E}$  and*

$$\delta(1 + \alpha + \alpha\lambda) = \alpha + \delta^2(1 + \lambda)$$

The proposition 3 implies that in general cases, except the very specific configuration of the values of the parameters (for example, symmetric country case, i.e.,  $\alpha = \delta = 1$ ), given by the above relationship, the welfare level of the country 1 can be improved by a movement into the differentiated standard case. Thus, it is possible to find a better arrangement (i.e. differentiated case with transfers) than the uniform case with transfers in terms of total welfare.

<sup>15</sup>See the appendix, section 6.8.1 for this comparison of the welfare levels.

**Proposition 4** *It is possible to find a differentiated standard case without transfers which can improve upon the uniform standard case without transfers, when  $\bar{E}_1 = \bar{E}_2 = \bar{E}$ ,  $\gamma = 1/2$ ,  $\alpha = 1$  and  $\delta \neq 1$ .*

The proposition 4 implies that an existing asymmetry on the abatement cost functions between countries makes possible to find a better arrangement (i.e. differentiated case without transfers) than the uniform case without transfers in terms of total welfare<sup>16</sup>.

### 3.7.3 General Treatment of the Welfare Levels

For a comparison of the welfare levels under a uniform standard (with or without transfers) and a differentiated standard (with or without transfers) cases in a general framework, we will especially refer to the concept of the *Pareto frontier*. In particular, it is interesting to verify if the *Nash bargaining solution with imperfect transfers* gives an *efficient* (in the sense of Pareto) *bargaining procedure*. We prove that the Nash bargaining solution with imperfect transfers derives from a Pareto efficient bargaining procedure<sup>17</sup>. Under this property, we can start to study the welfare comparisons.

We can first look at the performance in terms of welfare of a rule (uniform or differentiated) *with* transfers and another rule (uniform or differentiated) *without* transfers. We can claim that we dispose a larger choice set under a rule incorporating transfers compared to the one without transfers. With the rule incorporating transfers, we can either prefer not to use these transfers, in which case the benefit levels between the two rules are equal. Or we can prefer the rule with transfers to the one without transfers when the former gives a welfare improvement upon the latter. The preceding results can be claimed only when the bargaining procedure is efficient, which is a condition validated by the Nash bargaining solution with imperfect transfers. So we can conclude that it is possible to find a better arrangement in terms of efficiency, i.e. differentiated standards with transfers, than can improve upon the differentiated case without transfers. The same result is valid for the uniform standard case also.

In the framework of the Nash bargaining solution we used, according to the welfare criteria, we showed (proposition 4) that differentiated standards with transfers are generally (except the very specific configuration of the value of the parameters) better than uniform standards with transfers for asymmetric countries. We also showed (proposition 5) that differentiated standards without transfers are always better than uniform standards without transfers for asymmetric countries.

**Lemma 5** *The propositions 3 and 4, as well as the efficient character of the bargaining procedure together imply that differentiated rules with transfers are generally better than uniform rules without transfers in terms of welfare.*

<sup>16</sup>See the appendix, section 6.8.2 for this welfare analysis.

<sup>17</sup>See the appendix (section 6.7) for this proof.

It is an interesting result because we might think that for quasi similar countries, uniform standards without transfers are better than differentiated standards with imperfect transfers. For countries with a low degree of asymmetry, we could expect a low level of transfers between countries. So we might think that having uniform solutions without the possibility of transfers is not so constraining. We observe however, that costs related to the imperfection of transfers in the differentiated case are lower than the costs of having uniform solutions in the uniform standard case without transfers in this framework.

Now, we are interested in the comparison of the welfare levels under the uniform standard case with transfers and the differentiated standard case without transfers<sup>18</sup>.

**Proposition 6** *i) When the countries are close and the transfers are perfectly driven ( $\bar{E}_1 = \bar{E}_2, \gamma = 1/2, \alpha = 1, \delta = 1$  and  $\lambda = 0$ ), the two institutional arrangements are similar in terms of welfare, even though the countries become asymmetric in their abatement technology, in the neighborhood of  $\delta = 1$ .*

*ii) An increase of transfers ( $t$ ) from zero, considered as a parameter, implies that the uniform standard case with transfers will be always dominated by the differentiated standard case without transfers, when the countries have the same negotiation power.*

The first part of the proposition 6 could be explained by the fact that the countries need no transfers ( $t = 0$ ) when they are symmetric. The second part of the proposition 6 is implied by the use of the envelope theorem in the uniform standard case with transfers with regard to the transfers ( $t$ ), considered as a parameter and not as a control variable. We observe that the derivative of the value function with respect to ( $t$ ) is negative, when the countries have the same negotiation power. This result means that with an increase of transfers ( $t$ ) from zero, the uniform standard case with transfers will be always dominated by the differentiated standard case without transfers because of the negative effect of transfers on the value function, when the countries have the same negotiation power. We also remark that for perfect transfers ( $\lambda = 0$ ) and same negotiation power between the countries, it is similar from the social point of view to have transfers or not in the uniform standard case. This finding leads again to the superiority of the differentiated standard case without transfers on the uniform standard case with transfers. It is important to note that these findings are obtained in a framework where transfers are considered as a parameter and not as a variable.

### 3.8 Numerical Applications for the Comparison of Uniform Standards with Transfers and Differentiated Standards without Transfers

Our objective is to compare the welfare levels obtained with uniform standards with transfers and differentiated standards without transfers. Because this task

<sup>18</sup>See the appendix, sections 6.8.3 and 6.8.4 for two different analyses.

is difficult to achieve with general functional forms, we use the quadratic framework.

We assume that the abatement benefit functions for the countries 1 and 2 are:

$$\begin{aligned} B_1(a_1 + a_2) &= b(a_1 + a_2) - \frac{d}{2}(a_1 + a_2)^2 \\ B_2(a_1 + a_2) &= \alpha b(a_1 + a_2) - \alpha \frac{d}{2}(a_1 + a_2)^2 \end{aligned}$$

with  $b, d$  and  $\alpha > 0$ .

Similarly, the abatement cost functions for the countries 1 and 2 are:

$$\begin{aligned} C_1(a_1) &= fa_1^2 \\ C_2(a_2) &= \delta fa_2^2 \end{aligned}$$

with  $\delta > 0$ .

So, the net benefit functions of the countries 1 and 2 are:

$$\begin{aligned} NB_1 &= b(a_1 + a_2) - \frac{d}{2}(a_1 + a_2)^2 - fa_1^2 \\ NB_2 &= \alpha b(a_1 + a_2) - \alpha \frac{d}{2}(a_1 + a_2)^2 - \delta fa_2^2 \end{aligned}$$

For the sake of simplicity, we assume  $\gamma = 1/2$  to avoid non-linearity problem in the case of differentiated standards without transfers.

The appropriate solution of the system of equations in each case must validate the following conditions:

1) Concavity of the value functions: the determinant of the Hessian matrix must be positive and one of the elements of the diagonal must be negative.

2) Concavity of the abatement benefit function:  $B'_1 > 0$  and  $B'_2 > 0$  imply  $(a_1 + a_2) < \frac{b}{d}$ .

3) Positive payoff in the case of cooperation compared to the payoff at the Nash equilibrium:

$\left[ B_1 - C_1 - (1 + \lambda)t - \hat{N}B_1 \right] > 0$  and  $\left[ B_2 - C_2 + t - \hat{N}B_2 \right] > 0$  for uniform standards with transfers.

$\left[ B_1 - C_1 - \hat{N}B_1 \right] > 0$  and  $\left[ B_2 - C_2 - \hat{N}B_1 \right] > 0$  for differentiated standards without transfers.

4) Positive payoff in the case of cooperation:

$[B_1 - C_1 - (1 + \lambda)t] > 0$  and  $[B_2 - C_2 + t] > 0$  for uniform standards with transfers.

$[B_1 - C_1] > 0$  and  $[B_2 - C_2] > 0$  for differentiated standards without transfers.

5)  $0 < \beta < 1$  and  $t > 0$  (when the country 2 has less net benefits than the country 1, namely when either  $\alpha = 1$  and  $\delta > 1$  or,  $\delta = 1$  and  $0 < \alpha < 1$  or,  $\delta > 1$  and  $0 < \alpha < 1$ ).

### 3.8.1 Simulation Results

We observe that for the values of  $\lambda$  equal or higher than 0.1, the above conditions are not verified. So, we realize simulations for the values of  $\lambda$  equal or lower than 0.1. As we are obliged to choose the "good" solution in each case, we apply simulations for a specific combination of the values of the parameters.

**Case 1:**  $E_1 = E_2 = E = 1; b = d = f = 1; \alpha = 1; \delta = 1.01$  and  $\lambda = 0.0001$ .

$\beta$	$t$	$\beta_1$	$\beta_2$	$Dif = NB(UT) - NB(D)$
0.3328	0.0008	0.3341	0.3314	$-0.52884 \times 10^{-5}$

where  $NB(UT)$ ,  $NB(D)$  indicate respectively, total welfare for two countries in the cases of uniform standards with transfers and differentiated standards without transfers.

Since the country 2 has higher abatement costs than the country 1, it abates less in the differentiated case. The country 1 realizes positive transfers to the country 2 in order to compensate his abatement efforts in the uniform case. This configuration of the values of the parameters leads to the superiority of differentiated regime without transfers on the uniform regime with transfers.

**Case 2:**  $E_1 = E_2 = E = 1; b = d = f = 1; \alpha = 1; \delta = 1.01$  and  $\lambda = 0.01$ .

$\beta$	$t$	$\beta_1$	$\beta_2$	$Dif = NB(UT) - NB(D)$
0.3327	0.0007	0.3341	0.3314	$-0.52887 \times 10^{-5}$

Compared to the case 1, we remark that when there is higher loss in transfers between countries, the level of transfers decreases slightly. This finding could be related to the fact that the country which in a larger extent supports the burden of transfers (country 1 in our case) has less financial possibility to realize monetary transfers.

**Case 3:**  $E_1 = E_2 = E = 1; b = d = f = 1; \alpha = 1; \delta = 1.1$  and  $\lambda = 0.01$ .

$\beta$	$t$	$\beta_1$	$\beta_2$	$Dif = NB(UT) - NB(D)$
0.3278	0.0082	0.3405	0.3156	$-0.00049$

Compared to the case 2, we remark that when the difference in abatement costs of the two countries increases, the uniform abatement rate decreases, transfers increase, the abatement rate of the country 1 increases and the one of the country 2 decreases. Especially, the difference of the welfare levels between uniform and differentiated cases increases in favor of differentiated regime. The high level of abatement costs for the country 2 conducts him to abate less in the differentiated case. This fact also contributes to the reduction of the uniform abatement rate. Low level of abatement rate demanded from the country 1 in the uniform case helps him to compensate more the country 2.

Note that the maximum value of the parameter  $\delta$  which makes all conditions verified is equal to 2.49. At this point, with the values of all the other parameters unchanged, the difference between two regimes is highest and is equal to  $(-0.04705)$ .

**Case 4:**  $E_1 = E_2 = E = 1; b = d = f = 1; \alpha = 0.1; \delta = 1$  and  $\lambda = 0.01$ .

$\beta$	$t$	$\beta_1$	$\beta_2$	$Dif = NB(UT) - NB(D)$
0.2613	0.0932	0.4019	0.0748	<b>0.0558</b>

Since the country 2 has now lower abatement benefits than the country 1, it abates less in the differentiated case. The country 1 realize positive transfers to the country 2 in order to compensate his abatement efforts in the uniform case. This configuration of the values of the parameters leads now to the superiority of the uniform regime with transfers on the differentiated regime without transfers. This is an interesting result because the asymmetry on abatement benefits makes the uniform regime better in terms of total welfare than the differentiated regime, contrarily to an asymmetry on abatement costs.

**Case 5:**  $E_1 = E_2 = E = 1; b = d = f = 1; \alpha = 0.01; \delta = 1$  and  $\lambda = 0.01$ .

$\beta$	$t$	$\beta_1$	$\beta_2$	$Dif = NB(UT) - NB(D)$
0.2506	0.1021	0.3790	0.0150	<b>0.0780</b>

Here, we can express similar comments as in the case of the increase of abatements costs (see case 3). Compared to the case 4, we notice that when there is higher gap in abatement benefits between countries, the uniform abatement rate decreases, transfers increase and both the abatement rates of the countries 1 and 2 decrease. Especially, the difference in welfare levels between uniform and differentiated cases increases in favor of uniform regime. The low level of abatement benefits for the country 2 conducts him to abate less in the differentiated case. This fact also contributes to the reduction of the uniform abatement rate. Low level of abatement rate required for the country 1 in the uniform case helps him to compensate more the country 2.

Preliminary results of the numerical applications realized on quadratic functions show that there are two types of relationship between asymmetry parameters ( $\alpha$  and  $\delta$ ) and the comparison of uniform standards with transfers and differentiated standards without transfers. First, higher the parameter  $\delta$  is, i.e. the difference between the abatement cost functions increases, more the differentiated standard case without transfers dominates the uniform standard case with transfers. On the contrary, lower the parameter  $\alpha$  is, i.e. the difference the both abatement benefits functions increases, more the uniform standard case with transfers outperforms the differentiated standard case without transfers.

In fact, the parameter  $\alpha$  affects the gross abatement benefit function through the total abatement level, while the parameter  $\delta$  only affects the individual abatement level. The ranking of the two abatement standards changes whether the asymmetry is on the abatement cost or on the abatement benefit. Intuitively, when the heterogeneity comes from a common variable  $(a_1 + a_2)$ , the uniform rule is better than the differentiated one. When the heterogeneity arises from

an individual variable, the differentiated rule is better. These insights could be explained in the following way. On the one hand, when the countries have the same abatement costs but different total abatement benefits, transfers could help to compensate the difference of total abatement preferences between the two countries. In fact, the total abatement variable ( $a_1 + a_2$ ) is the same for the two countries whenever a standard is negotiated (uniform or differentiated). The only difference between the payoffs of the two countries is the preference parameter  $\alpha$ , and this difference could be easy to compensate by a side payment scheme. This could allow the uniform regime with the possibility of transfers to permit welfare level improvements, compared to the differentiated regime without transfers. On the other hand, when the countries have the same total abatement preferences but different abatement costs, transfers could fail to compensate the difference of costs between the countries. This problem is more severe if the individual abatement levels of the two countries are high and very different. In this case, the difference of payoffs levels of the countries could be very difficult to reduce by a side payment scheme, if we also take into account the losses implied by imperfect transfers. This could make better the differentiated regime without transfers in terms of welfare, compared to the uniform regime with transfers.

## 4 Summary

Our findings show that uniform standards are better than differentiated ones in the presence of symmetric countries, when fixed abatement costs are sufficiently low. This level of fixed costs guarantees the realization of the abatement effort by both countries at the Nash equilibrium. If we relax this condition of mutual abatement at the threat point, we show that differentiated standards outperform uniform ones when the level of these fixed costs is sufficiently high.

Our welfare analysis of both types of standards in the presence of asymmetric countries generally shows the superiority of differentiated standards on uniform ones. Numerical applications on specified functions contribute to the comparison of an agreement of uniform standards with the possibility of transfers with another agreement of differentiated standards without the possibility of transfers. We remark on the one hand that the asymmetry on abatement cost functions between countries causes the superiority of the latter agreement on the former one. So, the existence of differentiated rules respecting the different abatement costs of the countries, even without the possibility of a side payment scheme, permits welfare level improvements. On the other hand, the asymmetry on total abatement benefits makes better the constraining agreement with uniform standards, which has however the possibility to compensate the countries by side payments. In short, these preliminary results of the numerical applications firstly signify that an agreement of uniform standards with transfers can be welfare improving under certain conditions. These conditions are that the countries have identical abatement technology but different damages from global pollution. This situation applies well to the developed countries, having

similar advanced abatement technologies but different geographical situations, which implies a different degree of damages from global pollution.

## 5 Conclusion

This paper deals with the efficiency comparison of uniform versus differentiated pollution abatement standards in the case of a transboundary pollution problem. The emissions of two countries adversely affect each of them. One of the most recommended rule in international environmental agreements in the presence of such a pollution problem is the imposition of uniform percentage emission reduction rate for all countries. The efficiency criteria would recommend however, the use of differentiated emission reduction rates specified for each country. These differentiated rules would respect the different characteristics of the countries.

To understand the frequent use of uniform rules, we use a cooperative game theoretic framework given by the Nash bargaining solution. We assume that the countries cooperatively bargain over the percentage abatement rates and the transfers when there is possibility of transfers between countries. We also study all the cases where there is a imperfection of transfers between countries.

One of the arguments for the imposition of uniform solutions is that countries are similar. To investigate this issue, we first study the case of symmetric countries. We notice that uniform abatement standards can be worse than differentiated ones in terms of total welfare, when fixed costs in abatement technology are sufficiently high. However, we are constrained to choose a low level of fixed costs in order to have a symmetric Nash equilibrium. This constraint leads to the superiority of the symmetric Nash equilibrium on differentiated standards, as well as to the superiority of uniform standards on differentiated ones.

In the second case, we introduce another type of heterogeneity between the countries on their marginal abatement benefit and cost functions in order to understand the comparison of the two instruments in terms of efficiency. In the framework of the Nash bargaining solution we used, according to the welfare criteria, differentiated standards with transfers are generally better than uniform standards with transfers for asymmetric countries. We also show that differentiated standards without transfers are always better than uniform standards without transfers for asymmetric countries. An interesting result is that differentiated rules with imperfect transfers generally outperform uniform rules without transfers, even in the presence of quasi similar countries. This result is related to the efficiency of the bargaining procedure implied by the Nash bargaining solution.

Preliminary results of the numerical applications realized on quadratic functions show that there are two types of relationship between the asymmetry parameters and the comparison of uniform standards with transfers and differentiated standards without transfers. First, higher the parameter  $\delta$  is, i.e. the difference between the abatement cost functions increases, more the differentiated standard case without transfers dominates the uniform standard case with

transfers. On the contrary, lower the parameter  $\alpha$  is, i.e. the difference the both abatement benefits functions increases, more the uniform standard case with transfers outperforms the differentiated standard case without transfers. This is an interesting result in the sense that the asymmetry on abatement benefits between the countries makes the uniform regime with imperfect transfers better than the differentiated regime without transfers, while an asymmetry on abatement costs gives the opposite result.

The analysis has been limited in scope. Firstly, we have compared two particular instruments, namely uniform versus differentiated standards. We have not considered the efficiency of economic instruments like uniform or differentiated effluent taxes. Secondly, we have assumed that the negotiation process between the countries is efficient. Several inefficiencies, like a delay in negotiation process because of a stochastic character on the size of the cake to share, could change our superiority result of differentiated standards in the presence of asymmetric countries. Last, complete information is an essential prerequisite in order to determine the Nash bargaining solution. One should expect, however that governments try to influence a decision in its favor and therefore will not reveal their environmental preferences. So the problem of asymmetric information between the countries should be analyzed in a different context.

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## 6 Proofs

### 6.1 An Example of Introduction of Resource Constraint

The labor is split two sectors, namely the pollution abatement sector  $L_a$  and the production sector  $L_b$ . This relationship can be explained in the following way for the country 1 :

$$L_1 = L_{1a} + L_{1b}$$

We assume a decreasing returns to scale technology in labor and the existence of a fixed cost for the abatement sector:

$$\begin{aligned} a_1 &= \bar{E}_1 \times \beta_1 = (L_{1a})^{1/2} - co \\ \Leftrightarrow L_{1a} &= (\bar{E}_1 \times \beta_1 + co)^2 \end{aligned}$$

In the case of the consumption good production sector, we assume a constant returns to scale technology in labor:

$$b_1 = L_{1b}$$

where  $b_1$  denotes the consumption good production in the country 1. The production of this good leads to pollution.

So the utility function of a representative agent in the country  $i$  ( $i = 1, 2$ ) is:

$$U_i(b_i, [\varepsilon_1 - a_1 + \varepsilon_2 - a_2])$$

where  $\varepsilon_1$  and  $\varepsilon_2$  are respectively exogenously given emission levels for the countries 1 and 2. So the term in the bracket represents total net emissions of both countries. The agent has an utility from the consumption of the good  $b_i$  and has a disutility from total net emissions.

We use a very simple form for the utility function of the country 1:

$$\begin{aligned} U_1 &= b_1 - [\varepsilon_1 - a_1 + \varepsilon_2 - a_2]^2 \\ \Leftrightarrow U_1 &= L_1 - L_{1a} - \left[ \varepsilon_1 - \bar{E}_1 \times \beta_1 + \varepsilon_2 - \bar{E}_2 \times \beta_2 \right]^2 \\ \Leftrightarrow U_1 &= L_1 - \left[ \varepsilon_1 - \bar{E}_1 \times \beta_1 + \varepsilon_2 - \bar{E}_2 \times \beta_2 \right]^2 - (\bar{E}_1 \times \beta_1 + co)^2 \\ \Leftrightarrow U_1 &= L_1 - \left[ \begin{array}{l} (\varepsilon_1 + \varepsilon_2)^2 + (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)^2 \\ -2(\varepsilon_1 + \varepsilon_2)(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \end{array} \right] - (\bar{E}_1 \times \beta_1 + co)^2 \end{aligned}$$

$$\Leftrightarrow U_1 = \left\{ L_1 - (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)^2 + 2(\varepsilon_1 + \varepsilon_2)(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \right\} - \left\{ (\bar{E}_1 \times \beta_1)^2 + 2co\bar{E}_1 \times \beta_1 \right\} - co^2 - (\varepsilon_1 + \varepsilon_2)^2$$

where the first term represents the benefit part of our original payoff function, the second term represents the cost part and the last term is a constant.

The restriction that the marginal utility is non-negative implies that exogenous total emissions are higher than the total abatement, namely that:  $(\varepsilon_1 + \varepsilon_2) > (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)$ .

## 6.2 The Level of Fixed Costs: Symmetric Countries

We search for a specific level of the fixed cost  $co$  in order to satisfy two conditions. First, it must be sufficiently low to allow abatement of the two countries at the Nash equilibrium. Second, it could be high enough to obtain the superiority of the differentiated standard case on the uniform standard case with symmetric countries.

At the Nash equilibrium, when both countries abate, the net benefit levels are as follows:

$$\begin{aligned} NB_1 &= B(a_1 + a_2) - co - C(a_1) \\ NB_2 &= B(a_1 + a_2) - co - C(a_2) \end{aligned}$$

with  $B'(2a) = C'(a)$ . So, the net benefits are:

$$\begin{aligned} NB_1 &= B(2a) - co - C(a) \\ NB_2 &= B(2a) - co - C(a) \end{aligned}$$

At the Nash equilibrium, when only one of the countries abates, the first-order conditions become  $B'(a_1) = C'(a_1)$  for the country 1 and  $B'(a_2) = C'(a_2)$  for the country 2.

The payoff levels when only the country 1 abates are:

$$\begin{aligned} NB_1 &= B(a_1) - co - C(a_1) \\ NB_2 &= B(a_1) \end{aligned}$$

The payoffs in the case where only the country 2 abates are:

$$\begin{aligned} NB_1 &= B(a_2) \\ NB_2 &= B(a_2) - co - C(a_2) \end{aligned}$$

The two countries naturally get zero net benefit when no country abates.

We can represent these welfare levels in the tables below:

	country 1 abates
country 2 abates	$B(2a) - co - C(a); B(2a) - co - C(a)$
country 2 does not abate	$B(a_1) - co - C(a_1); B(a_1)$

	country 1 does not abate
country 2 abates	$B(a_2); B(a_2) - co - C(a_2)$
country 2 does not abate	0; 0

The conditions which ensure that the country 1 abates are the following:

$$\begin{aligned}
 B(2a) - co - C(a) &> B(a_2) \iff co < B(2a) - C(a) - B(a_2) \\
 B(a_1) - co - C(a_1) &> 0 \iff co < B(a_1) - C(a_1)
 \end{aligned}$$

The conditions which ensure that the country 2 abates are as follows:

$$\begin{aligned}
 B(2a) - co - C(a) &> B(a_1) \iff co < B(2a) - C(a) - B(a_1) \\
 B(a_2) - co - C(a_2) &> 0 \iff co < B(a_2) - C(a_2)
 \end{aligned}$$

We use one of the set of conditions (for example, for the country 2) and rewrite them as:

$$\begin{aligned}
 C1 &: co < B(2\bar{a}) - C(\bar{a}) - B(\bar{a}_1) \\
 C1' &: co < B(\bar{a}_2) - C(\bar{a}_2)
 \end{aligned}$$

where  $C1$  and  $C1'$  are arbitrarily chosen names for conditions.

The conditions which ensure that the differentiated standard case dominates in terms of total welfare the symmetric Nash equilibrium are:

$$2 \left[ B(a_1) - B(2a) \right] + \left[ C(\bar{a}) - \frac{C(a_1)}{2} \right] + co > 0$$

or

$$C2 : co > 2 \left[ B(2a) - B(a_1) \right] + \left[ \frac{C(a_1)}{2} - C(\bar{a}) \right]$$

with  $B'(a_1) = \frac{1}{2}C'(a_1)$  being the first-order condition for differentiated standards with perfect transfers ( $\lambda = 0$ ).

The conditions which ensure that the differentiated standard case dominates in terms of total welfare the uniform standard case are:

$$2 \left[ B(a_1) - B(2a) \right] + \left[ C(a) - \frac{C(a_1)}{2} \right] + co > 0$$

or

$$C3 : co > 2 [B(2a) - B(a_1)] + \left[ \frac{C(a_1)}{2} - C(a) \right]$$

with  $B'(2a) = \frac{1}{2}C'(a)$  being the first-order condition for uniform standards.

### 6.2.1 Numerical example

For the sake of simplicity, we assume simple quadratic form abatement benefit and cost functions:

$$B(a) = \alpha a - \frac{\beta}{2} a^2$$

with  $a < \frac{\alpha}{\beta}$ .

$$C(a) = \frac{\gamma}{2} a^2$$

where  $\alpha, \beta$  and  $\gamma$  are positive.

#### Abatement Levels

##### Symmetric Nash equilibrium

$$B'(2\bar{a}) = C'(\bar{a})$$

$$\Leftrightarrow \alpha - 2\beta\bar{a} = \gamma\bar{a}$$

$$\Leftrightarrow \bar{a} = \frac{\alpha}{\gamma + 2\beta}$$

##### Asymmetric Nash equilibrium

$$B'(\bar{a}_2) = C'(\bar{a}_2)$$

$$\Leftrightarrow \alpha - \beta\bar{a}_2 = \gamma\bar{a}_2$$

$$\Leftrightarrow \bar{a}_2 = \frac{\alpha}{\gamma + \beta} = \bar{a}_1$$

### Differentiated standards

$$\begin{aligned} B'(a_1) &= \frac{1}{2}C'(a_1) \\ \Leftrightarrow \alpha - \beta a_1 &= \frac{1}{2}\gamma a_1 \\ \Leftrightarrow a_1 &= \frac{\alpha}{\frac{\gamma}{2} + \beta} \end{aligned}$$

### Uniform standards

$$\begin{aligned} B'(2a) &= \frac{1}{2}C'(a) \\ \Leftrightarrow \alpha - 2\beta a &= \frac{1}{2}\gamma a \\ \Leftrightarrow a &= \frac{\alpha}{\frac{\gamma}{2} + 2\beta} \end{aligned}$$

**Numerical Results** The conditions  $C1, C1', C2$  and  $C3$  are summarized below:

$$\begin{aligned} C1 &: co < B(2\bar{a}) - C(\bar{a}) - B(\bar{a}_1) \\ C1' &: co < B(\bar{a}_2) - C(\bar{a}_2) \end{aligned}$$

$$C2 : co > 2 \left[ B(2\bar{a}) - B(a_1) \right] + \left[ \frac{C(a_1)}{2} - C(\bar{a}) \right]$$

$$C3 : co > 2 \left[ B(2a) - B(a_1) \right] + \left[ \frac{C(a_1)}{2} - C(a) \right]$$

First, we search a value of  $co$  such that the right-hand-sides (RHS's) of the conditions  $C1$  and  $C1'$  are positive. This allows the abatement of the two countries at the Nash equilibrium.

We allow the parameters  $\alpha, \beta$  and  $\gamma$  vary between 0.1 and 1 by an interval of 0.1. We observe that the conditions  $C1$  and  $C1'$  are verified in a majority of cases: 710 of 1000 cases.

Second, we look for a value of  $co$  such that the right-hand-sides (RHS's) of the conditions  $C1$  and  $C1'$  are higher than these of the conditions  $C2$  and  $C3$ . This will ensure the validity of these latter conditions. Applying the same combination of the values of the parameters, we remark that the conditions  $C2$  and  $C3$  are never verified. These preliminary numerical results points out that the differentiated standard case is always dominated by the symmetric Nash equilibrium. The same is true for the superiority of uniform standards on differentiated ones.

### 6.3 The Resolution of the NBS for Asymmetric Countries

#### 6.3.1 The Uniform Standard case without Transfers

If we note as  $V$  the value function, the first-order condition with respect to  $\beta$  is:

$$\begin{aligned}
 \frac{\partial V}{\partial \beta} &= 0 \iff \frac{\gamma \left[ \frac{\partial B}{\partial \beta}(\beta(\bar{E}_1 + \bar{E}_2)) - \frac{\partial C}{\partial \beta}(\beta\bar{E}_1) \right]}{\left[ B(\beta(\bar{E}_1 + \bar{E}_2)) - C(\beta\bar{E}_1) - \hat{N}B_1 \right]} \\
 &= \frac{(\gamma - 1) \left[ \alpha \frac{\partial B}{\partial \beta}(\beta(\bar{E}_1 + \bar{E}_2)) - \delta \frac{\partial C}{\partial \beta}(\beta\bar{E}_2) \right]}{\left[ \alpha B(\beta(\bar{E}_1 + \bar{E}_2)) - \delta C(\bar{E}_2 \times \beta) - \hat{N}B_2 \right]} \\
 &\iff \frac{\partial B}{\partial \beta}(\beta(\bar{E}_1 + \bar{E}_2)) \left[ \begin{array}{c} \alpha B - \gamma \delta C(\bar{E}_2 \times \beta) \\ +(\gamma - 1)\alpha C(\beta\bar{E}_1) - \gamma \hat{N}B_2 + (\gamma - 1)\alpha \hat{N}B_1 \end{array} \right] \\
 &= \frac{\partial C}{\partial \beta}(\beta\bar{E}_1) \left[ \gamma \alpha B - \gamma \delta C(\bar{E}_2 \times \beta) - \gamma \hat{N}B_2 \right] \\
 &\quad + \frac{\partial C}{\partial \beta}(\beta\bar{E}_2) \left[ -(\gamma - 1)\delta B + (\gamma - 1)\delta C(\beta\bar{E}_1) + (\gamma - 1)\delta \hat{N}B_1 \right] \\
 &\iff (\bar{E}_1 + \bar{E}_2)B'(A) \left[ \begin{array}{c} \alpha B - \gamma \delta C(a_2) \\ +(\gamma - 1)\alpha C(a_1) - \gamma \hat{N}B_2 + (\gamma - 1)\alpha \hat{N}B_1 \end{array} \right] \\
 &= \bar{E}_1 C'(a_1) \left[ \gamma \alpha B - \gamma \delta C(a_2) - \gamma \hat{N}B_2 \right] \\
 &\quad + \bar{E}_2 \delta C'(a_2) \left[ -(\gamma - 1)B + (\gamma - 1)C(a_1) + (\gamma - 1)\hat{N}B_1 \right]
 \end{aligned}$$

where  $A = \beta(\bar{E}_1 + \bar{E}_2)$ ,  $a_1 = \beta\bar{E}_1$ ,  $a_2 = \beta\bar{E}_2$ .

#### 6.3.2 The Uniform Standard case with Transfers

If we note as  $V$  the value function, the first-order condition with respect to  $\beta$  is:

$$\begin{aligned} \frac{\partial V}{\partial \beta} = 0 &\iff \frac{\gamma \left[ \frac{\partial B}{\partial \beta}(\beta(\bar{E}_1 + \bar{E}_2)) - \frac{\partial C}{\partial \beta}(\beta \bar{E}_1) \right]}{\left[ B(\beta(\bar{E}_1 + \bar{E}_2)) - C(\beta \bar{E}_1) - (1 + \lambda)t - \hat{N}B_1 \right]} \\ &= \frac{(\gamma - 1) \left[ \alpha \frac{\partial B}{\partial \beta}(\beta(\bar{E}_1 + \bar{E}_2)) - \delta \frac{\partial C}{\partial \beta}(\beta \bar{E}_2) \right]}{\left[ \alpha B(\beta(\bar{E}_1 + \bar{E}_2)) - \delta C(\bar{E}_2 \times \beta) + t - \hat{N}B_2 \right]} \end{aligned}$$

Similarly, the first-order condition with respect to  $t$  is:

$$\begin{aligned} \frac{\partial V}{\partial t} = 0 &\iff \frac{\gamma [-(1 + \lambda)]}{\left[ B(\beta(\bar{E}_1 + \bar{E}_2)) - C(\beta \bar{E}_1) - (1 + \lambda)t - \hat{N}B_1 \right]} \\ &= \frac{(\gamma - 1)}{\left[ \alpha B(\beta(\bar{E}_1 + \bar{E}_2)) - \delta C(\beta \bar{E}_2) + t - \hat{N}B_2 \right]} \end{aligned}$$

The ratio of these two first-order conditions is:

$$\begin{aligned} \frac{(\partial V / \partial \beta)}{(\partial V / \partial t)} &\iff \frac{\left[ \frac{\partial B}{\partial \beta}(\beta(\bar{E}_1 + \bar{E}_2)) - \frac{\partial C}{\partial \beta}(\beta \bar{E}_1) \right]}{-(1 + \lambda)} \\ &= \left[ \alpha \frac{\partial B}{\partial \beta}(\beta(\bar{E}_1 + \bar{E}_2)) - \delta \frac{\partial C}{\partial \beta}(\beta \bar{E}_2) \right] \\ \iff \frac{\partial B}{\partial \beta}(\beta(\bar{E}_1 + \bar{E}_2)) \times (1 + (1 + \lambda)\alpha) &= \frac{\partial C}{\partial \beta}(\beta \bar{E}_1) + (1 + \lambda)\delta \frac{\partial C}{\partial \beta}(\beta \bar{E}_2) \end{aligned}$$

$$\iff (\bar{E}_1 + \bar{E}_2)B'(A) = \frac{C'(a_1)\bar{E}_1}{(1 + (1 + \lambda)\alpha)} + \frac{(1 + \lambda)\delta C'(a_2)\bar{E}_2}{(1 + (1 + \lambda)\alpha)}$$

In order to calculate the value of transfers, we have to use the second of the first-order conditions, i.e.  $\frac{\partial V}{\partial t} = 0$ . This gives us:

$$\begin{aligned} t_{UT} &= B(A_{UT}) \times \left( \frac{\gamma(1 + \lambda)\alpha + (\gamma - 1)}{-(1 + \lambda)} \right) + \frac{(\gamma - 1)}{(1 + \lambda)} \times C(a_{1UT}) \\ &\quad + \gamma \delta C(a_{2UT}) + \frac{(\gamma - 1)}{(1 + \lambda)} \times \hat{N}B_1 + \gamma \hat{N}B_2 \end{aligned}$$

where  $A = \beta(\bar{E}_1 + \bar{E}_2)$ ,  $a_1 = \beta \bar{E}_1$ ,  $a_2 = \beta \bar{E}_2$ .

### 6.3.3 The Differentiated Standard case without Transfers

If we note as  $V$  the value function, the first-order condition with respect to  $\beta_1$  is:

$$\begin{aligned} \frac{\partial V}{\partial \beta_1} &= 0 \iff \frac{\gamma \left[ \frac{\partial B}{\partial \beta_1} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \frac{\partial C}{\partial \beta_1} (\beta_1 \bar{E}_1) \right]}{\left[ B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - C(\beta_1 \bar{E}_1) - \hat{N} B_1 \right]} \\ &= \frac{(\gamma - 1) \left[ \alpha \frac{\partial B}{\partial \beta_1} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \right]}{\left[ \alpha B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta C(\bar{E}_2 \times \beta_2) - \hat{N} B_2 \right]} \end{aligned}$$

Similarly, the first-order condition with respect to  $\beta_2$  is:

$$\begin{aligned} \frac{\partial V}{\partial \beta_2} &= 0 \iff \frac{\gamma \left[ \frac{\partial B}{\partial \beta_2} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \right]}{\left[ B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - C(\beta_1 \bar{E}_1) - \hat{N} B_1 \right]} \\ &= \frac{(\gamma - 1) \left[ \alpha \frac{\partial B}{\partial \beta_2} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta \frac{\partial C}{\partial \beta_2} (\beta_2 \bar{E}_2) \right]}{\left[ \alpha B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta C(\bar{E}_2 \times \beta_2) - \hat{N} B_2 \right]} \end{aligned}$$

The ratio of these first-order conditions is:

$$\begin{aligned} \frac{(\partial V / \partial \beta_1)}{(\partial V / \partial \beta_2)} &\iff \frac{\left[ \frac{\partial B}{\partial \beta_1} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \frac{\partial C}{\partial \beta_1} (\beta_1 \bar{E}_1) \right]}{\frac{\partial B}{\partial \beta_2} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)} \\ &= \frac{\alpha \frac{\partial B}{\partial \beta_1} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)}{\left[ \alpha \frac{\partial B}{\partial \beta_2} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta \frac{\partial C}{\partial \beta_2} (\beta_2 \bar{E}_2) \right]} \\ \iff 1 &= \frac{\frac{\partial B}{\partial \beta_1} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)}{\frac{\partial C}{\partial \beta_1} (\beta_1 \bar{E}_1)} + \frac{\alpha \frac{\partial B}{\partial \beta_2} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)}{\delta \frac{\partial C}{\partial \beta_2} (\beta_2 \bar{E}_2)} \\ &\iff 1 = \frac{B'(A)}{C'(a_1)} + \frac{\alpha B'(A)}{\delta C'(a_2)} \end{aligned}$$

where  $A = \bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2$ ,  $a_1 = \beta_1 \bar{E}_1$ ,  $a_2 = \beta_2 \bar{E}_2$ .

If we develop the first one of the first-order conditions, we obtain:

$$\begin{aligned}
 \frac{\partial V}{\partial \beta_1} &= 0 \iff \frac{\partial B}{\partial \beta_1} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \times \\
 &\quad \left[ \begin{array}{c} \alpha B - \gamma \delta C(\bar{E}_2 \times \beta_2) \\ +(\gamma - 1)\alpha C(\beta_1 \bar{E}_1) - \gamma \hat{N} B_2 + (\gamma - 1)\alpha \hat{N} B_1 \end{array} \right] \\
 &= \frac{\partial C}{\partial \beta_1} (\beta_1 \bar{E}_1) \left[ \gamma \alpha B - \gamma \delta C(\bar{E}_2 \times \beta_2) - \gamma \hat{N} B_2 \right] \\
 \iff & B'(A) \times \left[ \begin{array}{c} \alpha B - \gamma \delta C(a_2) \\ +(\gamma - 1)\alpha C(a_1) - \gamma \hat{N} B_2 + (\gamma - 1)\alpha \hat{N} B_1 \end{array} \right] \quad (1) \\
 &= C'(a_1) \left[ \gamma \alpha B - \gamma \delta C(a_2) - \gamma \hat{N} B_2 \right]
 \end{aligned}$$

If we develop the second one of the first-order conditions, we obtain:

$$\begin{aligned}
 \frac{\partial V}{\partial \beta_2} &= 0 \iff \frac{\partial B}{\partial \beta_2} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \times \\
 &\quad \left[ \begin{array}{c} \alpha B - \gamma \delta C(\bar{E}_2 \times \beta_2) \\ +(\gamma - 1)\alpha C(\beta_1 \bar{E}_1) - \gamma \hat{N} B_2 + (\gamma - 1)\alpha \hat{N} B_1 \end{array} \right] \\
 &= \frac{\partial C}{\partial \beta_2} (\beta_2 \bar{E}_2) \left[ -(\gamma - 1)\delta B + (\gamma - 1)\delta C(\beta_1 \bar{E}_1) + (\gamma - 1)\delta \hat{N} B_1 \right] \\
 \iff & B'(A) \times \left[ \begin{array}{c} \alpha B - \gamma \delta C(a_2) \\ +(\gamma - 1)\alpha C(a_1) - \gamma \hat{N} B_2 + (\gamma - 1)\alpha \hat{N} B_1 \end{array} \right] \quad (2) \\
 &= \delta C'(a_2) \left[ -(\gamma - 1)B + (\gamma - 1)C(a_1) + (\gamma - 1)\hat{N} B_1 \right]
 \end{aligned}$$

#### 6.3.4 The Differentiated Standard case with Transfers

If we note as  $V$  the value function, the first-order condition with respect to  $\beta_1$  is:

$$\begin{aligned} \frac{\partial V}{\partial \beta_1} = 0 &\iff \frac{\gamma \left[ \frac{\partial B}{\partial \beta_1} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \frac{\partial C}{\partial \beta_1} (\beta_1 \bar{E}_1) \right]}{\left[ B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - C(\beta_1 \bar{E}_1) - (1 + \lambda)t - \hat{N}B_1 \right]} \\ &= \frac{(\gamma - 1) \left[ \alpha \frac{\partial B}{\partial \beta_1} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \right]}{\left[ \alpha B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta C(\bar{E}_2 \times \beta_2) + t - \hat{N}B_2 \right]} \end{aligned}$$

Similarly, the first-order condition with respect to  $\beta_2$  is:

$$\begin{aligned} \frac{\partial V}{\partial \beta_2} = 0 &\iff \frac{\gamma \left[ \frac{\partial B}{\partial \beta_2} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \right]}{\left[ B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - C(\beta_1 \bar{E}_1) - (1 + \lambda)t - \hat{N}B_1 \right]} \\ &= \frac{(\gamma - 1) \left[ \alpha \frac{\partial B}{\partial \beta_2} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta \frac{\partial C}{\partial \beta_2} (\beta_2 \bar{E}_2) \right]}{\left[ \alpha B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta C(\bar{E}_2 \times \beta_2) + t - \hat{N}B_2 \right]} \end{aligned}$$

Finally, the first-order condition with respect to  $t$  is:

$$\begin{aligned} \frac{\partial V}{\partial t} = 0 &\iff \frac{\gamma [-(1 + \lambda)]}{\left[ B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - C(\beta_1 \bar{E}_1) - (1 + \lambda)t - \hat{N}B_1 \right]} \\ &= \frac{(\gamma - 1)}{\left[ \alpha B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta C(\bar{E}_2 \times \beta_2) + t - \hat{N}B_2 \right]} \end{aligned}$$

The ratio of the first and second conditions is:

$$\frac{(\partial V / \partial \beta_1)}{(\partial V / \partial \beta_2)} \iff 1 = \frac{B'(A)}{C'(a_1)} + \frac{\alpha B'(A)}{\delta C'(a_2)}$$

where  $A = \bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2$ ,  $a_1 = \beta_1 \bar{E}_1$ ,  $a_2 = \beta_2 \bar{E}_2$ .

The ratio of the first and third conditions is:

$$\begin{aligned} \frac{(\partial V / \partial \beta_1)}{(\partial V / \partial t)} &\iff \frac{\frac{\partial B}{\partial \beta_1} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \frac{\partial C}{\partial \beta_1} (\beta_1 \bar{E}_1)}{-(1 + \lambda)} \\ &= \alpha \frac{\partial B}{\partial \beta_1} (\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \end{aligned}$$

$$\begin{aligned} \Leftrightarrow \frac{\partial B}{\partial \beta_1}(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) &= \frac{\frac{\partial C}{\partial \beta_1}(\beta_1 \bar{E}_1)}{1 + (1 + \lambda)\alpha} \\ \Leftrightarrow B'(A) &= \frac{C'(a_1)}{1 + (1 + \lambda)\alpha} \end{aligned} \quad (1)$$

The ratio of the second and third conditions is:

$$\begin{aligned} \frac{(\partial V / \partial \beta_2)}{(\partial V / \partial t)} &\Leftrightarrow \frac{\frac{\partial B}{\partial \beta_2}(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)}{-(1 + \lambda)} \\ &= \alpha \frac{\partial B}{\partial \beta_2}(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta \frac{\partial C}{\partial \beta_2}(\bar{E}_2 \times \beta_2) \\ \Leftrightarrow \frac{\partial B}{\partial \beta_2}(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) &= \frac{1 + \lambda}{1 + (1 + \lambda)\alpha} \delta \frac{\partial C}{\partial \beta_2}(\beta_2 \bar{E}_2) \\ \Leftrightarrow B'(A) &= \frac{1 + \lambda}{1 + (1 + \lambda)\alpha} \delta C'(a_2) \end{aligned} \quad (2)$$

In order to calculate the value of transfers, we have to use the third of the first-order conditions, i.e.  $\frac{\partial V}{\partial t} = 0$ . This gives us:

$$\begin{aligned} t_{DT} &= B(A_{DT}) \times \left( \frac{\gamma(1 + \lambda)\alpha + (\gamma - 1)}{-(1 + \lambda)} \right) + \frac{(\gamma - 1)}{(1 + \lambda)} \times C(a_{1DT}) \\ &\quad + \gamma \delta C(a_{2DT}) + \frac{(\gamma - 1)}{(1 + \lambda)} \times \hat{N}B_1 + \gamma \hat{N}B_2 \end{aligned}$$

#### 6.4 The Direction of the Transfers between Countries

Here, we want to prove the inefficiency of the transfers from the country 2 to the country 1 (and not from the country 1 to the country 2) when there exists an asymmetry between the two countries on the abatement cost function like,  $\delta > 1$ .

For the uniform standard case with transfers from the country 2 to the country 1, the Nash bargaining problem can be expressed as follows:

$$\begin{aligned} \text{Max}_{\beta, t} &\left[ B(\beta(\bar{E}_1 + \bar{E}_2) - C(\beta\bar{E}_1) + t - \hat{N}B_1) \right]^\gamma \times \\ &\left[ \alpha B(\beta(\bar{E}_1 + \bar{E}_2) - \delta C(\beta\bar{E}_2) - (1 + \lambda)t - \hat{N}B_2) \right]^{(1-\gamma)} \end{aligned}$$

The first-order condition with respect to transfer ( $t$ ) is:

$$\begin{aligned}
 \frac{\partial V}{\partial t} = 0 &\iff \frac{\gamma}{B(A) - C(a_1) - \hat{N}B_1 + t} = \frac{(\gamma - 1)(-(1 + \lambda))}{\alpha B(A) - \delta C(a_2) - \hat{N}B_2 - (1 + \lambda)t} \\
 &\iff \gamma \alpha B(A) - \gamma \delta C(a_2) - \gamma \hat{N}B_2 - \gamma(1 + \lambda)t \\
 &= (1 + \lambda)(1 - \gamma)B(A) - (1 + \lambda)(1 - \gamma)C(a_1) \\
 &\quad - (1 + \lambda)(1 - \gamma)\hat{N}B_1 + (1 + \lambda)(1 - \gamma)t \\
 &\iff B(A) [\gamma \alpha + (1 + \lambda)(\gamma - 1)] - \gamma \delta C(a_2) + (1 + \lambda)(1 - \gamma)C(a_1) \\
 &\quad - \gamma \hat{N}B_2 + (1 + \lambda)(1 - \gamma)\hat{N}B_1 \\
 &= t [(1 + \lambda)(1 - \gamma) + \gamma(1 + \lambda)] \\
 &\iff t = B(A) \left( \frac{\gamma \alpha + (1 + \lambda)(\gamma - 1)}{1 + \lambda} \right) - \frac{\gamma}{1 + \lambda} \delta C(a_2) \\
 &\quad + (1 - \gamma)C(a_1) - \frac{\gamma}{1 + \lambda} \hat{N}B_2 + (1 - \gamma)\hat{N}B_1
 \end{aligned}$$

We make the following simplifying assumptions:

$$\gamma = 1/2; \alpha = 1; \lambda = 0.$$

Then, the level of transfers becomes:

$$\begin{aligned}
 &\iff t = (1/2) [C(a_1) - \delta C(a_2)] + (1/2) [\hat{N}B_1 - \hat{N}B_2] \\
 &\iff t - \frac{1}{2} C(a_1) + \frac{1}{2} \delta C(a_2) = (1/2) [\hat{N}B_1 - \hat{N}B_2] \\
 &\iff t + \frac{1}{2} [B(A) - C(a_1)] - \frac{1}{2} [B(A) - \delta C(a_2)] = (1/2) [\hat{N}B_1 - \hat{N}B_2] \\
 &\iff t + \frac{1}{2} [U_1 - U_2] = (1/2) [\hat{N}B_1 - \hat{N}B_2]
 \end{aligned}$$

where  $U_1$  and  $U_2$  represent respectively, the gross benefit functions (without considering transfers) of the countries 1 and 2.

We remark that if the difference  $[U_1 - U_2]$  is superior to the difference  $[\hat{N}B_1 - \hat{N}B_2]$ , then the transfers from the country 2 to the country 1 must

be negative. Or, expressed differently, if the difference  $\left[ U_1 - \hat{N}B_1 \right]$ , the gain of the country 1 in the new arrangement compared to the Nash equilibrium is superior to the difference  $\left[ U_2 - \hat{N}B_2 \right]$ , the additional gain of the country 2, then the transfers are negative.

$$\left[ U_1 - \hat{N}B_1 \right] >? \left[ U_2 - \hat{N}B_2 \right]$$

To see this, we make an additional simplifying assumption:  $\bar{E}_1 = \bar{E}_2 = \bar{E}$ . So we obtain:

$$\begin{aligned} \left[ B(A) - C(a) - \hat{N}B_1 \right] &>? \left[ B(A) - \delta C(a) - \hat{N}B_2 \right] \\ \iff [-C(a) + \delta C(a)] + \left[ \hat{N}B_2 - \hat{N}B_1 \right] &>? 0 \end{aligned}$$

We remark that if the countries are completely symmetric,  $\delta = 1$ , the above expression is equal to 0. When the asymmetry between the countries increases,  $\delta \nearrow$ , the first derivative of the above expression with respect to  $\delta$  is equal to  $\left[ C(a) - C(\hat{a}_2) \right]$  which is positive because  $a > \hat{a}_2$  (because of the assumption  $\delta > 1$ , see sections 3.6.3 and 3.6.4). This means that when the abatement costs of the country 2 are higher to the country 1's ( $\delta > 1$ ), the transfers from the country 2 to the country 1 are negative. So the natural direction of the transfers is from the country 1 to the country 2.

### 6.5 Comparison of the Abatement Levels under the Uniform versus Differentiated Standards with Transfers in the case of Asymmetric Countries

With the assumption iii)  $\bar{E}_1 = \bar{E}_2 = \bar{E}$ , we know that  $\beta_2 < \beta_1$  in the differentiated case with transfers. Now we can rank the parameter  $\beta$ , the emission reduction rate in the uniform case, resulting from the equation (1), (page 18), with the differentiated emission reduction rates in the differentiated case.

$$B'(A) = \frac{\bar{E}_1}{(\bar{E}_1 + \bar{E}_2)} \frac{C'(a_1)}{(1 + (1 + \lambda)\alpha)} + \frac{\bar{E}_2}{(\bar{E}_1 + \bar{E}_2)} \frac{(1 + \lambda)\delta C'(a_2)}{(1 + (1 + \lambda)\alpha)} \quad (1)$$

where  $A = \beta(\bar{E}_1 + \bar{E}_2)$ ,  $a_1 = \beta\bar{E}_1$ ,  $a_2 = \beta\bar{E}_2$ .

$$B'(A) = \frac{C'(a_1)}{1 + (1 + \lambda)\alpha} \quad (2)$$

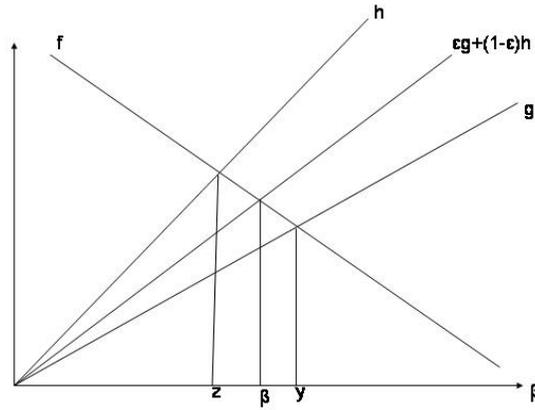


Figure 2:

$$B'(A) = \frac{1 + \lambda}{1 + (1 + \lambda)\alpha} \delta C'(a_2) \quad (3)$$

where  $A = \bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2$ ,  $a_1 = \beta_1 \bar{E}_1$ ,  $a_2 = \beta_2 \bar{E}_2$ .

The equations (2) and (3) will provide us a relation between  $\beta_1$  and  $\beta_2$ . We call  $z$  the intersection of the graph 3 with the  $45^\circ$  line. Similarly, we call  $y$  the intersection of the graph 2 with the  $45^\circ$  line. With these specifications, the equations (1), (2), (3) can be rewritten in the following way:

$$f(\beta) = \varepsilon g(\beta) + (1 - \varepsilon)h(\beta) \quad (1')$$

$$f(y) = g(y) \quad (2')$$

$$f(z) = h(z) \quad (3')$$

where  $\varepsilon = \frac{\bar{E}_1}{(\bar{E}_1 + \bar{E}_2)}$  and  $(1 - \varepsilon) = \frac{\bar{E}_2}{(\bar{E}_1 + \bar{E}_2)}$ .

We propose below a graphical presentation to illustrate the position of the parameters  $\beta$ ,  $z$  and  $y$  (see the figure 2).

Since the function in the RHS of the equation (1') is a weighted average of the functions in the RHS's of the equations (2') and (3'), the variable  $\beta$  is

located between those resulting from the equations (2') and (3'). The distance between  $\beta_1$  and  $\beta_2$  naturally depends on the respective slopes of the curves related to the equations (2') and (3').

We now proceed to the comparison of the slope of the curves related to the equations (2') and (3') in order to rank  $y, z$  and  $\beta$ . We take the total differential of the equation (2):

$$B''(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)(d\beta_1 \bar{E} + d\beta_2 \bar{E}_2) = \frac{1}{(1 + (1 + \lambda)\alpha)} C'''(\bar{E}_1 \times \beta_1) d\beta_1 \bar{E}_1$$

$$\Leftrightarrow \left(\frac{d\beta_2}{d\beta_1}\right)_2 = \frac{\left[\frac{1}{(1+(1+\lambda)\alpha)} C'''(\bar{E}_1 \times \beta_1) - B''(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)\right] \bar{E}_1}{B''(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \bar{E}_2}$$

Similarly, we take the total differential of the equation (3):

$$B''(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)(d\beta_1 \bar{E} + d\beta_2 \bar{E}_2) = \frac{(1 + \lambda)\delta}{(1 + (1 + \lambda)\alpha)} C'''(\bar{E}_2 \times \beta_2) d\beta_2 \bar{E}_2$$

$$\Leftrightarrow \left(\frac{d\beta_2}{d\beta_1}\right)_3 = \frac{B''(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) \bar{E}_1}{\left[\frac{(1+\lambda)\delta}{(1+(1+\lambda)\alpha)} C'''(\bar{E}_2 \times \beta_2) - B''(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2)\right] \bar{E}_2}$$

So the question is to know the sign of the term  $\left(\frac{d\beta_2}{d\beta_1}\right)_2 - \left(\frac{d\beta_2}{d\beta_1}\right)_3$ . Using the properties on the concavity of the benefit function and on the convexity of the cost function ( $B'' \leq 0$  and  $C''' \geq 0$ ), we find that  $\left(\frac{d\beta_2}{d\beta_1}\right)_2 < \left(\frac{d\beta_2}{d\beta_1}\right)_3$ . So the slope of the curve related to the equation (3') is higher than the one of the curve related to the equation (2'). So we have  $z < \beta < y$ , meaning that the percentage emission reduction rate in the uniform case with transfers, i.e.  $\beta$ , is between  $\beta_1$  and  $\beta_2$ , the ones in the differentiated case with transfers.

## 6.6 Comparison of the Welfare Levels in the case of Symmetric Countries

### 6.6.1 Uniform Standards / Nash Equilibrium

The difference of the total net benefits under the uniform standards and the Nash equilibrium is:

$$TNB_U - T\hat{N}B = 2 \times \left[ B(A_U) - C(A_U/2) - B(\hat{A}) + C(\hat{A}/2) \right]$$

We know that  $\hat{A} < A_U$ . So,

- $B(A_U) - B(\hat{A}) > 0$  because the benefit function is increasing.
- $C(A_U) > C(\hat{A}) \iff C(A_U/2) > C(\hat{A}/2) \iff C(\hat{A}/2) - C(A_U/2) < 0$  because the cost function is increasing.

We would like to know the sign of the following expression:

$$\left[ B(A_U) - B(\hat{A}) \right] + \left[ C(\hat{A}/2) - C(A_U/2) \right]$$

The proof is straightforward. We note  $A_U$  as  $x$  and  $\hat{A}$  as  $y$ . We would like to know under what conditions the expression  $[B(x) - B(y)]$  is superior to the expression  $[C(x/2) - C(y/2)]$ . We proceed in the following way:

$$\frac{[B(x) - B(y)]}{(1/2)(x - y)} > \frac{[C(x/2) - C(y/2)]}{(1/2)(x - y)}$$

given that  $x > y$ , i.e.  $A_U > \hat{A}$ .

$$\iff \frac{B(x) - B(y)}{(x - y)} > \frac{1}{2} \times \frac{C(x/2) - C(y/2)}{(\frac{x}{2} - \frac{y}{2})}$$

So we remark that the expression  $\left[ B(A_U) - B(\hat{A}) \right]$

+  $\left[ C(\hat{A}/2) - C(A_U/2) \right]$  is positive when the absolute value of the benefit function's slope between  $A_U$  and  $\hat{A}$  is higher than the half of the absolute value of the cost function's slope between  $(A_U/2)$  and  $(\hat{A}/2)$ .

We remark that the uniform emission reduction case can outperform the Nash equilibrium when the above condition is verified.

In the next section, we will compare the welfare levels under the differentiated standard case and the Nash equilibrium.

### 6.6.2 Differentiated Standards / Nash Equilibrium

The difference of the total net benefits under the differentiated standards and the Nash equilibrium, given the level of transfers  $t$ , is:

$$\begin{aligned}
 TNB_D - T\hat{N}B &= B(A_D) \times \left[ \frac{2 + 3\lambda - 2\lambda\gamma - \gamma\lambda^2 + \lambda^2}{1 + \lambda} \right] \\
 &\quad - B(\hat{A}) \times \left[ \frac{2 + 3\lambda - 2\lambda\gamma - \gamma\lambda^2 + \lambda^2}{1 + \lambda} \right] \\
 &\quad - C(A_D) \times (1 - \lambda(\gamma - 1)) \\
 &\quad + C(\hat{A}/2) \times \left[ \frac{2 + 3\lambda - 2\lambda\gamma - \gamma\lambda^2 + \lambda^2}{1 + \lambda} \right] \\
 &\quad + co \times \left[ \frac{1 + \lambda(1 - \gamma)}{1 + \lambda} \right]
 \end{aligned}$$

We know that  $\hat{A} > A_D$ . So,

·  $B(\hat{A}) > B(A_D) \iff B(A_D) - B(\hat{A}) < 0$  because the benefit function is increasing.

We know that the expression  $\left[ \frac{2+3\lambda-2\lambda\gamma-\gamma\lambda^2+\lambda^2}{1+\lambda} \right]$  is positive and superior to 2, when  $\gamma = 0.5$  (same negotiation power). So we have:

$$\left[ \frac{2+3\lambda-2\lambda\gamma-\gamma\lambda^2+\lambda^2}{1+\lambda} \right] \times \left[ B(A_D) - B(\hat{A}) \right] < 0.$$

·  $C(\hat{A}) > C(A_D)$  because the cost function is increasing.

The expression  $C(\hat{A}/2)$  is multiplied by  $\left[ \frac{2+3\lambda-2\lambda\gamma-\gamma\lambda^2+\lambda^2}{1+\lambda} \right]$  which is superior to 2 and the expression  $C(A_D)$  is multiplied by  $(1 - \lambda(\gamma - 1))$  which is superior to 1, when  $\gamma = 0.5$ . So the answer is not obvious.

· The expression  $co$  is multiplied by  $\left[ \frac{1+\lambda(1-\gamma)}{1+\lambda} \right]$  which is positive but inferior to 1, when  $\gamma = 0.5$ .

In order to simplify our task of comparison, we can assume that transfers are perfect, i.e.  $\lambda = 0$ . In this case, the difference term on cost functions becomes:

$$\left[ C(\hat{A}/2) \times 2 - C(A_D) \right].$$

If we divide this expression by 2, we obtain:

$\left[ C(\hat{A}/2) - C(A_D) \times \frac{1}{2} \right]$  which is *negative* under the assumptions that the cost function is convex and  $C(0) = 0$ .

So we can conclude that the Nash equilibrium can outperform the differentiated standards case, when the value of the fixed cost is not so high, and this even when transfers are perfect. The high level of fixed costs  $co$  can, however reverse this situation.

In the next section, we will compare the welfare levels under the differentiated standard and the uniform standard cases.

### 6.6.3 Differentiated Standards / Uniform Standards

The difference of the total net benefits under the differentiated and uniform standards, given the level of transfers  $t$ , is:

$$\begin{aligned}
 TNB_D - TNB_U &= B(A_D) \times \left[ \frac{2 + 3\lambda - 2\lambda\gamma - \gamma\lambda^2 + \lambda^2}{1 + \lambda} \right] \\
 &\quad - 2 \times B(A_U) + 2C(A_U/2) \\
 &\quad - C(A_D) \times (1 - \lambda(\gamma - 1)) \\
 &\quad + co \times [1 + \lambda(\gamma - 1)] \\
 &\quad + \hat{NB} \times \left[ \frac{\lambda}{1 + \lambda} (2\gamma + \gamma\lambda - 1 - \lambda) \right]
 \end{aligned}$$

We know that  $A_U > A_D$ . So

- $B(A_U) - B(A_D) > 0$  because the benefit function is increasing.

The expression  $B(A_D)$  is multiplied by  $\left[ \frac{2+3\lambda-2\lambda\gamma-\gamma\lambda^2+\lambda^2}{1+\lambda} \right]$  which is superior to 2 and the expression  $B(A_U)$  is multiplied by 2, when  $\gamma = 0.5$ . So the answer is not obvious.

- $C(A_U) > C(A_D)$  because the cost function is increasing.

The expression  $C(A_U/2)$  is multiplied by 2 and the expression  $C(A_D)$  is multiplied by  $(1 - \lambda(\gamma - 1))$  which is superior to 1, when  $\gamma = 0.5$ . So the answer is not obvious.

- The expression  $co$  is multiplied by  $[1 + \lambda(\gamma - 1)]$  which is positive but inferior to 1, when  $\gamma = 0.5$ .

- The expression  $\hat{NB}$  is multiplied by  $\left[ \frac{\lambda}{1+\lambda} (2\gamma + \gamma\lambda - 1 - \lambda) \right]$  which is *negative*, when  $\gamma = 0.5$ .

In order to simplify our task of comparison, we can assume again that transfers are perfect, i.e.  $\lambda = 0$ . In this case, the difference term on benefit functions becomes:

$2 \times [B(A_D) - B(A_U)]$  which is *negative*.

Similarly, the difference term on cost functions becomes:

$[C(A_U/2) \times 2 - C(A_D)]$ .

If we divide this expression by 2, we obtain:

$[C(A_U/2) - C(A_D) \times \frac{1}{2}]$  which is *negative* under the assumptions that the cost function is convex and  $C(0) = 0$ .

So we can conclude that the uniform standards can outperform the differentiated ones, when the value of the fixed cost is not so high, and this even when transfers are perfect. The high level of fixed costs  $co$  can, however reverse this situation.

## 6.7 Pareto Efficiency of a Nash Bargaining Solution with Imperfect Transfers

Let suppose that the countries 1 and 2 bargain over a variable ( $x$ ) and a transfer ( $t$ ). The aggregate utility levels of the two countries (without transfers) are noted as  $U_1(x)$  and  $U_2(x)$ . So the simple Nash bargaining solution consists on solving the following program:

$$\text{Max}_{x,t} [U_1(x) - (1 + \lambda)t] \times [U_2(x) + t]$$

The first-order conditions with respect to  $x$  and  $t$  are:

$$\frac{U_1'(x)}{U_1(x) - (1 + \lambda)t} = \frac{-U_2'(x)}{U_2(x) + t}$$

$$\frac{(1 + \lambda)}{U_1(x) - (1 + \lambda)t} = \frac{1}{U_2(x) + t}$$

The ratio of these first-order conditions gives us:

$$U_1'(x) + (1 + \lambda)U_2'(x) = 0$$

The *Pareto optimality* condition derives from the resolution of the following program:

$$\text{Max}_{x,t} [U_1(x) - (1 + \lambda)t]$$

$$\text{s.t.} \quad [U_2(x) + t] \geq \bar{U}_2$$

The first-order conditions with respect to  $x$  and  $t$  are:

$$U_1'(x) + \mu U_2'(x) = 0$$

$$-(1 + \lambda) + \mu = 0$$

with  $\mu$  the Lagrangian multiplier related to the constraint  $[U_2(x) + t] \geq \bar{U}_2$ . These conditions turn out to be the same as the preceding condition of the simple Nash bargaining solution:

$$U_1'(x) + (1 + \lambda)U_2'(x) = 0$$

So we remark that the Nash bargaining solution with imperfect transfers derives from an *efficient* bargaining procedure.

## 6.8 Comparison of the Welfare Levels in the case of Asymmetric Countries

### 6.8.1 The Uniform case *with* Transfers and the Differentiated case *with* Transfers

Suppose a situation where the countries 1 and 2 are in the uniform standard case. We would like to know if the total welfare level (for the two countries) can be increased if the country 1 moves into the differentiated standard case. So we maintain constant the welfare level of the country 2 (its indifference curve in the uniform case does not change) and the transfer level in the uniform standard case. We investigate the conditions under which the welfare of the country 1 can be elevated.

$$dNB_1 = \frac{\partial NB_1}{\partial \beta_1} d\beta_1 + \frac{\partial NB_1}{\partial \beta_2} d\beta_2 >? 0 \quad (1)$$

$$dNB_2 = \frac{\partial NB_2}{\partial \beta_1} d\beta_1 + \frac{\partial NB_2}{\partial \beta_2} d\beta_2 = 0 \quad (2)$$

The net benefit functions of the countries are:

$$NB_1 = B(\beta_1 \bar{E}_1 + \beta_2 \bar{E}_2) - C(\beta_1 \bar{E}_1) - (1 + \lambda)t$$

$$NB_2 = \alpha B(\beta_1 \bar{E}_1 + \beta_2 \bar{E}_2) - \delta C(\beta_1 \bar{E}_1) + t$$

The first derivatives of the net benefit functions with respect to percentage emission reduction levels  $\beta_1$  and  $\beta_2$  are:

$$\frac{\partial NB_1}{\partial \beta_1} = B'(A) \bar{E}_1 - C'(a_1) \bar{E}_1$$

$$\frac{\partial NB_1}{\partial \beta_2} = B'(A) \bar{E}_2$$

$$\frac{\partial NB_2}{\partial \beta_1} = \alpha B'(A) \bar{E}_1$$

$$\frac{\partial NB_2}{\partial \beta_2} = \alpha B'(A) \bar{E}_2 - \delta C'(a_2) \bar{E}_2$$

We know that  $\beta_{2DT} < \beta_{UT} < \beta_{1DT}$ ,  $\hat{\beta}_1 < \beta_{1DT}$  and  $\hat{\beta}_2 < \beta_{2DT}$  where the subscript *DT* indicates differentiated standards with transfers and *UT* indicates uniform standards with transfers. This ranking of the percentage emission reduction rates implies that we are in the decreasing side of the curve  $NB_2(\beta_2)$ , so we have  $\frac{\partial NB_2(\beta_1, \beta_2, t)}{\partial \beta_2} < 0$ . It is clear that the sign of the derivative  $\frac{\partial NB_2(\beta_1, \beta_2, t)}{\partial \beta_1}$  is positive.

From the equation 2, we can write:

$$d\beta_2 = -d\beta_1 \frac{\frac{\partial NB_2}{\partial \beta_1}}{\frac{\partial NB_2}{\partial \beta_2}}$$

Given the properties of the sign of the first derivatives  $\frac{\partial NB_2}{\partial \beta_2}$  and  $\frac{\partial NB_2}{\partial \beta_1}$ , the ratio  $\frac{d\beta_2}{d\beta_1}$  is positive. This means that the two percentage emission reduction levels  $\beta_1$  and  $\beta_2$  must increase together in order to make accept the country 2 the new arrangement ( $dNB_2 = 0$ ).

We can rewrite the equation 1 in the following way:

$$\begin{aligned} dNB_1 &= \frac{\partial NB_1}{\partial \beta_1} d\beta_1 + \frac{\partial NB_1}{\partial \beta_2} \left( -d\beta_1 \frac{\frac{\partial NB_2}{\partial \beta_1}}{\frac{\partial NB_2}{\partial \beta_2}} \right) > ? 0 \\ \Leftrightarrow d\beta_1 &\left[ \frac{\partial NB_1}{\partial \beta_1} - \frac{\partial NB_1}{\partial \beta_2} \left( \frac{\frac{\partial NB_2}{\partial \beta_1}}{\frac{\partial NB_2}{\partial \beta_2}} \right) \right] > ? 0 \end{aligned}$$

We remark that if the marginal substitution rates of the countries are equal, then  $dNB_1 = 0$ . So the following condition must not be validated, in order to obtain an improvement of the country 1's welfare:

$$\begin{aligned} \frac{\frac{\partial NB_1}{\partial \beta_1}}{\frac{\partial NB_1}{\partial \beta_2}} &= \frac{\frac{\partial NB_2}{\partial \beta_1}}{\frac{\partial NB_2}{\partial \beta_2}} \\ \Leftrightarrow \frac{B'(A)\bar{E}_1 - C'(a_1)\bar{E}_1}{B'(A)\bar{E}_2} &= \frac{\alpha B'(A)\bar{E}_1}{\alpha B'(A)\bar{E}_2 - \delta C'(a_2)\bar{E}_2} \end{aligned}$$

$$\Leftrightarrow \alpha B'(A)^2 - \alpha B'(A)C'(a_1) - B'(A)\delta C'(a_2) + \delta C'(a_1)C'(a_2) = \alpha B'(A)^2$$

$$\Leftrightarrow \delta C'(a_1)C'(a_2) = B'(A) [\alpha C'(a_1) + \delta C'(a_2)] \quad (3)$$

Now we will use the optimality conditions ( $\frac{\partial V/\partial \beta}{\partial V/\partial t}$ ) of the uniform standard case with transfers to replace the expression  $B'(A)$  in the equation 3. The optimality condition of the uniform standard case with transfers is:

$$(\bar{E}_1 + \bar{E}_2)B'(A) = \frac{C'(a_1)\bar{E}_1}{(1 + (1 + \lambda)\alpha)} + \frac{(1 + \lambda)\delta C'(a_2)\bar{E}_2}{(1 + (1 + \lambda)\alpha)}$$

To simplify our task, we assume that  $\bar{E}_1 = \bar{E}_2 = \bar{E}$ . Given this assumption, the abatement level of the country 1 will be naturally equal to the country 2's under the uniform standard case, i.e.  $a_1 = \beta \bar{E} = a_2$ . So we obtain:

$$B'(A) = \frac{C'(a)}{2(1 + (1 + \lambda)\alpha)} + \frac{(1 + \lambda)\delta C'(a)}{2(1 + (1 + \lambda)\alpha)}$$

If we replace the above optimality condition in the equation 3, we obtain:

$$\iff \delta C'(a)^2 = \left[ \frac{C'(a)}{2(1 + (1 + \lambda)\alpha)} + \frac{(1 + \lambda)\delta C'(a)}{2(1 + (1 + \lambda)\alpha)} \right] [(\alpha + \delta)C'(a)]$$

$$\iff \delta [2(1 + (1 + \lambda)\alpha)] = (1 + \delta(1 + \lambda))(\alpha + \delta)$$

$$\iff \delta(1 + \alpha + \alpha\lambda) = \alpha + \delta^2(1 + \lambda)$$

In order to simplify further our task, we assume that the transfers between the countries are made in a perfect manner, i.e.  $\lambda = 0$ . Then we obtain:

$$\delta(1 + \alpha) = \alpha + \delta^2$$

$$\iff \delta = \alpha$$

We can conclude that in a case where the asymmetry parameters between the countries, the one on the damage function  $\alpha$  and the other on the abatement cost function  $\delta$  have the same value, the welfare level of the country 1 cannot be increased by a movement from the uniform standard case to the differentiated one, when the transfers are perfect. In all the cases where the asymmetry parameters are not equal ( $\alpha \neq \delta$ ), the total welfare of the countries can be improved by a movement from the uniform standard case to the differentiated one.

### 6.8.2 The Uniform case *without* Transfers and the Differentiated case *without* Transfers

We do the same reasoning as the preceding section. We suppose a situation where the countries 1 and 2 are in the uniform standard case. We maintain constant the welfare level of the country 2 (leaving its surplus unchanged in the uniform case). We would like to know if the total welfare level (for the two countries) can be increased if the country 1 moves into the differentiated standard case.

$$dNB_1 = \frac{\partial NB_1}{\partial \beta_1} d\beta_1 + \frac{\partial NB_1}{\partial \beta_2} d\beta_2 >? 0 \tag{4}$$

$$dNB_2 = \frac{\partial NB_2}{\partial \beta_1} d\beta_1 + \frac{\partial NB_2}{\partial \beta_2} d\beta_2 = 0 \tag{5}$$

The net benefit functions of the countries are:

$$NB_1 = B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - C(\bar{E}_1 \times \beta_1)$$

$$NB_2 = \alpha B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta C(\bar{E}_2 \times \beta_2)$$

We can rewrite the equation 4 in the following way (as in the preceding section):

$$\begin{aligned} dNB_1 &= \frac{\partial NB_1}{\partial \beta_1} d\beta_1 + \frac{\partial NB_1}{\partial \beta_2} (-d\beta_1 \frac{\frac{\partial NB_2}{\partial \beta_1}}{\frac{\partial NB_2}{\partial \beta_2}}) >? 0 \\ &\iff d\beta_1 \left[ \frac{\partial NB_1}{\partial \beta_1} - \frac{\partial NB_1}{\partial \beta_2} \left( \frac{\frac{\partial NB_2}{\partial \beta_1}}{\frac{\partial NB_2}{\partial \beta_2}} \right) \right] >? 0 \end{aligned}$$

We remark again that if the marginal substitution rates of the countries are equal, then  $dNB_1 = 0$ . So the following condition must not be validated, in order to obtain an improvement of the country 1's welfare:

$$\frac{\frac{\partial NB_1}{\partial \beta_1}}{\frac{\partial NB_1}{\partial \beta_2}} = \frac{\frac{\partial NB_2}{\partial \beta_1}}{\frac{\partial NB_2}{\partial \beta_2}}$$

which implies:

$$\iff \delta C'(a_1)C'(a_2) = B'(A) [\alpha C'(a_1) + \delta C'(a_2)] \quad (6)$$

Now we will use the optimality condition ( $\partial V/\partial \beta = 0$ ) of the uniform standard case without transfers to replace the expression  $B'(A)$  in the equation 6. The optimality condition of the uniform standard case without transfers is:

$$\begin{aligned} &\iff (\bar{E}_1 + \bar{E}_2)B'(\beta(\bar{E}_1 + \bar{E}_2)) \\ &\quad \left[ \begin{aligned} &\alpha B(\beta(\bar{E}_1 + \bar{E}_2)) - \gamma \delta C(\bar{E}_2 \times \beta) \\ &+ (\gamma - 1)\alpha C(\beta \bar{E}_1) - \gamma \hat{N}B_2 + (\gamma - 1)\alpha \hat{N}B_1 \end{aligned} \right] \\ &= \bar{E}_1 C'(\beta \bar{E}_1) \\ &\quad \left[ \begin{aligned} &\gamma \alpha B(\beta(\bar{E}_1 + \bar{E}_2)) - \gamma \delta C(\bar{E}_2 \times \beta) - \gamma \hat{N}B_2 \\ &+ \bar{E}_2 \delta C'(\bar{E}_2 \times \beta) \\ &- (\gamma - 1)B(\beta(\bar{E}_1 + \bar{E}_2)) + (\gamma - 1)C(\beta \bar{E}_1) + (\gamma - 1)\hat{N}B_1 \end{aligned} \right] \end{aligned}$$

To simplify our task, we assume that  $\bar{E}_1 = \bar{E}_2 = \bar{E}$ . Given this assumption, the abatement level of the country 1 will be naturally equal to the country 2's under the uniform standard case, i.e.  $a_1 = \beta \bar{E} = a_2$ . So we obtain:

$$\begin{aligned}
 \Leftrightarrow B'(A) &= \frac{1}{2}C'(a) \\
 &\times \frac{\left[ \gamma\alpha B(A) - \gamma\delta C(a) - \gamma\hat{N}B_2 \right]}{\left[ \alpha B(A) - \gamma\delta C(a) + (\gamma - 1)\alpha C(a) - \gamma\hat{N}B_2 + (\gamma - 1)\alpha\hat{N}B_1 \right]} \\
 &+ \frac{1}{2}\delta C'(a) \\
 &\times \frac{\left[ -(\gamma - 1)B(A) + (\gamma - 1)C(a) + (\gamma - 1)\hat{N}B_1 \right]}{\left[ \alpha B(A) - \gamma\delta C(a) + (\gamma - 1)\alpha C(a) - \gamma\hat{N}B_2 + (\gamma - 1)\alpha\hat{N}B_1 \right]}
 \end{aligned}$$

If we replace the above optimality condition in the equation 6 and we use two simplifying assumptions  $\gamma = 1/2$  and  $\alpha = 1$ , we obtain:

$$\Leftrightarrow 2\delta = (1 + \delta)$$

$$\Leftrightarrow \delta = 1$$

We remark that in case of the symmetry between the countries ( $\alpha = 1, \delta = 1$ ), the welfare level of the country 1 cannot be increased by a movement from the uniform standard case to the differentiated one. In all the *general* cases where there exists an asymmetry on the abatement cost levels between the countries ( $\alpha = 1, \delta \neq 1$ ), the total welfare of the countries can be improved by a movement from the uniform standard case to the differentiated one.

### 6.8.3 The Uniform case *with* Transfers and the Differentiated case *without* Transfers (1)

We know that the Nash bargaining solution in the differentiated standard case with transfers can be written in the following way:

$$Max_{\beta_1, \beta_2, t} \left[ \begin{aligned} &(B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - C(\beta_1 \bar{E}_1) - \hat{N}B_1 - (1 + \lambda)t)^\gamma \\ &\times (\alpha B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta C(\beta_2 \times \bar{E}_2) - \hat{N}B_2 + t)^{1-\gamma} \end{aligned} \right]$$

We denote the expression within the bracket by a function  $f$  depending on several variables and parameters:  $f(\beta_1, \beta_2, t, \bar{E}_1, \bar{E}_2, \lambda, \gamma, \alpha, \delta)$ .

We have two types of constraints:

- 1)  $\beta_1 = \beta_2$  : the case of uniform standards.
- 2)  $t = 0$  : the absence of transfers.

We write below the Lagrangian expressions of both constraints:

$$V_1 = \text{Max}_{\beta_1, \beta_2, t} [f_1(\cdot) - \lambda_1(p)(\beta_1 - \beta_2)]$$

$$V_2 = \text{Max}_{\beta_1, \beta_2, t} [f_2(\cdot) - \lambda_2(p)(t)]$$

where  $p = (\bar{E}_1, \bar{E}_2, \lambda, \gamma, \alpha, \delta)$  is the vector of parameters.

The first-order conditions of the first maximization problem imply:

$$\frac{\partial f_1}{\partial \beta_1} = \lambda_1$$

$$\frac{\partial f_1}{\partial \beta_2} = -\lambda_1$$

$$\frac{\partial f_1}{\partial t} = 0$$

We can show that (envelope theorem):

$$\frac{\partial V_1}{\partial p} = \frac{\partial f_1}{\partial p}$$

**Proof.**  $\frac{\partial V_1}{\partial p} = \frac{\partial f_1}{\partial \beta_1} \frac{\partial \beta_1}{\partial p} + \frac{\partial f_1}{\partial \beta_2} \frac{\partial \beta_2}{\partial p} + \frac{\partial f_1}{\partial t} \frac{\partial t}{\partial p} + \frac{\partial f_1}{\partial p} - \frac{\partial(\lambda_1(p))}{\partial p}(\beta_1 - \beta_2) - \lambda_1(p) \frac{\partial(\beta_1 - \beta_2)}{\partial p}$

Since  $\frac{\partial f_1}{\partial \beta_1} = \lambda_1$ ,  $\frac{\partial f_1}{\partial \beta_2} = -\lambda_1$ ,  $\frac{\partial f_1}{\partial t} = 0$ , we obtain:

$$\frac{\partial V_1}{\partial p} = \lambda_1 \frac{\partial \beta_1}{\partial p} - \lambda_1 \frac{\partial \beta_2}{\partial p} + \frac{\partial f_1}{\partial p} - \lambda_1(p) \frac{\partial \beta_1}{\partial p} + \lambda_1(p) \frac{\partial \beta_2}{\partial p} = \frac{\partial f_1}{\partial p} \quad \blacksquare$$

Our objective is to measure the difference between the value functions  $V_1$  and  $V_2$ , the former representing the welfare in the uniform standard case with transfers and the latter being the welfare level in the differentiated standard case without transfers. One simple way to evaluate this difference is to look at the sensibility of this difference when the countries become asymmetric in their abatement technology, in the neighborhood of the following points (symmetric countries)  $\bar{E}_1 = \bar{E}_2, \lambda = 0, \gamma = 1/2, \alpha = 1$  and  $\delta = 1$ .

$$\frac{\partial(V_1 - V_2)}{\partial \delta} = \frac{\partial V_1}{\partial \delta} - \frac{\partial V_2}{\partial \delta} = \frac{\partial f_1}{\partial p} - \frac{\partial f_2}{\partial p} = 0$$

$$\text{where } \frac{\partial f_1}{\partial p} = \frac{\partial f_2}{\partial p} = \frac{1}{2} N B_1^{1/2} N B_2^{-1/2} \left[ -C(\beta \bar{E}) - C(\hat{\beta} \bar{E}) \right]$$

This results shows under some conditions the uniform standard case with transfers gives the same welfare level as the differentiated standard case without transfers.

**6.8.4 The Uniform case *with* Transfers and the Differentiated case *without* Transfers (2)**

The *simple* Nash bargaining solution is written in the following way, respectively for the differentiated standard case without transfers and the uniform standard case with transfers:

$$\begin{aligned}
 &Max_{\beta_1, \beta_2} \left[ \begin{array}{l} (B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - C(\beta_1 \bar{E}_1) - \hat{N}B_1) \\ \times (\alpha B(\bar{E}_1 \times \beta_1 + \bar{E}_2 \times \beta_2) - \delta C(\beta_2 \times \bar{E}_2) - \hat{N}B_2) \end{array} \right] \\
 &Max_{\beta} \left[ \begin{array}{l} (B(\beta(\bar{E}_1 + \bar{E}_2)) - C(\beta \bar{E}_1) - \hat{N}B_1 - (1 + \lambda)t) \\ \times (\alpha B(\beta(\bar{E}_1 + \bar{E}_2)) - \delta C(\beta \times \bar{E}_2) - \hat{N}B_2 + t) \end{array} \right]
 \end{aligned}$$

It is clear that for  $t = 0$ , the differentiated standard case without transfers dominates in terms of welfare the uniform standard case with transfers because it exists a larger choice set for the former case,  $\beta_1$  and  $\beta_2$ , than the latter case,  $\beta$ .

Using the envelope theorem in the uniform standard case with transfers with regard to the transfers  $t$ , which is now a **parameter**, we can show that the derivative of the value function with respect to ( $t$ ) is negative:

$$\frac{\partial V}{\partial t} = -\lambda NB_1 NB_2 \leq 0$$

where  $V$  is the value function in the Nash bargaining solution. We also have:

$$\begin{aligned}
 NB_1 &= B(\beta(\bar{E}_1 + \bar{E}_2)) - C(\beta \bar{E}_1) - \hat{N}B_1 \\
 NB_2 &= \alpha B(\beta(\bar{E}_1 + \bar{E}_2)) - \delta C(\beta \times \bar{E}_2) - \hat{N}B_2
 \end{aligned}$$

Applying the same procedure we can prove that, for *different* negotiation powers between the countries, transfers have a negative impact on the value function in the uniform standard case with transfers if:

$$\begin{aligned}
 &\frac{NB_1}{NB_2} < \frac{\gamma(1 + \lambda)}{(1 - \gamma)} \\
 \Leftrightarrow &\frac{(B(\beta(\bar{E}_1 + \bar{E}_2)) - C(\beta \bar{E}_1) - \hat{N}B_1)}{(\alpha B(\beta(\bar{E}_1 + \bar{E}_2)) - \delta C(\beta \times \bar{E}_2) - \hat{N}B_2)} < \frac{\gamma(1 + \lambda)}{(1 - \gamma)}
 \end{aligned}$$

This comparison naturally depends on the levels of the net payoffs and the negotiation powers of two countries, as well as on the level of imperfection of transfers.